Making Sense of the Cosmos by a Fractal Geometry Experiment

Updated from first published [1]Sept 21st, 2014 paper.

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Abstract

'Fractals are everywhere'; but for the large-scale galaxy distribution in observable universe, by the 2012 WiggleZ Dark Energy Survey, they were ruled out. They were however, granted on small cosmic scales, as claimed by proponents, and notwithstanding the ensuing enigma of explaining the standard model of cosmology 'very large' cosmic structures (LQGs) have indeed – as predicted – have been observed in the large-scale universe since the WiggleZ survey. Can a (different) fractal model explain all these cosmological observations and conjectures, and if so, are we are modeling the fractal the universe incorrectly? An experiment was conducted on a 'simple' (Koch snowflake) fractal from the perspective of an observer within a growing/emergent fractal – 'looking' in iteration-time. The fractal was inverted – to show what the fractal looks like back to its origin; new triangle sizes were held constant allowing earlier triangles in the set to expand as the set iterated. Classical kinematic equations of velocities and accelerations were calculated for total area total and the distance between points. Hubble-Lemaitre's Law and other cosmological observations and conjectures was tested for. Results showed area(s) expanded exponentially from an arbitrary starting position; and as a consequence, the distances between points – from any location within the set – receded away from the 'observer' at increasing velocities and accelerations. It was concluded that the fractal is a geometrical match to the cosmological problems. It explains: Hubble-Lemaitre and accelerated expansion; inhomogeneous (and said fractal) galaxy distribution on scale and large scales; and other problems – including the cosmological catastrophe and the early inflationary expansion epoch of the universe. Also, the fractal offers a direct mechanism to the cosmological problem and can further explain the quantum problem – unifying the two realities as being two aspects of the same geometry.

Keywords Fractal-cosmology, Dark Energy, Inflation, Hubble- Lemaitre's Law, Cosmological Constant

Preface

The idea to conduct an experiment on a fractal – testing for cosmological relevance – has its roots long before I knew much about cosmology. It started with my day job where I teach economics at secondary school level; I had for a long time been interested in the natural sciences and also in fractal geometry. It was while teaching the common 'supply demand' model I noticed that it seemed to behave as a fractal where it is produced and consumed and grows and develops, and it seems to have a shared equilibrium; and so, I questioned this, is this a coincidence? The more I thought about the fractal the stranger – even weirder – it became to me, especially an isolated one – it is very strange. In an infinite fractal there is no location and it appears to be infinite in size. This had me thinking. Once, when I talked about my problem to my (interested) class, one of my students said, 'that sounds like quantum mechanics'. I said yes, 'I think so too', but for a time I was afraid to investigate – 'no one understands quantum mechanics' – right?! I have since done an investigation and experiment and am totally satisfied the geometry of the fractal offers a solution the great physics problem of our time: to make sense of the 'small scale quantum world' and unify it with the large.

When my mind turned to thinking about what an observer would experience if they were within a fractal and looking back in time I immediately released this had relevance to cosmology; especially when I found the observer would experience acceleration. In 2013 I made a start and wrote up this fractal cosmology experiment and then went onto write up my economics (supply demand) fractal experiment. When I am finished with this update I aim to write up my fractal experiment pointing to making sense of the quantum world.

Table of Contents

1 INTRODUCTION

At the time of writing this (update) – 2019, one of the key architects of the cosmological standard model, Professor Jim Peebles, was awarded the Nobel Prize for physics; a prize to a model that by all accounts remains in a self-titled 'state of crisis'– no disrespect to the Professor and others. Nobody, it is claimed, has any idea how to make sense it, let alone – and importantly – be able to marry it with the equally enigmas quantum reality. One geometric candidate has been fractal geometry. Though fractals are well accepted as a real and exciting geometric description of our Earthly reality, explaining the likes of market prices, clouds and trees [2]; for cosmology [3],[4], [5],[6], [7],[8] – based on the small-scale astronomical observations of clustering and super-clustering – 'fractals are out', and have been totally dismissed by the mainstream cosmologists and reduced to the trivial. If this cosmology problem – and the quantum – were a murder investigation and we were pinning up facts on the suspect board, forming a profile (a model) towards a solution: this dismissing of fractal properties may be ignoring vital evidence. The reinstatement of this geometry is the aim of this paper. The 2012 WiggleZ Dark Energy Survey[9] and others[10] concluded – but however, granted – that the universe does indeed show direct evidence of small-scale fractal galaxy distribution for distances less than 70 to 100 Mega parsecs away (give light years) ; however overall the universe is assumed overall 'smooth', homogenous and isotropic, on large scales [11],[12], and that the cosmological principle and thus general relativity and the standard (ΛCDM) model holds. Soon after the Wigglez paper was released – in direct contradiction to all of the above fractal rebuttals – large-scale surveys discovered 'very large', 'thin' and old structures in the assumed smooth universe – the 4 billion light years in sized Huge 'Large Quasar Group'[13] and the 10 billion light years sized Hercules–Corona Borealis Great Wall [14] – have indeed been discovered; just where they were predicted to be by the fractal cosmologists, in the large scale universe.

'The gap in the knowledge' in cosmology is aptly quoted to be the largest in science: how to make sense of what we cosmologically observe and unify this in a coherent way that corresponds to our reality, namely, the equally baffling quantum enigma. Can we marry these small (cosmic) scale fractal like observations with the already observed

CMB[15] singularity beginning; Hubble-Lemaitre [16],[17] – and now accelerating 'dark energy'[18],[19] – expansion; the apparent evolving galaxy demographic history and finally the observed varying distribution dark matter halo merger trees [20] clusters with time, in the aim of forming a more coherent model than the current standard model?

The model should not take away from what has a has already been achieved – namely general relativity – it should complement it and be a simple geometric, akin to the Copernican Heliocentric solution of the solar system. And it should open the door to a quantum unification.

Solving this problem may lie in how we understand how the fractal grows. Current cosmological – and other – fractal studies all assume a traditional or classical fractal perspective of growth where the growth of a fractal emerges from the addition of new diminishing sized bits convergent upon the 'snowflake' structure – in and around 7 ± 2 iterations – as demonstrated with the Koch snow flake in Figure 1 (A). In this, the initial triangle bit remains constant in size.

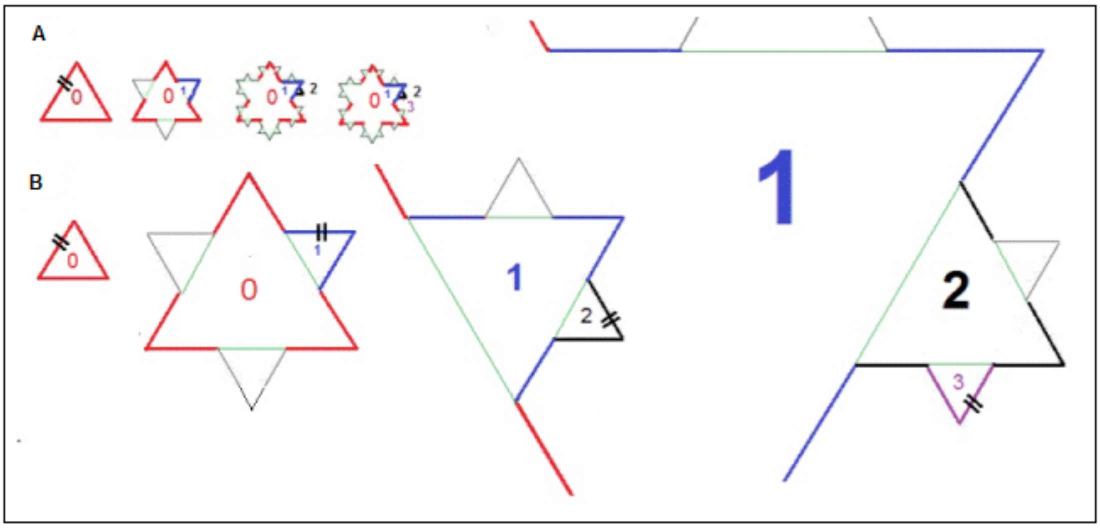

Figure 1. Dual Perspectives of (Koch Snowflake) Fractal Growth. The schematics above demonstrate fractal growth by: (A) the classical convergent snowflake perspective, where the original sized triangle bit (thatched iteration '0') remains constant, and the following triangles diminish in size from colour red iteration 0 to colour purple iteration 3; and (B) the inverted, 'expanding' perspective where the new (thatched) triangle is the focus and held at standard size and the original red iteration 0 triangle expands in area – as the fractal iterates.

However, the fractal can – paradoxically – be modelled (simultaneously) from an alternative 'backward looking' production perspective (Figure 1 B) – as opposed to the classical 'forward looking' perspective. It places the 'observer' within the fractal (in situ) – observing back in time and 'measuring' as the fractal grows with time (iteration time). To model this *in situ* perspective of the fractal the classical emergent fractal methods were corrected to grow rather than diminish. The fractal was 'inverted': it assumed that it is the new bit sizes that remain constant size (the same size as the original bit size '0') and older generations of bit sizes grow with iteration-time as demonstrated with colours red (the original size) blue (the 1^{st} iteration), black (the 2^{nd}), and so on. With iteration-time, the size of the initial red iteration 0 triangle expands relative to the size of the new blue triangle.

As a practical example of this fractal model is to think of the growth of a tree: follow the first (new growth) stem size – keeping this stem/branch size at a constant size – as the rest of the tree grows. To grow more branches, the volume of the earlier/older branches must expand. Now think of sitting/observing on one the branches of a tree that is infinitely large, infinitely growing. What would you see in front? What would you see behind? In a recent paper – and also an enigma – it was found trees – a contemporary example of a 'natural' fractal – volumetric growth accelerates with age [21].

In this paper I hypothesised that the conjectured and observed behaviour of the universe are all inextricable features of a growing iterating fractal observed from an in-situ position. This model will explain and predict: an observed 'singularity' (Big Bang) beginning, (section 4.1); the presence, and dominance of a 'uniform' Cosmic Microwave Background like origin (section 4.1.1);an inflation epoch (section 4.2); a Hubble-Lemaitre Law expansion (section 4.1); accelerating (exponential) 'dark energy' expansion (section 4.5); a cosmological constant (section 4.5); and the demographics evolution and transition in large- and small-scale galaxy distribution with time; offer a solution to a unified (quantum) theory and solve the cosmological catastrophe.

The model should not take away from what has a has already been achieved – namely general relativity – it should complement it and be a simple geometric, akin to the Copernican Heliocentric solution of the solar system.

2 METHODS

To create a quantitative data series for analysis of the inverted fractal, the classical Koch Snowflake area equations were adapted to account for this perspective, and a spreadsheet model [22] was developed to trace area expansion with iteration.
The scope of this investigation was limited to the two-dimensional – as a demonstration; three-dimensional space or volume can be inferred from this initial assumption.

