Mathematical connections between various Ramanujan's equations, values of mass and electric charges of fundamental particles and physical data of Kerr Supermassive Black Hole M87

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Abstract

In this research thesis, we have described some mathematical connections between various Ramanujan's equations, values of mass and electric charges of fundamental particles and physical data of Kerr Supermassive Black Hole M87. We have obtained some very interesting results concerning a possible mathematical unification between some sectors of particle and string physics and some sectors of black hole physics, through the use and development of some formulas discovered by S. Ramanujan

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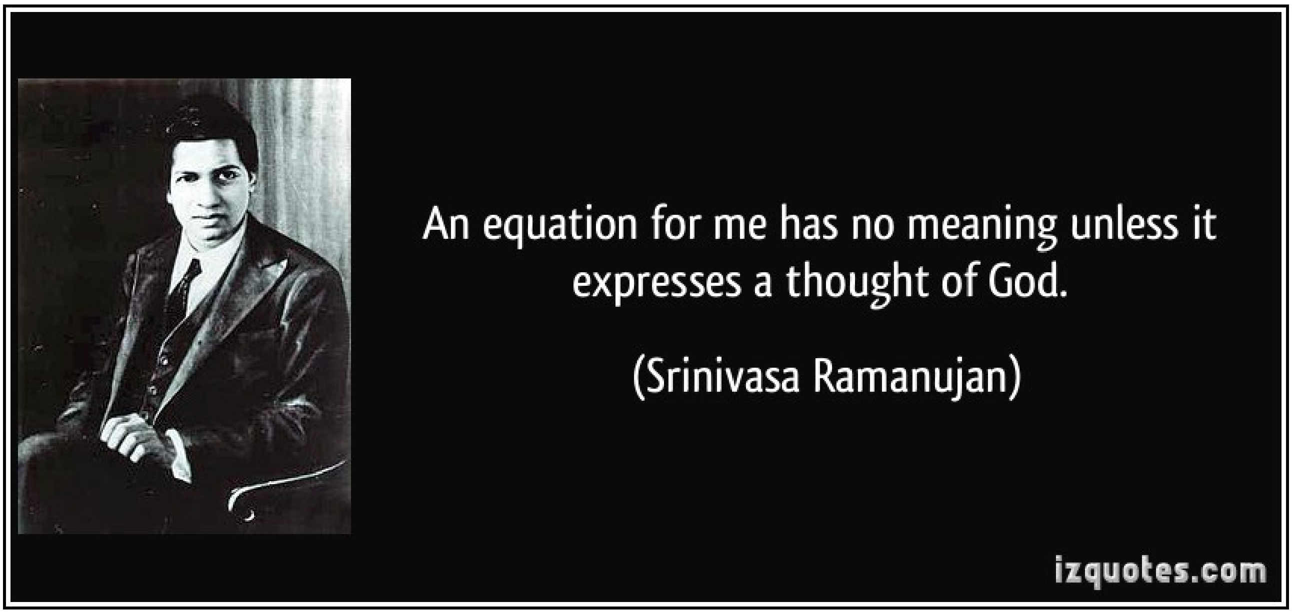
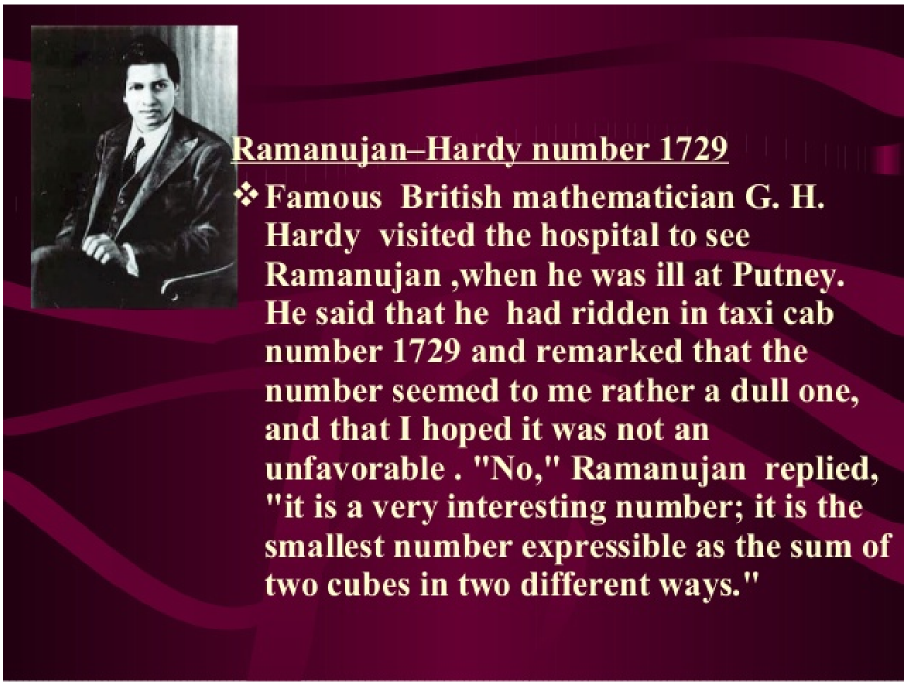


https://quotesgram.com/img/equation-quotes/5286174/



https://www.slideshare.net/SSridhar2/talk-on-ramanujan

From:

ELEVEN-DIMENSIONAL SUPERGRAVITY ON A MANIFOLD WITH BOUNDARY

Petr Horava - Joseph Henry Laboratories, Princeton University - Jadwin Hall, Princeton, NJ 08544, USA **and Edward Witten**⋆ School of Natural Sciences, Institute for Advanced Study * Olden Lane, Princeton, NJ 08540, USA

The supergravity multiplet consists of the metric g, the gravitino $\psi_{I\alpha}$, and a three-form C (with field strength G, normalized as in a previous footnote). The supergravity Lagrangian, up to terms quartic in the gravitino (which we will not need), is [8]

$$L_S = \frac{1}{\kappa^2}\int_{M^{11}} d^{11}x\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K - \frac{1}{48}G_{IJKL}G^{IJKL}\right.$$
$$- \frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N + 12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM} \qquad (2.1)$$
$$\left.\frac{\sqrt{2}}{3456}\epsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}C_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\right).$$

Note that: 3456 = 1728 * 2 and that $\frac{\sqrt{2}}{3456} = \frac{1}{1728\sqrt{2}}$

From Polchinski book "String Theory vol. I", we have that:

$$S = \frac{1}{2\kappa^2}\int d^DX\,(-\tilde{G})^{1/2}\Big[-\frac{2(D-26)}{3\alpha'}e^{4\tilde{\Phi}/(D-2)} + \tilde{R}$$
$$- \frac{1}{12}e^{-8\tilde{\Phi}/(D-2)}H_{\mu\nu\lambda}\tilde{H}^{\mu\nu\lambda} - \frac{4}{D-2}\partial_\mu\tilde{\Phi}\tilde{\partial}^\mu\tilde{\Phi} + O(\alpha')\Big], \qquad (3.7.25)$$

where tildes have been inserted as a reminder that indices here are raised with $\tilde{G}^{\mu\nu}$. In terms of $\tilde{G}_{\mu\nu}$, the gravitational Lagrangian density takes the standard Hilbert form $(-\tilde{G})^{1/2}\tilde{R}/2\kappa^2$. The constant $\kappa = \kappa_0 e^{\Phi_0}$ is the gravitational coupling, which in four-dimensional gravity has the value

$$\kappa = (8\pi G_N)^{1/2} = \frac{(8\pi)^{1/2}}{M_P} = (2.43\times10^{18}\ \text{GeV})^{-1}. \qquad (3.7.26)$$

Thence: $(8\pi G_N)^{1/2} = \kappa = (2.43 * 10^{18}\ \text{GeV})^{-1} = 4{,}115226337448 * 10^{-19}$;

and $\kappa^2 = 1{,}693508780843 * 10^{-37}$.

We have that:

$$\left(-\frac{1}{2}R-\frac{1}{2}\overline{\psi}_I\Gamma^{IJK}D_J\psi_K-\frac{1}{48}G_{IJKL}G^{IJKL}-\frac{\sqrt{2}}{192}\left(\overline{\psi}_I\Gamma^{IJKLMN}\psi_N+12\overline{\psi}^J\Gamma^{KL}\psi^M\right)G_{JKLM}\right.$$
$$\left.-\frac{\sqrt{2}}{3456}\epsilon^{I_1I_2\ldots I_{11}}C_{I_1I_2I_3}G_{I_4\ldots I_7}G_{I_8\ldots I_{11}}\right).$$

$$-\frac{1}{2}-\frac{1}{2}-\frac{1}{48}-\frac{\sqrt{2}}{192}-\frac{12\sqrt{2}}{192}-\frac{\sqrt{2}}{3456}=$$

$$=\frac{-1728-1728-72-18\sqrt{2}-216\sqrt{2}-\sqrt{2}}{3456}=$$

$$=\frac{1728}{3456}-\frac{1728}{3456}-\frac{72}{3456}-\frac{18\sqrt{2}}{3456}-\frac{216\sqrt{2}}{3456}-\frac{\sqrt{2}}{3456}=$$

-3860,34018656 / 3456 = – 1,11699658

$1/\kappa^2 = 5{,}9049 * 10^{36}$ that multiplied to – 1,11699658 = $-6{,}59575311 * 10^{36}$

Now the gravitational coupling is:

$$\alpha_G=\frac{Gm_e^2}{\hbar c}=\left(\frac{m_e}{m_P}\right)^2\approx 1.751751596\times 10^{-45}$$

(1) In weakly coupled heterotic string theory, the gauge and gravitational couplings unify at tree level to form one dimensionless string coupling constant g_{string} [10]

$$k_Y g_Y^2 = k_2 g_2^2 = k_3 g_3^2 = 8\pi\frac{G_N}{\alpha'} = g_{\text{string}}^2\ , \qquad (1)$$

where g_Y, g_2, and g_3 are the gauge couplings for the $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, G_N is the gravitational coupling and α' is the string tension. Here, k_Y, k_2 and k_3 are the levels of the corresponding Kac-Moody algebras; k_2 and k_3 are positive integers while k_Y is a rational number in general [10].

In the paper "INTRODUCTION TO STRING THEORY∗ version 14-05-04

of Gerard 't Hooft" α' appeared to be universal, approximately 1 GeV^{-2} . Thence:

$g^2 = 8\pi\,(1{,}751751596 * 10^{-45}) = 4{,}40263196 * 10^{-44}$; $g = 2{,}09824497 * 10^{-22}$;

$\sqrt{g} = 1{,}44853201 * 10^{-11}$.

Now, we calculate the following integral:

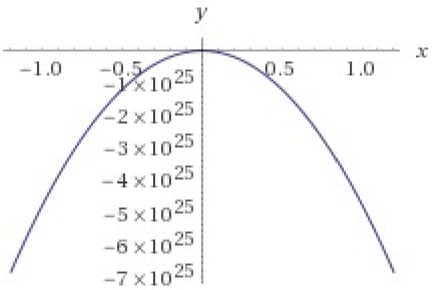
5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x

$$5.9049 \times 10^{36} \int \left(\frac{1.44853201}{10^{11}} \times (-1.11699658)\right) x \, dx$$

Result:

$-4.77708 \times 10^{25} x^2$

Plot:



(x from -1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.59236 \times 10^{25} x^3$ + constant

Now:

(1/10^54) * 1.08643^2 * 5.9049 * (10^36) integrate [(1.44853201 * 10^-11)*(-1.11699658)] x, [0, 34/(2Pi)]

$$\frac{1}{10^{54}} \times 1.08643^2 \times 5.9049 \times 10^{36} \int_0^{\frac{34}{2\pi}} \left(\frac{1.44853201}{10^{11}} \times (-1.11699658)\right) x \, dx$$

Result:

-1.65106×10^{-27}

We note that:

$$\sqrt{g}\left(-\frac{1}{2}R - \frac{1}{2}\overline{\psi}_I \Gamma^{IJK} D_J \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} \right.$$
$$- \frac{\sqrt{2}}{192}\left(\overline{\psi}_I \Gamma^{IJKLMN} \psi_N + 12 \overline{\psi}^J \Gamma^{KL} \psi^M\right) G_{JKLM}$$
$$\left. - \frac{\sqrt{2}}{3456} \epsilon^{I_1 I_2 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_7} G_{I_8 \ldots I_{11}} \right).$$

is equal to

$$\frac{1.44853201}{10^{11}} \times (-1.11699658)$$

Result:

$-1.6180053011905258 \times 10^{-11}$

Now, we calculate the following double integral:

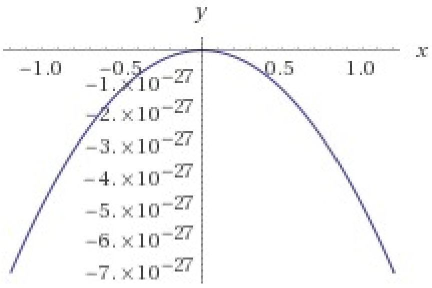
(1*10^-52) * (2*0.618)^3 * 1.08643 integrate integrate [-4.77708*10^25]

$$1 \times 10^{-52} (2 \times 0.618)^3 \times 1.08643 \int\left(\int -4.77708 \times 10^{25}\, dx\right) dx$$

Result:

$-4.89993 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

Indefinite integral assuming all variables are real:

$-1.63331 \times 10^{-27} x^3$ + constant

Now:

From "Ramanujan - "Twelve Lectures on subjects suggested by his life and work" – by G. H. Hardy – Cambridge at the University Press - 1940

10.5. The congruences of § 10.4 are satisfied by all n of certain arithmetical progressions. There are also congruences satisfied by "almost all" n. For example

$$(10.5.1) \qquad \tau(n) \equiv 0 \pmod 5$$

for almost all n (in the sense of § 3.4).

We begin by proving that

$$(10.5.2) \qquad \tau(n) \equiv n\sigma(n) \pmod 5,$$

where $\sigma(n)$ is the sum of the divisors of n, for all n. This depends on two identities in the theory of the modular functions, viz.

$$(10.5.3) \qquad Q^3 - R^2 = 1728 g(x),$$

and

$$(10.5.4) \qquad Q - P^2 = 288 \Sigma \frac{n^2 x^n}{(1-x^n)^2},$$

where

$$(10.5.5) \qquad P = 1 - 24\left(\frac{x}{1-x} + \frac{2x^2}{1-x^2} + \frac{3x^3}{1-x^3} + \dots\right),$$

$$(10.5.6) \qquad Q = 1 + 240\left(\frac{x}{1-x} + \frac{2^3 x^2}{1-x^2} + \frac{3^3 x^3}{1-x^3} + \dots\right)$$

$$(10.5.7) \qquad R = 1 - 504\left(\frac{x}{1-x} + \frac{2^5 x^2}{1-x^2} + \frac{3^5 x^3}{1-x^3} + \dots\right).$$

The identity (10.5.3) is familiar, but I have not seen (10.5.4) anywhere except in Ramanujan's work.

We have that $Q = 241$, $P = -23$ $\quad Q - P^2 = 241 - 529 = -288$

$Q^3 - R^2 = (1 + 240)^3 - (1 - 504)^2 = 13997521 - 253009 = 13744512$;

Where $13744512 = 1728 * 7954$; $1728 * 7954 = 13744512$;

$1728 = 13744512 / 7954$

We observe that 1728 is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We calculate the following simple integral:

integrate [13744512] x, [0, 1/(1.644934^13*Pi)]

$$\int_0^{\frac{1}{1.644934^{13}\pi}} 13\,744\,512\, x\, dx = 1.67086$$

where 1,644934 = ζ(2) = $\pi^2/6$

And the following simple double integral:

1/(10^34) * 1.08643 * (Pi/sqrt(2)) integrate integrate [13744512]

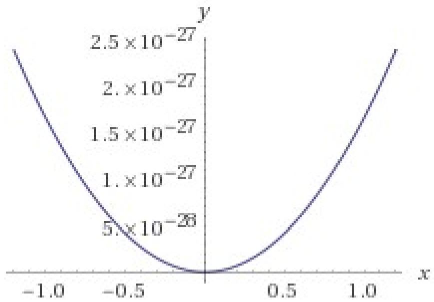
$$\frac{1}{10^{34}} \times 1.08643 \times \frac{\pi}{\sqrt{2}} \int \left(\int 13\,744\,512\, dx \right) dx$$

Result:

$$1.65858 \times 10^{-27}\, x^2$$

Result that is a good approximation to the proton mass

Plot:



(x from −1.2 to 1.2)

From:

INTRODUCTION TO STRING THEORY¤
version 14-05-04
Gerard ’t Hooft

12.2. Computing the spectrum of states.

The general method to compute the number of states consists of calculating, for the entire Hilbert space,

$$G(q) = \sum_{n=0}^{\infty} W_n q^n = \mathrm{Tr}\, q^N \ , \tag{12.12}$$

where q is a complex number corresponding to $1/z = e^{-i\tau}$, as in Eq. (10.3), W_n is the degree of degeneracy of the n^{th} level, and N is the number operator,

$$N = \sum_{\mu=1}^{D-2} \Big(\sum_{n=1}^{\infty} \alpha_{-n}\alpha_n + \sum_{r>0} r d_{-r} d_r \Big) = \sum_{\mu=1}^{D-2} \Big(\sum_{n=1}^{\infty} n\, N^{\mathrm{Bos}}_{\mu,n} + \sum_{r>0} r\, N^{\mathrm{Ferm}}_{\mu,r} \Big) \ , \tag{12.13}$$

where the sum over the fermionic operators is either over integers (Ramond) or integers $+\frac{1}{2}$ (Neveu-Schwarz). Since N receives its contributions independently from each mode, we can write $G(q)$ as a product:

$$G(q) = \prod_{\mu=1}^{D-2} \prod_{n=1}^{\infty} \prod_{r>0} f_n(q) g_r(q) \ , \tag{12.14}$$

with

$$f_n(q) = \sum_{m=0}^{\infty} q^{nm} = \frac{1}{1-q^n} \ , \tag{12.15}$$

while

$$g_r(q) = \sum_{m=0}^{1} q^{rm} = 1 + q^r \ . \tag{12.16}$$

We find that, for the purely bosonic string in 24 transverse dimensions:

$$G(q) = \prod_{n=1}^{\infty} (1-q^n)^{-24} \ . \tag{12.17}$$

The Taylor expansion of this function gives us the level density functions W_n. There are also many mathematical theorems concerning functions of this sort.

For the superstring in 8 transverse dimensions, we have

$$\begin{aligned} G(q) &= \prod_{n=1}^{\infty} \left(\frac{1+q^{n-\frac{1}{2}}}{1-q^n} \right)^8 \quad (\mathrm{NS}) \ ; \\ G(q) &= 16 \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n} \right)^8 \quad (\mathrm{Ramond}) \ , \end{aligned} \tag{12.18}$$

where, in the Ramond case, the overall factor 16 comes from the 16 spinor elements of the ground state.

Now let us impose the GSO projection. In the Ramond case, it simply divides the result by 2, since we start with an 8 component spinor in the ground state. In the NS case, we have to remove the states with even fermion number. This amounts to

$$G(q) = \tfrac{1}{2}\mathrm{Tr}\left(q^N - (-1)^F q^N \right) \ , \tag{12.19}$$

where F is the fermion number. Multiplying with $(-1)^F$ implies that we replace $g(r)$ in Eq. (12.16) by

$$\tilde{g}(r) = \sum_{m=0}^{1} (-q)^{rm} = 1 - q^r \ . \tag{12.20}$$

This way, Eq. (12.18) turns into

$$\begin{aligned} G_{\mathrm{NS}}(q) &= \frac{1}{2\sqrt{q}} \left[\prod_{n=1}^{\infty} \left(\frac{1+q^{n-\frac{1}{2}}}{1-q^n} \right)^8 - \prod_{n=1}^{\infty} \left(\frac{1-q^{n-\frac{1}{2}}}{1-q^n} \right)^8 \right] \quad \text{(NS)} \ ; \\ G_{\mathrm{R}}(q) &= 8 \prod_{n=1}^{\infty} \left(\frac{1+q^n}{1-q^n} \right)^8 \qquad \text{(Ramond)} \ . \end{aligned} \tag{12.21}$$

Here, in the NS case, we divided by $\sqrt{q}$ because the ground state can now be situated at $N = -\frac{1}{2}$, and it cancels out.

The mathematical theorem alluded to in the previous subsection says that, in Eq. (12.21), $G_{\mathrm{NS}}(q)$ and $G_{\mathrm{R}}(q)$ are equal. Mathematica gives for both:

$$\begin{aligned} G(q) &= 8 + 128\,q + 1152\,q^2 + 7680\,q^3 + 42112\,q^4 + 200448\,q^5 \\ &\quad + 855552\,q^6 + 3345408\,q^7 + 12166272\,q^8 + \cdots \ . \end{aligned} \tag{12.22}$$

We note that:

1152 / 288 = 4   12166272 / 288 = 42244   200448 / 1728 = 116

3345408 / 1728 = 1936;   where   288 * 6 = 1728

The number 1728 is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

We calculate the following simple double integral:

1/(10^38) * 1.08643^2 * (Pi/2) integrate integrate [1728]

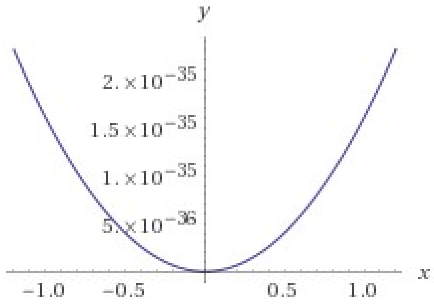
$$\frac{1}{10^{38}} \times 1.08643^2 \times \frac{\pi}{2} \int \left(\int 1728\, dx \right) dx$$

Result:

$$1.60191 \times 10^{-35}\, x^2$$

Result very near to the Planck length about equal to 1.6 * 10^{-35}

Plot:



(x from −1.2 to 1.2)

And:

1.08643 integrate [1728] x, [0, Pi/((1.618)^9)]

Definite integral:

$$1.08643 \int_0^{0.0413374} 1728\, x\, dx = 1.60399$$

Now we take some parts of the following very interesting paper: "RAMANUJAN'S UNPUBLISHED MANUSCRIPT ON THE PARTITION AND TAU FUNCTIONS WITH PROOFS AND COMMENTARY - Bruce C. Berndt and Ken Ono"

PROPERTIES OF $p(n)$ AND $\tau(n)$ DEFINED BY THE FUNCTIONS $\sum_{n=0}^{\infty} p(n)q^n = (q;q)_{\infty}^{-1}$, $\sum_{n=1}^{\infty} \tau(n)q^n = q(q;q)_{\infty}^{24}$

S. RAMANUJAN

We take:

$$\sum_{n=1}^{\infty} \tau(n)q^n = q(q;q)_{\infty}^{24}$$

Modulus 5

1. Let

$$P := 1 - 24 \sum_{n=1}^{\infty} \frac{nq^n}{1-q^n},$$

$$Q := 1 + 240 \sum_{n=1}^{\infty} \frac{n^3 q^n}{1-q^n}$$

and

$$R := 1 - 504 \sum_{n=1}^{\infty} \frac{n^5 q^n}{1-q^n},$$

so that[2]

$$Q^3 - R^2 = 1728q(q;q)_\infty^{24}. \tag{1.1}$$

Let $\sigma_s(n)$ denote the [sum of the] s^{th} powers of the divisors of n. Then it is easy to see that

$$Q = 1 + 5J; \qquad R = P + 5J. \tag{1.2}$$

Hence,

$$Q^3 - R^2 = Q - P^2 + 5J. \tag{1.3}$$

But[3]

$$Q - P^2 = 288 \sum_{n=1}^{\infty} n\sigma_1(n) q^n; \tag{1.4}$$

and it is obvious that

$$(q;q)_\infty^{24} = \frac{(q^{25};q^{25})_\infty}{(q;q)_\infty} + 5J. \tag{1.5}$$

Now:

$$Q^3 - R^2 = 1728q(q;q)_\infty^{24}.$$

where

$$\sum_{n=1}^{\infty} \tau(n) q^n = q(q;q)_\infty^{24}$$

We note that: Q = 1 + 240 = 241; R = 1 – 504 = – 503; thence

$Q^3 - R^2 = 241^3 - (-503^2) = 13997521 - 253009 = 13744512;$

We have that: 13744512 / 1728 = 7954; thence $q(q;q)_{\infty}^{24} = 7954$. Indeed:

13744512 = 1728 * 7954.

Now, we calculate the following simple double integral:

1/(10^33) * 1/((sqrt(e))^3) * 1.08643 * integrate integrate [13744512]

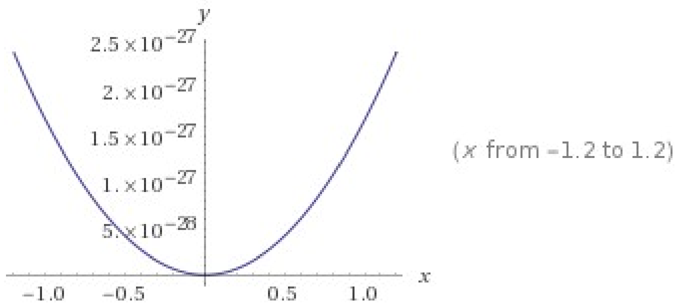
$$\frac{1}{10^{33}} \times \frac{1}{\sqrt{e}^3} \times 1.08643 \int\left(\int 13744512\,dx\right)dx$$

Result:

$1.66594 \times 10^{-27} x^2$

Also this result is very near to the proton mass

Plot:



$$(1.4) \qquad Q - P^2 = 288 \sum_{n=1}^{\infty} n\sigma_1(n)q^n;$$

We note that 288 / 144 = 2 and that 288 * 6 = 1728 and 1728 / 144 = 12

$$(3.2) \qquad Q^2 - PR = 1008 \sum_{n=1}^{\infty} n\sigma_5(n)q^n;$$

We note that 1008 / 144 = 7

We have:

$$(5.2) \qquad Q^2 = P + 7J; \qquad R = 1 + 7J;$$

and so

$$(5.3) \qquad (Q^3 - R^2)^2 = P^3 - 2PQ + R + 7J.$$

But[8]

$$(5.4) \qquad \begin{cases} PQ - R = 720 \sum_{n=1}^{\infty} n\sigma_3(n)q^n, \\ P^3 - 3PQ + 2R = -1728 \sum_{n=1}^{\infty} n^2\sigma_1(n)q^n; \end{cases}$$

We note that 720 / 144 = 5; 1728 / 288 = 6

Then: R = 1 + 7J; for R = - 503; 1 + 7J = - 503; 7J = - 504; J = - 504 / 7 = - 72.

