# On the Ramanujan's equations applied to various sectors of Particle Physics and Cosmology: new possible mathematical connections with the values of Pion mesons and other baryons and mesons. 

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#### Abstract

In this research thesis, we have analyzed further Ramanujan formulas and described new possible mathematical connections with some sectors of Particle Physics (values of Pion mesons and other baryons and mesons) and Cosmology


[^0]
https://www.britannica.com/biography/Srinivasa-Ramanujan

https://biografieonline.it/foto-enrico-fermi

## Summary

In this research thesis, we have analyzed further Ramanujan formulas and described new mathematical connections with some sectors of Particle Physics and Cosmology. We have described, as in previous papers, the possible and new connections between different formulas of Ramanujan's mathematics and some formulas concerning particle physics and cosmology. In the course of the discussion we describe and highlight the connections between some developments of Ramanujan equations and particles type solutions such as the mass of the Higgs boson and the masses of proton (or neutron), and other baryons and mesons. Principally solutions of Ramanujan equations, connected with the masses of the $\pi$ mesons ( 139.57 and 134.9766 MeV ) have been described and highlighted.

Further, we have described the connections between the mathematics of Ramanujan and different equations concerning some areas of theoretical physics and cosmology (see part "Replica Wormholes and the Entropy of Hawking Radiation"). Is our opinion, that the possible connections between the mathematical developments of some Rogers-Ramanujan continued fractions, various mathematical Ramanujan's expressions, the value of the dilaton and that of "the dilaton mass calculated as a type of Higgs boson that is equal about to $125 \mathrm{GeV}^{\prime}$ ", the Higgs boson mass itself and the masses of the $\boldsymbol{\pi}$ mesons (139.57 and 134.9766 MeV ) are fundamental.

All the results of the most important connections are highlighted in blue throughout the drafting of the paper

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For $\mathrm{x}=0.24$, we obtain:
$1 / 0.24^{\wedge} 2-3 /\left(1+0.24^{\wedge} 2\right)+5 /\left(3+0.24^{\wedge} 2\right)-7 /\left(6+0.24^{\wedge} 2\right)+9 /\left(10+0.24^{\wedge} 2\right)$
Input:
$\frac{1}{0.24^{2}}-\frac{3}{1+0.24^{2}}+\frac{5}{3+0.24^{2}}-\frac{7}{6+0.24^{2}}+\frac{9}{10+0.24^{2}}$

## Result:

15.89904193290744865691890961750664151215726023917181323420...
15.8990419329...

For $\mathrm{x}=1 / 12=0.083$, we obtain:

$$
1 / 0.083^{\wedge} 2-3 /\left(1+0.083^{\wedge} 2\right)+5 /\left(3+0.083^{\wedge} 2\right)-7 /\left(6+0.083^{\wedge} 2\right)+9 /\left(10+0.083^{\wedge} 2\right)
$$

## Input:

$\frac{1}{0.083^{2}}-\frac{3}{1+0.083^{2}}+\frac{5}{3+0.083^{2}}-\frac{7}{6+0.083^{2}}+\frac{9}{10+0.083^{2}}$

## Result:

143.5763746029481662180096635360300782826003184852433469694...
143.576374602948...

$$
1 / 0.083^{\wedge} 2-3 /\left(1+0.083^{\wedge} 2\right)+5 /\left(3+0.083^{\wedge} 2\right)-7 /\left(6+0.083^{\wedge} 2\right)+9 /\left(10+0.083^{\wedge} 2\right)-4
$$

Where 4 is a Lucas number and the dimensions of a D4-brane

## Input:

$\frac{1}{0.083^{2}}-\frac{3}{1+0.083^{2}}+\frac{5}{3+0.083^{2}}-\frac{7}{6+0.083^{2}}+\frac{9}{10+0.083^{2}}-4$

## Result:

139.5763746029481662180096635360300782826003184852433469694.
139.576374602948... result practically equal to the rest mass of Pion meson 139.57

## when $C=\frac{1}{2}+\frac{1}{3}+\sqrt{2}-\frac{1}{5}+\sqrt{22}+\frac{1}{7}+\sqrt{43}$

$1 / 2+1 /(3+$ sqrt 8$)-1 /(5+\operatorname{sqrt} 24)+1 /(7+$ sqrt 48$)=\mathrm{C}$

## Input:

$\frac{1}{2}+\frac{1}{3+\sqrt{8}}-\frac{1}{5+\sqrt{24}}+\frac{1}{7+\sqrt{48}}$

## Result:

$\frac{1}{2}+\frac{1}{3+2 \sqrt{2}}+\frac{1}{7+4 \sqrt{3}}-\frac{1}{5+2 \sqrt{6}}$

## Decimal approximation:

0.642349130544656924681405334968897159021330195317921598288
0.64234913054...

## Alternate forms:

$\frac{1}{2}(11-4 \sqrt{2}-8 \sqrt{3}+4 \sqrt{6})$
$\frac{11}{2}-2 \sqrt{2}-4 \sqrt{3}+2 \sqrt{6}$
$\frac{1}{2}(11-8 \sqrt{3}+8 \sqrt{2-\sqrt{3}})$

## Minimal polynomial:

$16 x^{4}-352 x^{3}+344 x^{2}+5224 x-3407$


For $\mathrm{x}=0.083$, we obtain:

$$
1 /\left(2 \mathrm{Pi}^{*} 0.083\right)+\left(\mathrm{Pi}^{*} 0.083\right) / 6-0.64234913054
$$

## Input interpretation:

$$
\frac{1}{2 \pi \times 0.083}+\frac{\pi \times 0.083}{6}-0.64234913054
$$

## Result:

1.31864...
1.31864...

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
& -0.642349130540000+\frac{14.94^{\circ}}{6}+\frac{1}{29.88^{\circ}} \\
& \frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
& -0.642349130540000-\frac{1}{6} \times 0.083 i \log (-1)+-\frac{1}{0.166 i \log (-1)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
& -0.642349130540000+\frac{1}{6} \times 0.083 \cos ^{-1}(-1)+\frac{1}{0.166 \cos ^{-1}(-1)}
\end{aligned}
$$

## Series representations:

$$
\left.\begin{array}{l}
\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
\frac{0.0553333\left(-8.34863+\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)\left(-3.26009+\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}\right)}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{1+2 k}} \\
\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
0.0276667\left(-17.6973+\sum_{k=1}^{\infty}\left(\frac{2^{k}}{\binom{k}{k}}\right)\left(-7.52019+\sum_{k=1}^{\infty} \frac{2^{k}}{2 k}\right)\right. \\
\left.-1+\sum_{k=1}^{\infty} \frac{2^{k}}{(2 k}\right) \\
k
\end{array}\right) .
$$

## Integral representations:

$$
\begin{aligned}
& \frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
& \frac{0.0276667\left(-16.6973+\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)\left(-6.52019+\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t} \\
& \frac{\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000=}{0.0276667\left(-16.6973+\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)\left(-6.52019+\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)} \\
& \int_{0}^{\infty} \frac{\sin (t)}{t} d t
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000= \\
& \frac{0.0553333\left(-8.34863+\int_{0}^{1} \sqrt{1-t^{2}} d t\right)\left(-3.26009+\int_{0}^{1} \sqrt{1-t^{2}} d t\right)}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

$\left.\left(\left(\left(\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 0.083\right)+\left(\mathrm{Pi}^{*} 0.083\right) / 6-0.64234913054\right)\right)\right)^{\wedge} 2+\mathrm{sqrt} 2\right)\right)\right)\right)^{*} 521 / 10^{\wedge} 3$
Where 521 is a Lucas number. Note that $521=496+25$, where 496 is the dimension of Lie's Group $\mathrm{E}_{8} \mathrm{X}_{8}$ and 25 corresponding to the dimensions of a D- 25 brane

## Input interpretation:

$$
\left(\left(\frac{1}{2 \pi \times 0.083}+\frac{\pi \times 0.083}{6}-0.64234913054\right)^{2}+\sqrt{2}\right) \times \frac{521}{10^{3}}
$$

## Result:

1.642724660893565725916220256844860859141606394336521851856...
$1.64272466 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Series representations:

$$
\begin{aligned}
& \frac{\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521}{0.301804+\frac{18.907}{\pi^{2}}-\frac{10^{3}}{4.0321}}= \\
& \frac{521 \sqrt{z_{0}}-0.00925903 \pi+0.0000996991 \pi^{2}+}{\sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k!}} \text { for not }\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right) \\
& \frac{\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521}{1000}= \\
& 0.301804+\frac{18.907}{\pi^{2}}-\frac{10^{3}}{4.0321} \\
& \pi \\
& 0.0 .00925903 \pi+0.0000996991 \pi^{2}+ \\
& \frac{521 \exp \left(i \pi\left\lfloor\frac{\arg (2-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!}}{1000} \text { for }(x \in \mathbb{R} \text { and } x<0)
\end{aligned}
$$

$$
\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521
$$

$$
0.301804+\frac{18.907}{\pi^{2}}-\frac{10^{3}}{\pi}-0.00925903 \pi+0.0000996991 \pi^{2}+
$$

$$
\frac{521\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(2-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1 / 2\left(1+\left\lfloor\arg \left(2-z_{0}\right) /(2 \pi)\right\rfloor\right)} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}^{\left(2-z_{0}\right)^{k} z_{0}^{-k}}}{k!}}{1000}
$$

$1 / 10^{\wedge} 27^{*}\left(\left(()\left(\left(()\left(1 /\left(2 \mathrm{Pi}^{*} 0.083\right)+\left(\mathrm{Pi}^{*} 0.083\right) / 6-\right.\right.\right.\right.\right.$
$0.64234913054)))^{\wedge} 2+$ sqrt2 $\left.\left.)\right)\right)^{\left.\left.\left.* 521 / 10^{\wedge} 3+29 / 10^{\wedge} 3\right)\right)\right) ~}$

## Input interpretation:

$\frac{1}{10^{27}}\left(\left(\left(\frac{1}{2 \pi \times 0.083}+\frac{\pi \times 0.083}{6}-0.64234913054\right)^{2}+\sqrt{2}\right) \times \frac{521}{10^{3}}+\frac{29}{10^{3}}\right)$

Where 521 and 29 are Lucas numbers. Note that $521=496+25$, where 496 is the dimension of Lie's Group $\mathrm{E}_{8} \mathrm{X}_{8}$ and 25 corresponding to the dimensions of a D-25 brane

## Result:

$1.67172 \ldots \times 10^{-27}$
$1.67172 \ldots * 10^{-27} \mathrm{~kg}$
result practically equal to the value of the formula:
$m_{p^{\prime}}=2 \times \frac{\eta}{R} m_{P}=1.6714213 \times 10^{-24} \mathrm{gm}$
that is the holographic proton mass (N. Haramein)

## Series representations:

$\frac{\frac{\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521}{10^{3}}+\frac{29}{10^{3}}}{10^{27}}=$ $3.30804 \times 10^{-28}+\frac{1.8907 \times 10^{-26}}{\pi^{2}}-\frac{4.0321 \times 10^{-27}}{\pi}-9.25903 \times 10^{-30} \pi+$ $9.96991 \times 10^{-32} \pi^{2}+5.21 \times 10^{-28} \sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(2-z_{0}\right)^{k} z_{0}^{-k}}{k!}$ for $\operatorname{not}\left(\left(z_{0} \in \mathbb{R}\right.\right.$ and $\left.\left.-\infty<z_{0} \leq 0\right)\right)$

$$
\frac{\frac{\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521}{10^{3}}+\frac{29}{10^{3}}}{10^{27}}=
$$

$$
3.30804 \times 10^{-28}+\frac{1.8907 \times 10^{-26}}{\pi^{2}}-\frac{4.0321 \times 10^{-27}}{\pi}-9.25903 \times 10^{-30} \pi+
$$

$$
9.96991 \times 10^{-32} \pi^{2}+5.21 \times 10^{-28} \exp \left(i \pi\left\lfloor\frac{\arg (2-x)}{2 \pi}\right\rfloor\right)
$$

$$
\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}(2-x)^{k} x^{-k}\left(-\frac{1}{2}\right)_{k}}{k!} \text { for }(x \in \mathbb{R} \text { and } x<0)
$$

$$
\frac{\frac{\left(\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)^{2}+\sqrt{2}\right) 521}{10^{3}}+\frac{29}{10^{3}}}{10^{27}}=3.30804 \times 10^{-28}+
$$

$$
\frac{1.8907 \times 10^{-26}}{\pi^{2}}-\frac{4.0321 \times 10^{-27}}{\pi}-9.25903 \times 10^{-30} \pi+9.96991 \times 10^{-32} \pi^{2}+
$$

$$
521\left(\frac{1}{z_{0}}\right)^{1 / 2\left\lfloor\arg \left(2-z_{0}\right) /(2 \pi)\right\rfloor} z_{0}^{1 / 2\left(1+\left\lfloor\arg \left(2-z_{0}\right) /(2 \pi)\right\rfloor\right)} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}^{\left(2-z_{0}\right)^{k} z_{0}^{-k}}}{k!}
$$

$$
10^{\wedge} 2\left(\left(\left(1 /\left(2 \mathrm{Pi}^{*} 0.083\right)+\left(\mathrm{Pi}^{*} 0.083\right) / 6-0.64234913054\right)\right)\right)+\mathrm{Pi}
$$

Where 10 is the number of dimensions in superstring theory

## Input interpretation:

$$
10^{2}\left(\frac{1}{2 \pi \times 0.083}+\frac{\pi \times 0.083}{6}-0.64234913054\right)+\pi
$$

## Result:

135.005
$135.005 \ldots$. result very near to the rest mass of Pion meson 139.57

## Alternative representations:

$$
\begin{aligned}
& 10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi= \\
& 180^{\circ}+10^{2}\left(-0.642349130540000+\frac{14.94^{\circ}}{6}+\frac{1}{29.88^{\circ}}\right) \\
& 10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi= \\
& -i \log (-1)+10^{2}\left(-0.642349130540000-\frac{1}{6} \times 0.083 i \log (-1)+-\frac{1}{0.166 i \log (-1)}\right) \\
& 10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi= \\
& \cos ^{-1}(-1)+10^{2}\left(-0.642349130540000+\frac{1}{6} \times 0.083 \cos ^{-1}(-1)+\frac{1}{0.166 \cos ^{-1}(-1)}\right)
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& 10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi= \\
& \frac{4.76667\left(63.1898-13.4759 \int_{0}^{\infty} \frac{1}{1+t^{2}} d t+\left(\int_{0}^{\infty} \frac{1}{1+t^{2}} d t\right)^{2}\right)}{\int_{0}^{\infty} \frac{1}{1+t^{2}} d t} \\
& \frac{10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi=}{4.76667\left(63.1898-13.4759 \int_{0}^{\infty} \frac{\sin (t)}{t} d t+\left(\int_{0}^{\infty} \frac{\sin (t)}{t} d t\right)^{2}\right)} \\
& \int_{0}^{\infty} \frac{\sin (t)}{t} d t
\end{aligned}
$$

$$
\begin{aligned}
& 10^{2}\left(\frac{1}{2 \pi 0.083}+\frac{\pi 0.083}{6}-0.642349130540000\right)+\pi= \\
& \frac{9.53333\left(15.7975-6.73793 \int_{0}^{1} \sqrt{1-t^{2}} d t+\left(\int_{0}^{1} \sqrt{1-t^{2}} d t\right)^{2}\right)}{\int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

For $\mathrm{x}=2$, from the following expression, considering the symbol \& , we obtain:
$1 / 2^{\wedge} 2-3 /\left(1+2^{\wedge} 2\right)+5 /\left(3+2^{\wedge} 2\right)-7 /\left(6+2^{\wedge} 2\right)+9 /\left(10+2^{\wedge} 2\right)+\ldots$

## Input interpretation:

$\frac{1}{2^{2}}-\frac{3}{1+2^{2}}+\frac{5}{3+2^{2}}-\frac{7}{6+2^{2}}+\frac{9}{10+2^{2}}+\cdots$

## Infinite sum:

$$
\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2 n-1)}{\frac{1}{2}(n-1) n+4}=2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)
$$

## Decimal approximation:

0.001999617057621813260053366854580340306777114236479440058...
0.0019996170576...

## Convergence tests:

By the alternating series test, the series converges.

## Partial sum formula:

$$
\begin{aligned}
& \sum_{n=1}^{m} \frac{(-1)^{1+n}(-1+2 n)}{4+\frac{1}{2}(-1+n) n}=2\left((-1)^{m+1} \Phi\left(-1,1, m+\frac{1}{2}(-1-i \sqrt{31})+1\right)+\right. \\
& (-1)^{m+1} \Phi\left(-1,1, m+\frac{1}{2}(-1+i \sqrt{31})+1\right)+ \\
& \left.\Phi\left(-1,1,1+\frac{1}{2}(-1+i \sqrt{31})\right)+\Phi\left(-1,1,1+\frac{1}{2}(-1-i \sqrt{31})\right)\right)
\end{aligned}
$$

## Partial sums:


$1 /\left(\left(\left(\left(1+2^{\wedge} 2 / 1\right)\left(1+2^{\wedge} 2 / 3\right)\left(1+2^{\wedge} 2 / 6\right)\left(1+2^{\wedge} 2 / 10\right)^{*} . ..\right)\right)\right)$

## Input interpretation:

$\frac{1}{\left(1+2^{2}\right)\left(1+\frac{2^{2}}{3}\right)\left(1+\frac{2^{2}}{6}\right)\left(\left(1+\frac{2^{2}}{10}\right) \times \cdots\right)}$

## Result: <br> $\frac{9}{175 \times \prod_{n=1}^{\infty} \frac{7}{5}}$

$1 /\left(\left(\left(\left(1+2^{\wedge} 2 / 1\right)\left(1+2^{\wedge} 2 / 3\right)\left(1+2^{\wedge} 2 / 6\right)\left(1+2^{\wedge} 2 / 10\right)^{*} \ldots\right)\right)\right)=1 / 2^{\wedge} 2-3 /\left(1+2^{\wedge} 2\right)+5 /\left(3+2^{\wedge} 2\right)-$ $7 /\left(6+2^{\wedge} 2\right)+9 /\left(10+2^{\wedge} 2\right)-\ldots$

## Input interpretation:

$\frac{1}{\left(1+2^{2}\right)\left(1+\frac{2^{2}}{3}\right)\left(1+\frac{2^{2}}{6}\right)\left(\left(1+\frac{2^{2}}{10}\right) \times \cdots\right)}=\frac{1}{2^{2}}-\frac{3}{1+2^{2}}+\frac{5}{3+2^{2}}-\frac{7}{6+2^{2}}+\frac{9}{10+2^{2}}-\cdots$

## Result:

$\frac{9}{175 \times \prod_{n=1}^{\infty} \frac{7}{5}}=2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)$

## Input:

$2 \pi \operatorname{sech}\left(\frac{1}{2}(\sqrt{31} \pi)\right)$

## Exact result:

$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)$

## Decimal approximation:

0.001999617057621813260053366854580340306777114236479440058...
$0.0019996170576 \ldots$

## Alternate forms:

$\frac{2 \pi}{\cosh \left(\frac{\sqrt{31} \pi}{2}\right)}$
$4 \pi \cosh \left(\frac{\sqrt{31} \pi}{2}\right)$
$1+\cosh (\sqrt{31} \pi)$
$\frac{4 \pi}{e^{-(\sqrt{31} \pi) / 2}+e^{(\sqrt{31} \pi) / 2}}$
$\cosh (x)$ is the hyperbolic cosine function

## Alternative representations:

$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=\frac{2 \pi}{\cosh \left(\frac{\pi \sqrt{31}}{2}\right)}$
$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=2 \pi \csc \left(\frac{\pi}{2}+\frac{1}{2} i \pi \sqrt{31}\right)$
$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=\frac{2 \pi}{\cos \left(\frac{1}{2} i \pi \sqrt{31}\right)}$

## Series representations:

$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=2 \sum_{k=0}^{\infty} \frac{(-1)^{k}(1+2 k)}{8+k+k^{2}}$
$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=-4 \pi \sum_{k=1}^{\infty}(-1)^{k} q^{-1+2 k}$ for $q=e^{(\sqrt{31} \pi) / 2}$
$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=4 e^{-(\sqrt{31} \pi) / 2} \pi \sum_{k=0}^{\infty}(-1)^{k} e^{-\sqrt{31} k \pi}$

## Integral representation:

$2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)=4 \int_{0}^{\infty} \frac{t^{i \sqrt{31}}}{1+t^{2}} d t$
$(1 / 4) * 1 /(((2 \pi \operatorname{sech}((\operatorname{sqrt}(31) \pi) / 2))))+11+3+1 /$ golden ratio
Where 11 and 3 are Lucas numbers

## Input:

$\frac{1}{4} \times \frac{1}{2 \pi \operatorname{sech}\left(\frac{1}{2}(\sqrt{31} \pi)\right)}+11+3+\frac{1}{\phi}$
$\operatorname{sech}(x)$ is the hyperbolic secant function
$\phi$ is the golden ratio

## Exact result:

$\frac{1}{\phi}+14+\frac{\cosh \left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$

## Decimal approximation:

139.6419724709162115699630652093636492431933614860570506324...
$139.64197247 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$\frac{1}{2}(27+\sqrt{5})+\frac{\cosh \left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$
$14+\frac{2}{1+\sqrt{5}}+\frac{\cosh \left(\frac{\sqrt{31} \pi}{2}\right)}{8 \pi}$
$\frac{1}{\phi}+14+\frac{e^{-(\sqrt{31} \pi) / 2}}{16 \pi}+\frac{e^{(\sqrt{31} \pi) / 2}}{16 \pi}$

## Alternative representations:

$\frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{1}{\frac{4(2 \pi)}{\cosh \left(\frac{\pi \sqrt{31}}{2}\right)}}$

$$
\begin{aligned}
& \frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{1}{\frac{4(2 \pi)}{\cos \left(\frac{1}{2} i \pi \sqrt{31}\right)}} \\
& \frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{1}{4\left(2 \pi \csc \left(\frac{\pi}{2}+\frac{1}{2} i \pi \sqrt{31}\right)\right)}
\end{aligned}
$$

## Series representations:

$\frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{\sum_{k=0}^{\infty} \frac{\left(\frac{31}{4} k^{2} \pi^{2 k}\right.}{(2 k)!}}{8 \pi}$
$\frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+\sqrt{31}) \pi\right)^{1+2 k}}{(1+2 k)!}}{8 \pi}$
$\frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{\sum_{k=0}^{\infty} I_{2 k}\left(\frac{1}{2}\right) T_{2 k}(\sqrt{31} \pi)\left(2-\delta_{k}\right)}{8 \pi}$

## Integral representations:

$$
\begin{aligned}
& \frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{1}{\phi}+\frac{1}{8 \pi} \int_{\frac{i \pi}{2}}^{\frac{\sqrt{31}}{2} \pi} \sinh (t) d t \\
& \frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}=14+\frac{2}{1+\sqrt{5}}+\frac{1}{8 \pi}+\frac{\sqrt{31}}{16} \int_{0}^{1} \sinh \left(\frac{1}{2} \sqrt{31} \pi t\right) d t \\
& \frac{1}{\left(2 \pi \operatorname{sech}\left(\frac{\sqrt{31} \pi}{2}\right)\right) 4}+11+3+\frac{1}{\phi}= \\
& 14+\frac{2}{1+\sqrt{5}}-\frac{i}{16 \pi^{3 / 2}} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{\left(31 \pi^{2}\right) /(16 s)+s}}{\sqrt{s}} d s \text { for } \gamma>0
\end{aligned}
$$

$\left(\left(\left(1 / 2^{\wedge} 2-3 /\left(1+2^{\wedge} 2\right)+5 /\left(3+2^{\wedge} 2\right)-7 /\left(6+2^{\wedge} 2\right)+9 /\left(10+2^{\wedge} 2\right)+\ldots\right)\right)\right) /(((2 \pi \operatorname{sech}((\operatorname{sqrt}(31)$ $\pi) / 2)$ ))

## Input interpretation:

$$
\frac{\frac{1}{2^{2}}-\frac{3}{1+2^{2}}+\frac{5}{3+2^{2}}-\frac{7}{6+2^{2}}+\frac{0}{10+2^{2}}+\cdots}{2 \pi \operatorname{sech}\left(\frac{1}{2}(\sqrt{31} \pi)\right)}
$$

## Result:

1
1 result that can be interpreted as the photon spin

We have that:

## $=1-\frac{\pi}{7}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}$

For $\mathrm{x}=1 / 12=0.083$, we obtain:
$1-\mathrm{Pi} / 2+1 /\left(6(3+\mathrm{sqrt} 8)^{\wedge} 2\right)-1 /\left(10(5+\mathrm{sqrt24})^{\wedge} 2\right)+1 /\left(14(7+\mathrm{sqrt48})^{\wedge} 2\right)$

## Input:

$1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}$

## Result:

$1+\frac{1}{6(3+2 \sqrt{2})^{2}}+\frac{1}{14(7+4 \sqrt{3})^{2}}-\frac{1}{10(5+2 \sqrt{6})^{2}}-\frac{\pi}{2}$

## Decimal approximation:

$-0.56654243434547778978801159476609237831534974246486940743$
-0.566542434.....

