# A proof of Twin Prime Conjecture 

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#### Abstract

I proved the Twin Prime Conjecture. The probability twin prime approximately is slightly lower than $4 / 3$ times the square of the probability that a prime will appear in.

I investigated up to $5 \times 10^{12}$. When the number grows to the limit, the primes to be produced rarely, but since Twin Primes are slightly lower than $4 / 3$ times the square of the distribution of primes, the frequency of production of Twin Primes is very equal to 0 .

However, it is not 0. Because, primes continue to be produced. Therefore, Twin Primes continue to be produced.

If the Twin Primes is finite, the primes is finite. This is because slightly lower than $4 / 3$ times the square of the probability of primes is the probability of Twin Primes.

This is contradiction. Because there are an infinite of primes.


[Probability of the Existence of primes] $]^{2} \times 4 / 3=$ (Probability of the Existence of Twin Primes)

When the number becomes extreme, the generation of prime numbers becomes extremely small. However, it is not 0 .
Very few, but prime numbers are generated.
Therefore, even if the number reaches the limit, twin prime numbers are also generated.
That is, Twin Primes exist forever.

## key words

Hexagonal circulation, Twin Primes, slightly lower than $4 / 3$ times the square of the probability of primes

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## Introduction

In this paper, it is written in advance that 2 and 3 are omitted from primes.
The prime number is represented as $(6 n-1)$ or $(6 n+1)$. And, $n$ is positive integer.
All Twin Primes are combination of $(6 n-1)$ and $(6 n+1)$.
That is, all Twin Primes are a combination of 5th-angle and 1th-angle.
[ n is positive integer]
5th-angle is $(6 n-1)$.
1 th-angle is $(6 n+1)$.
$(6 \mathrm{n}-2),(6 \mathrm{n}),(6 \mathrm{n}+2)$ in are even numbers.
$(6 n-1),(6 n+1),(6 n+3)$ are odd numbers.
Primes are $(6 n-1)$ or $(6 n+1)$.
The following is a prime number.
There are no primes that are not $(6 n-1)$ or $(6 n+1)$.
$5-6 \mathrm{n}-1$ (Twin prime)
$7-6 n+1$
$11-6 \mathrm{n}-1$ (Twin prime)
$13-6 \mathrm{n}+1$
$17-6 \mathrm{n}-1$ (Twin prime)
$19-6 n+1$
$23-6 n-1$
$29-6 \mathrm{n}-1$ (Twin prime)
$31-6 n+1$
$\qquad$
$\qquad$

## Part 1

There are 164 primes from 5 to 1000 .
Probability is $\frac{164}{996}$.
In this, there are 34 Twin Primes. Probability is $\frac{34}{996}=0.034136546 \ldots$
and $\left[\frac{164}{996}\right]^{2} \times \frac{5}{4}=0.0338905824 \ldots$
$\left[\frac{164}{996}\right]^{2} \times \frac{4}{3}=0.0361499546 \ldots$

There are 455052507 primes from 5 to $10000000000=1 \times 10^{10}$.
Probability is $\frac{455052507}{9999999996}$.

In this, there are 27412678 Twin Primes. Probability is $\frac{27412678}{999999996}=0.0027412678 \ldots$ and $\left[\frac{455052507}{9999999996}\right]^{2} \times \frac{4}{3}=0.0027609704572 \ldots$

There are 37607912014 primes from 5 to $1 \times 10^{12}$.
Probability is $\frac{37607912014}{9999999996}$.
In this, there are 1870585218 Twin Primes. Probability is $\frac{1870585218}{999999999996}=0.001870585218007 \ldots$ and $\left[\frac{37607912014}{99999999996}\right]^{2} \times \frac{4}{3}=0.00188580672808544 \ldots$