Q^2 = P – 504; P – 504 = Q^2; P = Q^2 + 504 = 241^2 + 504 = 58585;

PQ – R = 58585 * 241 - (- 503) = 14118985 + 503 = 14119488;

Indeed: 720 * 19610,4 = 141194888;

P^3 – 3PQ + 2R = 201075567351625 – 3(14118985) – 1006 = 201075524993664;

Indeed: -116363151038 * -1728 = 201075524993664

Now, we calculate the following double integral:

1.1056^2 * 1/(10^40) * 1/(e)^2 integrate integrate [201075524993664]

where 1.1056 is the value of the cosmological constant (Planck 2018)

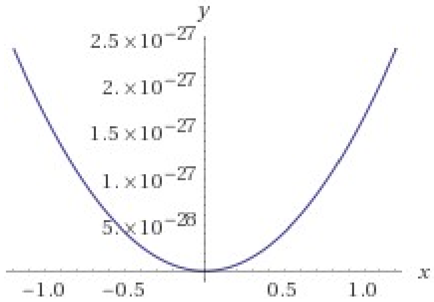
$$1.1056^2 \times \frac{1}{10^{40}} \times \frac{1}{e^2} \int\left(\int 201\,075\,524\,993\,664\, dx\right) dx$$

Result:

$1.66317 \times 10^{-27}\, x^2$

Plot:

A result similar to the previous one



(x from -1.2 to 1.2)

Now:

$$(7.1)\qquad \begin{cases} Q^3 - R^2 = 1728 \sum_{n=1}^{\infty} \tau(n) q^n, \\ 3Q^3 + 2R^2 - 5PQR = 1584 \sum_{n=1}^{\infty} n\sigma_9(n) q^n, \\ 5Q^3 + 4R^2 - 18PQR + 9P^2Q^2 = 8640 \sum_{n=1}^{\infty} n^2 \sigma_7(n) q^n; \end{cases}$$

We have that:

$5 * 241^3 + 4 * (-503)^2 - 18(58585*241* -503) + 9(58585^2 * 241^2) =$

= 69987605 + 1012036 + 127833290190 + 1794111636872025 =

= 1794239541161856.   We have that

1038333067802 * (8640/5) = 1794239541161856 and 1728 * 5 = 8640

Now, we calculate the following double integral, where -0.165421 is ζ'(-1):

1/(10^33) * (-0.165421) * 1.08643 * integrate integrate [1794239541161856]

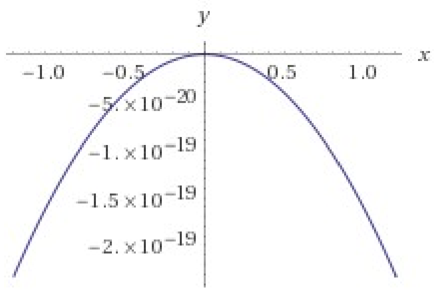
$$\frac{1}{10^{33}} \times (-0.165421) \times 1.08643 \int \left(\int 1794239541161856 \, dx \right) dx$$

Result:

$$-1.61229 \times 10^{-19} x^2$$

Plot:

Result very near to the electric charge of electron



(x from -1.2 to 1.2)

Now:

$$(9.3)\quad \begin{cases} P^5 - 10P^3Q + 20P^2R - 15PQ^2 + 4QR = -20736 \sum_{n=1}^{\infty} n^4\sigma_1(n)q^n, \\ P^3Q - 3P^2R + 3PQ^2 - QR = 3456 \sum_{n=1}^{\infty} n^3\sigma_3(n)q^n, \\ P^2R - 2PQ^2 + QR = -1728 \sum_{n=1}^{\infty} n^2\sigma_5(n)q^n, \\ PQ^2 - QR = 720 \sum_{n=1}^{\infty} n\sigma_7(n)q^n; \end{cases}$$

58585^2 * (-503) – 2(58585)$(241)^2$ + 241* (-503) =

= 1726397719175 – 6805350770 – 121223 = 1719592247182:

We have that - 995134402,304398148 * - 1728 = 1719592247182

Now, we calculate the following double integral:

1.08643^2 * 1/(10^37) * 1/(4e^e) integrate integrate [1719592247182]

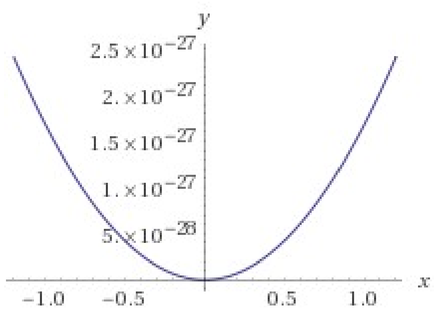
$$1.08643^2 \times \frac{1}{10^{37}} \times \frac{1}{4\, e^e} \int \left(\int 1719592247182\, dx \right) dx$$

Result:

$$1.67419 \times 10^{-27}\, x^2$$

Result practically equal to the neutron mass

Plot:



(x from -1.2 to 1.2)

And:

$$(13.3)\quad \begin{cases} 5(P^6 - 15P^4Q + 40P^3R - 45P^2Q^2 + 24PQR) \\ \quad - (9Q^3 + 16R^2) = -248832 \sum_{n=1}^{\infty} n^5 \sigma_1(n) q^n, \\ 7(P^4Q - 4P^3R + 6P^2Q^2 - 4PQR) + (3Q^3 + 4R^2) = 41472 \sum_{n=1}^{\infty} n^4 \sigma_3(n) q^n, \\ 2(P^3R - 3P^2Q^2 + 3PQR) - (Q^3 + R^2) = -5184 \sum_{n=1}^{\infty} n^3 \sigma_5(n) q^n, \\ 9(PQ - R)^2 + 5(Q^3 - R^2) = 8640 \sum_{n=1}^{\infty} n^3 \sigma_7(n) q^n, \\ 5PQR - (3Q^3 + 2R^2) = -1584 \sum_{n=1}^{\infty} n \sigma_9(n) q^n, \\ Q^3 - R^2 = 1728 \sum_{n=1}^{\infty} \tau(n) q^n; \end{cases}$$

Where – 248832 / 1728 = – 144

Thence:

$-(9*241^3 + 16*(-503)^2) = -(125977689 + 4048144) = - 130025833$

-248832 * 522,544660654578189300411522633374 = - 130025833

(1728 * (-144)) * 522,544660654578189300411522633374 = - 130025833.

Now, we calculate the following double integral:

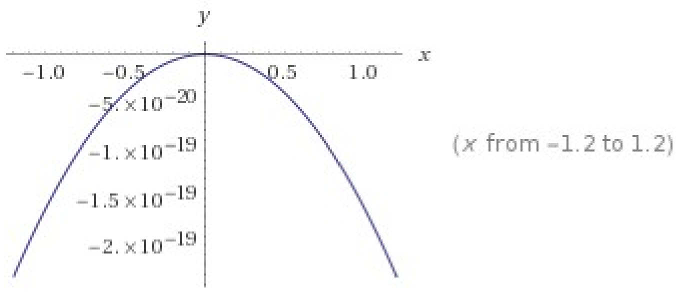
1.08643^2 * 1/(10^25) * 1/(Pi*e^e) integrate integrate [-130025833]

$$1.08643^2 \times \frac{1}{10^{25}} \times \frac{1}{\pi\, e^e} \int \left(\int -130\,025\,833\, dx \right) dx$$

Result:

$-1.61183 \times 10^{-19}\, x^2$

Plot:



Now:

$$6912 \sum_{n=1}^{\infty} n^3 \sigma_1(n) q^n = 6P^2Q - 8PR + 3Q^2 - P^4.$$

$6*58585^2 * 241 – 8*58585*(-503) + 3*241^2 – 58585^4 =$

= 4962964417350 + 235746040 + 174243 – 11780012113294950625 =

= -11780007150094769992

(4*1728) * -1704283441853988,7141203703703704 = -11780007150094769992

Now, we calculate the following double integral:

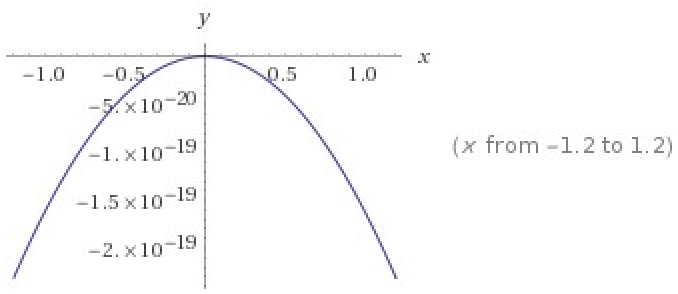
1.08643^2 * 1/(10^36) * 1/(Pi^2*1.61803398*e) integrate integrate [-11780007150094769992]

$$1.08643^2 \times \frac{1}{10^{36}} \times \frac{1}{\pi^2 \times 1.61803398\, e} \int\left(\int -11\,780\,007\,150\,094\,769\,992\, dx\right) dx$$

Result:

$-1.60154 \times 10^{-19}\, x^2$

Plot:



$-1.61183 \times 10^{-19}\, x^2$

$-1.60154 \times 10^{-19}\, x^2$

Results both very close to the value of the electron electric charge

From: Canad. Math. Bull. Vol. **42** (4), 1999 pp. 427–440
"Ramanujan and the Modular *j*-Invariant" - Bruce C. Berndt and Heng Huat Chan

Now, we have the following Ramanujan function:

Except for four entries, the last two pages in Ramanujan's third notebook, pages 392 and 393 in the pagination of [21, vol. 2], are devoted to values of the modular j-invariant. Recall [14, p. 81], [15, p. 224] that the invariants $J(\tau)$ and $j(\tau)$, for $\tau \in \mathbb{H} := \{\tau : \operatorname{Im}\tau > 0\}$, are defined by

$$J(\tau) = \frac{g_2^3(\tau)}{\Delta(\tau)} \quad \text{and} \quad j(\tau) = 1728\,J(\tau), \tag{1.1}$$

where

$$\Delta(\tau) = g_2^3(\tau) - 27g_3^2(\tau), \tag{1.2}$$

$$g_2(\tau) = 60 \sum_{\substack{m,n=-\infty \\ (m,n)\neq(0,0)}}^{\infty} (m\tau + n)^{-4},$$

and

$$g_3(\tau) = 140 \sum_{\substack{m,n=-\infty \\ (m,n)\neq(0,0)}}^{\infty} (m\tau + n)^{-6}.$$

Furthermore, the function $\gamma_2(\tau)$ is defined by [15, p. 249]

$$\gamma_2(\tau) = \sqrt[3]{j(\tau)}, \tag{1.3}$$

Theorem 1.1 *For* $q = \exp(-\pi\sqrt{n})$, *define*

$$t := t_n := \sqrt{3}q^{1/18}\frac{f(q^{1/3})f(q^3)}{f^2(q)}. \tag{1.13}$$

Then

$$t_n = \left(2\sqrt{64J_n^2 - 24J_n + 9} - (16J_n - 3)\right)^{1/6}. \tag{1.14}$$

Ramanujan then gives a table of polynomials satisfied by t_n, for five values of n.

Theorem 1.2 *For the values of n given below, we have the following table of polynomials* $p_n(t)$ *satisfied by* t_n.

n	$p_n(t)$
11	$t-1$
35	t^2+t-1
59	t^3+2t-1
83	t^3+2t^2+2t-1
[illegible]	[illegible]

Proof of Theorem 1.2 It is well known that $J_{11} = \mathbf{1}$ [15, p. 261]. Thus, we find that

$$t_{11} = (2\cdot 7 - 13)^{1/6} = 1,$$

as desired.

Secondly, from a paper of W. E. Berwick [6],

$$J_{35} = \sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4.$$

Hence,

$$t_{35} = \left(2\sqrt{64\cdot 5\left(\frac{\sqrt{5}+1}{2}\right)^8 - 24\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 + 9} - \left(16\sqrt{5}\left(\frac{\sqrt{5}+1}{2}\right)^4 - 3\right)\right)^{1/6}$$

$$= \left(2\sqrt{7349 + 3276\sqrt{5}} - 117 - 56\sqrt{5}\right)^{1/6}.$$

We have:

$J_{35} = 15{,}3262379$; and

$t_{35} = (2*121{,}1377674149 - 117 - 125{,}219806739)^{1/6} = (0{,}0557280908)^{1/6} =$

$= 0{,}618033990227 = (\sqrt{5} - 1) / 2$

For

$$j(\tau) = 1728J(\tau),$$

we have: 1728 * 15,3262379 = 26483,7390912

We calculate the following double integral:

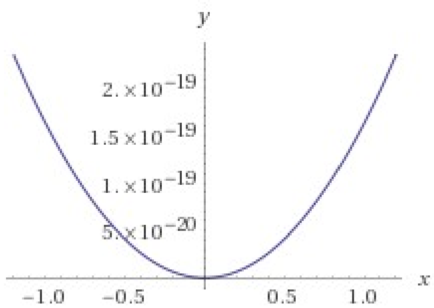
1.08643^2 * 1/(10^20) * 1/(Pi^6) integrate integrate [26483.7390912]

$$1.08643^2 \times \frac{1}{10^{20}} \times \frac{1}{\pi^6} \int \left(\int 26483.7390912 \, dx \right) dx$$

Result:

$$1.62575 \times 10^{-19} x^2$$

Plot:



(x from −1.2 to 1.2)

It is easy to verify that if $a^2 - db^2 = c^2$, then

$$\sqrt{a \pm b\sqrt{d}} = \sqrt{\frac{a+c}{2}} \pm \sqrt{\frac{a-c}{2}}. \tag{4.1}$$

Now, since

$$7349^2 - 5 \cdot 3276^2 = 589^2,$$

we find that

$$\sqrt{7349 + 3276\sqrt{5}} = \sqrt{\frac{7349+589}{2}} + \sqrt{\frac{7349-589}{2}} = \sqrt{3969} + \sqrt{3380} = 63 + 26\sqrt{5},$$

by (4.1). Hence,

$$t_{35} = \left(2\left(63 + 26\sqrt{5}\right) - 117 - 56\sqrt{5}\right)^{1/6} = (9 - 4\sqrt{5})^{1/6} = \frac{\sqrt{5}-1}{2}.$$

Hence, t_{35} is a root of $t^2 + t - 1$, and the second result is established.

For $n = 59$, Greenhill [18] showed that u_{59}, defined by (1.4), is a root of the equation

$$u - 392 \cdot 2^{1/3} u^{2/3} + 1072 \cdot 4^{1/3} u^{1/3} - 2816 = 0.$$

We have that: $63 + 26\sqrt{5} = 121{,}137767414994532106638515387 01$;

Note that $(121{,}1377674149)^{1/10} = 1{,}61557809657...$

We calculate the following double integral:

1.08643^2 * 1/(10^20) * 0.226 integrate integrate [121.13776741499453210663851538701]

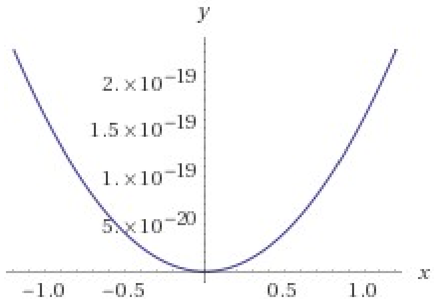
where 0.225791 = log(sqrt(π/2) = 0.226

$$1.08643^2 \times \frac{1}{10^{20}} \times 0.226 \int\left(\int 121.13776741499453210663851538701\, dx\right) dx$$

Result:

$$1.6157 \times 10^{-19}\, x^2$$

Plot:



(x from −1.2 to 1.2)

$$1.62575 \times 10^{-19}\, x^2$$
$$1.6157 \times 10^{-19}\, x^2$$

Result very close to the value of the electron electric charge

From Wikipedia

The **Dirac sea** is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930[1] to explain the anomalous negative-energy quantum states predicted by the Dirac equation for relativistic electrons.[2] The positron, the antimatter counterpart of the electron, was originally conceived of as a hole in the Dirac sea, well before its experimental discovery in 1932.[nb 1]

Upon solving the free Dirac equation,

$$i\hbar\frac{\partial\Psi}{\partial t} = (c\hat{\boldsymbol{\alpha}}\cdot\hat{\boldsymbol{p}} + mc^2\hat{\beta})\Psi,$$

one finds

$$\Psi_{\mathbf{p}\lambda} = N\begin{pmatrix} U \\ \frac{(c\hat{\sigma}\cdot p)}{mc^2+\lambda E_p}U \end{pmatrix}\frac{\exp[i(\mathbf{p}\cdot\mathbf{x}-\varepsilon t)/\hbar]}{\sqrt{2\pi\hbar}^3},$$

where

$$\varepsilon = \pm E_p, \quad E_p = +c\sqrt{\mathbf{p}^2 + m^2c^2}, \quad \lambda = \operatorname{sgn}\varepsilon$$

for plane wave solutions with 3-momentum **p**. This is a direct consequence of the relativistic energy-momentum relation

$$E^2 = p^2c^2 + m^2c^4$$

upon which the Dirac equation is built. The quantity U is a constant 2×1 column vector and N is a normalization constant. The quantity ε is called the *time evolution factor*, and its interpretation in similar roles in, for example, the plane wave solutions of the Schrödinger equation, is the energy of the wave (particle). This interpretation is not immediately available here since it may acquire negative values. A similar situation prevails for the Klein–Gordon equation. In that case, the *absolute value* of ε can be interpreted as the energy of the wave since in the canonical formalism, waves with negative ε actually have *positive* energy E_p. But this is not the case with the Dirac equation. The energy in the canonical formalism associated with negative ε is $-E_p$. In hole theory, the solutions with negative time evolution factors are reinterpreted as representing the positron, discovered by Carl Anderson. The interpretation of this result requires a Dirac sea, showing that the Dirac equation is not merely a combination of special relativity and quantum mechanics, but it also implies that the number of particles cannot be conserved

$$\beta mc^2 \rightarrow \begin{bmatrix} mc^2 & 0 & 0 & 0 \\ 0 & mc^2 & 0 & 0 \\ 0 & 0 & -mc^2 & 0 \\ 0 & 0 & 0 & -mc^2 \end{bmatrix} \qquad (16)$$

For eigenvalues and considering $p = 0$ (in equation (4)), we will have[9]:

$$E_+ = mc^2, \; E_- = -mc^2 \qquad (17)$$

From the Dirac equation to the photon structure

In pair production of "electron-positron", one photon with spin 1 and at least energy $E = 1.022\, MeV$ is converted to two fermions, electron and positron with spin $\frac{1}{2}$, each of them with context of energy 0.511 MeV in vicinity of a heavy nucleus so that we have the following relation:

$$\gamma \rightarrow e^- + e^+ \qquad (18)$$

Relation (18) is justifiable according to Dirac equation by relations (16) and (17), (Figure 1.A). In pair decay, an electron is combined with a positron and is produced two photons (Figure 1.B).

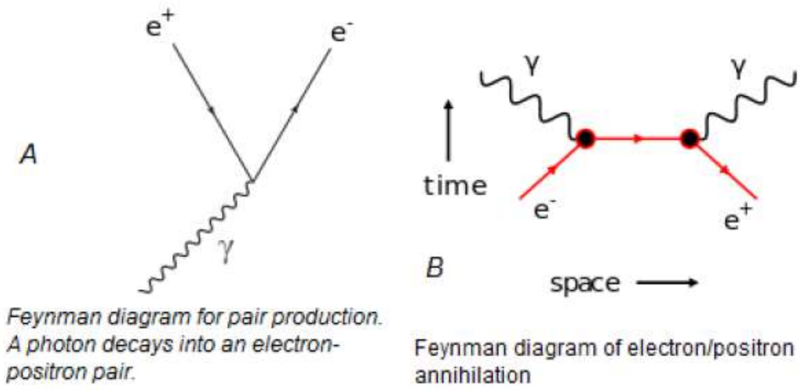


Fig1: Production and decay of pair "electron-positron"

In pair decay, reverse of relation (18) takes place and we will have:

$$e^- + e^+ \rightarrow 2\gamma \qquad (19)$$

In all physical processes including pair production and decay, it must be held the following conservation laws:

1- Electric charge conservation law, pure charge before and after the process must be equal.

2- Linear momentum and total energy conservation laws: These rules has made forbidden production of just one photon (Gamma ray). As it is seen in Figure (2), two photon with the same energy move but in two opposite directions. Angular momentum conservation law must be held too. In fact, in the process of "electron-positron" decay, these following relations hold:

$$e^- + e^+ \rightarrow 2\gamma$$
$$E_{2\gamma} = 2m_0c^2 + E_{e^-} + E_{e^+}$$
$$m_0c^2 = 0.511\ MeV$$

In which m_0c^2 is zero rest mass of electron (also positron) and E_{e^-} , E_{e^+} are kinetic energy of electron and positron that are converted to energy of photons ($E_{2\gamma}$) at the time of pair decay.

We take the value E = 1.022 MeV and calculate the following integral:

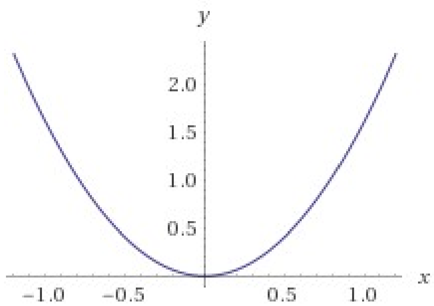
1.08643 * [(2*0.61803398)^5] integrate integrate [1.022]

$$1.08643\,(2 \times 0.61803398)^5 \int \left(\int 1.022\, dx \right) dx$$

Result:

$1.6019\, x^2$

Plot:



(x from -1.2 to 1.2)

result that is practically equal to the electric charge of the positron.

From: *RAMANUJAN'S EXPLICIT VALUES FOR THE CLASSICAL THETA-FUNCTION* - BRUCE C. BERNDT AND HENG HUAT CHAN

§2. *Evaluations of* $\varphi(q)$.

All page numbers below refer to Ramanujan's first notebook.

THEOREM 1.

$$\frac{\varphi(e^{-5\pi})}{\varphi(e^{-\pi})}=\frac{1}{\sqrt{5\sqrt{5}-10}}.$$

THEOREM 2 (p. 284).

$$\frac{\varphi(e^{-3\pi})}{\varphi(e^{-\pi})}=\frac{1}{\sqrt[4]{6\sqrt{3}-9}}.$$

From Theorem 1, we have:

$$\frac{1}{\sqrt{5\sqrt{5}-10}}=\frac{1}{1,0864344837582}=$$

$$=0,920442065259$$

We note that 0,920442 is a value very near to the spin of the Kerr black hole SMBH87 that is a = 0,9375

$(0,920442065259)^6 = 0,6081052449363...$

We note that 1 / 0,920442065259 = 1,08643448... and that

$(1,08643448...)^6 = 1,6444521529221 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

With regard 1.08643448, we have that:

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)}=1+\int_0^\infty \frac{e^{-t^2/2}}{\sqrt{2\pi}}\left[\frac{4e^{\pi}\left(e^{2\pi}-\cos\left(\sqrt{2\pi}t\right)\right)}{e^{4\pi}-2e^{2\pi}\cos\left(\sqrt{2\pi}t\right)+1}\right]dt$$

where:

$$\Gamma\left(\frac{3}{4}\right)=\frac{\pi\sqrt{2}}{\Gamma\left(\frac{1}{4}\right)}=\frac{4,44288293815}{3,625609908}=1,2254167025$$

$$\frac{\pi^{1/4}}{\Gamma\left(\frac{3}{4}\right)} = \frac{1,3313353638}{1,2254167025} = 1,08643481 \ldots$$

We calculate the following double integral:

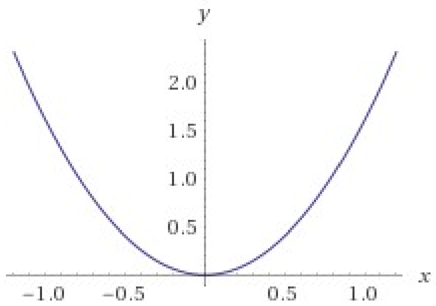
(11/(Pi)) * integrate integrate [0.920442065259]

$$\frac{11}{\pi} \int \left(\int 0.920442065259 \, dx \right) dx$$

Result:

$1.61142194967 \, x^2$

Plot:



(x from −1.2 to 1.2)

From Theorem 2, we have:

$$\frac{1}{\sqrt[4]{6\sqrt{3} - 9}} = 0,920590346$$

We note that 0,920590346 is a value very near to the spin of the Kerr black hole SMBH87 that is a = 0,9375

We note that 1 / 0,920590346 = 1,08625949027.... We have that $(0,920590346)^6$ = = 0,6086932664295... and $(1,08625949027)^6$ = 1,64286358189...

From Theorem 5, we have:

$$\frac{1 + \sqrt[3]{2\left(\sqrt{3} + 1\right)}}{3} = 0,920441787836$$

We note that 0,92044178 is a value very near to the spin of the Kerr black hole SMBH87 that is a = 0,9375

We note that 1 / 0,920441787836 = 1,0864348112… and that $(0.920441787836)^6$ =

= 0,6081041452326... and $(1.0864348112)^6$ = 1,6444551607959...

From the Theorem 7, we have:

$$\frac{3+\sqrt{5}+\left(\sqrt{3}+\sqrt{5}+(60)^{1/4}\right)\sqrt[3]{2+\sqrt{3}}}{3\sqrt{10+10\sqrt{5}}}=$$

$$=\frac{5{,}23606797+(1{,}732050807+5{,}0192256612)(1{,}551133518071245)}{17{,}06593443017349}=\frac{15{,}70819918959}{17{,}06593443017349}=$$

= 0,920441787343196...

We note that 0,92044178 is a value very near to the spin of the Kerr black hole SMBH87 that is a = 0,9375

We note that 1 / 0,920441787343196 = 1,0864348117945... and that $(0{,}920441787343196)^6$ = 0,608104143279149... and $(1{,}0864348117945...)^6$ = = 1,644455166195...

From the Theorem 8, we have:

$$\frac{\sqrt{13+\sqrt{7}}+\sqrt{7+3\sqrt{7}}}{14}(28)^{1/8}=0{,}558596102528\cdot 1{,}516682772959=$$

$$=0{,}84721308574758....$$

We note that 1 / 0,84721308574758 = 1,180340597687... and that $(0{,}84721308574758)^3$ = 0,60810414728439... and $(1{,}180340597687)^3$ = = 1,64445515536166478

From the Theorem 9, we have:

$$G=G_{169}$$

$$=\frac{1}{3}\left((\sqrt{13}+2)+\left(\frac{13+3\sqrt{13}}{2}\right)^{1/3}\right.$$

$$\left.\times\left\{\left(\frac{11+\sqrt{13}}{2}+3\sqrt{3}\right)^{1/3}+\left(\frac{11+\sqrt{13}}{2}-3\sqrt{3}\right)^{1/3}\right\}\right)$$

$$a=(G-G^{-1})^3+7(G-G^{-1}).$$

$$\frac{\varphi(e^{-13\pi})}{\varphi(e^{-\pi})}=\left(G^{-3}\left(\frac{a+\sqrt{a^2+52}}{2}\right)\right)^{-1/2}.$$

$$G = \frac{1}{3}(5{,}60555127546 + 2{,}283583604339 \cdot 3{,}60265210495172565) =$$

$= 1/3\ (5{,}60555127546 + 8{,}2269572790051469) =$

$= 4{,}610836184821...$

$a = (4{,}610836184821 - 1 / 4{,}610836184821)^3 + 7\ (4{,}393955783974) =$

$= 84{,}8334339420727 + 30{,}7576904878238$

$= 115{,}5911244298...$

Thence:

$$\left(G^{-3}\left(\frac{a+\sqrt{a^2+52}}{2}\right)\right)^{-1/2}.$$

$(4{,}610836184821^{-3}(115{,}70348060040195))^{-1/2} =$

$= (0{,}010201426896501439614925349770 45(115{,}70348060040195))^{-1/2} =$

$= 1{,}1803405990157729898986354312672^{-1/2} =$

$= 0{,}920441787835717....$

We note that 0,92044178 is a value very near to the spin of the Kerr black hole SMBH87 that is $a = 0{,}9375$

Note that $1 / 0{,}920441787835717 = 1{,}0864348112....$ and $(0{,}920441787835717)^6 =$

$= 0{,}60810414523149....$; $(1{,}0864348112)^6 = 1{,}6444551607959....$

From Theorem 10, we have:

$$\frac{\varphi(e^{-27\pi})}{\varphi(e^{-3\pi})} = \frac{1}{3}\left(1+(\sqrt{3}-1)\left(\frac{\sqrt[3]{2(\sqrt{3}+1)}+1}{\sqrt[3]{2(\sqrt{3}-1)}-1}\right)^{1/3}\right).$$

$$\frac{1}{3}\left(1 + 0{,}732050807568 \cdot \left(\frac{2{,}7613253635096}{0{,}135508544551208}\right)^{1/3}\right) =$$

$$= \frac{1}{3}(1 + 0{,}732050807568 \cdot 2{,}731389488758629) =$$

= 0,9998386270095...

We note that 0,9998386 is a good approximation to the spin of the Kerr black hole SMBH87 that is a = 0,9375

We note that 1 / 0,9998386270095 = 1,0001613990359... and that

$(1{,}0001613990539)^{512}$ = 1,08613947969... and $(1{,}08613947969)^{6}$ = 1,64177485549...

1 / 1,08613947969 = 0,9206920646...

From Theorem 11, we have:

$$\frac{\varphi(e^{-63\pi})}{\varphi(e^{-7\pi})} = \frac{1}{3}\left(1+\left(\frac{\sqrt{4+\sqrt{7}}-7^{1/4}}{2}\right)^{3}\sqrt{\sqrt{3}+\sqrt{7}}(2+\sqrt{3})^{1/6} \times \sqrt{\frac{2+\sqrt{7}+\sqrt{7+4\sqrt{7}}}{2}}\sqrt{\frac{\sqrt{3+\sqrt{7}}+(6\sqrt{7})^{1/4}}{\sqrt{3+\sqrt{7}}-(6\sqrt{7})^{1/4}}}\right).$$

Thence:

$$\frac{1}{3}\left[1 + (0{,}10763244) \cdot 2{,}60586945 \cdot 2{,}102256018 \sqrt{\frac{4{,}3721457616}{0{,}3800121949}}\right] =$$

$$\frac{1}{3}[1 + (0{,}10763244) \cdot 2{,}60586945 \cdot 2{,}102256018 \cdot 3{,}391943] =$$

= 0,99999999144589576154611990169733.