## Property:

$1+\frac{1}{6(3+2 \sqrt{2})^{2}}+\frac{1}{14(7+4 \sqrt{3})^{2}}-\frac{1}{10(5+2 \sqrt{6})^{2}}-\frac{\pi}{2}$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& \frac{1}{210}(1231-420 \sqrt{2}-840 \sqrt{3}+420 \sqrt{6}-105 \pi) \\
& \frac{1231}{210}-2 \sqrt{2}-4 \sqrt{3}+2 \sqrt{6}-\frac{\pi}{2} \\
& \frac{1}{210}(1231-840 \sqrt{3}+840 \sqrt{2-\sqrt{3}})-\frac{\pi}{2}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}= \\
& 1-\frac{1}{2}+\frac{1}{6\left(3+\sqrt{7} \sum_{k=0}^{\infty} 7^{-k}\binom{\frac{1}{2}}{k}\right)^{2}}- \\
& \frac{1}{10\left(5+\sqrt{23} \sum_{k=0}^{\infty} 23^{-k}\left(\frac{1}{2}\right)\right)^{2}}+\frac{1}{14\left(7+\sqrt{47} \sum_{k=0}^{\infty} 47^{-k}\binom{\frac{1}{2}}{k}\right)^{2}} \\
& 1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}= \\
& 1-\frac{\pi}{2}+\frac{1}{6\left(3+\sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}- \\
& \frac{1}{10\left(5+\sqrt{23} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{23}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}+\frac{1}{14\left(7+\sqrt{47} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{47}\right)\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}
\end{aligned}
$$

$$
\begin{aligned}
1-\frac{\pi}{2}+ & \frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}= \\
1 & -\frac{1}{2}+\frac{1}{6\left(3+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(8-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2}}- \\
& \frac{1}{10\left(5+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(24-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2}}+ \\
& \frac{1}{14\left(7+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(48-z_{0}\right)^{k} z_{0}^{k}}{k!}\right)^{2}} \text { for not }\left(\left(z_{0} \in \mathbb{R} \text { and }-\infty<z_{0} \leq 0\right)\right)
\end{aligned}
$$

```
\(-1 /\left(\left(\left(1-\mathrm{Pi} / 2+1 /\left(6(3+\mathrm{sqrt} 8)^{\wedge} 2\right)-\right.\right.\right.\)
\(\left.\left.\left.1 /\left(10(5+\text { sqrt } 24)^{\wedge} 2\right)+1 /\left(14(7+\text { sqrt48 })^{\wedge} 2\right)\right)\right)\right)^{*} 76+1 /\) golden ratio
```

Where 76 is a Lucas number

## Input:

$-\frac{76}{1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}}+\frac{1}{\phi}$

## Result:

$\frac{1}{\phi}-\frac{76}{1+\frac{1}{6(3+2 \sqrt{2})^{2}}+\frac{1}{14(7+4 \sqrt{3})^{2}}-\frac{1}{10(5+2 \sqrt{6})^{2}}-\frac{\pi}{2}}$

## Decimal approximation:

134.7650905773745897871094135928998181319929018953260271255...
$134.765090577 \ldots$ result practically equal to the rest mass of Pion meson 134.976

## Property:

$\frac{1}{\phi}-\frac{76}{1+\frac{1}{6(3+2 \sqrt{2})^{2}}+\frac{1}{14(7+4 \sqrt{3})^{2}}-\frac{1}{10(5+2 \sqrt{6})^{2}}-\frac{\pi}{2}}$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& (33151-420 \sqrt{2}-840 \sqrt{3}-1231 \sqrt{5}+420 \sqrt{6}+ \\
& 420 \sqrt{10}+840 \sqrt{15}-420 \sqrt{30}-105 \pi+105 \sqrt{5} \pi) / \\
& (2(-1231+420 \sqrt{2}+840 \sqrt{3}-420 \sqrt{6}+105 \pi)) \\
& \frac{1}{\phi}-\frac{76}{\frac{1231}{210}-2 \sqrt{2}-4 \sqrt{3}+2 \sqrt{6}-\frac{\pi}{2}} \\
& \frac{1}{\phi}+\frac{76}{-\frac{1231}{210}+2 \sqrt{2}+4 \sqrt{3}-2 \sqrt{6}+\frac{\pi}{2}}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{76(-1)}{1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}}+\frac{1}{\phi}= \\
& \left.\frac{1}{\phi}-\frac{76}{\left.1-\frac{\pi}{2}+\frac{1}{6\left(3+\sqrt{7} \sum_{k=0}^{\infty} 7^{-k}\left(\frac{1}{2}\right)\right)^{2}}-\frac{1}{10\left(5+\sqrt{23} \sum_{k=0}^{\infty} 23^{-k}\left(\frac{1}{2}\right.\right.} \begin{array}{l}
k
\end{array}\right)^{2}}+\frac{14\left(7+\sqrt{47} \sum_{k=0}^{\infty} 47^{-k}\left(\frac{1}{2}\right)\right)^{2}}{k}\right)^{2} \\
& \frac{76(-1)}{1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}}+\frac{1}{\phi}= \\
& \frac{1}{\phi}-76 /\left(1-\frac{\pi}{2}+\frac{1}{6\left(3+\sqrt{7} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{7}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}-\right. \\
& \frac{1}{10\left(5+\sqrt{23} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{23}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}\right)^{2}}+\frac{14\left(7+\sqrt{47} \sum_{k=0}^{\infty} \frac{\left.\left(-\frac{1}{47}\right)^{k}\left(-\frac{1}{2}\right)_{k}\right)^{2}}{k!}\right)}{1}
\end{aligned}
$$

$$
\left.\begin{array}{l}
\frac{76(-1)}{1-\frac{\pi}{2}+\frac{1}{6(3+\sqrt{8})^{2}}-\frac{1}{10(5+\sqrt{24})^{2}}+\frac{1}{14(7+\sqrt{48})^{2}}}+\frac{1}{\phi}= \\
\left.\frac{\frac{1}{\phi}-76 /\left(1-\frac{\pi}{2}+\frac{1}{6\left(3+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(8-z_{0}\right)^{k} z_{0}^{-k}}{k!}\right)^{2}}-\right.}{10\left(5+\sqrt{z_{0}} \sum_{k=0}^{\infty} \frac{1}{(-1)^{k}\left(-\frac{1}{2}\right)_{k}\left(24-z_{0}\right)^{k} z_{0}^{-k}}\right.} k\right)^{2}
\end{array}\right] .
$$

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For $\mathrm{x}=2$, we obtain:
$\mathrm{Pi} /(4 \mathrm{sqrt} 3) *\left(\left(\left(\sinh \left(2 \mathrm{Pi}^{*} \mathrm{sqrt} 3\right)-\mathrm{sqrt} 3 * \sin (2 \mathrm{Pi})\right)\right)\right) /(((\cosh (2 \mathrm{Pi} * \operatorname{sqrt} 3)-\cos (2 \mathrm{Pi}))))$

## Input:

$\frac{\pi}{4 \sqrt{3}} \times \frac{\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)}{\cosh (2 \pi \sqrt{3})-\cos (2 \pi)}$

## Exact result:

$\frac{\pi \sinh (2 \sqrt{3} \pi)}{4 \sqrt{3}(\cosh (2 \sqrt{3} \pi)-1)}$

## Decimal approximation:

$0.453466871624258724623634815745739322304887984526058956146 \ldots$
$0.4534668716242587 \ldots$

## Alternate forms:

$\frac{\pi \operatorname{coth}(\sqrt{3} \pi)}{4 \sqrt{3}}$
$\frac{\left(e^{2 \sqrt{3} \pi}-e^{-2 \sqrt{3} \pi}\right) \pi}{8 \sqrt{3}\left(\frac{1}{2}\left(e^{-2 \sqrt{3} \pi}+e^{2 \sqrt{3}} \pi\right)-1\right)}$

## Alternative representations:

$\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\frac{\left(-e^{-2 i \pi}+e^{2 i \pi}\right) \sqrt{3}}{2 i}\right)}{\left(-\cosh (-2 i \pi)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}$
$\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)+\cos \left(\frac{5 \pi}{2}\right) \sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}$
$\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\frac{\left(-e^{-2 i \pi}+e^{2 i \pi}\right) \sqrt{3}}{2 i}\right)}{\left(\cos (-2 i \pi \sqrt{3})+\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)\right)(4 \sqrt{3})}$

## Series representations:

$\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{i \pi \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{2 k}}{(2 k)!}}{4 \sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}$

$$
\begin{aligned}
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{4 \sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)} \\
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi \sum_{k=0}^{\infty} \frac{3^{1 / 2(1+2 k)}(2 \pi)^{1+2 k}}{(1+2 k)!}}{4 \sqrt{3}\left(-1+i \sum_{k=0}^{\infty} \frac{\left(-\frac{i \pi}{2}+2 \sqrt{3} \pi\right)^{1+2 k}}{(1+2 k)!}\right)}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi \int_{0}^{1} \cosh (2 \sqrt{3} \pi t) d t}{4 \sqrt{3} \int_{0}^{1} \sinh (2 \sqrt{3} \pi t) d t} \\
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi^{2} \int_{0}^{1} \cosh (2 \sqrt{3} \pi t) d t}{2\left(-1+\int_{\frac{i \pi}{2}}^{2 \sqrt{3} \pi} \sinh (t) d t\right)} \\
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=-\frac{i \sqrt{\frac{\pi}{3}} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{\left(3 \pi^{2}\right) / s+s}}{s^{3 / 2}} d s}{16 \int_{0}^{1} \sinh (2 \sqrt{3} \pi t) d t} \text { for } \gamma>0
\end{aligned}
$$

## Multiple-argument formulas:

$$
\begin{aligned}
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi \operatorname{coth}(\sqrt{3} \pi)}{4 \sqrt{3}} \\
& \frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi)(4 \sqrt{3})}=\frac{\pi \cosh (\sqrt{3} \pi) \sinh (\sqrt{3} \pi)}{2 \sqrt{3}\left(-2+2 \cosh ^{2}(\sqrt{3} \pi)\right)}
\end{aligned}
$$

$$
\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}=\frac{\pi \operatorname{csch}^{2}(\sqrt{3} \pi)\left(3 \sinh \left(\frac{2 \pi}{\sqrt{3}}\right)+4 \sinh ^{3}\left(\frac{2 \pi}{\sqrt{3}}\right)\right)}{8 \sqrt{3}}
$$

$\left(\left(\left(\exp \left(\left(\left(\mathrm{Pi} /(4 \mathrm{sqrt} 3) *\left(\left(\left(\sinh \left(2 \mathrm{Pi}^{*} \mathrm{sqrt} 3\right)-\mathrm{sqrt} 3 * \sin (2 \mathrm{Pi})\right)\right)\right) /(((\cosh (2 \mathrm{Pi} *\right.\right.\right.\right.\right.\right.$ sqrt3)$)-$ $\cos (2 \mathrm{Pi}))))))))))^{\wedge} 16-29+1 /$ golden ratio

Where 29 is a Lucas number and 16 is the difference between 26 and 10 , where in bosonic string theory, spacetime is 26 -dimensional, while in superstring theory it is 10-dimensional

## Input:

$$
\exp ^{16}\left(\frac{\pi}{4 \sqrt{3}} \times \frac{\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)}{\cosh (2 \pi \sqrt{3})-\cos (2 \pi)}\right)-29+\frac{1}{\phi}
$$

$\sinh (x)$ is the hyperbolic sine function
$\cosh (x)$ is the hyperbolic cosine function

## Exact result:

$\frac{1}{\phi}-29+e^{\frac{4 \pi \sinh (2 \sqrt{3} \pi)}{\sqrt{3}(\cosh (2 \sqrt{3} \pi)-1)}}$

## Decimal approximation:

1387.446243586492327751485699773040770815595398403027547115...
1387.4462435.... result practically equal to the rest mass of Sigma baryon 1387.2

## Alternate forms:

$\frac{1}{\phi}-29+e^{(4 \pi \operatorname{coth}(\sqrt{3} \pi)) / \sqrt{3}}$
$\frac{1}{2}(\sqrt{5}-59)+e^{\frac{4 \pi \sinh (2 \sqrt{3} \pi)}{\sqrt{3}(\cosh (2 \sqrt{3} \pi)-1)}}$
$-29 \phi+e^{\frac{4\left(1+e^{2 \sqrt{3} \pi}\right) \pi}{\sqrt{3}\left(e^{2 \sqrt{3} \pi-1}\right)}} \phi+1$

## Alternative representations:

$$
\begin{aligned}
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\frac{\left(-e^{-2 i \pi}+e^{2 i \pi}\right) \sqrt{3}}{2 i}\right)}{\left(-\cosh (-2 i \pi)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right) \\
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)+\cos \left(\frac{5 \pi}{2}\right) \sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right) \\
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\cos \left(-\frac{3 \pi}{2}\right) \sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -27-29 \sqrt{5}+\exp \left(\frac{\left.4 i \pi \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{2 k}}{\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}\right)+\sqrt{5} \exp \left(\frac{4 i \pi \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{2 k}}{(2 k)!}}{\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}\right)}{}\right)
\end{aligned}
$$

$$
1+\sqrt{5}
$$

$$
\begin{aligned}
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& \frac{-27-29 \sqrt{5}+\exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}\right)+\sqrt{5} \exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}\right)}{1+\sqrt{5}}
\end{aligned}
$$

$$
\begin{gathered}
\exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi)(4 \sqrt{3})}\right)-29+\frac{1}{\phi}=\frac{1}{1+\sqrt{5}} \\
\left(-27-29 \sqrt{5}+\exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{1+2 k}}{(1+2 k)!}\right)}\right)+\right. \\
\left.\sqrt{5} \exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{1+2 k}}{(1+2 k)!}\right.}\right)\right)
\end{gathered}
$$

## Multiple-argument formulas:

$$
\begin{aligned}
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}=-29+e^{(4 \pi \operatorname{coth}(\sqrt{3} \pi)) / \sqrt{3}}+\frac{1}{\phi} \\
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -29+\exp \left(\frac{2 \pi \operatorname{csch}^{2}(\sqrt{3} \pi)\left(3 \sinh \left(\frac{2 \pi}{\sqrt{3}}\right)+4 \sinh ^{3}\left(\frac{2 \pi}{\sqrt{3}}\right)\right)}{\sqrt{3}}\right)+\frac{1}{\phi} \\
& \exp ^{16}\left(\frac{(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)) \pi}{(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))(4 \sqrt{3})}\right)-29+\frac{1}{\phi}= \\
& -29+\exp \left(\frac{8 \pi \cosh (\sqrt{3} \pi) \sinh (\sqrt{3} \pi)}{\sqrt{3}\left(-2+2 \cosh ^{2}(\sqrt{3} \pi)\right)}\right)+\frac{1}{\phi}
\end{aligned}
$$

$1 / 10\left(\left(\left(\exp \left(\left(\left(\mathrm{Pi} /(4 \mathrm{sqrt} 3) *\left(\left(\left(\sinh \left(2 \mathrm{Pi}^{*} \mathrm{sqrt} 3\right)-\mathrm{sqrt} 3 * \sin (2 \mathrm{Pi})\right)\right)\right) /\left(\left(\left(\cosh \left(2 \mathrm{Pi}^{*} \mathrm{sqrt} 3\right)-\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\cos (2 \mathrm{Pi})))))))$ )) ) ${ }^{\wedge} 16-2$

Where 10 is the number of dimensions in superstring theory. In bosonic string theory, spacetime is 26 -dimensional, while in superstring theory it is 10 -dimensional, and in M-theory it is 11-dimensional. Note that $26-10=16$

## Input:

$\frac{1}{10} \exp ^{16}\left(\frac{\pi}{4 \sqrt{3}} \times \frac{\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi)}{\cosh (2 \pi \sqrt{3})-\cos (2 \pi)}\right)-2$
$\sinh (x)$ is the hyperbolic sine function
$\cosh (x)$ is the hyperbolic cosine function

## Exact result:

$\frac{1}{10} e^{\frac{4 \pi \sinh (2 \sqrt{3} \pi)}{\sqrt{3}(\cosh (2 \sqrt{3} \pi)-1)}}-2$

## Decimal approximation:

139.5828209597742432903281112938675132697875089223221784253...
$139.5828209 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$\frac{1}{10} e^{(4 \pi \operatorname{coth}(\sqrt{3} \pi)) / \sqrt{3}}-2$
$\frac{1}{10}\left(e^{\frac{4 \pi \sinh (2 \sqrt{3} \pi)}{\sqrt{3}(\cosh (2 \sqrt{3} \pi)-1)}}-20\right)$
$\frac{1}{10}\left(e^{\left.\frac{4\left(1+e^{2 \sqrt{3}} \pi\right.}{\sqrt{3}\left(e^{2 \sqrt{3} \pi}-1\right.}\right)}-20\right)$

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& -2+\frac{1}{10} \exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\frac{\left(-e^{-2 i \pi}+e^{2 i \pi}\right) \sqrt{3}}{2 i}\right)}{\left(-\cosh (-2 i \pi)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& -2+\frac{1}{10} \exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)+\cos \left(\frac{5 \pi}{2}\right) \sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right) \\
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& -2+\frac{1}{10} \exp ^{16}\left(\frac{\pi\left(\frac{1}{2}\left(-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)-\cos \left(-\frac{3 \pi}{2}\right) \sqrt{3}\right)}{\left(\frac{1}{2}\left(-e^{-2 i \pi}-e^{2 i \pi}\right)+\frac{1}{2}\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right)\right)(4 \sqrt{3})}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& \frac{1}{10}\left(-20+\exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right.}\right)\right)
\end{aligned}
$$

$$
\frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2=
$$

$$
\frac{1}{10}\left(-20+\exp \left(\frac{4 \pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{\sqrt{3}\left(-1+i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{1+2 k}}{(1+2 k)!}\right.}\right)\right)
$$

$$
\frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2=
$$

$$
\frac{1}{10}\left(-20+\exp \left(\frac{4 \pi^{5 / 2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-s} \pi^{-2 s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}}\right)\right)
$$

## Multiple-argument formulas:

$$
\begin{aligned}
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2=-2+\frac{1}{10} e^{(4 \pi \operatorname{coth}(\sqrt{3} \pi)) / \sqrt{3}} \\
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& -2+\frac{1}{10} \exp \left(\frac{8 \pi \cosh (\sqrt{3} \pi) \sinh (\sqrt{3} \pi)}{\sqrt{3}\left(-2+2 \cosh ^{2}(\sqrt{3} \pi)\right)}\right) \\
& \frac{1}{10} \exp ^{16}\left(\frac{\pi(\sinh (2 \pi \sqrt{3})-\sqrt{3} \sin (2 \pi))}{(4 \sqrt{3})(\cosh (2 \pi \sqrt{3})-\cos (2 \pi))}\right)-2= \\
& -2+\frac{1}{10} \exp \left(-\frac{4 i \pi \operatorname{csch}^{2}(\sqrt{3} \pi) \prod_{k=0}^{1} \sinh \left(\left(\sqrt{3}+\frac{i k}{2}\right) \pi\right)}{\sqrt{3}}\right)
\end{aligned}
$$

Or:
$1 /\left(1^{\wedge} 2+2^{\wedge} 2+2^{\wedge} 4 / 1^{\wedge} 2\right)+1 /\left(2^{\wedge} 2+2^{\wedge} 2+2^{\wedge} 4 / 2^{\wedge} 2\right)+1 /\left(3^{\wedge} 2+2^{\wedge} 2+2^{\wedge} 4 / 3^{\wedge} 2\right)+\ldots$

## Input interpretation:

$$
\frac{1}{1^{2}+2^{2}+\frac{2^{4}}{1^{2}}}+\frac{1}{2^{2}+2^{2}+\frac{2^{4}}{2^{2}}}+\frac{1}{3^{2}+2^{2}+\frac{2^{4}}{3^{2}}}+\cdots
$$

## Infinite sum:

$$
\sum_{n=1}^{\infty} \frac{1}{n^{2}+\frac{16}{n^{2}}+4}=-\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}
$$

## Decimal approximation:

$0.453466871624258724623634815745739322304887984526058956146 \ldots$
$0.4534668716242587 \ldots$.

## Convergence tests:

The ratio test is inconclusive.
The root test is inconclusive.
By the comparison test, the series converges.

## Partial sum formula:

$$
\begin{aligned}
& \sum_{n=1}^{m} \frac{1}{4+\frac{16}{n^{2}}+n^{2}}= \\
& \left(i \left(i m^{4}-\sqrt{3} m^{4} \psi^{(0)}(m-i \sqrt{3})+\sqrt{3} m^{4} \psi^{(0)}(m+i \sqrt{3})-\sqrt{3} m^{4} \psi^{(0)}(i \sqrt{3})+\right.\right. \\
& \\
& \quad \sqrt{3} m^{4} \psi^{(0)}(-i \sqrt{3})+8 i m^{3}-2 \sqrt{3} m^{3} \psi^{(0)}(m-i \sqrt{3})+ \\
& \quad 2 \sqrt{3} m^{3} \psi^{(0)}(m+i \sqrt{3})-2 \sqrt{3} m^{3} \psi^{(0)}(i \sqrt{3})+2 \sqrt{3} m^{3} \psi^{(0)}(-i \sqrt{3})+ \\
& \\
& \quad 10 i m^{2}-7 \sqrt{3} m^{2} \psi^{(0)}(m-i \sqrt{3})+7 \sqrt{3} m^{2} \psi^{(0)}(m+i \sqrt{3})- \\
& \\
& \\
& 7 \sqrt{3} m^{2} \psi^{(0)}(i \sqrt{3})+7 \sqrt{3} m^{2} \psi^{(0)}(-i \sqrt{3})+21 i m- \\
& \\
& 6 \sqrt{3} m \psi^{(0)}(m-i \sqrt{3})+6 \sqrt{3} m \psi^{(0)}(m+i \sqrt{3})-6 \sqrt{3} m \psi^{(0)}(i \sqrt{3})+ \\
& \\
& \\
& 6 \sqrt{3} m \psi^{(0)}(-i \sqrt{3})-12 \sqrt{3} \psi^{(0)}(m-i \sqrt{3})+12 \sqrt{3} \psi^{(0)}(m+i \sqrt{3})- \\
& \\
& \\
& \left.\left.12 \sqrt{3} \psi^{(0)}(i \sqrt{3})+12 \sqrt{3} \psi^{(0)}(-i \sqrt{3})\right)\right) /\left(12\left(m^{2}+3\right)\left(m^{2}+2 m+4\right)\right)
\end{aligned}
$$

## Partial sums:



## Alternate forms:

$\frac{\pi \operatorname{coth}(\sqrt{3} \pi)}{4 \sqrt{3}}$

$$
-\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}
$$

$\frac{\left(e^{-\sqrt{3} \pi}+e^{\sqrt{3} \pi}\right) \pi}{\sqrt{3}(\sqrt{3}+-i)(\sqrt{3}+i)\left(e^{\sqrt{3} \pi}-e^{-\sqrt{3} \pi}\right)}$

## Series representations:

$$
\begin{aligned}
& -\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}=\frac{\pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{4 \sqrt{3}\left(-1+\sum_{k=0}^{\infty} \frac{12^{k} \pi^{2 k}}{(2 k)!}\right)} \\
& -\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}= \\
& \frac{\pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{4 \sqrt{3}\left(-1+i \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}(-i+4 \sqrt{3}) \pi\right)^{1+2 k}}{(1+2 k)!}\right)} \\
& -\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}=\frac{\pi^{5 / 2} \sum_{j=0}^{\infty} \operatorname{Res}_{s=-j} \frac{(-3)^{-s} \pi^{-2 s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}}{4\left(-1+\sum_{k=0}^{\infty} \frac{\left.\frac{12^{k} \pi^{2 k}}{(2 k)!}\right)}{4}\right.} \\
& -\frac{\pi \sinh (2 \sqrt{3} \pi)}{16 \sqrt{3}(1-\cosh (2 \sqrt{3} \pi))}-\frac{\sqrt{3} \pi \sinh (2 \sqrt{3} \pi)}{16(1-\cosh (2 \sqrt{3} \pi))}= \\
& \frac{\pi \sum_{k=0}^{\infty} \frac{3^{1 / 2+k}(2 \pi)^{1+2 k}}{(1+2 k)!}}{4 \sqrt{3}\left(-1+\sqrt{\pi} \sum_{j=0}^{\infty} \operatorname{Res}_{s-j} \frac{(-3)^{-s} \pi^{-2 s} \Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\right)}
\end{aligned}
$$



For $\mathrm{n}=2$, we obtain:

$$
1 /\left(12^{*} 2^{\wedge} 2\right)+1 / 2^{*}\left(1 /\left(1^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+1 /\left(2^{\wedge} 2+3 * 2^{\wedge} 2\right)+1 /\left(3^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+\ldots\right)
$$

## Input interpretation:

$\frac{1}{12 \times 2^{2}}+\frac{1}{2}\left(\frac{1}{1^{2}+3 \times 2^{2}}+\frac{1}{2^{2}+3 \times 2^{2}}+\frac{1}{3^{2}+3 \times 2^{2}}+\cdots\right)$

## Result:

$\frac{1}{48}+\frac{1}{48}(2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi)-1)$

## Alternate forms:

$$
\begin{aligned}
& \frac{\pi \operatorname{coth}(2 \sqrt{3} \pi)}{8 \sqrt{3}} \\
& -\frac{\pi \sinh (4 \sqrt{3} \pi)}{8 \sqrt{3}(1-\cosh (4 \sqrt{3} \pi))} \\
& \frac{\pi \tanh (\sqrt{3} \pi)}{16 \sqrt{3}}+\frac{\pi \operatorname{coth}(\sqrt{3} \pi)}{16 \sqrt{3}}
\end{aligned}
$$

$1 / 48+1 / 48(-1+2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi))$

## Input:

$$
\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))
$$

## Decimal approximation:

0.226724920689178751345059994437316352094407237779531520754
0.22672492...

## Alternate forms:

$$
\begin{aligned}
& \frac{\pi \operatorname{coth}(2 \sqrt{3} \pi)}{8 \sqrt{3}} \\
& -\frac{\pi \sinh (4 \sqrt{3} \pi)}{8 \sqrt{3}(1-\cosh (4 \sqrt{3} \pi))}
\end{aligned}
$$

$\frac{\pi \tanh (\sqrt{3} \pi)}{16 \sqrt{3}}+\frac{\pi \operatorname{coth}(\sqrt{3} \pi)}{16 \sqrt{3}}$

## Alternative representations:

$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{1}{48}+\frac{1}{48}(-1-2 i \pi \cot (-2 i \pi \sqrt{3}) \sqrt{3})$
$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{1}{48}+\frac{1}{48}(-1+2 i \pi \cot (2 i \pi \sqrt{3}) \sqrt{3})$
$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{1}{48}+\frac{1}{48}\left(-1+2 \pi\left(1+\frac{2}{-1+e^{4 \pi \sqrt{3}}}\right) \sqrt{3}\right)$

## Series representations:

$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{1}{48}+\frac{1}{2} \sum_{k=1}^{\infty} \frac{1}{12+k^{2}}$
$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{1}{4} \pi \sum_{k=-\infty}^{\infty} \frac{1}{12 \pi+k^{2} \pi}$
$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=\frac{\pi}{8 \sqrt{3}}+\frac{\pi \sum_{k=0}^{\infty} e^{-4 \sqrt{3}}(1+k) \pi}{4 \sqrt{3}}$

## Integral representation:

$\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))=-\frac{\pi}{8 \sqrt{3}} \int_{\frac{i \pi}{2}}^{2 \sqrt{3}} \pi \operatorname{csch}^{2}(t) d t$
$(((\exp (((1 / 48+1 / 48(-1+2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi))))))))^{\wedge} 32-29+1 /$ golden ratio
Where 29 is a Lucas number

## Input:

$\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}$
$\operatorname{coth}(x)$ is the hyperbolic cotangent function

## Exact result:

$\frac{1}{\phi}-29+e^{2 / 3+2 / 3(2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi)-1)}$

## Decimal approximation:

1387.060505701553890257491110080389692406376704143735732815...
$1387.0605057 \ldots$. result practically equal to the rest mass of Sigma baryon 1387.2

## Alternate forms:

$\frac{1}{2}(\sqrt{5}-59)+e^{(4 \pi \operatorname{coth}(2 \sqrt{3} \pi)) / \sqrt{3}}$
$-29+\frac{2}{1+\sqrt{5}}+e^{(4 \pi \operatorname{coth}(2 \sqrt{3} \pi)) / \sqrt{3}}$
$\frac{1}{\phi}-29+e^{-\frac{4 \pi \sinh (4 \sqrt{3} \pi)}{\sqrt{3}(1-\cosh (4 \sqrt{3} \pi))}}$
$\cosh (x)$ is the hyperbolic cosine function
$\sinh (x)$ is the hyperbolic sine function

## Alternative representations:

$$
\begin{aligned}
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1-2 i \pi \cot (-2 i \pi \sqrt{3}) \sqrt{3})\right) \\
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}\left(-1+2 \pi\left(1+\frac{2}{-1+e^{4 \pi \sqrt{3}}}\right) \sqrt{3}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}\left(-1+\frac{2 \pi\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right) \sqrt{3}}{-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}}\right)\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& \frac{-27-29 \sqrt{5}+(1+\sqrt{5}) e^{2 / 3+16 \times \sum_{k=1}^{\infty} 1 /\left(12+k^{2}\right)}}{1+\sqrt{5}} \\
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -29+\exp \left(\frac{2}{3}+\frac{2}{3}\left(-1+12 \pi^{2} \sum_{k=-\infty}^{\infty} \frac{1}{\left(12+k^{2}\right) \pi^{2}}\right)\right)+\frac{1}{\phi} \\
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -\frac{59}{2}+\frac{\sqrt{5}}{2}+e^{8 \pi \sum_{k=-\infty}^{\infty} 1 /\left(12 \pi+k^{2} \pi\right)}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}= \\
& -29+\exp \left(\frac{2}{3}+\frac{2}{3}\left(-1-2 \sqrt{3} \pi \int_{\frac{i \pi}{2}}^{2 \sqrt{3} \pi} \operatorname{csch}^{2}(t) d t\right)\right)+\frac{1}{\phi}
\end{aligned}
$$

$1 / 10\left(\left(\left((((\exp (((1 / 48+1 / 48(-1+2 \operatorname{sqrt}(3) \pi \operatorname{coth}(2 \operatorname{sqrt}(3) \pi))))))))^{\wedge} 32-29+1 /\right.\right.\right.$ golden ratio $))+1 /$ golden ratio

Where 10 is the numbers of dimensions in superstring theory and 29 is a Lucas number

## Input:

$$
\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}
$$

## Exact result:

$\frac{1}{\phi}+\frac{1}{10}\left(\frac{1}{\phi}-29+e^{2 / 3+2 / 3(2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi)-1)}\right)$

## Decimal approximation:

139.3240845589052838739536978424046073583579795941793361436...
$139.32408455 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$\frac{11}{10 \phi}-\frac{29}{10}+\frac{1}{10} e^{(4 \pi \operatorname{coth}(2 \sqrt{3} \pi)) / \sqrt{3}}$
$\frac{1}{20}\left(-69+11 \sqrt{5}+2 e^{(4 \pi \operatorname{coth}(2 \sqrt{3} \pi)) / \sqrt{3}}\right)$
$-\frac{29}{10}+\frac{11}{5(1+\sqrt{5})}+\frac{1}{10} e^{(4 \pi \operatorname{coth}(2 \sqrt{3} \pi)) / \sqrt{3}}$

## Expanded form:

$$
\frac{11}{10 \phi}-\frac{29}{10}+\frac{1}{10} e^{2 / 3+2 / 3(2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi)-1)}
$$

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}= \\
& \frac{1}{\phi}+\frac{1}{10}\left(-29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1-2 i \pi \cot (-2 i \pi \sqrt{3}) \sqrt{3})\right)\right)
\end{aligned}
$$

$$
\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}=
$$

$$
\frac{1}{\phi}+\frac{1}{10}\left(-29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}\left(-1+2 \pi\left(1+\frac{2}{-1+e^{4 \pi \sqrt{3}}}\right) \sqrt{3}\right)\right)\right)
$$

$$
\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}=
$$

$$
\frac{1}{\phi}+\frac{1}{10}\left(-29+\frac{1}{\phi}+\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}\left(-1+\frac{2 \pi\left(e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}\right) \sqrt{3}}{-e^{-2 \pi \sqrt{3}}+e^{2 \pi \sqrt{3}}}\right)\right)\right)
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}= \\
& \frac{-7-29 \sqrt{5}+(1+\sqrt{5}) e^{2 / 3+16 \times \sum_{k=1}^{\infty} 1 /\left(12+k^{2}\right)}}{10(1+\sqrt{5})}
\end{aligned}
$$

$$
\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}=
$$

$$
-\frac{69}{20}+\frac{11}{4 \sqrt{5}}+\frac{1}{10} e^{8 \pi \sum_{k=-\infty}^{\infty} 1 /\left(12 \pi+k^{2} \pi\right)}
$$

$$
\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}=
$$

$$
-\frac{29}{10}+\frac{1}{10} \exp \left(\frac{2}{3}+\frac{2}{3}\left(-1+12 \pi^{2} \sum_{k=-\infty}^{\infty} \frac{1}{\left(12+k^{2}\right) \pi^{2}}\right)\right)+\frac{11}{10 \phi}
$$

## Integral representation:

$\frac{1}{10}\left(\exp ^{32}\left(\frac{1}{48}+\frac{1}{48}(-1+2 \sqrt{3} \pi \operatorname{coth}(2 \sqrt{3} \pi))\right)-29+\frac{1}{\phi}\right)+\frac{1}{\phi}=$

$$
-\frac{29}{10}+\frac{1}{10} \exp \left(\frac{2}{3}+\frac{2}{3}\left(-1-2 \sqrt{3} \pi \int_{\frac{i \pi}{2}}^{2 \sqrt{3} \pi} \operatorname{csch}^{2}(t) d t\right)\right)+\frac{11}{10 \phi}
$$

Or:

```
\(1 /\left(12^{*} 2^{\wedge} 2\right)+\)
\(1 / 2^{*}\left(1 /\left(1^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+1 /\left(2^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+1 /\left(3^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+1 /\left(4^{\wedge} 2+3^{*} 2^{\wedge} 2\right)+1 /\left(5^{\wedge} 2+3^{*} 2^{\wedge} 2\right)\right.\)
\(\left.+1 /\left(6^{\wedge} 2+3 * 2^{\wedge} 2\right)+1 /\left(7^{\wedge} 2+3 * 2^{\wedge} 2\right)\right)\)
```


## Input:

$$
\begin{aligned}
& \frac{1}{12 \times 2^{2}}+\frac{1}{2}\left(\frac{1}{1^{2}+3 \times 2^{2}}+\frac{1}{2^{2}+3 \times 2^{2}}+\right. \\
&\left.\frac{1}{3^{2}+3 \times 2^{2}}+\frac{1}{4^{2}+3 \times 2^{2}}+\frac{1}{5^{2}+3 \times 2^{2}}+\frac{1}{6^{2}+3 \times 2^{2}}+\frac{1}{7^{2}+3 \times 2^{2}}\right)
\end{aligned}
$$

## Exact result:

$\frac{231449}{1408368}$

## Decimal approximation:

0.164338439953194051554707292412210444997330243231882576144...
$1 /(48)+$
$1 / 2 *(1 /(64+12)+1 /(81+12)+1 /(100+12)+1 /(121+12)+1 /(144+12)+1 /(169+12)+1 /(196$ $+12)+1 /(225+12)+1 /(256+12)+1 /(289+12)+1 /(324+12)+1 /(361+12)+1 /(400+12)+1 /($ $441+12)+1 /(496)+1 /(541))$

## Input:

$$
\begin{array}{r}
\frac{1}{48}+\frac{1}{2}\left(\frac{1}{64+12}+\frac{1}{81+12}+\frac{1}{100+12}+\frac{1}{121+12}+\frac{1}{144+12}+\right. \\
\\
\frac{1}{169+12}+\frac{1}{196+12}+\frac{1}{225+12}+\frac{1}{256+12}+\frac{1}{289+12}+ \\
\left.\frac{1}{324+12}+\frac{1}{361+12}+\frac{1}{400+12}+\frac{1}{441+12}+\frac{1}{496}+\frac{1}{541}\right)
\end{array}
$$

## Exact result:

2950867038919393320551
47519195324227082625936

## Decimal approximation:

0.062098421885837989402622956253925345596783182155262208075 .
0.06209842...
$0.164338439+1 /(48)+$
$1 / 2 *(1 /(64+12)+1 /(81+12)+1 /(100+12)+1 /(121+12)+1 /(144+12)+1 /(169+12)+1 /(196$
$+12)+1 /(225+12)+1 /(256+12)+1 /(289+12)+1 /(324+12)+1 /(361+12)+1 /(400+12)+1 /($ $441+12)+1 /(496)+1 /(541))$

## Input interpretation:

$0.164338439+\frac{1}{48}+$

$$
\begin{gathered}
\frac{1}{2}\left(\frac{1}{64+12}+\frac{1}{81+12}+\frac{1}{100+12}+\frac{1}{121+12}+\frac{1}{144+12}+\frac{1}{169+12}+\right. \\
\frac{1}{196+12}+\frac{1}{225+12}+\frac{1}{256+12}+\frac{1}{289+12}+\frac{1}{324+12}+ \\
\left.\frac{1}{361+12}+\frac{1}{400+12}+\frac{1}{441+12}+\frac{1}{496}+\frac{1}{541}\right)
\end{gathered}
$$

## Result:

0.226436860885837989402622956253925345596783182155262208075...
0.22643686...