2.1 Koch Snowflake Model

Changes in the areas of triangles, and distances between points in the fractal set were measured and analysed to determine whether the fractal area and distance between points expand. To do this a data table was produced (Table 1) to calculate the area growth at each, and every iteration of a single triangle. Area was calculated from the following equation (1) measured in standard (arbitrary) centimetres (***cm***)

$$A = \frac{l^2\sqrt{3}}{4} \tag{1}$$

where (***A***) is the area of a single triangle, and where ***l*** is the triangle's base length. ***l*** was placed in Table 1 and was set to 1.51967128766173 ***cm*** so that the area of the first triangle (i_0) approximated an arbitrary area of 1 cm^2. To expand the triangle with iteration the base length was multiplied by a factor of 3. The iteration number was placed in a column, followed by the base length of the equilateral triangle, and in the final column the formula to calculate the area of the triangle. Calculations were made to the 10th iteration, and the results graphed.

2.2 Iteration-Time

The iterating fractal exposes the issue of time. In isolation the fractal grows with the passing of time, and in isolation this time can only be the iteration-time. For the purposes of this investigation the iteration count was assumed to be equal to time, called: iteration-time, and denoted ***i***.

2.3 Distance and Displacement

To measure and analyse the expansion behaviour of the inverted fractal set, changes in the distance between points in the set after each iteration were calculated in a second data table (table 2) on the spreadsheet. The triangle's geometric centre points were chosen as the points to measure as demonstrated in Figure 2. The line traces distances between triangle centre points; and the blue line, the displacement from an arbitrary observation point.

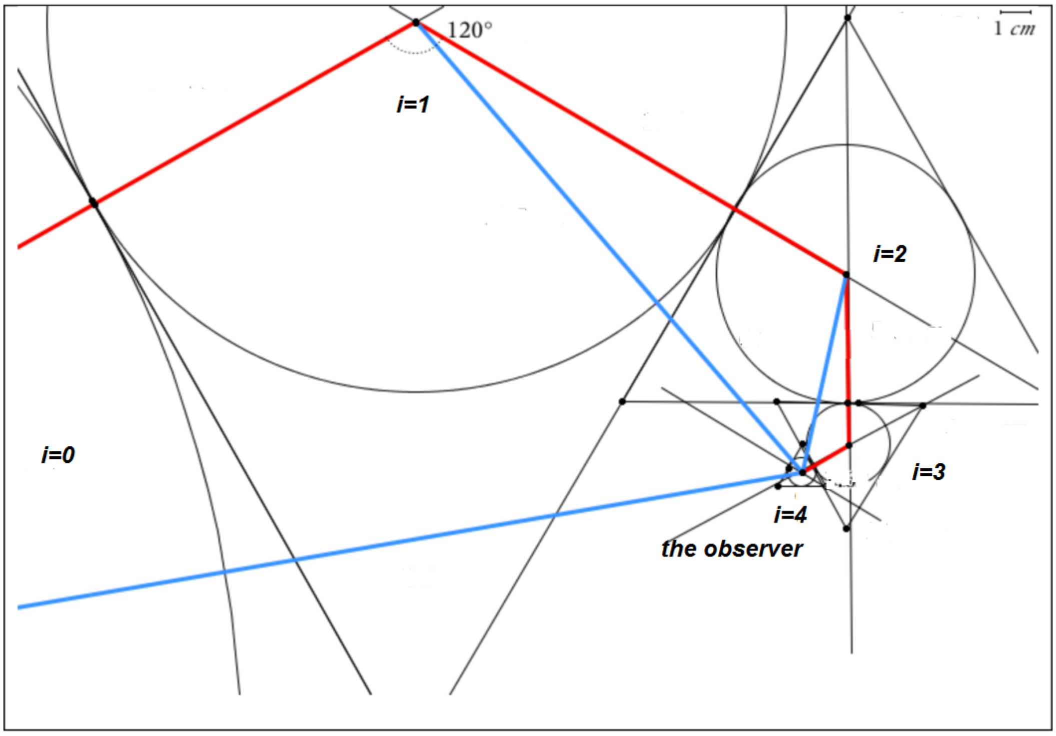

Figure 2. Displacement (blue line) and Distance (red line) Between 'Observer' and Triangle Centre Points on an Iterating Koch Snowflake Fractal.

Equation (2) below calculated the inscribed radius of an equilateral triangle. Distance between points was calculated by adding the inscribed radius of the first triangle (***i_0***) to the inscribed radius of the next expanded triangle (***i_1***).

$$r = \frac{\sqrt{3}}{6} l \tag{2}$$

From the radius distance measurements; displacement, displacement expansion ratio, velocity, acceleration, and expansion acceleration ratio for each and every iteration-time were calculated using classical mechanics equations. The change in distance between points was recorded, as was the change in displacement (distance from ***i_0***).

2.4 Spiral Propagation

The propagation of triangles in the (inverted) Koch Snowflake fractal, is not linear but in the form of a logarithmic spiral – as shown in Figure 1 B (above), and in Appendix Figure 15. The method given thus far assumes, and calculates the linear circumference

of this spiral and not the true displacement (the radius). This method was justified by arguing the required radius (or displacement) of the logarithmic spiral calculation was too complex to calculate, (and beyond the scope of this investigation), and that expansion inferences from inverted fractal could be made from the linear circumference alone. This been said, a spiral model was created independently, and radii measured to test whether spiral results were consistent with the linear results in the investigation. Measurements were made using TI – Nspire™ geometric software (see Appendix I Figure 15). Displacements, and the derived Hubble diagram from this radius model were expected to show significantly lower values than the above (calculated) circumference non-vector method, but nonetheless share the same (exponential) behaviour. Appendix Figure 1 shows in the distance between centre points, and in blue the displacement.
See Appendix Figure 16, and Figure 17, and Table 1 for results.

2.5 Area Expansion of the Total Inverted Fractal with Iteration-time

With iteration, new triangles are (in discrete quantities) introduced into the set – at an exponential rate. While the areas of new triangles remain constant, the earlier triangles expand, and by this the total fractal set expands. To calculate the area change of a total inverted fractal (as it iterated), the area of the single triangle (at each iteration-time) was multiplied by its corresponding quantity of triangles (at each iteration-time).
Two data tables (tables 3 and 4 in the spreadsheet file) were developed. Table 3 columns were filled with the calculated triangle areas at each of the corresponding iteration-time – beginning with the birth of the triangle and continuing to iteration ten. Table 4 triangle areas of table 3 were multiplied by the number of triangles in the series corresponding with their iteration-time.
Values calculated in table 3 and 4 were totalled and analysed in a new table (table 5). Analysed were: total area expansion per iteration, expansion ratio, expansion velocity, expansion acceleration, and expansion acceleration ratio. Calculations in the columns used kinematic equations developed below.

2.6 Kinematics Analysis

Classical physics equations were used to calculate velocity and acceleration of the receding points (table 2) and the increasing area (table 5).

2.6.1 Velocity

Velocity (v) was calculated by the following equation

$$\boldsymbol{v} = \frac{\Delta \boldsymbol{d}}{\Delta \boldsymbol{i}} \tag{3}$$

where classical time was exchanged for iteration-time (***i***). Velocity is measured in standard units per iteration $cm^{-1}\, i^{-1}$ for receding points and $cm^{-2}\, i^{-1}$ for increasing area.

2.6.2 Acceleration

Acceleration (a) was calculated by the following equation

$$a = \frac{\Delta \boldsymbol{v}}{\Delta \boldsymbol{i}} \tag{4}$$

Acceleration is measured in standard units per iteration $\boldsymbol{cm^{-1}\, i^{-2}}$ and $\boldsymbol{cm^{-2}\, i^{-2}}$.

2.6.3 Ratios

Ratios of displacement expansion and acceleration were calculated by dividing the outcome of $\boldsymbol{i_1}$ by the outcome of $\boldsymbol{i_0}$.

The same method of ratio calculation was used to determine change or expansion of area.

2.7 Hubble's Law and Diagram

To test for Hubble's Law, a Hubble (like) a scatter graph titled 'The Fractal/Hubble diagram' was constructed from the results of the recession velocity and distance calculations (in table 2 of inverted fractal spread sheet file). On the x-axis was the displacement (total distance) of triangle centre points at each iteration-time from $\boldsymbol{t_0}$; and on the y-axis the expansion velocity at each iteration-time. A best fitting linear regression line was calculated and a Hubble's Law equation (5) was derived

$$\boldsymbol{v} = \boldsymbol{H_0 D} \tag{5}$$

where $\boldsymbol{H_0}$ the (present) Hubble constant (the gradient), and $\boldsymbol{D}$ the distance.

2.8 Acceleration vs. Distance

Using the same methods as used to develop the Hubble diagram (as described above in 2.7) an 'acceleration vs. distance' diagram was created, regressed, and an expansion constant derived.

2.9 Small Scale Long Scale Point Distribution Analysis

To analyse point distribution on the fractal-Hubble diagram:

1. The quantity of triangle sizes per total distance increment on the fractal-Hubble diagram was calculated by: counting the quantity of triangle sizes (in distance column in table 2) and dividing this by the distance increments measured in the sample. See Table 2a of spreadsheet model.
2. The quantity of triangles at each increment was calculated by totalling the quantity of triangles (from table 4) for each respective iteration-distance.
3. An amended Fractal-Hubble diagram – combining (recessional) velocity with the quantity of triangles at every distance – was created. See table 7 of spreadsheet model.

3 RESULTS

Figure 3 to 10 show graphically the results of the experiment.

3.1 Area Expansion of Initial Triangle

The area of the initial triangle of the inverted Koch Snowflake fractal increased exponentially – shown here in

Figure 3.

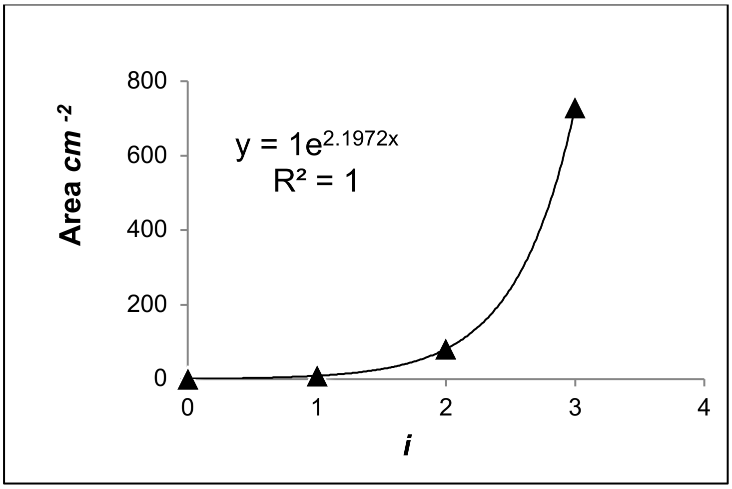

Figure 3. Initial triangle exponential area expansion. The area of the initial triangle bite on the inverted Koch Snowflake fractal increases exponentially with iteration-time. cm = centimetres. *i* = iteration-time.

This expansion with respect to iteration-time is written as

$$\boldsymbol{A = 1e^{2.197i}} \tag{6}$$

3.2 Area Expansion of Total Inverted-Fractal

The area of the total fractal (Figure 4 A) and the distance between points (Figure 4 B) of the inverted fractal increased exponentially.

