# Speed Of Light In FG5 gravimeter 

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#### Abstract

The absolute gravimeter measures the gravitational constant by dropping a corner cube retroreflector in a vacuum. The light reflected by the corner cube interferes with another light from the same emission. The interference pattern can not be explained by the theory if the speed of light remains constant upon reflection. Two research teams were obliged to propose new definition of acceleration to match their test data. Neither team understands that the speed of light actually changes upon reflection by a moving mirror. The definition of acceleration should remain intact. The speed of reflected light should increase to match the observed fringe pattern from the gravimeter.


## I. INTRODUCTION

Various types of gravimeters are used to measure the gravitational constant. One variation, absolute gravimeter, drops a corner cube retro-reflector in a vacuum to create interference pattern. The light from the laser is divided by a beam splitter into two beams. One beam aims at the falling corner cube and continues to the photo detector. The other beam aims directly at the photo detector. The combination of two light beams creates a fringe pattern that varies with time.

Many research teams attempted to apply the Doppler effect with constant speed of light to data of the gravimeter. They found minor discrepancy between the equation and the data. By assuming the speed of light to remain constant, they choose to add extra term to the definition of acceleration. One research team names the extra term as "speed of light perturbation" by stating that "the perturbation due to the finite speed of light was among the most inconsistent in corner-cube absolute gravimeters" [1].

No team understands that the speed of light can change upon reflection off a mirror in motion. The new speed after reflection produces a new phase shift corresponding to the interference pattern. The definition of acceleration should always stay intact.

## II. PROOF

## A. FG5 Gravimeter

Van Camp, M., Camelbeeck, T. and Richard, P. published a paper in 2003, "The FG5 absolute gravimeter: metrology and geophysics" [2]. The original equations in this paper are restated with modification on the name of one variable, $T_{i}$.

$$
\begin{gather*}
x_{i}=x_{0}+v_{0} T_{i}+\frac{g_{0}}{2}\left(T_{i}^{2}+\frac{1}{12} \gamma T_{i}^{4}\right)  \tag{1}\\
T_{i}=t_{i}-\frac{x_{i}-x_{0}}{c} \tag{2}
\end{gather*}
$$

The original statement from the team is:
"Where the three unknowns are $x_{0}, v_{0}$ and $g_{0}$ are the initial position, velocity and acceleration at $t=0, \gamma$ is the vertical gravity gradient and $c$ is the speed of light. Methods to extract the gravity gradient and trajectory parameters simultaneously have proved difficult to implement as signal-to-noise levels are low."


FIG. 1. FG5 gravimeter

Van Camp and the team decided to alter the definition of acceleration with equation (1) instead of considering the possibility that the speed of light can become different upon reflection.

## B. IMGC02 Gravimeter

G D'Agostino, A Germak, S Desogus, C Origlia and G Barbato published a paper in 2005, "A method to estimate the time-position coordinates of a free-falling testmass in absolute gravimetry" [3]. The equation (2) in the original paper is restated below with a different derivative convention as

$$
\begin{equation*}
\frac{d^{2}}{d t^{2}} z=g_{0}+\gamma\left(z-z_{0}\right)-\phi \frac{d}{d t} z \tag{3}
\end{equation*}
$$

with original comment as: "where $g_{0}$ is the acceleration at the level $z_{0}, \gamma$ is the vertical linearized gradient and $\phi$ is the friction coefficient of residual air."

Based on the simulation results, the original conclusion from G D'Agostino and the team is:
"The results are strictly correlated to the algorithm that estimates the falling object acceleration from the trajectory. In particular, the total least-squares algorithm used by the IMGC02 was tested, which includes the vertical gradient $\gamma$ and the friction coefficient of residual air $\phi$ as estimating parameters.

G D'Agostino's team also decided to alter the definition of acceleration with equation (3) instead of considering the possibility that the speed of light can become different upon reflection.

## C. Micro-g LaCoste FG5 Gravimeter

Neither team understands that the speed of light changes upon reflection off a mirror in motion. The speed of light can be altered by the speed of the mirror $[4,5]$.

Let $C_{1}$ be the speed of light before reflection. $C_{2}$ is the speed of light after reflection off a mirror approaching at the speed of $v$.

$$
\begin{equation*}
C_{2}=C_{1}+2 v \tag{4}
\end{equation*}
$$

The FG5 gravimeter works by releasing a corner cube retro-reflector in the vacuum. The reference beam passes


FIG. 2. Beam Path in FG5 interferometer
straight through the first splitter and is then split again
to enter the detector and the optical devices for alignment. The test beam leaves vertically from the first splitter, travels through the dropper and Superspring, and is recombined with the reference beam at splitter $\# 2$.