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$1^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)$

## Input:

$$
\frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}
$$

## Decimal approximation:

0.001984126912823947830626260402638897891829286043471033054...
$0.001984126912 \ldots$

## Property:

$\frac{1}{-1+e^{2 \pi}}+\frac{32}{-1+e^{4 \pi}}+\frac{243}{-1+e^{6 \pi}}+\frac{1024}{-1+e^{8 \pi}}$ is a transcendental number

## Alternate forms:

$\frac{1}{2}\left(-1+\frac{64}{e^{4 \pi}-1}+\frac{486}{e^{6 \pi}-1}+\frac{2048}{e^{8 \pi}-1}+\operatorname{coth}(\pi)\right)$
$\frac{177}{e^{\pi}-1}-\frac{177}{1+e^{\pi}}-\frac{272}{1+e^{2 \pi}}+\frac{81\left(e^{\pi}-2\right)}{2\left(1-e^{\pi}+e^{2 \pi}\right)}-\frac{81\left(2+e^{\pi}\right)}{2\left(1+e^{\pi}+e^{2 \pi}\right)}-\frac{512}{1+e^{4 \pi}}$
$\frac{1300+1301 e^{2 \pi}+1334 e^{4 \pi}+278 e^{6 \pi}+34 e^{8 \pi}+e^{10 \pi}}{\left(e^{\pi}-1\right)\left(1+e^{\pi}\right)\left(1+e^{2 \pi}\right)\left(1-e^{\pi}+e^{2 \pi}\right)\left(1+e^{\pi}+e^{2 \pi}\right)\left(1+e^{4 \pi}\right)}$

## Alternative representations:

$$
\begin{aligned}
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}= \\
& \frac{1^{5}}{-1+e^{360^{\circ}}}+\frac{2^{5}}{-1+e^{720^{\circ}}}+\frac{3^{5}}{-1+e^{1080^{\circ}}}+\frac{4^{5}}{-1+e^{1440^{\circ}}} \\
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{e^{4 \pi}}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}= \\
& \frac{4^{5}}{-1+e^{-8 i \log (-1)}}+\frac{3^{5}}{-1+e^{-6 i \log (-1)}}+\frac{2^{5}}{-1+e^{-4 i \log (-1)}}+\frac{1^{5}}{-1+e^{-2 i \log (-1)}} \\
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}= \\
& \frac{1^{5}}{\exp ^{2 \pi}(z)-1}+\frac{2^{5}}{\exp ^{4 \pi}(z)-1}+\frac{3^{5}}{\exp ^{6 \pi}(z)-1}+\frac{4^{5}}{\exp ^{8 \pi}(z)-1} \text { for } z=1
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}=\frac{1}{-1+e^{8 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+ \\
& \frac{32}{-1+e^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{1024}{-1+e^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{}{-1+e^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}
\end{aligned}
$$

$$
\frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}=
$$

$$
\frac{1}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{32}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{16} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+
$$

$$
\frac{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}{}+\frac{1024}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}=\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+ \\
& \frac{32}{-1+e^{8} \int_{0}^{\infty 01 /\left(1+t^{2}\right) d t}+\frac{1024}{-1+e^{12} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{10}{-1+e^{16} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}} \\
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}= \\
& \frac{32}{-1+e^{4} \int_{0}^{\infty} \sin (t) / t d t}+\frac{-1+e^{8} \int_{0}^{\infty} \sin (t) / t d t}{}+\frac{243}{-1+e^{12 \int_{0}^{\infty} \sin (t) / t d t}}+\frac{1024}{-1+e^{16} \int_{0}^{\infty} \sin (t) / t d t} \\
& \frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}= \\
& \frac{32}{-1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}{-1+e^{24} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{243}{-1+e^{32} \int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

1/504

## Input:

$\frac{1}{504}$

## Exact result:

$\frac{1}{504}$ (irreducible)

## Decimal approximation:

0.001984126984126984126984126984126984126984126984126984126...
0.001984126984....
$1^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)$
Input:
$\frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}$

## Decimal approximation:

0.003787833999809716424483550438828375181491636367211553105
0.0037878339...

## Property:

$\frac{1}{-1+e^{2 \pi}}+\frac{512}{-1+e^{4 \pi}}+\frac{19683}{-1+e^{6 \pi}}+\frac{262144}{-1+e^{8 \pi}}$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& \frac{512}{e^{4 \pi}-1}+\frac{1}{2}\left(-1+\frac{39366}{e^{6 \pi}-1}+\frac{524288}{e^{8 \pi}-1}+\operatorname{coth}(\pi)\right) \\
& \frac{36177}{e^{\pi}-1}-\frac{36177}{1+e^{\pi}}-\frac{65792}{1+e^{2 \pi}}+\frac{6561\left(e^{\pi}-2\right)}{2\left(1-e^{\pi}+e^{2 \pi}\right)}-\frac{6561\left(2+e^{\pi}\right)}{2\left(1+e^{\pi}+e^{2 \pi}\right)}-\frac{131072}{1+e^{4 \pi}}
\end{aligned}
$$

$$
\frac{282340+282341 e^{2 \pi}+282854 e^{4 \pi}+20198 e^{6 \pi}+514 e^{8 \pi}+e^{10 \pi}}{\left(e^{\pi}-1\right)\left(1+e^{\pi}\right)\left(1+e^{2 \pi}\right)\left(1-e^{\pi}+e^{2 \pi}\right)\left(1+e^{\pi}+e^{2 \pi}\right)\left(1+e^{4 \pi}\right)}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function

## Alternative representations:

$$
\begin{aligned}
& \frac{1^{\circ}}{e^{2 \pi}-1}+\frac{2^{\circ}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{\circ}}{e^{8 \pi}-1}= \\
& \frac{1^{9}}{-1+e^{360^{\circ}}}+\frac{2^{9}}{-1+e^{720^{\circ}}}+\frac{3^{9}}{-1+e^{1080^{\circ}}}+\frac{4^{9}}{-1+e^{1440^{\circ}}}
\end{aligned}
$$

$$
\frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}=
$$

$$
\frac{4^{9}}{-1+e^{-8 i \log (-1)}}+\frac{3^{9}}{-1+e^{-6 i \log (-1)}}+\frac{2^{9}}{-1+e^{-4 i \log (-1)}}+\frac{1^{9}}{-1+e^{-2 i \log (-1)}}
$$

$$
\frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}=
$$

$$
\frac{1^{9}}{\exp ^{2 \pi}(z)-1}+\frac{2^{9}}{\exp ^{4 \pi}(z)-1}+\frac{3^{9}}{\exp ^{6 \pi}(z)-1}+\frac{4^{9}}{\exp ^{8 \pi}(z)-1} \text { for } z=1
$$

## Series representations:

$$
\begin{aligned}
& \frac{1^{\rho}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}=\frac{1}{-1+e^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+ \\
& \frac{512}{-1+e^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{19683}{-1+e^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{262144}{-1+e^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{1^{\circ}}{e^{2 \pi}-1}+\frac{2^{\circ}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}=\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+ \\
& \frac{512}{-1+e^{8} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{19683}{-1+e^{12} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{262144}{-1+e^{16} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}= \\
& \frac{1}{-1+e^{4} \int_{0}^{\infty \sin (t) / t / d t}}+\frac{512}{-1+e^{8} \int_{0}^{\infty} \sin (t) / t d t}+\frac{19683}{-1+e^{12} \int_{0}^{\infty \sin (t) / t d t}}+\frac{262144}{-1+e^{16} \int_{0}^{\omega \sin (t)) / t d t}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{\rho}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{\circ}}{e^{8 \pi}-1}= \\
& -1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t \\
& \frac{512}{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{19683}{-1+e^{24 \int_{0}^{1} \sqrt{1-t^{2}} d t}}+\frac{262144}{-1+e^{32} \int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

## Input:

$\frac{1}{264}$

$$
\begin{aligned}
& \frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}= \\
& \frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{512}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+ \\
& 19683262144 \\
& -1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k) \quad+\overline{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}\right) .} \\
& \frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}=
\end{aligned}
$$

$$
\begin{aligned}
& -1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\cdots-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{32}
\end{aligned}
$$

## Exact result:

## $\frac{1}{264}$ (irreducible)

## Decimal approximation:

$0.003787878787878787878787878787878787878787878787878787878 \ldots$
0.003787878...
$1^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)$

## Input:

$\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}$

## Decimal approximation:

0.041638381585443662182517651348977915286722918403784080981...
0.0416383815854......

## Property:

$\frac{1}{-1+e^{2 \pi}}+\frac{8192}{-1+e^{4 \pi}}+\frac{1594323}{-1+e^{6 \pi}}+\frac{67108864}{-1+e^{8 \pi}}$ is a transcendental number
Alternate forms:
$\frac{8192}{e^{4 \pi}-1}+\frac{1594323}{e^{6 \pi}-1}+\frac{67108864}{e^{8 \pi}-1}+\frac{1}{2}(\operatorname{coth}(\pi)-1)$

$$
\begin{aligned}
& \frac{8656377}{e^{\pi}-1}-\frac{8656377}{1+e^{\pi}}-\frac{16781312}{1+e^{2 \pi}}+ \\
& \frac{531441\left(e^{\pi}-2\right)}{2\left(1-e^{\pi}+e^{2 \pi}\right)}-\frac{531441\left(2+e^{\pi}\right)}{2\left(1+e^{\pi}+e^{2 \pi}\right)}-\frac{33554432}{1+e^{4 \pi}}
\end{aligned}
$$

$$
\frac{68711380+68711381 e^{2 \pi}+68719574 e^{4 \pi}+1602518 e^{6 \pi}+8194 e^{8 \pi}+e^{10 \pi}}{\left(e^{\pi}-1\right)\left(1+e^{\pi}\right)\left(1+e^{2 \pi}\right)\left(1-e^{\pi}+e^{2 \pi}\right)\left(1+e^{\pi}+e^{2 \pi}\right)\left(1+e^{4 \pi}\right)}
$$

## Alternative representations:

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{e^{4 \pi}}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{4^{13}}{-1+e^{-8 i \log (-1)}}+\frac{3^{13}}{-1+e^{-6 i \log (-1)}}+\frac{2^{13}}{-1+e^{-4 i \log (-1)}}+\frac{1^{13}}{-1+e^{-2 i \log (-1)}} \\
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{1^{13}}{\exp ^{2 \pi}(z)-1}+\frac{2^{13}}{\exp ^{4 \pi}(z)-1}+\frac{3^{13}}{\exp ^{6 \pi}(z)-1}+\frac{4^{13}}{\exp ^{8 \pi}(z)-1} \text { for } z=1
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=\frac{1}{-1+e^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+ \\
& \frac{8192}{-1+e^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{1594323}{-1+e^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{67108864}{-1+e^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}{1594323}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+ \\
& \frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}{-1}
\end{aligned}
$$

$$
\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=
$$

$$
\frac{1}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{8192}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{(-1)^{k}}{k!}}\right)^{16} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+
$$

$$
\frac{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{\left(-1 k^{k}\right.}{k!}}\right)^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}{}+\frac{67108864}{-1+\left(\frac{1}{\sum_{k=0}^{\infty} \frac{\left(-1 k^{k}\right.}{k!}}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+ \\
& \frac{8192}{-1+e^{8} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{1594323}{-1+e^{12} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{67108864}{-1+e^{16} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{1}{-1+e^{4} \int_{0}^{\infty \sin (t) / t} d t}+\frac{8192}{-1+e^{8} \int_{0}^{\infty \sin (t) / t d t}}+\frac{1594323}{-1+e^{12} \int_{0}^{\infty \sin (t) / t d t}}+\frac{67108864}{-1+e^{16} \int_{0}^{\infty \sin (t) / t d t}} \\
& \frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& \frac{1}{-1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{1594323}{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{159108864}{-1+e^{24} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{672}{-1+e^{32} \int_{0}^{1} \sqrt{1-t^{2}} d t}
\end{aligned}
$$

$1 / 24$
Input:
$\frac{1}{24}$

Exact result:
$\frac{1}{24}$ (irreducible)

## Decimal approximation:

$0.041666666666666666666666666666666666666666666666666666666 \ldots$
0.0416666666666.....

We note that:

From:

## SUPERSYMMETRY AND STRING THEORY - Beyond the Standard Model

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First, we give a general formula for the normal ordering constant. This is related to the algebra of the energy-momentum tensor we have discussed in Section 21.4. For a left- or right-moving boson, with modes which differ from an integer by $\eta$ (e.g. modes are $1-\eta, 2-\eta$, etc.), the contribution to the normal ordering constant is:

$$
\begin{equation*}
\Delta=-\frac{1}{24}+\frac{1}{4} \eta(1-\eta) . \tag{22.30}
\end{equation*}
$$

For fermions, the contribution is the opposite. So we can recover some familiar results. In the bosonic string, with 24 transverse degrees of freedom, we see that the normal ordering constant is -1 . For the superstring, in the NS-NS sector, we have a contribution of $-1 / 24$ for each boson, and $1 / 24-1 / 16$ for each of the eight fermions on the left (and similarly on the right). So the normal ordering constant is $-1 / 2$. For the RR sector, the normal ordering vanishes.

Thence 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

We have that:
$\left(\left(\left(1^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 5 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-\right.\right.\right.\right.$
$1)))+\left(\left(\left(1^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 9 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)\right)\right)\right)$

## Input:

$$
\left(\frac{1^{5}}{e^{2 \pi}-1}+\frac{2^{5}}{e^{4 \pi}-1}+\frac{3^{5}}{e^{6 \pi}-1}+\frac{4^{5}}{e^{8 \pi}-1}\right)+\left(\frac{1^{9}}{e^{2 \pi}-1}+\frac{2^{9}}{e^{4 \pi}-1}+\frac{3^{9}}{e^{6 \pi}-1}+\frac{4^{9}}{e^{8 \pi}-1}\right)
$$

## Exact result:

$\frac{2}{e^{2 \pi}-1}+\frac{544}{e^{4 \pi}-1}+\frac{19926}{e^{6 \pi}-1}+\frac{263168}{e^{8 \pi}-1}$

## Decimal approximation:

0.005771960912633664255109810841467273073320922410682586159...
0.0057719609126336... Partial Result

## Property:

$\frac{2}{-1+e^{2 \pi}}+\frac{544}{-1+e^{4 \pi}}+\frac{19926}{-1+e^{6 \pi}}+\frac{263168}{-1+e^{8 \pi}}$ is a transcendental number
$0.0057719609126336642551098+1^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-\right.$ $1)+3^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-1\right)+4^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)$

## Input interpretation:

$0.0057719609126336642551098+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}$

## Result:

0.0474103424980773264376275...
0.047410342498.....

## Alternative representations:

$0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=$ $0.00577196091263366425510980000+$

$$
\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}
$$

$0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=$

$$
\begin{aligned}
& 0.00577196091263366425510980000+\frac{4^{13}}{-1+e^{-8 i \log (-1)}}+ \\
& \frac{3^{13}}{-1+e^{-6 i \log (-1)}}+\frac{2^{13}}{-1+e^{-4 i \log (-1)}}+\frac{1^{13}}{-1+e^{-2 i \log (-1)}}
\end{aligned}
$$

$0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=$ $0.00577196091263366425510980000+\frac{1^{13}}{\exp ^{2 \pi}(z)-1}+$

$$
\frac{2^{13}}{\exp ^{4 \pi}(z)-1}+\frac{3^{13}}{\exp ^{6 \pi}(z)-1}+\frac{4^{13}}{\exp ^{8 \pi}(z)-1} \text { for } z=1
$$

Series representations:

$$
\begin{aligned}
& 0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}= \\
& 0.0057719609126336642551098000+ \\
& \frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{8192}{1594323}+ \\
& \frac{1+\left(\sum_{k=0}^{\infty} \frac{1}{0}\right)^{16} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}{67108864} \\
& -1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)
\end{aligned}+\frac{1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}{-1}+
$$

$0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=$ $0.00577196091263366425510980000+$
$\frac{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}{1594323}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+$
$-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+$
$0.00577196091263366425510980000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}=$ $0.00577196091263366425510980000+$

$$
\begin{aligned}
& \frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+ \\
& \frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}
\end{aligned}
$$

We observe that:
$1 /\left(\left(\left(\left(0.00577196091263+1^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.1)+4^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)\right)\right)\right)\right)$

## Input interpretation:

$0.00577196091263+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}$

## Result:

21.09244412315...
21.09244412315...

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}= \\
& \frac{1}{0.005771960912630000+\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}}
\end{aligned}
$$


$\frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}=$
$1 /\left(0.005771960912630000+\frac{4^{13}}{-1+e^{-8 i \log (-1)}}+\right.$

$$
\left.\frac{3^{13}}{-1+e^{-6 i \log (-1)}}+\frac{2^{13}}{-1+e^{-4 i \log (-1)}}+\frac{1^{13}}{-1+e^{-2 i \log (-1)}}\right)
$$

## Series representations:

$\frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi-1}}+\frac{2^{13}}{e^{4 \pi-1}}+\frac{3^{13}}{e^{6 \pi-1}}+\frac{4^{13}}{e^{8 \pi-1}}}=$
$1 /(0.005771960912630000+$
$\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+$
$\frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+\frac{67108864}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}\right)}$

$$
\begin{aligned}
& \frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi-1}}+\frac{2^{13}}{e^{4 \pi-1}}+\frac{3^{13}}{e^{6 \pi-1}}+\frac{4^{13}}{e^{8 \pi-1}}}= \\
& 1 /\left(0.005771960912630000+\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\right. \\
& \frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+ \\
& \left.\frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}\right) \\
& \frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}= \\
& 1 /\left(0.005771960912630000+\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+\right. \\
& \frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+ \\
& \left.\frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) s(1+k)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) s(1+k)}\right)
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}= \\
& 1 /\left(0.005771960912630000+\frac{1}{-1+e^{4 \int_{0}^{\infty} \sin (t) / t d t}}+\right. \\
& \left.\frac{8192}{-1+e^{8} \int_{0}^{\infty \sin (t) / t d t}}+\frac{1594323}{-1+e^{12 \int_{0}^{\infty} \sin (t) / t d t}}+\frac{67108864}{-1+e^{16} \int_{0}^{\infty \sin (t) / t d t}}\right)
\end{aligned}
$$

$\frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}=$

$$
\begin{aligned}
& 1 /\left(0.005771960912630000+\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\right. \\
& \left.\frac{8192}{-1+e^{8} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\frac{1594323}{-1+e^{12 \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}}+\frac{67108864}{-1+e^{16} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}\right)
\end{aligned}
$$

$\frac{1}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}=$
$1 /\left(0.005771960912630000+\frac{1}{-1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\right.$

$$
\left.\frac{8192}{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{1594323}{-1+e^{24} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{67108864}{-1+e^{32} \int_{0}^{1} \sqrt{1-t^{2}} d t}\right)
$$

$6 /\left(\left(\left(0.00577196091263+1^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-\right.\right.\right.\right.$ $\left.\left.\left.1)+4^{\wedge} 13 /\left(e^{\wedge}(8 \mathrm{Pi})-1\right)\right)\right)\right)$ )-golden ratio

## Input interpretation:

$\frac{6}{0.00577196091263+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi$

## Result:

124.9366307501...
$124.936630 \ldots$ result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternative representations:

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$

$$
-2 \cos \left(\frac{\pi}{5}\right)+\frac{6}{0.005771960912630000+\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$
$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$
$\frac{6}{0.005771960912630000+\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}}-$
root of $-1-x+x^{2}$ near $x=1.61803$

## Series representations:

$$
\begin{aligned}
& \frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi= \\
& -\phi+6 /\left(\begin{array}{l}
0.005771960912630000+ \\
\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+ \\
\frac{1594323}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}\right)}
\end{array}=\right.
\end{aligned}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$

$$
-\phi+6 /(0.005771960912630000+
$$

$$
\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+
$$

$$
\left.\frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}\right)
$$

$$
\begin{aligned}
& \frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi-1}}}-\phi= \\
& -\phi+6 /\left(\begin{array}{l}
0.005771960912630000+\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2 \times} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+ \\
\frac{8192 \quad}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4 \times \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+} 1594323 \\
\frac{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6 \times \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+}{\left.\frac{67108864}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}\right)}
\end{array} .\right.
\end{aligned}
$$

## Integral representations:

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$
$6 /\left(0.005771960912630000+\frac{1}{-1+e^{4} \int_{0}^{\infty} \sin (t) / t d t}+\right.$
$\left.\frac{8192}{\left.-1+e^{8} \int_{0}^{\infty} \sin (t)\right) t d t}+\frac{1594323}{\left.-1+e^{12} \int_{0}^{\infty} \sin (t)\right) t d t}+\frac{67108864}{-1+e^{16} \int_{0}^{\infty} \sin (t) / t d t}\right)-\phi$
$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi-1}}}-\phi=$

$$
6 /\left(0.005771960912630000+\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\right.
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}-\phi=$
$6 /\left(0.005771960912630000+\frac{1}{-1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\right.$

$$
\left.\frac{8192}{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{1594323}{-1+e^{24} \int_{0}^{1 \sqrt{1-t^{2}} d t}}+\frac{67108864}{-1+e^{32} \int_{0}^{1 \sqrt{1-t^{2}} d t}}\right)-\phi
$$

$6 /\left(\left(\left(0.00577196091263+1^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(2 \mathrm{Pi})-1\right)+2^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(4 \mathrm{Pi})-1\right)+3^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(6 \mathrm{Pi})-\right.\right.\right.\right.$ 1) $\left.\left.\left.\left.+4^{\wedge} 13 /\left(\mathrm{e}^{\wedge}(8 \mathrm{Pi})-1\right)\right)\right)\right)\right)+11+$ golden ratio

Where 11 is a Lucas number and are the number of dimensions of bulk in M-theory (hyperspace) and 6 are the extra dimensions (compactified toroidal dimensions) of the superstring theory in 10 D

## Input interpretation:

$\frac{6}{0.00577196091263+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi$

## Result:

139.1726987276
139.1726987276.... result practically equal to the rest mass of Pion meson 139.57

## Alternative representations:

$$
\begin{aligned}
& \frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi=11+2 \cos \left(\frac{\pi}{5}\right)+ \\
& \frac{6}{0.005771960912630000+\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}}
\end{aligned}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi=$

$$
11+\frac{6}{0.005771960912630000+\frac{1^{13}}{-1+e^{2 \pi}}+\frac{2^{13}}{-1+e^{4 \pi}}+\frac{3^{13}}{-1+e^{6 \pi}}+\frac{4^{13}}{-1+e^{8 \pi}}}+
$$

$$
\text { root of }-1-x+x^{2} \text { near } x=1.61803
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi=$

$$
11+\frac{6}{0.005771960912630000+\frac{1^{13}}{-1+e^{360^{\circ}}}+\frac{2^{13}}{-1+e^{720^{\circ}}}+\frac{3^{13}}{-1+e^{1080^{\circ}}}+\frac{4^{13}}{-1+e^{1440^{\circ}}}}+
$$

$$
\text { root of }-1-x+x^{2} \text { near } x=1.61803
$$

## Series representations:

$$
\begin{aligned}
& \frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi-1}}}+11+\phi= \\
& 11+\phi+6 /(0.005771960912630000+ \\
& \frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}}+ \\
& \frac{1594323}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}+\frac{67108864}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=0}^{\infty}(-1)^{k} /(1+2 k)}\right)}\right)}
\end{aligned}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi-1}}}+11+\phi=$

$$
\begin{aligned}
11+\phi+ & 6 /(0.005771960912630000+ \\
& \frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8} \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}+\frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{16 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+ \\
& \frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{24 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}}+\frac{67108864}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{32 \sum_{k=1}^{\infty} \tan ^{-1}\left(1 / F_{1+2 k}\right)}\right)}
\end{aligned}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi-1}}}+11+\phi=$

$$
\begin{aligned}
11+\phi+ & 6 /\left(0.005771960912630000+\frac{1}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+\right. \\
& \frac{8192}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{4} \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+ \\
& \frac{1594323}{-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{6 \times \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}+} \\
& \frac{67108864}{\left.-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{8 \times \sum_{k=1}^{\infty} 4^{-k}\left(-1+3^{k}\right) \zeta(1+k)}\right)}
\end{aligned}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi-1}}+\frac{4^{13}}{e^{8 \pi-1}}}+11+\phi= \\
& 11+6 /\left(0.005771960912630000+\frac{1}{-1+e^{4} \int_{0}^{\infty \sin (t) / t d t}}+\right. \\
& \left.\frac{8192}{-1+e^{8} \int_{6}^{\infty \sin (t) / t d t}}+\frac{1594323}{-1+e^{12} \int_{6}^{\infty \sin (t) / t d t}}+\frac{67108864}{-1+e^{16} \int_{6}^{\infty \sin (t) / t d t}}\right)+\phi
\end{aligned}
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi=$

$$
11+6 /\left(0.005771960912630000+\frac{1}{-1+e^{4} \int_{0}^{\infty} 1 /\left(1+t^{2}\right) d t}+\right.
$$

$\frac{6}{0.005771960912630000+\frac{1^{13}}{e^{2 \pi}-1}+\frac{2^{13}}{e^{4 \pi}-1}+\frac{3^{13}}{e^{6 \pi}-1}+\frac{4^{13}}{e^{8 \pi}-1}}+11+\phi=$

$$
\begin{aligned}
11+6 /( & 0.005771960912630000+\frac{1}{-1+e^{8} \int_{0}^{1} \sqrt{1-t^{2}} d t}+ \\
& \left.\frac{8192}{-1+e^{16} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{1594323}{-1+e^{24} \int_{0}^{1} \sqrt{1-t^{2}} d t}+\frac{67108864}{-1+e^{32} \int_{0}^{1} \sqrt{1-t^{2}} d t}\right)+\phi
\end{aligned}
$$

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For $\mathrm{x}=0.5$, we obtain:
$1-0.5^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4-0.5^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-0.5^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-\right.\right.\right.\right.\right.$ $\left.0.5^{\wedge} 4\right)$ ))))

## Input:

$1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)$

## Result:

0.0314354...
$0.0314354 \ldots$

## Alternative representations:

$$
\begin{array}{r}
1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)= \\
1-4 \pi 0.5^{4}\left(\frac{i \cot (i \pi)}{-0.5^{4}+1^{4}}+\frac{2 i \cot (2 i \pi)}{-0.5^{4}+2^{4}}+\frac{3 i \cot (3 i \pi)}{-0.5^{4}+3^{4}}\right)
\end{array}
$$

$$
\begin{aligned}
& 1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)= \\
& 1-4 \pi 0.5^{4}\left(-\frac{i \cot (-i \pi)}{-0.5^{4}+1^{4}}-\frac{2 i \cot (-2 i \pi)}{-0.5^{4}+2^{4}}-\frac{3 i \cot (-3 i \pi)}{-0.5^{4}+3^{4}}\right)
\end{aligned}
$$

$$
1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)=
$$

$$
1-4 \pi 0.5^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{-0.5^{4}+1^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{-0.5^{4}+2^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{-0.5^{4}+3^{4}}\right)
$$

## Series representations:

$$
\begin{array}{r}
1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)= \\
0.714558+\sum_{k=1}^{\infty}\left(-\frac{0.533333}{1+k^{2}}-\frac{0.12549}{4+k^{2}}-\frac{0.0555985}{9+k^{2}}\right)
\end{array}
$$

$$
1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)=
$$

$$
1+\sum_{k=-\infty}^{\infty} \frac{-10.2759-4.23311 k^{2}-0.357211 k^{4}}{36+49 k^{2}+14 k^{4}+k^{6}}
$$

$$
1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)=
$$

$$
0.714558+\sum_{k=-\infty}^{\infty}\left(\left\{\begin{array}{ll}
-\frac{(0.357211 i)\left(5.16175-(4.6687 i) k-k^{2}\right)}{(-3+i k)(-2+i k)(-1+i k) k} & k \neq 0 \\
0 & \text { otherwise }
\end{array}\right)\right.
$$

## Integral representation:

$$
\begin{aligned}
& 1-0.5^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.5^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.5^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.5^{4}}\right)= \\
& 1+\int_{\frac{i \pi}{2}}^{3 \pi} \frac{1}{-6+i} \pi\left((-0.0555985+0.00926641 i) \operatorname{csch}^{2}(t)+\right. \\
& (-0.12549+0.0313725 i) \operatorname{csch}^{2}\left(\frac{-i \pi-4 t+i t}{-6+i}\right)+ \\
& \left.(-0.533333+0.266667 i) \operatorname{csch}^{2}\left(\frac{-2 i \pi-2 t+i t}{-6+i}\right)\right) d t
\end{aligned}
$$

For $x=1 / 12=0.083 \ldots$, we obtain:
$\left(\left(\left(1-0.083^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4-0.083^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.0.083^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-0.083^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)^{\wedge} 16$

## Input:

$\left(1-0.083^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-0.083^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-0.083^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-0.083^{4}}\right)\right)^{16}$
$\operatorname{coth}(x)$ is the hyperbolic cotangent function

## Result:

0.9889334 ..
$0.9889334 \ldots$... result very near to the value of the following Rogers-Ramanujan continued fraction:
$\frac{\mathrm{e}^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^{5 \sqrt[4]{5^{3}}}-1}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-2 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-3 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-4 \pi \sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684 .10 .}$
and to the dilaton value $\mathbf{0 . 9 8 9 1 1 7 3 5 2 2 4 3 = \boldsymbol { \phi }}$

For $\mathrm{x}=12$, we obtain:
$1-12^{\wedge} 4 * 4^{*} \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4-12^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-12^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-\right.\right.\right.\right.\right.\right.$ 12^4))))))

## Input:

$1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)$

## Exact result:

$1-82944 \pi\left(-\frac{\operatorname{coth}(\pi)}{20735}-\frac{\operatorname{coth}(2 \pi)}{10360}-\frac{\operatorname{coth}(3 \pi)}{6885}\right)$

## Decimal approximation:

76.61327686396115476033877181540069163017090611360142200794...
76.6132768639...