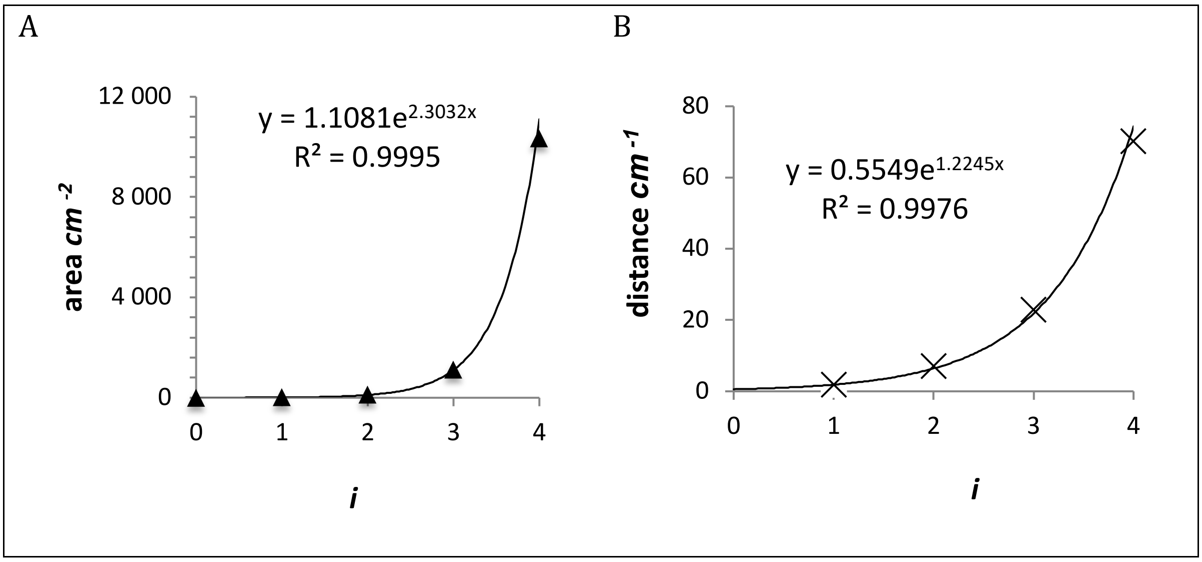

Figure 4. Area / distance expansion per iteration-time on the Inverted Koch Snowflake fractal. (A) total area expansion and (B) distance between points. cm = centimetres. *i* = iteration-time.

The expansion of the total area ($\boldsymbol{A}^{\boldsymbol{T}}$) is described as

$$A^T = 1.1081e^{2.3032i} \tag{7}$$

The expansion of distance between points ($\boldsymbol{D}$) is described by the equation

$$D = 0.5549e^{1.2245i} \tag{8}$$

3.3 Expansion Ratios

The expansion ratios for the given 10 iteration sample (shown below in Figure 5A and Figure 5B) were initially high (12 and 4 respectively), followed by a decreasing range, to settle finally at the stable ratio of expansion of 9 and 3 respectively (for the tested 10 iterations).

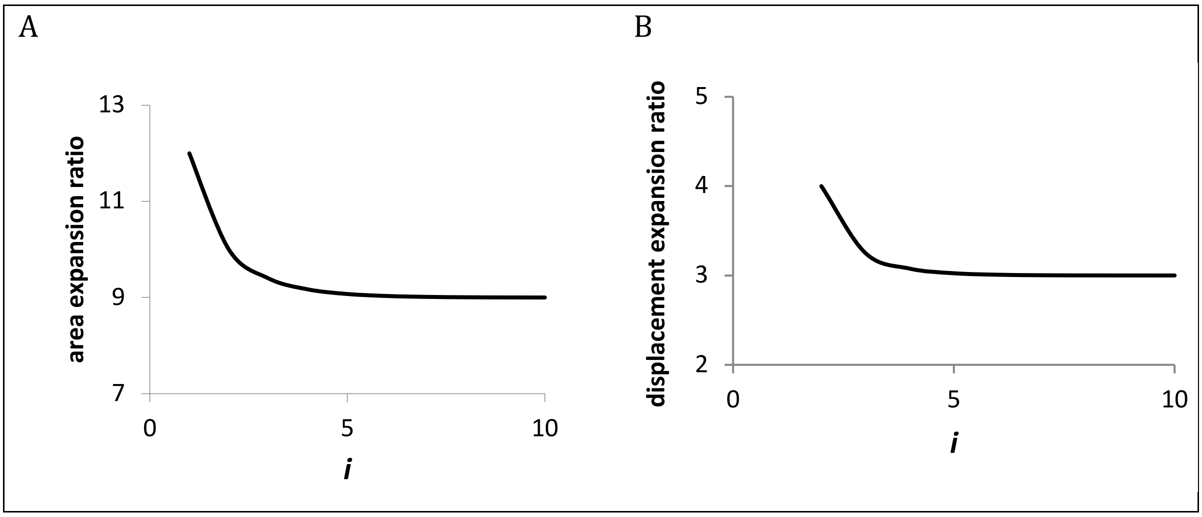

Figure 5. Expansion ratios on the Inverted Koch Snowflake fractal. Results corresponding to each iteration-time (i) of (A) total area; and (B) between points. *i* = iteration-time.

3.4 Velocity

The (recession) velocities for both total area and distance between points (Figure 6 A and B respectively) increased exponentially per iteration-time.

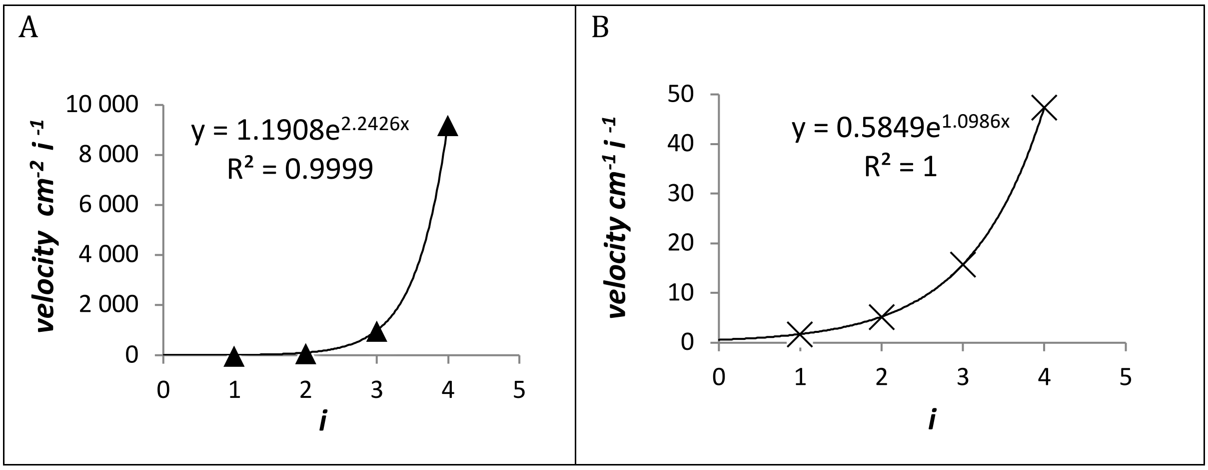

Figure 6. (Expansion) velocity of Area and Points on the Inverted Koch Snowflake Fractal. Expansion velocity of the inverted fractal at each corresponding iteration-time (*i*): (A) expansion of total area, and (B) distance between points. cm = centimetres. *i* = iteration-time.

Velocity is described by the following equations respectively

$$\boldsymbol{v = 1.1908e^{2.2426i}} \tag{9}$$

$$\boldsymbol{v^T = 0.5849e^{1.0986i}} \tag{10}$$

where $\boldsymbol{v^T}$ is the (recession) velocity of the total area; and $\boldsymbol{v}$ the (recession) velocity of distance between points.

3.5 The Fractal/ Hubble Diagram

As the distance between centre points increases (at each corresponding iteration-time), so too does the recession velocity of the points – as shown in Figure 7 below.

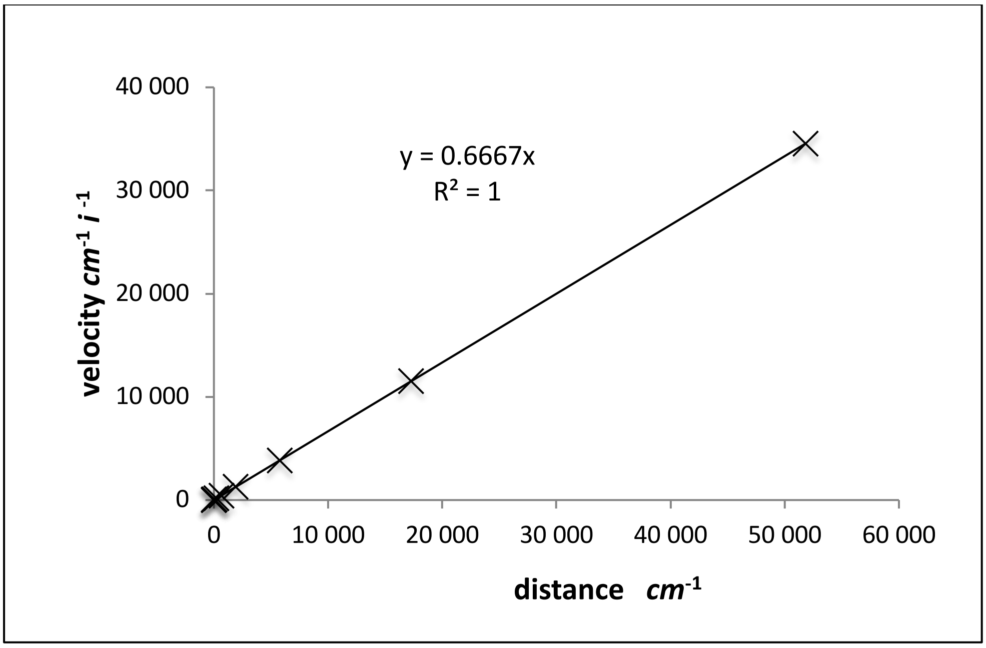

Figure 7. The Fractal-Hubble diagram. As the (exponential) distance between triangle geometric centres increases with iteration-time, the recession velocity of the points increases. cm = centimetres. i = iteration-time.

Recession velocity vs. distance of the fractal is described by the equation

$$\boldsymbol{v = 0.6679D} \tag{11}$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\, i^{-1}\, cm^{-1}}$.

The spiral radius (see Appendix Figure 16 and Appendix Table 1 for details) – where the centre is the observation point – resulted in a Fractal-Hubble equation of

$$\boldsymbol{v = 0.6581D} \tag{12}$$

3.6 Acceleration of Area and Distance Between Points

The accelerations for both total area and (recession) distance between points (Figure 8A and B respectively) increased exponentially per iteration-time.