## D. Speed Of Light in FG5 Gravimeter

The light path in FG5 gravimeter can be summarized with a simplified diagram in figure 3 .


FIG. 3. Direct Measurement of g
As the optical fringes go through zero, the precise time is recorded by an atomic clock. A least-squares fit to the time and distance pairs is used to determine $g$.
let $t_{1}$ be the time the corner cube is released. The following table shows a list of the subsequent events of one divided beam based on figure 3 .

TABLE I. Time and Event

| Time | Event |
| :--- | :--- |
| $t_{1}$ | corner cube is released |
| $t_{2}$ | light leaves the laser |
| $t_{3}$ | light is reflected by the splitter |
| $t_{4}$ | light is reflected off the corner cube |
| $t_{5}$ | light is reflected off the reference cube |
| $t_{6}$ | light is reflected by the splitter |
| $t_{7}$ | light reaches the photo detector |

The distance travelled by the light is shown together with the speed of light in the next table.
$L_{1}$ is the distance between the splitter and the laser.
$L_{2}$ is the distance between the splitter and the photo detector.

TABLE II. Distance and Speed Elapsed Time $\mid$ Distance $\mid$ Light Speed

| $t_{3}-t_{2}$ | $L_{1}$ | $C_{1}$ |
| :--- | :--- | :--- |
| $t_{4}-t_{3}$ | $Z_{1}-\mathrm{x}$ | $C_{1}$ |
| $t_{5}-t_{4}$ | $Z_{1}-\mathrm{x}+Z_{2}$ | $C_{2}$ |
| $t_{6}-t_{5}$ | $Z_{2}$ | $C_{2}$ |
| $t_{7}-t_{6}$ | $L_{2}$ | $C_{2}$ |

x is the distance moved by the corner cube.
$C_{1}$ is the speed of light at the time of emission.
$C_{2}$ is the speed of light after reflection off the corner cube.
$Z_{1}$ is the distance between the splitter and the initial position of the corner cube.
$Z_{2}$ is the distance between the splitter and the reference cube.

The other divided beam that passes through the splitter travels the distance of $L_{1}+L_{2}$ to reach the photo detector at $t_{8}$.

The phase of each beam is $\phi_{0}$ initially from the laser. The phase of a beam depends on the elapsed time and distance.

$$
\begin{equation*}
\phi=\phi_{0}+\omega \delta t-k \delta L \tag{5}
\end{equation*}
$$

Let $\phi_{2}$ be the phase of the beam that is redirected toward the falling corner cube. Let $\phi_{1}$ be the phase of the beam that goes directly to the photo detector.

$$
\begin{equation*}
\phi_{1}\left(t_{8}\right)=\phi_{0}+\omega_{1}\left(t_{8}-t_{2}\right)-k_{1}\left(L_{1}+L_{2}\right)=\phi_{0} \tag{6}
\end{equation*}
$$

$\omega_{1}$ is the angular frequency of beam 1 that goes directly to the photo detector. $k_{1}$ is the wave number of beam 1 .

From the table 2,

$$
\begin{gather*}
t_{7}=\frac{L_{2}}{C_{2}}+t_{6}  \tag{7}\\
t_{6}=\frac{Z_{2}}{C_{2}}+t_{5}  \tag{8}\\
t_{5}=\frac{Z_{1}-x+Z_{2}}{C_{2}}+t_{4}  \tag{9}\\
t_{4}=\frac{Z_{1}-x}{C_{1}}+t_{3}  \tag{10}\\
t_{3}=\frac{L_{1}}{C_{1}}+t_{2} \tag{11}
\end{gather*}
$$

The phase of beam 2 that is redirected toward the falling corner cube is $\phi_{2}$. $\omega_{2}$ is the angular frequency of beam 2 after reflection from the corner cube. $k_{2}$ is the wave number of beam 2 after reflection from the corner cube.

$$
\begin{equation*}
\phi_{2}=\phi_{0}+\left(\omega_{1}\left(t_{3}-t_{2}\right)-k_{1} L_{1}\right)+\left(\omega_{1}\left(t_{4}-t_{3}\right)\right. \tag{12}
\end{equation*}
$$

$$
\begin{gather*}
\left.-k_{1}\left(Z_{1}-x\right)\right)+\left(\omega_{2}\left(t_{5}-t_{4}\right)-k_{2}\left(Z_{1}-x+Z_{2}\right)\right)  \tag{13}\\
+\left(\omega_{2}\left(t_{6}-t_{5}\right)-k_{2}\left(Z_{2}\right)\right)+\left(\omega_{2}\left(t_{7}-t_{6}\right)-k_{2} L_{2}\right)  \tag{14}\\
=\phi_{0} \tag{15}
\end{gather*}
$$