## Alternate forms:

$\frac{1}{91296205}(91296205+365202432 \pi \operatorname{coth}(\pi)+730933632 \pi \operatorname{coth}(2 \pi)+$
$1099850752 \pi \operatorname{coth}(3 \pi))$
$1+\frac{82944 \pi \operatorname{coth}(\pi)}{20735}+\frac{10368 \pi \operatorname{coth}(2 \pi)}{1295}+\frac{1024}{85} \pi \operatorname{coth}(3 \pi)$
$\frac{5370365+21482496 \pi \operatorname{coth}(\pi)+42996096 \pi \operatorname{coth}(2 \pi)}{5370365}+\frac{1024}{85} \pi \operatorname{coth}(3 \pi)$

## Alternative representations:

$1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)=$
$1-4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}-12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}-12^{4}}\right)$
$1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)=$
$1-4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}-12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}-12^{4}}\right)$

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1-4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}-12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{2^{4}-12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{3^{4}-12^{4}}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1+\sum_{k=-\infty}^{\infty} \frac{768\left(51435289+46698002 k^{2}+6675289 k^{4}\right)}{91296205\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)} \\
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& \frac{3565747111}{273888615}+\sum_{k=1}^{\infty}\left(\frac{165888}{20735\left(1+k^{2}\right)}+\frac{41472}{1295\left(4+k^{2}\right)}+\frac{6144}{85\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
\begin{array}{r}
1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)=1+\frac{2195986816 \pi}{91296205}+ \\
\sum_{k=0}^{\infty} \frac{256 e^{-6(1+k) \pi}\left(8592584+5710419 e^{2(1+k) \pi}+2853144 e^{4(1+k) \pi}\right) \pi}{91296205}
\end{array}
$$

## Integral representation:

$1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)=$

$$
\begin{aligned}
& 1+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{1024}{85} \pi \operatorname{csch}^{2}(t)+\left(\frac{13}{37}-\frac{4 i}{37}\right)\right. \\
& \left(-\frac{82944 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{20735}-\left(\frac{93312}{6475}+\frac{20736 i}{6475}\right) \pi\right. \\
& \left.\operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)}{\pi}\right)\right) d t
\end{aligned}
$$

$8 / 5^{*}\left(\left(\left(1-12^{\wedge} 4^{*} 4^{*} \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4-12^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)+\mathrm{Pi}$

## Input:

$\frac{8}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right)+\pi$

## Exact result:

$\pi+\frac{8}{5}\left(1-82944 \pi\left(-\frac{\operatorname{coth}(\pi)}{20735}-\frac{\operatorname{coth}(2 \pi)}{10360}-\frac{\operatorname{coth}(3 \pi)}{6885}\right)\right)$

## Decimal approximation:

$125.7228356359276408550046782879206094924706191811373810336 \ldots$
$125.72283563 \ldots$ result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternate forms:

$\frac{8}{5}+\pi+\frac{663552 \pi \operatorname{coth}(\pi)}{103675}+\frac{82944 \pi \operatorname{coth}(2 \pi)}{6475}+\frac{8192}{425} \pi \operatorname{coth}(3 \pi)$

$$
\begin{aligned}
& \frac{1}{456481025}(730369640+456481025 \pi+2921619456 \pi \operatorname{coth}(\pi)+ \\
& 5847469056 \pi \operatorname{coth}(2 \pi)+8798806016 \pi \operatorname{coth}(3 \pi))
\end{aligned}
$$

$\frac{8}{5}+\pi\left(1+\frac{663552 \operatorname{coth}(\pi)}{103675}+\frac{82944 \operatorname{coth}(2 \pi)}{6475}+\frac{8192}{425} \operatorname{coth}(3 \pi)\right)$

## Alternative representations:

$\frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi=$
$\pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}-12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}-12^{4}}\right)\right)$
$\frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi=$
$\pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}-12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}-12^{4}}\right)\right)$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi= \\
& \pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}-12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{2^{4}-12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{3^{4}-12^{4}}\right)\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi= \\
& \frac{8}{5}+\pi+\sum_{k=-\infty}^{\infty} \frac{6144\left(51435289+46698002 k^{2}+6675289 k^{4}\right)}{456481025\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi= \\
& \frac{28525976888}{1369443075}+\pi+\sum_{k=1}^{\infty}\left(\frac{1327104}{103675\left(1+k^{2}\right)}+\frac{331776}{6475\left(4+k^{2}\right)}+\frac{49152}{425\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
\frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi=\frac{8}{5}+\frac{18024375553 \pi}{456481025}+
$$

$$
\sum_{k=0}^{\infty} \frac{2048 e^{-6(1+k) \pi}\left(8592584+5710419 e^{2(1+k) \pi}+2853144 e^{4(1+k) \pi}\right) \pi}{456481025}
$$

## Integral representation:

$\frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi=$

$$
\frac{8}{5}+\pi+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{8192}{425} \pi \operatorname{csch}^{2}(t)+\left(\frac{13}{37}-\frac{4 i}{37}\right)\right.
$$

$$
\left(-\frac{663552 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{103675}-\left(\frac{746496}{32375}+\frac{165888 i}{32375}\right)\right.
$$

$$
\left.\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)}{\pi}\right)\right) d d
$$

Where 11 and 3 are Lucas number (furthermore 11 is also the number of dimensions of M-Theory)

## Input:

$\frac{8}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right)+\pi+11+3$

## Exact result:

$14+\pi+\frac{8}{5}\left(1-82944 \pi\left(-\frac{\operatorname{coth}(\pi)}{20735}-\frac{\operatorname{coth}(2 \pi)}{10360}-\frac{\operatorname{coth}(3 \pi)}{6885}\right)\right)$

## Decimal approximation:

$139.7228356359276408550046782879206094924706191811373810336 \ldots$
$139.72283563 \ldots$. result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$$
\frac{78}{5}+\pi+\frac{663552 \pi \operatorname{coth}(\pi)}{103675}+\frac{82944 \pi \operatorname{coth}(2 \pi)}{6475}+\frac{8192}{425} \pi \operatorname{coth}(3 \pi)
$$

$$
\begin{aligned}
& \frac{1}{456481025}(7121103990+456481025 \pi+2921619456 \pi \operatorname{coth}(\pi)+ \\
& 5847469056 \pi \operatorname{coth}(2 \pi)+8798806016 \pi \operatorname{coth}(3 \pi))
\end{aligned}
$$

$$
\frac{78}{5}+\pi\left(1+\frac{663552 \operatorname{coth}(\pi)}{103675}+\frac{82944 \operatorname{coth}(2 \pi)}{6475}+\frac{8192}{425} \operatorname{coth}(3 \pi)\right)
$$

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \quad 14+\pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}-12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}-12^{4}}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \quad 14+\pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}-12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}-12^{4}}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-\frac{12^{4}}{}}\right)\right) 8+\pi+11+3= \\
& 14+\pi+\frac{8}{5}\left(1-4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}-12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{2^{4}-12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{3^{4}-12^{4}}\right)\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \frac{78}{5}+\pi+\sum_{k=-\infty}^{\infty} \frac{6144\left(51435289+46698002 k^{2}+6675289 k^{4}\right)}{456481025\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \frac{47698179938}{1369443075}+\pi+\sum_{k=1}^{\infty}\left(\frac{1327104}{103675\left(1+k^{2}\right)}+\frac{331776}{6475\left(4+k^{2}\right)}+\frac{49152}{425\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \frac{78}{5}+\frac{18024375553 \pi}{456481025}+ \\
& \quad \sum_{k=0}^{\infty} \frac{2048 e^{-6(1+k) \pi}\left(8592584+5710419 e^{2(1+k) \pi}+2853144 e^{4(1+k) \pi}\right) \pi}{456481025}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{5}\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}-12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}-12^{4}}\right)\right) 8+\pi+11+3= \\
& \frac{78}{5}+\pi+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{8192}{425} \pi \operatorname{csch}^{2}(t)+\left(\frac{13}{37}-\frac{4 i}{37}\right)\right. \\
& \left(-\frac{663552 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{103675}-\left(\frac{746496}{32375}+\frac{165888 i}{32375}\right)\right. \\
& \left.\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)}{\pi}\right)\right) d t
\end{aligned}
$$

Now, we have that:

$1+12^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge} 4+1\right.\right.\right.\right.\right.\right.$ $\left.2^{\wedge} 4\right)$ )) )) )

## Input:

$1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)$

## Exact result:

$1+82944 \pi\left(\frac{\operatorname{coth}(\pi)}{20737}+\frac{\operatorname{coth}(2 \pi)}{10376}+\frac{\operatorname{coth}(3 \pi)}{6939}\right)$

## Decimal approximation:

76.27874609711877953712482478244915518016178203760089714270 .
76.278746097....

## Alternate forms:

$\frac{1}{6912243473}(6912243473+27647640576 \pi \operatorname{coth}(\pi)+55255312512 \pi \operatorname{coth}(2 \pi)+$ $82624171008 \pi \operatorname{coth}(3 \pi))$
$1+\frac{82944 \pi \operatorname{coth}(\pi)}{20737}+\frac{10368 \pi \operatorname{coth}(2 \pi)}{1297}+\frac{3072}{257} \pi \operatorname{coth}(3 \pi)$
$\frac{26895889+107578368 \pi \operatorname{coth}(\pi)+215001216 \pi \operatorname{coth}(2 \pi)}{26895889}+\frac{3072}{257} \pi \operatorname{coth}(3 \pi)$

## Alternative representations:

$$
\begin{array}{r}
1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)= \\
\quad 1+4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}+12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}+12^{4}}\right)
\end{array}
$$

$$
\begin{aligned}
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1+4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}+12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}+12^{4}}\right) \\
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1+4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}+12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{2^{4}+12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{3^{4}+12^{4}}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1+\sum_{k=-\infty}^{\infty}\left(\frac{82944}{20737\left(1+k^{2}\right)}+\frac{20736}{1297\left(4+k^{2}\right)}+\frac{9216}{257\left(9+k^{2}\right)}\right) \\
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& \frac{8972893641}{6912243473}+\sum_{k=1}^{\infty}\left(\frac{165888}{20737\left(1+k^{2}\right)}+\frac{41472}{1297\left(4+k^{2}\right)}+\frac{18432}{257\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)=1+\frac{165527124096 \pi}{6912243473}+
$$

$$
\sum_{k=0}^{\infty}\left(\frac{6144}{257} e^{-6(1+k) \pi} \pi+\frac{20736 e^{-4(1+k) \pi} \pi}{1297}+\frac{165888 e^{-2(1+k) \pi} \pi}{20737}\right)
$$

## Integral representation:

$1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)=$

$$
\begin{aligned}
& 1+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{3072}{257} \pi \operatorname{csch}^{2}(t)+\left(\frac{13}{37}-\frac{4 i}{37}\right)\right. \\
& \left(-\frac{82944 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{20737}-\left(\frac{93312}{6485}+\frac{20736 i}{6485}\right) \pi\right. \\
& \left.\operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)}{\pi}\right)\right) d t
\end{aligned}
$$

$\left(\left(\left(\left(1+12^{\wedge}\right) * 4^{*} \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge}\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)$ )) )) $+47+$ golden ratio

Where 47 is a Lucas number

## Input:

$\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi$

## Exact result:

$\phi+48+82944 \pi\left(\frac{\operatorname{coth}(\pi)}{20737}+\frac{\operatorname{coth}(2 \pi)}{10376}+\frac{\operatorname{coth}(3 \pi)}{6939}\right)$

## Decimal approximation:

124.8967800858686743853294116168147932978820912174066600048...
124.89678008586.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternate forms:

$$
\begin{aligned}
& \frac{1}{13824486946}(670487616881+6912243473 \sqrt{5}+55295281152 \pi \operatorname{coth}(\pi)+ \\
& 110510625024 \pi \operatorname{coth}(2 \pi)+165248342016 \pi \operatorname{coth}(3 \pi))
\end{aligned}
$$

$$
\frac{97}{2}+\frac{\sqrt{5}}{2}+\frac{82944 \pi \operatorname{coth}(\pi)}{20737}+\frac{10368 \pi \operatorname{coth}(2 \pi)}{1297}+\frac{3072}{257} \pi \operatorname{coth}(3 \pi)
$$

$$
\frac{1}{2}(97+\sqrt{5})+
$$

$$
\frac{384 \pi(71999064 \operatorname{coth}(\pi)+143894043 \operatorname{coth}(2 \pi)+215167112 \operatorname{coth}(3 \pi))}{6912243473}
$$

## Alternative representations:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\
& \quad 48+\phi+4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}+12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}+12^{4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\
& 48+\phi+4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}+12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}+12^{4}}\right)
\end{aligned}
$$

$$
\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi=
$$

$$
48+\phi+4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}+12^{4}}+\frac{2\left(1+\frac{2}{-1+e^{4 \pi}}\right)}{2^{4}+12^{4}}+\frac{3\left(1+\frac{2}{-1+e^{6 \pi}}\right)}{3^{4}+12^{4}}\right)
$$

## Series representations:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\
& 48+\phi+\sum_{k=-\infty}^{\infty} \frac{2304\left(1294010737+1173562562 k^{2}+167548081 k^{4}\right)}{6912243473\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)} \\
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\
& \frac{414604373872}{6912243473}+\phi+\sum_{k=1}^{\infty}\left(\frac{165888}{20737\left(1+k^{2}\right)}+\frac{41472}{1297\left(4+k^{2}\right)}+\frac{18432}{257\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi=
$$

$$
48+\phi+\frac{165527124096 \pi}{6912243473}+
$$

$$
\sum_{k=0}^{\infty}\left(\frac{6144}{257} e^{-6(1+k) \pi} \pi+\frac{20736 e^{-4(1+k) \pi} \pi}{1297}+\frac{165888 e^{-2(1+k) \pi} \pi}{20737}\right)
$$

## Integral representation:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right)+47+\phi= \\
& 48+\phi+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{3072}{257} \pi \operatorname{csch}^{2}(t)+\right. \\
& \left(\frac{13}{37}-\frac{4 i}{37}\right)\left(-\frac{82944 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{20737}-\left(\frac{93312}{6485}+\frac{20736 i}{6485}\right)\right. \\
& \left.\pi \operatorname{csch}^{2}\left(\frac{\left(\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)\right.}{\pi}\right)\right) d t
\end{aligned}
$$

$\left(\left(\left(\left(1+12^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{coth}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+(2 \operatorname{coth}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{coth}(3 \mathrm{Pi})) /\left(3^{\wedge}\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.4+12^{\wedge} 4\right)\right)\right)\right)\right)()\right)\right)\right)\right)^{*} 2-13$

Where 13 is a Fibonacci number

## Input:

$\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) \times 2-13$

## Exact result:

$2\left(1+82944 \pi\left(\frac{\operatorname{coth}(\pi)}{20737}+\frac{\operatorname{coth}(2 \pi)}{10376}+\frac{\operatorname{coth}(3 \pi)}{6939}\right)\right)-13$

## Decimal approximation:

139.5574921942375590742496495648983103603235640752017942854...
$139.557492 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$$
\begin{aligned}
& \frac{1}{6912243473}(-76034678203+55295281152 \pi \operatorname{coth}(\pi)+ \\
& 110510625024 \pi \operatorname{coth}(2 \pi)+165248342016 \pi \operatorname{coth}(3 \pi))
\end{aligned}
$$

$-11+\frac{165888 \pi \operatorname{coth}(\pi)}{20737}+\frac{20736 \pi \operatorname{coth}(2 \pi)}{1297}+\frac{6144}{257} \pi \operatorname{coth}(3 \pi)$
$\frac{-295854779+215156736 \pi \operatorname{coth}(\pi)+430002432 \pi \operatorname{coth}(2 \pi)}{26895889}+\frac{6144}{257} \pi \operatorname{coth}(3 \pi)$

## Alternative representations:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& \quad-13+2\left(1+4 \pi 12^{4}\left(\frac{i \cot (i \pi)}{1^{4}+12^{4}}+\frac{2 i \cot (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \cot (3 i \pi)}{3^{4}+12^{4}}\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& \quad-13+2\left(1+4 \pi 12^{4}\left(-\frac{i \cot (-i \pi)}{1^{4}+12^{4}}-\frac{2 i \cot (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \cot (-3 i \pi)}{3^{4}+12^{4}}\right)\right)
\end{aligned}
$$

$$
\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13=
$$

$$
-13+2\left(1+4 \pi 12^{4}\left(\frac{1+\frac{2}{-1+e^{2 \pi}}}{1^{4}+12^{4}}+\frac{2\left(1+\frac{\frac{1}{2}}{-1+e^{4 \pi}}\right)}{2^{4}+12^{4}}+\frac{3\left(1+\frac{2}{\left.-1+e^{6 \pi}\right)}\right)}{3^{4}+12^{4}}\right)\right)
$$

## Series representations:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& -11+\sum_{k=-\infty}^{\infty}\left(\frac{165888}{20737\left(1+k^{2}\right)}+\frac{41472}{1297\left(4+k^{2}\right)}+\frac{18432}{257\left(9+k^{2}\right)}\right) \\
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& \frac{8959869133}{6912243473}+\sum_{k=1}^{\infty}\left(\frac{331776}{20737\left(1+k^{2}\right)}+\frac{82944}{1297\left(4+k^{2}\right)}+\frac{36864}{257\left(9+k^{2}\right)}\right) \\
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& -11+\frac{331054248192 \pi}{6912243473}+ \\
& \sum_{k=0}^{\infty}\left(\frac{12288}{257} e^{-6(1+k) \pi} \pi+\frac{41472 e^{-4(1+k) \pi} \pi}{1297}+\frac{331776 e^{-2(1+k) \pi} \pi}{20737}\right)
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{coth}(\pi)}{1^{4}+12^{4}}+\frac{2 \operatorname{coth}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{coth}(3 \pi)}{3^{4}+12^{4}}\right)\right) 2-13= \\
& -11+\int_{\frac{i \pi}{2}}^{3 \pi}\left(-\frac{6144}{257} \pi \operatorname{csch}^{2}(t)+\left(\frac{13}{37}-\frac{4 i}{37}\right)\right. \\
& \left(-\frac{165888 \pi \operatorname{csch}^{2}\left(\frac{\left(\frac{12}{37}+\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)}{\pi}\right)}{20737}-\left(\frac{186624}{6485}+\frac{41472 i}{6485}\right)\right. \\
& \\
& \left.\pi \operatorname{csch}^{2}\left(\frac{\left(\frac{4}{5}+\frac{2 i}{5}\right)\left(\frac{i \pi^{2}}{2}+\left(\frac{25}{37}-\frac{2 i}{37}\right)\left(-i \pi^{2}-\left(1-\frac{i}{2}\right) \pi t\right)\right)}{\pi}\right)\right) d t
\end{aligned}
$$

And:

$1+12^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{cosech}(\mathrm{Pi}) /\left(1^{\wedge} 4-12^{\wedge} 4\right)-(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)$

## Input:

$1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)$

## Exact result:

$1+82944 \pi\left(-\frac{\operatorname{csch}(\pi)}{20735}+\frac{\operatorname{csch}(2 \pi)}{10360}-\frac{\operatorname{csch}(3 \pi)}{6885}\right)$

## Decimal approximation:

-0.00033643634739567899698155811973395443437640261934855899...
$-0.000336436347 .$.

## Alternate forms:

$\frac{1}{91296205}(91296205-365202432 \pi \operatorname{csch}(\pi)+730933632 \pi \operatorname{csch}(2 \pi)-$ $1099850752 \pi \operatorname{csch}(3 \pi))$
$1-\frac{82944 \pi \operatorname{csch}(\pi)}{20735}+\frac{10368 \pi \operatorname{csch}(2 \pi)}{1295}-\frac{1024}{85} \pi \operatorname{csch}(3 \pi)$
$\frac{5370365-21482496 \pi \operatorname{csch}(\pi)+42996096 \pi \operatorname{csch}(2 \pi)}{5370365}-\frac{1024}{85} \pi \operatorname{csch}(3 \pi)$

## Alternative representations:

$$
\begin{aligned}
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1+4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}-12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}-12^{4}}\right) \\
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1+4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}-12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}-12^{4}}\right) \\
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1+4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}-12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}-12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}-12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& 1+\sum_{k=-\infty}^{\infty}-\frac{768(-1)^{k}\left(17172775+8628542 k^{2}+2868343 k^{4}\right)}{91296205\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)} \\
& 1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)= \\
& -\frac{165033797}{54777723}+\sum_{k=1}^{\infty} \frac{1536(-1)^{k}\left(-\frac{475524}{1+k^{2}}+\frac{1903473}{4+k^{2}}-\frac{4296292}{9+k^{2}}\right)}{91296205}
\end{aligned}
$$

$$
1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)=
$$

$$
1+\sum_{k=0}^{\infty}-\frac{256 e^{-3(\pi+2 k \pi)}\left(8592584-5710419 e^{\pi+2 k \pi}+2853144 e^{2 \pi+4 k \pi}\right) \pi}{91296205}
$$

$-1 /\left(\left(\left(\left(\left(1+12^{\wedge} 4^{*} 4^{*} \operatorname{Pi}\left(\left(\left(\left(\left(\operatorname{cosech}(\mathrm{Pi}) /\left(1^{\wedge} 4-12^{\wedge} 4\right)-(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)+11$
Where 11 is a Lucas number and the number of dimensions of M-Theory

## Input:

$-\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11$

## Exact result:

$$
11-\frac{1}{1+82944 \pi\left(-\frac{\operatorname{csch}(\pi)}{20735}+\frac{\operatorname{csch}(2 \pi)}{10360}-\frac{\operatorname{csch}(3 \pi)}{6885}\right)}
$$

## Decimal approximation:

2983.330450443011345236626372998106078723434944496179850604...
2983.330450443 ... result very near to the rest mass of Charmed eta meson 2980.3

## Alternate forms:

$11+91296205 /(-91296205+365202432 \pi \operatorname{csch}(\pi)-730933632 \pi \operatorname{csch}(2 \pi)+$ $1099850752 \pi \operatorname{csch}(3 \pi))$

$$
\begin{aligned}
& 11-\frac{1}{1+82944 \pi\left(\operatorname{csch}(\pi)\left(\frac{\operatorname{sech}(\pi)}{20720}-\frac{1}{20735}\right)-\frac{\operatorname{csch}(3 \pi)}{6885}\right)} \\
& 11-\frac{1}{1-\frac{82944 \pi \operatorname{csch}(\pi)}{20735}-\frac{1024 \pi}{85\left(\sinh ^{3}(\pi)+3 \sinh (\pi) \cosh ^{2}(\pi)\right)}+\frac{5184 \pi \operatorname{csch}(\pi) \operatorname{sech}(\pi)}{1295}}
\end{aligned}
$$

## Alternative representations:

$$
\begin{gathered}
-\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
11-\frac{11}{1+4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}-12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}-12^{4}}\right)} \\
-\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
11-\frac{1}{1+4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}-12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}-12^{4}}\right)}
\end{gathered}
$$

$$
\begin{aligned}
& -\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
& 11-\frac{1}{1+4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}-12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}-12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}-12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
& 11-\frac{1{ }^{2}-82944 \pi \sum_{k=-\infty}^{\infty}-\frac{(-1)^{k}\left(17172775+8628542 k^{2}+2868343 k^{4}\right)}{9859990140\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right) \pi}}{1+8} \\
& -\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
& 11-\frac{1}{1+82944 \pi \sum_{k=0}^{\infty}\left(-\frac{2 e^{-3 \pi-6 k \pi}}{6885}+\frac{e^{-2 \pi-4 k \pi}}{5180}-\frac{2 e^{-\pi-2 k \pi}}{20735}\right)} \\
& -\frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11= \\
& 11-1 /\left(1+82944 \pi \sum_{k=0}^{\infty}-\left(\left(( \mathrm { Li } _ { - k } ( - e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( e ^ { z _ { 0 } } ) ) \left(2853144\left(\pi-z_{0}\right)^{k}-5710419\right.\right.\right.\right. \\
& \left.\left.\left(2 \pi-z_{0}\right)^{k}+8592584\left(3 \pi-z_{0}\right)^{k}\right)\right) / \\
& (59159940840 k!))) \text { for } \frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

$-1 / 24^{*} 1 /\left(\left(\left(\left(\left(1+12^{\wedge} 4^{*} 4^{*} \operatorname{Pi}\left(\left(\left(\left((\operatorname{cosech}(\mathrm{Pi})) /\left(1^{\wedge} 4-12^{\wedge} 4\right)-(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)+11$

Where 11 is a Lucas number and the number of dimensions of M-Theory and 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

$$
\begin{aligned}
& \text { Input: } \\
& -\frac{1}{24} \times \frac{1}{1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)}+11
\end{aligned}
$$

## Exact result:

$11-\frac{1}{24\left(1+82944 \pi\left(-\frac{\operatorname{csch}(\pi)}{20735}+\frac{\operatorname{csch}(2 \pi)}{10360}-\frac{\operatorname{csch}(3 \pi)}{6885}\right)\right)}$

## Decimal approximation:

134.8471021017921393848594322082544199468097893540074937751...
$134.847102101 \ldots$ result practically equal to the rest mass of Pion meson 134.9766

## Alternate forms:

$11+91296205 /(24(-91296205+365202432 \pi \operatorname{csch}(\pi)-$
$730933632 \pi \operatorname{csch}(2 \pi)+1099850752 \pi \operatorname{csch}(3 \pi)))$
$11-\frac{1}{24\left(1+82944 \pi\left(\operatorname{csch}(\pi)\left(\frac{\operatorname{scch}(\pi)}{20720}-\frac{1}{20735}\right)-\frac{\operatorname{csch}(3 \pi)}{6885}\right)\right)}$
$11-\frac{1}{24\left(1-\frac{82944 \pi \operatorname{csch}(\pi)}{20735}-\frac{1024 \pi}{85\left(\sinh ^{3}(\pi)+3 \sinh (\pi) \cosh ^{2}(\pi)\right)}+\frac{5184 \pi \operatorname{csch}(\pi) \operatorname{sech}(\pi)}{1295}\right)}$

## Alternative representations:

$$
\begin{aligned}
& -\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11= \\
& 11-\frac{11}{24\left(1+4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}-12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}-12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}-12^{4}}\right)\right)}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{coch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11= \\
& 11-\frac{24\left(1+4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}-12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}-12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}-12^{4}}\right)\right)}{24}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11= \\
& 11-\frac{1}{24\left(1+4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}-12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}-12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}-12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)\right)}
\end{aligned}
$$

## Series representations:

$-\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11=$
$11+\frac{91296205}{8\left(825168985+\sum_{k=1}^{\infty} \frac{4608(-1)^{k}\left(17172775+8628542 k^{2}+2868343 k^{4}\right)}{\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right)}\right)}$

$$
\begin{aligned}
& -\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11= \\
& 11-\frac{1}{24\left(1+82944 \pi \sum_{k=-\infty}^{\infty}-\frac{(-1)^{k}\left(17172775+8628542 k^{2}+2868343 k^{4}\right)}{9859990140\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right) \pi}\right)}
\end{aligned}
$$

$$
-\frac{1}{\left(1+12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}-12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}-12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}-12^{4}}\right)\right) 24}+11=
$$

$$
11-\frac{1}{24\left(1+82944 \pi \sum_{k=0}^{\infty}\left(-\frac{2 e^{-3 \pi-6 k \pi}}{6885}+\frac{e^{-2 \pi-4 k \pi}}{5180}-\frac{2 e^{-\pi-2 k \pi}}{20735}\right)\right)}
$$

And:

$1-12^{\wedge} 4^{*} 4 * \operatorname{Pi}\left(\left(\left(() \operatorname{cosech}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.$
$\left.\left.\left.\left.\left.(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)$

## Input:

$1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)$
$\operatorname{csch}(x)$ is the hyperbolic cosecant function

## Exact result:

$1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)$

## Decimal approximation:

-0.00032880867775530301888073140480179229202636145261724743...
-0.000328808677...