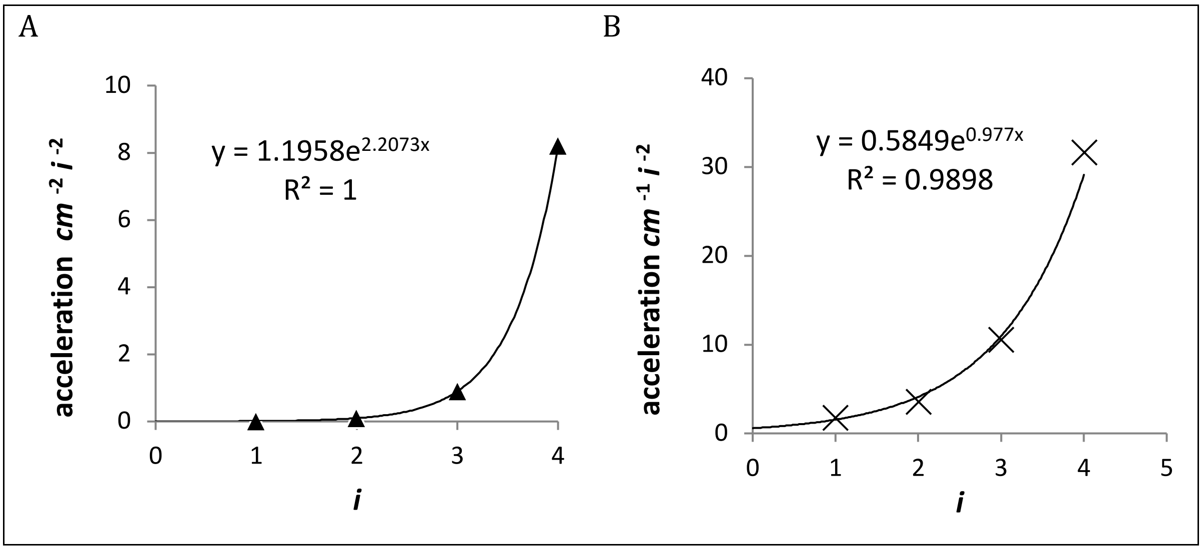

Figure 8. (Expansion) Acceleration of Area and Points on the Inverted Koch Snowflake Fractal. Acceleration of the inverted fractal at each corresponding iteration-time (i): (A) expansion of total area, and (B) distance between points. cm = centimetres. *i* = iteration-time.

Acceleration is described by the following equations respectively

$$\boldsymbol{a^T = 1.1958e^{2.2073i}} \tag{13}$$

$$\boldsymbol{a = 0.5849e^{0.977i}} \tag{14}$$

where $\boldsymbol{a^T}$ is the (recession) acceleration of the total area, and $\boldsymbol{a}$ the (recession) acceleration of distance between points.

As function of distance from the observer, the distance of centre points increases (at each corresponding iteration-time) from an observer, so does the recession acceleration of the points (expanding away) – as shown in Figure 9 below.

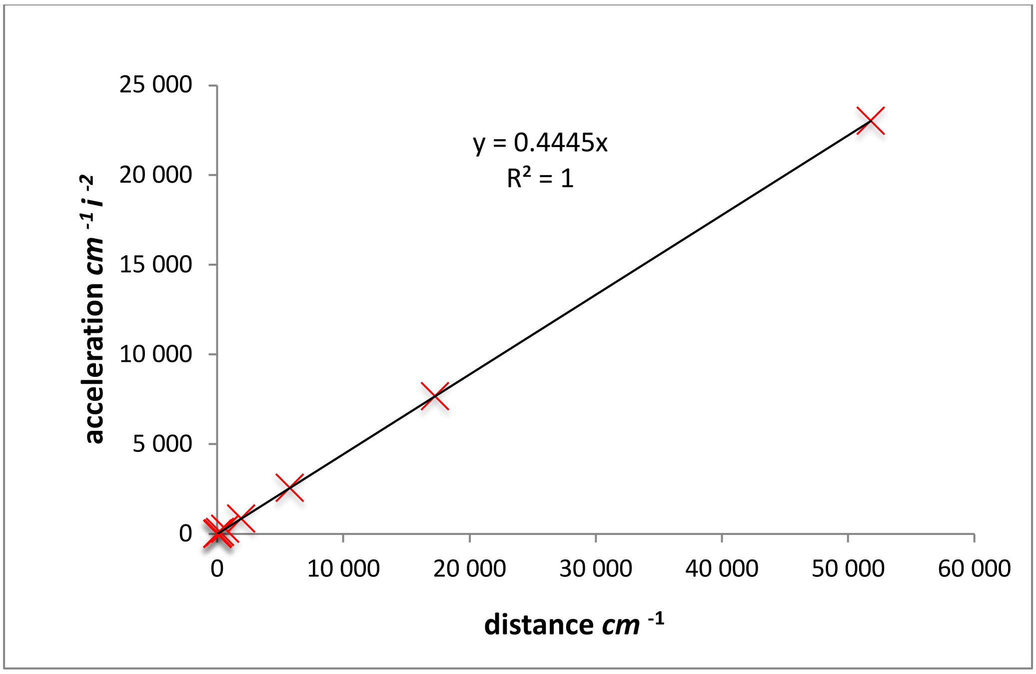

Figure 9. Recessional Acceleration vs. distance on the Inverted Koch Snowflake Fractal. As distance between triangle geometric centres increases with iteration, the recession acceleration of the points increases. cm = centimetres. i = iteration-time.

The recession acceleration of points at each iteration-time at differing distances on the inverted fractal is described by the equation

$$\boldsymbol{a} = \mathbf{0.4447}\boldsymbol{D} \qquad (15)$$

where the constant factor is measured in units of $\boldsymbol{cm^{-1}\, i^{-2}\, cm^{-1}}$. $\boldsymbol{a}$ = acceleration; $\boldsymbol{D}$ = distance.

The spiral radius (see Appendix Figure 17 and Appendix Table 1 for details) – where the centre is the observation point – resulted in an acceleration equation of

The spiral radius – where the centre is the observation point – equation resulted (see Appendix Figure 17 for details)

$$\boldsymbol{a} = \mathbf{0.4295}\boldsymbol{D}. \qquad (16)$$

3.7 Distribution of Points and Triangles with Iteration-Time

8 of the 10 measurement points are located inside the first (1.20E+4cm^{-1}) increment distance. The remaining 2 measurement points are outside this range.

Figure 10 below shows the quantity of triangles by distance – between geometric centres from the observer. The quantity of triangles decreased exponentially from 7.86E+05, at iteration-distance 0, to a quantity of 1 at distance 51800cm-1 (iteration-10).

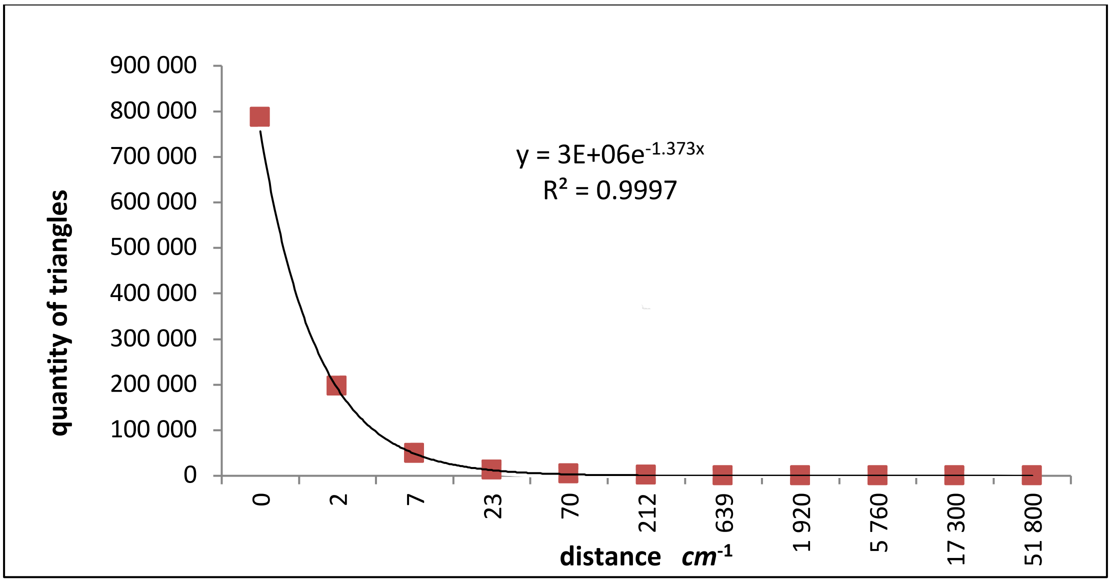

Figure 10. Quantity of Triangles at each Distance (Point) from the Observer on the Inverted Koch Snowflake Fractal. As the distance between triangle geometric centres increases (exponentially) with iteration, and so increasing the distance from the observer, the quantity of triangles per iteration decreases exponentially to a quantity of one – at time 0. cm = centimetre.

4 DISCUSSIONS

In this paper I hypothesised that the conjectured and observed behaviour of the universe are all features of a growing iterating fractal observed from an in-situ position. This model explains and predicts: an observed 'singularity' (Big Bang) beginning, (section 4.1); the presence, and dominance of a 'uniform' Cosmic Microwave Background like origin (section 4.1.1);an inflation epoch (section 4.2); a expansion (section 4.1); accelerating (exponential) 'dark energy' expansion (section 4.5); a cosmological constant (section 4.5); and the demographics evolution and transition in large- and small-scale galaxy distribution with time; offer a solution to a unified (quantum) theory and solve the cosmological catastrophe.

I found that the in-site fractal model – a model of looking back in time from the fractals origin – is an exquisite and inextricable fit to what is observed and conjectured in the cosmos.

My work brings together what is currently 'granted and accepted' about the fractal universe – that the universe appears to be fractal on small cosmic scales – with the many gaps in our knowledge: why is the universe smooth on larger scales (4.5); and have directly addressed its singularity 4.1 and 'inflationary' origin (4.2) ; along with its Hubble- Lemaitre Law (4.3) and accelerating expansion behaviour with age. It may also 'shed light' on the dark matter distribution by the model's prediction of galaxy distribution (4.8).

The universe, by observation, it behaving exactly how like a growing fractal. If we had no cosmological observations, and only fractal geometry to work from to make predictions on what the structure and evolution of our universe might be – derived from its apparent universal ubiquity – it would match. The problem is solved by a geometry, in this case fractals. This is not the first time a geometry has solved observational discrepancies or paradoxes; one only has to look at how circles and later ellipses explained ended the paradigm of epicycles.

Maybe the most remarkable thing I have discovered about the fractal from this study is what is not covered directly in it; that the mechanism of fractal development, growth and emergence points to how quantum mechanics – the wave particle duality of light and matter – is described by experts. I have found that looking 'back' – as I have with this investigation – models the cosmos; and I hypothesis that looking 'forward' into the fractal from an in-situ observation point – models 'the quantum'. Together, fractal geometry will complete the gap of knowledge.

The following discussions attempts to follow the order production of the fractal and not always the important 'fractal cosmology' issues at hand.

4.1 Singularity Beginning

The expansion of the single – first – (inverted) triangle bit (
Figure 3) in this model demonstrates a singularity 'Big Bang' beginning. Its area begins from an arbitrary small size; it could be set to any size value, one akin to the Planck area.