We have that 1 / 0,99999999144589576154611990169733 = 1,0000000085541...

and 1,0000000171082 = $1{,}0000000085541^{2}$

Note that:

1,0743942252863497655136433325161 = $1{,}0000000085541^{8388608}$

$(1{,}0743942252863497655136433325161)^{7}$ = 1,6525159416742578....

Now, we have

$$(28)^{1/4}(\sqrt{13+\sqrt{7}}+\sqrt{7+3\sqrt{7}})^2((2\sqrt{7}+9)\sqrt{4\sqrt{7}+7}+\sqrt{1820+688\sqrt{7}})$$
$$=(14)^2\left(\frac{7^{1/4}+\sqrt{4+\sqrt{7}}}{2}\right)^6. \qquad (3.45)$$

(196) (86,320436234837760525804749984845) = 16918,8055;

$(16918,8055)^{1/19}$ = 1,66934374266... and $(1,66934374266...)^{1/6}$ = 1,0891581918...

Now:

$$56+23\sqrt{7}=\sqrt{7}(4+\sqrt{7})^2$$

(56 + 60,85228015448) = 116,85228... $(116,85228)^{1/10}$ = 1,609769590768... and $(1,609769590768)^{1/6}$ = 1,08258154636...

Now:

$$53172+20097\sqrt{7}=63(844+319\sqrt{7})=63(4+\sqrt{7})(127+48\sqrt{7})$$

(418,682332597) (253.9960629311) = 106343,664098465 $(106343,664098465)^{1/24}$ = 1,61974376738679... and $(1,61974376738679...)^{1/6}$ = 1,08369662189088...

Now we calculate the following double integral:

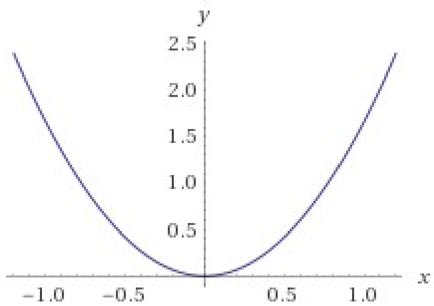
1.086434811 * integrate integrate [sqrt(106343.664)] [Pi/336]

$$1.086434811 \int\left(\int \sqrt{106\,343.664} \times \frac{\pi}{336}\, dx\right) dx$$

Result:

$1.6563\, x^2$

Plot:



(x from −1.2 to 1.2)

Now:

$$\sqrt{455+172\sqrt{7}}=\frac{5 \, . \, 7^{3/4}}{\sqrt{2}}+\frac{91}{7^{3/4}\sqrt{2}} \, . \qquad (3.51)$$

That is equal to 30,1673536377175... $(30,1673536377175...)^{1/7}$ = 1,626905993875... and $(1,626905993875)^{1/6}$ = 1,0844938075...

Now:

$$A=4\sqrt{2}(133+59\sqrt{7})\left(\sqrt{\frac{7}{2}}+\sqrt{\frac{1}{2}}\right)+4 \, . \, 7^{1/4}(14+9\sqrt{7})(4+\sqrt{7})\left(\sqrt{\frac{7}{2}}+\sqrt{\frac{1}{2}}\right)$$
$$+2 \, . \, 7^{1/4}(20+4\sqrt{7})(5 \, . \, 7^{3/4}+91 \, . \, 7^{-3/4})$$
$$+24 \, . \, 7^{3/4}\left(\sqrt{\frac{7}{2}}+\sqrt{\frac{1}{2}}\right)(8+3\sqrt{7}). \qquad (3.52)$$

(4215,93700689) + (4214,8055475) + (4244,6010469) + (4243,461483) =

= 16918,80508429 result that is equal to (3.45)

And

$$B=(14)^2\left(\frac{7^{1/4}+\sqrt{\frac{7}{2}}+\sqrt{\frac{1}{2}}}{2}\right)^6.$$

That is: (196) (86,320436234837760525804749984845) = 16918,805502028... result that is equal to (3.45) and (3.52)

We note that $(16918,805)^{1/20}$ = 1,62711588227593... and that A + B = 33837,61

$(33837,61)^{1/21}$ = 1,6431821143123... and $(1,6431821143123)^{1/6}$ = 1,0862945896...

Further, we note that: 126 (1,086294589) = 136,8731 and 126 = 21 * 3 * 2 and is the mass of the Higgs boson.

Now, we calculate the following double integral of the result 16918,805

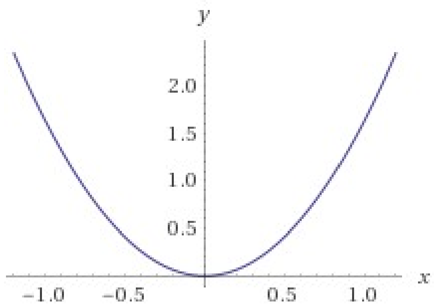
1.086434811 * integrate integrate [sqrt(16918.805)] [Pi/137.035]

$$1.086434811 \int \left(\int \sqrt{16918.805} \times \frac{\pi}{137.035}\, dx \right) dx$$

Result:

$1.61986\, x^2$

Plot:



(x from −1.2 to 1.2)

We note also that 1,64318211 * 4 = 6,57272844 ; 1,64318211 * 8 = 13,14545688 and 13,14545688 – 0,2 = 12,94545688.

Further: 1,64318211 * 5 = 8,21591055.

And: 16918,805 * 8 or 33837,61 * 4 = 135350,44 = 13,535044 * 10^4 where 13,5350 is a value very near to the value of SMBH87

We note that 6,57 is the value of the supermassive black hole M87 equal to about 6,5 milliard of solar masses, i.e. 12,92915 * 10^{39} kg (the mass is about 6,5 – 6,6 * 10^9 solar masses, thence 12,92915 * 10^{39} - 13,12806 * 10^{39} kg or 1,292915 - 1,312806 * 10^{40} kg) that is practically equal to 13,14545688 – 0,2 = 12,94545688. While 8,2159 is equal to the supermassive black hole of our galaxy the Milky Way that is 8,2 * 10^{36} kg. It is wonderful to see how from the number 1,64318211 we can obtain different and precise results, concerning observed and measured phenomena, such as the mass of supermassive black holes as M87.

https://www.livescience.com/65196-black-hole-event-horizon-image.html

This is the first-ever image of a black hole.

Credit: NSF

M87 is 53 million light-years away, deep in the center of a distant galaxy, surrounded by clouds of dust and gas and other matter, so no visible light telescope could see the black hole through all that gunk. It's not the nearest black hole, or even the nearest supermassive black hole. But it's so huge (as wide as our entire solar system, and 6.5 billion times the mass of the sun) that it's one of the two biggest-appearing in Earth's sky. (The other is Sagittarius A* at the center of the Milky Way.) To make this image, astronomers networked radio telescopes all over the world to magnify M87 to unprecedented resolution. They called the combined network the Event Horizon Telescope.

The Fine Structure Constant

From various papers of Michael John Sarnowski:

http://www.vixra.org/author/michael_john_sarnowski

proton-neutron mass ratio

$m_{\mathrm{p}}/m_{\mathrm{n}}$

Value 0.998 623 478 44

y2 is exactly the ratio of the mass of the neutron minus the mass of the electron divided my the mass of the neutron

Equation 4 $$\frac{1.674927351*10^{-27}-9.10938291*10^{-31}}{1.674927351*10^{-27}}=0.999456133$$

Why is the ratio of the proton divided by the neutron mass important? It is important since has been found in other examples to be important. In "The Aether Found, Discrete Calculations of Charge and Gravity with Planck Spinning Spheres and Kaluza Spinning Spheres" (5) In the calculation of the Force of Charge $q^2 = T\pi^3 hc\varepsilon(Me)/2Mn$ where $T^2 = \frac{((Mp-Me)^2 + Mn^2 + Mn^2)}{Mn^2}$ which uses the value described above of the mass of the proton minus the mass of the electron all over the mass of the neutron.

We have:

$$T^2 = (\frac{Mp-Me}{Mn})^2 + (\frac{Mn}{Mn})^2 + (\frac{Mn}{Mn})^2].$$

$$T^2 = 2.99616291064$$

$$T = 1.73094278087$$

$$\sigma = \frac{1}{\sqrt{1-(\frac{\pi Me}{3*3Mn})^2}} T\pi^3 \frac{Me}{4Mn}$$

T^2 is equal to:

$$((1.67262171\times 10^{-27} - 9.10938356\times 10^{-31})^2 + (1.674927351\times 10^{-27})^2 + (1.674927351\times 10^{-27})^2)/(1.674927351\times 10^{-27})^2$$

2.9961628296920973886784071889135415100832169385132916844 23...

$$\sqrt{2.9961628296920973886 78}$$

Where T is equal to

1.7309427574856706938 00...

Now we put the values that have obtained from the Ramanujan's expressions with the mass of the electron:

$$((1.6444551589\times 10^{-27} - 9.10938356\times 10^{-31})^2 + (1.6444551589\times 10^{-27})^2 + (1.6444551589\times 10^{-27})^2)/(1.6444551589\times 10^{-27})^2$$

2.998892416073470474394905578056746545000319947267420695234...

$$\sqrt{(((1.6444551589 \times 10^{-27} - 9.10938356 \times 10^{-31})^2 + (1.6444551589 \times 10^{-27})^2 + (1.6444551589 \times 10^{-27})^2)/(1.6444551589 \times 10^{-27})^2)}$$

1.7317310461...

This number 1,7317310461 is given from the value 1,6444551589 i.e. $(1{,}086434811)^6$ and after we calculate the fine structure constant with a specific formula.

We have also

$$((1.6444551589 \times 10^{-27} - 8.9933557229 \times 10^{-31})^2 + (1.6444551589 \times 10^{-27})^2 + (1.6444551589 \times 10^{-27})^2)/(1.6444551589 \times 10^{-27})^2$$

2.998906519708065270565036692990759754483997671319239990574...

and

$$\sqrt{(((1.6444551589 \times 10^{-27} - 8.9933557229 \times 10^{-31})^2 + (1.6444551589 \times 10^{-27})^2 + (1.6444551589 \times 10^{-27})^2)/(1.6444551589 \times 10^{-27})^2)}$$

1.7317351182...

Where $8{,}9933557229 = 1{,}7317310461^4$

electron-proton mass ratio m_e/m_p	
Value	5.446 170 213 52 x 10^{-4}
Standard uncertainty	0.000 000 000 52 x 10^{-4}
Relative standard uncertainty	9.5 x 10^{-11}
Concise form	5.446 170 213 52(52) x 10^{-4}

((1.6444551589*10^-27 - 8.99*10^-31))^2 + (1.6444551589*10^-27)^2 + (1.6444551589*10^-27)^2)/(1.6444551589*10^-27)^2

= 2.99890693 and 2.99890693^(0.5) = 1.73173524

Also this number can be utilized for to obtain the fine structure constant with the usual formula

((1.6513017728*10^-27 - 8.99*10^-31))^2 + (1.6513017728*10^-27)^2 + (1.6513017728*10^-27)^2)/(1.6513017728*10^-27)^2 = 2.99891146

2.99891146^(0.5) = 1.73173654

Where 1.6513017728 = 115.591124102 / 70 and 1.6444551589 is $(1.086434811)^6$

Further, we note that 115.591124102 / 72 = 1,605432279194

We have also 4,610836180 / (0,920442 * 3) = 1,66976059… and

0,99999999144589576154611990169733 * 9 = 8,99999992301306185391 4 = 8,99...

$$\sqrt{(((1.66979059 \times 10^{-27} - 8.99 \times 10^{-31})^2 + (1.66979059 \times 10^{-27})^2 + (1.66979059 \times 10^{-27})^2)/(1.66979059 \times 10^{-27})^2)}$$

1.73174002...

With this number 1,73174002 after we calculate the fine structure constant with a usual specific formula.

Now:

$$\sigma = \frac{1}{\sqrt{1-(\frac{\pi Me}{3 * 3Mn})^2}} T\pi^3 \frac{Me}{4Mn}$$

We have that:

3. Mass of Neutron = Mn= 1.674 927 471(21) x 10^{-27} kg1.674 927 351(74) x 10^{-27} kg

5. Mass of Electron = Me = 9.109 383 56(11) x 10^{-31} kg 9.109 382 91(40) x 10^{-31} kg.

Thence:

$$\sqrt{1 - \left(\frac{9.1093829140 \times 10^{-31}\,\pi}{3 \times 3 \times 1.67492735174 \times 10^{-27}}\right)^2}$$

0.999999981979339639...

$$\frac{1}{\sqrt{1-\left(\frac{9.1093829140\times 10^{-31}\,\pi}{3\times 3\times 1.67492735174\times 10^{-27}}\right)^2}}$$

1.00000001802066069...

$$T^2 = 2.99616291064$$

$$T = 1.73094278087$$

Now: (1.73094278087*Pi^3) * [((9.1093829140*10^-31))/((4*1.67492735174*10^-27))]

$$(1.73094278087\,\pi^3)\times\frac{9.1093829140\times 10^{-31}}{4\times 1.67492735174\times 10^{-27}}$$

0.0072973524411...

Thence:

1,00000001802066069 * 0,0072973524411 = 0,0072973525726031122764063 10359

and 1 / 0,0072973525726 = 137,03599....

Indeed:

$$\sigma^{-1} = 1.00000001802066067 / (1.73094278087 * pi^3 * 0.00054386734442 / 4)$$
$$\sigma^{-1} = 137.035999098$$

From the first formula, for T obtained with the same values for the mass of proton and/or neutron:

$$\sqrt{(((1.6444551589\times 10^{-27}-9.10938356\times 10^{-31})^2+(1.6444551589\times 10^{-27})^2+(1.6444551589\times 10^{-27})^2)/(1.6444551589\times 10^{-27})^2)}$$

1.7317310461...

We have, for 0,000544617 that is the electron-proton mass ratio:

1,0000000171082 / (1,7317310461 * π^3 * 0,000544617 / 4) =

= 1,0000000171082 / 0,0073107387269223424515704978 4385 = 136,78508.. We note that 137,035 – 0.25 = 136,785.

Further, with the following values:

$$\sqrt{(((1.66979059 \times 10^{-27} - 8.99 \times 10^{-31})^2 + (1.66979059 \times 10^{-27})^2 + (1.66979059 \times 10^{-27})^2)/(1.66979059 \times 10^{-27})^2)}$$

1.73174002...

and √35 (0,920442) = 5,445408 where 5,445408 / 10^4 = 0,0005445408 a value practically equal to the electron-proton mass ratio, we obtain:

1,0000000171082 / (1,73174002 * π^3 * 0,0005445408 / 4) =

1,0000000171082 / 0,00730975372533 = 136,8035168;

136,8035168 + 0,25 = 137,0535168 that is practically equal to 137.0359... = α^{-1}

This result has been obtained only with the values of the various Ramanujan's expressions that we have previously analyzed.

From:

ACTA ARITHMETICA LXXIII.1 (1995)

Ramanujan's class invariants and cubic continued fraction

By - Bruce C. Berndt (Urbana, Ill.), Heng Huat Chan (Princeton, N.J.) and Liang-Cheng Zhang (Springfield, Mo.)

We have the following expression:

THEOREM 6.

(3.9) G_{441}

$$= \sqrt{\frac{2+\sqrt{7}+\sqrt{7+4\sqrt{7}}}{2}}\sqrt{\frac{\sqrt{3}+\sqrt{7}}{2}}(2+\sqrt{3})^{1/6}\sqrt{\frac{\sqrt{3+\sqrt{7}}+6^{1/4}7^{1/8}}{\sqrt{3+\sqrt{7}}-6^{1/4}7^{1/8}}}.$$

We have that:

2,102256018 * 1,479493514 * 1,2454451084 * 3,3919430295 =

= 13,139287349123... $(13,139287349123)^{1/5}$ = 1,67384161897728 and

13,139287349123 / 2 = 6,56964367...

We note that 13,1392873 – 0,2 = 12,9392873 value practically equal to the mass of the supermassive black hole M87 equal to about 6,5-6,6 milliard of solar masses, i.e. 12,92915 * 10^{39} kg - 13,12806 * 10^{39} kg a distance measurement of $16.8^{+0.8}_{-0.7}$ Mpc gives a black hole mass of $M = 6.5 \pm 0.2|_{\text{stat}} \pm 0.7|_{\text{sys}} \times 10^9\, M_{\odot}$. This measurement from lensed emission near the event horizon is consistent with the presence of a central Kerr black hole, as predicted by the general theory of relativity.

Also 1,67384161 * 8 = 13,39073288 – 0,5 = 12,89073288

Now, we have:

THEOREM 7.

$$g_{90} = (2+\sqrt{5})^{1/6}(\sqrt{5}+\sqrt{6})^{1/6}\left(\sqrt{\frac{3+\sqrt{6}}{4}}+\sqrt{\frac{\sqrt{6}-1}{4}}\right). \tag{3.15}$$

1,272019649514 * 1,293580763607 * 1,76918089239 = 2,9111166557...

We note that: (2,9111166557) / e = $(1,070940688876...)^7$ = 1,61568962048...

2,9111166557 * 43 = 125,1780161951 result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0

We observe that 43 is the 14th smallest prime number. The previous is 41, with which it comprises a twin prime, and the next is 47. 43 is the smallest prime that is not a Chen prime. It is also the third Wagstaff prime. In number theory, a Wagstaff prime is a prime number p of the form:

$$p = \frac{2^q+1}{3}$$

where q is an odd prime. Indeed: $43 = \frac{2^7+1}{3} = \frac{128+1}{3} = \frac{129}{3}$. The number 43 is also the sum of 34 + 8 + 1 that are Fibonacci's number. Further, 43 is the smallest prime number expressible as the sum of 2, 3, 4, or 5 different primes:

43 = 41 + 2; 43 = 11 + 13 + 19; 43 = 2 + 11 + 13 + 17; 43 = 3 + 5 + 7 + 11 + 17.

Now, we have:

THEOREM 8.

$$g_{198} = \sqrt{1+\sqrt{2}}(4\sqrt{2}+\sqrt{33})^{1/6}\left(\sqrt{\frac{9+\sqrt{33}}{8}}+\sqrt{\frac{1+\sqrt{33}}{8}}\right). \tag{3.20}$$

1,55377397403 (1,50023676572) (2,2757858778) = 5,304922518... We note that:

5,304922518 * 26 = 137,927985468 and that 2e / 5,304922518 = 1,0248142138...

$(1,0248142138)^{21}$ = 1,673200347... result very near to the proton mass

Note that (5,304922518 * 24) / 10 = 12,7318140432 + 0,2 = 12,9318140432 value very near to the mass of the supermassive black hole M87 that is 12,92915 * 10^{39} kg - 13,12806 * 10^{39} kg .

Now, we have:

EXAMPLE 2.

$$G(-e^{-\pi\sqrt{5}}) = \frac{(\sqrt{5}-3)(\sqrt{5}-\sqrt{3})}{4}.$$

(-0,7639320225002) (0,5040171699309) / 4 = - 0,096258714.

Note that 0,096258714 * 10 = 0,96258714 is a value very near to the spin of the Kerr black hole SMBH87 that is a = 0,9375

We have that:

(- 0,096258714 * 13 * 10^2) = - 125,1363282; - $(0,096258714)^{1/5}$ = - 0,6261638787

- 0,096258714 * 2π = - 0,6048113374928 and 1 / (-0,6048113374928) =

= - 1,65340815889... Practically, we have used the formula of a circle A = 2πr where r = 0,096258714 and after we have calculated the inverse.

Now, we have:

EXAMPLE 3.

$$G(-e^{-\pi\sqrt{13}}) = -\sqrt{\frac{\sqrt{13}-2\sqrt{3}}{\sqrt{13}+3}}\left(\sqrt{\frac{11+6\sqrt{3}}{2}}-\sqrt{\frac{9+6\sqrt{3}}{2}}\right).$$

that is equal to:

-0,1463343962452094 (0,156632726623639) = - 0,02292075548271117

0,02292 * 41 = 0,93975097... practically equal to the spin of SMBH87. Furthermore, (1 / 0,02292075) * 17 = 43,6285898 * 17 = 741,686 value that is a good approximation to the value of the energy of SMBH87 that is 737,4497.

We have also that √43,6285898 = 6,6051941; 6,6051941 * 2 = 13,21038 where 6,6051941 is about the value of the reduced Planck's constant, while 13,21038 is about the mass of the SMBH87 that is 13,128 . And $(43,6285898)^{1/8}$ = 1,6031398028... that is the electric charge of the positron.

We have:

EXAMPLE 4.

$$G(-e^{-\pi\sqrt{37}}) = -\sqrt{\frac{2\sqrt{37}-7\sqrt{3}}{2\sqrt{37}+12}}\left(\sqrt{\frac{146+84\sqrt{3}}{2}}-\sqrt{\frac{144+84\sqrt{3}}{2}}\right).$$

that is equal to: - 0,00171240918; 0,00171240918 * 55 * 10 = 0,941825 value very near to the spin of SMBH87

We note that 1 / 0,0017124 = 583,9757066 and $(583,9757066)^{1/13}$ = 1,6322985638...

1,6322985638 * 4 = 6,5291942552; 1,6322985638 * 8 = 13,058388... very near to the values of the mass of the proton, the reduced Planck's constant and the mass of the SMBH87.

We calculate the following double integrals:

integrate integrate [(583.9757066)^0.33333333333] [Pi/8]

$$\int\left(\int 583.9757066^{0.33333333333} \times \frac{\pi}{8}\, dx\right) dx$$

Result:
$1.6412\, x^2$

integrate integrate [sqrt(583.9757066)] [Pi/23] 23 = 5 + 7 + 11 è la somma di tre numeri primi consecutivi

$$\int\left(\int \sqrt{583.9757066} \times \frac{\pi}{23}\, dx\right) dx$$

Result:
$1.6504\, x^2$

Results that are very near to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

We have:

EXAMPLE 6.

$$G(e^{-\pi\sqrt{10}}) = \frac{\sqrt{9+3\sqrt{6}} - \sqrt{7+3\sqrt{6}}}{(1+\sqrt{5})\sqrt{\sqrt{6}+\sqrt{5}}}.$$

that is equal to: 0,0364586006; 0,0364586006 * 26 = 0,947923 value very near to the spin of SMBH87.

We have that 1 / 0,0364586006 = 27,42837035 and $(27.42837035)^{1/7}$ = 1,604933874428... (1,604933874428) * 4 = 6,419735497; (1,604933874428) * 8 = 12,8394709... very near to the values of the electric charge of the positron, the reduced Planck's constant and the mass of the SMBH87.

We have:

EXAMPLE 7.

$$G(e^{-\pi\sqrt{22}}) = \frac{\sqrt{36+6\sqrt{33}} - \sqrt{34+6\sqrt{33}}}{(2+2\sqrt{2})\sqrt{\sqrt{33}+\sqrt{32}}}.$$

that is equal to: 0,0073592864817; 0,0073592864 * 13 * 10 = 0,956707 value that is a good approximation to the spin of SMBH87.

We have that 1 / 0,0073592864 = 135,882739 and $(135,882739)^{1/10}$ = 1,63424223... (1,634242239) * 4 = 6,53696; (1,634242239) * 8 = 13,07393 all values very near to the values of the fine structure constant, the mass of the proton, the reduced Planck's constant and the mass of the SMBH87.

We have:

EXAMPLE 8.

$$G(e^{-\pi\sqrt{58}}) = \frac{\sqrt{729+297\sqrt{6}} - \sqrt{727+297\sqrt{6}}}{(5+\sqrt{29})\sqrt{11\sqrt{6}+5\sqrt{29}}}.$$

$3{,}438796051186 * 10^{-4} = 0{,}0003438796051186$

that is equal to: $3,438796051 * 10^{-4}$; $0,0003438796051 * 27 * 10^2 = 0,92847493...$ value very near to the spin of SMBH87.

We have that $1 / 0,0003438796051 = 2907,99449...$ and $(2907,99449)^{1/16} =$

$= 1,646169681609$; $(1,646169681609) * 4 = 6,584678...$; $(1,646169681609) * 8 =$

$= 13,169357$ all values very near to the values of the mass of the proton, the reduced Planck's constant and the mass of the SMBH87.

We remember that for the Kerr black hole SMBH87 the spin is $a = 0,9375$ and the mass is about $6,5 – 6,6 * 10^9$ solar masses, thence $12,92915 * 10^{39}$ - $13,12806 * 10^{39}$ kg or $1,292915 - 1,312806 * 10^{40}$ kg

From:

First M87 Event Horizon Telescope Results. V. Physical Origin of the Asymmetric Ring - The Event Horizon Telescope Collaboration - (See the end matter for the full list of authors.) - Received 2019 March 4; revised 2019 March 12; accepted 2019 March 12; published 2019 April 10

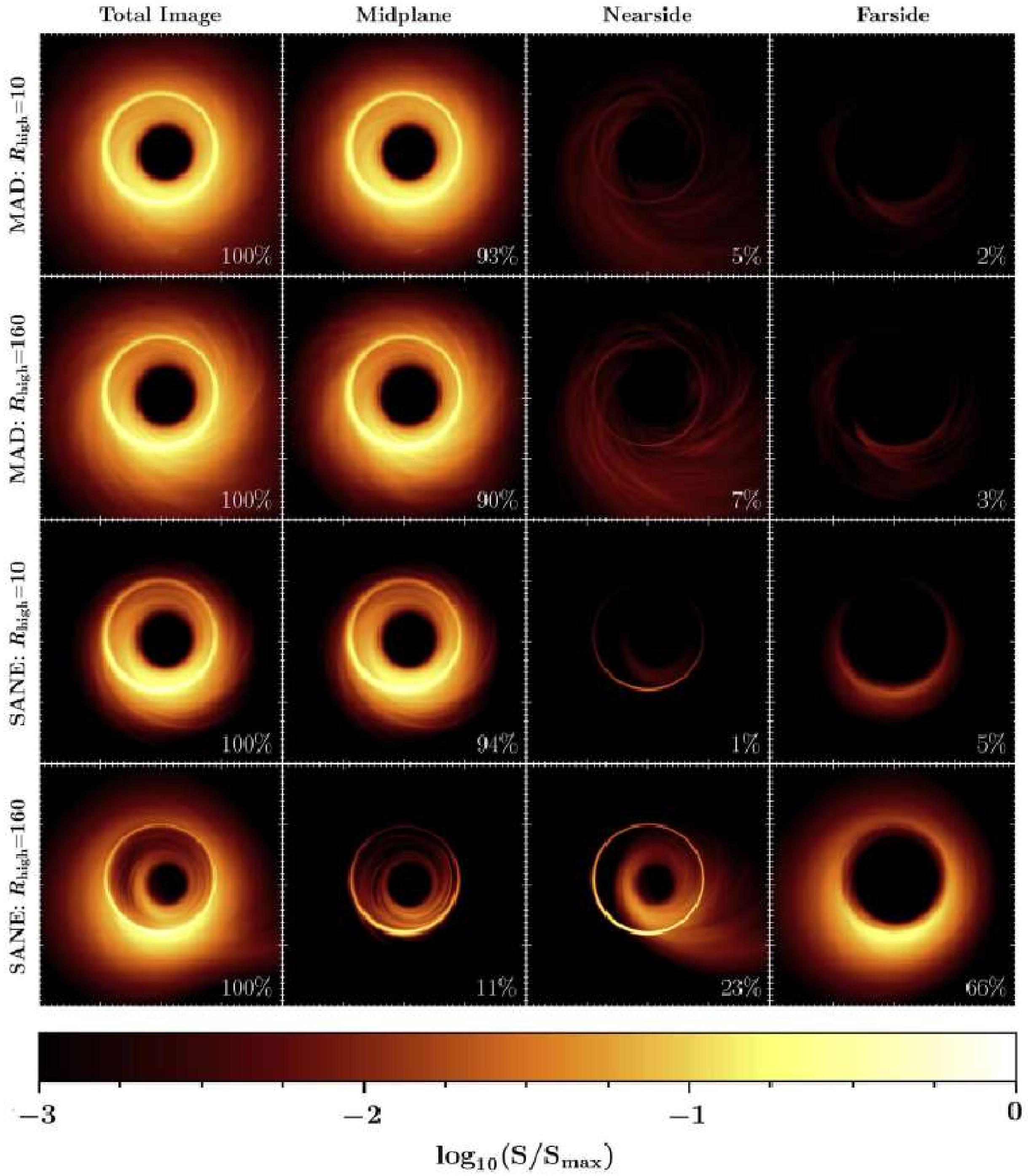


Figure 11. Decomposition of time-averaged 1.3 mm images from Figure 4 into midplane, nearside, and farside components (MAD and SANE models with $a* = 0.94$). Each model (row) of the figure corresponds to a simulation in Figure 4. The percentage of the total image flux from each component is indicated in the bottom right of each panel. The color scale is logarithmic and spans three decades in total flux with respect to the total image from each model, chosen in order to emphasize both nearside and farside components, which are nearly invisible when shown in a linear scale. The field of view is 80 μas

From:

Two-temperature, Magnetically Arrested Disc simulations of the jet from the supermassive black hole in M87 – 09/04/2019 - Andrew Chael, Ramesh Narayan and Michael D. Johnson - Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138, USA

3 NUMERICAL SIMULATIONS

3.1 Units

In both simulations presented in this work, we fix the distance to M87 as $D = 16.7$ Mpc (Mei et al. 2007) and fix the black hole mass to $6.2 \times 10^9 M_\odot$ (Gebhardt et al. 2011, scaled for this distance). We take the dimensionless black hole spin in both simulations as $a = 0.9375$.