## Alternate forms:

$\frac{1}{6912243473}(6912243473-27647640576 \pi \operatorname{csch}(\pi)+55255312512 \pi \operatorname{csch}(2 \pi)-$ $82624171008 \pi \operatorname{csch}(3 \pi))$
$1-\frac{82944 \pi \operatorname{csch}(\pi)}{20737}+\frac{10368 \pi \operatorname{csch}(2 \pi)}{1297}-\frac{3072}{257} \pi \operatorname{csch}(3 \pi)$
$\frac{26895889-107578368 \pi \operatorname{csch}(\pi)+215001216 \pi \operatorname{csch}(2 \pi)}{26895889}-\frac{3072}{257} \pi \operatorname{csch}(3 \pi)$

## Alternative representations:

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{\left.2^{4}+12^{4} i \pi\right)}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1-4 \pi 12^{4}\left(\frac{i \operatorname{scc}(i \pi)}{1^{4}+12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}+12^{4}}\right) \\
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1-4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}+12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}+12^{4}}\right)
\end{aligned}
$$

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1-4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}+12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}+12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}+12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1+\sum_{k=-\infty}^{\infty}\left(-\frac{82944(-1)^{k}}{20737\left(1+k^{2}\right)}+\frac{20736(-1)^{k}}{1297\left(4+k^{2}\right)}-\frac{9216(-1)^{k}}{257\left(9+k^{2}\right)}\right) \\
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& -\frac{20649131183}{6912243473}+\sum_{k=1}^{\infty}\left(-\frac{165888(-1)^{k}}{20737\left(1+k^{2}\right)}+\frac{41472(-1)^{k}}{1297\left(4+k^{2}\right)}-\frac{18432(-1)^{k}}{257\left(9+k^{2}\right)}\right)
\end{aligned}
$$

$$
\begin{aligned}
& 1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)= \\
& 1+\sum_{k=0}^{\infty}\left(-\frac{6144}{257} e^{-3 \pi-6 k \pi} \pi+\frac{20736 e^{-2 \pi-4 k \pi} \pi}{1297}-\frac{165888 e^{-\pi-2 k \pi} \pi}{20737}\right)
\end{aligned}
$$

$-1 /\left(\left(\left(1-12^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(()\left(\operatorname{cosech}(\operatorname{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.\left.\left.\left.(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)^{+47+7+\text { golden ratio }}$

## Input:

$$
-\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi
$$

## Exact result:

$\phi+54-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)}$

## Decimal approximation:

3096.900298273126807801702180739848133876304011281727955975
3096.90029827... result practically equal to the rest mass of J/Psi meson 3096.916

## Alternate forms:

$\phi+54-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(3 \pi)}{6939}+\operatorname{csch}(\pi)\left(\frac{1}{20737}-\frac{\operatorname{sech}(\pi)}{20752}\right)\right)}$
$\frac{1}{2}(109+\sqrt{5})+6912243473 /(-6912243473+27647640576 \pi \operatorname{csch}(\pi)-$
$55255312512 \pi \operatorname{csch}(2 \pi)+82624171008 \pi \operatorname{csch}(3 \pi))$
$54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6039 \sinh (3 \pi)}\right)}$

## Alternative representations:

$$
\begin{aligned}
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-\frac{1}{1-4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}+12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}+12^{4}}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-\frac{1}{1-4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}+12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}+12^{4}}\right)} \\
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-\frac{1}{1-4 \pi 12^{4}\left(\frac{2 \pi^{\pi}}{\left(1^{4}+12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}+12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}+12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-\frac{1}{1-82944 \pi \sum_{k=-\infty}^{\infty} \frac{(-1)^{k}\left(430646479+214268942 k^{2}+71618719 k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right) \pi}} \\
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-\frac{1}{1-82944 \pi \sum_{k=0}^{\infty}\left(\frac{2 e^{-3 \pi-6 k \pi}}{6939}-\frac{e^{-2 \pi-4 k \pi}}{5188}+\frac{2 e^{-\pi-2 k \pi}}{20737}\right)} \\
& -\frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+47+7+\phi= \\
& 54+\phi-1 /\left(1-82944 \pi \sum_{k=0}^{\infty}\left(( \mathrm { Li } _ { - k } ( - e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( e ^ { z _ { 0 } } ) ) \left(71999064\left(\pi-z_{0}\right)^{k}-\right.\right.\right. \\
& \left.\left.143894043\left(2 \pi-z_{0}\right)^{k}+215167112\left(3 \pi-z_{0}\right)^{k}\right)\right) /
\end{aligned}
$$

## $-1 / 24^{*} 1 /\left(\left(\left(\left(1-12^{\wedge} 4^{*} 4 * \operatorname{Pi}\left(\left(()\left(\operatorname{cosech}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.\right.\right.\right.\right.$

$\left.\left.\left.\left.\left.\left.\left.\left.\left.(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)-1$

$$
\begin{aligned}
& \text { Input: } \\
& -\frac{1}{24} \times \frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}-1
\end{aligned}
$$

## Exact result:

$-1-\frac{1}{24\left(1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)\right)}$

## Decimal approximation:

125.7200943451823713730623997460617706566076542542467580463...
125.7200943451... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternate forms:

## 6912243473/

(24 (-6912 $243473+27647640576 \pi \operatorname{csch}(\pi)-55255312512 \pi \operatorname{csch}(2 \pi)+$ $82624171008 \pi \operatorname{csch}(3 \pi)))-1$
$-1-\frac{1}{24\left(1-82944 \pi\left(\frac{\operatorname{scch}(3 \pi)}{6939}+\operatorname{csch}(\pi)\left(\frac{1}{20737}-\frac{\operatorname{sech}(\pi)}{20752}\right)\right)\right)}$
$-1-\frac{1}{24\left(1-\frac{82944 \pi \operatorname{csch}(\pi)}{20737}-\frac{3072 \pi}{257\left(\sinh ^{3}(\pi)+3 \sinh (\pi) \cosh ^{2}(\pi)\right)}+\frac{5184 \pi \operatorname{csch}(\pi) \operatorname{sch}(\pi)}{1297}\right)}$

## Alternative representations:

$$
\begin{aligned}
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1= \\
& -1-\frac{1}{24\left(1-4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}+12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}+12^{4}}\right)\right)} \\
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1= \\
& -1-\frac{1}{24\left(1-4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}+12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}+12^{4}}\right)\right)}
\end{aligned}
$$

$$
-\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1=
$$

$$
-1-\frac{1}{24\left(1-4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}+12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}+12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}+12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)\right)}
$$

## Series representations:

$$
\begin{aligned}
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1= \\
& -1-\frac{1}{24\left(1-82944 \pi \sum_{k=-\infty}^{\infty} \frac{(-1)^{k}\left(430646479+214268942 k^{2}+71618719 k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right) \pi}\right)} \\
& \begin{array}{l}
-\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1= \\
-1-\frac{1 l^{24}\left(1-82944 \pi \sum_{k=0}^{\infty}\left(\frac{2 e^{-3 \pi-6 k \pi}}{6939}-\frac{e^{-2 \pi-4 k \pi}}{5188}+\frac{2 e^{-\pi-2 k \pi}}{20737}\right)\right)}{}
\end{array} \\
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}-1= \\
& -1-1 / 24\left(1-82944 \pi \sum_{k=0}^{\infty}\left(( \mathrm { Li } _ { - k } ( - e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( e ^ { z _ { 0 } } ) ) \left(71999064\left(\pi-z_{0}\right)^{k}-\right.\right.\right. \\
& \left.\left.143894043\left(2 \pi-z_{0}\right)^{k}+215167112\left(3 \pi-z_{0}\right)^{k}\right)\right) / \\
& (1493044590168 k!)) \text { for } \frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

$-12^{\wedge} 4 * 4 * \operatorname{Pi}\left(\left(()\left(\operatorname{cosech}(\mathrm{Pi}) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.\left.\left.\left.(2 \operatorname{cosech}(2 \mathrm{Pi})) /\left(2^{\wedge} 4+12^{\wedge} 4\right)+(3 \operatorname{cosech}(3 \mathrm{Pi})) /\left(3^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)\right)+13$
Input:
$-\frac{1}{24} \times \frac{1}{1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)}+13$
$\operatorname{csch}(x)$ is the hyperbolic cosecant function

## Exact result:

$13-\frac{1}{24\left(1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)\right)}$

## Decimal approximation:

139.7200943451823713730623997460617706566076542542467580463...
$139.7200943451 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternate forms:

$13+6912243473$ /

$$
(24(-6912243473+27647640576 \pi \operatorname{csch}(\pi)-55255312512 \pi \operatorname{csch}(2 \pi)+
$$ $82624171008 \pi \operatorname{csch}(3 \pi)))$

$$
13-\frac{1}{24\left(1-82944 \pi\left(\frac{\operatorname{csch}(3 \pi)}{6939}+\operatorname{csch}(\pi)\left(\frac{1}{20737}-\frac{\operatorname{sech}(\pi)}{20752}\right)\right)\right)}
$$

$$
13-\frac{1}{24\left(1-\frac{82944 \pi \operatorname{csch}(\pi)}{20737}-\frac{3072 \pi}{257\left(\sinh ^{3}(\pi)+3 \sinh (\pi) \cosh ^{2}(\pi)\right)}+\frac{5184 \pi \operatorname{csch}(\pi) \operatorname{sech}(\pi)}{1297}\right)}
$$

## Alternative representations:

$$
\begin{aligned}
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}+13= \\
& 13-\frac{1}{24\left(1-4 \pi 12^{4}\left(\frac{i \csc (i \pi)}{1^{4}+12^{4}}-\frac{2 i \csc (2 i \pi)}{2^{4}+12^{4}}+\frac{3 i \csc (3 i \pi)}{3^{4}+12^{4}}\right)\right)} \\
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}+13= \\
& 13-\frac{1}{24\left(1-4 \pi 12^{4}\left(-\frac{i \csc (-i \pi)}{1^{4}+12^{4}}+\frac{2 i \csc (-2 i \pi)}{2^{4}+12^{4}}-\frac{3 i \csc (-3 i \pi)}{3^{4}+12^{4}}\right)\right)} \\
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}+13= \\
& 13-\frac{1}{24\left(1-4 \pi 12^{4}\left(\frac{2 e^{\pi}}{\left(1^{4}+12^{4}\right)\left(-1+e^{2 \pi}\right)}-\frac{4 e^{2 \pi}}{\left(2^{4}+12^{4}\right)\left(-1+e^{4 \pi}\right)}+\frac{6 e^{3 \pi}}{\left(3^{4}+12^{4}\right)\left(-1+e^{6 \pi}\right)}\right)\right)}
\end{aligned}
$$

## Series representations:

$-\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}+13=$
$\left.13-\frac{24\left(1-82944 \pi \sum_{k=-\infty}^{\infty} \frac{(-1)^{k}\left(430646479+214268942 k^{2}+71618719 k^{4}\right)}{248840765028\left(1+k^{2}\right)\left(4+k^{2}\right)\left(9+k^{2}\right) \pi}\right)}{2}\right)$

$$
-\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csch}(3 \pi)}{1^{4}+12^{4}}\right)\right) 24}+13=
$$

$$
13-\frac{1}{24\left(1-82944 \pi \sum_{k=0}^{\infty}\left(\frac{2 e^{-3 \pi-6 k \pi}}{6939}-\frac{e^{-2 \pi-4 k \pi}}{5188}+\frac{2 e^{-\pi-2 k \pi}}{20737}\right)\right)}
$$

$$
\begin{aligned}
& -\frac{1}{\left(1-12^{4} \times 4 \pi\left(\frac{\operatorname{csch}(\pi)}{1^{4}+12^{4}}-\frac{2 \operatorname{csch}(2 \pi)}{2^{4}+12^{4}}+\frac{3 \operatorname{csh}(3 \pi)}{3^{4}+12^{4}}\right)\right) 24}+13= \\
& 13-1 /\left(2 4 \left(1-82944 \pi \sum_{k=0}^{\infty}\left(( \mathrm { Li } _ { - k } ( - e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( e ^ { z _ { 0 } } ) ) \left(71999064\left(\pi-z_{0}\right)^{k}-\right.\right.\right.\right. \\
& \left.\left.143894043\left(2 \pi-z_{0}\right)^{k}+215167112\left(3 \pi-z_{0}\right)^{k}\right)\right) / \\
& (1493044590168 k!))) \text { for } \frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$


$\left(\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4-12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3 \wedge 4-12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4-\right.\right.\right.\right.\right.\right.$ 12^4))))

## Input:

$$
\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}-12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}-12^{4}}
$$

## Exact result:

$$
-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}
$$

## Decimal approximation:

-0.00024116380692975031845195053018781805637144094896405406...
-0.0002411638...

## Property:

$-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}$ is a transcendental number

## Alternate forms:

$\underline{-250614 \tanh \left(\frac{\pi}{2}\right)-377377 \tanh (3 \pi)-645975 \tanh (5 \pi)}$

$$
\begin{aligned}
& \frac{-2754 \tanh \left(\frac{\pi}{2}\right)-4147 \tanh (3 \pi)}{57104190}-\frac{5 \tanh (5 \pi)}{40222} \\
& -\frac{\sinh (\pi)}{20735(1+\cosh (\pi))}-\frac{\sinh (6 \pi)}{13770(1+\cosh (6 \pi))}-\frac{5 \sinh (10 \pi)}{40222(1+\cosh (10 \pi))}
\end{aligned}
$$

## Alternative representations:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{5\left(-1+\frac{2}{1+e^{-10 \pi}}\right)}{2\left(5^{4}-12^{4}\right)}+\frac{3\left(-1+\frac{2}{1+e^{-6 \pi}}\right)}{2\left(3^{4}-12^{4}\right)}+\frac{-1+\frac{2}{1+e^{-\pi}}}{1^{4}-12^{4}} \\
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{1}{\operatorname{coth}\left(\frac{\pi}{2}\right)\left(1^{4}-12^{4}\right)}+\frac{3}{2 \operatorname{coth}(3 \pi)\left(3^{4}-12^{4}\right)}+\frac{5}{2 \operatorname{coth}(5 \pi)\left(5^{4}-12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{\operatorname{coth}\left(\frac{\pi}{2}-\frac{i \pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \operatorname{coth}\left(3 \pi-\frac{i \pi}{2}\right)}{2\left(3^{4}-12^{4}\right)}+\frac{5 \operatorname{coth}\left(5 \pi-\frac{i \pi}{2}\right)}{2\left(5^{4}-12^{4}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \sum_{k=1}^{\infty}-\frac{\frac{4}{20735\left(1+(1-2 k)^{2}\right)}+\frac{100}{2295\left(37-4 k+4 k^{2}\right)}+\frac{10011\left(101-4 k+4 k^{2}\right)}{20}}{\pi} \\
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& -\frac{636983}{2598240645}+\sum_{k=0}^{\infty}\left(\frac{e^{(-6-(6-i) k) \pi}}{6885}+\frac{2 e^{(-1-(1-i) k) \pi}}{20735}+\frac{5(-1)^{k} e^{-10(1+k) \pi}}{20111}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \sum_{k=0}^{\infty}\left(\frac{\left(\delta_{k}+\frac{2^{2+k} \mathrm{Li}_{k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(\frac{\pi}{2}-z_{0}\right)^{k}}{20735}+\frac{\left(\delta_{k}+\frac{2^{1+k_{\mathrm{Li}}}-\left(-e^{2 z_{0}}\right)}{k!}\right)\left(3 \pi-z_{0}\right)^{k}}{13770}+\right. \\
& \left.\quad \frac{5\left(\delta_{k}+\frac{2^{1+k_{\mathrm{Li}}}-k\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5 \pi-z_{0}\right)^{k}}{40222}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \quad \int_{0}^{5 \pi}\left(\frac{1}{10}\left(-\frac{\operatorname{sech}^{2}\left(\frac{t}{10}\right)}{20735}-\frac{\operatorname{sech}^{2}\left(\frac{3 t}{5}\right)}{2295}\right)-\frac{5 \operatorname{sech}^{2}(t)}{40222}\right) d t
\end{aligned}
$$

$\left(\left(\left(-1 /\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4-12^{\wedge}\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4-12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.$ 12^4)))))))))-521-4

Where 521 and 4 are Lucas numbers. Note that $521=496+25$, where 496 is the dimension of Lie's Group $\mathrm{E}_{8} \times \mathrm{E}_{8}$ and 25 corresponding to the dimensions of a D-25 brane

## Input:

$-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}-12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}-12^{4}}}-521-4$

## Exact result:

$-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}}$

## Decimal approximation:

3621.559190331965785566481981872280066466747509278894151676...
$3621.55919 \ldots$ result practically equal to the rest mass of double charmed Xi baryon 3621.40

## Property:

$$
-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}} \text { is a transcendental number }
$$

## Alternate forms:

$\frac{5196481290}{250614 \tanh \left(\frac{\pi}{2}\right)+377377 \tanh (3 \pi)+645975 \tanh (5 \pi)}-525$
$\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}+\frac{\tanh (3 \pi)}{13770}+\frac{5 \tanh (5 \pi)}{40222}}-525$
$-\frac{105\left(-49490298+1253070 \tanh \left(\frac{\pi}{2}\right)+1886885 \tanh (3 \pi)+3229875 \tanh (5 \pi)\right)}{250614 \tanh \left(\frac{\pi}{2}\right)+377377 \tanh (3 \pi)+645975 \tanh (5 \pi)}$

## Alternative representations:

$$
\begin{aligned}
-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}}-521-4= \\
-525-\frac{1}{\frac{1}{\operatorname{coth}\left(\frac{\pi}{2}\right)\left(1^{4}-12^{4}\right)}+\frac{3}{2 \operatorname{coth}(3 \pi)\left(3^{4}-12^{4}\right)}+\frac{5}{2 \operatorname{coth}(5 \pi)\left(5^{4}-12^{4}\right)}}
\end{aligned}
$$

$$
\begin{aligned}
-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}}+\frac{\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right)^{2}}}{}-521-4= \\
-525-\frac{1}{\frac{5\left(-1+\frac{2}{1+e^{-10 \pi}}\right)}{2\left(5^{4}-12^{4}\right)}+\frac{3\left(-1+\frac{2}{1+e^{-6 \pi}}\right)}{2\left(3^{4}-12^{4}\right)}+\frac{-1+\frac{2}{1+e^{-\pi}}}{1^{4}-12^{4}}}
\end{aligned}
$$

$$
-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}-12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}-12^{4}\right) 2}}-521-4=
$$

$$
-525-\frac{1}{\frac{\operatorname{coth}\left(\frac{\pi}{2}-\frac{i \pi}{2}\right)}{1^{4}-12^{4}}+\frac{3 \operatorname{coth}\left(3 \pi-\frac{i \pi}{2}\right)}{2\left(3^{4}-12^{4}\right)}+\frac{5 \operatorname{coth}\left(5 \pi-\frac{i \pi}{2}\right)}{2\left(5^{4}-12^{4}\right)}}
$$

1/golden ratio $+1 / 29^{*}\left(\left(\left(\left(\left(-1 /\left(\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4-12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)-521-4\right)\right)$

Where 29 is a Lucas numbers

## Input:

$\frac{1}{\phi}+\frac{1}{29}\left(-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}-12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}-12^{4}}}-521-4\right)$

## Exact result:

$\frac{1}{\phi}+\frac{1}{29}\left(-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}}\right)$

## Decimal approximation:

125.4993853795073357298074137954787438579529819135607336096...
125.49938537... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Property:

$\frac{1}{\phi}+\frac{1}{29}\left(-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}}\right)$ is a transcendental number
$8+2+1 / 29^{*}\left(\left(\left(\left(\left(-1 /\left(\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4-12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4-\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)\right)\right)-521-4\right)\right)$
Where 8 and 2 are Fibonacci numbers

## Input:

$8+2+\frac{1}{29}\left(-\frac{1}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}-12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}-12^{4}}}-521-4\right)$

## Exact result:

$10+\frac{1}{29}\left(-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}}\right)$

## Decimal approximation:

134.8813513907574408816028269611131057402326727337549707474...
$134.88135139 \ldots$ result practically equal to the rest mass of Pion meson 134.9766

## Property:

$10+\frac{1}{29}\left(-525-\frac{1}{-\frac{\tanh \left(\frac{\pi}{2}\right)}{20735}-\frac{\tanh (3 \pi)}{13770}-\frac{5 \tanh (5 \pi)}{40222}}\right)$ is a transcendental number

$\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4+12^{\wedge} 4\right)\right.\right.\right.\right.$ )))

## Input:

$\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}+12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}+12^{4}}$

## Exact result:

$$
\frac{\tanh \left(\frac{\pi}{2}\right)}{20737}+\frac{\tanh (3 \pi)}{13878}+\frac{5 \tanh (5 \pi)}{42722}
$$

## Decimal approximation:

0.000233320032211875296176516082527934356176673416489630365 .
$0.0002333200322 \ldots$

## Property:

$\frac{\tanh \left(\frac{\pi}{2}\right)}{20737}+\frac{\tanh (3 \pi)}{13878}+\frac{5 \tanh (5 \pi)}{42722}$ is a transcendental number

## Alternate forms:

$296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)$ 6147441305046
$\frac{13878 \tanh \left(\frac{\pi}{2}\right)+20737 \tanh (3 \pi)}{287788086}+\frac{5 \tanh (5 \pi)}{42722}$
$\frac{\sinh (\pi)}{20737(1+\cosh (\pi))}+\frac{\sinh (6 \pi)}{13878(1+\cosh (6 \pi))}+\frac{5 \sinh (10 \pi)}{42722(1+\cosh (10 \pi))}$

Alternative representations:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{5\left(-1+\frac{2}{1+e^{-10 \pi}}\right)}{2\left(5^{4}+12^{4}\right)}+\frac{3\left(-1+\frac{2}{1+e^{-6 \pi}}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{-1+\frac{2}{1+e^{-\pi}}}{1^{4}+12^{4}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{1}{\operatorname{coth}\left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}+\frac{3}{2 \operatorname{coth}(3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5}{2 \operatorname{coth}(5 \pi)\left(5^{4}+12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{\operatorname{coth}\left(\frac{\pi}{2}-\frac{i \pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \operatorname{coth}\left(3 \pi-\frac{i \pi}{2}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \operatorname{coth}\left(5 \pi-\frac{i \pi}{2}\right)}{2\left(5^{4}+12^{4}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \sum_{k=1}^{\infty} \frac{\frac{4}{20737\left(1+(1-2 k)^{2}\right)}+\frac{4}{2313\left(37-4 k+4 k^{2}\right)}+\frac{100}{21361\left(101-4 k+4 k^{2}\right)}}{\pi}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{729440615}{3073720652523}+\sum_{k=0}^{\infty}\left(-\frac{e^{(-6-(6-i) k) \pi}}{6939}-\frac{2 e^{(-1-(1-i) k) \pi}}{20737}-\frac{5(-1)^{k} e^{-10(1+k) \pi}}{21361}\right)
\end{aligned}
$$

$$
\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}=
$$

$$
\sum_{k=0}^{\infty}\left(-\frac{\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(\frac{\pi}{2}-z_{0}\right)^{k}}{20737}-\frac{\left(\delta_{k}+\frac{2^{1+k} \mathrm{~L}_{-}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(3 \pi-z_{0}\right)^{k}}{13878}-\right.
$$

$$
\left.\frac{5\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5 \pi-z_{0}\right)^{k}}{42722}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \int_{0}^{5 \pi}\left(\frac{1}{10}\left(\frac{\operatorname{sech}^{2}\left(\frac{t}{10}\right)}{20737}+\frac{\operatorname{sech}^{2}\left(\frac{3 t}{5}\right)}{2313}\right)+\frac{5 \operatorname{sech}^{2}(t)}{42722}\right) d t
\end{aligned}
$$

## $0.256 /\left(\left(\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2)) /\left(5^{\wedge} 4\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.+12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)+18$

Where 18 is a Lucas number and $0.256=(64 * 4) / 10^{3}$

## Input:

$$
\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}+12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}+12^{4}}}+18
$$

## Result:

1115.21.
$1115.21 \ldots$ result practically equal to the rest mass of Lambda baryon 1115.683

## Alternative representations:

$\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18=$
$18+\frac{0.256}{\frac{1}{\operatorname{coth}\left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}+\frac{3}{2 \operatorname{coth}(3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5}{2 \operatorname{coth}(5 \pi)\left(5^{4}+12^{4}\right)}}$
$\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18=18+\frac{0.256}{\frac{5\left(-1+\frac{2}{1+e^{-10 \pi}}\right)}{2\left(5^{4}+12^{4}\right)}+\frac{3\left(-1+\frac{2}{1+e^{-6 \pi}}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{-1+\frac{2}{1+e^{-\pi}}}{1^{4}+12^{4}}}$
$\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18=18+\frac{0.256}{\frac{\operatorname{coth}\left(\frac{\pi}{2}-\frac{i \pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \operatorname{coth}\left(3 \pi-\frac{i \pi}{2}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \operatorname{coth}\left(5 \pi-\frac{i \pi}{2}\right)}{2\left(5^{4}+12^{4}\right)}}$

## Series representations:

$$
\begin{aligned}
& \frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18= \\
& 18-\frac{1093.68}{-1.01386+\sum_{k=0}^{\infty}(-1)^{k} e^{-10(1+k) \pi}\left(1+0.615679 e^{4(1+k) \pi}+0.412036 e^{9(1+k) \pi}\right)} \\
& \frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18=18+\frac{1327.17}{\pi} \\
& \frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18=18+ \\
& \frac{\pi \sum_{k=1}^{\infty} \frac{1+(1-2 k)^{2}}{}+\frac{6.06742}{25.25-k+k^{2}}+\frac{8.96541}{37-4 k+4 k^{2}}}{\pi^{2}} \\
& \sum_{k=0}^{\infty}-\frac{\left(k!\delta_{k}+2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)\right)\left(296447958\left(\frac{\pi}{2}-z_{0}\right)^{k}+442963057\left(3 \pi-z_{0}\right)^{k}+719470215\left(5 \pi-z_{0}\right)^{k}\right)}{6147441305046 k!} \\
& \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \sum_{\mathbb{Z}}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{\left(3^{4}+12^{4}\right) 2}+\frac{5 \tanh (5 \pi)}{\left(5^{4}+12^{4}\right) 2}}+18= \\
& 18+\frac{5308.67}{\int_{0}^{5 \pi}\left(0.1 \operatorname{sech}^{2}\left(\frac{t}{10}\right)+0.896541 \operatorname{sech}^{2}\left(\frac{3 t}{5}\right)+2.42697 \operatorname{sech}^{2}(t)\right) d t}
\end{aligned}
$$

$1 / 7^{*}\left(\left(\left(0.256 /\left(\left(\left(\left(\left(\tanh (\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)+((3 \tanh (3 \mathrm{Pi}) / 2)) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+((5 \tanh (5 \mathrm{Pi}) / 2\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.)) /\left(5^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)\right)-76-2\right)\right)\right)-11$

Where 7, 76, 2 and 11 are Lucas numbers ( 11 is also the number of dimensions of M Theory)

## Input:

$$
\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3\left(\frac{1}{2} \tanh (3 \pi)\right)}{3^{4}+12^{4}}+\frac{5\left(\frac{1}{2} \tanh (5 \pi)\right)}{5^{4}+12^{4}}}-76-2\right)-11
$$

## Result:

134.601..
134.601... result practically equal to the rest mass of Pion meson 134.9766

## Percent decrease:

$\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11=134.601$ is 7.55491
\% smaller than $\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)=145.601$.