4.1.1 The Cosmic Microwave Background

This simplest of demonstrations is consistent with the observed very cool cosmic microwave background (CMB). It is not an explosion: it is an infinite exponential expansion of area – consistent with descriptions that 'space itself that is expanding'. The fractal in isolation is expanding into 'nothing', just as space is claimed to be. It has a frontier; however, any position beyond this is unattainable.

4.1.2 The Fractal Distance - Magnification/Zoom Back in Iteration-time

To an observer anywhere in the set, this initial triangle (t_0) will dominate the extreme horizon, but it will not be seen by all observers. If an observer is more than 7 ± 2 iterations distant from (triangle) bit t^0 and observing without any form of technology – to 'zoom' back in iteration-time, the said bit will not be seen: this is the fractal distance. The 7 ± 2 is derived from the classical emergent development of the fractal as shown in Figure 1 may be termed the equilibrium iteration count, or observable fractal distance.

4.1.3 Increasing Wavelength with Time and Distance from Observer

As demonstrated, the new bit rotates and spirals through iteration-time: thus – if interpreted as a transverse, electromagnetic like wave, the wavelength increases, while its frequency decreases. This is all consistent with observations and conjectures surrounding the CMB. To see where the original bit size goes, see 4.11.

The size and thus wavelength (due to the spiralling propagation) increases, while its frequency decreases. This is consistent with electromagnetism theory and the observed microwave light; it was once ultra-high frequency. More on this is beyond the scope of this investigation, but will be addressed in an investigation into the 'classical' fractal and light.

The expansion of the initial (triangle) bit (t_0) is due to iteration and coming into existence of the new t_1 bits; the original size area 'branches' and remains in another new position – see Figure 1B. It's area behind – as a result – expands exponentially with iteration-time.

4.2 Fractal Growth Consistent with Inflation Epoch Expansion

The expansion rate of the isolated (unbounded) fractal – by fractal-expansion – may be able to demonstrate – or is at least be consistent with – conjected early 'inflation' epoch expansion [23]. From equation (21), the iteration-time to expand from one size to another. The time taken was calculated by setting the initial triangle area (the Planck area) using the Planck length constant (1.61619926 ×10^{-35}).

$$\boldsymbol{i} = \frac{\mathbf{1}}{\mathbf{2.2073}} \ln\left(\mathbf{2.61223} \times \mathbf{10^{70}}\right) \tag{17}$$

It takes the inverted fractal 72.59 (2s.f.) iteration-times to expand from this arbitrary small area to the arbitrary large area of 1 $\boldsymbol{cm^{-2}}$.

4.2.1 Opportunity to Verify a Fractal Universe

From the above equations, if the iteration speed of triangle bits of the (inverted Koch) fractal is set so as to correspond to the frequency or 'clock' of photons of 'light' propagation this 72.59 iterations if consistent, will verify the said fractal claims. It is expected it will correspond, 73 iterations is very fast, as was the inflation epoch. Such a finding will improve our understanding of both light and the geometry of space.

4.3 Expansion in Excess of Light Speed and the Cosmological Principle

Following from the above (4.2 and 4.12.1), the modelled inverted fractal-expansion demonstrates – and is consistent with – space's ability to expand very fast. If we think about the production of the fractal from the classical fractal perspective (Figure 1 A) and that this production has a speed, a rate of production that is propagated akin to the propagation of a light photon, then if we compare this speed with inverted expanding area behind the fractal then the complete model makes sense and the claim 'space expands faster than the speed of light' as proposed by Albert Einstein in his General Theory of Relativity and as conjectured by inflation theory.

Arbitrary points on the surface of the original – iteration 0 – triangle may be assumed to be close enough to assumed to have 'causal contact'; however, with the exponential expansion of the fractal object, this contact will not remain and the points will exponentially expand apart at a rate demonstrated from this experiment (4.2) With respect to the speed of light; the fractal has a constant propagation speed, this speed can be assumed, in principle, to be able to be surpassed by the (accelerating) area expansion 'speed' of the fractal itself. This fractal expansion speed claim is also consistent with and addresses issues surrounding the particle horizon problem and the cosmological principle (axiom) as discussed in 4.8.

4.4 Hubble-Lemaitre Law

The shape of the fractal-Hubble curve (Figure 7) has direct significance Georges Lemaître's conjecture surrounding the expanding universe [17] and Edwin Hubble and Humason's 1929 concurring observations [24]. The fractal model demonstrates, without any talk of 'rising raisin bread' or 'rubber sheets', the Hubble-Lemaitre Law

where from any observation point within a fractal the recession speed of points increases (exponentially –
Figure 6) with distance.
When velocity ($\boldsymbol{v}$) is plotted against distance of points ($\boldsymbol{D}$) (Figure 7, and Appendix Figure 16) the inverted fractal demonstrates Hubble's Law described by the equation

$$\boldsymbol{v} = \boldsymbol{F_v D} \tag{18}$$

where ($\boldsymbol{F_v}$) is the slope of the line of best fit – the fractal (Hubble) recession velocity constant.
The scale invariance of the Fractal-Hubble diagram concurs with the historical development of the curve: from its 1929 original, to the improved 1931 curve the deeper we look (back), the shape will remain constant.

4.4.1 Measurement Points: Exponential Separation

Figure 7 also shows the distance between measurement points on the fractal Hubble curve is not linear but increases in what appears to be exponential. Of course, this is as a result of the increasing size of the triangles with growth. This observation has significance on the concentration of points – or galaxies – within these triangles. This further supports the decreasing distribution – the smoothness – of galaxies looking back in time.

4.5 Accelerating Expansion of the Fractal Explains 'Dark Energy' Conjecture

Not only does the fractal expand in area with distance, this expansion accelerates with respect to iteration-time. Both the total area and the distance between points increases exponentially – Figure 8. This property of the fractal is consistent with the 1998 astronomical discovery (by observation) of the accelerating expanding universe and conjectures surrounding the term 'dark energy' and the cosmological constant (lambda). It can be inferred (from this inverted fractal model) that the accelerating expansion of the universe with respect to distance (Figure 9) is a property of fractal geometry, and can be described by the equation

$$\boldsymbol{a} = \boldsymbol{F_a D} \tag{19}$$

where $\boldsymbol{F_a}$ is the fractal (cosmological) recession acceleration constant measured in units of $\boldsymbol{cm^{-1}\ i^{-2}\ cm^{-1}}$.

The constant $\boldsymbol{F_a}$ (in equation (19) may be interpreted as a fractal a 'cosmological constant' – lambda – with respect to point acceleration and distance.

The acceleration between points with respect to time (from equation (14) is described as

$$\boldsymbol{a = a_0 e^{F\lambda i}} \tag{20}$$

where the constant $\boldsymbol{F\lambda}$ may be interpreted as a fractal 'Cosmological Constant' Lambda with respect to point acceleration and iteration-time.

With continual entry (or birth) of new triangles into the fractal set, the total fractal area (Figure 9 above) the total universe, growths exponentially. The total area expansion with respect to time is described by the function

$$\boldsymbol{A^T = A_0\, e^{F\Lambda i}} \tag{21}$$

where $\boldsymbol{F\Lambda}$ is a fractal constant with respect to total area expansion and time.

4.6 Galaxy Distribution – Clustering of Measurement Points – Explains Small Scale Fractality

The claimed 'small-cosmic scale fractality of galaxy distribution' (Figure 11A) as identified and concluded by the WiggleZ survey Figure 11B – is from this analysis of the inverted fractal model what one would expect to observe if one were observing within a greater fractal.

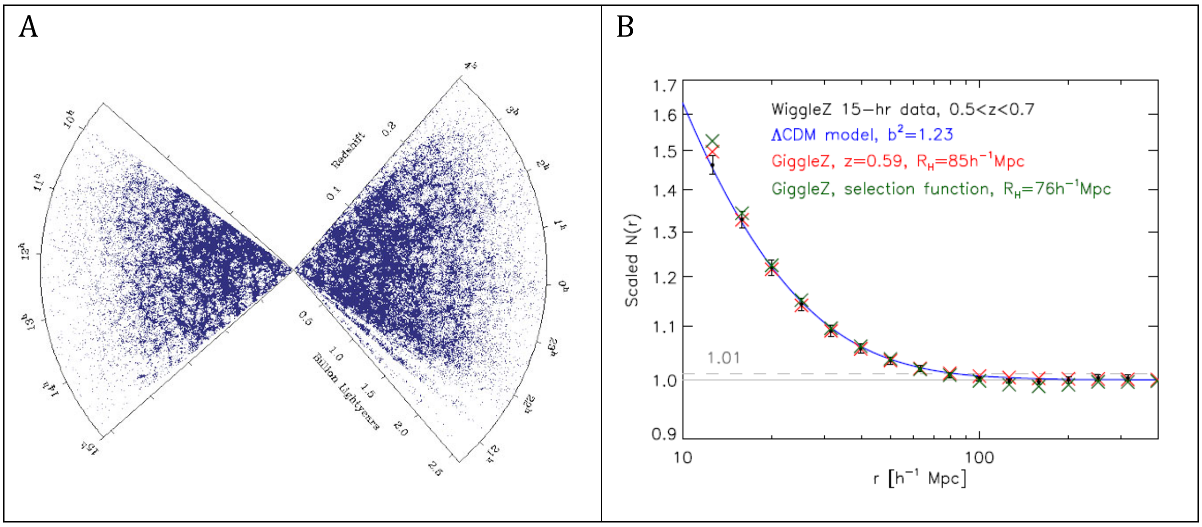

Figure 11. 2dF Galaxy Redshift Surveys and Evolution. A, 2003 2df Redshift Survey map showing small scale 'fractal' clustering[25]; and B, the WiggleZ Dark Energy Survey figure 13, page 16, corresponding to 'A' and revealing changing galaxy distributions from small-scale to large-scale [9].

Figure 10 shows that from the origin on the Fractal-Hubble diagram a quantity of 786,432 triangles is first observed, all of which are the same size as the observer's triangle viewing position. This quantity of bits also corresponds to the clustering of the measurement points near the origin of the diagram and this is due to the location the observer is within the emergent (inverted) fractal and the relative size of these triangle-bits near the observer. The observer is 'in the branches' so to speech. Indeed, the best analogy fractal to visualise this geometric perspective is the tree plant fractal. It is as if the observer is on a branch of the tree (see section 4.10) surrounded by branches of similar age and size and is able to looking back – down – to the trunk of the tree, which was the origin of the tree, and has now expanded.

The observer will not see all these triangles, how many they will see is beyond of the scope of this investigation, but it will be many. As we view further out, the quantity of triangles decreases and while the area of the respective triangles increases. This property of clustering near the origin is scale invariant: no matter the distance, this pattern of clustering near the origin will remain.

Using a tree as a metaphor to model the fractal universe is not to say the universe is a fractal tree structure; it is to say, that just as a tree is a fractal structure, the universe is a fractal structure.

4.6.1 Clustering and the Fractal-Hubble Law

Combing the fractal-Hubble diagram (Figure 7) with the quantity of triangles at each distance point (Figure 10) produces fractal Hubble point distribution diagram (Figure 12). The diagram reveals the relationship between the clustering of measurement points close to the (low recessional velocity) origin, and the smooth distribution (high recessional velocity) at large distances – towards the origin of the set.