There are 177291661645 primes from 5 to
$5000000000000=5 \times 10^{12}$.
Probability is $\frac{177291661645}{4999999999996}$.
In this, there are 8312493001 Twin Primes. Probability is
$\frac{8312493001}{4999999999996}=0.00166249860020133 \ldots$
and
$\left[\frac{177291661645}{4999999999996}\right]^{2} \times \frac{4}{3}=0.00167639110874109 \ldots$

## Part 2

There are $455052507-50847530=404204977$ primes from $1 \times 10^{9}$ to $1 \times 10^{10}=9 \times 10^{9}$. Probability is $\frac{404204977}{900000000}=0.04491166411 \ldots$
In this, there are $27412678-3424505=23988173$ Twin Primes. Probability is
$\frac{23988173}{9000000000}=0.00266535255 \ldots$
and
$\left[\frac{404204977}{9000000000}\right]^{2} \times \frac{4}{3}=0.00268941009764 \ldots$

There are $4118054809-455052507=3663002302$ primes from $1 \times 10^{10}$ to $1 \times 10^{11}=9 \times 10^{10}$.
Probability is $\frac{3663002302}{9000000000}=0.0407000255777 \ldots$...
In this, there are $224376047-27412678=196963369$ Twin Primes. Probability is $\frac{196963369}{90000000000}=0.0021884818777 \ldots$
and
$\left[\frac{3663002302}{90000000000}\right]^{2} \times \frac{4}{3}=0.00220865610937 \ldots$

There are $37607912016-4118054809=33489857207$ primes from $1 \times 10^{11}$ to $1 \times 10^{12}=9 \times 10^{11}$. Probability is $\frac{33489857207}{900000000000}=0.0372109524522 \ldots$
In this, there are 1870585219-224376047=1646209172 Twin Primes. Probability is $\frac{1646209172}{90000000000}=0.00182912130222 \ldots$
and
$\left[\frac{33489857207}{90000000000}\right]^{2} \times \frac{4}{3}=0.0018462066432020 \ldots$

There are $17729166164-3760791201=13968374963$ primes from $1 \times 10^{12}$ to $5 \times 10^{12}=4 \times 10^{12}$. Probability is $\frac{13968374963}{40000000000}=0.0349209374075$
In this, there are $8312493001-1870585219=6441907782$ Twin Primes. Probability is $\frac{6441907782}{400000000000}=0.0016104769455$
and
$\left[\frac{13968374963}{400000000000}\right]^{2} \times \frac{4}{3}=0.001625962492558 \ldots$

When I didn't know that it can get the number of twin primes in Wolfram Cloud, I did it by hand, the constant was $6 / 5$.
It seems that there was a mistake in calculation.
It gradually changed to $5 / 4$ and changed to $4 / 3$.
And it settled in $4 / 3$, but I know it's possible to calculate the number of twin primes in Wolfram Cloud, and it shows a slightly lower value than $4 / 3$.

Calculation depends on Wolfram Alpha and Wolfram Cloud.


## Discussion

At this stage, it is slightly lower than $4 / 3$, but it is unpredictable how it will change as the number grows.
However, it is expected to settle at a slightly lower value than $4 / 3$.
The need for a constant suggest that the prime numbers $(6 n-1)$ and $(6 n+1)$ do not occur independently.