For this mass, the gravitational length scale of M87 is $r_\mathrm{g} = GM/c^2 = 9.2 \times 10^{14}\,\mathrm{cm} = 61\,\mathrm{AU}$. The corresponding angular scale is $r_\mathrm{g}/D = 3.7\,\mu\mathrm{as}$. The gravitational time-scale is $t_\mathrm{g} = r_\mathrm{g}/c = 3 \times 10^4\,\mathrm{s} = 8.5\,\mathrm{hr}$.

M87's Eddington luminosity is $L_\mathrm{Edd} = 7.8 \times 10^{47}$ erg s^{-1}. The Eddington accretion rate is $\dot{M}_\mathrm{Edd} = L_\mathrm{Edd}/\eta c^2 = 77\,M_\odot\,\mathrm{yr}^{-1}$, where for our chosen value of spin, we set the efficiency $\eta = 0.18$, as expected for a thin accretion disc with $a = 0.9375$ (Novikov & Thorne 1973).

Model	Spin	Heating	$\langle \dot{M}/\dot{M}_\mathrm{Edd} \rangle$	$\langle \Phi_\mathrm{BH}/(\dot{M}c)^{1/2} r_\mathrm{g} \rangle$	$\langle P_J(100) \rangle$ [erg s^{-1}]	ϵ_J	$\langle P_{J,\mathrm{rad}}(100) \rangle$ [erg s^{-1}]	$\epsilon_{J,\mathrm{rad}}$
H10	0.9375	Turb. Cascade	3.6×10^{-6}	55	6.6×10^{42}	0.5	8.8×10^{42}	0.7
R17	0.9375	Mag. Reconnection	2.3×10^{-6}	63	1.3×10^{43}	1.6	1.4×10^{43}	1.6

Table 1. Time-averaged quantities for both simulations. From left to right, the quantities presented are the model name, the spin and heating prescription used, the average mass accretion rate through the black hole horizon measured in Eddington units, the magnetic flux threading the horizon measured in natural units, the mechanical jet power P_J measured at a radius of $100\,r_\mathrm{g}$ (Eq. 15), the corresponding jet efficiency $\epsilon_J = \langle P_J \rangle / \langle \dot{M} c^2 \rangle$, the jet power including radiation $P_{J,\mathrm{rad}}$ (Eq. 16) and the corresponding efficiency, both measured at the same radius.

We measure the thermal, magnetic, and jet mechanical power in both simulations as a function of radius using the definition (Tchekhovskoy et al. 2011; Ryan et al. 2018)

$$P_J = -\int \left(T^r{}_t + \rho u^r \right) \sqrt{-g}\, \mathrm{d}\theta \mathrm{d}\phi, \qquad (15)$$

where the integral is at a fixed r is over the jet cap, which we define by the criterion Be > 0.05 (Narayan et al. 2012; Sądowski et al. 2013b). The time-averaged jet power measured by Eq. 15 is roughly constant with radius from around $r = 10\,r_g$ out to $r = 1000\,r_g$. We measure the average jet powers at $100\,r_g$ from the averaged data to be 6.6×10^{42} erg s^{-1} for model H10 and 1.3×10^{43} erg s^{-1} for R17 (Table 1).

While the jet powers obtained from the two simulations agree to within a factor of two, the value obtained for model R17 is more consistent with the measured values for M87 of $\sim 10^{43} - 10^{44}$erg s^{-1} (Reynolds et al. 1996; Stawarz et al. 2006). Comparing the jet power to the accretion rate gives a jet efficiency $\epsilon_J = P_J/\dot{M}c^2$ of 1.6 and 0.5 for R17 and H10, respectively, indicating that spin energy is being extracted from the black hole. This is especially true in model R17, which has greater than 100 per cent efficiency (Tchekhovskoy et al. 2011).

Because much of H10's energy and momentum is converted to radiation in the jet, it has a correspondingly lower mechanical jet power. Including radiation in the jet power measurement, we define

$$P_{J,\mathrm{rad}} = -\int \left(T^r{}_t + R^r{}_t - \rho u^r\right) \sqrt{-g}\,\mathrm{d}\theta \mathrm{d}\phi, \qquad (16)$$

This increases the measured jet powers to $P_{J,\mathrm{rad}} = 8.8 \times 10^{42}$ erg s^{-1} for H10 and $P_{J,\mathrm{rad}} = 1.4 \times 10^{43}$ erg s^{-1} for R17, and increases the jet efficiencies in the two models to 0.7 and 1.6, respectively.

these preferred parameter values in this study. Unlike in the present work, at $M = 6.2 \times 10^9 M_\odot$ and $a = 0.9375$, they obtain a compact, counterjet-dominated 230 GHz image that is consistent with past EHT measurements of the overall image size. However, the jet powers produced in their simulations

highly magnetized regions. The matter content of the jet is still unknown; it may be populated by a pair plasma of electrons and positrons (Mościbrodzka et al. 2011; Broderick & Tchekhovskoy 2015). Further work with our simulations us-

La luminosità dell'anello e la sua asimmetria possono essere spiegata in termini di "*radiazione relativistica dell'emissione di un plasma che ruota quasi alla velocità della luce attorno al buco nero*"

For the Kerr black hole SMBH87 the spin is $a = 0{,}9375$ and the mass is about 6,5 – 6,6 * 10^9 solar masses, thence 12,92915 * 10^{39} - 13,12806 * 10^{39} kg or 1,292915 - 1,312806 * 10^{40} kg. With regard the mass of the SMBH87, we have calculated the equivalent energy utilizing the formula $E = mc^2$. We obtain, for $c = 9 * 10^{16}$ m^2/s^2 :
M = 13.12806 * 10^{39} kg; E = 118,15254 * 10^{55} kg m^2/s^2 = 118,15254 J.
Now 118,15254 * 10^{55} J = 737,4497 * 10^{73} eV. Thence, the energy of the SMBH87 is equal to 737,4497 * 10^{73} eV.

From:
RAMANUJAN'S CLASS INVARIANTS,
KRONECKER'S LIMIT FORMULA,
AND MODULAR EQUATIONS
BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG{CHENG ZHANG

We have:

$$\left(\sqrt{\frac{18+9\sqrt{3}}{4}}+\sqrt{\frac{14+9\sqrt{3}}{4}}\right)^3=\frac{1}{\sqrt{2}}\sqrt{7855+4536\sqrt{3}}+\frac{9}{\sqrt{2}}(7+4\sqrt{3}),$$

The result is: 0,707106781186 (125,34585139) + 6,36396103067 (13,9282032302) = 88,63290151 + 88,63854258 = 177,27144409;
Note that √177,27144409 = 13,31433062 and that $(177{,}27144409)^{1/11}$ = 1,60111198.. and 1,60111198 * 8 = 12,8088958... values very near to the mass of SMBH87 and to the charge of the positron.

We have:

$$2\sqrt{29}+5\sqrt{5}+\sqrt{240+20\sqrt{145}}=\left(\sqrt{\frac{17+\sqrt{145}}{8}}+\sqrt{\frac{9+\sqrt{145}}{8}}\right)^3,$$

The result is $(1{,}90530819615 + 1{,}62178892657)^3$ = 43,8785488241248
Note that √43,87854882241248 = 6,62408 that is a value practically equal to the Planck's constant h = 6,62606957. Further, 6,62408 * 2 = 13,24817... We have also that $(43{,}8785488241248)^{1/8}$ = 1,604285033 and 1,604285033 * 8 = 12,834280269... that are values very near to the charge of the positron and to the mass of SMBH87 13,12806
We have also that $(145)^{1/10}$ = 1,644889772; 1,644889772 * 8 = 13,159118... and that 13,159118... / 2 = 6,579559... Further: $(1 / 1{,}644889772)^{1/8}$ = 0,939686...
The number 1729 = (145 * 12) – 11

Ramanujan said on the number 1729: "...it is a very interesting number; it is the smallest number expressible as the sum of two cubes in two different ways."

The two different ways are:

$1729 = 1^3 + 12^3 = 9^3 + 10^3$

We have:

$$P^2 := (G_{205}G_{41/5})^4 = \left(\frac{\sqrt{5}+1}{2}\right)^8 \left(\frac{43+3\sqrt{205}}{2}\right) \\ -\left(\frac{7+3\sqrt{5}}{2}\right)^2 \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2.$$

that is equal to: 46,9787137637477918 (42,9767315949145297) = 2018,991572...
And $(2018{,}991572)^{1/16}$ = 1,6090550645269... 1,6090550645 * 8 = 12,8724405162
Further: $(1 / 1{,}6090550645269)^{1/8}$ = 0,942277... that are values very near to the charge of the positron and to the mass and spin of SMBH87.

We have:

$$\left(\sqrt{\frac{7+\sqrt{41}}{8}}+\sqrt{\frac{\sqrt{41}-1}{8}}\right)^2 = \frac{\sqrt{41}+3}{4}+\sqrt{\frac{17+3\sqrt{41}}{8}}.$$

that is equal to: 2,35078105935821217 + 2,127480103088467959 = 4,47826116244668 and 4,47826116244668 * 3 = 13,43478348734...
we have that $(4{,}47826116244668)^{1/3}$ = 1,648300811203... and 1,648300811203 * 8 = 13,186406489... Further: $(1 / 1{,}648300811203)^{1/8}$ = 0,9394430... that are values very near to the charge of the proton and to the mass and spin of SMBH87.

We have:

$$\sqrt{\frac{1}{2}(135619+78300\sqrt{3})}+\frac{3}{\sqrt{2}}(87+50\sqrt{3}) = \left(\sqrt{\frac{21+12\sqrt{3}}{2}}+\sqrt{\frac{19+12\sqrt{3}}{2}}\right)^3,$$

that is equal to: 736,53184348... We have that $(736{,}53184348)^{1/13}$ = 1,661702198...
And 1,661702198 * 4 = 6,646808; 1,661702198 * 8 = 13,2936175...
Further: $(1 / 1{,}661702198)^{1/8}$ = 0,9384925... that are values very near to the charge of the proton and to the mass and spin of SMBH87.

We have:

$$\frac{1}{2\sqrt{2}}\sqrt{6917+425\sqrt{265}}+\frac{1}{4}(85+5\sqrt{265})=\left(\sqrt{\frac{89+5\sqrt{265}}{8}}+\sqrt{\frac{81+5\sqrt{265}}{8}}\right)^2,$$

that is equal to: 83,185030096... We have that $(83,185030096)^{1/9}$ = 1,6343247329... and 1,6343247329 * 4 = 6,53729893... 1,6343247329 * 8 = 13,0745978... Further: $(1 / 1,6343247329)^{1/8}$ = 0,94044349... that are values very near to the charge of the proton and to the mass and spin of SMBH87. We note that: 83,185 * 9 – 11 = = 737,665

We have:

$$\frac{1}{\sqrt{2}}(301+46\sqrt{43})+\frac{1}{\sqrt{2}}\sqrt{7(25941+3956\sqrt{43})}$$
$$=\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3,$$

that is equal to: 852,2635597... We have that $(852,2635597)^{1/14}$ = 1,6192977355292 and 1,6192977355292 * 4 = 6,47719094... 1,6192977355292 * 8 = 12,9543818... Further: $(1 / 1,6192977355292)^{1/8}$ = 0,941529... that are values very near to the charge of the positron and to the mass and spin of SMBH87. We note that: $(1,6192977355292)^7$ * 25 + 8 = 737,838834 value very near to the value of the energy of the SMBH87

We have that:

$$189+20\sqrt{89}+\sqrt{71320+7560\sqrt{89}}=\left(\sqrt{\frac{13+\sqrt{89}}{8}}+\sqrt{\frac{5+\sqrt{89}}{8}}\right)^6.$$

that is equal to: 755,3579214... We have that $(755,3579214)^{1/14}$ = 1,605396611576... and 1,605396611576 * 4 = 6,4215864463... 1,605396611576 * 8 = 12,8431728... Further: $(1 / 1,605396611576)^{1/8}$ = 0,9425452... that are values very near to the charge of the positron and to the mass and spin of SMBH87.

Let $Q = (G_{505}/G_{101/5})^3$. Then, by Lemma 3.4 and (4.35),

$$Q = (P^{-1} - P) + \sqrt{(P^{-1} - P)^2 - 1}$$

$$= (130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}}. \tag{4.36}$$

Therefore, by (4.35) and (4.36),

$$G_{505} = P^{-1/4}Q^{1/6} = (\sqrt{5}+2)^{1/2}\left(\frac{\sqrt{5}+1}{2}\right)^{1/4}(\sqrt{101}+10)^{1/4}$$
$$\times\left((130\sqrt{5} + 29\sqrt{101}) + \sqrt{169440 + 7540\sqrt{505}}\right)^{1/6}.$$

Thus, it remains to show that

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440 + 7540\sqrt{505}}=\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

which is straightforward. $\square$

We have:

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3.$$

that is equal to 1164,269601267364. We note that $(1164{,}269601267364)^{1/14}$ = 1,655784548804 and 1,655784548804 * 8 = 13,2462763904...
Further: $(1 / 1{,}655784548804)^{1/8} = 0{,}60394330936)^{1/8}$ = 0,93891120462... that are values very near to the charge of the proton and to the mass and spin of SMBH87.
We note that $((1164{,}2696)^{1/2} * 21) + 21$ = 737,549296 a value very near to the value of the energy of SMBH87. We have also that $((1{,}655784548804)^7 * 21) + 21$ =
= 737,549296

From:
CHAPTER 3. WEBER-RAMANUJAN'S CLASS INVARIANTS

We have:

$$\frac{1}{A} - 2A = 2\sqrt{17409 + 6580\sqrt{7}}.$$

and

$$\frac{1}{A} = 94 + 35\sqrt{7} + \sqrt{17409 + 6580\sqrt{7}}.$$

that are equal to: 373,19187358143 and 373,19723267797
We note that 373,19187 / 28 = 13,32828... and 373,19723267 / 28 = 13,32847... (note that 13.328 / 2 = 6,664). Further, we have that $(373,19187)^{1/12}$ = 1,638051988... and 1,638051988 * 8 = 13,1044159... (note that 13,1044159 / 2 = 6,5522...) all values very near to the mass of the proton and of the SMBH87 that is 13,12806. Further, we have that: $((1,638051988)^7 * 23) + 10$ = 737,8160089 value very near to the energy of SMNH87

We have that:

$$\frac{94 + 35\sqrt{7}}{\sqrt{2}} + \sqrt{\frac{17409 + 6580\sqrt{7}}{2}} = \left(\sqrt{\frac{11 + 4\sqrt{7}}{2}} + \sqrt{\frac{9 + 4\sqrt{7}}{2}}\right)^3$$

that is equal to: $(3,2850422557600657 + 3,12913768027697)^3$ = 263,89029394... we have that 263,89029394 / 20 = 13,194514697. Further, $(263,89029394)^{1/11}$ = 1,66008146838 and 1,66008146838 * 8 = 13,280651747... all values very near to the mass of the proton and of the SMBH87. We note that $(1,66008146838)^7$ = 34,746079 and that (34,746079 * 21) + 8 = 737,667659 value practically equal to the value of the energy of the SMBH87.

We have that:

$$\left(\frac{3 + \sqrt{13}}{4} + \sqrt{\frac{3 + 3\sqrt{13}}{8}}\right)^{1/4} = \sqrt{\frac{5 + \sqrt{13}}{8}} + \sqrt{\frac{\sqrt{13} - 3}{8}}.$$

is equal to: 1,3122819078... We have that 1,3122819078 * 10 = 13,122819... and that $(1 / 1,3122819078)^{1/4}$ = 0,93431477... Further, (1,3122819078) / 2 = 0,6561409539
We have that 13,122819 and 0,93431477 are values very near to the mass and spin of the SMBH87.

We have that:

$$\left(\frac{9+5\sqrt{5}}{4}+\sqrt{\frac{95+45\sqrt{5}}{8}}\right)^{1/6}=\sqrt{\frac{7+\sqrt{5}}{8}}+\sqrt{\frac{\sqrt{5}-1}{8}}.$$

Is equal to: 1,46755625975... We have that 1,46755625975 * 9 = 13,2080... and that (1 / 1,46755625975) = 0,681404881997 and $(0,681404881997)^{1/6}$ = 0,93806775...
We note that that 13,2080 and 0,93806775 are values very near to the mass and spin of the SMBH87

We have that:

$$G_{65}^2=\left(\sqrt{\frac{7+\sqrt{65}}{8}}+\sqrt{\frac{\sqrt{65}-1}{8}}\right)\left(\sqrt{\frac{9+\sqrt{65}}{8}}+\sqrt{\frac{1+\sqrt{65}}{8}}\right).$$

and

$$G_{13/5}^2=\left(\sqrt{\frac{7+\sqrt{65}}{8}}-\sqrt{\frac{\sqrt{65}-1}{8}}\right)\left(\sqrt{\frac{9+\sqrt{65}}{8}}+\sqrt{\frac{1+\sqrt{65}}{8}}\right).$$

that are equal to: 2,311710024865895 (2,5247272355065468) = 5,83643726037...
0,432580206532612651 (2,5247272355065468) = 1,09214702897...
We note that 1 / 1,09214702897 = 0,91562763389... and that $(5,83643726)^{1/7}$ * 10 = 12,86618194; and 1,09214702897 * 12 = 13,105764347.
Further: 5,83643726037 – 1,09214702897 = 4,7442902314;
$(4,7442902314)^{1/\pi}$ = 1,6414663847751.
All values very near to the mass of the proton and to the mass and spin of the SMBH87.

We have said that:

We have:

$$2\sqrt{29}+5\sqrt{5}+\sqrt{240+20\sqrt{145}}=\left(\sqrt{\frac{17+\sqrt{145}}{8}}+\sqrt{\frac{9+\sqrt{145}}{8}}\right)^3,$$

The result is $(1,90530819615 + 1,62178892657)^3$ = 43,8785488241248
Note that √43,87854882241248 = 6,62408 that is a value practically equal to the Planck's constant h = 6,62606957. Further, 6,62408 * 2 = 13,24817... We have also that $(43,8785488241248)^{1/8}$ = 1,604285033 and 1,604285033 * 8 = 12,834280269... that are values very near to the charge of the positron and to the mass of SMBH87

13,12806 (The black hole mass has been measured to be 6.6 * 10^9 solar masses, that is 13,12806 * 10^{39}).
We have also that $(145)^{1/10}$ = 1,644889772; 1,644889772 * 8 = 13,159118... and that 13,159118... / 2 = 6,579559... Further: $(1 / 1,644889772)^{1/8}$ = 0,939686... value very near to the SMBH87 spin, that is 0.9375

The Rydberg constant represents the limiting value of the highest wavenumber (the inverse wavelength) of any photon that can be emitted from the hydrogen atom, or, alternatively, the wavenumber of the lowest-energy photon capable of ionizing the hydrogen atom from its ground state. The spectrum of hydrogen can be expressed simply in terms of the Rydberg constant, using the Rydberg formula.

The **Rydberg unit of energy**, symbol Ry, is closely related to the Rydberg constant. It corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

This constant is often used in atomic physics in the form of the Rydberg unit of energy:

$$1\,\mathrm{Ry} \equiv hcR_\infty = \frac{m_e e^4}{8\varepsilon_0^2 h^2} = 13.605\ 693\ 009(84)\,\mathrm{eV} \approx 2.179 \times 10^{-18}\,\mathrm{J}.$$

Indeed, we have:

$(9,109 * 10^{-31})\ (1,602 * 10^{-19})^4 / 8 * (8,854 * 10^{-12})^2\ (6,626 * 10^{-34})^2 =$
$= 2,17992866 * 10^{-18}$ J = 13,606 eV.
We note that 13,248 is very near to the value of the Rydberg unit of energy.

With regard the mass of the SMBH87, we have calculated the equivalent energy utilizing the formula $E = mc^2$. We obtain, for $c = 9 * 10^{16}\ m^2/s^2$:
M = 13.12806 * 10^{39} kg; E = 118,15254 * 10^{55} kg m^2/s^2 = 118,15254 J.
Now 118,15254 * 10^{55} J = 737,4497 * 10^{73} eV. Thence, the energy of the SMBH87 is equal to 737,4497 * 10^{73} eV.

From the above formula, we have that $(43,8785488241248)^{1/8}$ = 1,604285033 and $(1,604)^7 * (11 + 16)$ = 737,55483. Further, we have also: 2 * $\sqrt{43,87854882241248}$ = 6,62408; 6,62408 * 2 = 13,24817. Now: (13,24817 * 55) + 9 = 737,64935. Thence, we obtain values very near to the energy of the SMBH87

On the numbers 1728 and 1729

From:

Illinois Journal of Mathematics - Volume 45, Number 1, Spring 2001, Pages 75{90 S 0019-2082

EISENSTEIN SERIES AND APPROXIMATIONS TO π

BRUCE C. BERNDT AND HENG HUAT CHAN

Dedicated to K. Venkatachaliengar

2. Eisenstein series and the modular j–invariant

Recall the definition of the modular j–invariant $j(\tau)$,

$$j(\tau) = 1728\frac{Q^3(q)}{Q^3(q) - R^2(q)}, \qquad q = e^{2\pi i\tau}, \qquad \operatorname{Im}\tau > 0. \tag{2.1}$$

In particular, if n is a positive integer,

$$j\left(\frac{3+\sqrt{-n}}{2}\right) = 1728\frac{Q_n^3}{Q_n^3 - R_n^2}, \tag{2.2}$$

where, for brevity, we set

$$Q_n := Q(-e^{-\pi\sqrt{n}}) \qquad \text{and} \qquad R_n := R(-e^{-\pi\sqrt{n}}). \tag{2.3}$$

In his third notebook, at the top of page 392 in the pagination of [17], Ramanujan defined a certain function J_n of singular moduli, which, as the authors [4] easily showed, has the representation

$$J_n = -\frac{1}{32}\sqrt[3]{j\left(\frac{3+\sqrt{-n}}{2}\right)}. \tag{2.4}$$

Hence, from (2.2) and (2.4),

$$(-32J_n)^3 = 1728\frac{Q_n^3}{Q_n^3 - R_n^2}. \tag{2.5}$$

After a simple manipulation of (2.5), we deduce the following theorem.

We note that $\sqrt[3]{1728} = 12$ and 1728 / 32 = 54

THEOREM 2.1. *For each positive integer n,*

$$\left(\left(\frac{8}{3}J_n\right)^3 + 1\right)Q_n^3 - \left(\frac{8}{3}J_n\right)^3 R_n^2 = 0, \tag{2.6}$$

where J_n is defined by (2.4), and Q_n and R_n are defined by (2.3).

EXAMPLES 2.2 [19, p. 211]. *We have*

$$\begin{aligned}
539Q_{11}^3 - 512R_{11}^2 &= 0,\\
(8^3+1)Q_{19}^3 - 8^3R_{19}^2 &= 0,\\
(40^3+9)Q_{27}^3 - 40^3R_{27}^2 &= 0,\\
(80^3+1)Q_{43}^3 - 80^3R_{43}^2 &= 0,\\
(440^3+1)Q_{67}^3 - 440^3R_{67}^2 &= 0,\\
(53360^3+1)Q_{163}^3 - 53360^3R_{163}^2 &= 0,\\
((60+28\sqrt{5})^3+27)Q_{35}^3 - (60+28\sqrt{5})^3R_{35}^2 &= 0,
\end{aligned}$$

and

$$((4(4+\sqrt{17})^{2/3}(5+\sqrt{17}))^3+1)Q_{51}^3 - (4(4+\sqrt{17})^{2/3}(5+\sqrt{17}))^3R_{51}^2 = 0.$$

Proof. In [4], [3, pp. 310, 311], we showed that

$$\begin{cases}
J_{11} = 1, & J_{19} = 3,\\
J_{27} = 5\cdot 3^{1/3}, & J_{43} = 30,\\
J_{67} = 165, & J_{163} = 20,010,\\
J_{35} = \sqrt{5}\left(\dfrac{1+\sqrt{5}}{2}\right)^4, & J_{51} = 3(4+\sqrt{17})^{2/3}\left(\dfrac{5+\sqrt{17}}{2}\right).
\end{cases} \tag{2.7}$$

Using (2.7) in (2.6), we readily deduce all eight equations in Q_n and R_n. □

The expression J_{35} = 15,3262379… and J_{51} = 55,298756114.... We note that (55 * 32) – 32 = 1728

We have already determined the value of t_{3315} in Section 4. It suffices to compute $\mathbf{J}_{3315}$ in order to write down a series for $1/\pi$ associated with $n = 3315$. We first quote the identity [4] [5],

$$j\left(\frac{3+\sqrt{-3n}}{2}\right) = -27\frac{(\lambda_n^2-1)(9\lambda_n^2-1)^3}{\lambda_n^2}, \tag{5.9}$$

where

$$\lambda_n = \frac{e^{\pi\sqrt{n}/2}}{3\sqrt{3}}\frac{f^6(e^{-\pi\sqrt{n/3}})}{f^6(e^{-\pi\sqrt{3n}})},$$

where $f(-q)$ is defined prior to (4.25). Since [5]

$$\lambda_{1105}^2 = \left(\frac{\sqrt{5}+1}{2}\right)^{24}(4+\sqrt{17})^6\left(\frac{15+\sqrt{221}}{2}\right)^6(8+\sqrt{65})^6,$$

the value of $\mathbf{J}_{3315}$ follows immediately from (5.9). The values $\mathbf{J}_{3315}$ and t_{3315}, when substituted into (5.8), yield the series which we mentioned at the end of Section 4.

The expression:

$$\lambda_{1105}^2 = \left(\frac{\sqrt{5}-1}{2}\right)^{24} (4+\sqrt{17})^6 \left(\frac{15+\sqrt{221}}{2}\right)^6 (8+\sqrt{65})^3.$$

is equal to

5672353980974662982450422,1841

We have that:

$$\left(\sqrt[10]{\frac{(5672353980974662982450422,1841)}{(1728)^7}}\right) = 1,6184366917....$$

Indeed:

$$\left(\frac{5.6723539809746629824504221841 \times 10^{24}}{1728^7}\right)^{0.1}$$

1.61844...

where 1,6184366917 * 4 = 6,4737467668 and 1,6184366917 * 8 = 12,9474935336

Now, we calculate the following double integral:

integrate integrate [(5672353980974662982450422.1841)/(1728^7)] [1/(12Pi)]

$$\int\left(\int \frac{5.6723539809746629824504221841 \times 10^{24}}{1728^7} \times \frac{1}{12\,\pi}\,dx\right)dx$$

Result:

$1.6352947541809757012654459871\,x^2$

We note that 1,63529475418 * 4 = 6,54117901672 and
1,63529475418 * 8 = 13,082358

The values 6,473746 6,541179 12,94749 and 13,082358 are very near to the reduced Planck's constant 6,582119 and to the mass of SMBH87 that is 13,12806

From:

The Fate of Massive F-Strings

Bin Chen, Miao Li, and Jian-Huang She - https://arxiv.org/abs/hep-th/0504040v2

In the following, we set $\alpha' = \frac{1}{2}$. Energy conservation gives

$$M = \sqrt{M_1^2 + k^2} + \sqrt{M_2^2 + k^2}, \tag{2.5}$$

with k the momentum in the noncompact dimension.