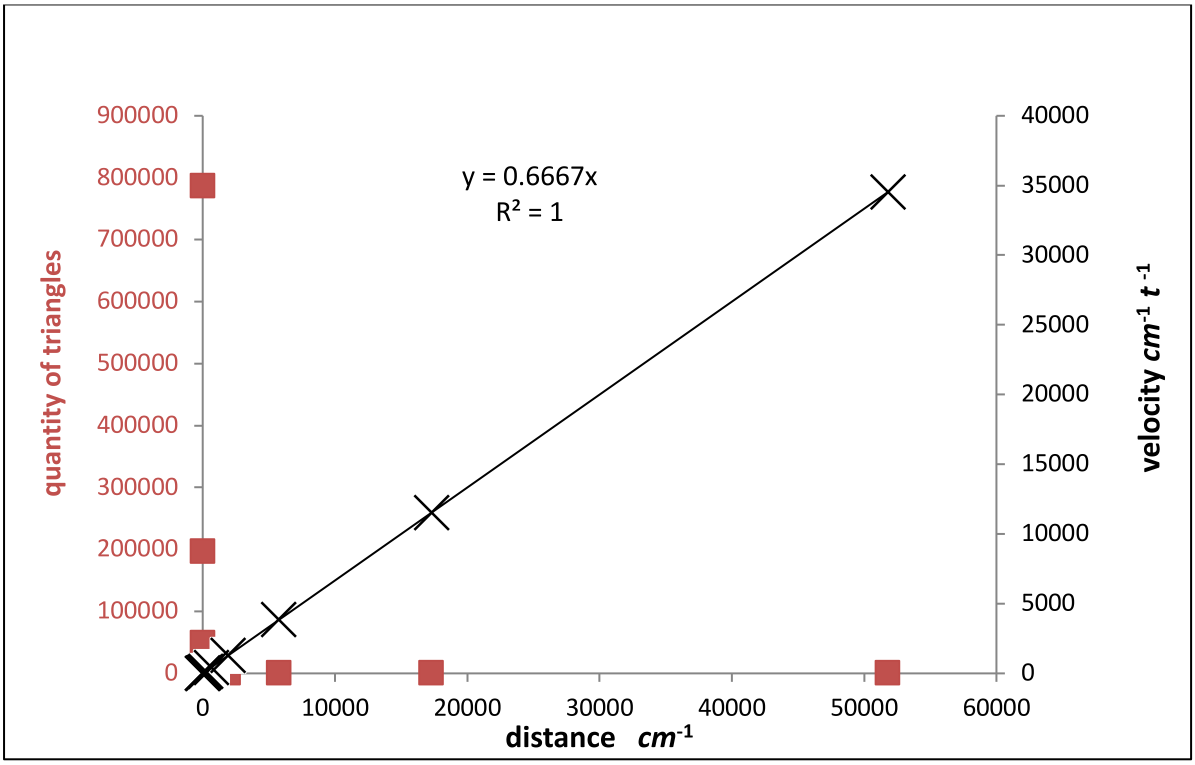

Figure 12. Fractal-Hubble Point Distribution Diagram. As distance increases from the observer with respect to iteration-time: the recession velocity of distance between geometric points increases; while the quantity of triangles at each distance decreases. cm = centimetre.

4.6.2 'Super Clusters' are Limited to the Small-Scale and not Beyond

At the time of writing up my discoveries with the inverted fractal model in the first verson of this paper 2014, the proponents of fractal cosmology – Luciano Pietronero, Francesco Sylos Labini and others – claimed they expected to see even larger galactic clusters further out into the large-scale homogeneous region [5],[6],[20],[7],[8],[2]. In this paper I explained:

"...the distance (in principle) to the next cluster (next larger branch or node) may be beyond the age of the universe and or may not exist at all. This is to say there will be no larger 'super clusters' beyond the current clustering and the remaining space will be smooth – like the trunk of the tree in the tree analogy – until the singularity (CMB)."

In 2016 my meet Pietronnero in person and explained to him my model fits his 'small-scale' fractal predictions and that this matches the lambda CMB expansion. I had no thoughts of structures between.

4.6.3 LQGs and Large-scale Structure Observations

As it turns out, based on recent discoveries, I was wrong about this, and so too were the fractal proponent's sceptics – WiggleZ team. In direct contradiction to all of the above fractal rebuttals, the large-scale surveys discovered 'very large', 'thin' and old structures in the assumed smooth universe were discovered. They are the 4 billion light years in sized Huge 'Large Quasar Group'[13] and the 10 billion light years sized Hercules–Corona Borealis Great Wall [14].

These large structures concur totally with my model. They represent the 'first branches' away from the 'trunk' or CMB of the fractal structure. To support this claim the structures are very large, they are also old – being composed of quasars; and are rather thinly distributed, compared to the small-scale clustered region.

4.6.4 Decreasing Fractal Dimension looking Back

Recent studies have shown fractal dimension decreases with increased *z* values [26]. This complements my model and claims as complexity of the fractal system 'develops' with iteration-time.

4.7 Addressing Dark Flow, the Great Attractor and Dissenting 'Dark Energy' Papers

At the time of this update there have been papers published [27] – based flow of 'dark flow' and the Great Attractor which appears to be 'flowing' in the opposite direction as to 'dark energy accelerating observations – that challenge the observations pointing to an accelerating universe (and thus the existence of dark energy).

I believe the fractal model can address these rebuttals as being part of the fractal system. If an observer is assumed to be within the fractal set (the universe), which I am assuming we are in my model; then a flow in the opposite direction to the early and older parts of the fractal – as claimed in the paper – is to be expected, even predicted as part of the continued growth of the system. To use my analogy of the fractal tree, the former and older branches – even trunk – are expanding and acceleration behind us, while in front of us new branches are forming. There will appear to be a flow in the opposite direction, and it would be reasonable to think that the 'flow' of growth point back to the observer. Of course – again – the I am not suggesting the universe is a tree, but I am suggesting it acts like one.

4.8 The Fractal Refutes the – Homogeneity and Isotropy – Cosmological Principle

Observations concurring with an in-situ fractal perspective reveals the Universe to be neither homogenous, nor isotropic. The standard model of cosmology's key assumption – the cosmological principle – maybe, as it stands today, be a mare illusion, a false paradigm.

Before explaining how this fractal model does not conform to the status quo – something that has been continually explained throughout this paper – it should be made clear it is already claimed and granted by cosmologists in their explanations of the cosmological principle that based on modern observation it only holds on large scales – scales larger a redshift z factor of .25 (about 4 billion light years) – and that on small cosmic scales it does not. The recent discovery of – thinner and older – large quasar groups (LQGs) and the Hercules–Corona Borealis Great Wall (4.6.3) that are beyond this z factor distance (beyond a *z* factor of 4) add strength to the large scale rebuttal [13],[28].

The following deals with this cosmological principle rebuttal.

1. On homogeneity: from this fractal experiment, distributions is not be the same in all directions but rather the galaxy distribution will diminish with distance and time as explained in section 4.5. Smoothness will be observed on large – older – scales (towards the trunk), and clustered fractal activity on small – newer – scales (the branches); just as observed looking out from the Earth's position towards the singularity CMB smoothness. More on this is section 4.5. Also, as we look back in time, the fractal model concurs with observations and claims made about the evolution of galaxies – evidenced by dark matter halo merger trees structures (section 4.8). This corresponds to the 'old' LQGs discovery also.
2. On isotropic: in a fractal, observations will not be the same in all directions; points will be very different from different locations. As with a fractal tree modelled here, there is an obvious trunk to the structure and there are obvious clusters of branches, and these will not be observed isotopically in all directions. The view will be different if viewed from the perspective of the trunk, and if viewed from within the branches. In this fractal model it remains true everything is receding away from any observer, but the view will be different – depending on the position of the observer – and thus not necessarily the same in all directions. There is a 'strange' fractal edge that has grown since the fractal's origin, and this edge appears – by the model – to also be 'the centre', though this has expanded and is viewed today – in part and consistent with the standard

model – as the CMB. All space between this 'edge' and Earth observation is newer, and this is again supported by the evolution of galaxies.

But still the cosmological principle persists. It is as if that even with the many said observational facts the cosmological principle must be saved; saved in order to save General Relativity. And this claim has been made by some.

4.9 Dark Matter Halo Trees and the Evolution of Stars and Galaxies

Something that is rather beyond the scope of the investigation – but important enough to mention as it is seen by the author to be inextricable to the fractal model – is the evolution of galaxy demographics and distribution in the form of Dark Matter halo trees. From a presentation given by Sandra Faber on this subject, a fractal interpretation of the universe would give rise to this 'fractal' tree structure; again, from smooth and thin at far and early distances, to rough and clustered nearby.

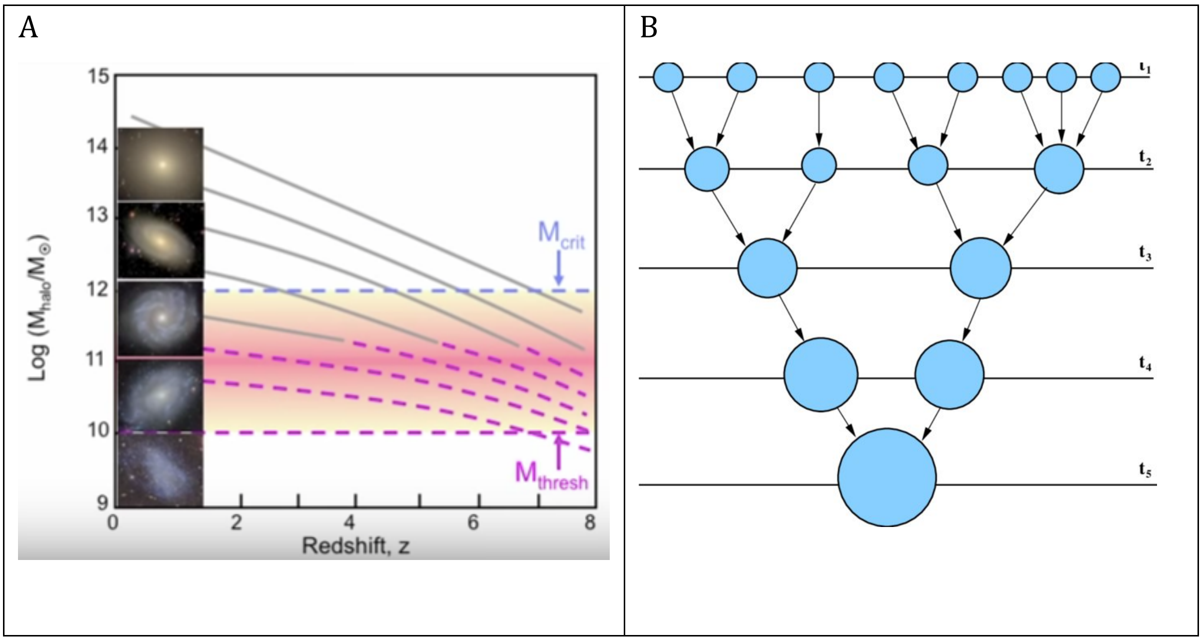

Figure 13. Fractal Dark Matter Halo Trees and the Evolution of Atoms, Stars and Galaxies. 'A' Diagram showing the age and size structure of the galaxies – we (Earth) are surrounded by large and old galaxy clusters. 'B', the classic Dark Matter Halo tree – evolving from early t_1 (top) to large clusters t_5 (bottom).