First, say $6 n-1=6 n+5$
$(6 n-1) \times 5=6(5 n-1)+1=1$ th-angle.
$(6 n+1) \times 5=6(5 n)+5=5$ th-angle.
and
$(6 n-1) \times 7=6(7 n-2)+5=5$ th-angle.
$(6 n+1) \times 7=6(7 n+1)+1=1$ th-angle.
and
$(6 n-1) \times 11=6(11 \mathrm{n}-2)+1=1$ th-angle.
$(6 n+1) \times 11=6(11 n+1)+5=5$ th-angle.
and
$(6 n-1) \times 13=6(13 n-3)+5=5$ th-angle.
$(6 n+1) \times 13=6(13 n+2)+1=1$ th-angle.
and
$(6 n-1) \times 17=6(17 \mathrm{n}-3)+1=1$ th-angle.
$(6 n+1) \times 17=6(17 n+2)+1=5$ th-angle.
and
$(6 n-1) \times 19=6(19 n-4)+5=5$ th-angle.
$(6 n+1) \times 19=6(19 n+3)+1=1$ th-angle.
and
$(6 n-1) \times(6 n-1)=6\left(6 n^{2}-2 n\right)+1=1$ th-angle.
$(6 \mathrm{n}-1) \times(6 n+1)=6\left(6 n^{2}\right)-1=6\left(6 n^{2}-1\right)+5=5$ th-angle.
and
$(6 n+1) \times(6 n-1)=6\left(6 n^{2}\right)-1=6\left(6 n^{2}-1\right)+5=5$ th-angle.
$(6 n+1) \times(6 n+1)=6\left(6 n^{2}+2 n\right)+1=1$ th-angle.
In this way, prime multiples of $(6 n-1)$ or $(6 n+1)$ of primes fill 5 th-angle, 1 th-angle, and the location of primes becomes little by little narrower.

However, every time the hexagon is rotated once, the number of locations where the primes exists increases by two.

The probability that a Twin Prime will be produced slightly lower than $4 / 3$ times the square of the probability that a prime will be produced in a huge number, where the probability that a prime production is low from the equation (1).

And, as can be seen from the equation below, even if the number becomes large, the degree of production of primes only decreases little by little.

$$
\begin{equation*}
\pi(x) \sim \frac{x}{\log x} \quad(x \rightarrow \infty) \tag{1}
\end{equation*}
$$

$\log \left(10^{20}\right)=20 \log (10) \approx 46.0517018$
$\log \left(10^{200}\right)=200 \log (10) \approx 460.517018$
$\log \left(10^{2000}\right)=2000 \log (10) \approx 4605.17018$
$\log \left(10^{20000}\right)=20000 \log (10) \approx 46051.7018$
$\log \left(10^{200000}\right)=200000 \log (10) \approx 460517.018$

As x in $\log (x)$ grows to the limit, the denominator of the equation also grows extremely large. Even if primes are produced, the frequency of production is extremely low.
The production of Twin Primes is slightly lower than $4 / 3$ times the square of the production frequency of primes, and the production frequency is extremely low.

The production of Twin Primes equal the existence of Twin Primes. And, the production of Primes equal the existence of Primes.
The formula[3] below shows that primes are generated even if the number goes to infinity.

$$
\begin{equation*}
\liminf _{n \rightarrow \infty} \frac{p_{n+1}-p_{n}}{\log p_{n}}=0 \tag{2}
\end{equation*}
$$

Use a contradiction method.
If the Twin Primes is finite, the primes is finite.
This is because slightly lower than $4 / 3$ times the square of the probability of primes is the probability of Twin Primes.
This is contradiction. Because there are an infinite of primes.
[Probability of the Existence of primes] ${ }^{2} \times 4 / 3=$ (Probability of the Existence of Twin Primes)

However, I am at a loss as to whether it can be said that extremely low is produced mathematically.
However, it can be said that it is produced because it is greater than zero.
The contradiction method is decisive.

That is, Twin Primes exist forever.
Proof end.

## References

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## Postscript

I thank Prof. S. Saitoh for his many advices.
And fried-turnip's Yahoo Answers, for a Wolfram Cloud program that you have me tell you, the last of the stuffing was able at once.
Thanks to fried-turnip, it was decided whether $4 / 3$ would be a constant.
In the early days of manual calculations, the constant was $6 / 5$.
There was a mistake in the hand calculation, and at the beginning it became such a stance.
After 200,000 by hand calculation, the constant changed to $4 / 3$, and I was thinking what value this would change in the future. However, Wolfram Cloud can easily calculate the number of twin primes. Knew. This was taught by fried-turnip in Yahoo!


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