What we want to consider is the averaged semi-inclusive two-body decay rate. That is, for the initial string, we average over all states of some given mass, winding and KK momentum. For one of the two final strings, we sum over all states with some given mass, winding and KK momentum; only the other string's state is fully specified (by keeping explicit its vertex operator).

This decay rate can be written as

$$\Gamma_{semi-incl} = \frac{A_{D-d_c}}{M^2} g_c^2 \frac{F_L}{\mathcal{G}(N_L)} \frac{F_R}{\mathcal{G}(N_R)} k^{D-3-d_c} \prod_i^{d_c} R_i^{-1} \tag{2.6}$$

with closed string coupling g_c, compactification radius R_i, and numerical coefficient $A_p = \frac{2^{-p}\pi^{\frac{3-p}{2}}}{\Gamma(\frac{p-1}{2})}$, and F_L and F_R are given by

$$F_L = \sum_{\Phi_i|N_L} \sum_{\Phi_f|N_{2L}} \left|\langle\Phi_f|V_L(n_{1i}, w_{1i}, k)|\Phi_i\rangle\right|^2 \tag{2.7}$$

$$F_R = \sum_{\Phi_i|N_R} \sum_{\Phi_f|N_{2R}} \left|\langle\Phi_f|V_R(n_{1i}, w_{1i}, k)|\Phi_i\rangle\right|^2. \tag{2.8}$$

$D = 26$ is the full space-time dimension.

In the following, we shall calculate the decay rate (2.6). In the above discussions, we have fixed the levels of the incoming string states and one of the outgoing string states, which are $N_{L,R}, N_{2L,2R}$ respectively. From the mass-shell conditions, we know that once we fix the quanta of the incoming string states, the outgoing string states could have various kinds of masses, KK-momenta and windings, with respect to the energy condition (2.5), and conservations:

$$Q_- = Q_{1-} + Q_{2-}, \qquad Q_+ = Q_{1+} + Q_{2+}. \tag{2.40}$$

One important observation is that we have inequality

$$\sqrt{N_{1L}} + \sqrt{N_{2L}} \leq \sqrt{N_L}. \tag{2.41}$$

The equality saturate when

$$k = 0, \qquad \frac{M}{Q_-} = \frac{M_2}{Q_{2-}}, \tag{2.42}$$

where k is the momentum in the noncompact directions and M_2, Q_{2-} are the quanta of the outgoing string states with fixed level N_{2L}. The same inequality holds in the right-mover.

Given a very massive initial string of high level, its state density has the asymptotic form

$$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}}, \qquad a = 2\pi\sqrt{\frac{D-2}{6}}. \tag{2.43}$$

The ratio between the first two terms in (2.37) can be estimated to be

$$\frac{\mathcal{G}(N_L - n)}{\mathcal{G}(N_L - 2n + m_L^2)} \sim e^{a\left(\sqrt{N_L - n} - \sqrt{N_L - 2n + m_L^2}\right)}. \tag{2.44}$$

Using the inequality(2.41), it is at most of order

$$\exp\left(a(\sqrt{N_L} - \sqrt{N_{2L}})\right), \quad \sqrt{N_{2L}} \geq \frac{\sqrt{N_L}}{2} \tag{2.45}$$

or

$$\exp(a(3\sqrt{N_{2L}} - \sqrt{N_L})), \quad \sqrt{N_{2L}} \leq \frac{\sqrt{N_L}}{2}. \tag{2.46}$$

In the extremal case $N_{2L} = N_L$, one can try to calculate $\mathcal{I}$ directly. Thus if generically $N_L > N_{2L} > \frac{N_L}{3}$, the first term dominates the whole summation in (2.37), and the other terms will be neglected to get

$$F_L \approx (N_L - N_{2L} - \frac{1}{2}m_L^2)\mathcal{G}(N_{2L}). \tag{2.47}$$

F_R can be carried out in the same way.

Note that in our approximation, $F_{L,R}$ do not depend on the details of the state specified in eq.(2.7) and (2.8) by the vertex operators, and all states of the same level are emitted with the same probability. Taking advantage of this, we can get the total decay rate for decays into arbitrary states of given mass, winding and KK momentum by simply multiplying eq.(2.6) by the state density $\mathcal{G}(N_1)$

$$\Gamma[(M, n_i, w_i) \rightarrow (m, n_{1i}, w_{1i}) + (M_2, n_{2i}, w_{2i})] \approx A_{D-d_c} \frac{g_c^2}{M^2} \mathcal{N}_L \mathcal{N}_R \mathcal{G}_L \mathcal{G}_R k^{D-3-d_c} \prod_i^{d_c} R_i^{-1}, \tag{2.48}$$

where

$$\mathcal{N}_L = N_L - N_{2L} - \frac{1}{2} m_L^2 \,, \quad \mathcal{N}_R = N_R - N_{2R} - \frac{1}{2} m_R^2, \tag{2.49}$$

and

$$\mathcal{G}_L = \frac{\mathcal{G}(N_{1L})\mathcal{G}(N_{2L})}{\mathcal{G}(N_L)} \,, \quad \mathcal{G}_R = \frac{\mathcal{G}(N_{1R})\mathcal{G}(N_{2R})}{\mathcal{G}(N_R)}. \tag{2.50}$$

Remember that

$$m_L^2 = \frac{1}{4}(m^2 - Q_{1-}^2) \approx 2N_{1L}. \tag{2.51}$$

As long as $N_{1L} \gg 1$ and $N_{2L} \gg 1$, we can use eq.(2.43) to write

$$\mathcal{G}_L \sim (2\pi T_H)^{-\frac{D-1}{2}} (\frac{N_{1L} N_{2L}}{N_L})^{-\frac{D+1}{4}} e^{-\sqrt{2} t_L / T_H}, \tag{2.52}$$

with the Hagedorn temperature $T_H = \frac{1}{\pi}\sqrt{\frac{3}{D-2}}$ and $t_L = \sqrt{N_L} - \sqrt{N_{1L}} - \sqrt{N_{2L}}$ in a sense coming from the kinetic energy released in the decay process. We have in the above restored the multiplicative constant in front of the state density. Note that for later convenience, in our notation we set $\mathcal{G}(M)dN = \mathcal{G}(N)dN$, a little different from usual sense $\mathcal{G}(M)dM = \mathcal{G}(N)dN$.

We can also compactify type II strings on the torus. According to the above calculations, we will obtain the same formulas as in (2.37), (2.47)-(2.50). Now the state density has the asymptotic form

$$\mathcal{G}(N_L) \approx 2^{-\frac{13}{4}} N_L^{-\frac{11}{4}} e^{\pi\sqrt{8N_L}}. \tag{4.1}$$

Consider first open strings and NS sector only. The state degeneracy $\mathcal{G}_{NS}(n)$ is given by

$$f_{NS}(w) = \mathrm{Tr}\frac{1+e^{i\pi F}}{2}w^N = \sum_{n=0}^{\infty} \mathcal{G}_{NS}(n)w^n 8\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^8, \tag{A.1}$$

with N the summation of the bosonic and fermionic number operators. Generalization of Hardy-Ramanujan formula gives

$$\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^{-1} = \vartheta_4(0|w) = (-\frac{\ln w}{\pi})^{-\frac{1}{2}}\vartheta_2(0|e^{\frac{\pi}{\ln w}}), \tag{A.2}$$

where the modular transformation of ϑ function

$$\vartheta_4(0|\tau) = (-i\tau)^{-\frac{1}{2}}\vartheta_2(0|-\frac{1}{\tau}) \tag{A.3}$$

has been used, with

$$\tau = -\frac{i\ln w}{\pi}. \tag{A.4}$$

As $w \to \mathbf{1}$, the second argument of ϑ_2, which now reads

$$\tau' = -\frac{1}{\tau} = -\frac{i\pi}{\ln w}, \tag{A.5}$$

approaches ∞. We know from the expansion

$$\vartheta_2(0|\tau') = \sum_{n=-\infty}^{\infty} e^{i\pi(n-\frac{1}{2})^2\tau'} \tag{A.6}$$

that

$$\vartheta_2(0|\tau' \to \infty) \to 2e^{\frac{i\pi}{4}\tau'} = 2e^{\frac{\pi^2}{4\ln w}}. \tag{A.7}$$

Thus (A.2) is asymptotically

$$\prod_{n=1}^{\infty}(\frac{1+w^n}{1-w^n})^{-1} \to (-\frac{\ln w}{\pi})^{-\frac{1}{2}} 2\exp(\frac{\pi^2}{4\ln w}). \tag{A.8}$$

¿From (A.1), the state degeneracy $\mathcal{G}_{NS}(n)$ can be expressed as a contour integral on a small circle around $w = 0$

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint \frac{f_{NS}}{w^{n+1}}dw. \tag{A.9}$$

To compute the above integration, we make a saddle point approximation near $w = 1$. The power of w can be put on the exponential

$$\mathcal{G}_{NS}(n) = \frac{1}{2\pi i}\oint 8(-\frac{\ln w}{\pi})^4 2^{-8}\exp[-\frac{2\pi^2}{\ln w} - (n+1)\ln w]dw, \tag{A.10}$$

to get the saddle point at

$$\ln w_0 = \frac{\sqrt{2}\pi}{\sqrt{n+1}}, \tag{A.11}$$

where expansion can be made

$$\ln w = \ln w_0 + iu. \tag{A.12}$$

Then $\mathcal{G}_{NS}(n)$ is approximately

$$\mathcal{G}_{NS}(n) \sim \frac{1}{2\pi}\frac{1}{32}(\frac{\sqrt{2}}{\sqrt{n}})^4 e^{\pi\sqrt{8n}}\int_{-\infty}^{\infty}\exp(-\frac{\sqrt{2}n^{\frac{3}{2}}}{\pi}u^2)du. \tag{A.13}$$

Carrying out the integration over u we find

$$\mathcal{G}_{NS}(n) \sim 2^{-\frac{13}{4}}n^{-\frac{11}{4}}e^{\pi\sqrt{8n}}. \tag{A.14}$$

Or using $n \sim \alpha' m^2$, write it out in terms of mass

$$\mathcal{G}_{NS}(m) \sim 2^{-\frac{13}{4}} \alpha'^{-\frac{11}{4}} m^{-\frac{11}{2}} e^{\pi\sqrt{8\alpha'}m}. \tag{A.15}$$

Here we use the convention $\mathcal{G}_{NS}(m)dn = \mathcal{G}_{NS}(n)dn$, different from [17].

At this point, we also note that R sector has the same expression. And combine the left and right pieces together we arrive at the expression for closed strings

$$\mathcal{G}^{cl}(n) = [\mathcal{G}^{op}(n)]^2 \sim 2^{-\frac{9}{2}} n^{-\frac{11}{2}} e^{4\pi\sqrt{2n}}. \tag{A.16}$$

Taking care of the difference between the mass shell conditions of open and closed strings ($\alpha' m^2 \sim 4n$ for closed strings), the state degeneracy for closed string as a function of mass reads

$$\mathcal{G}^{cl}(m) \sim 2^{\frac{13}{2}} \alpha'^{-\frac{11}{2}} m^{-11} e^{\pi\sqrt{8\alpha'}m}. \tag{A.17}$$

Thus open and closed strings have the same Hagedorn temperature

$$T_H = \frac{1}{\pi\sqrt{8\alpha'}}. \tag{A.18}$$

We know that (The Legacy of Srinivasa Ramanujan, RMS-Lecture Notes Series No. 20, 2013, pp. 261–279.**The Partition Function Revisited** - M. Ram Murty):

The partition function, denoted $p(n)$, is the number of ways of writing n as a non-decreasing sum of positive integers. Thus, $p(1) = 1$, $p(2) = 2$, $p(3) = 3$ and $p(4) = 5$ since

$$4, \quad 1+3, \quad 2+2, \quad 1+1+2, \quad 1+1+1+1+1$$

are the five partitions of 4. Thus, each partition can be "factored" uniquely as

$$1^{k_1} 2^{k_2} \ldots$$

where the notation symbolizes

$$n = \underbrace{1+1+\cdots+1}_{k_1} + \underbrace{2+2+\cdots+2}_{k_2} + \cdots$$

and that:

The question of the asymptotic behaviour of $p(n)$ was first answered in the 1918 paper of Hardy and Ramanujan [9]. They proved that

$$p(n) \sim \frac{e^{\pi\sqrt{2n/3}}}{4n\sqrt{3}}, \quad n \to \infty. \tag{4}$$

In their proof, they discovered a new method called *the circle method* which made fundamental use of the modular property of the Dedekind η-function. We see from the Hardy-Ramanujan formula that $p(n)$ has exponential growth.

We have, from (2.41), that:

$$\sqrt{N_{1L}} + \sqrt{N_{2L}} \leq \sqrt{N_L}.$$

$\sqrt{3} + \sqrt{5} \leq \sqrt{8}$; 1,732050807 + 2,23606797 ≤ 2,82842712;

3,968118777 ≤ 2,82842712

Thence, we have that:

$$p(n) \sim \frac{1}{4n\sqrt{3}} \cdot e^{\pi\sqrt{2n/3}} \quad (4)$$

$$\mathcal{G}(N) \sim \frac{1}{N^{(D+1)/4}} \cdot e^{2\pi\sqrt{(D-2)/6}\sqrt{N}} \quad (2.43)$$

We observe that the eq. (2.43) and the (4) is practically very similar.

Now, we have, from (2.43), for N = 10, α' = 1/2, and D = 26:

$$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}}, \qquad a = 2\pi\sqrt{\frac{D-2}{6}}.$$

that is:

$$\mathcal{G}(N) \sim \frac{1}{N^{(D+1)/4}} \cdot e^{2\pi\sqrt{(D-2)/6}\sqrt{N}}$$

$\mathcal{G}(N) \sim N^{-\frac{D+1}{4}} e^{a\sqrt{N}} = 10^{-27/4} e^{4\pi\sqrt{10}} = 10^{-6.75} e^{4\pi(3,1622776)} = (1,778279410038 * 10^{-7})$ * (181195519824656285,625) = 32221626209,548572. Given a very massive initial string of high level, its state density has the above asymptotic form. We note that $(32221626209,548572)^{1/48}$ = 1,65546399123955

From the paper "**RAMANUJAN'S CLASS INVARIANTS,KRONECKER'S LIMIT FORMULA, AND MODULAR EQUATIONS** BRUCE C. BERNDT, HENG HUAT CHAN, AND LIANG-CHENG ZHANG", we have various expressions that can be related with some sectors of the string theory.

We take:

$$(130\sqrt{5}+29\sqrt{101})+\sqrt{169440+7540\sqrt{505}}=\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3,$$

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3.$$

that is equal to 1164,269601267364. We note that $(1164{,}269601267364)^{1/14} =$ 1,655784548804.

We have that:

$$\sqrt[48]{\frac{1}{N^{(26+1)/4}}\cdot e^{2\pi\sqrt{(26-2)/6}\sqrt{10}}} = 1{,}6554639\ldots$$

and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}6557845$$

Thence a new possible mathematical connection between the asymptotic form of the state density of a very massive initial string of high level, and the above Ramanujan's class invariant. We have indeed:

$$\sqrt[48]{\frac{1}{N^{(26+1)/4}}\cdot e^{2\pi\sqrt{(26-2)/6}\sqrt{10}}} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

$$1{,}65546 \approx 1{,}65578$$

results that are very near to the mass of the proton ($1{,}672 * 10^{-27}$)

We have, from (2.52), that:

$$\mathcal{G}_L \sim (2\pi T_H)^{-\frac{D-1}{2}}(\frac{N_{1L}N_{2L}}{N_L})^{-\frac{D+1}{4}}e^{-\sqrt{2}t_L/T_H},$$

with the Hagedorn temperature $T_H = \frac{1}{\pi}\sqrt{\frac{3}{D-2}}$ and $t_L = \sqrt{N_L} - \sqrt{N_{1L}} - \sqrt{N_{2L}}$

76,1095894446859531 * 0,01436287608 * 1659103,33006 = 1813653,1217337...
that is the total decay rate for decays into arbitrary states of given mass. We note that $(1813653{,}1217337)^{1/30}$ = 1,6166591858....

We have that:

$$\frac{1}{\sqrt{2}}(301+46\sqrt{43})+\frac{1}{\sqrt{2}}\sqrt{7(25941+3956\sqrt{43})}$$
$$=\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3,$$

that is equal to: 852,2635597... We have that $(852{,}2635597)^{1/14}$ = 1,6192977355292

Thence, we have the following new mathematical connection:

$$\mathcal{G}_L \sim (2\pi T_H)^{-\frac{D-1}{2}}(\frac{N_{1L}N_{2L}}{N_L})^{-\frac{D+1}{4}}e^{-\sqrt{2}t_L/T_H},$$

$$\sqrt[30]{\mathcal{G}_L} \sim \sqrt[30]{(2\pi\cdot 0{,}1125395)^{-25/2}\left(\frac{15}{8}\right)^{-27/4} e^{-\sqrt{2}(-1{,}1396916/0{,}1125395)}} =$$

$$\sqrt[30]{1813653{,}1217337} = 1{,}6166591858\ldots$$

$$\sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3} = 1{,}6192977\ldots$$

Thence:

$$\sqrt[30]{(2\pi\cdot 0{,}1125395)^{-2\ /2}\left(\frac{15}{8}\right)^{-27/4} e^{-\sqrt{2}(-1{,}1396916/0{,}1125395)}} \cong$$

$$\cong \sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3}$$

$1,61665 \approx 1,61929$

values very near to the electric charge of the electron.

Now, we have:

$$\mathcal{G}(N_L) \approx 2^{-\frac{13}{4}} N_L^{-\frac{11}{4}} e^{\pi\sqrt{8N_L}}.$$

That is equal to: 28390031,9 we note that $(28390031,9)^{1/34} = 1,65657369...$ and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1,6557845$$

Thence, we have:

$$\sqrt[34]{(2^{-1\ /4}\cdot 8^{-11/4}\cdot e^{8\pi})} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

$$1,65657 \approx 1,65578$$

results that are very near to the mass of the proton.

We have:

$$\mathcal{G}_{NS}(n) \sim 2^{-\frac{13}{4}} n^{-\frac{11}{4}} e^{\pi\sqrt{8n}}$$

That, for n = 4, is equal to: 121357,2462164 and $(121357,2462164)^{1/23} =$ 1,66359013

and:

$$\sqrt{\frac{1}{2}(135619+78300\sqrt{3})}+\frac{3}{\sqrt{2}}(87+50\sqrt{3}) = \left(\sqrt{\frac{21+12\sqrt{3}}{2}}+\sqrt{\frac{19+12\sqrt{3}}{2}}\right)^3,$$

that is equal to: 736,53184348... We have that $(736{,}53184348)^{1/13} = 1{,}661702198...$
Thence:

$$\sqrt[23]{\left(2^{-13/4} \cdot 4^{-11/4} \cdot e^{\pi\sqrt{32}}\right)} = \sqrt[13]{\left(\sqrt{\frac{21+12\sqrt{3}}{2}} + \sqrt{\frac{19+12\sqrt{3}}{2}}\right)^3}$$

$$1{,}66359 \approx 1{,}66170$$

results that are very near to the mass of the proton.

Now, we have:

$$\mathcal{G}_{NS}(m) \sim 2^{-\frac{13}{4}} \alpha'^{-\frac{11}{4}} m^{-\frac{11}{2}} e^{\pi\sqrt{8\alpha' m}}.$$

That is equal to: 121357,180534 with results similar as the expression obtained above.

We have:

$$\mathcal{G}^{cl}(n) = [\mathcal{G}^{op}(n)]^2 \sim 2^{-\frac{9}{2}} n^{-\frac{11}{2}} e^{4\pi\sqrt{2n}}.$$

That is equal to: 58910324836,98435 and $(58910324836{,}98435)^{1/49} =$ 1,65882214636257.

We have that:

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3.$$

that is equal to 1164,269601267364. We note that $(1164{,}269601267364)^{1/14} =$ 1,655784548804

We obtain:

$$\sqrt[49]{2^{-9/2} \cdot 4^{-11/2} \cdot e^{4\pi\sqrt{8}}} = \sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}} + \sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3}$$

$$1{,}65882 \approx 1{,}65578$$

results that are very near to the mass of the proton.

In conclusion, we have:

$$\mathcal{G}^{cl}(m) \sim 2^{\frac{13}{2}} \alpha'^{-\frac{11}{2}} m^{-11} e^{\pi\sqrt{8\alpha'}m}.$$

That is equal to: 2309101,7209 and $(2309101{,}7209)^{1/29} = 1{,}657406627\ldots$

We have that:

$$\sqrt[29]{2^{13/2} \cdot 0{,}5^{-11/2} \cdot 2{,}828427^{-11} \cdot e^{2\pi \cdot 2{,}828427}} = \sqrt[14]{\left(\sqrt{\frac{113 + 5\sqrt{505}}{8}} + \sqrt{\frac{105 + 5\sqrt{505}}{8}}\right)^3}$$

$$1{,}65740 \approx 1{,}65578$$

values that are very near to the mass of proton.

From:

Brane World of Warp Geometry: An Introductory Review
Yoonbai Kim, Chong Oh Lee, Ilbong Lee

https://arxiv.org/abs/hep-th/0307023v2

3.1 Pure anti-de Sitter spacetime

When the bulk is filled only with negative vacuum energy $\Lambda < 0$ without other matters $S_{\rm matter} = 0$ so that $T_{AB} = 0$, then the Einstein equations (2.14)$\sim$(2.15) are

$$A^{''} = 0 \text{ and } A^{'2} = -\frac{2\Lambda}{p(p+1)}. \tag{3.1}$$

Notice that $A(Z)$ can have a real solution only when Λ is nonpositive. General solution of Eq. (3.1) is given by

$$A_{\pm}(Z) = \pm\sqrt{\frac{2|\Lambda|}{p(p+1)}}Z + A_0, \tag{3.2}$$

where the integration constant A_0 can be removed by rescaling of the spacetime variables of p-brane, i.e., $dx^\mu \rightarrow d\bar{x}^\mu = e^{A_0}dx^\mu$. The resultant metric is

$$ds^2 = e^{\pm 2kZ}\eta_{\mu\nu}d\bar{x}^\mu d\bar{x}^\nu - dZ^2, \tag{3.3}$$

where $k = \sqrt{2|\Lambda|/p(p+1)}$ and a schematic shape of the metric $e^{2A(Z)}$ is shown in Fig. 2. Since the metric function $e^{2A_\pm}$ vanishes or is divergent at spatial infinity $Z = \mp\infty$ respectively, there exists coordinate singularity at those points. Despite of the coordinate singularity, the spacetime is physical-singularity-free everywhere as expected

$$R^{ABCD}R_{ABCD} = \frac{8(p+2)}{p^2(p+1)}|\Lambda|^2. \tag{3.4}$$

4.2 Gauge hierarchy from model I

As we explained briefly in the introduction, the gauge hierarchy problem is a notorious fine tuning problem in particle phenomenology of which the basic language is quantum field theory. So the readers unfamiliar to field theories may skip this subsection.

Let us assume that we live on the p-brane at $Z = r_c\pi$ and try a dimensional reduction of the Einstein gravity from the $D = p + 2$-dimensional gravity to $p + 1$-dimensional gravity on the p-brane at $Z = r_c\pi$. Then we have

$$
\begin{aligned}
S_{\mathrm{EH}\,D} &= -\frac{M_*^p}{16\pi}\int d^Dx\sqrt{|g_D|}\,R && (4.23)\\
&= -\frac{M_*^p}{16\pi}\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\int_{-r_c\pi}^{r_c\pi} dZ e^{-(p-1)k|Z|}(R_{p+1}+\cdots) && (4.24)\\
&= -\frac{M_*^p}{16k\pi}\left[1-e^{-(p-1)kr_c\pi}\right]\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\,(R_{p+1}+\cdots) && (4.25)\\
&\equiv -\frac{M_{\mathrm{Planck}}^2}{16\pi}\int d^{p+1}x\sqrt{|\det g_{\mu\nu}|}\,(R_{p+1}+\cdots) && (4.26)\\
&= S_{\mathrm{EH}\,p+1}+\cdots. && (4.27)
\end{aligned}
$$

We used $g_D = e^{-2(p+1)k|Z|}\det g_{\mu\nu}$ and $R = e^{2k|Z|}g^{\mu\nu}R_{\mu\nu} + \cdots = e^{2k|Z|}R_{p+1} + \cdots$ when we calculated the second line (4.24) from the first line (4.23). By comparing the third line (4.25) with the fourth line (4.27), we obtain a relation for 3-brane among three scales M_{Planck}, M_*, $|\Lambda|$ ($p = 3$):

$$
M_{\mathrm{Planck}}^2 = \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1-\exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right]M_*^{p=3}. \qquad (4.28)
$$

A natural choice for the bulk theory is to bring up almost the same scales for two bulk mass scales, i.e., $M_* \approx \sqrt{|\Lambda|}$. Suppose that the exponential factor in the relation (4.28) is negligible to the unity, which means r_c is slightly larger than $1/\sqrt{|\Lambda|}$. Then we reach

$$
M_{\mathrm{Planck}} \approx M_* \approx \sqrt{|\Lambda|}. \qquad (4.29)
$$

A striking character of this Randall-Sundrum compactification I is that it provides an explanation for gauge hierarchy problem that *why is so large the mass gap between the Planck scale $M_{\mathrm{Planck}} \sim 10^{19}\mathrm{GeV} \sim 10^{-38}\,M_\odot$ and the electroweak scale $M_{\mathrm{EW}} \sim 10^3\mathrm{GeV} \sim 10^{-54}\,M_\odot$* without assuming supersymmetry or others. As a representative example, let us consider a massive neutral scalar field H which lives on our 3-brane at $Z = r_c\pi$:

$$
S_{\mathrm{scalar}} = \int_{-r_c\pi}^{r_c\pi} dZ\delta(Z-r_c\pi)\int d^4x\sqrt{g_5}\left[\frac{1}{2}g^{AB}\partial_A H\partial_B H - \frac{1}{2}M_{\mathrm{Planck}}^2 H^2\right]
$$

$$
\begin{aligned}
&= \int_{-r_c\pi}^{r_c\pi} dZ e^{-4k|Z|}\delta(Z - r_c\pi) \int d^4x\sqrt{-\hat{g}_4} \\
&\qquad \times \left[\frac{1}{2}e^{2k|Z|}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}M_{\text{Planck}}^2 H^2 - \frac{1}{2}\hat{g}^{ZZ}\left(\partial_Z H\right)^2\right] \\
&= e^{-2r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}(e^{-r_c\pi k}M_{\text{Planck}})^2 H^2\right] \qquad (4.30) \\
&= e^{-2r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu H\partial_\nu H - \frac{1}{2}M_{\text{EW}}^2 H^2\right], \qquad (4.31)
\end{aligned}
$$

where $ds^2 = g_{AB}dx^A dx^B = e^{-2k|Z|}\hat{g}_{\mu\nu}dx^\mu dx^\nu - dZ^2$. The last two lines give us a relation:

$$
\frac{M_{\text{EW}}}{M_{\text{Planck}}} = \exp\left(-\sqrt{\frac{2|\Lambda|}{p(p+1)}}r_c\pi\right) \sim 10^{-16}. \qquad (4.32)
$$

Therefore, the radius r_c of compactified extra dimension of the Randall-Sundrum brane world model I is determined nearly by the Planck scale :

$$
\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\text{Planck}}}{30}. \qquad (4.33)
$$

All the scales such as the fundamental scale of the bulk M_*, the bulk cosmological constant $\sqrt{|\Lambda|}$, the inverse size of the compactification $1/r_c$, are almost the Planck scales $M_{\text{Planck}} \sim 10^{19}$ GeV together. The masses of matter particles on our visible brane at $Z = r_c\pi$ are in electroweak scale $M_{\text{EW}} \sim 10^3$ GeV, however those on the hidden brane at $Z = 0$ in the Planck scale. Though the gauge hierarchy problem seems to be solved, it is actually not because a fine-tuning condition was urged in Eq. (3.25). However, it becomes much milder than that before.