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11= \\
& -11+\frac{1}{7}\left(-78+\frac{0.256}{\frac{3}{\operatorname{coth}\left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}+\frac{1}{2 \operatorname{coth}(3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5}{2 \operatorname{coth}(5 \pi)\left(5^{4}+12^{4}\right)}}\right)
\end{aligned}
$$

$\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11=$

$$
-11+\frac{1}{7}\left(-78+\frac{0.256}{\frac{5\left(-1+\frac{2}{1+e^{-10 \pi}}\right)}{2\left(5^{4}+12^{4}\right)}+\frac{3\left(-1+\frac{2}{1+e^{-6 \pi}}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{-1+\frac{2}{1+e^{-\pi}}}{1^{4}+12^{4}}}\right)
$$

$$
\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11=
$$

$$
-11+\frac{1}{7}\left(-78+\frac{0.256}{\frac{\operatorname{coth}\left(\frac{\pi}{2}-\frac{i \pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \operatorname{coth}\left(3 \pi-\frac{i \pi}{2}\right)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \operatorname{coth}\left(5 \pi-\frac{i \pi}{2}\right)}{2\left(5^{4}+12^{4}\right)}}\right)
$$

## Series representations:

$$
\frac{\frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11=-\frac{155}{7}-}{\frac{156.24}{-1.01386+\sum_{k=0}^{\infty}(-1)^{k} e^{-10(1+k) \pi}\left(1+0.615679 e^{4(1+k) \pi}+0.412036 e^{9(1+k) \pi}\right)}}
$$

$$
\begin{aligned}
& \frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11= \\
& -\frac{155}{7}+ \\
& \sum_{k=1}^{\infty} \frac{\frac{2}{20737\left(1-2 k+2 k^{2}\right)}+\frac{4}{2313\left(37-4 k+4 k^{2}\right)}+\frac{100}{21361\left(101-4 k+4 k^{2}\right)}}{\pi} \\
& \frac{1}{7}\left(\frac{0.256}{\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}}-76-2\right)-11=-\frac{155}{7}+ \\
& 0.0365714 \\
& \begin{array}{l}
\sum_{k=0}^{\infty}-\frac{\left(k!\delta_{k}+2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)\right)\left(296447958\left(\frac{\pi}{2}-z_{0}\right)^{k}+442963057\left(3 \pi-z_{0} k^{k}+719470215\left(5 \pi-z_{0}\right)^{k}\right)\right.}{6147441305046 k!} \\
\text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{array}
\end{aligned}
$$

Integral representation:

$$
\begin{aligned}
& \frac{1}{7}\left(\frac{0.256}{\left.\frac{\tanh \left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}+\frac{3 \tanh (3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5 \tanh (5 \pi)}{2\left(5^{4}+12^{4}\right)}-76-2\right)-11=}\right. \\
& -\frac{155}{7}+\frac{758.382}{\int_{0}^{5 \pi}\left(0.1 \operatorname{sech}^{2}\left(\frac{t}{10}\right)+0.896541 \operatorname{sech}^{2}\left(\frac{3 t}{5}\right)+2.42697 \operatorname{sech}^{2}(t)\right) d t}
\end{aligned}
$$

Now, we have that:

$\left(\left(\left(\left(\left(1^{\wedge} 3 \operatorname{sech}(\mathrm{Pi} / 2) /\left(1^{\wedge} 4-12^{\wedge} 4\right)-\left(\left(3^{\wedge} 3 \operatorname{sech}(3 \mathrm{Pi}) / 2\right)\right) /\left(3^{\wedge} 4-\right.\right.\right.\right.\right.\right.$
$\left.\left.\left.\left.\left.\left.12^{\wedge} 4\right)+\left(\left(5^{\wedge} 3 \operatorname{sech}(5 \mathrm{Pi}) / 2\right)\right) /\left(5^{\wedge} 4-12^{\wedge} 4\right)\right)\right)\right)\right)\right)$

## Input:

$$
1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3}\left(\frac{1}{2} \operatorname{sech}(3 \pi)\right)}{3^{4}-12^{4}}+\frac{5^{3}\left(\frac{1}{2} \operatorname{sech}(5 \pi)\right)}{5^{4}-12^{4}}
$$

## Exact result:

$$
-\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20735}+\frac{\operatorname{sech}(3 \pi)}{1530}-\frac{125 \operatorname{sech}(5 \pi)}{40222}
$$

## Decimal approximation:

$-0.00001911593496126075908503136511058224125590211372808061$
-0.00001911593496126....

## Property:

$-\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20735}+\frac{\operatorname{sech}(3 \pi)}{1530}-\frac{125 \operatorname{sech}(5 \pi)}{40222}$ is a transcendental number

## Alternate forms:

$$
\frac{-27846 \operatorname{sech}\left(\frac{\pi}{2}\right)+377377 \operatorname{sech}(3 \pi)-1794375 \operatorname{sech}(5 \pi)}{577386810}
$$

$$
\frac{4147 \operatorname{sech}(3 \pi)-306 \operatorname{sech}\left(\frac{\pi}{2}\right)}{6344910}-\frac{125 \operatorname{sech}(5 \pi)}{40222}
$$

$$
-\frac{1}{20735 \cosh \left(\frac{\pi}{2}\right)}+\frac{1}{1530 \cosh (3 \pi)}-\frac{125}{40222 \cosh (5 \pi)}
$$

Alternative representations:

$$
\begin{aligned}
& \frac{\frac{1}{}^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{1}{\cosh \left(\frac{\pi}{2}\right)\left(1^{4}-12^{4}\right)}-\frac{27}{2 \cosh (3 \pi)\left(3^{4}-12^{4}\right)}+\frac{5^{3}}{2 \cosh (5 \pi)\left(5^{4}-12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{1}{\cos \left(\frac{i \pi}{2}\right)\left(1^{4}-12^{4}\right)}-\frac{27}{2 \cos (3 i \pi)\left(3^{4}-12^{4}\right)}+\frac{5^{3}}{2 \cos (5 i \pi)\left(5^{4}-12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \frac{\csc \left(\frac{\pi}{2}+\frac{i \pi}{2}\right)}{1^{4}-12^{4}}-\frac{27 \csc \left(\frac{\pi}{2}+3 i \pi\right)}{2\left(3^{4}-12^{4}\right)}+\frac{\csc \left(\frac{\pi}{2}+5 i \pi\right) 5^{3}}{2\left(5^{4}-12^{4}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \sum_{k=0}^{\infty} \frac{2(-1)^{k}(1+2 k)\left(-\frac{13923}{1+2 k+2 k^{2}}+\frac{377377}{37+4 k+4 k^{2}}-\frac{1794375}{101+4 k+4 k^{2}}\right)}{288693405 \pi}
\end{aligned}
$$

$$
\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}=
$$

$$
\sum_{k=0}^{\infty}-\frac{e^{(-5-(10-i) k) \pi}\left(1794375-377377 e^{2 \pi+4 k \pi}+27846 e^{(9 \pi) / 2+9 k \pi}\right)}{288693405}
$$

$$
\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}=
$$

$$
\sum_{k=0}^{\infty}-\frac{1}{577386810 k!} i\left(\mathrm{Li}_{-k}\left(-i e^{z_{0}}\right)-\mathrm{Li}_{-k}\left(i e^{z_{0}}\right)\right)
$$

$$
\left(27846\left(\frac{\pi}{2}-z_{0}\right)^{k}-377377\left(3 \pi-z_{0}\right)^{k}+1794375\left(5 \pi-z_{0}\right)^{k}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}-12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}-12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}-12^{4}\right) 2}= \\
& \int_{0}^{\infty}-\frac{\left(27846-377377 t^{5 i}+1794375 t^{9 i}\right) t^{i}}{288693405 \pi\left(1+t^{2}\right)} d t
\end{aligned}
$$

$\left(\left(()\left(1^{\wedge} 3 \operatorname{sech}(\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.$
$\left.\left.\left.\left.\left.\left(\left(3^{\wedge} 3 \operatorname{sech}(3 \mathrm{Pi}) / 2\right)\right) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+\left(\left(5^{\wedge} 3 \operatorname{sech}(5 \mathrm{Pi}) / 2\right)\right) /\left(5^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)$

## Input:

$1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3}\left(\frac{1}{2} \operatorname{sech}(3 \pi)\right)}{3^{4}+12^{4}}+\frac{5^{3}\left(\frac{1}{2} \operatorname{sech}(5 \pi)\right)}{5^{4}+12^{4}}$

## Exact result:

$\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{sech}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}$

## Decimal approximation:

0.000019114847340277282671102750452267872320911492891002346...
$0.00001911484734027 \ldots .$.

## Property:

$\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{sech}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}$ is a transcendental number

## Alternate forms:

$32938662 \operatorname{sech}\left(\frac{\pi}{2}\right)-442963057 \operatorname{sech}(3 \pi)+1998528375 \operatorname{sech}(5 \pi)$
683049033894
$\frac{1542 \operatorname{sech}\left(\frac{\pi}{2}\right)-20737 \operatorname{sech}(3 \pi)}{31976454}+\frac{125 \operatorname{sech}(5 \pi)}{42722}$
$\frac{1}{20737 \cosh \left(\frac{\pi}{2}\right)}-\frac{1}{1542 \cosh (3 \pi)}+\frac{125}{42722 \cosh (5 \pi)}$

## Alternative representations:

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{1}{\cosh \left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cosh (3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cosh (5 \pi)\left(5^{4}+12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{1}{\cos \left(\frac{i \pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos (3 i \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos (5 i \pi)\left(5^{4}+12^{4}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \frac{\csc \left(\frac{\pi}{2}+\frac{i \pi}{2}\right)}{1^{4}+12^{4}}-\frac{27 \csc \left(\frac{\pi}{2}+3 i \pi\right)}{2\left(3^{4}+12^{4}\right)}+\frac{\csc \left(\frac{\pi}{2}+5 i \pi\right) 5^{3}}{2\left(5^{4}+12^{4}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \sum_{k=0}^{\infty}\left(\frac{125 e^{(-5-(10-i) k) \pi}}{21361}-\frac{1}{771} e^{(-3-(6-i) k) \pi}+\frac{2 e^{(-1 / 2-(1-i) k) \pi}}{20737}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \sum_{k=0}^{\infty} \frac{2(-1)^{k}(1+2 k)\left(\frac{16469331}{1+2 k+2 k^{2}}-\frac{449263057}{37+4 k+4 k^{2}}+\frac{1908528375}{101+4 k+4 k^{2}}\right)}{341524516947 \pi}
\end{aligned}
$$

$$
\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}=
$$

$$
\sum_{k=0}^{\infty} \frac{1}{683049033894 k!} i\left(\mathrm{Li}_{-k}\left(-i e^{z_{0}}\right)-\mathrm{Li}_{-k}\left(i e^{z_{0}}\right)\right)\left(32938662\left(\frac{\pi}{2}-z_{0}\right)^{k}-\right.
$$

$$
\left.442963057\left(3 \pi-z_{0}\right)^{k}+1998528375\left(5 \pi-z_{0}\right)^{k}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{\left(5^{4}+12^{4}\right) 2}= \\
& \int_{0}^{\infty} \frac{\left(32938662-442963057 t^{5 i}+1998528375 t^{9 i}\right) t^{i}}{341524516947 \pi\left(1+t^{2}\right)} d t
\end{aligned}
$$

$1 / 24^{*} 1 /\left(\left(()\left(1^{\wedge} 3 \operatorname{sech}(\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.$
$\left.\left.\left.\left.\left.\left(\left(3^{\wedge} 3 \operatorname{sech}(3 \mathrm{Pi}) / 2\right)\right) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+\left(\left(5^{\wedge} 3 \operatorname{sech}(5 \mathrm{Pi}) / 2\right)\right) /\left(5^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)-64-\mathrm{Pi}$
Where 24 can be identified with the number of the transverse degrees of freedom in the bosonic string

Input:
$\frac{1}{24} \times \frac{1}{1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3}\left(\frac{1}{2} \operatorname{sech}(3 \pi)\right)}{3^{4}+12^{4}}+\frac{5^{3}\left(\frac{1}{2} \operatorname{sech}(5 \pi)\right)}{5^{4}+12^{4}}}-64-\pi$

## Exact result:

$-64-\pi+\frac{1}{24\left(\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{sech}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}\right)}$

## Decimal approximation:

2112.664812541705066184005071570661311862410928268875704164...
2112.66481254 $\qquad$ result practically equal to the rest mass of strange D meson 2112.3

## Alternate forms:

$-64-\pi+\frac{113841505649}{4\left(32938662 \operatorname{sech}\left(\frac{\pi}{2}\right)-442963057 \operatorname{sech}(3 \pi)+1998528375 \operatorname{sech}(5 \pi)\right)}$
$-64-\pi+\frac{1}{\frac{24 \operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{4}{257} \operatorname{sech}(3 \pi)+\frac{1500 \operatorname{sech}(5 \pi)}{21361}}$
$-64-\pi+\frac{1}{24\left(\frac{1}{20737 \cosh \left(\frac{\pi}{2}\right)}-\frac{1}{1542 \cosh (3 \pi)}+\frac{125}{42722 \cosh (5 \pi)}\right)}$

$\cosh (x)$ is the hyperbolic cosine function

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi= \\
& -64-\pi+\frac{1}{24\left(\frac{1}{\cosh \left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cosh (3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cosh (5 \pi)\left(5^{4}+12^{4}\right)}\right)}
\end{aligned}
$$

$$
\frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi=
$$

$$
-64-\pi+\frac{1}{24\left(\frac{1}{\cos \left(\frac{i \pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos (3 i \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos (5 i \pi)\left(5^{4}+12^{4}\right)}\right)}
$$

$$
\frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi=
$$

$$
-64-\pi+\frac{1}{24\left(\frac{1}{\cos \left(-\frac{i \pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos (-3 i \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos (-5 i \pi)\left(5^{4}+12^{4}\right)}\right)}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi= \\
& -64-\pi+\frac{1}{24 \sum_{k=0}^{\infty} \frac{2(-1)^{k}(1+2 k)\left(\frac{16469331}{1+2 k+2 k^{2}}-\frac{442963057}{37+4 k+4 k^{2}}+\frac{1998528375}{101+4 k+4 k^{2}}\right)}{341524516947 \pi}} \\
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi= \\
& -64-\pi+\frac{1}{24 \sum_{k=0}^{\infty}\left(\frac{125(-1)^{k} e^{-5 \pi-10 k \pi}}{21361}-\frac{1}{771}(-1)^{k} e^{-3 \pi-6 k \pi}+\frac{2(-1)^{k} e^{-\pi / 2-k \pi}}{20737}\right)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi= \\
& -64-\pi+1 /\left(2 4 \sum _ { k = 0 } ^ { \infty } \left(i ( \mathrm { Li } _ { - k } ( - i e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( i e ^ { z _ { 0 } } ) ) \left(32938662\left(\frac{\pi}{2}-z_{0}\right)^{k}-\right.\right.\right. \\
& \left.\left.442963057\left(3 \pi-z_{0}\right)^{k}+1998528375\left(5 \pi-z_{0}\right)^{k}\right)\right) / \\
& (683049033894 k!)) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 24}-64-\pi= \\
& -64-\pi+\frac{1}{24 \int_{0}^{\infty} \frac{\left(32938662-442963057 t^{5 i}+1998528375 t^{9}\right) t^{i}}{341524516947 \pi\left(1+t^{2}\right)} d t}
\end{aligned}
$$

$1 /(256)^{*} 1 /\left(\left(()\left(1^{\wedge} 3 \operatorname{sech}(\mathrm{Pi} / 2) /\left(1^{\wedge} 4+12^{\wedge} 4\right)-\right.\right.\right.$
$\left.\left.\left.\left.\left.\left(\left(3^{\wedge} 3 \operatorname{sech}(3 \mathrm{Pi}) / 2\right)\right) /\left(3^{\wedge} 4+12^{\wedge} 4\right)+\left(\left(5^{\wedge} 3 \operatorname{sech}(5 \mathrm{Pi}) / 2\right)\right) /\left(5^{\wedge} 4+12^{\wedge} 4\right)\right)\right)\right)\right)\right)-64-1 /$ golden ratio

## Input:

$\frac{1}{256} \times \frac{1}{1^{3} \times \frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3}\left(\frac{1}{2} \operatorname{sech}(3 \pi)\right)}{3^{4}+12^{4}}+\frac{5^{3}\left(\frac{1}{2} \operatorname{sech}(5 \pi)\right)}{5^{4}+12^{4}}}-64-\frac{1}{\phi}$
$\operatorname{sech}(x)$ is the hyperbolic secant function $\phi$ is the golden ratio

## Exact result:

$-\frac{1}{\phi}-64+\frac{1}{256\left(\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{scch}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}\right)}$

## Decimal approximation:

139.7388164983089982226517614425663132647741999765927505739...
139.738816498... result practically equal to the rest mass of Pion meson 139.57

## Property:

$-64-\frac{1}{\phi}+\frac{1}{256\left(\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{sech}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}\right)}$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& -\frac{1}{\phi}-64+\frac{1}{\frac{256 \operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{128}{771} \operatorname{sech}(3 \pi)+\frac{16000 \operatorname{sech}(5 \pi)}{21361}} \\
& \frac{1}{2}(-127-\sqrt{5})+\frac{1}{256\left(\frac{\operatorname{sech}\left(\frac{\pi}{2}\right)}{20737}-\frac{\operatorname{sech}(3 \pi)}{1542}+\frac{125 \operatorname{sech}(5 \pi)}{42722}\right)}
\end{aligned}
$$

$$
-64-\frac{1}{\phi}+\frac{1}{256\left(\frac{1}{20737 \cosh \left(\frac{\pi}{2}\right)}-\frac{1}{1542 \cosh (3 \pi)}+\frac{125}{42722 \cosh (5 \pi)}\right)}
$$

$\cosh (x)$ is the hyperbolic cosine function

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}= \\
& -64-\frac{1}{\phi}+\frac{1}{256\left(\frac{1}{\cosh \left(\frac{\pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cosh (3 \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cosh (5 \pi)\left(5^{4}+12^{4}\right)}\right)}
\end{aligned}
$$

$$
\frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}=
$$

$$
-64-\frac{1}{\phi}+\frac{1}{256\left(\frac{1}{\cos \left(\frac{i \pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos (3 i \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos (5 i \pi)\left(5^{4}+12^{4}\right)}\right)}
$$

$$
\frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}=
$$

$$
-64-\frac{1}{\phi}+\frac{1}{256\left(\frac{1}{\cos \left(-\frac{i \pi}{2}\right)\left(1^{4}+12^{4}\right)}-\frac{27}{2 \cos (-3 i \pi)\left(3^{4}+12^{4}\right)}+\frac{5^{3}}{2 \cos (-5 i \pi)\left(5^{4}+12^{4}\right)}\right)}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}= \\
& -64-\frac{1}{\phi}+\frac{1}{256 \sum_{k=0}^{\infty} \frac{2(-1)^{k}(1+2 k)\left(\frac{16469331}{1+2 k+2 k^{2}}-\frac{442963057}{37+4 k+4 k^{2}}+\frac{1998528375}{101+4 k+4 k^{2}}\right)}{341524516947 \pi}} \\
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}= \\
& -64-\frac{1}{\phi}+\frac{1}{256 \sum_{k=0}^{\infty}\left(\frac{125(-1)^{k} e^{-5 \pi-10 k \pi}}{21361}-\frac{1}{771}(-1)^{k} e^{-3 \pi-6 k \pi}+\frac{2(-1)^{k} e^{-\pi / 2-k \pi}}{20737}\right)} \\
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}= \\
& -64-\frac{1}{\phi}+1 /\left(2 5 6 \sum _ { k = 0 } ^ { \infty } \left(i ( \operatorname { L i } _ { - k } ( - i e ^ { z _ { 0 } } ) - \operatorname { L i } _ { - k } ( i e ^ { z _ { 0 } } ) ) \left(32938662\left(\frac{\pi}{2}-z_{0}\right)^{k}-\right.\right.\right. \\
& \left.\left.442963057\left(3 \pi-z_{0}\right)^{k}+1998528375\left(5 \pi-z_{0}\right)^{k}\right)\right) / \\
& (683049033894 k!) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{\left(\frac{1^{3} \operatorname{sech}\left(\frac{\pi}{2}\right)}{1^{4}+12^{4}}-\frac{3^{3} \operatorname{sech}(3 \pi)}{2\left(3^{4}+12^{4}\right)}+\frac{5^{3} \operatorname{sech}(5 \pi)}{2\left(5^{4}+12^{4}\right)}\right) 256}-64-\frac{1}{\phi}= \\
& -64-\frac{1}{\phi}+\frac{1}{256 \int_{0}^{\infty} \frac{\left(32938662-442963057 t^{5 i}+1998528375 t^{9 i}\right) t^{i}}{341524516947 \pi\left(1+t^{2}\right)} d t}
\end{aligned}
$$

From the sum of the results, we obtain:
$(76.6132768639+76.278746097-0.000336436347-0.000328808677-0.0002411638$ $+0.0002333200322-0.00001911593496126+0.00001911484734027)$

## Input interpretation:

```
76.6132768639 + 76.278746097 - 0.000336436347 -
    0.000328808677-0.0002411638+0.0002333200322 -
    0.00001911593496126 +0.00001911484734027
```


## Result:

152.89134987102057901
152.891349871......
$(76.6132768639+76.278746097-0.000336436347-0.000328808677-0.0002411638$ $+0.0002333200322-0.00001911593496126+0.00001911484734027)$-18-7-golden ratio^2

Where 18 and 7 are Lucas numbers

## Input interpretation:

```
(76.6132768639 + 76.278746097-0.000336436347 -
    0.000328808677-0.0002411638+0.0002333200322-
    0.00001911593496126+0.00001911484734027) - 18-7- - }\mp@subsup{}{}{2
```


## Result:

125.27331588...
125.27331588... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternative representations:

```
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 +0.00023332-0.000019115934961260000 +
    0.000019114847340270000)-18-7-\mp@subsup{\phi}{}{2}=127.891-(2\operatorname{sin}(54\mp@subsup{4}{}{\circ})\mp@subsup{)}{}{2}
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 + 0.00023332-0.000019115934961260000 +
    0.000019114847340270000)-18-7-\mp@subsup{\phi}{}{2}=127.891-(-2\operatorname{cos}(216 %))}\mp@subsup{)}{}{2
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 +0.00023332-0.000019115934961260000 +
    0.000019114847340270000)-18-7-\mp@subsup{\phi}{}{2}=127.891-(-2\operatorname{sin}(66\mp@subsup{6}{}{\circ})\mp@subsup{)}{}{2}
```


## Input interpretation:

```
(76.6132768639 + 76.278746097-0.000336436347-
    0.000328808677-0.0002411638+0.0002333200322 -
    0.00001911593496126+0.00001911484734027) - 11- - }\mp@subsup{}{}{2
```


## Result:

139.27331588...
$139.27331588 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Alternative representations:

```
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 + 0.00023332-0.000019115934961260000 +
    0.000019114847340270000)-11- - }\mp@subsup{}{}{2}=141.891-(2\operatorname{sin}(5\mp@subsup{4}{}{\circ})\mp@subsup{)}{}{2
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 +0.00023332-0.000019115934961260000 +
    0.000019114847340270000) - 11- - $}=141.891-(-2\operatorname{cos}(21\mp@subsup{6}{}{\circ})\mp@subsup{)}{}{2
(76.61327686390000 + 76.2787460970000-0.000336436-0.000328809 -
    0.000241164 +0.00023332-0.000019115934961260000 +
    0.000019114847340270000) - 11- - $ }=141.891-(-2\operatorname{sin}(666\mp@subsup{6}{}{\circ})\mp@subsup{)}{}{2
```

$($ sqrt10-3)(1/76.6132768639 * $1 / 76.278746097 * 1 /-0.000336436347 * 1 /-$
$0.000328808677 * 1 /-0.0002411638 * 1 / 0.0002333200322 * 1 /-$
$0.00001911593496126 * 1 / 0.00001911484734027$ )

## Input interpretation:

$$
\begin{array}{r}
(\sqrt{10}-3)\left(\frac{1}{76.6132768639} \times \frac{1}{76.278746097}\left(-\frac{1}{0.000336436347}\right)\right. \\
\left(-\frac{1}{0.000328808677}\right)\left(-\frac{1}{0.0002411638}\right) \times \frac{1}{0.0002333200322} \\
\left.\left(-\frac{1}{0.00001911593496126}\right) \times \frac{1}{0.00001911484734027}\right) \\
105
\end{array}
$$

## Result:

$1.220884 \ldots \times 10^{19}$
$1.220884 \ldots * 10^{19} \approx 1.2209 * 10^{19} \mathrm{GeV}$ that is the value of Planck energy

Example of Ramanujan mathematics applied to the physics

From:

## Replica Wormholes and the Entropy of Hawking Radiation

Ahmed Almheiri, Thomas Hartman, Juan Maldacena,
Edgar Shaghoulian and Amirhossein Tajdini - arXiv:1911.12333v1 [hep-th] 27 Nov 2019

From chapter "Geometry of the black hole", is described the following formula:

$$
\begin{equation*}
S_{\mathrm{gen}}([-a, b])=S_{0}+\frac{2 \pi \phi_{r}}{\beta} \frac{1}{\tanh \left(\frac{2 \pi a}{\beta}\right)}+\frac{c}{6} \log \left(\frac{2 \beta \sinh ^{2}\left(\frac{n}{\beta}(a+b)\right)}{\pi \epsilon \sinh \left(\frac{2 \pi a}{\beta}\right)}\right) \tag{3.10}
\end{equation*}
$$

From the previous Ramanujan expressions
$54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6039 \sinh (3 \pi)}\right)}$
$296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)$
6147441305046

We obtain:
$1 / \tanh ((((296447958(\pi / 2)+442963057(3 \pi)+719470215(5 \pi)) / 6147441305046)))$

## Input:

$\frac{1}{\frac{296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)}{6147441305046}}$
$\tanh (x)$ is the hyperbolic tangent function

## Exact result:

6147441305046
$296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)$

## Decimal approximation:

4285.958605954208213734361862548850123878070152765655347630...
4285.9586

## Property:

6147441305046
$296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)$ is a transcendental number

## Alternate forms:

6147441305046
$296447958 \tanh \left(\frac{\pi}{2}\right)+20737(21361 \tanh (3 \pi)+34695 \tanh (5 \pi))$
$\frac{6147441305046}{\frac{296447958 \sinh (\pi)}{1+\cosh (\pi)}+\frac{442963057 \sinh (6 \pi)}{1+\cosh (6 \pi)}+\frac{719470215 \sinh (10 \pi)}{1+\cosh (10 \pi)}}$
$\frac{6147441305046}{\frac{296447958 \sinh \left(\frac{\pi}{2}\right)}{\cosh \left(\frac{\pi}{2}\right)}+\frac{442963057 \sinh (3 \pi)}{\cosh (3 \pi)}+\frac{719470215 \sinh (5 \pi)}{\cosh (5 \pi)}}$

## Alternative representations:

$\frac{1}{\frac{296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)}{6147441305046}}=\frac{1}{\frac{296447958}{\frac{\operatorname{coth}\left(\frac{\pi}{2}\right)}{2}+\frac{442963057}{\operatorname{coth}(3 \pi)}+\frac{719470215}{\operatorname{coth}(5 \pi)}}}$


## Series representations:


$\frac{1}{\frac{296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)}{6147441305046}}=$
$296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)=$

$$
6147441305046
$$

$$
\begin{aligned}
& 6147441305046 /\left(\sum _ { k = 0 } ^ { \infty } \left(-296447958\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(\frac{\pi}{2}-z_{0}\right)^{k}-\right.\right. \\
& 442963057\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(3 \pi-z_{0}\right)^{k}- \\
& \left.\left.719470215\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5 \pi-z_{0}\right)^{k}\right)\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

## Integral representation:

```
\(\frac{1}{\frac{296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)}{6147441305046} 6147441305046}=\)
    \(\int_{0}^{\frac{\pi}{2}}\left(296447958 \operatorname{sech}^{2}(t)+124422\left(21361 \operatorname{sech}^{2}(6 t)+57825 \operatorname{sech}^{2}(10 t)\right)\right) d t\)
```

$54+$ golden ratio $-1 /(1-82944 \pi(1 /(20737 \sinh (\pi))-1 /(10376 \sinh (2 \pi))+1 /(6939$ $\sinh (3 \pi))$ )

## Input:

$54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}$
$\sinh (x)$ is the hyperbolic sine function
$\phi$ is the golden ratio

## Exact result:

$\phi+54-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)}$

## Decimal approximation:

3096.900298273126807801702180739848133876304011281727955975
3096.9002982... result practically equal to the rest mass of J/Psi meson 3096.916

