A toroidal or disk-like *Zitterbewegung* electron?

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Summary: We present Oliver Consa's classical calculations of the anomalous magnetic moment of an electron, pointing out some of what we perceive to be weaker arguments, and adding comments and questions with a view to possibly arrive at a more elegant approach to the problem on hand in the future.

Keywords: *Zitterbewegung*, mass-energy equivalence, wavefunction interpretations, realist interpretation of quantum mechanics.

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Introduction

Oliver Consa is a promising young scientist[1] who further builds on Hestenes' *Zitterbewegung*[2] to calculate the anomalous magnetic moment of an electron in a pretty classical way, thereby confirming a realist interpretation of quantum physics is possible.[3] We want to trace the main argument in this paper, while pointing out some of what we perceive to be weaker arguments – adding comments and questions with a view to possibly arrive at a more elegant approach to the problem on hand in the future.

Consa's model is based on the so-called ring electron model. He summarizes it as follows:

> "The Ring Electron Model proposes that the electron has an extremely thin, ring-shaped geometry that is about 2000 times larger than a proton. A unitary charge flows through the ring at the speed of light, generating an electric current and an associated magnetic field. This model allows us to combine experimental evidence that the electron has an extremely small size (corresponding to the thickness of the ring) as well as a relatively large size (corresponding to the circumference of the ring)."

Consa obviously refers to the radii one gets from elastic versus inelastic scattering of photons by electrons (Thomson versus Compton scattering). However, he gets the right result from the wrong formula. Assuming, correctly, that the rotational (tangential) velocity of the electric charge ($v_r = c$) will match the speed of light and – incorrectly – that the angular momentum will match $\hbar$ (the reduced Planck constant), he calculates the Compton radius as follows:

$$\mathrm{L} = \hbar = \mathrm{m_e} \cdot \mathrm{R} \cdot v_r = \mathrm{m_e} \cdot \mathrm{R} \cdot c \Leftrightarrow \mathrm{R} = \frac{\hbar}{\mathrm{m_e} \cdot c}$$

While he gets the correct result ($\mathrm{R} = r_\mathrm{C}$) Planck constant, he uses the wrong mass (the electron mass $\mathrm{m_e}$) and the wrong formula for the angular momentum:

1. The electron mass ($\mathrm{m_e}$) is the electron's rest mass. It is non-zero and, therefore, such mass cannot travel around at the speed of light.

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[1] See his profile on: https://upc.academia.edu/OliverConsa. The paper we will be discussing here is his *Helical Solenoid Model of the Electron* (http://www.ptep-online.com/2018/PP-53-06.PDF).

[2] David Hestenes revived the *Zitterbewegung* theory of an electron in the 1970s and 1980s. It is worth reminding ourselves that it was Erwin Schrödinger who stumbled upon the idea of a *Zitterbewegung* when he was exploring solutions to Dirac's wave equation for free electrons, which Dirac summarized as follows: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, Theory of Electrons and Positrons, Nobel Lecture, December 12, 1933)

[3] See my papers on: http://vixra.org/author/jean_louis_van_belle.

2. The angular momentum of an electron is not $\hbar$ but $\hbar/2$: that's why electrons are considered to be spin-1/2 particles.

The two errors cancel each other out, which is why Consa does get the right result. What are the formulas to be used? Consa should have used the concept of the *effective* mass of an electron (which is *half* of the electron's rest mass and the L = $\hbar/2$ formula. We can then calculate R = r_C as:

$$\mathrm{L} = \frac{\hbar}{2} = \mathrm{m} \cdot \mathrm{R} \cdot v_r = \frac{\mathrm{m_e}}{2} \cdot \mathrm{R} \cdot c \Leftrightarrow \mathrm{R} = \frac{\hbar}{\mathrm{m_e} \cdot c}$$

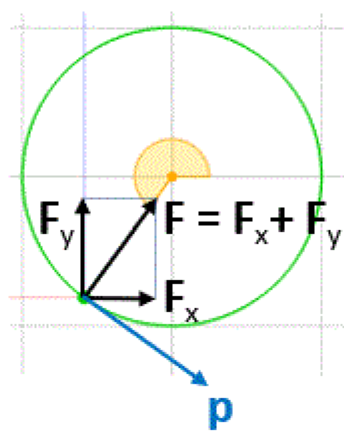
The concept of the *effective* mass of an electron is key to truly understanding what might be going on here, so let us explore that first.