Finally let us consider a fermionic field of which mass is provided by spontaneous symmetry breaking and its Lagrangian is

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}\gamma^A \nabla_A \Psi + g\phi\bar{\Psi}\Psi, \tag{4.36}$$

where g is the coupling constant of Yukawa interaction. If we neglect the quantum fluctuation $\delta\phi$ of ϕ, i.e. $\phi \equiv \langle\phi\rangle + \delta\phi$, the Lagrangian (4.36) becomes

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}\gamma^A \nabla_A \Psi + g\langle\phi\rangle\bar{\Psi}\Psi + \cdots, \tag{4.37}$$

where the second term is identified as mass term, and we neglected the vertex term $g\delta\phi\bar{\Psi}\Psi$ because we are not interested in quantum fluctuation. Again the fermion lives on our 3-brane at $Z = r_c\pi$, and then the action is

$$S_{\text{fermion}} = \int_{r_c\pi}^{r_c\pi} dZ\delta(Z - r_c\pi)\int d^4x\sqrt{g_5}\left[\bar{\Psi}\gamma^a e_a^A \nabla_A \Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right], \tag{4.38}$$

where e_a^A is vielbein defined by $g_{AB} = \eta_{ab}e_A^a e_B^b$ and $M_{\text{Planck}} = g\langle\phi\rangle$ since the symmetry breaking scale should coincide with the fundamental scale. Subsequently, the action (4.38) becomes

$$\begin{aligned} S_{\text{fermion}} &= \int_{r_c\pi}^{r_c\pi} dZ\delta(Z - r_c\pi)\int d^4x\sqrt{g_5}\left[\bar{\Psi}\gamma^a e_a^A \nabla_A \Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right] \\ &= \int_{-r_c\pi}^{r_c\pi} dZ e^{-4k|Z|}\delta(Z - r_c\pi)\int d^4x\sqrt{-\hat{g}_4} \\ &\qquad \times\left[e^{k|Z|}\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu \Psi - \bar{\Psi}\gamma^a \hat{e}_a^Z \nabla_Z \Psi + M_{\text{Planck}}\bar{\Psi}\Psi\right] \\ &= e^{-3r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu \Psi + (e^{-r_c\pi k}M_{\text{Planck}})\bar{\Psi}\Psi\right] \qquad (4.39) \\ &= e^{-3r_c\pi k}\int d^4x\sqrt{-\hat{g}_4}\left[\bar{\Psi}\gamma^a \hat{e}_a^\mu \nabla_\mu \Psi + m_{\text{fermion}}\bar{\Psi}\Psi\right]. \qquad (4.40) \end{aligned}$$

Once again we obtain the same mass hierarchy relation $m_{\text{fermion}} = e^{-r_c\pi k}M_{\text{Planck}} = M_{\text{EW}}$ for the fermion from Eq. (4.39) and Eq. (4.40) with the help of Eq. (4.32).

We know that the mass Planck is defined as:

$$m_P = \sqrt{\frac{\hbar c}{G}},$$

where c is the speed of light in a vacuum, G is the gravitational constant, and $\hbar$ is the reduced Planck constant.

Substituting values for the various components in this definition gives the approximate equivalent value of this unit in terms of other units of mass:

1 $m_P \approx 1.220910 \times 10^{19}$ GeV/c^2 = $2.176470(51) \times 10^{-8}$ kg.

Particle physicists and cosmologists often use an alternative normalization with the **reduced Planck mass**, which is

$$M_P = \sqrt{\frac{\hbar c}{8\pi G}}$$

$M_P \approx 2.435\times10^{18}$ GeV/c^2 = 4.341×10^{-9} kg

Now, from (4.28)

$$M^2_{\rm Planck} = \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1 - \exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}}r_c\pi\right)\right] M_*^{p=3}.$$

we have that:

$M^2_{Planck} = 1{,}4906212281 \times 10^{38}$ and E = 1,098819 * 10^{36} or:

$M^2_{Planck} = 5{,}929225 \times 10^{36}$ and E = 2,1915 * 10^{35}

And, from (4.33), we have that:

$$\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\rm Planck}}{30}.$$

Thence: $\frac{1}{r_c} = 4{,}0697 * 10^{17}$ and $r_c = 2{,}4571835761 * 10^{-18}$

$r_c = 2{,}45718357618497677961520505196 94 * 10^{-18}$ or:

$\frac{1}{r_c} = 8{,}11666 * 10^{16}$ and $r_c = 1{,}2320328542 * 10^{-17}$

We have that:

$M_{\rm EW} \sim 10^3{\rm GeV}$

and

$m_{\rm fermion} = e^{-r_c\pi k} M_{\rm Planck} = M_{\rm EW}$

We have that:

$m_{\rm fermion}$ = 1,22091 * 10^{19} GeV/c^2 or

$m_{\rm fermion}$ = 2,435 × 10^{18} GeV/c^2. The energy E is: 2,1915 * 10^{35}

Now:

$(2{,}435 \times 10^{18})^{1/88} = 1{,}61784715017$ or $(2{,}435 \times 10^{18})^{1/89} = 1{,}609125347$ and for the value of the fermion energy E (for $E = mc^2$):
$(2{,}1915 * 10^{35})^{1/168} = 1{,}6231608397$ or $(2{,}1915 * 10^{35})^{1/169} = 1{,}6185153159$.
Further, we have:
$(1{,}2320328542 * 10^{-17})^{1/80} = 0{,}614656924537$ and the reciprocal is 1,626923833;
and
$(5{,}929225 \times 10^{36})^{1/168} = 1{,}65533879..$ and $(5{,}929225 \times 10^{36})^{1/178} = 1{,}609125347$.

Now, we have that:

$$P^{-2} := (G_{205}G_{41/5})^4 = \left(\frac{\sqrt{5}+1}{2}\right)^8 \left(\frac{43+3\sqrt{205}}{2}\right)$$
$$= \left(\frac{7+3\sqrt{5}}{2}\right)^2 \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2.$$

that is equal to: 46,9787137637477918 (42,9767315949145297) = 2018,991572...
and $(2018{,}991572)^{1/16} = 1{,}6090550645269$

Thence:

$$M^2_{\text{Planck}} = \sqrt{\frac{p(p+1)}{2|\Lambda|}}\left[1 - \exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}} r_c \pi\right)\right] M_*^{p=3}.$$

$= 5{,}929225 \times 10^{36}$ and $(5{,}929225 \times 10^{36})^{1/178} = 1{,}609125347$.

We have the following interesting mathematical connection:

$$\sqrt[178]{\left(\sqrt{\frac{p(p+1)}{2|\Lambda|}}\left(1 - exp\left(-\sqrt{\frac{8|\Lambda|}{p(p+1)}} r_c \pi\right)\right) M_*^{p=3}\right)} =$$

$$= \sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2 + \left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2}$$

$1{,}6091253 \approx 1{,}609055$

Values that are very near to the electric charge of positron.

Now:

$$\frac{1}{r_c} \sim \frac{\pi}{16\sqrt{6}\ln 10}\sqrt{|\Lambda|} \sim \frac{M_{\text{Planck}}}{30}.$$

$= 8{,}11666 * 10^{16}$ and $r_c = 1{,}2320328542 * 10^{-17}$

and $1 / (1{,}2320328542 * 10^{-17})^{1/80} = 1{,}626923833.$

We have that:

$$\frac{1}{\sqrt{2}}(301+46\sqrt{43})+\frac{1}{\sqrt{2}}\sqrt{7(25941+3956\sqrt{43})}$$
$$=\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3,$$

$= 852{,}2635597...$ and $(852{,}2635597)^{1/14} = 1{,}6192977355292.$

Thence:

$$\left(1/\sqrt[80]{\frac{M_{Planck}}{30}}\right) = \sqrt[14]{\left(\sqrt{\frac{46+7\sqrt{43}}{4}}+\sqrt{\frac{42+7\sqrt{43}}{4}}\right)^3};$$

$1{,}626923 \approx 1{,}619297$

Values that are a good approximations to the electric charge of the positron.

Now:

$m_{\text{fermion}} = e^{-r_c \pi k} M_{\text{Planck}} = M_{\text{EW}}$

$= 2{,}435 \times 10^{18}$ and $(2{,}435 \times 10^{18})^{1/89} = 1{,}609125347.$

We have that:

$$\sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2+\left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2} = 1{,}609055..$$

Thence:

$$\sqrt[89]{e^{-r_c\pi k}\cdot M_{Planck}} = \sqrt[16]{\left(\frac{7+3\sqrt{5}}{2}\right)^2+\left(\frac{3\sqrt{5}+\sqrt{41}}{2}\right)^2}$$

$1{,}60912 \approx 1{,}60905$

values very similar and very near to the electric charge of the positron.

We calculate the following double integrals:

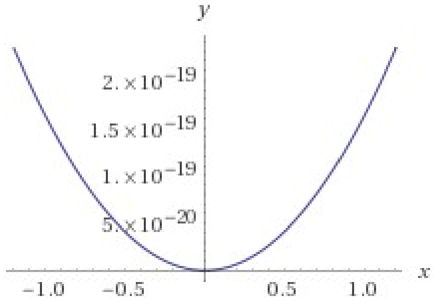
(4Pi/23) 1/(10^55) integrate integrate [5.929225 *10^36]

$$\left(4\times\frac{\pi}{23}\right)\times\frac{1}{10^{55}}\int\left(\int 5.929225\times 10^{36}\,dx\right)dx$$

Result:

$1.61976\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

and

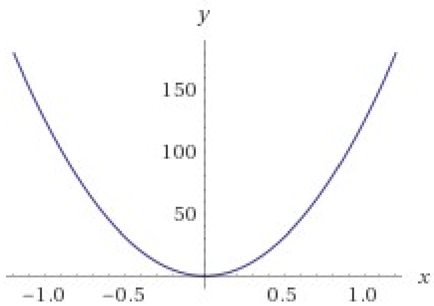
(2*21) * integrate integrate [5.929225]

$$2\times 21\int\left(\int 5.929225\,dx\right)dx$$

Result:

$124.514\,x^2$

Plot:



(x from −1.2 to 1.2)

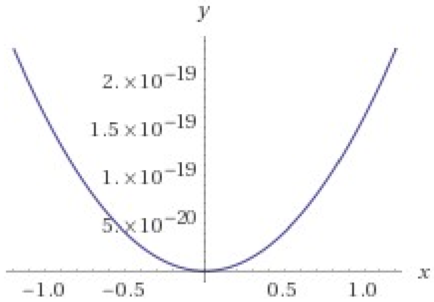
(4Pi/86) 1/(10^53) integrate integrate [2.1915 * 10^35]

$$\left(4\times\frac{\pi}{86}\right)\times\frac{1}{10^{53}}\int\left(\int 2.1915\times 10^{35}\,dx\right)dx$$

Result:

$1.60112\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

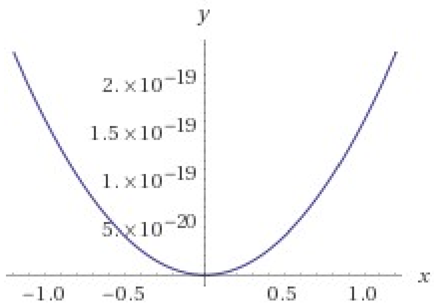
(4Pi/95) 1/(10^36) integrate integrate [2.435 × 10^18]

$$\left(4\times\frac{\pi}{95}\right)\times\frac{1}{10^{36}}\int\left(\int 2.435\times 10^{18}\,dx\right)dx$$

Result:

$1.61048\times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

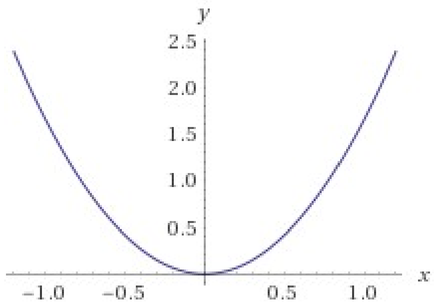
(24Pi/5) 1/(10^36) integrate integrate [2.1915 * 10^35]

$$\left(24\times\frac{\pi}{5}\right)\times\frac{1}{10^{36}}\int\left(\int 2.1915\times 10^{35}\,dx\right)dx$$

Result:

$1.65235\,x^2$

Plot:



(x from −1.2 to 1.2)

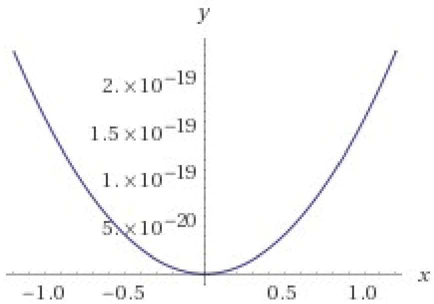
(3Pi^2/2) * 1/(10^55) * integrate integrate [2.1915 * 10^35]

$$\left(3\times\frac{\pi^2}{2}\right)\times\frac{1}{10^{55}}\int\left(\int 2.1915\times10^{35}\,dx\right)dx$$

Result:

$1.62219\times10^{-19}\,x^2$

Plot:



(x from -1.2 to 1.2)

All values very near to the electric charge of the positron.

From:

Modular Relations for J-invariant and Explicit evaluations
M.S. Mahadeva Naika, D.S. Gireesh and N.P. Suman
Communicated by P. K. Banerji - Palestine Journal of Mathematics - Vol. 5(2) (2016), 83–95

Theorem 4.2. *We have*

$$\text{(i)}\ J_{63} = \frac{5}{32}\left(\frac{3}{4}(a_1 + b_1)\right)^{\frac{1}{3}}, \tag{4.12}$$

$$\text{(ii)}\ J_{75} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}}, \tag{4.13}$$

$$\textit{(iii)}\ J_{99} = \left(\frac{1147951079}{2} + \frac{199832633\sqrt{33}}{2}\right)^{\frac{1}{3}}, \tag{4.14}$$

$$\textit{(iv)}\ J_{147} = \left(531745995375 + 116036489250\sqrt{21}\right)^{\frac{1}{3}}, \tag{4.15}$$

$$\textit{(v)}\ J_{171} = \frac{1}{4}(a_2 + b_2)^{\frac{1}{3}}, \tag{4.16}$$

where

$$a_1 = 180040533 + 39288067\sqrt{21},$$
$$b_1 = 273\sqrt{2\left(434925969567 + 94908627499\sqrt{21}\right)},$$
$$a_2 = 21187806942033 + 2806393586997\sqrt{57},$$

and

$$b_2 = 2\sqrt{\frac{448923163012861107298413933}{2} + \frac{59461325524651981512667761\sqrt{57}}{2}}.$$

We have:

$$\text{(ii)}\ J_{75} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}},$$

that is equal to: 0,6239645246738... we have that: (1 / 0,6239645246738...) = = 1,60265521589... and (1,60265521589) * 8 = 12,821241..; (1,60265521589) * 4 = = 6,41062.. values that are good approximations to the mass of the SMBH87 and to the reduced Planck's constant.

$$J_{99} = \left(\frac{1147951079}{2} + \frac{199832633\sqrt{33}}{2}\right)^{\frac{1}{3}}$$

that is equal to: 1047,06697; $(1047{,}06697)^{1/14}$ = 1,64328337... and (1,64328337) * 8 = 13,146266997; (1,64328337) * 4 = 6,573133... values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

$$J_{147} = \left(531745995375 + 116036489250\sqrt{21}\right)^{\frac{1}{3}},$$

that is equal to: 10207,312425; $(10207{,}312425)^{1/19}$ = 1,6255313... and (1,6255313) * 8 = 13,00425; (1,6255313) * 4 = 6,5021252; values that are very near to the mass of the SMBH87 and to the reduced Planck's constant .

Now:

$$a_2 = 21187806942033 + 2806393586997\sqrt{57},$$
and
$$b_2 = 2\sqrt{\frac{448923163012861107298413933}{2} + \frac{59461325524651981512667761\sqrt{57}}{2}}.$$

$$(v)\ \ J_{171} = \frac{1}{4}(a_2 + b_2)^{\frac{1}{3}}$$

a_2 = 42375613884065,949055701411614262
b_2 = 42375613884065,967937986172212435

The result is : 10981,3400827; $(10981,3400827)^{1/19}$ = 1,63179677... and (1,63179677) * 8 = 13,0543742; (1,63179677) * 4 = 6,5271871...; values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

Now:

$$a_1 = 180040533 + 39288067\sqrt{21},$$
$$b_1 = 273\sqrt{2\left(434925969567 + 94908627499\sqrt{21}\right)},$$

$$\text{(i)}\ \ J_{63} = \frac{5}{32}\left(\frac{3}{4}(a_1 + b_1)\right)^{\frac{1}{3}},$$

a_1 = 360081073,935996
b_1 = 360081088,577444

The result is: 127,2478775... $(127,2478775)^{1/10}$ = 1,6235477... (1,6235477) * 8 = 12,9883816; (1,6235477) * 4 = 6,4941908... values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

Now, we have that:

Theorem 4.3.

$$(i)\ \ J_{243} = 5\left(8704c_1 + \frac{72603983653110c_1}{1447318030184058288801} + 151022371885959\right)^{\frac{1}{3}}, \quad (4.22)$$

$$(ii)\ \ J_{275} = \frac{544127}{2} + 121667\sqrt{5} + \frac{\sqrt{592131660065 + 264809328784\sqrt{5}}}{2}, \quad (4.23)$$

where $c_1 = 5223567527018075142287271005853^{1/3}$.

$$(i)\ \ J_{243} = 5\left(8704c_1 + \frac{72603983653110c_1}{1447318030184058288801} + 151022371885959\right)^{\frac{1}{3}},$$

$5\,(151022371885959{,}00037 + 8703{,}999 + 151022371885959)^{1/3} =$
$= 5\,(67095{,}0417540786) = 335475{,}2087...$
We have that $(335475{,}2087)^{1/27} = 1{,}6019689349...$ $(1{,}6019689349) * 8 = 12{,}81575...$ $(1{,}6019689349) * 4 = 6{,}407875$ values that are very near to the mass of the SMBH87 and to the reduced Planck's constant.

We now calculate the following double integral:

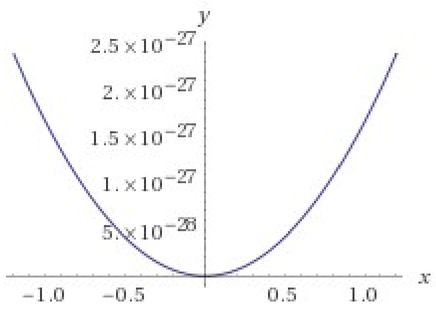
(Pi)^2 * (1/(10)^33) * integrate integrate [335475.2087]

$$\pi^2 \times \frac{1}{10^{33}} \int\left(\int 335\,475.2087\, dx\right) dx$$

Result:

$$1.6555 \times 10^{-27}\, x^2$$

Plot:



(x from −1.2 to 1.2)

Now:

$$(ii)\quad J_{275} = \frac{544127}{2} + 121667\sqrt{5} + \frac{\sqrt{592131660065 + 264809328784\sqrt{5}}}{2},$$

We have that: $272063{,}5 + 272055{,}682618 + 544119{,}316028 = 1088238{,}498646$

We obtain: $(1088238{,}498646)^{1/29} = 1{,}61496420014$; $(1{,}61496420014) * 8 = 12{,}9197$ and $(1{,}61496420014) * 4 = 6{,}4598568$ values that are a good approximation to the reduced Planck's constant and to the mass of the SMBH87

We calculate the following double integral:

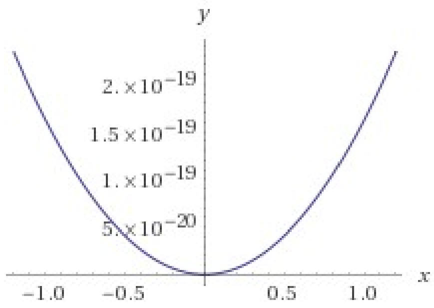
(Pi)/(13*0.25)^2 * (1/(10)^24) * integrate integrate [1088238.498646]

$$\frac{\pi}{(13 \times 0.25)^2} \times \frac{1}{10^{24}} \int\left(\int 1.088238498646 \times 10^6\, dx\right) dx$$

Result:

$$1.61837 \times 10^{-19}\, x^2$$

Plot:



(x from -1.2 to 1.2)

Theorem 4.4. *We have*

$$\text{(i)}\ \ J_{363} = 15\left(\frac{a_3}{4} + \frac{9\sqrt{2(b_3+c_3)}}{4}\right)^{\frac{1}{3}}, \tag{4.25}$$

$$\text{(ii)}\ \ J_{387} = \left(\frac{a_4}{4} + \frac{1}{2}\sqrt{\frac{b_4}{2} + \frac{c_4}{2}}\right)^{\frac{1}{3}}, \tag{4.26}$$

$$\text{(iii)}\ \ J_{475} = \frac{127580541}{2} + 28527876\sqrt{5} + a_5, \tag{4.27}$$

$$\text{(iv)}\ \ J_{603} = \left(\frac{a_6}{4} + \frac{1}{2}\sqrt{\frac{b_6}{2} + \frac{c_6}{2}}\right)^{\frac{1}{3}}, \tag{4.28}$$

where $a_3 = 8935875480904000075 + 1555536257627776261\sqrt{33}$,

$b_3 = 98580087173112722442256274921544461$,

$c_3 = 17160590499003817972086593347646335\sqrt{33}$,

$a_4 = 211345136395511192813125 + 18607901688694106 11875\sqrt{129}$,

$b_4 = 44666766678037540637472435599838341224 1203125$,

$c_4 = 39326895204313325954377906531132680150421875\sqrt{129}$,

$a_6 = 973319388123934741480720976 25 + 6865265632433907880859325375\sqrt{201}$,

$b_6 = 947350631297950717475266928949327772335258966254882001 5625$,

and

$c_6 = 668209614466884909039855025792091189769949745940542671875\sqrt{201}$.

We have that:

$$(ii)\ \ J_{387} = \left(\frac{a_4}{4} + \frac{1}{2}\sqrt{\frac{b_4}{2} + \frac{c_4}{2}}\right)^{\frac{1}{3}},$$

$$a_4 = 21134513639551192813125 + 1860790168869410611875\sqrt{129},$$
$$b_4 - 446667666780375406374724355998383412241203125,$$
$$c_4 = 39326895204313325954377906531132680150421875\sqrt{129},$$

We obtain:

(((((21134513639551192813125+(1860790168869410611875(sqrt(129)))/4)+0.5(sqrt((446667666780375406374724355998383412241203125)/2+((39326895204313325954377906531132680150421875*sqrt(129))/2)))))^0.33333

$$\left(\left(\frac{1}{4} \times 21\,134\,513\,639\,551\,192\,813\,125 + 1\,860\,790\,168\,869\,410\,611\,875 \times 11.3578\right) + 0.5\sqrt{\left(\frac{446\,667\,666\,780\,375\,406\,374\,724\,355\,998\,383\,412\,241\,203\,125}{2} + \frac{1}{2}\,39\,326\,895\,204\,313\,325\,954\,377\,906\,531\,132\,680\,150\,421\,875 \times 11.3578\right)}\right)^{0.33333}$$

$3.33121... \times 10^7$

33.312.064;

Note that: 3,33121 * 4 = 13,32484 and 3,33121 * 2 = 6,66242. Furthermore, $(33312064)^{1/35}$ = 1,6403309248 and (1,6403309248) * 8 = 13,12264739; (1,6403309248 * 4) = 6,56132369 values very near to the Planck's constant and to the mass of SMBH87.

We now calculate the following double integral:

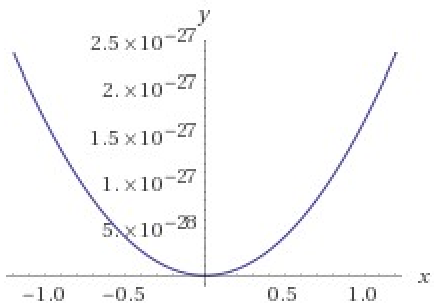
(Pi)^2 * (1/(10)^35) * integrate integrate [3.33121*10^7]

$$\pi^2 \times \frac{1}{10^{35}} \int \left(\int 3.33121 \times 10^7 \, dx \right) dx$$

Result:

$1.64389 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

We have:

$$\text{(i)}\quad J_{363} = 15\left(\frac{a_3}{4} + \frac{9\sqrt{2\,(b_3+c_3)}}{4}\right)^{\frac{1}{3}}$$

$$\textit{where } a_3 = 893587548090400075 + 155553625762776261\sqrt{33},$$
$$b_3 = 9858008717311272244225627492154461,$$
$$c_3 = 1716059049900381797208659334764635\sqrt{33},$$

We obtain:

$15\,((0.25 \times 893\,587\,548\,090\,400\,075 + 155\,553\,625\,762\,776\,261 \times 1.43614) + 2.25\,\sqrt{(2\,(9\,858\,008\,717\,311\,272\,244\,225\,627\,492\,154\,461 + 1\,716\,059\,049\,900\,381\,797\,208\,659\,334\,764\,635 \times 5.74456)))^{0.3333}$

$1.44280\ldots \times 10^7$

14428000;

Now: 1,4428 * 9 = 12,9852 and 12,9852 / 2 = 6,4926. Furthermore:

$(14428000)^{1/33}$ = 1,6479561079... and 1,6479561079 * 8 = 13,183648; 1,6479561079 * 4 = 6,591824... all values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

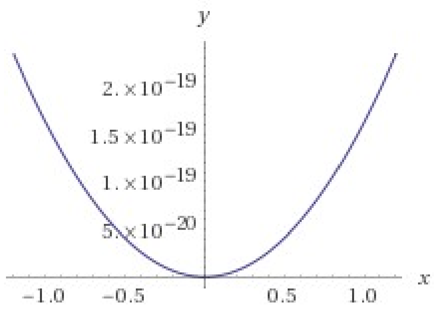
(Pi)/14 * (1/(10)^25) * integrate integrate [1.44280*10^7]

$$\frac{\pi}{14} \times \frac{1}{10^{25}} \int\left(\int 1.44280 \times 10^7\,dx\right)dx$$

Result:

$1.61882 \times 10^{-19}\,x^2$

Plot:



(x from −1.2 to 1.2)

Now, we have:

$$(iv)\;\; J_{603} = \left(\frac{a_6}{4} + \frac{1}{2}\sqrt{\frac{b_6}{2} + \frac{c_6}{2}}\right)^{\frac{1}{3}},$$

$$a_6 = 97331938812393474148072097625 + 6865265632433907880859325375\sqrt{201},$$
$$b_6 = 9473506312979507174752669289493277723352589662548820015625,$$

and

$$c_6 = 668209614466884909039855025792091189769949745940542671875\sqrt{201}.$$

We obtain:

$$0.5\sqrt{\left(9\,473\,506\,312\,979\,507\,174\,752\,669\,289\,493\,277\,723\,352\,589\,662\,548\,820\,015\,625 / 2 + \frac{1}{2}\, 668\,209\,614\,466\,884\,909\,039\,855\,025\,792\,091\,189\,769\,949\,745\,940\,542\,671\,875 \times 14.1774\right)}$$

$4.86659... \times 10^{28}$

$$(0.25\,(97\,331\,938\,812\,393\,474\,148\,072\,097\,625 + 6\,865\,265\,632\,433\,907\,880\,859\,325\,375 \times 14.1774) + 4.86659 \times 10^{28})^{0.33333333}$$

$4.59993... \times 10^{9}$

4.599.930.000;

We note that 4,59993 * 3 = 13,79979; 13,79979 / 2 = 6,899895. Furthermore: $(4.599.930.000)^{1/46}$ = 1,62203346153.. and 1,62203346153 * 8 = 12,976267692...; 1,62203346153 * 4 = 6,4881338; value very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

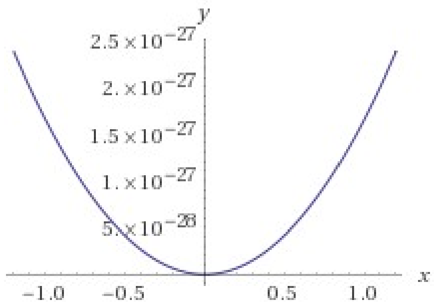
(Pi)/44 * (1/(10)^35) * integrate integrate [4.59993*10^9]

$$\frac{\pi}{44} \times \frac{1}{10^{35}} \int \left(\int 4.59993 \times 10^{9} \, dx \right) dx$$

Result:

$1.64217 \times 10^{-27} x^2$

Plot:



(x from -1.2 to 1.2)

We have:

Theorem 4.5. *We have*

$$\text{(i)}\ J_{\frac{11}{9}} = -\left(\frac{11}{2}\right)^{\frac{1}{3}} \left(-104359189 + 18166603\sqrt{33}\right)^{\frac{1}{3}}, \tag{4.32}$$

$$\text{(ii)}\ J_{\frac{3}{25}} = 3\left(369830 - 165393\sqrt{5}\right)^{\frac{1}{3}}, \tag{4.33}$$

$$\text{(iii)}\ J_{\frac{11}{25}} = \frac{544127}{2} + 121667\sqrt{5} - \frac{\sqrt{11\left(53830150915 + 24073575344\sqrt{5}\right)}}{2}, \tag{4.34}$$

$$\text{(iv)}\ J_{\frac{3}{49}} = 15\left(157554369 - 34381182\sqrt{21}\right)^{\frac{1}{3}}. \tag{4.35}$$

Now:

$$\text{(iv)}\ J_{\frac{3}{49}} = 15\left(157554369 - 34381182\sqrt{21}\right)^{\frac{1}{3}}.$$

We obtain: $15(157554369 - 157554368{,}9970532)^{1/3} = 2{,}150505873...$ and

$5\ (2{,}150505873) = 10{,}7525293...$ $(10{,}7525293)^{1/5} = 1{,}60805954....$ Thence: $(1{,}60805954) * 8 = 12{,}864476$; $(1{,}60805954) * 4 = 6{,}432238...$ values that are a good approximation to the reduced Planck's constant and to the mass of the SMBH87.