## Alternate forms:

$\phi+54-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(3 \pi)}{6939}+\operatorname{csch}(\pi)\left(\frac{1}{20737}-\frac{\operatorname{sech}(\pi)}{20752}\right)\right)}$
$\frac{1}{2}(109+\sqrt{5})+6912243473 /(-6912243473+27647640576 \pi \operatorname{csch}(\pi)-$
$55255312512 \pi \operatorname{csch}(2 \pi)+82624171008 \pi \operatorname{csch}(3 \pi))$
$54+\frac{1}{2}(1+\sqrt{5})-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)}$
$\operatorname{sech}(x)$ is the hyperbolic secant function

## Alternative representations:

$$
\begin{aligned}
& 54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}= \\
& 54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{\left.\frac{20737}{\operatorname{csch}(\pi)}-\frac{1}{\frac{10376}{\operatorname{csch}(2 \pi)}}+\frac{1}{\frac{6939}{\operatorname{csch}(3 \pi)}}\right)}\right.}=
\end{aligned}
$$

$$
54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}=
$$

$$
54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}=
$$

## Series representations:


$54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}=$

$$
54+\phi-\frac{1}{1-82944 \pi \sum_{k=0}^{\infty}\left(\frac{2 e^{-3 \pi-6 k \pi}}{6939}-\frac{e^{-2 \pi-4 k \pi}}{5188}+\frac{2 e^{-\pi-2 k \pi}}{20737}\right)}
$$

$$
\begin{aligned}
& 54+\phi-\frac{1}{1-82944 \pi\left(\frac{1}{20737 \sinh (\pi)}-\frac{1}{10376 \sinh (2 \pi)}+\frac{1}{6939 \sinh (3 \pi)}\right)}= \\
& 54+\phi-1 /\left(1-82944 \pi \sum_{k=0}^{\infty}\left(( \mathrm { Li } _ { - k } ( - e ^ { z _ { 0 } } ) - \mathrm { Li } _ { - k } ( e ^ { z _ { 0 } } ) ) \left(71999064\left(\pi-z_{0}\right)^{k}-\right.\right.\right. \\
& \left.\left.143894043\left(2 \pi-z_{0}\right)^{k}+215167112\left(3 \pi-z_{0}\right)^{k}\right)\right) / \\
& (1493044590168 k!)) \text { for } \frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

## Integral representations:




$$
1 /\left(1-82944 \pi\left(\frac{4 i}{20737 \sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{\pi^{2} /(4 s)+s}}{s^{3 / 2}} d s}-\frac{i}{5188 \sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{\pi^{2} / s+s}}{s^{3 / 2}} d s}+\right.\right.
$$

$$
\left.\left.\frac{4 i}{20817 \sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{\left(9 \pi^{2}\right) /(4 s)+s}}{s^{3 / 2}} d s}\right)\right) \text { for } \gamma>0
$$

If we put:
$\left(\frac{1}{\tanh \left(\frac{2 \pi a}{\beta}\right)}\right)=\left(\frac{1}{\frac{296447958 \tanh \left(\frac{\pi}{2}\right)+442963057 \tanh (3 \pi)+719470215 \tanh (5 \pi)}{6147441305046}}\right)=4285.9586$
and
$\left(\left(\frac{2 \beta \sinh ^{2}\left(\frac{\pi}{\beta}(a+b)\right)}{\pi \epsilon \sinh \left(\frac{2 \pi a}{\beta}\right)}\right)\right)=\left(\phi+54-\frac{1}{1-82944 \pi\left(\frac{\operatorname{csch}(\pi)}{20737}-\frac{\operatorname{csch}(2 \pi)}{10376}+\frac{\operatorname{csch}(3 \pi)}{6939}\right)}\right)=$
$=3096.9002982 \ldots$

We obtain from
$S_{0}+\frac{2 \pi \phi_{r}}{\beta} \frac{1}{\tanh \left(\frac{2 \pi a}{\beta}\right)}+\frac{c}{6} \log \left(\frac{2 \beta \sinh ^{2}\left(\frac{\pi}{\beta}(a+b)\right)}{\pi \epsilon \sinh \left(\frac{2 \pi a}{\beta}\right)}\right)$
For
$\phi_{r} /(\beta c) \gtrsim 1=0.98911$ or 1.0864055
$\frac{\text { Area }}{4 G_{N}}=S_{0}+\phi$.
$4 G_{N}=1$
$\mathrm{S}_{0}=4 \mathrm{Pi}-0.98911$
$\mathrm{c}=1$
$\mathrm{S}_{0}+2 \mathrm{Pi}^{*} 0.98911 * 4285.9586+\frac{c}{6} \ln (3096.9002982)$

4Pi-0.98911+2Pi*0.98911*4285.9586+1/6* $\ln (3096.9002982)$

## Input interpretation:

$4 \pi-0.98911+2 \pi \times 0.98911 \times 4285.9586+\frac{1}{6} \log (3096.9002982)$

## Result:

26649.1...
26649.1...

## Alternative representations:

$$
\begin{gathered}
4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
-0.98911+8482.57 \pi+\frac{\log _{e}(3096.90029820000)}{6}
\end{gathered}
$$

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+8482.57 \pi+\frac{1}{6} \log (a) \log _{a}(3096.90029820000)
\end{aligned}
$$

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+8482.57 \pi-\frac{\mathrm{Li}_{1}(-3095.90029820000)}{6}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+8482.57 \pi+\frac{\log (3095.90029820000)}{6}-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k} e^{-8.03783403076730 k}}{k}
\end{aligned}
$$

$$
\begin{array}{r}
4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
-0.98911+8482.57 \pi+\frac{1}{3} i \pi\left[\left.\frac{\arg (3096.90029820000-x)}{2 \pi} \right\rvert\,+\right. \\
\frac{\log (x)}{6}-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}(3096.90029820000-x)^{k} x^{-k}}{k} \text { for } x<0
\end{array}
$$

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+8482.57 \pi+\frac{1}{6}\left\lfloor\frac{\arg \left(3096.90029820000-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+ \\
& \frac{\log \left(z_{0}\right)}{6}+\frac{1}{6}\left\lfloor\frac{\arg \left(3096.90029820000-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)- \\
& \frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(3096.90029820000-z_{0}\right)^{k} z_{0}^{k}}{k}
\end{aligned}
$$

## Integral representations:

$$
\begin{gathered}
4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
-0.98911+8482.57 \pi+\frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} d t
\end{gathered}
$$

$$
\begin{array}{r}
4 \pi-0.98911+2 \pi 0.98911 \times 4285.96+\frac{\log (3096.90029820000)}{6}=-0.98911+ \\
8482.57 \pi+\frac{1}{12 i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{-8.03783403076730 s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
\end{array}
$$

Inserting the entropy value 26649.1 in the Hawking radiation calculator, we obtain:
Mass $=0.00000100229$
Radius $=1.48856 \mathrm{E}-33$
Temperature $=1.22416 \mathrm{E} 29$
Entropy $=26649.1$

From the Ramanujan-Nardelli mock formula, we obtain:

$$
\begin{aligned}
& \operatorname{sqrt}\left[\left[\left[\left[1 /\left(\left(\left(\left(\left(\left(\left(4^{*} 1.962364415 \mathrm{e}+19\right) /\left(5^{*} 0.0864055^{\wedge} 2\right)\right)\right)^{*} 1 /(0.00000100229)^{*}\right.\right.\right.\right. \text { sqrt[[]- }\right.\right.\right.\right. \\
& \left.\left.\left.\left.\left.\left.\left(\left(\left(\left(1.22416 \mathrm{e}+29 * 4^{*} \mathrm{Pi}^{*}(1.48856 \mathrm{e}-33)^{\wedge} 3-(1.48856 \mathrm{e}-33)^{\wedge} 2\right)\right)\right)\right)\right) /\left(\left(6.67^{*} 10^{\wedge}-11\right)\right)\right]\right]\right]\right]\right]
\end{aligned}
$$

## Input interpretation:

$$
\begin{aligned}
& \sqrt{ }\left(1 /\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{1.00229 \times 10^{-6}}\right.\right. \\
& \left.\sqrt{-\frac{1.22416 \times 10^{29} \times 4 \pi\left(1.48856 \times 10^{-33}\right)^{3}-\left(1.48856 \times 10^{-33}\right)^{2}}{6.67 \times 10^{-11}}}\right)
\end{aligned}
$$

## Result:

1.618081735392146230436561397898828941494902451109365297284...
1.61808173539...

And:
1/sqrt[[[[1/(((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(0.00000100229)* $\operatorname{sqrt[[-(((1.22416e+29*4*Pi*(1.48856e-33)^{\wedge }3-(1.48856e-33)^{\wedge }2)))))/((6.67^{*}10^{\wedge }-~.~}$ 11))]]J]]

## Input interpretation:



## Result:

$0.618015751693561668331267490642891547545081526820311348060 \ldots$
$0.61801575169 \ldots$
Practically we obtain the values of the golden ratio and his conjugate
Or:
$4 \mathrm{Pi}-0.98911+2 \mathrm{Pi}^{*} 1.0864055 * 4285.9586+1 / 6 * \ln (3096.9002982)$

## Input interpretation:

$4 \pi-0.98911+2 \pi \times 1.0864055 \times 4285.9586+\frac{1}{6} \log (3096.9002982)$

## Result:

29269.244..
29269.244...

## Alternative representations:

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+9316.58 \pi+\frac{\log _{e}(3096.90029820000)}{6}
\end{aligned}
$$

$4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}=$
$-0.98911+9316.58 \pi+\frac{1}{6} \log (a) \log _{a}(3096.90029820000)$

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+9316.58 \pi-\frac{\operatorname{Li}_{1}(-3095.90029820000)}{6}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+9316.58 \pi+\frac{\log (3095.90029820000)}{6}-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k} e^{-8.03783403076730 k}}{k} \\
& 4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
& -0.98911+9316.58 \pi+\frac{1}{3} i \pi\left|\frac{\arg (3096.90029820000-x)}{2 \pi}\right|+ \\
& \quad \frac{\log (x)}{6}-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}(3096.90029820000-x)^{k} x^{-k}}{k} \text { for } x<0
\end{aligned}
$$

$$
4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}=
$$

$$
-0.98911+9316.58 \pi+\frac{1}{6}\left\lfloor\frac{\arg \left(3096.90029820000-z_{0}\right)}{2 \pi}\right\rfloor \log \left(\frac{1}{z_{0}}\right)+
$$

$$
\frac{\log \left(z_{0}\right)}{6}+\frac{1}{6}\left[\frac{\arg \left(3096.90029820000-z_{0}\right)}{2 \pi}\right\rfloor \log \left(z_{0}\right)-
$$

$$
\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(3096.90029820000-z_{0}\right)^{k} z_{0}^{-k}}{k}
$$

## Integral representations:

$$
\begin{gathered}
4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}= \\
-0.98911+9316.58 \pi+\frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} d t
\end{gathered}
$$

$$
\begin{array}{r}
4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}=-0.98911+ \\
9316.58 \pi+\frac{1}{12 i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{-8.03783403076730 s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s \text { for }-1<\gamma<0
\end{array}
$$

Inserting the entropy value 29269.244 in the Hawking radiation calculator, we obtain:
Mass $=0.00000105040$
Radius $=1.56002 \mathrm{e}-33$
Temperature $=1.16808 \mathrm{e}+29$
From the Ramanujan-Nardelli mock formula, we obtain:
$\operatorname{sqrt}\left[\left[\left[\left[1 /\left(\left(\left(\left(\left(\left(4^{*} 1.962364415 \mathrm{e}+19\right) /\left(5^{*} 0.0864055^{\wedge} 2\right)\right)\right)^{*} 1 /(0.00000105040)^{*} \operatorname{sqrt}[[-\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left(\left(\left(\left(1.16808 \mathrm{e}+29 * 4^{*} \mathrm{Pi}^{*}(1.56002 \mathrm{e}-33)^{\wedge} 3-(1.56002 \mathrm{e}-33)^{\wedge} 2\right)\right)\right)\right)\right) /\left(\left(6.67^{*} 10^{\wedge}-11\right)\right)\right]\right]\right]\right]\right]$

Input interpretation:
$\sqrt{ } \left\lvert\, 1 /\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{1.05040 \times 10^{-6}}\right.\right.$

$$
\left.\left.\sqrt{-\frac{1.16808 \times 10^{29} \times 4 \pi\left(1.56002 \times 10^{-33}\right)^{3}-\left(1.56002 \times 10^{-33}\right)^{2}}{6.67 \times 10^{-11}}}\right)\right)
$$

## Result:

1.618077063491289140603706176247888824149668700084618992874...
1.618077063...

We have also that:
$\left(\left(\left(4 \mathrm{Pi}-0.98911+2 \mathrm{Pi}^{*} 1.0864055 * 4285.9586+1 / 6 * \ln (3096.9002982)\right)\right)\right)^{\wedge} 1 / 2-29-$ golden ratio^2

## Input interpretation:

$$
\sqrt{4 \pi-0.98911+2 \pi \times 1.0864055 \times 4285.9586+\frac{1}{6} \log (3096.9002982)}-29-\phi^{2}
$$

## Result:

139.46453...
139.46453... result practically equal to the rest mass of Pion meson 139.57

## Alternative representations:

$$
\begin{aligned}
& \sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}
\end{aligned}-29-\phi^{2}=
$$

$$
\begin{aligned}
& \sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}= \\
& -29-\phi^{2}+\sqrt{-0.98911+9316.58 \pi+\frac{1}{6} \log (a) \log _{a}(3096.90029820000)}
\end{aligned}
$$

$$
\sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}=
$$

$$
-29-\phi^{2}+\sqrt{-0.98911+9316.58 \pi-\frac{\mathrm{Li}_{1}(-3095.90029820000)}{6}}
$$

## Series representations:

$$
\begin{aligned}
& \sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}= \\
& -29-\phi^{2}+ \\
& \frac{\sqrt{-5.93466+55899.5 \pi+\log (3095.90029820000)-\sum_{k=1}^{\infty} \frac{(-1)^{k} e^{-8.03783403076730 k}}{k}}}{\sqrt{6}}
\end{aligned}
$$

$$
\sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}=
$$

$$
-29-\phi^{2}+\sqrt{\left(-0.98911+9316.58 \pi+\frac{1}{6}\left(2 i \pi\left\lfloor\frac{\arg (3096.90029820000-x)}{2 \pi}\right)+\right.\right.}
$$

$$
\left.\left.\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}(3096.90029820000-x)^{k} x^{-k}}{k}\right)\right) \text { for } x<0
$$

$$
\begin{aligned}
& \sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}-29-\phi^{2}=} \\
& -29-\phi^{2}+\sqrt{\left(-0.98911+9316.58 \pi+\frac{1}{6}\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(3096.90029820000-z_{0}\right)}{2 \pi}\right\rfloor\right.\right.} \\
& \left.\left.\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(3096.90029820000-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)\right)
\end{aligned}
$$

## Integral representations:

```
\(\sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}=\)
\(-29-\phi^{2}+\sqrt{-0.98911+9316.58 \pi+\frac{1}{6} \int_{1}^{3096.90029820000} \frac{1}{t} d t}\)
\(\sqrt{4 \pi-0.98911+2 \pi 1.08641 \times 4285.96+\frac{\log (3096.90029820000)}{6}}-29-\phi^{2}=\)
\(-29-\phi^{2}+\)
    \(\sqrt{-0.98911+9316.58 \pi+\frac{1}{12 i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{-8.03783403076730 s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}\)
for \(-1<\gamma<0\)
```

solve this equation, we must impose the same condition on the right-hand side. The $k=1$ mode requires

$$
\begin{equation*}
\int_{0}^{2 \pi} d \tau e^{-i \tau}\left(\frac{c}{12 \phi_{r}} \mathcal{F}-\partial_{\tau} R(\tau)\right)=0 \tag{3.29}
\end{equation*}
$$

Doing the integrals, this gives the condition

$$
\begin{equation*}
\frac{c}{6 \phi_{r}} \frac{\sinh \frac{a-b}{2}}{\sinh \frac{b+a}{2}}=\frac{1}{\sinh a} \tag{3.30}
\end{equation*}
$$

For
$a, b>0$
$\mathrm{a}=5, \mathrm{~b}=2, \mathrm{c}=1$ we obtain, from (3.30):
$(((1 /(\sinh (5))))) /\left(\left(\left(1 /\left(6^{*} 0.98911\right) *(\sinh (3 / 2) / \sinh (7 / 2))\right)\right)\right)$

## Input:



## Result:

0.621362...
0.621362...

## Alternative representations:

$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{Z}{2}\right)}}=\frac{1}{\frac{1}{\frac{\operatorname{csch}(5)\left(5.93466 \operatorname{csch}\left(\frac{3}{2}\right)\right)}{\operatorname{csch}\left(\frac{7}{2}\right)}}}$
$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{Z}{2}\right)}}=\frac{1}{\frac{\left(-\frac{1}{e^{5}}+e^{5}\right)\left(-\frac{1}{e^{3 / 2}}+e^{3 / 2}\right)}{\frac{2}{2}\left(2 \times 5.93466\left(-\frac{1}{e^{7 / 2}}+e^{7 / 2}\right)\right)}}$

$$
\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{Z}{2}\right)}}=-\frac{1}{\frac{i(-i)}{\frac{\csc (5 i)\left(5.93466 \csc \left(\frac{3 i}{2}\right)(-i)\right)}{\csc \left(\frac{Z i}{2}\right)}}}
$$

## Series representations:

$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{Z}{2}\right)}}=\frac{5.93466 \sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{1+2 k}}{(1+2 k)!}}{\left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{1+2 k}}{(1+2 k)!}\right) \sum_{k=0}^{\infty} \frac{5^{1+2 k}}{(1+2 k)!}}$
$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}}=\frac{2.96733 \sum_{k=0}^{\infty} I_{1+2 k}\left(\frac{7}{2}\right)}{\left(\sum_{k=0}^{\infty} I_{1+2 k}\left(\frac{3}{2}\right)\right) \sum_{k=0}^{\infty} I_{1+2 k}(5)}$
$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}}=\frac{5.93466 \sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}-\frac{i \pi}{2}\right)^{2 k}}{(2 k)!}}{i\left(\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-\frac{i \pi}{2}\right)^{2 k}}{(2 k)!}\right) \sum_{k=0}^{\infty} \frac{\left(5-\frac{i \pi}{2}\right)^{2 k}}{(2 k)!}}$

## Integral representations:

$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}}=\frac{2.76951 \int_{0}^{1} \cosh \left(\frac{7 t}{2}\right) d t}{\left(\int_{0}^{1} \cosh \left(\frac{3 t}{2}\right) d t\right) \int_{0}^{1} \cosh (5 t) d t}$
$\frac{1}{\frac{\sinh \left(\frac{3}{2}\right) \sinh (5)}{(6 \times 0.98911) \sinh \left(\frac{Z}{2}\right)}}=\frac{11.078 i \pi \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{49 /(16 s)+s}}{s^{3 / 2}} d s}{\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{9 /(16 s)+s}}{s^{3 / 2}} d s\right)\left(\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{25 /(4 s)+s}}{s^{3 / 2}} d s\right) \sqrt{\pi}}$ for $\gamma>0$
$0.62136239751766^{*}\left(\left(\left(1 /\left(6^{*} 0.98911\right) *(\sinh (3 / 2) / \sinh (7 / 2))\right)\right)\right)$

## Input interpretation:

$0.62136239751766\left(\frac{1}{6 \times 0.98911} \times \frac{\sinh \left(\frac{3}{2}\right)}{\sinh \left(\frac{7}{2}\right)}\right)$
$\sinh (x)$ is the hyperbolic sine function

## Result:

0.0134765...
0.0134765...

## Alternative representations:

$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.621362397517660000}{\frac{5.93466 \operatorname{csch}\left(\frac{3}{2}\right)}{\operatorname{csch}\left(\frac{7}{2}\right)}}$
$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.310681198758830000\left(-\frac{1}{e^{3 / 2}}+e^{3 / 2}\right)}{\frac{1}{2} \times 5.93466\left(-\frac{1}{e^{7 / 2}}+e^{7 / 2}\right)}$
$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=-\frac{0.621362397517660000 i}{\frac{5.93466 \csc \left(\frac{3 i}{2}\right)(-i)}{\csc \left(\frac{7 i}{2}\right)}}$

## Series representations:

$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.104701 \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)^{1+2 k}}{(1+2 k)!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}\right)^{1+2 k}}{(1+2 k)!}}$
$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.104701 \sum_{k=0}^{\infty} I_{1+2 k}\left(\frac{3}{2}\right)}{\sum_{k=0}^{\infty} I_{1+2 k}\left(\frac{7}{2}\right)}$
$\frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.104701 \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}-i \frac{\pi}{2}\right)^{2 k}}{(2 k)!}}{\sum_{k=0}^{\infty} \frac{\left(\frac{7}{2}-\frac{\pi}{2}\right)^{2 k}}{(2 k)!}}$

## Integral representations:

$$
\begin{aligned}
& \frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.0448717 \int_{0}^{1} \cosh \left(\frac{3 t}{2}\right) d t}{\int_{0}^{1} \cosh \left(\frac{7 t}{2}\right) d t} \\
& \frac{0.621362397517660000 \sinh \left(\frac{3}{2}\right)}{(6 \times 0.98911) \sinh \left(\frac{7}{2}\right)}=\frac{0.0448717 \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{9 /(16 s)+s}}{s^{3 / 2}} d s}{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{49 /(16 s)+s}}{s^{3 / 2}} d s} \text { for } \gamma>0
\end{aligned}
$$

$(((1 /(\sinh (5)))))$

## Input:

$\frac{1}{\sinh (5)}$
$\sinh (x)$ is the hyperbolic sine function

## Exact result:

$\operatorname{csch}(5)$

## Decimal approximation:

$0.013476505830589086655381881284337964618035455336483814697 \ldots$
$0.013476505 \ldots$

## Property:

csch(5) is a transcendental number

Alternate forms:

$$
\begin{aligned}
& \frac{2 e^{5}}{e^{10}-1} \\
& \frac{2}{e^{5}-\frac{1}{e^{5}}} \\
& -\frac{2 \sinh (5)}{1-\cosh (10)}
\end{aligned}
$$

Alternative representations:
$\frac{1}{\sinh (5)}=\frac{1}{\frac{1}{\operatorname{csch}(5)}}$
$\frac{1}{\sinh (5)}=\frac{1}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}$
$\frac{1}{\sinh (5)}=-\frac{1}{\frac{i}{\csc (5 i)}}$

## Series representations:

$\frac{1}{\sinh (5)}=\frac{2 \sum_{k=0}^{\infty} e^{-10 k}}{e^{5}}$
$\frac{1}{\sinh (5)}=-2 \sum_{k=1}^{\infty} q^{-1+2 k}$ for $q=e^{5}$
$\frac{1}{\sinh (5)}=5 \sum_{k=-\infty}^{\infty} \frac{(-1)^{k}}{25+k^{2} \pi^{2}}$

## Integral representations:

$\frac{1}{\sinh (5)}=\frac{1}{5 \int_{0}^{1} \cosh (5 t) d t}$
$\frac{1}{\sinh (5)}=\frac{4 i \pi}{5 \sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{25 /(4 s)+s}}{s^{3 / 2}} d s}$ for $\gamma>0$

The fundamental result in this expression is 0.62136239751766 . Note that the inverse of this value is $1.6093667785417 \ldots$...: these are "golden numbers"
at $t=0$. The generalized entropy, including the island, is

$$
\begin{equation*}
S_{\operatorname{gen}}(I \cup R)=\frac{\phi_{r}}{a}+\frac{c}{6} \log \frac{(a+b)^{2}}{a} . \tag{4.3}
\end{equation*}
$$

Setting $\partial_{a} S_{\text {gen }}=0$ gives the position of the QES,

$$
\begin{equation*}
a=\frac{1}{2}\left(k+b+\sqrt{b^{2}+6 b k+k^{2}}\right), \quad k \equiv \frac{6 \phi_{r}}{c} . \tag{4.4}
\end{equation*}
$$

For $\mathrm{a}=5, \mathrm{~b}=2$
$5=1 / 2\left(x+2+\operatorname{sqrt}\left(4+12 x+x^{\wedge} 2\right)\right)$

## Input:

$5=\frac{1}{2}\left(x+2+\sqrt{4+12 x+x^{2}}\right)$

Plot:


## Alternate forms:

$\sqrt{x^{2}+12 x+4}+x=8$
$5=\frac{1}{2}(x+\sqrt{x(x+12)+4}+2)$

## Alternate form assuming $\mathbf{x}$ is positive:

$x+\sqrt{x(x+12)+4}=8$

## Expanded form:

$5=\frac{1}{2} \sqrt{x^{2}+12 x+4}+\frac{x}{2}+1$

## Solution:

$x=\frac{15}{7}$
$15 / 7=\mathrm{k}$
$15 / 7=6 x$

## Input:

$$
\frac{15^{2}}{7}=6 x
$$

## Plot:



Alternate form:

$$
\frac{15}{7}-6 x=0
$$

## Solution:

$$
x=\frac{5}{14}
$$

$5 / 14=\phi_{r}$
$\mathrm{a}=5, \mathrm{~b}=2$

$$
S_{\text {gen }}(I \cup R)=\frac{\phi_{r}}{a}+\frac{c}{6} \log \frac{(a+b)^{2}}{a}
$$

$5 / 14 * 1 / 5+1 / 6 * \ln (49 / 5)$

## Input:

$$
\frac{5}{14} \times \frac{1}{5}+\frac{1}{6} \log \left(\frac{49}{5}\right)
$$

## Exact result:

$\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)$

## Decimal approximation:

$0.451825635707992467839752930371933398529523255577772204405 \ldots$
$0.451825635 \ldots$

## Property:

$\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)$ is a transcendental number

## Alternate forms:

$\frac{1}{42}\left(3+7 \log \left(\frac{49}{5}\right)\right)$
$\frac{1}{14}-\frac{\log (5)}{6}+\frac{\log (7)}{3}$
$\frac{1}{42}(3-7 \log (5)+14 \log (7))$

## Alternative representations:

$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{\log _{e}\left(\frac{49}{5}\right)}{6}+\frac{5}{5 \times 14}$
$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{6} \log (a) \log _{a}\left(\frac{49}{5}\right)+\frac{5}{5 \times 14}$
$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=-\frac{1}{6} \mathrm{Li}_{1}\left(1-\frac{49}{5}\right)+\frac{5}{5 \times 14}$

## Series representations:

$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{14}+\frac{1}{6} \log \left(\frac{44}{5}\right)-\frac{1}{6} \sum_{k=1}^{\infty} \frac{\left(-\frac{5}{44}\right)^{k}}{k}$

$$
\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{14}+\frac{1}{3} i \pi\left[\frac{\arg \left(\frac{49}{5}-x\right)}{2 \pi} \left\lvert\,+\frac{\log (x)}{6}-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-x\right)^{k} x^{-k}}{k}\right.\right.
$$

$$
\begin{aligned}
& \frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{14}+\frac{1}{6}\left[\left.\frac{\arg \left(\frac{49}{5}-z_{0}\right)}{2 \pi} \right\rvert\, \log \left(\frac{1}{z_{0}}\right)+\right. \\
& \frac{\log \left(z_{0}\right)}{6}+\frac{1}{6}\left[\frac{\arg \left(\frac{49}{5}-z_{0}\right)}{2 \pi} \left\lvert\, \log \left(z_{0}\right)-\frac{1}{6} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-z_{0}\right)^{k} z_{0}^{-k}}{k}\right.\right.
\end{aligned}
$$

## Integral representations:

$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{14}+\frac{1}{6} \int_{1}^{\frac{49}{5}} \frac{1}{t} d t$
$\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)=\frac{1}{14}-\frac{i}{12 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\left(\frac{5}{44}\right)^{s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$ for $-1<\gamma<0$

Note that:
$64 /(((5 / 14 * 1 / 5+1 / 6 * \ln (49 / 5))))-16$

## Input:

$\frac{64}{\frac{5}{14} \times \frac{1}{5}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16$
$\log (x)$ is the natural logarithm

## Exact result:

$$
\frac{64}{\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16
$$

## Decimal approximation:

125.6475625596466933973543735598565493271424256496263802118...
$125.64756255 \ldots$ result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Property:

$-16+\frac{64}{\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}$ is a transcendental number

## Alternate forms:

$\frac{2688}{3+7 \log \left(\frac{49}{5}\right)}-16$
$-\frac{16\left(7 \log \left(\frac{49}{5}\right)-165\right)}{3+7 \log \left(\frac{49}{5}\right)}$
$-\frac{16(165+7 \log (5)-14 \log (7))}{-3+7 \log (5)-14 \log (7)}$

## Alternative representations:

$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{64}{\frac{\log \left(\frac{49}{5}\right)}{6}+\frac{5}{5 \times 14}}$
$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{64}{\frac{1}{6} \log (a) \log _{a}\left(\frac{49}{5}\right)+\frac{5}{5 \times 14}}$
$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{64}{-\frac{1}{6} \operatorname{Li}\left(1-\frac{49}{5}\right)+\frac{5}{5 \times 14}}$

## Series representations:

$$
\begin{aligned}
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{2688}{3+7 \log \left(\frac{44}{5}\right)-7 \sum_{k=1}^{\infty} \frac{\left(-\frac{5}{44}\right)^{k}}{k}} \\
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16= \\
& -16+\frac{64}{\left.\frac{1}{14}+\frac{1}{6}\left(2 i \pi \left\lvert\, \frac{\arg \left(\frac{49}{5}-x\right)}{2 \pi}\right.\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-x\right)^{k} x^{-k}}{k}\right)} \text { for } x<0
\end{aligned}
$$

$$
\begin{aligned}
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16= \\
& -16+\frac{64}{\frac{1}{14}+\frac{1}{6}\left(\log \left(z_{0}\right)+\left\lfloor\frac{\arg \left(\frac{49}{5}-z_{0}\right)}{2 \pi}\right]\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-z_{0}\right)^{k} z_{0}^{-k}}{k}\right)}
\end{aligned}
$$