From equations (7,8,9,10,11),

$$
\begin{equation*}
t_{7}=\frac{L_{2}}{C_{2}}+\frac{Z_{2}}{C_{2}}+\frac{Z_{1}-x+Z_{2}}{C_{2}}+\frac{Z_{1}-x}{C_{1}}+\frac{L_{1}}{C_{1}}+t_{2} \tag{16}
\end{equation*}
$$

Beam 2 reaches the photo detector at $t_{7}$ which is later than $t_{8}$. Therefore, the interference takes place at $t_{7}$.

$$
\begin{equation*}
\phi_{1}\left(t_{7}\right)=\phi_{0}+\omega_{1}\left(t_{7}-t_{2}\right)-k_{1}\left(L_{1}+L_{2}\right) \tag{17}
\end{equation*}
$$

The phase shift between two beams at the time of $t_{7}$ is

$$
\begin{equation*}
\delta \phi=\phi_{2}\left(t_{7}\right)-\phi_{1}\left(t_{7}\right) \tag{18}
\end{equation*}
$$

The FG5 gravimeter counts and times the fringes when the fringe signal becomes zero. From equations $(15,17,18)$,

$$
\begin{equation*}
2(n+A) \pi=\delta \phi=\omega_{1}\left(t_{7}-t_{2}\right)-k_{1}\left(L_{1}+L_{2}\right) \tag{19}
\end{equation*}
$$

$n=n(t)$ is an integer function of time. A is a constant.

$$
\begin{equation*}
t_{7}-t_{2}=\frac{L_{1}+L_{2}}{C_{1}}+(n+A) \frac{\lambda_{1}}{C_{1}} \tag{20}
\end{equation*}
$$

$\lambda_{1}$ is the wavelength of beam 1 . From equations $(16,20)$,

$$
\begin{equation*}
\frac{L_{2}}{C_{2}}+\frac{Z_{2}}{C_{2}}+\frac{Z_{1}-x+Z_{2}}{C_{2}}+\frac{Z_{1}-x}{C_{1}}=\frac{L_{2}}{C_{1}}+(n+A) \frac{\lambda_{1}}{C_{1}} \tag{21}
\end{equation*}
$$

From equations $(4,21)$,

$$
\begin{align*}
& x=Z_{1}+\frac{Z_{2} C_{1}-L_{2} v-\frac{\lambda_{1}}{2}(n+A)\left(C_{1}+2 v\right)}{C_{1}+v}  \tag{22}\\
= & Z_{1}+Z_{2}-\frac{\lambda_{1}}{2}(n+A)-v \frac{Z_{2}+L_{2}+\frac{\lambda_{1}}{2}(n+A)}{C_{1}+v} \tag{23}
\end{align*}
$$

From initial condition, $\mathrm{x}=0=\mathrm{v}$ at $\mathrm{t}=0$,

$$
\begin{equation*}
n(0)+A=2 \frac{Z_{1}+Z_{2}}{\lambda_{1}} \tag{24}
\end{equation*}
$$

As x increases, the phase shift decreases. n decreases by an integer i.

$$
\begin{equation*}
n+A=2 \frac{Z_{1}+Z_{2}}{\lambda_{1}}-i \tag{25}
\end{equation*}
$$

From equations $(23,25)$,

$$
\begin{equation*}
x_{i}=i \frac{\lambda_{1}}{2}-\frac{v_{i}}{C_{1}+v_{i}}\left(Z_{1}+2 Z_{2}+L_{2}-i \frac{\lambda_{1}}{2}\right) \tag{26}
\end{equation*}
$$

As the corner cube falls, both $x_{i}$ and $v_{i}$ increase. i also increases as a positive integer.

## E. Experimental Data

The manufacturer of Micro-g LaCroste FG5 gravimeter provides an equation[6] to calculate the gravitational acceleration with the consideration of the gravity gradient.

$$
\begin{equation*}
x_{i}=x_{0}+v_{0} t_{i}+\frac{1}{2} g_{0} t_{i}^{2}+\gamma\left(x_{0}+v_{0} \frac{t_{i}}{3}+g_{0} \frac{t_{i}^{2}}{12}\right) \frac{t_{i}^{2}}{2} \tag{27}
\end{equation*}
$$

The reason for the gradient is: "The finite value of the Earth's gravity gradient, $\gamma$ approximately $-3 \mu G a l / \mathrm{cm}$ causes a measurable change in $g$ over the small length of the drop, and this complicates the 'standard' equation." The "standard" equation is

$$
\begin{equation*}
x_{i}=x_{0}+v_{0} t_{i}+\frac{1}{2} g_{0} t_{i}^{2} \tag{28}
\end{equation*}
$$