We have:

$$(iii)\ \ J_{\frac{11}{25}} = \frac{544127}{2} + 121667\sqrt{5} - \frac{\sqrt{11\left(538301509 15 + 240735753 44\sqrt{5}\right)}}{2},$$

We obtain:

$(272063{,}5 + 272055{,}6826184 - 544119{,}31602868) = -0{,}13341028$; $-(0{,}13341028)^{1/4} = 0{,}60436224239$. Now we note that $1 / 0{,}60436224239 = 1{,}6546367887...$ and $1{,}6546367887 * 8 = 13{,}237094309$; $1{,}6546367887 * 4 = 6{,}618547...$ that are values very near to the Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

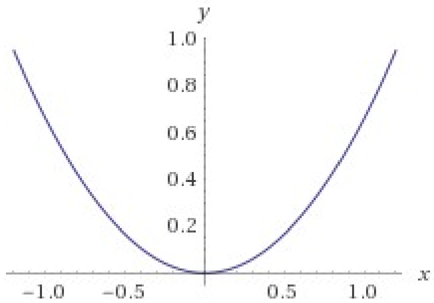
Pi^2 * integrate integrate [0.13341028]

$$\pi^2 \int\left(\int 0.13341028\, dx\right) dx$$

Result:

$0.658353\, x^2$

Plot:



(x from −1.2 to 1.2)

We note that the integral is given utilizing the following simple rules:

$$\int a\, dx = ax + C$$

$$\int x^a\, dx = \frac{x^{a+1}}{a+1} + C \qquad (\text{for } a \neq -1)\ \text{(Cavalieri's quadrature formula)}$$

We have:

$$\text{(i)}\ J_{\frac{11}{9}} = -\left(\frac{11}{2}\right)^{\frac{1}{3}} \left(-104359189 + 18166603\sqrt{33}\right)^{\frac{1}{3}},$$

We obtain: -0,35718823192 and $(0,35718823192)^{1/e}$ = 0,6847309205...

(0,6847309205) * 19 = 13,00988; 13,00988 / 2 = 6,504943 that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

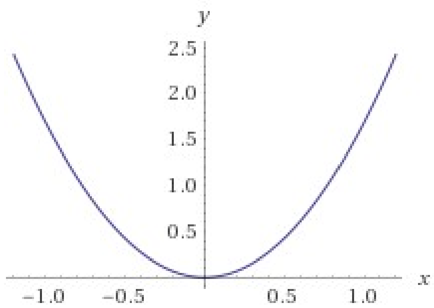
Pi * 3 integrate integrate [0.35718823192]

$$\pi \times 3 \int \left(\int 0.35718823192\, dx \right) dx$$

Result:

$1.6832098880\, x^2$

Plot:



(x from −1.2 to 1.2)

We note that (1,6832098) * 8 = 13,4656784; (1.6832098) * 4 = 6,7328392

We have that:

Theorem 4.6. *We have*

$$\text{(i)}\quad \left[\left(13\sqrt{5}-25\right)^3+6^3\right]L_5^3-\left(13\sqrt{5}-25\right)^3 R_5^2=0, \tag{4.38}$$

$$\text{(ii)}\quad \left(5^3+4^3\right)L_7^3-5^3R_7^2=0, \tag{4.39}$$

$$\text{(iii)}\quad \left(m_{13}^3+2^3\right)L_{13}^3-m_{13}^3R_{13}^2=0, \tag{4.40}$$

where $m_{13}=-155+45\sqrt{13}$,

$$\text{(iv)}\quad \left(45\left(a_1+b_1\right)+4^4\right)L_{63}^3-45\left(a_1+b_1\right)R_{63}^2=0, \tag{4.41}$$

$$\text{(v)}\quad \left(m_{75}+8^{-3}\right)L_{75}^3-m_{75}R_{75}^2=0, \tag{4.42}$$

where $m_{75}=369830-165393\sqrt{5}$,

$$\text{(vi)}\quad \left(8^3m_{147}+3^3\right)L_{147}^3-8^3m_{147}R_{147}=0, \tag{4.43}$$

where $m_{147}=531745995375+116036489250\sqrt{21}$,

$$\text{(vii)}\quad \left[2^7\left(a_2+b_2\right)+3^2\right]L_{171}^3-\left[2^7\left(a_2+b_2\right)\right]R_{171}^2=0, \tag{4.44}$$

$$\text{(viii)}\quad \left[40^3\left(a_3+9\sqrt{2\left(b_3+c_3\right)}\right)+4\right]L_{363}^3-40^3\left(a_3+9\sqrt{2\left(b_3+c_3\right)}\right)R_{363}^2=0, \tag{4.45}$$

$$\text{(ix)}\quad \left(m_{3/25}+8^{-3}\right)L_{3/25}^3-m_{3/25}R_{3/25}^2=0, \tag{4.46}$$

$$\text{(x)}\quad \left[40^3m_{3/49}+1\right]L_{3/49}^3-40^3m_{3/49}R_{3/49}^2. \tag{4.47}$$

where $m_{3/25}=369830-165393\sqrt{5}$ *and* $m_{3/49}=\left(157554369-34381182\sqrt{21}\right)$.

We have:

$$m_{147}=531745995375+116036489250\sqrt{21},$$

We obtain: 1063491990740,054608; we note that: $(1063491990740{,}054608)^{1/55}$ = 1,6545042800359... and (1,6545042800359) * 8 = 13,2360342; (1,6545042800359)*4 = 6,61801712 that are values very near to the Planck's constant and to the mass of the SMBH87.

Now, we calculate the following double integral:

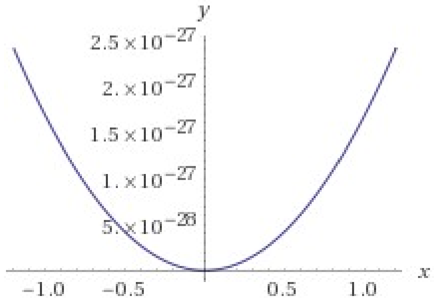
Pi * (1/(10)^39) * integrate integrate [1063491990740.054608]

$$\pi\times\frac{1}{10^{39}}\int\left(\int 1.063491990740054608\times10^{12}\,dx\right)dx$$

Result:

$$1.670529312630269987\times10^{-27}\,x^2$$

Plot:



(x from -1.2 to 1.2)

From:

MODULAR EQUATIONS FOR THE RATIOS OF RAMANUJAN'S THETA FUNCTION ψ AND EVALUATIONS

M. S. MAHADEVA NAIKA, S. CHANDANKUMAR AND K. SUSHAN BAIRY

(Received August 2010)

We have:

$$l_{9,8} = \frac{\sqrt{13+5\sqrt{6}+7\sqrt{3}+9\sqrt{2}}}{\sqrt{2}} + \frac{\sqrt{15+6\sqrt{6}+9\sqrt{3}+12\sqrt{2}}}{\sqrt{2}}, \qquad (4.6)$$

That is equal to

(5,004983837549 + 5,5792454002) = 10,5842292; $(10,5842292)^{1/5}$ = 1,60299381...
and (1,60299381) * 8 = 12,82395; (1,60299381) * 4 = 6,41197... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

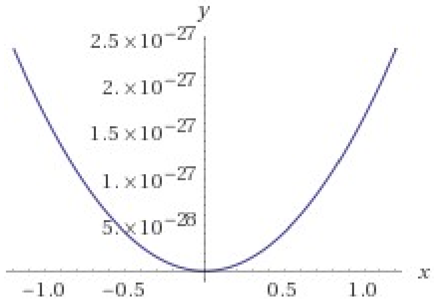
Pi * (1/(10)^28) * integrate integrate [10.5842292]

$$\pi \times \frac{1}{10^{28}} \int \left(\int 10.5842292\, dx \right) dx$$

Result:

$$1.66257 \times 10^{-27}\, x^2$$

Plot:



(x from −1.2 to 1.2)

And (1,66257) * 8 = 13,30056; (1,66257) * 4 = 6,65028 values very near to the Planck's constant and to the mass of the SMBH87.

Now:

$$l'_{9,7} = \frac{3 + 2\sqrt{7} + 4\sqrt{3} + \sqrt{21} + \sqrt{9 + 2\sqrt{21}}(2 + 3\sqrt{3} - \sqrt{7})}{4}. \qquad (4.15)$$

That is equal to: 27,37560109; we note that $(27,37560109)^{1/7}$ = 1,604492407... and (1,604492407) * 8 = 12,835939; (1,604492407) * 4 = 6,41796... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

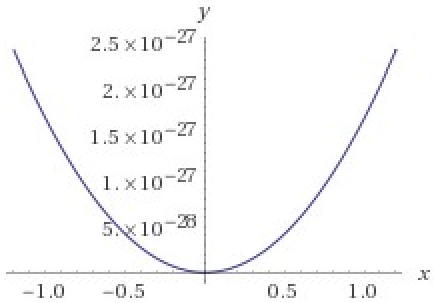
Pi^4/8 * (1/(10)^29) * integrate integrate [27.37560109]

$$\frac{\pi^4}{8} \times \frac{1}{10^{29}} \int\left(\int 27.37560109\, dx\right) dx$$

Result:

$1.66665 \times 10^{-27} x^2$

Plot:



(x from −1.2 to 1.2)

Now:

$$l'_{9,28} = 7\sqrt{7} + 11\sqrt{3} + 4\sqrt{21} + 18 + (2 + \sqrt{7})(2 + \sqrt{3})\sqrt{9 + 2\sqrt{21}}. \quad (4.21)$$

18,52025917 + 19,052558 + 18,33030 + 18 + (4,64575131)(3,7320508)(4,2620595) = 147,79947... we note that $(147,79947)^{1/10}$ = 1,64803825... and (1,64803825) * 8 = 13,184306; (1,64803825) * 4 = 6,592153... that are values very near to the reduced Planck's constant and to the mass of the SMBH87.

We calculate the following double integral:

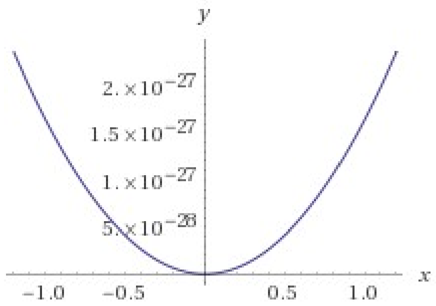
Pi^3/14 * (1/(10)^29) * integrate integrate [147.79947]

$$\frac{\pi^3}{14} \times \frac{1}{10^{29}} \int\left(\int 147.79947\, dx\right) dx$$

Result:

$1.63668 \times 10^{-27}\, x^2$

Plot:



(x from −1.2 to 1.2)

M87 is known to host a supermassive black hole (SMBH) of mass~ 6.6×10^9 solar mass. Models of the stellar velocity distribution imply a mass for the central core M ≈ 6.2 *10^9 solar mass. 22/04/2019 Astrophysics, said April 12 at a talk at MIT. "Easy check, we can see whether one or the other of these [mass measuring methods] is correct." The shadow of M87's black hole yielded a diameter of 38 billion kilometers, which let astronomers calculate a mass of 6.5 billion suns — very close to the mass suggested by the motion of stars.

Corollary 4.5. *We have*

$$l_{9,7} = \sqrt{10+2\sqrt{21}} + \sqrt{9+2\sqrt{21}}, \tag{4.33}$$

$$l_{9,1/7} = \sqrt{10+2\sqrt{21}} - \sqrt{9+2\sqrt{21}}. \tag{4.34}$$

Corollary 4.6. *We have*

$$l_{9,13} = \sqrt{\frac{77+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}}{2}} + \sqrt{\frac{75+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}}{2}}, \quad (4.39)$$

$$l_{9,1/13} = \sqrt{\frac{77+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}}{2}} - \sqrt{\frac{75+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}}{2}}. \quad (4.40)$$

We note that:

((sqrt(10+2sqrt(21))) + ((sqrt(9+2sqrt(21)))

Input:

$$\sqrt{10+2\sqrt{21}}+\sqrt{9+2\sqrt{21}}$$

Open code

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Decimal approximation:

- More digits

8.639861643078390510710112906734320169572685560382258845765...

Open code

Alternate forms:

- Step-by-step solution

$$\sqrt{9+2\sqrt{21}}+\sqrt{3}+\sqrt{7}$$

Open code

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$$\sqrt{19+4\sqrt{21}+2\sqrt{2\left(87+19\sqrt{21}\right)}}$$

Open code

$$\frac{\sqrt{9-i\sqrt{3}}+\sqrt{2}\left(\sqrt{3}+\sqrt{7}+\sqrt{\frac{1}{2}\,i\left(\sqrt{3}+-9\,i\right)}\right)}{\sqrt{2}}$$

Minimal polynomial:

$$x^8-76x^6+102x^4-76x^2+1$$

Continued fraction:

- Linear form

$$8+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{11+\cfrac{1}{8+\cfrac{1}{21+\cfrac{1}{1+\cfrac{1}{48+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}$$

((sqrt(10+2sqrt(21))) - ((sqrt(9+2sqrt(21)))

Input:

$$\sqrt{10+2\sqrt{21}}-\sqrt{9+2\sqrt{21}}$$

Open code

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Decimal approximation:

- More digits

0.115742594188545257348011283555945415733443313403402771082...

Open code

Alternate forms:

- More forms
- Step-by-step solution

$$-\sqrt{9+2\sqrt{21}}+\sqrt{3}+\sqrt{7}$$

Open code

$$\sqrt{2\left(5+\sqrt{21}\right)}-\sqrt{9+2\sqrt{21}}$$

Open code

$$\sqrt{19+4\sqrt{21}-2\sqrt{2\left(87+19\sqrt{21}\right)}}$$

Open code

Minimal polynomial:

$$x^8 - 76 x^6 + 102 x^4 - 76 x^2 + 1$$
Open code

Continued fraction:

- Linear form

$$\cfrac{1}{8+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{11+\cfrac{1}{8+\cfrac{1}{21+\cfrac{1}{1+\cfrac{1}{48+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}}$$

Note that the sum of two results is:

8,639861643 + 0,1157425941 = 8,7556042371 and

(8,7556042371) / 2 = 4,37780211855

8,639861643 – 0,1157425941 = 8,5241190489 and

(8,5241190489) / 2 = 4,26205952445

1/2 * $(8,7556042371)^2$ = 38,3303027.....

The value 4,3778 is very near to the value of mass of the dark atom ≈ 5 GeV = 4.5 * 10^{17} , while 38,33 is a good approximation to the M87's black hole diameter of 38 billion kilometres. The value 4,262059 is very near to the range of DM particle mass 4,2

[[[[[((((((((sqrt[((77+21sqrt(13)+44sqrt(3)+12sqrt(39))/2]))))))))]]]] + [[[[[((((((((sqrt[((75+21sqrt(13)+44sqrt(3)+12sqrt(39))/2]))))))))]]]]]

Input:

$$\sqrt{\frac{1}{2}\left(77+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}\right)}+\sqrt{\frac{1}{2}\left(75+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}\right)}$$

Open code

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Decimal approximation:

- More digits

24.61162176090378001010289077274321226594063424808258109346...

Open code

Alternate forms:

- More forms
- Step-by-step solution

$$\frac{1}{2}\left(\sqrt{2\left(75+44\sqrt{3}+21\sqrt{13}+12\sqrt{39}\right)}+\sqrt{2\left(77+44\sqrt{3}+21\sqrt{13}+12\sqrt{39}\right)}\right)$$

Open code

$$\frac{1}{2}\left(\left(2+\sqrt{3}\right)\left(3+\sqrt{13}\right)+\sqrt{150+88\sqrt{3}+42\sqrt{13}+24\sqrt{39}}\right)$$

Open code

$$\frac{1}{2}\left(\sqrt{2\left(7+4\sqrt{3}\right)\left(11+3\sqrt{13}\right)}+\sqrt{150+88\sqrt{3}+42\sqrt{13}+24\sqrt{39}}\right)$$

Open code

Minimal polynomial:

$$x^8-24x^7-16x^6+24x^5-18x^4+24x^3-16x^2-24x+1$$

Open code

Continued fraction:

- Linear form

$$24+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{9+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{12+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{13+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}$$

The result is:
24.61162176090378001010289077274321226594063424808258109346

This result is very near to the range of black hole entropy 24,24 – 24,78

Furthermore, we have:

32 * (24.61162176090378001010289077274321226594063424808258109346)

Input interpretation:

$32 \times 24.61162176090378001010289077274321226594063424808258109346$

Open code

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Result:

- More digits

787.5718963489209603232925047277827925101002959386425949907...

787.5718963489209603232925047277827925101002959386425949907

Continued fraction:

- Linear form

$$787+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{42+\cfrac{1}{1+\cfrac{1}{10+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{6+\cfrac{1}{2+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{7+\cfrac{1}{\ddots}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$\frac{32\,141}{40}-\frac{53\,187}{320\,\pi}+\frac{941\,\pi}{80}\approx 787.5718963489209603202942$$

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$$\sec\left(\cosh\left(\frac{1\,849\,670\,411}{828\,644\,147}\right)\right)\approx 787.5718963489209603111420$$

$$\frac{7485\,\pi\,\pi!+23\,851-6614\,\pi+5125\,\pi^2}{90\,\pi}\approx 787.5718963489209603228173$$

72 * (24.61162176090378001010289077274321226594063424808258109346) - (36+8)

Input interpretation:

$72\times 24.61162176090378001010289077274321226594063424808258109346-(36+8)$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1728.036766785072160727408135637511283147725665861945838729...

Open code

1728.036766785072160727408135637511283147725665861945838729

Continued fraction:

- Linear form

$$1728+\cfrac{1}{27+\cfrac{1}{5+\cfrac{1}{25+\cfrac{1}{1+\cfrac{1}{4+\cfrac{1}{8+\cfrac{1}{4+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{17+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$$-4524-\frac{11}{\pi}+\frac{173}{\sqrt{\pi}}-1735\sqrt{\pi}+2939\pi \approx 1728.036766785072160734490$$

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$$\frac{1}{22}\left(1441 e^{\pi}-481\pi-230\log(\pi)+3011\log(2\pi)+722\tan^{-1}(\pi)\right) \approx$$
$$1728.0367667850721607250018$$

$$\frac{7485\pi\pi!+23851-8374\pi+5125\pi^2}{40\pi} \approx 1728.0367667850721607263391$$

We note that 787,57 is very near to the rest mass of Omega meson 782.65±0.12 and 1728,036 is very near to the range of the mass of $f_0(1710)$ candidate glueball.

[[[[[((((((((sqrt[((77+21sqrt(13)+44sqrt(3)+12sqrt(39))/2]))))))))]]]] - [[[[[((((((((sqrt[((75+21sqrt(13)+44sqrt(3)+12sqrt(39))/2]))))))))]]]]]

Input:

$$\sqrt{\frac{1}{2}\left(77+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}\right)}-\sqrt{\frac{1}{2}\left(75+21\sqrt{13}+44\sqrt{3}+12\sqrt{39}\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Decimal approximation:

- More digits

0.040631211129228662564783907655191188463334639030709524571...

Open code

Alternate forms:

- More forms

- Step-by-step solution

$$\frac{1}{2}\left(\sqrt{2\left(77+44\sqrt{3}+21\sqrt{13}+12\sqrt{39}\right)}-\sqrt{2\left(75+44\sqrt{3}+21\sqrt{13}+12\sqrt{39}\right)}\right)$$

Open code

$$\frac{1}{2}\left(\left(2+\sqrt{3}\right)\left(3+\sqrt{13}\right)-\sqrt{150+88\sqrt{3}+42\sqrt{13}+24\sqrt{39}}\right)$$

Open code

$$\frac{1}{2}\left(\sqrt{2\left(7+4\sqrt{3}\right)\left(11+3\sqrt{13}\right)}-\sqrt{150+88\sqrt{3}+42\sqrt{13}+24\sqrt{39}}\right)$$

Open code

Minimal polynomial:

$$x^8-24x^7-16x^6+24x^5-18x^4+24x^3-16x^2-24x+1$$

Open code

Continued fraction:

- Linear form

$$\cfrac{1}{24+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{3+\cfrac{1}{9+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{12+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{13+\cfrac{1}{6+\cfrac{1}{10+\cfrac{1}{\ldots}}}}}}}}}}}}}}}}}}}}}}}$$

The result is:

0.040631211129228662564783907655191188463334639030709524571

1/((((((((sqrt(10+2sqrt(21))) + ((sqrt(9+2sqrt(21))))))))))^1/5

Input:

$$\frac{1}{\sqrt[5]{\sqrt{10+2\sqrt{21}}+\sqrt{9+2\sqrt{21}}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Decimal approximation:

- More digits

0.649678720896300756321777877443674502662270857429030871299...

Open code

Alternate forms:

- More forms
- Step-by-step solution

$$\left(\sqrt{9+2\sqrt{21}}+\sqrt{3}+\sqrt{7}\right)^{-1/5}$$

Open code

$$\frac{1}{2}\sqrt{1+\sqrt{21}-\sqrt{2\left(3+\sqrt{21}\right)}}$$

Open code

$$\frac{1}{\sqrt[5]{\sqrt{2\left(5+\sqrt{21}\right)}+\sqrt{9+2\sqrt{21}}}}$$

Minimal polynomial:

$$x^8-x^6-3x^4-x^2+1$$

Open code

Continued fraction:

- Linear form

$$\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{13+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\cdots}}}}}}}}}}}}}}}}}}}}}}}$$

The result is:
0.64967872089630075632177787744367450266227085742903087129 9

Or:

(((((((sqrt(10+2sqrt(21))) - ((sqrt(9+2sqrt(21)))))))))^1/5

Input:

$$\sqrt[5]{\sqrt{10+2\sqrt{21}}-\sqrt{9+2\sqrt{21}}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Decimal approximation:

- More digits

0.64967872089630075632177787744367450266227085742903087129 9...

Open code

Alternate forms:

- More forms
- Step-by-step solution

$$\sqrt[5]{-\sqrt{9+2\sqrt{21}}+\sqrt{3}+\sqrt{7}}$$

Open code

$$\frac{1}{2}\sqrt{1+\sqrt{21}-\sqrt{2\left(3+\sqrt{21}\right)}}$$

Open code

root of $x^8 - x^6 - 3x^4 - x^2 + 1$ near $x = 0.649679$

Open code

Minimal polynomial:

$x^8 - x^6 - 3x^4 - x^2 + 1$

Open code

(24.61162176090378001010289077274321226594063424808258109346)^1/(2Pi)

Input interpretation:

$$\sqrt[2\pi]{24.61162176090378001010289077274321226594063424808258109346}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.664971655506222039609151378640104829589714954098698575824...

Series representations:

- More

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = 24.611621760903780010102890772743212265940634248082581093460000 \wedge \left(\frac{1}{8\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}}\right)$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = 24.611621760903780010102890772743212265940634248082581093460000 \wedge \left(\frac{1}{4\left(-1+\sum_{k=1}^{\infty}\frac{2^k}{\binom{2k}{k}}\right)}\right)$$

Open code

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = 24.611621760903780010102890772743212265940634248082581093460000 \wedge \left(\frac{1}{2x + 4\sum_{k=1}^{\infty}\frac{\sin(kx)}{k}}\right) \text{ for } (x \in \mathbb{R} \text{ and } x > 0)$$

Open code

Integral representations:

- More

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = e^{0.800804690170939832995755937326278007467228299823501793619865 43 / \left(\int_0^\infty \frac{1}{1+t^2}\, dt\right)}$$

Open code

Enlarge Data Customize A Plaintext Interactive

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = e^{0.400402345085469916497877968663139003733614149911750896809932 72 / \left(\int_0^1 \sqrt{1-t^2}\, dt\right)}$$

Open code

$$\sqrt[2\pi]{24.611621760903780010102890772743212265940634248082581093460000} = e^{0.800804690170939832995755937326278007467228299823501793619865 43 / \left(\int_0^\infty \frac{\sin(t)}{t}\, dt\right)}$$

Open code

Possible closed forms:

- More

$$\frac{1}{6}\left(147 e^{\pi} - 997\pi + 890\log(\pi) - 677\log(2\pi) - 27\tan^{-1}(\pi)\right) \approx 1.664971655506222043167$$

Enlarge Data Customize A Plaintext Interactive

$$\frac{1237063231\pi}{2334183135} \approx 1.66497165550622203951214$$

$$\frac{-169 + 1213\pi + 80\pi^2}{3(468 + 124\pi + 3\pi^2)} \approx 1.66497165550622203988377$$

-

The result is:

1.6649716555062220396091513786401048295897149540986985758 24

Or:

1 / ((((((0.040631211129228662564783907655191188463334639030709524571)^1/(2Pi))))))

Input interpretation:

$$\frac{1}{\sqrt[2\pi]{0.040631211129228662564783907655191188463334639030709524571}}$$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

1.664971655506222039609151378640104829589714954098698 5758...

1.66497165550622203960915137864010482958971495409869857 5824 * 8

Input interpretation:

$1.664971655506222039609151378640104829589714954098698575824 \times 8$

Open code

Enlarge Data Customize A Plaintext Interactive

Result:

- More digits

13.319773244049776316873211029120838636717719632789588 60659...

Open code

Continued fraction:

- Linear form

$$13+\cfrac{1}{3+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{6+\cfrac{1}{5+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{7+\cfrac{1}{1+\cfrac{1}{3+\cfrac{1}{5+\cfrac{1}{7+\cfrac{1}{6+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{4+\cfrac{1}{13+\cfrac{1}{216+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{\dots}}}}}}}}}}}}}}}}}}}}}$$

Possible closed forms:

- More

$\sqrt{-5521+6676 e-5203 \pi+5622 \log(2)} \approx 13.31977324404977628100$

Enlarge Data Customize A Plaintext Interactive

π root of $67 x^5-192 x^4-400 x^3-74 x^2+666 x-758$ near $x=4.23982$ $\approx$

13.31977324404977631 52377

$$\frac{4894016655\,\pi}{1154299438} \approx 13.319773244049776317263 1$$

•

The range for the mass of SMBH87 is 6.5 – 6.7 * 10^9 solar masses, thence:

12,92915 – 13,12806 – 13,3269 * 10^{39}

We note that the result 13,319 is in the range of the mass of SMBH87

We have also that:

1.6649716555062220396091513786401048295897149540986985758 24 * e =

= 4.525862196061936816672713582829299539188265746766292064495.

This result is practically equal to the value of mass of the dark atom ≈ 5 GeV = 4.5 * 10^{17} .