The significance of this merger halo structures is that they concur and correspond with an evolving emerging universe as revealed in this fractal model.

4.10 Vacuum Catastrophe

Continuing from the above (4.3) the 'vacuum catastrophe discrepancy' may also be resolved by understanding the universe as a fractal and that we, the observer, are in one. As described in the introduction, the fractal shares a duality of perspectives from an observer in one; the classical (forward) view and the expanding (back) view, together they are different aspects of the one. This investigation focused on the expansion and has claimed this to be the dark energy cosmological constant. The classical aspect – outside this investigation – can be shown to behave as the quantum problem. The classical fractal demonstrates wave like spiralling, smaller and smaller (wavelengths), higher and higher frequencies; while the expansion (behind) is in terms of exponentials.

In detail, focusing on the unit used to calculate the total area of the inverted fractal set at any iteration-time. If the standard fixed area size (the area of iteration 0 triangle) is used to calculate the total area of the set, the result will be a very large number; however, if the total area of the inverted fractal set is divided by the area sizes of the expanded triangles (allowing for their expansion at each iteration-time) the number will equate to a lower and more realistic number. The total area will equate to the total number of triangles propagated in the set. In principle all triangles are as identical as the iteration 0 standard triangle, and only differ in scale due to the fractal-expansion.

4.11 (Accelerating) Growth and Development of the (Fractal) Tree

The growing tree – or any plant for that matter – is the perfect example of a fractal and stands as the perfect real-life metaphor of the inverted fractal model, they have similar properties. Figure 14 reveals the reality of the fundamental assumption of this paper; that it is the original bit size that remains constant.

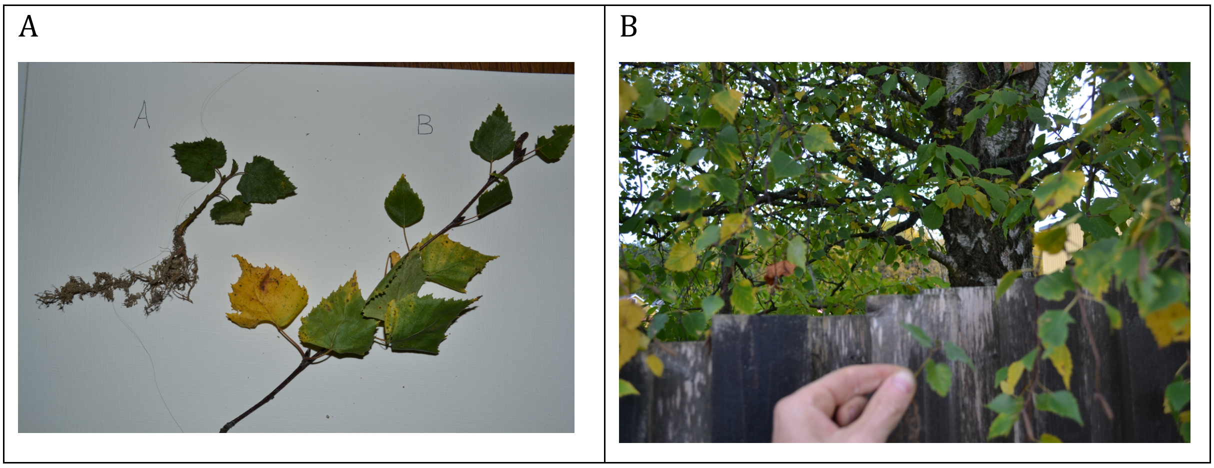

Figure 14. Fractal Tree Growth from a Constant Leaf Size. Figure 14A shows the one constant on an iterating tree fractal, the leaf size: A-A as a seedling size, and A-B the leaf size at the outer branches of the fully-grown tree. Figure 14B shows this same seedling size is same as the outer branch size of a fully developed tree,

and shows the trunk of the same tree and branches thereof. The trunk of the tree, it can be deduced, was once the same size as the seedling in hand. They are all much the same; demonstrating uniformity.

A-A show the leaf size as a seedling, and A-B the size of leaf of a fully grown/developed tree. This property maybe analogous to the constant 'Planck' size of matter.

4.11.1 Clustering of Branches

If the observation from deep within a snowflake fractal is substituted with observation from high within a common branching tree, the clustering of points on the Fractal-Hubble diagram would equally correspond to the clustering of self-similar (sized) branches – in the tree – surrounding the observer. If the observer were to look down, inwards from the outer branches – towards the trunk of the tree – the branch (nodes) quantity would decrease, the volume of the single branches would increase, and the branch 'clustering' would smooth out.

4.11.2 Accelerating Tree (Plant) Growth

In a recent publication it was found trees were found to be growing at an accelerating rate [21],[29]. The study measured up to 80 years of tree growth, on more than 600,000 trees, over 6 continents and found that the growth of 97 percent of the trees were accelerating with age. This accelerated growth rate with time is a mystery to biologists.

Trees, and all plants are the perfect example of fractals. A tree's growth is generally described as being of a 'natural' fractal geometry (or L systems). This phenomenon of acceleration of plant growth may be explained by the plant's growth being fractal. If the productive leafy stem of the emergent tree (Figure 14A-A) becomes the focus of the tree's growth, and held constant in size – just as with the standard triangle size is to the inverted Koch snowflake – then the older branches, and the load bearing trunk of the tree will grow exponentially with iteration-time – again just as the snowflake did.

4.11.3 Uniformity and the (tree) Fractal

An important insight from the inverted fractal is that – just as demonstrated on the inverted snowflake – the shape and size of the original branch can be found on the fully developed tree. There is uniformity; taken directly from the Geological Principle. The leaves in Figure 14A-B look the same as the seedling of the same species of tree (Figure 14A-A) only they were taken from the outer branches of a fully developed tree (Figure

14B). In Figure 14B the fully expanded tree trunk can be viewed Along with the smallest of branches.

4.11.4 Fractal Age of a Tree.

The tree grows in terms of iteration-time, and not solar time. As trees grow they lay down tree rings, these rings do not show exponential growth. Trees can generally – by counting the tree rings – age several hundreds of years old, but in terms of fractal age may only be some 4 to 7 iteration-times old. One can imagine that more iteration-times would result in an exponentially growing, exponentially large base trunk.

4.12 Raised Questions

There are many questions and issues arising from this finding – all of which, at this point, are beyond the scope of this investigation, but not beyond the scope of reason.

4.12.1 The Fractal and the Speed of Light.

From the 'classical view' of the fractal Figure 1A there may be strong insights gained on the nature and behaviour of light – it seems to pointing towards that. If this is so, this may help understand why the universe expands and behaves the way it does and also help unify the large-scale universe with the quantum nature of the universe. One question that may need addressing for a fractal understanding is that light may not be constant. If light is by nature following a fractal geometry, then it may mean light is not constant at large scales. Current experiments I am running on the fractal and its light characteristics are not – so far – pointing to any concept of constant 'light speed', but the fractal fits many of its other properties. This may have implications on the age we perceive the universe to be: why is it so young – relative to the age of our solar system? Could it be there is a distortion to how we receive the light information.

4.12.2 High Initial Expansion Ratios – Inflation?

Notwithstanding the discussion on inflation theory above, the early fractal reveals an anomaly period (Figure 5 A and B) of high expansion ratio for both area expansion and distance between points. Though the ratio values shown are minimal in comparison to Allan Guth's inflation theory's actual predictions, the presence of this anomaly – in the context of the other observed cosmic similarities with the fractal – may well strengthen the theory and cannot be over looked, and will demand explanation.

4.12.3 Multiverse,

With some trepidation, fractal-expansion and the fractal itself is consistent with conjectures surrounding a multiverse as it demonstrates multiple beginnings. An isolated fractal, by definition, has no arbitrary single beginnings, and is an infinity of beginnings.

4.12.4 Emergent History and the Big Bang

A fractal universe would imply an emergent structure – the whole is made of many parts – just as the tree is made of many branches. It may force us to question the initial conditions of the big bang beginning. Namely, whether all mass (in the universe) was together in one place and at one time. It could now be argued – from the principles of fractal emergence – the universe developed/evolved mass from the bottom up, with the passing of time. It started small, from a seedling and developed structure. However, this does not explain the extreme temperatures claimed. There is a begging question from the hot dense 'Big Bang'; how can there be dense and heat before the time of – at least – photons? I think it is emergent all the way.

4.12.5 General Relativity

What a fractal universe means for the future of General Relativity theory is unclear and beyond the scope of the author – though it is conceivable it may have to be adapted to take account the geometry of the fractal. Work has already begun in this area: from noted theorist Laurent Nottale [30],[31] and others [32].

It should be made clear; this fractal model does not point to anything to do with gravity.

4.12.6 On Blackholes and Galaxy Formation

I have personally not invested any thought to blackholes and galaxy formation directly, but a colleague in fractal theory – Lori Gardi [33] – and it seems very promising.

4.12.7 A Fractal Force

The final word. If it is accepted that the said cosmological observations are as a result of a fractal geometry or are as a result of a growing fractal, then this may open the discussion as to if there is a new force at play, a fractal force. This force may well be directly related to the electromagnetic force – the topic of my next paper. Whatever the case, the movement of these galaxies are akin to the movement of the planets in solar systems where they are described by the mechanics of gravity. The cosmic fractal is not the same orbital law but is a geometry that carries matter.

4.13 Limitations to the Model

There are many issues arising from this model and insights from it impact many fields of knowledge.

4.13.1 Addressing Gravity and the Fractal

The universe may by this investigation – and by the observations made – turn out to behave as a fractal; however, this is not to say the universe behaves as a regular-regularity fractal as the Koch snowflake demonstrates. Reality seems to point to regular-irregularity (roughness or chaos) as best demonstrated by the Mandelbrot diagram. This irregular property gives credence to how we currently observe the universe, it is varied and is not a linear straight-line thing as modelled here, gravity plays an import role as a tropism as currently claimed. On that note, the model gives no insight into gravity.

4.13.2 Deceleration of the Universe

Following from the above, the expanding (regular inverted Koch snowflake) fractal does not demonstrate or offer any insight to deceleration (whether observed post inflation epoch early universe, or conjectured pre-inflation epoch). The current theories on gravity are plausible enough to explain this.

4.13.3 Quintessence?

While the fractal constant $\boldsymbol{F\lambda}$ is in this investigation constant and relevant for only the Koch Snowflake fractal, in reality it may well be dynamic – able to change with changes of other trophic stimuli such as gravity, as posited in quintessence theory [34].

4.13.4 Which Fractal Shape?

This investigation also does not in any way suggest the universe has the shape of a tree, or a snowflake: fractal-expansion could have equally been demonstrated using the Sierpinski triangle. The universe shares a feature special to fractals: fractals come in many forms, what that form is is beyond the scope of this paper.