The concept of the effective mass

The 'unitary charge' that is whizzing around the center is a naked charge: it has no properties but its charge. Its *rest* mass is, therefore, zero, and it acquires all of its mass from its velocity. As such, some refer to it as some kind of toroidal photon, or an electron photon – but I don't like these terms because they are not only imprecise but also misleading: photons are not supposed to carry any charge.

Of course, the question is: how does a naked charge acquire mass? Just from whizzing around? The answer is positive. To keep an object with some momentum in a circular orbit, a centripetal force is needed, as shown in Figure 1. What is the nature of this force? Because the force can only grab onto the charge, it must be electromagnetic. We will come back to the force in a moment – because Consa's calculations are most interesting in this regard – but, at this point, we will want to think about the nature of the momentum of the charge (**p**).

Figure 1: The *Zitterbewegung* model of an electron

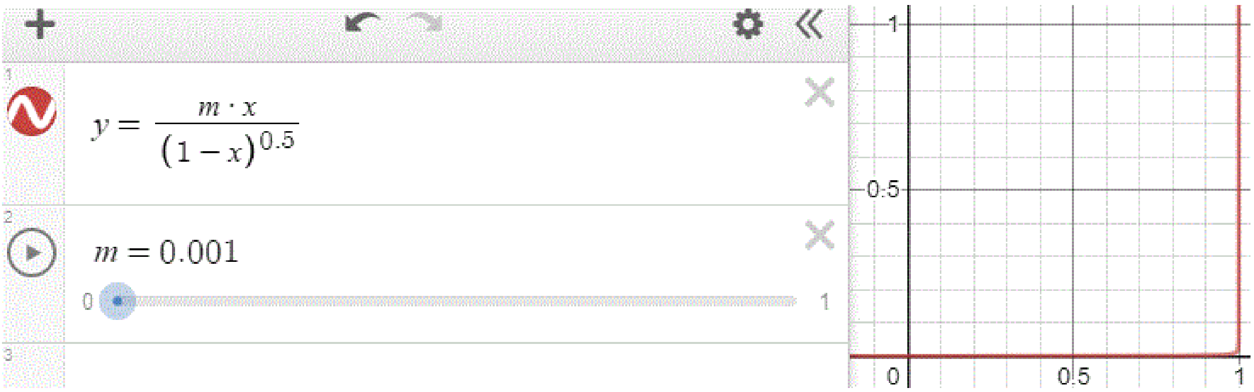


The momentum **p** is relativistic momentum, of course, so its magnitude |**p**| = p is equal to:

$$p = mc = \gamma m_0 c$$

How should we calculate this? The Lorentz factor (γ) goes to infinity as the velocity goes to c and, as mentioned above, we assume the pointlike charge has zero rest mass, so $m_0 = 0$. So we are multiplying zero by infinity. What do we get? The behavior of the p = $\gamma m_0 v$ function is quite weird. The graph in Figure 2 shows what happens with the p = $m_v v = \gamma m_0 v$ for $m_0 = 0.001$ and v/c ranging between 0 and 1.[4]

Figure 2: p = $m_v v = \gamma m_0 v$ for $m_0 \rightarrow 0$



It is quite enlightening: p is (very close to) zero for v/c going from 0 to (very close to) 1 but then becomes infinity near or at v/c = 1 itself. What can we say about this? Perhaps we should say that the momentum of an object with zero rest mass is a nonsensical concept? Perhaps we should associate a tiny but non-zero rest mass with the pointlike charge? If it is *something*, then it should have some mass, shouldn't it?

Maybe. Maybe not. We are not in a position to say much about this right now, and so we won't. The discussion is, in any case, quite philosophical here and, therefore, not so relevant. What we want to do is to find some value for the *effective* mass and, preferable, a value that is expressed in terms of the actual rest mass of our *electron*: note that we distinguish the electron, as a whole, from the pointlike charge that (we think) is part of it !