We note that 1,66497 is is a good approximation to the value of the fourteenth root of Ramanujan's class invariant 1164.2696 and very near to the mass of the proton. Indeed:

We have the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 =$ 1164,2696

$$\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3 = 1164{,}269601267364$$

and

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\,\ldots$$

Chronology of the universe
From Wikipedia

Electroweak symmetry breaking[edit]

10^{-12} seconds after the Big Bang

As the universe's temperature continued to fall below a certain very high energy level, a third symmetry breaking occurs. So far as we currently know, it was the final symmetry breaking event in the formation of our universe. It is believed that below some energies unknown yet, the Higgs field spontaneously acquires a vacuum expectation value. When this happens, it breaks electroweak gauge symmetry. This has two related effects:

1. Via the Higgs mechanism, all elementary particles interacting with the Higgs field become massive, having been massless at higher energy levels.
2. As a side-effect, the weak force and electromagnetic force, and their respective bosons (the W and Z bosons and photon) now begin to manifest differently in the present universe. Before electroweak symmetry breaking these bosons were all massless particles and interacted over long distances, but at this point the W and Z bosons abruptly become massive particles only interacting over distances smaller than the size of an atom, while the photon remains massless and remains a long-distance interaction.

After electroweak symmetry breaking, the fundamental interactions we know of – gravitation, electromagnetism, the strong interaction and the weak interaction – have all taken their present forms, and fundamental particles have mass, but the temperature of the universe is still too high to allow the formation of many fundamental particles we now see in the universe.

The quark epoch

Between 10^{-12} seconds and 10^{-6} seconds after the Big Bang

The quark epoch began approximately 10^{-12} seconds after the Big Bang. This was the period in the evolution of the early universe immediately after electroweak symmetry breaking, when the fundamental interactions of gravitation, electromagnetism, the strong interaction and the weak interaction had taken their present forms, but the temperature of the universe was still too high to allow quarks to bind together to form hadrons.

During the quark epoch the universe was filled with a dense, hot quark–gluon plasma, containing quarks, leptons and their antiparticles. Collisions between particles were too energetic to allow quarks to combine into mesons or baryons.

The quark epoch ended when the universe was about 10^{-6} seconds old, when the average energy of particle interactions had fallen below the binding energy of hadrons.

W and Z bosons

From Wikipedia, the free encyclopedia

The **W and Z bosons** are together known as the **weak** or more generally as the intermediate vector bosons**.** These elementary particles mediate the weak interaction; the respective symbols are W^+, W^-, and Z. The W bosons have either a positive or negative electric charge of 1 elementary charge and are each other's antiparticles. The Z boson is electrically neutral and is its own antiparticle. The three particles have a spin of 1. The W bosons have a magnetic moment, but the Z has none. All three of these particles are very short-lived, with a half-life of about 3×10^{-25} s. Their experimental discovery was a triumph for what is now known as the Standard Model of particle physics.

These bosons are among the heavyweights of the elementary particles. With masses of 80.4 GeV/c^2 and 91.2 GeV/c^2, respectively, the W and Z bosons are almost 80 times as massive as the proton:

W: 80.379±0.012 GeV/c^2 Z: 91.1876±0.0021 GeV/c^2

We note that: 80,379 / 48 = 1,6745625; 80,379 / 50 = 1,60758 and 91,1876 / 55 = 1,6579563; 91.1876 / 56 = 1,62835 values very near to the mass of the proton and the electric charge of the electron.

β^- decay (electron emission)

An unstable atomic nucleus with an excess of neutrons may undergo β^- decay, where a neutron is converted into a proton, an electron, and an electron antineutrino (the antiparticle of the neutrino).

This process is mediated by the weak interaction. The neutron turns into a proton through the emission of a virtual W^- boson. At the quark level, W^- emission turns a down quark into an up quark, turning a neutron (one up quark and two down quarks) into a proton (two up quarks and one down quark). The virtual W^- boson then decays into an electron and an antineutrino.

β^+ decay (positron emission)

Unstable atomic nuclei with an excess of protons may undergo β^+ decay, also called positron decay, where a proton is converted into a neutron, a positron, and an electron neutrino.

Beta Decay

β-decay, radioactive decay of an atomic nucleus accompanied by the escape of an electron or positron from the nucleus. This process is caused by a spontaneous transformation of one of the nucleons in the nucleus into a nucleon of another type—specifically, a transformation either of a neutron (n) into a proton (p) or of a proton into a neutron. In the former case, with an electron (e^+) escaping from the nucleus, so-called β^-decay takes place. In the latter case, with a positron (e^+) escaping from the nucleus, β^+ -decay takes place. The electrons and positrons emitted in beta decay are termed beta particles. The mutual transformations of the nucleons are accompanied by the appearance of still another particle—the neutrino (v) in the case of β^+ -decay, the antineutrino (v) in the case of β^- -decay. In β^- -decay, the number of protons (Z) in the nucleus increases by a unit and the number of neutrons decreases by a unit. The mass number *A* of the nucleus—equal to the total number of nucleons present in the nucleus—does not vary, and the product nucleus is an isobar of the original nucleus, standing on the right of the latter in the periodic table of the elements. Conversely, the number of protons in β^+ -decay decreases by a unit and the number of neutrons increases by a unit, so that an isobar standing to the left of the original nucleus is formed. The two beta decay processes are written symbolically as

$$^{A}_{Z}X \rightarrow {}_{Z+1}^{A}X + e^- + \bar{\nu}$$

$$^{A}_{Z}X \rightarrow {}_{Z-1}^{A}X + e^+ + \nu$$

where $^{A}_{Z}X$ is the symbol of the nucleus, consisting of Z protons and *A* - *Z* neutrons.

The simplest example of β^- -decay is the transformation of a free neutron into a proton with the emission of an electron and an antineutrino (neutron half-life ≈ 13 min)

$$^{1}_{0}n \xrightarrow{\beta^-} {}^{1}_{1}p + e^- + \bar{\nu}$$

Higgs Boson

The Higgs boson is an elementary, massive and scalar boson that plays a fundamental role within the standard model. It was theorized in 1964 and detected for the first time in 2012 in the ATLAS and CMS experiments, conducted with the LHC accelerator of CERN. Its importance is to be the particle associated with the Higgs field, which according to the theory permeates the universe by giving the mass to elementary particles.

Since the Higgs field is scalar, the Higgs boson has no spin. The Higgs boson is also its own antiparticle and is CP-even, and has zero electric and colour charge.

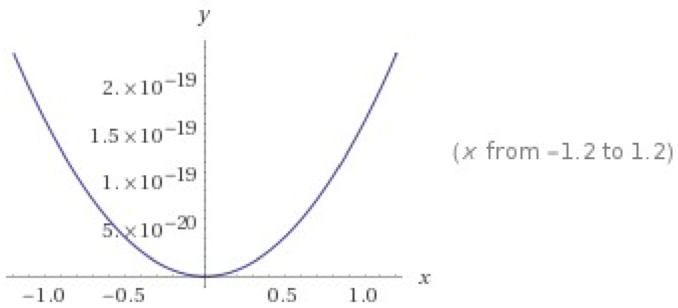
The Standard Model does not predict the mass of the Higgs boson. If that mass is between 115 and 180 GeV/c^2 (consistent with empirical observations of 125 GeV/c^2), then the Standard Model can be valid at energy scales all the way up to the Planck scale (10^{19} GeV).

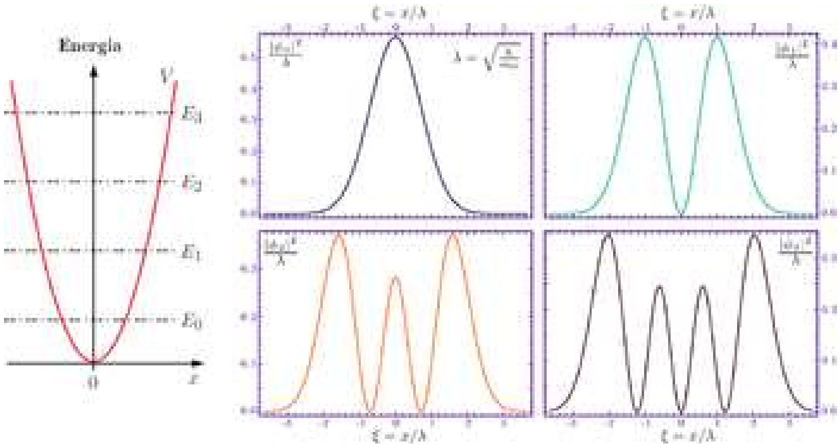
We note that 1,602176 * 78 = 124,969728 and 1,672621 * 74 = 123,773954 where 1,602176 and 1,672621 are the electric charge of the positron and the mass of the proton respectively.

Furthermore, we have that:

$125{,}09 * 9 * 10^{16} = 11258100000000000000$; $(11258100000000000000)^{1/93} =$
$= 1{,}6027082167167$

Now, we want to analyze the parabola plots concerning the results of the various double integrals. Indeed, for all values i.e. for the electric charge of positron, the mass of the proton and the mass of the Higgs boson, the plot is always a parabola of this type:





From Wikipedia:

Quantum harmonic oscillator

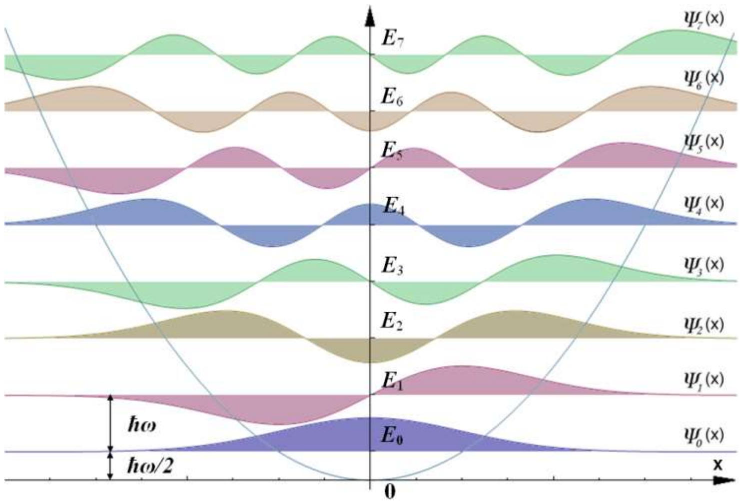


Fig.: Wavefunction representations for the first eight bound eigenstates, n = 0 to 7. The horizontal axis shows the position x. Note: The graphs are not normalized, and the signs of some of the functions differ from those given in the text.

This suggests that the graphs representing the parabolas associated with the particle-like solutions of the integrals performed could mean that electrons / positrons, protons / neutrons and massive bosons, such as Higgs, are open strings. It is important to underline that from the above mentioned graphs, it is highlighted that these strings are in a sort of "ground state". Subsequently, due to the quantum fluctuations of the false vacuum, these strings pass from the "ground state" to a dynamic state, in which they begin to vibrate and behave like waves, as happens for the quantum harmonic oscillator.
Thus, this could mean that the static parabola represented in the graphs is the corpuscular nature of the electron, the proton and the Higgs boson, while the graph, again of the parabola type, of the harmonic oscillator, their undulatory nature (dualism wave-particle)

One-dimensional harmonic oscillator

Hamiltonian and energy eigenstates

The Hamiltonian of the particle is:

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2}k\hat{x}^2 = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega^2\hat{x}^2 \, ,$$

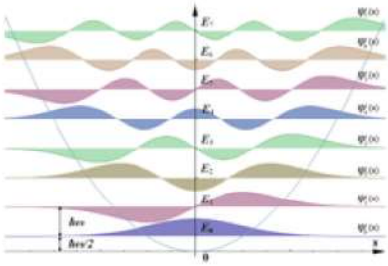
where m is the particle's mass, k is the force constant, $\omega = \sqrt{\frac{k}{m}}$ is the angular frequency of the oscillator, $\hat{x}$ is the position operator (given by x), and $\hat{p}$ is the momentum operator (given by $\hat{p} = -i\hbar\frac{\partial}{\partial x}$). The first term in the Hamiltonian represents the kinetic energy of the particle, and the second term represents its potential energy, as in Hooke's law.

One may write the time-independent Schrödinger equation,

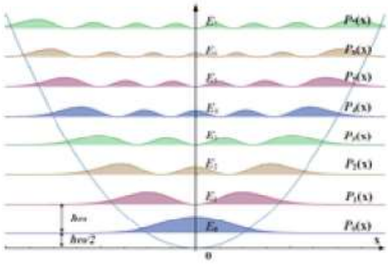
$$\hat{H}|\psi\rangle = E|\psi\rangle\ ,$$

where E denotes a to-be-determined real number that will specify a time-independent energy level, or eigenvalue, and the solution $|\psi\rangle$ denotes that level's energy eigenstate.

One may solve the differential equation representing this eigenvalue problem in the coordinate basis, for the wave function $\langle x|\psi\rangle = \psi(x)$, using a spectral method. It turns out that there is a family of solutions. In this basis, they amount to Hermite functions,



Wavefunction representations for the first eight bound eigenstates, $n = 0$ to 7. The horizontal axis shows the position x. Note: The graphs are not normalized, and the signs of some of the functions differ from those given in the text.



Corresponding probability densities.

$$\psi_n(x) = \frac{1}{\sqrt{2^n\, n!}} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right), \qquad n = 0, 1, 2, \ldots.$$

The functions H_n are the physicists' Hermite polynomials,

$$H_n(z) = (-1)^n\, e^{z^2} \frac{d^n}{dz^n}\left(e^{-z^2}\right).$$

The corresponding energy levels are

$$E_n = \hbar\omega\left(n + \frac{1}{2}\right) = (2n+1)\frac{\hbar}{2}\omega\,.$$

This energy spectrum is noteworthy for three reasons. First, the energies are quantized, meaning that only discrete energy values (integer-plus-half multiples of $\hbar\omega$) are possible; this is a general feature of quantum-mechanical systems when a particle is confined. Second, these discrete energy levels are equally spaced, unlike in the Bohr model of the atom, or the particle in a box. Third, the lowest achievable energy (the energy of the $n = 0$ state, called the ground state) is not equal to the minimum of the potential well, but $\hbar\omega/2$ above it; this is called zero-point energy. Because of the zero-point energy, the position and momentum of the oscillator in the ground state are not fixed (as they would be in a classical oscillator), but have a small range of variance, in accordance with the Heisenberg uncertainty principle.

The ground state probability density is concentrated at the origin, which means the particle spends most of its time at the bottom of the potential well, as one would expect for a state with little energy. As the energy increases, the probability density peaks at the classical "turning points", where the state's energy coincides with the potential energy. (See the discussion below of the highly excited states.) This is consistent with the classical harmonic oscillator, in which the particle spends more of its time (and is therefore more likely to be found) near the turning points, where it is moving the slowest. The correspondence principleis thus satisfied. Moreover, special nondispersive wave packets, with minimum uncertainty, called coherent states oscillate very much like classical objects, as illustrated in the figure; they are*not* eigenstates of the Hamiltonian.

From:
http://www.umich.edu/~chem461/Ex5.pdf

1. For a classical harmonic oscillator, the particle can not go beyond the points where the total energy equals the potential energy. Identify these points for a quantum-mechanical harmonic oscillator in its ground state. Write an integral giving the probability that the particle will go beyond these classically-allowed points. (You need not evaluate the integral.)

2. Evaluate the average (expectation) values of potential energy and kinetic energy for the ground state of the harmonic oscillator. Comment on the relative magnitude of these two quantities.

3. Apply the Heisenberg uncertainty principle to the ground state of the harmonic oscillator. Applying the formula for expectation values, calculate

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} \qquad \text{and} \qquad \Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2}$$

and find the product $\Delta x \Delta p$.

1. The turning points for quantum number occur where the kinetic energy equals 0, so that the potential energy equals the total energy. For quantum number n, this is determined by

$$\frac{1}{2}kx_{\max}^2 = \left(n + \frac{1}{2}\right)\hbar\omega$$

recalling that $\omega = \sqrt{k/m}$ and $\alpha = \sqrt{mk}/\hbar$, we find

$$x_{\max}^2 = (2n+1)\frac{\hbar}{\sqrt{km}} = \frac{(2n+1)}{\alpha}$$

Therefore

$$P(x_{\max} \leq x \leq \infty) = P(-\infty \leq x \leq -x_{\max}) = \int_{x_{\max}}^{\infty} |\psi_n(x)|^2\, dx$$

[Optional: For $n = 0$,

$$P_{\text{outside}} = 2\int_{1/\sqrt{\alpha}}^{\infty} \left(\frac{\alpha}{\pi}\right)^{1/2} e^{-\alpha x^2}\, dx = \frac{2}{\sqrt{\pi}}\int_{1}^{\infty} e^{-\xi^2}\, d\xi = \mathrm{erfc}(1) \approx 0.158$$

where erfc is the complementary error function. This result means that in the ground state, there is a 16% chance that the oscillator will "tunnel" outside its classical allowed region.]

2.

$$\psi_0(x) = (\alpha/\pi)^{1/4} e^{-\alpha x^2/2}, \qquad \alpha = (mk/\hbar^2)^{1/2}$$

Using integrals in Supplement 5,

$$\langle V \rangle = \int_{-\infty}^{\infty} \psi_0(x)\left(\frac{1}{2}kx^2\right)\psi_0(x)\, dx = \frac{k}{4\alpha} = \frac{1}{4}\hbar\omega = \frac{1}{2}E_0$$

$$\langle T \rangle = \int_{-\infty}^{\infty} \psi_0(x) \left(-\frac{\hbar^2}{2m}\right) \psi_0''(x)\, dx = \frac{1}{2} E_0$$

Thus the average values of potential and kinetic energies for the harmonic oscillator are equal. This is an instance of the virial theorem, which states that for a potential energy of the form $V(x) = \text{const}\, x^n$, the average kinetic and potential energies are related by

$$\langle T \rangle = \frac{n}{2} \langle V \rangle$$

3. The expectation values $\langle x \rangle$ and $\langle p \rangle$ are both equal to zero since they are integrals of odd functions, such that $f(-x) = -f(x)$, over a symmetric range of integration. You have already calculated the expectation values $\langle x^2 \rangle$ and $\langle p^2 \rangle$ in Exercise 2, namely

$$\langle x^2 \rangle = \frac{1}{2\alpha} \qquad \text{and} \qquad \langle p^2 \rangle = \frac{\hbar^2 \alpha}{2}$$

Therefore

$$\Delta x \Delta p = \frac{\hbar}{2}$$

which is its minimum possible value.

We know that the reduced Planck's constant is:

$\hbar = 1{,}054\,571\,726(47) \times 10^{-34}\ \mathrm{J \cdot s} = 6{,}582\,119\,28(15) \times 10^{-16}\ \mathrm{eV \cdot s}$

Thence, we have that the minimum possible value is:

$$\Delta x \Delta p = \frac{\hbar}{2} = \frac{6{,}582119 \times 10^{-16}}{2} = 3{,}2910595 \times 10^{-16}$$

Now, we calculate the following double integrals:

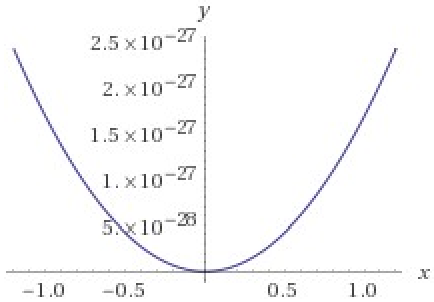
(2*1.618*Pi) * 1/(10^12) integrate integrate [0.00000000000000032910595]

$$(2 \times 1.618\, \pi) \times \frac{1}{10^{12}} \int \left(\int 3.2910595 \times 10^{-16}\, dx \right) dx$$

Result:

$1.67288 \times 10^{-27}\, x^2$

Plot:



(x from -1.2 to 1.2)

Or:

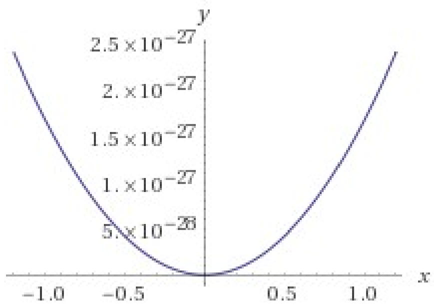
1.08643 * (8Pi/27) * 1/(10^11) integrate integrate [0.00000000000000032910595]

$$1.08643\left(8\times\frac{\pi}{27}\right)\times\frac{1}{10^{11}}\int\left(\int 3.2910595\times10^{-16}\,dx\right)dx$$

Result:

$$1.66412\times10^{-27}\,x^2$$

Plot:



(x from -1.2 to 1.2)

Or:

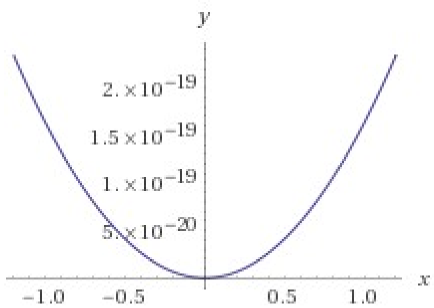
(Pi^2) * 1/(10^4) integrate integrate [0.00000000000000032910595]

$$\pi^2\times\frac{1}{10^4}\int\left(\int 3.2910595\times10^{-16}\,dx\right)dx$$

Result:

$$1.62407\times10^{-19}\,x^2$$

Plot:



(x from -1.2 to 1.2)

And

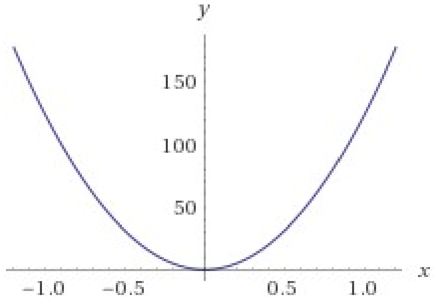
1.08643 (7*Pi^2) (10^16) integrate integrate [0.00000000000000032910595]

$$1.08643\,(7\pi^2)\times 10^{16}\int\left(\int 3.2910595\times 10^{-16}\,dx\right)dx$$

Result:

$$123.511\,x^2$$

Plot:



(x from −1.2 to 1.2)

Value very near to the mass of the Higgs boson, while the energy from the E = mc^2, considering the value 125,09 is 11,2581 * 10^{18} . Note that (11,2581 * 10^{18})$^{1/89}$ = 1,63704797... value very near to the mass of the proton.

We now calculate the following double integral:

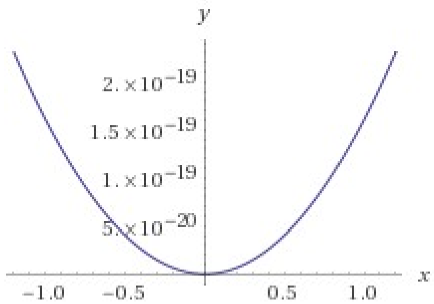
1.08643 (Pi/12) * 1/(10^37) * integrate integrate [11.2581*10^18]

$$1.08643\times\frac{\pi}{12}\times\frac{1}{10^{37}}\int\left(\int 11.2581\times 10^{18}\,dx\right)dx$$

Result:

$$1.60105\times 10^{-19}\,x^2$$

Plot:



(x from −1.2 to 1.2)

Open code

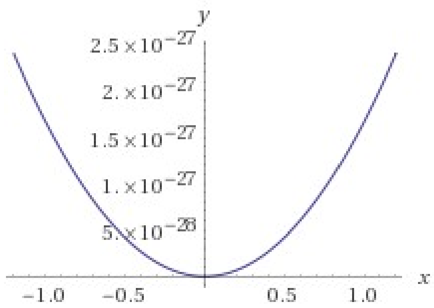
1.08643 (Pi^5/(3*37)) * 1/(10^46) * integrate integrate [11115990000000000000]

That is the value of the energy obtained from 123,511 (123,511 * 9 * 10^{16} =

= 11115990000000000000)

$$1.08643\times\frac{\pi^5}{3\times 37}\times\frac{1}{10^{46}}\int\left(\int 11\,115\,990\,000\,000\,000\,000\,dx\right)dx$$

Result:
$1.66474\times 10^{-27}\,x^2$



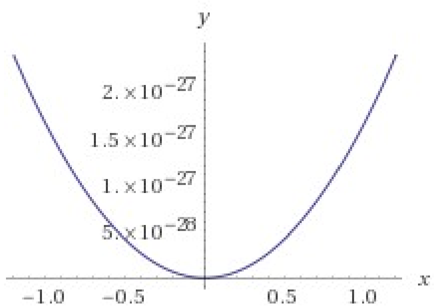
(x from –1.2 to 1.2)

Or:

1.08643 * (e) * 1/(10^46) * integrate integrate [11115990000000000000]

$$1.08643\, e \times \frac{1}{10^{46}} \int\left(\int 11\,115\,990\,000\,000\,000\,000\, dx\right) dx$$

Result:
$1.6414\times 10^{-27}\,x^2$



(x from –1.2 to 1.2)

Composite Twin Dark Matter - John Terning , Christopher B. Verhaaren , and Kyle Zora **-** arXiv:1902.08211v2

If the recombination of these particles into twin atoms is not sufficiently efficient then the DM remains primarily a plasma, which can develop instabilities that affect galaxy collisions, like the Bullet Cluster [2]. This translates into a bound on the twin fine structure constant α' as a function of m_D [67]:

$$\frac{\alpha'^4}{\xi}\left(\frac{\Omega_D h^2}{0.11}\right)\left(\frac{\text{GeV}}{m_D}\right)^2\left[\frac{(1+R)^2}{R}-\frac{1}{2}\alpha'^2\right]^2 \gtrsim 7.5\times 10^{\ 11}, \tag{4.4}$$

where $\Omega_D h^2$ is the relic density of dark matter and ξ is the ratio of the present day temperature of the dark radiation to the CMB temperature

$$\xi - \left(\frac{T_D}{T_{\text{CMB}}}\right)\bigg|_{z-0}. \tag{4.5}$$

Recall from Fig. 1 that larger values of $m_{t'}$ also lead to larger $m_{\Delta'}$. In addition, it is clear that if $\lambda_{b'} > \lambda_b$ then even larger values of $m_{t'}$ and $m_{\Delta'}$ would be required to agree with experiment. Thus, direct detection and naturalness (preferring lighter $m_{t'}$) push us toward twin bottom Yukawas that are smaller than the SM value. This, in turn, reduces $m_{\Delta'}$, pushing it toward the naive ADM expectation of $\sim$5 GeV.

asymmetric dark matter (ADM)

The mass of the dark atom m_D

In short, twin atoms can make up an interesting ADM population. To have m_D values closest to 5 GeV, the τ' mass should be close to $m_{\Delta'}$, so that $R \sim 1$. These lightest mass atoms also require the α' coupling be somewhat stronger than in the visible sector. In addition, the velocity dependence of the self-interaction of these twin atoms agrees with self-interaction estimates better than DM with a velocity independent self-interaction cross section.

Thence, we have the following value: ≈ 5 GeV = $4.5 * 10^{17}$

Observations

The Universe, according to the most accredited theories, would have been born of a singularity (Big Bang), even the rotating black holes of Kerr, like every black hole, are singularities. From the study of them it is therefore possible to understand the origin of the universe.
In fact the black hole of the M87 type, which is an asymmetric ring singularity, can be compared to the Hilbert space, which we foresee in our cosmological model, which is in fact also a singularity, represented as a toroid of infinite dimensions. As it is logical to deduce such geometric entities we must represent at a level lower than the Planck scale, as we are describing a period inherent to the universe before the Big Bang, then in a phase prior to the clash of the strings / branes. At that time, therefore, there was only one point of infinite density and energy which, at a later time, with a well-defined collapse of wave function, gave way to the formal phase in which a pair of particles (massive bosons) it annihilates (collision between branes), giving rise to the particles of matter and energy (electrons, protons, neutrons and photons), therefore to the explosion defined Big Bang. The "no-boundary proposal" by Hartle-Hawking appears more and more likely, in which the universe simply "is", where therefore the Big Bang is nothing but a phase of a cycle, eternal in time and infinite in space. S. Ramanujan was right when he said that "an equation makes no sense if it does not express a thought of God". The Universe "is", just as God called himself "I Am", this affirmation, which reinforces a pantheistic vision of a Universe as a manifestation-thought of an immanent and transcendent Cosmic Intelligence. (Antonio Nardelli)

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References

The references relating to the papers consulted for the development of the formulas presented here are present during the discussion of the topics