4.13.5 Quantum Mechanics (Like) Properties of the Fractal

Viewed from (arbitrary) position outside the set a fractal will grow at a decreasing rate to form the classical fractal shape – a snowflake as shown in Figure 1A. But from the perspective of an observer within the fractal set the same expansion will appear to expand. This assumption of observation from within the set, looking forward from a fixed position has an uncanny resemblance to properties and problems shared with objects described only by the quantum mechanics and the electromagnetic spectrum.

When isolated, the iterating (snowflake) fractal is an infinitely of discrete triangles (bits). The snowflake is a superposition of all triangles, in one place, at one time. The production of new triangles propagates in the geometry of a spiral: rotating in an arbitrary direction to form – when viewed from a side elevation – a logarithmic sinusoidal wave, comparable to the described electromagnetic spectrum. This spiralling wave like propagation is illustrated below in Figure 1B and in Appendix Figure 15. Location or position within this infinite set is only known when observed or measured; otherwise all positions are possible – at the same time. These quantum-like features of the fractal are an essential background to this investigation – one that will not be taken further in this publication, but cannot be over looked and will be the topic of my next paper. Together the dual perspectives will make sense of the universe.

5 CONCLUSIONS

By simple experiment, in this investigation it was found the simple (inverted) fractal model reveals an exquisite fit to what is observed and conjectured in the cosmos. From a fixed (but arbitrary) location within a (Koch snowflake) fractal set – and from its beginning – the areas of triangles bits expand exponentially and marked points (on triangles) recession velocity from 'the observers' perspective also increased exponentially as a function of distance and time. This (exponential) expansion is a property shared by all (irregular/chaotic) fractal objects.

A fractal-expansion model demonstrates and addresses problems directly associated with the ΛCMB model: the expansion of space, and reveals directly both a Hubble-Lemaitre Law and a cosmological constant. Fractal-expansion offers a geometric mechanism that explains the presence of the CMB, and deals and concurs with conjectures surrounding early inflationary expansion of the universe. There is an opportunity to test and tie the fractal to the speed of light and this (inflationary) expansion. The fractal model explains the conjectured dark energy and explains and concurs with the distribution and demographics of galaxies in the observable universe – from granted 'rough and fractal' on small cosmic scales to the old large and thin LQGs structures on large-cosmic-scales.
As a by-product to this experiment, fractal geometry by fractal-expansion explains why trees – and thus all plants, and arguably other living systems – grow at accelerating

rates with age. The iterating fractal model may also offer a direct solution to the quantum properties of matter and light, time, and reality itself; the model opens the door to a unified theory.

6 APPENDIX

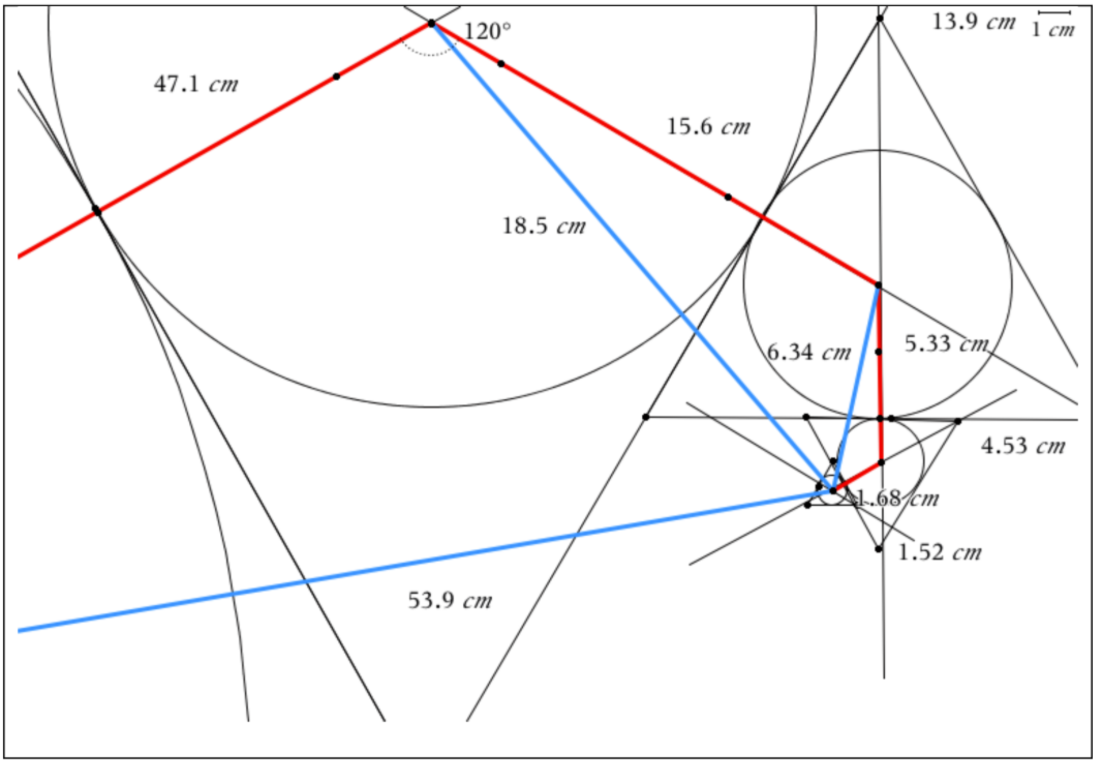

Figure 15. Displacement measurements from radii on the iterating Koch Snowflake created with TI-Nspire ™ software. Displacement is measured between (discrete) triangle centres and used in the calculation of the fractal/Hubble constant. The red line traces the circumference (the distance) of the fractal spiral, and the blue line the displacement of the fractal spiral from an arbitrary centre of observation. cm = centimetres.

Table 1. Displacement taken from radius measurements and calculations from the iterating Koch Snowflake fractal spiral (Appendix Figure 15).

i	Displace-ment:: cm	Total Displacement: cm	Expansion Ratio	Velocity: $cm\ i^{-1}$	Acceleration: $cm\ i^{-2}$	Acceleration Ratio
0			-			
1	1.68	1.68	-	1.68	1.68	
2	4.66	6.34	3.77	4.66	2.98	1.773809524
3	12.16	18.5	2.92	12.16	7.50	2.516778523
4	35.4	53.9	2.91	35.40	23.24	3.098666667

cm = centimetres. i = iteration-time.

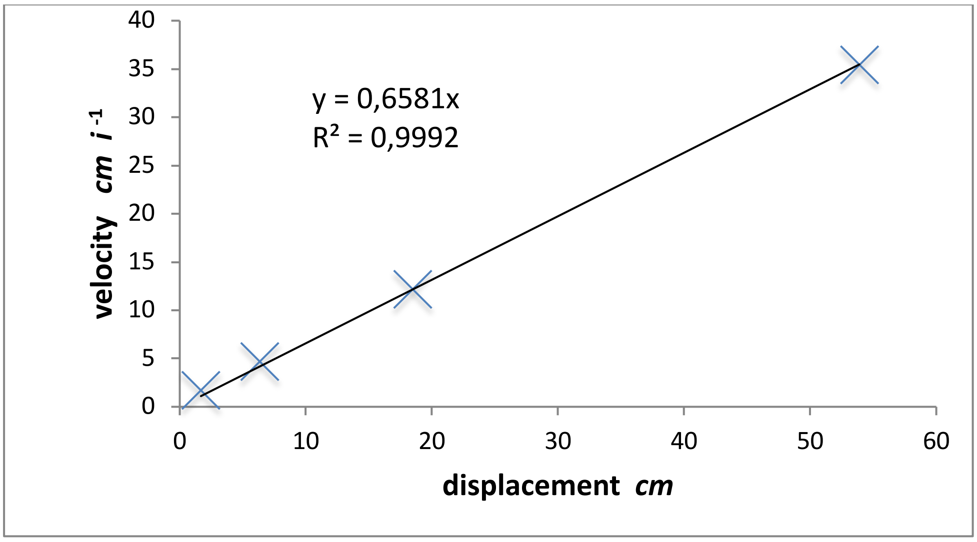

Figure 16. The Hubble Fractal Diagram (recessional velocity vs. distance) from radius measurements (Appendix Figure 15). From an arbitrary observation point on the inverted (Koch Snowflake) fractal: as the distance between triangle geometric centres points increases, the recession velocity of the points receding away increases. cm = centimetres. i = iteration-time.

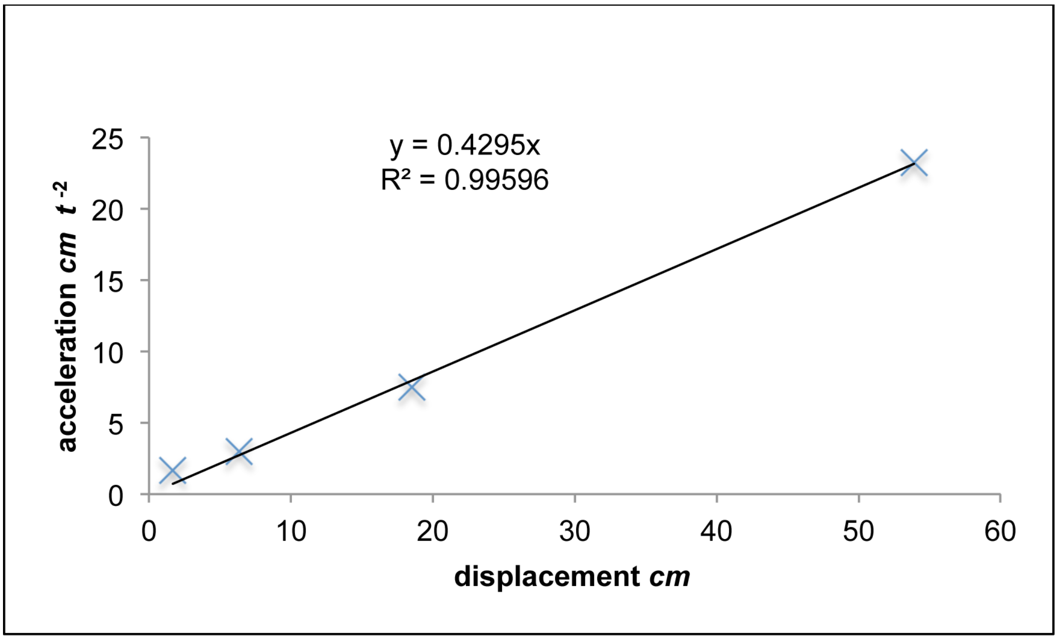

Figure 17. Recessional acceleration with distance on the inverted Koch Snowflake fractal. from a fixed central observation point. Using radius measurements (Appendix Figure 1): as the distance between triangle geometric centres points increases, the recession acceleration of the points receding away increases. cm = centimetres. t = iteration-time.

7 Acknowledgments

Firstly, I would like to thank my wife and children for their patience and support.

Thank you to my student's and colleague's in the International Baccalaureate programmes in Stockholm Sweden – Åva, Sodertalje, and Young Business Creatives – for their help and support. Without the guiding words, support and supervision of Homayoun Tabeshnia, this work may never have begun. For their direct belief and moral support, I would also like to thank Maria Waern and Dr. Ingegerd Rosborg. Mathematicians Rolf Oberg, Peter Morris, and Tosun Ertan helped and guided me no end.

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