Calculating the effective mass of the charge

Let us distinguish the components of the momentum vector **p** in the *x*- and *y*-direction respectively. We write:

$$\mathbf{p} = \mathbf{p_x} + \mathbf{p_y}$$

The *magnitude* of these vectors can then be written as $|\mathbf{p}|$ = p, $|\mathbf{p_x}|$ = p_x, and $|\mathbf{p_s}|$ = p_y respectively. If we then write the effective mass as m_v or – even simpler – as m (as opposed to me), then we can write p_x and p_y as:

$$p_x = m v_x = \gamma m_0 v_x \text{ and } p_y = m v_y = \gamma m_0 v_y$$

The *origin* of both the force and momentum vectors is the position vector ***r***, which we can write using the elementary wavefunction, i.e. Euler's function:

$$\boldsymbol{r} = a \cdot e^{i\theta} = x + i \cdot y = a \cdot \cos(\theta) + i \cdot a \cdot \sin(\theta) = a \cdot \cos(\omega t) + i \cdot a \cdot \sin(\omega t) = (x, y)$$

We can also calculate the centripetal acceleration: it's equal to $a_c = v_t^2/a = a \cdot \omega^2$. This formula is relativistically correct. It might be useful to remind ourselves how we get this result. The position vector

[4] We used the online desmos.com graphing tool to produce the graph.

$\boldsymbol{r}$ has a horizontal and a vertical component: $x = a\cdot\cos(\omega t)$ and $y = a\cdot\sin(\omega t)$. We can now calculate the two components of the (tangential) velocity vector $\boldsymbol{v} = d\boldsymbol{r}/dt$ as $v_x = -a\cdot\omega\cdot\sin(\omega t)$ and $v_x\ y = -a\cdot\ \omega\cdot\cos(\omega t)$ and, in the next step, the components of the (centripetal) acceleration vector $\boldsymbol{a_c}$: $a_x = -a\cdot\omega^2\cdot\cos(\omega t)$ and $a_y = -a\cdot\omega^2\cdot\sin(\omega t)$. The magnitude of this vector is then calculated as follows:

$$a_c^2 = a_x^2 + a_y^2 = a^2\cdot\omega^4\cdot\cos^2(\omega t) + a^2\cdot\omega^4\cdot\sin^2(\omega t) = a^2\cdot\omega^4 \Leftrightarrow a_c = a\cdot\omega^2 = v_t^2/a$$

Now, Newton's force law tells us that the magnitude of the centripetal force $|\mathbf{F}| = F$ will be equal to:

$$F = m\cdot a_c = m\cdot a\cdot\omega^2$$

However, we again have this problem of determining what the mass of our pointlike charge actually is: the m_0 in our $m = \gamma m_0$ is zero ! We should find another way !

We may note the horizontal and vertical force component behave like the restoring force causing linear harmonic oscillation. This restoring force depends linearly on the (horizontal or vertical) displacement from the center, and the (linear) proportionality constant is usually written as k. In case of a mechanical spring, this constant will be the *stiffness* of the spring. We don't have a spring here so it is tempting to think it models some elasticity of space itself. However, we should probably not engage in such philosophical thought. Let us just write down the formulas:

$$F_x = dp_x/dt = -k\cdot x = -k\cdot a\cdot\cos(\omega t) = -F\cdot\cos(\omega t)$$

$$F_y = dp_y/dt = -k\cdot y = -k\cdot a\cdot\sin(\omega t) = -F\cdot\sin(\omega t)$$

Now, it is quite straightforward to show that the constant (k) can always be written as:

$$k = m\cdot\omega^2$$

We get that from the *solution* we find for ω when solving the differential equations $F_x = dp_x/dt = -k\cdot x$ and $F_y = dp_y/dt = F_y = dp_y/dt = -k\cdot y$ and assuming there is nothing particular about p and m. In other words, we assume there is nothing wrong with this $p = m\cdot v = \gamma m_0 v$ relation. So we just don't think about the weird behavior of that function. It's a bit like what Dirac did when he *defined* his rather (in)famous Dirac function: the function doesn't make sense mathematically but it works – i.e. we get the right answers – when we use it.