## Integral representations:

$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{2688}{3+7 \int_{1}^{\frac{49}{5}} \frac{1}{t} d t}$

$$
\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-16=-16+\frac{5376 \pi}{6 \pi-7 i \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\left(\frac{5}{44}\right)^{5} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s} \text { for }-1<\gamma<0
$$

And:
$64 /\left(\left(\left(5 / 14^{*} 1 / 5+1 / 6 * \ln (49 / 5)\right)\right)\right)$-sqrt5

## Input:

$\frac{64}{\frac{5}{14} \times \frac{1}{5}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}$

## Exact result:

$\frac{64}{\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}$

## Decimal approximation:

139.4114945821469037009451998911252730917018072900148544875...
$139.41149458 \ldots$ result practically equal to the rest mass of Pion meson 139.57

## Property:

$-\sqrt{5}+\frac{64}{\frac{1}{14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}$ is a transcendental number

## Alternate forms:

$$
\begin{aligned}
& \frac{2688}{3+7 \log \left(\frac{49}{5}\right)}-\sqrt{5} \\
& -\frac{-2688+3 \sqrt{5}+7 \sqrt{5} \log \left(\frac{49}{5}\right)}{3+7 \log \left(\frac{49}{5}\right)} \\
& -\frac{2688-3 \sqrt{5}+7 \sqrt{5} \log (5)-14 \sqrt{5} \log (7)}{-3+7 \log (5)-14 \log (7)}
\end{aligned}
$$

## Alternative representations:

$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=\frac{64}{\frac{\log _{e}\left(\frac{49}{5}\right)}{6}+\frac{5}{5 \times 14}}-\sqrt{5}$
$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=\frac{64}{\frac{1}{6} \log (a) \log _{a}\left(\frac{49}{5}\right)+\frac{5}{5 \times 14}}-\sqrt{5}$
$\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=\frac{64}{-\frac{1}{6} \operatorname{Li}\left(1-\frac{49}{5}\right)+\frac{5}{5 \times 14}}-\sqrt{5}$

## Series representations:

$$
\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=-\sqrt{5}+\frac{2688}{3+7 \log \left(\frac{44}{5}\right)-7 \sum_{k=1}^{\infty} \frac{\left(-\frac{5}{44}\right)^{k}}{k}}
$$

$$
\begin{aligned}
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}= \\
& -\sqrt{5}+\frac{64}{\frac{1}{14}+\frac{1}{6}\left(2 i \pi\left[\frac{\arg \left(\frac{49}{5}-x\right)}{2 \pi}\right]+\log (x)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-x\right)^{k} x^{-k}}{k}\right)} \text { for } x<0
\end{aligned}
$$

$$
\frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=
$$

$$
-\sqrt{5}+\frac{2688}{3+7 \log \left(z_{0}\right)+7\left[\frac{\arg \left(\frac{49}{5}-z_{0}\right)}{2 \pi}\right]\left(\log \left(\frac{1}{z_{0}}\right)+\log \left(z_{0}\right)\right)-7 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(\frac{49}{5}-z_{0}\right)^{k} z_{0}^{-k}}{k}}
$$

## Integral representations:

$$
\begin{aligned}
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=-\sqrt{5}+\frac{2688}{3+7 \int_{1}^{\frac{49}{5}} \frac{1}{t} d t} \\
& \frac{64}{\frac{5}{5 \times 14}+\frac{1}{6} \log \left(\frac{49}{5}\right)}-\sqrt{5}=-\sqrt{5}+\frac{5376 \pi}{6 \pi-7 i \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\left(\frac{5}{44}\right)^{5} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s} \text { for }-1<\gamma<0
\end{aligned}
$$

Inserting the entropy value 0.451826 in the Hawking radiation calculator, we obtain:

Mass $=4.12701 \mathrm{e}-9$
Radius $=6.12930 \mathrm{e}-36$

Temperature $=2.97299 \mathrm{e}+31$
From the Ramanujan-Nardelli mock formula, we obtain:
$\operatorname{sqrt}\left[\left[\left[\left[1 /\left(\left(\left(\left(\left(\left(\left(4^{*} 1.962364415 \mathrm{e}+19\right) /\left(5^{*} 0.0864055^{\wedge} 2\right)\right)\right)^{*} 1 /(4.12701 \mathrm{e}-9)^{*} \operatorname{sqrt}[[-\right.\right.\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.\left(\left(\left(\left(2.97299 \mathrm{e}+31 * 4^{*} \mathrm{Pi}^{*}(6.12930 \mathrm{e}-36)^{\wedge} 3-(6.12930 \mathrm{e}-36)^{\wedge} 2\right)\right)\right)\right)\right) /\left(\left(6.67 * 10^{\wedge}-11\right)\right)\right]\right]\right]\right]\right]$

## Input interpretation:

$$
\begin{aligned}
& \sqrt{\left(1 /\left(\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{4.12701 \times 10^{-9}}\right.\right.} \\
& \left.\quad \sqrt{\left.-\frac{2.97299 \times 10^{31} \times 4 \pi\left(6.12930 \times 10^{-36}\right)^{3}-\left(6.12930 \times 10^{-36}\right)^{2}}{6.67 \times 10^{-11}}\right)}\right)
\end{aligned}
$$

## Result:

1.618077245318552386950716639328104478879882410161156440606...
1.618077245...

And:
$1 /$ sqrt[[[[[1/(((()((4*1.962364415e+19)/(5*0.0864055^2)))*1/(4.12701e-9)* $\operatorname{sqrt}[[-$ $\left.\left.\left.\left.\left.\left.\left(\left(\left(\left(2.97299 \mathrm{e}+31^{*} 4^{*} \mathrm{Pi}^{*}(6.12930 \mathrm{e}-36)^{\wedge} 3-(6.12930 \mathrm{e}-36)^{\wedge} 2\right)\right)\right)\right)\right) /\left(\left(6.67 * 10^{\wedge}-11\right)\right)\right]\right]\right]\right]\right]$

## Input interpretation:

$\sqrt{\frac{4 \times 1.962364415 \times 10^{19}}{5 \times 0.0864055^{2}} \times \frac{1}{4.12701 \times 10^{-9}} \sqrt{-\frac{2.97299 \times 10^{31} \times 4 \pi\left(6.12930 \times 10^{-36}\right)^{3}-\left(6.12930 \times 10^{-36}\right)^{2}}{6.67 \times 10^{-11}}}}$

## Result:

0.618017466652606600879908700049928924823645848704609289180...
0.61801746...

From:

$$
S_{\text {fermions }}(I \cup R)=\frac{c}{3} \log \left[\frac{2 \cosh t_{a} \cosh t_{b}\left|\cosh \left(t_{a}-t_{b}\right)-\cosh (a+b)\right|}{\sinh a \cosh \left(\frac{a+b-t_{a}-t_{b}}{2}\right) \cosh \left(\frac{a+b+t_{a}+t_{b}}{2}\right)}\right]
$$

we obtain:
$1 / 3 \ln (((2-\cosh (5+2)) /(\sinh (5) \cosh (7 / 2) \cosh (7 / 2))))$

## Input:

$\frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)$

## Exact result:

$\frac{1}{3}\left(\log \left(-(2-\cosh (7)) \operatorname{csch}(5) \operatorname{sech}^{2}\left(\frac{7}{2}\right)\right)+i \pi\right)$

## Decimal approximation:

$-1.2063788441890901037158798352081118020154307200752687721 \ldots+$ 1.0471975511965977461542144610931676280657231331250352736... i

## Polar coordinates:

```
r\approx1.59749 (radius), }0\approx139.04\mp@subsup{4}{}{\circ}\mathrm{ (angle)
```

1.59749

## Alternate forms:

$\frac{1}{3}\left(\log \left((\cosh (7)-2) \operatorname{csch}(5) \operatorname{sech}^{2}\left(\frac{7}{2}\right)\right)+i \pi\right)$
$\frac{1}{3} \log \left((\cosh (7)-2) \operatorname{csch}(5) \operatorname{sech}^{2}\left(\frac{7}{2}\right)\right)+\frac{i \pi}{3}$
$\frac{1}{3}\left(i \pi+2 \log \left(\operatorname{sech}\left(\frac{7}{2}\right)\right)+\log (\cosh (7)-2)+\log (\operatorname{csch}(5))\right)$

## Alternative representations:

$\frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)=\frac{1}{3} \log _{e}\left(\frac{2-\cosh (7)}{\cosh ^{2}\left(\frac{7}{2}\right) \sinh (5)}\right)$
$\frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)=\frac{1}{3} \log (a) \log _{a}\left(\frac{2-\cosh (7)}{\cosh ^{2}\left(\frac{7}{2}\right) \sinh (5)}\right)$
$\frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)=\frac{1}{3} \log \left(\frac{2-\cos (7 i)}{\frac{1}{2} \cos ^{2}\left(\frac{7 i}{2}\right)\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)$

## Series representation:

$$
\begin{aligned}
& \frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)= \\
& \frac{i \pi}{3}-\frac{1}{3} \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-1+(-2+\cosh (7)) \operatorname{csch}(5) \operatorname{sech}^{2}\left(\frac{7}{2}\right)\right)^{k}}{k}
\end{aligned}
$$

## Integral representation:

$\frac{1}{3} \log \left(\frac{2-\cosh (5+2)}{\sinh (5) \cosh \left(\frac{7}{2}\right) \cosh \left(\frac{7}{2}\right)}\right)=\frac{i \pi}{3}+\frac{1}{3} \int_{1}^{(-2+\cosh (7)) \operatorname{csch}(5) \operatorname{sech}^{2}\left(\frac{Z}{2}\right) \frac{1}{t} d t . t .}$

We have that:

$$
\begin{equation*}
S_{\text {gen }}^{\text {island }}=2 S_{0}+\frac{2 \phi_{r}}{\tanh a}+\frac{c}{3} \log \left(\frac{4 \tanh ^{2} \frac{a+b}{2}}{\sinh a}\right) . \tag{5.10}
\end{equation*}
$$

$2 *(4 \mathrm{Pi}-0.98911)+(((2 * 0.98911) /(\tanh (5))))+1 / 3 * \ln ((((4 \tanh \wedge 2(7 / 2)) /(\sinh (5))))$

## Input:

$2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)$

## Result:

### 24.15820...

$24.15820 \ldots$ result very near to the black hole entropy 24.2477 (see Table)

## Alternative representations:

$$
\begin{aligned}
& 2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)= \\
& 2(-0.98911+4 \pi)+\frac{1}{3} \log _{e}\left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}} \\
& 2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)= \\
& 2(-0.98911+4 \pi)+\frac{1}{3} \log (a) \log _{a}\left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}
\end{aligned}
$$

$$
2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)=
$$

$$
2(-0.98911+4 \pi)+\frac{1}{3} \log \left(\frac{4\left(-1+\frac{2}{1+\frac{1}{e^{7}}}\right)^{2}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}
$$

## Series representations:

$2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)=$
$\frac{1}{\sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}} 8\left(0.00618194-0.247278 \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}+\right.$

$$
\left.\pi \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}-0.0416667 \sum_{k_{1}=1}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{(-1)^{k_{2}}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k_{2}}}{\left(100+\pi^{2}\left(1-2 k_{1}\right)^{2}\right) k_{2}}\right)
$$

$$
\begin{aligned}
& 2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)= \\
& \left(8 \left(-0.247278+0.5 \pi-0.247278 \sum_{k=1}^{\infty}(-1)^{k} q^{2 k}+\right.\right. \\
& \pi \sum_{k=1}^{\infty}(-1)^{k} q^{2 k}-0.0208333 \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k}}{k}- \\
& \left.0.0416667 \sum_{k_{1}=1}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{\left.(-1)^{k_{1}+k_{2}} q^{2 k_{1}}\left(-1+\frac{4 \tanh 2\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k_{2}}\right)}{k_{2}}\right) / \\
& \left(0.5+\sum_{k=1}^{\infty}(-1)^{k} q^{2 k}\right) \text { for } q=e^{5}
\end{aligned}
$$

$$
2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)=
$$

$$
\left(8 \left(-0.247278-0.247278 \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}+\right.\right.
$$

$$
\pi \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}-0.0416667
$$

$$
\left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{\left.\left.(-1)^{k_{2}}\left(\delta_{k_{1}}+\frac{2^{1+k_{1}} \mathrm{Li}_{-k_{1}}\left(-e^{2 z_{0}}\right)}{k_{1}!}\right)\left(5-z_{0}\right)^{k_{1}}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k_{2}}\right)\right)}{k_{2}}\right)
$$

$$
/\left(\sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
$$

From:
Three-dimensional AdS gravity and extremal CFTs at $\mathbf{c}=\mathbf{8 m}$
Spyros D. Avramis, Alex Kehagiasb and Constantina Mattheopoulou
Received: September 7, 2007 -Accepted: October 28, 2007 - Published: November 9, 2007

| $m$ | $L_{0}$ | $d$ | $S$ | $S_{B H}$ | $m$ | $L_{0}$ | d | S | $S_{B H}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 1 | 196883 | 12.1904 | 12.5664 | 6 | 1 | 42987519 | 17.5764 | 17.7715 |
|  | 2 | 21296876 | 16.8741 | 17.7715 |  | 2 | 40448921875 | 24.4233 | 25.1327 |
|  | 3 | 8426093326 | 20.5520 | 21.7656 |  | 3 | 846:351171):3277 | 29.7668 | 30.7812 |
| 4 | $2 / 3$ | 139503 | 11.8458 | 11.8177 | 7 | 2/3 | 7102775 | 15.8174 | 15.6730 |
|  | 5/3 | 69193488 | 18.0524 | 18.7328 |  | 5/3 | 33934039437 | 24.2477 | 24.7812 |
|  | 8/3 | 6928824200 | 22.6589 | 23.6954 |  | 8/3 | 16953652012291 | 30.4615 | 31.3460 |
| 5 | 1/3 | 20619 | 9.9340 | 9.3664 | 8 | 1/3 | 278511 | 12.5372 | 11.8477 |
|  | 4/3 | 86645620 | 18.2773 | 18.7328 |  | 4/3 | 13996384631 | 23.3621 | 23.6954 |
|  | 7/3 | 24157197490 | 23.9078 | 24.7812 |  | 7/3 | 19400406113385 | 30.5963 | 31.3460 |

Table 1: Degeneracies, microscopic entropies and semiclassical entropies for the first few values of $m$ and $L_{0}$.
$5 *((2 *(4 \mathrm{Pi}-0.98911)+(((2 * 0.98911) /(\tanh (5))))+1 / 3 *$
$\left.\left.\ln \left(\left(\left(\left(4 \tanh ^{\wedge} 2(7 / 2)\right) /(\sinh (5))\right)\right)\right)\right)\right)+18+1 /$ golden ratio

## Input:

$5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}$

## Result:

139.4090...
139.4090... result practically equal to the rest mass of Pion meson 139.57

## Alternative representations:

$5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}=$

$$
18+\frac{1}{\phi}+5\left(2(-0.98911+4 \pi)+\frac{1}{3} \log _{e}\left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}\right)
$$

$$
\begin{aligned}
& 5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}= \\
& 18+\frac{1}{\phi}+5\left(2(-0.98911+4 \pi)+\frac{1}{3} \log (a) \log _{a}\left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}\right) \\
& 5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}= \\
& 18+\frac{1}{\phi}+5\left(2(-0.98911+4 \pi)+\frac{1}{3} \log \left(\frac{4\left(-1+\frac{2}{1+\frac{1}{e^{7}}}\right)^{2}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)+\frac{1.97822}{-1+\frac{2}{1+\frac{1}{e^{10}}}}\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& 5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}= \\
& 40\left(\begin{array}{l}
0.00618194 \phi+0.025 \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}+ \\
0.202723 \phi \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}+\phi \pi \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}-0.0416667 \\
\left.\left.\phi \sum_{k_{1}=1 k_{2}=1}^{\infty} \sum_{(-1)^{k_{2}}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\left.\sinh ^{2}\right)}\right)^{k_{2}}}^{\left(100+\pi^{2}\left(1-2 k_{1}\right)^{2}\right) k_{2}}\right)\right) /\left(\phi \sum_{k=1}^{\infty} \frac{1}{100+(1-2 k)^{2} \pi^{2}}\right)
\end{array}\right.
\end{aligned}
$$

$$
\begin{aligned}
& 5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}= \\
& 40\left(0.0125-0.0222775 \phi+0.5 \phi \pi+0.025 \sum_{k=1}^{\infty}(-1)^{k} q^{2 k}+\right. \\
& 0.202723 \phi \sum_{k=1}^{\infty}(-1)^{k} q^{2 k}+\phi \pi \sum_{k=1}^{\infty}(-1)^{k} q^{2 k}- \\
& 0.0208333 \phi \sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k}}{k}- \\
& \left.0.0416667 \phi \sum_{k_{1}=1}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{(-1)^{k_{1}+k_{2}} q^{2 k_{1}}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k_{2}}}{k_{2}}\right) / \\
& \left(\phi\left(0.5+\sum_{k=1}^{\infty}(-1)^{k} q^{2 k}\right)\right) \text { for } q=e^{5} \\
& 5\left(2(4 \pi-0.98911)+\frac{2 \times 0.98911}{\tanh (5)}+\frac{1}{3} \log \left(\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)\right)+18+\frac{1}{\phi}= \\
& \left(4 0 \left(-0.247277 \phi+0.025 \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} L_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}+\right.\right. \\
& 0.202723 \phi \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}+ \\
& \phi \pi \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}-0.0416667 \phi \\
& \left.\sum_{k_{1}=0}^{\infty} \sum_{k_{2}=1}^{\infty} \frac{(-1)^{k_{2}}\left(\delta_{k_{1}}+\frac{2^{1+k_{1}} \mathrm{Li}_{k_{1}}\left(-e^{2 z_{0}}\right)}{k_{1}!}\right)\left(5-z_{0}\right)^{k_{1}}\left(-1+\frac{4 \tanh ^{2}\left(\frac{7}{2}\right)}{\sinh (5)}\right)^{k_{2}}}{k_{2}}\right) \\
& /\left(\phi \sum_{k=0}^{\infty}\left(\delta_{k}+\frac{2^{1+k} \mathrm{Li}_{-k}\left(-e^{2 z_{0}}\right)}{k!}\right)\left(5-z_{0}\right)^{k}\right) \text { for } \frac{1}{2}+\frac{i z_{0}}{\pi} \notin \mathbb{Z}
\end{aligned}
$$

Now, we have that:

$$
S_{\text {matter }}(I \cup R) \approx 2 S_{\text {matter }}\left(\left[P_{1}, P_{2}\right]\right)-\frac{c}{3} \log \left(\frac{2\left|\cosh (a+b)-\cosh \left(t_{c}-t_{b}\right)\right|}{\sinh a}\right)
$$

$1 / 3 \ln ((((2 \cosh (5+2)-\cosh (0))) /((\sinh (5)))))$

## Input:

$\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)$

## Exact result:

$\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))$

## Decimal approximation:

0.897427038608265865479582877913152494054097509045630356825...
0.8974270386082....

## Alternate forms:

$\frac{1}{3}(\log (2 \cosh (7)-1)+\log (\operatorname{csch}(5)))$
$\frac{1}{3} \log \left(\frac{2\left(-1+\frac{1}{e^{7}}+e^{7}\right)}{e^{5}-\frac{1}{e^{5}}}\right)$
$\frac{1}{3}\left(-2+\log (2)-\log \left(e^{10}-1\right)+\log \left(1-e^{7}+e^{14}\right)\right)$

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=\frac{1}{3} \log \left(\frac{-1+\frac{1}{e^{7}}+e^{7}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right) \\
& \frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=\frac{1}{3} \log _{e}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right) \\
& \frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=\frac{1}{3} \log (a) \log _{a}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)
\end{aligned}
$$

## Series representation:

$\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=$
$\frac{1}{3} \log (-1+(-1+2 \cosh (7)) \operatorname{csch}(5))-\frac{1}{3} \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1+(-1+2 \cosh (7)) \operatorname{csch}(5)}\right)^{k}}{k}$

## Integral representations:

$\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=\frac{1}{3} \int_{1}^{\left(-1+2 \cosh (7) \operatorname{csch}(5) \frac{1}{t} d t . t .\right.}$
$\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)=$
$-\frac{i}{6 \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{(-1+(-1+2 \cosh (7)) \operatorname{csch}(5))^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s$ for $-1<\gamma<0$
$((((1 / 3 \ln ((((2 \cosh (5+2)-\cosh (0))) /((\sinh (5))))))))))^{\wedge} 1 / 16$

## Input:

$\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}$

## Exact result:

$\sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))}$

## Decimal approximation:

0.993258858131342001248394167369224755984632041799723686055...
$0.993258858131342 \ldots$. result very near to the value of the following RogersRamanujan continued fraction:
$\frac{\mathrm{e}^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^{5 \sqrt[4]{5^{3}}}}-1}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-2 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-3 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-4 \pi \sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$
and to the dilaton value $\mathbf{0 . 9 8 9 1 1 7 3 5 2 2 4 3 = \boldsymbol { \phi }}$

## Alternate forms:

$\sqrt[16]{\frac{1}{3}(\log (2 \cosh (7)-1)+\log (\operatorname{csch}(5)))}$
$\sqrt[16]{\frac{1}{3}} \log \left(\frac{2\left(-1+\frac{1}{e^{7}}+e^{7}\right)}{e^{5}-\frac{1}{e^{5}}}\right)$
$\frac{1}{\sqrt[16]{\frac{3}{-2-\log \left(\frac{e^{10}-1}{2\left(1-e^{7}+e^{14}\right)}\right)}}}$

## All 16th roots of $1 / 3 \log ((2 \cosh (7)-1) \operatorname{csch}(5))$ :

$e^{0} \sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))} \approx 0.99326$ (real, principal root)
$e^{(i \pi) / 8} \sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))} \approx 0.91765+0.38010 i$
$e^{(i \pi) / 4} \sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))} \approx 0.70234+0.70234 i$
$e^{(3 i \pi) / 8} \sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))} \approx 0.38010+0.91765 i$
$e^{(i \pi) / 2} \sqrt[16]{\frac{1}{3} \log ((2 \cosh (7)-1) \operatorname{csch}(5))} \approx 0.99326 i$

## Alternative representations:

$$
\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=\sqrt[16]{\frac{1}{3} \log \left(\frac{-1+\frac{1}{e^{7}}+e^{7}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)}
$$

$$
\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=\sqrt[16]{\frac{1}{3} \log _{e}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)}
$$

$$
\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=\sqrt[16]{\frac{1}{3} \log (a) \log _{a}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)}
$$

## Series representation:

$\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=$
$\frac{\sqrt[16]{\log (-1+(-1+2 \cosh (7)) \operatorname{csch}(5))-\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-1+(-1+2 \cosh (7)) \operatorname{csch}(5)}\right)^{k}}{k}}}{\sqrt[16]{3}}$

## Integral representations:

$\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=\frac{\sqrt[16]{\int_{1}^{\left(-1+2 \cosh (7) \operatorname{csch}(5) \frac{1}{t} d t\right.}}}{\sqrt[16]{3}}$
$\sqrt[16]{\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)}=\frac{\sqrt[16]{-i \int_{-i \infty \infty+\gamma}^{i \infty+\gamma} \frac{(-1+(-1+2 \cosh (7)) \operatorname{csch}(5))^{-s} \Gamma(-s)^{2} \Gamma(1+s)}{\Gamma(1-s)} d s}}{\sqrt[16]{6 \pi}}$
for $-1<\gamma<0$
$8 \log$ base $0.993258858131342((((1 / 3 \ln ((((2 \cosh (5+2)-\cosh (0))) /((\sinh (5)))))))))-$ $\mathrm{Pi}+1 /$ golden ratio

Where 8 is a Fibonacci number

## Input interpretation:

$8 \log _{0.903258858131342}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}$

## Result:

125.4764413352...
$125.4764413352 \ldots$ result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18

## Alternative representations:

$$
\begin{aligned}
& 8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}= \\
& -\pi+8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{-1+\frac{1}{e^{7}}+e^{7}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)\right)+\frac{1}{\phi}
\end{aligned}
$$

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$ $-\pi+8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log _{e}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)\right)+\frac{1}{\phi}$
$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$

$$
-\pi+\frac{1}{\phi}+\frac{8 \log \left(\frac{1}{3} \log \left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)\right)}{\log (0.9932588581313420000)}
$$

## Series representations:

$8 \log _{0.9032588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$

$$
\frac{1}{\phi}-\pi-\frac{8 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^{k}\left(-3+\log \left(-\frac{\cosh (0)-2 \cosh (7)}{\sinh (5)}\right)\right\}^{k}}{k}}{\log (0.9932588581313420000)}
$$

$8 \log _{0.9032588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$

$$
\frac{1}{\phi}-\pi+8 \log _{0.9032588581313420000}(
$$

$$
\left.\frac{1}{3}\left(\log \left(-\frac{\cosh (0)-2 \cosh (7)+\sinh (5)}{\sinh (5)}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-\frac{\cosh (0)-2 \cosh (7)+\sinh (5)}{\sinh (5)}\right)^{-k}}{k}\right)\right)
$$

## Integral representations:

$$
\begin{aligned}
& 8 \log _{0.0032588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}= \\
& \frac{1}{\phi}-\pi+8 \log _{0.9932588581313420000}\left(\frac{1}{3} \int_{1}^{-\frac{\cosh (0)-2 \cosh (7)}{\sinh (5)}} \frac{1}{t} d t\right)
\end{aligned}
$$

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$ $-1+\phi \pi-8 \phi \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{1+14 \int_{0}^{1} \sinh (7 t) d t}{5 \int_{0}^{1} \cosh (5 t) d t}\right)\right)$
$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)-\pi+\frac{1}{\phi}=$ $-\frac{-1+\phi \pi-8 \phi \log _{0.9032588581313420000}\left(\frac{1}{3} \log \left(-\frac{\sqrt{\pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{s}-2 e^{49 /(4 s)+s}}{\sqrt{s}} d s}{10 i \pi \int_{0}^{1} \cosh (5 t) d t}\right)\right)}{\phi}$ for
$8 \log$ base $0.993258858131342((((1 / 3 \ln )(((2 \cosh (5+2)-\cosh (0))) /((\sinh$ $(5))))$ )) )) ) $)+11+1 /$ golden ratio

Where 8 is a Fibonacci number and 11 is a Lucas number

## Input interpretation:

$8 \log _{0.993258858131342}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}$

## Result:

139.6180339887...
139.61803398... result practically equal to the rest mass of Pion meson 139.57

## Alternative representations:

$$
\begin{aligned}
& 8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}= \\
& 11+8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{-1+\frac{1}{e^{7}}+e^{7}}{\frac{1}{2}\left(-\frac{1}{e^{5}}+e^{5}\right)}\right)\right)+\frac{1}{\phi}
\end{aligned}
$$

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$ $11+8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log _{e}\left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)\right)+\frac{1}{\phi}$
$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$

$$
11+\frac{1}{\phi}+\frac{8 \log \left(\frac{1}{3} \log \left(\frac{-\cosh (0)+2 \cosh (7)}{\sinh (5)}\right)\right)}{\log (0.9932588581313420000)}
$$

## Series representations:

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$

$$
11+\frac{1}{\phi}-\frac{8 \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{3}\right)^{k}\left(-3+\log \left(-\frac{\cosh (0)-2 \cosh (77)}{\sinh (5)}\right)\right)^{k}}{k}}{\log (0.9932588581313420000)}
$$

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$ $11+\frac{1}{\phi}+8 \log _{0.9932588581313420000}($

$$
\left.\frac{1}{3}\left(\log \left(-\frac{\cosh (0)-2 \cosh (7)+\sinh (5)}{\sinh (5)}\right)-\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(-\frac{\cosh (0)-2 \cosh (7)+\sinh (5)}{\sinh (5)}\right)^{-k}}{k}\right)\right)
$$

## Integral representations:

$$
\begin{gathered}
8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}= \\
11+\frac{1}{\phi}+8 \log _{0.9032588581313420000}\left(\frac{1}{3} \int_{1}^{-\frac{\cosh (0)-2 \cosh (7)}{\sinh (5)}} \frac{1}{t} d t\right)
\end{gathered}
$$

$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$ $1+11 \phi+8 \phi \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{1+14 \int_{0}^{1} \sinh (7 t) d t}{5 \int_{0}^{1} \cosh (5 t) d t}\right)\right)$
$\phi$
$8 \log _{0.9932588581313420000}\left(\frac{1}{3} \log \left(\frac{2 \cosh (5+2)-\cosh (0)}{\sinh (5)}\right)\right)+11+\frac{1}{\phi}=$
$\frac{1+11 \phi+8 \phi \log _{0.0932588581313420000}\left(\frac{1}{3} \log \left(-\frac{\sqrt{\pi} \int_{-i \infty \infty+\gamma}^{i \infty+\gamma} \frac{e^{s-2} e^{49 /(4 s)+s}}{\sqrt{s}} d s}{10 i \pi \int_{0}^{1} \cosh (5 t) d t}\right)\right)}{\phi}$ for $\gamma>0$

## Acknowledgments

I would like to thank Prof. George E. Andrews Evan Pugh Professor of Mathematics at Pennsylvania State University for his great availability and kindness towards me

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