The user's manual of Micro-g LaCoste FG5 states[6]: "The interferometer generates an optical interference fringe each time the test mass falls $1 / 2$ the wavelength of the laser light. These fringes are counted and timed with an atomic clock to obtain precise time and distance pairs. A least-squares fit to these data are used to determine the value of $g$."

$$
\begin{equation*}
i \frac{\lambda_{1}}{2}=\iint^{t_{i}} g d t \tag{29}
\end{equation*}
$$

Theoretically, the data will generate a constant for $g$ to fit the "standard" equation.

$$
\begin{equation*}
i \frac{\lambda_{1}}{2}=x_{0}+v_{0} t_{i}+\frac{1}{2} g_{0} t_{i}^{2} \tag{30}
\end{equation*}
$$

However, the least-squares fit to the data shows

$$
\begin{equation*}
x_{i}=x_{0}+v_{0} t_{i}+\frac{1}{2} g_{0} t_{i}^{2}+\gamma\left(x_{0}+v_{0} \frac{t_{i}}{3}+g_{0} \frac{t_{i}^{2}}{12}\right) \frac{t_{i}^{2}}{2} \tag{31}
\end{equation*}
$$

There is a minor difference between the theory and the actual data. From equations $(30,31)$,

$$
\begin{equation*}
x_{i} \neq i \frac{\lambda_{1}}{2} \tag{32}
\end{equation*}
$$

This discrepancy is the source of puzzle and difficulty experienced by most researchers on absolute gravimeter.

## F. Experimental Verification

The common mistake by most research teams is to assume that the speed of light remains constant upon reflection off a moving object.

$$
\begin{equation*}
C_{2}=C_{1} \tag{33}
\end{equation*}
$$

From equation $(21,33)$,

$$
\begin{equation*}
Z_{2}+Z_{1}-x+Z_{2}+Z_{1}-x=(n+A) \lambda_{1} \tag{34}
\end{equation*}
$$

$$
\begin{equation*}
Z_{2}+Z_{1}-x=(n+A) \frac{\lambda_{1}}{2} \tag{35}
\end{equation*}
$$

From initial condition $x=0$ at $t=0$,

$$
\begin{equation*}
n(0)+A=2 \frac{Z_{2}+Z_{1}}{\lambda_{1}} \tag{36}
\end{equation*}
$$

As x increases, the phase shift decreases. n decreases by an integer i.

$$
\begin{equation*}
n+A=2 \frac{Z_{2}+Z_{1}}{\lambda_{1}}-i \tag{37}
\end{equation*}
$$

From equations $(35,37)$,

$$
\begin{equation*}
x_{i}=i \frac{\lambda_{1}}{2} \tag{38}
\end{equation*}
$$

Equation (38) contradicts equation (32).
Most research teams recognize that equation (38) does not match the data collected by FG5 gravimeter. Few realize that the assumption of constant speed of light upon reflection from a moving mirror is invalid and is the source of problem.

The manufacturer of FG5 gravimeter applies leastsquares fit to the data and proposes a more realistic equation (31).

Under the initial condition $x_{0}=0=v_{0}$ at $\mathrm{t}=0$, the non-standard equation (31) becomes

$$
\begin{equation*}
x_{i}=\frac{1}{2} g_{0} t_{i}^{2}\left(1+\gamma \frac{t_{i}^{2}}{12}\right) \tag{39}
\end{equation*}
$$

From equations $(30,39)$,

$$
\begin{equation*}
x_{i}=i \frac{\lambda_{1}}{2}\left(1+\gamma \frac{t_{i}^{2}}{12}\right) \tag{40}
\end{equation*}
$$

In better agreement with equation (26) under the fact that the speed of light increases upon reflection.

$$
\begin{equation*}
C_{2}=C_{1}+2 v \tag{41}
\end{equation*}
$$

The fringe pattern serves as an excellent experimental evidence that the speed of light increases upon reflection off the falling corner cube retro-reflector.

## III. CONCLUSION

The FG5 gravimeter provides experimental evidence that the speed of light changes upon reflection off a moving mirror. Several researchers reported great puzzlement on the data from the absolute gravimeter while assuming that the speed of light remains constant.

Instead of looking for the correct speed of light, most researchers choose to augment the definition of acceleration. Without any theoretical proof, many physicists speculate that the velocity of light remains the same in different inertial reference frame. The speculation surely leads to incorrect conclusion.

The theoretical proof that the speed of light depends on reference frame finally became available in 2019[4,5]. The proof states that the speed of light is not conserved in reference frame while the wavelength of the light is
conserved. The experimental proof was available from the FG5 gravimeter for decades. However, the data has not been understood correctly due to the lack of the theoretical proof.
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