So now we have the $k = m\cdot\omega^2$ equation and we know m is *not* the rest mass of our electron here. We referred to it as the *effective* mass of our pointlike charge as it's whizzing around at the speed of light. We need to remember mass is a measure of inertia and, hence, we can measure that inertia along the horizontal and vertical axis respectively. Hence, we can write something like this: $m = m_\gamma = m_x = m_y$, in line with the distinction we made between p, p_x and p_y. Why m_γ? The notation is just a placeholder: we need to remind ourselves it is a relativistic mass concept and so I used γ (the symbol for the Lorentz factor) to remind ourselves of that. So let us write this:

$$k = m_\gamma\cdot\omega^2$$

From the equations for F_x and F_x, we know that $k\cdot a = F$, so $k = F/a$. Hence, the following equality must hold:

$$F/a = m_\gamma\cdot\omega^2 \Leftrightarrow F = m_\gamma\cdot a\cdot\omega^2 \Leftrightarrow F/a = m_\gamma\cdot a^2\cdot\omega^2 = \Leftrightarrow F/a\cdot m_\gamma = a^2\cdot\omega^2$$

We know the sum of the potential and kinetic energy in a linear oscillator adds up to $E = m \cdot a^2 \cdot \omega^2/2$. We have *two* independent linear oscillations here so we can just add their energies and the ½ factor vanishes. Now I am going to ask you to accept Einstein's mass-energy equivalence relation should apply, so I am asking you to accept that **the total energy in this oscillation must be equal to $E = m \cdot c^2$**. The mass factor here is the *rest mass of our electron*, so it's *not* that weird relativistic m_γ concept. *However*, we did equate c to $a \cdot \omega^2$. Hence, we can now write the following:

$$E = m \cdot c^2 = m \cdot a^2 \cdot \omega^2 = m \cdot F/a \cdot m_\gamma$$

The force is, therefore, equal to:

$$F = (m_\gamma/m) \cdot (E/a)$$

Now what can we say about the m_γ/m ratio? We know m_γ is sort of undefined—but it shouldn't be zero and it shouldn't be infinity. It is also quite sensible to think m_γ should be smaller than m. It cannot be larger because than the energy of the oscillation would be larger than $E = mc^2$. What could it be? 1/2, 1/2π? Rather than guessing, we may want to remind ourselves that we know the angular momentum: $L = \hbar/2$. We calculated it using the $L = I \cdot \omega$ formula and using an educated guess for the moment of inertia ($I = m \cdot a^2/2$), but we also have the $\mathbf{L} = \boldsymbol{r} \times \mathbf{p}$ formula, of course! The lever arm is the radius here, so we can write:

1. $L = \hbar/2 \Leftrightarrow p = L/a = (\hbar/2)/a = (\hbar/2) \cdot mc/\hbar = mc/2$
2. $p = m_\gamma c$

$$\Rightarrow m_\gamma c = mc/2 \Leftrightarrow m_\gamma = m/2$$

We found the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is half of the (rest) mass of the electron.

Basic calculations

We can now calculate the force using the $F = (m_\gamma/m) \cdot (E/a)$ formula:

$$F = \frac{1}{2}\frac{E}{a} \approx \frac{8.187 \times 10^{-14}\ \text{J}}{\frac{2}{2\pi} \cdot 2.246\ \times 10^{-12}\ \text{m}} \approx 0.115\ \text{N}$$

This force is equivalent to a force that gives a mass of about 115 *gram* ($1\ \text{g} = 10^{-3}$ kg) an acceleration of 1 m/s per second. This is *huge* at the sub-atomic scale. Does it make sense? We think it does, because Oliver Consa also gets rather enormous values for *his* calculations, to which we shall turn in a minute. For example, Consa gets the same value for the electric current as I do[5]:

$$I = q_e f = q_e \frac{E}{h} \approx (1.6 \times 10^{-19}\ \text{C}) \frac{8.187 \times 10^{-14}\ \text{J}}{6.626 \times 10^{-34}\ \text{Js}} \approx 19.8\ \text{A}\ (ampere)$$

[5] See: Jean Louis Van Belle, *The Electron as a Harmonic Electromagnetic Oscillator*, 1 June 2019 (http://vixra.org/abs/1905.0521).

This is huge: a household-level current at the sub-atomic scale. However, this result is consistent with the calculation of the magnetic moment, which is equal to the current times the area of the loop and which is, therefore, equal to:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e} \frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2} \frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar$$

Is it also consistent with the presumed angular momentum of an electron, which is that of a spin-1/2 particle? Here we must make some assumption as to how the effective mass of the electron will be spread over the disk. If we assume it is spread uniformly over the whole disk[6], then we can use the 1/2 form factor for the moment of inertia (*I*). We write:

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2} \frac{c}{a} = \frac{mc}{2} \frac{\hbar}{mc} = \frac{\hbar}{2}$$

We now get the correct g-factor for the pure spin moment of an electron:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}} \hbar = \mathrm{g} \frac{\mathrm{q_e}}{2\mathrm{m}} \frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

But why would the mass be spread uniformly over the area of our circle with radius *a*? In fact, why would be think of our electron as some disk? This is where some of Consa's other introductory calculations are interesting.

Consa's calculations

Unsurprisingly, Consa's gets *twice* the value for the force holding the pointlike charge in orbit (0.23 N instead of 0.115 N). It's the ½ factor once again. In contrast, he does some calculations I didn't do. He calculates, among other things, the magnetic field at the center of the ring, using the Biot-Savart Law:

$$\mathrm{B} = \frac{\mu_0 \mathrm{I}}{2\mathrm{R}} \approx 3.23 \times 10^7 \ \mathrm{T}$$

It's another humongous value[7] but – again – quite in line the other humongous values. Hence, Consa's calculations are all essential correct and in line with Hestenes' views on what an electron might actually *be*. Indeed, because the force can only grab onto the charge, it must be an electromagnetic.

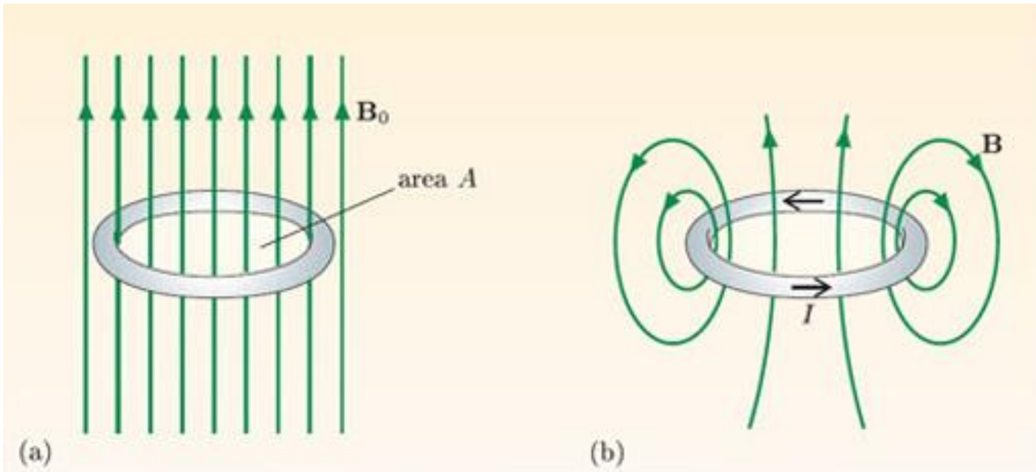
Hestenes, who revived the *Zitterbewegung* theory in the 1980s and 1990s, thinks that the nature of the *zbw* current is the same as that of a superconducting current, as illustrated in Figure 3. If we have some magnetic field – let us denote it by $\mathbf{B}_0$, as in the left-hand side (a) of the illustration below – going through a ring made of superconducting material, we can then cool the ring below the critical temperature and switch off the field. Lenz's law – which is nothing but a consequence of Faradays' law of induction – then tells us the *change* (because of the switch-off) in the magnetic field will induce an electromotive force. Hence, we get an *induced* electric current, and its direction and magnitude will be

[6] This is a very essential point. It is also very deep and philosophical. We say the energy is in the motion, but it's also in the oscillation. It is difficult to capture this in a mathematical formula. In fact, we think this is the key paradox in the model.

[7] Consa dutifully notes the largest artificial magnetic field created by man is only 90 T (*tesla*).

such that the magnetic flux it generates will compensate for the flux change: the induced current in the superconducting circuit will just maintain the flux through the ring at the same value.

Figure 3: A perpetual current in a superconducting ring[8]



This may sound very complicated but it's just yet another application of Maxwell's equations. The hypothesis gives rise to Hestenes' interpretation of the *zbw* model of an electron, which he summarizes as follows:

> "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field."[9]

There is only problem with this interpretation: in free space, we do not have any ring to hold and guide our charge. So what holds it in place? What makes it *stable*? Neither Hestenes nor Consa answer that question. Do I? I think I do: the conceptualization of an electron as a two-dimensional oscillation of a pointlike charge does not require any 'machinery' or the idea of some 'wire' in space. It may, therefore, be more appropriate.

As for the *form factor* – toroid or disk? – we note the magnetic energy must have an equivalent mass. That consideration should also lend more credibility to our assumption of the electron having a disk-like structure – as opposed to being a simple ring current only.

We won't dwell on that here. Let us return to what we wanted to write about: Consa's classical calculations of the anomalous magnetic moment. How does he get it?

[8] Source: Open University, Superconductivity, https://www.open.edu/openlearn/science-maths-technology/engineering-and-technology/engineering/superconductivity/content-section-2.2#. The reader who's interested in the detailed equations proving this fact will find them there.
[9] Email from Dr. David Hestenes to the author dated 17 March 2019.

Calculating the anomalous magnetic moment

Consa's calculations are based on a very alternative interpretation of the ring electron model. He refers to it as the *Helical Electron Model*. The basic assumptions are the following:

1. All of the electron's charge is concentrated in a single infinitesimal point, which is referred to as the center of charge, and which rotates at the speed of light around a point in space called the center of mass.
2. As it moves around the center of mass (CM), the center of charge (CC) follows a helical path.

These two hypotheses are best illustrated in Fig. 3 and 4 of his paper, which we copy below so as to illustrate the main ideas.

Figure 4: Consa's Helical Electron Model (toroidal versus poloidal currents)

The rest of the argument then becomes somewhat confusing. As shown above, Consa distinguishes between a toroidal versus a poloidal current, but he seems to conveniently forget the charge is supposed to move at the speed of light around the center of mass and, therefore, that the pointlike charge cannot move in any other direction – except around the center ! The calculations become even more confusing because Consa needs to assume a helical motion within the helical motion, which he motivates as follows: "The universe generally behaves in a fractal way, so the most natural solution assumes that the electron's substructure is similar to the main structure, that is, a helix in a helix."

I personally like the idea of a fractal structure but, at this point, the assumption comes across as fairly random. To make a long story short, Consa obtains the following result:

$$\frac{1}{2}\left(\frac{r \cdot \mathrm{N}}{\mathrm{R}}\right)^2 = \frac{\alpha}{2\pi}$$

The N, R and *r* in this equality are the number of loops (N), the diameter of the ring (R) and its thickness (*r*) respectively. Of course, we recognize Schwinger's factor ($\alpha/2\pi$), but the whole argument feels convoluted and, therefore, artificial. This is why we prefer our own initial approach to the calculations, which is based on a purely geometrical approach.[10]

[10] See: Jean Louis Van Belle, *The Anomalous Magnetic Moment: Classical Calculations* (http://vixra.org/abs/1906.0007).

However, because Consa's calculations are more precise, we should probably play with them a bit more. The *r*/*R* ratio is generally assumed to be equal to α, i.e. the ratio of the *Thomson* and *Compton* radius of an electron. Substituting this value, we get:

$$\frac{1}{2}(\alpha \cdot \mathrm{N})^2 = \frac{\alpha}{2\pi} \Leftrightarrow \mathrm{N} = \frac{1}{\sqrt{\alpha\pi}} \approx 0.1514110604 \ldots$$

We find it hard to make sense of this result (a charge which turns about 1/6.6 of a turn only?) but the reader may have other interpretations and, therefore, judge otherwise.

The important thing is that Consa shows an anomalous magnetic moment may not be anomalous at all. For that, he should be appreciated – even if the detail of the calculations (and the model) raise as many questions as they're supposed to solve.

Jean Louis Van Belle, 23 January 2020