On some Ramanujan functions applied to various sectors of String Theory and Particle Physics: new possible mathematical connections II.

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#### Abstract

In this research thesis, we have analyzed and deepened various Ramanujan functions applied to some sectors of String Theory and Particle Physics. We have therefore described further new possible mathematical connections.


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From:

## Aspects of SUSY Breaking in String Theory

Augusto Sagnotti
Scuola Normale Superiore and INFN -Pisa - OKC Colloquium
Stockholm, September 2018
a. "Climbing" solution ( $\varphi$ climbs, then descends):

$$
\dot{\varphi}=\frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)-\sqrt{\frac{1+\gamma}{1-\gamma}} \tanh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)\right]
$$

b. "Descending" solution ( $\varphi$ only descends ):

$$
\dot{\varphi}=\frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}} \tanh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)-\sqrt{\frac{1+\gamma}{1-\gamma}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)\right]
$$

## $\gamma<1$

From:

## Pre - Inflationary Clues from String Theory ?

N. Kitazawa a and A. Sagnotti - https://arxiv.org/abs/1402.1418v2

For $0<\gamma<1$ there are actually two classes of such solutions, which describe respectively a scalar that emerges from the initial singularity while climbing or descending the potential. To begin with, the climbing solutions for the $\tau$-derivatives of $\varphi$ and $\mathcal{A}$ are

$$
\begin{align*}
\dot{\varphi} & =\frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)-\sqrt{\frac{1+\gamma}{1-\gamma}} \tanh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)\right] \\
\dot{\mathcal{A}} & =\frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)+\sqrt{\frac{1+\gamma}{1-\gamma}} \tanh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)\right] \tag{2.12}
\end{align*}
$$

and the reader should appreciate that these expressions do not involve any initial-value constants other than the Big-Bang time, here set for convenience at $\tau=0$. On the other hand, the corresponding fields read

$$
\begin{align*}
\varphi & =\varphi_{0}+\frac{1}{1+\gamma} \log \sinh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)-\frac{1}{1-\gamma} \log \cosh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right) \\
\mathcal{A} & =\frac{1}{1+\gamma} \log \sinh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)+\frac{1}{1-\gamma} \log \cosh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right) \tag{2.13}
\end{align*}
$$

and do involve an important integration constant, $\varphi_{0}$. This determines the value of $\varphi$ at a reference "parametric time" $\tau>0$ or, what is more interesting for us, bounds from above the largest value that it can attain during the cosmological evolution. Strictly speaking, $\mathcal{A}$ would also involve an additive constant, but one can set it to zero up to a rescaling of the spatial coordinates. On the other hand, $\varphi_{0}$ has interesting effects on the dynamics that become particularly pronounced in the two-exponential potentials

$$
\begin{equation*}
V(\varphi)=V_{0}\left(e^{2 \varphi}+e^{2 \gamma \varphi}\right) \tag{2.14}
\end{equation*}
$$

## From the following Ramanujan mock theta function:

## MOCK THETA ORDER 6

## (https://en.wikipedia.org/wiki/Mock modular form\#Order 6)

$$
\sigma(q)=\sum_{n \geq 0} \frac{q^{(n+1)(n+2) / 2}(-q ; q)_{n}}{\left(q ; q^{2}\right)_{n+1}}
$$

That is:
(A053271 sequence OEIS)
Sum_\{n>=0\} $q^{\wedge}((n+1)(n+2) / 2)(1+q)\left(1+q^{\wedge} 2\right) \ldots\left(1+q^{\wedge} n\right) /\left((1-q)\left(1-q^{\wedge} 3\right) \ldots(1-\right.$
$\left.\left.q^{\wedge}(2 n+1)\right)\right)$

## We have that:

$\left.\left.\operatorname{sum} q^{\wedge}((n+1)(n+2) / 2)(1+q)\left(1+q^{\wedge} 2\right)\left(1+q^{\wedge} n\right)\right)\right) /\left((1-q)\left(1-q^{\wedge} 3\right)\left(1-q^{\wedge}(2 n+1)\right)\right), n=0$ to $k$
$\sum_{n=0}^{k} \frac{q^{1 / 2(n+1)(n+2)}(1+q)\left(1+q^{2}\right)\left(1+q^{n}\right)}{(1-q)\left(1-q^{3}\right)\left(1-q^{2 n+1}\right)}$
$\sum_{n=0}^{k} \frac{q^{1 / 2(n+1)(n+2)}(1+q)\left(1+q^{2}\right)\left(1+q^{n}\right)}{(1-q)\left(1-q^{3}\right)\left(1-q^{2 n+1}\right)}$

For $\mathrm{q}=0.5$ and $\mathrm{n}=2$, we develop the above formula in the following way:
$\left(\left(\left(0.5^{\wedge}((2+1)(2+2) / 2)(1+0.5)\left(1+0.5^{\wedge} 2\right)\left(1+0.5^{\wedge} 2\right)\right)\right) /\left(\left((1-0.5)\left(1-0.5^{\wedge} 3\right)(1-\right.\right.\right.$ $\left.\left.0.5^{\wedge}\left(2^{*} 2+1\right)\right)\right)$
$\frac{0.5^{(2+1) \times(2+2) / 2}(1+0.5)\left(1+0.5^{2}\right)\left(1+0.5^{2}\right)}{(1-0.5)\left(1-0.5^{3}\right)\left(1-0.5^{2 \times 2+1}\right)}$
0.086405529953917050691244239631336405529953917050691244239...
0.0864055...
$1+\left(\left(\left(0.5^{\wedge}((2+1)(2+2) / 2)(1+0.5)\left(1+0.5^{\wedge} 2\right)\left(1+0.5^{\wedge} 2\right)\right)\right) /\left(\left((1-0.5)\left(1-0.5^{\wedge} 3\right)(1-\right.\right.\right.$ $\left.\left.0.5^{\wedge}(2 * 2+1)\right)\right)$

## Input:

$$
1+\frac{0.5^{(2+1)(2+2) / 2}(1+0.5)\left(1+0.5^{2}\right)\left(1+0.5^{2}\right)}{(1-0.5)\left(1-0.5^{3}\right)\left(1-0.5^{2 \times 2+1}\right)}
$$

1.086405529953917050691244239631336405529953917050691244239
1.0864055...

From the formula (b), for $\gamma=0.0864055$, we obtain:

$$
\begin{aligned}
& 1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \tanh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-\right.\right.\right. \\
& \left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}\left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)
\end{aligned}
$$

## Input interpretation:

$$
\begin{gathered}
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{gathered}
$$

$\tanh (x)$ is the hyperbolic tangent function
$\operatorname{coth}(x)$ is the hyperbolic cotangent function

## Result:

-0.9724368...
-0.9724368...

## Alternative representations:

$$
\begin{gathered}
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
\frac{1}{2}\left(\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)-\right. \\
\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)
\end{gathered}
$$

$$
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.
$$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
$$

$$
\frac{1}{2}\left(i \cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}-\right.
$$

$$
\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)
$$

$$
\begin{gathered}
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
\frac{1}{2}\left(i \cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}+\right. \\
\left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& 0.0867299+\sum_{k=1}^{\infty}\left(1.09048-0.917024(-1)^{k}\right) q^{2 k} \text { for } q=1.64564
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& 0.545242+\sum_{k=1}^{\infty}\left(1.09048 q^{2 k}+\frac{\left.1.83405 \sqrt{0.992534}_{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right) \text { for } q=1.64564}{}\right.
\end{aligned}
$$

$$
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.
$$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
$$

$$
-0.458512-\frac{1.09048}{\sqrt{0.992534}}+\sum_{k=1}^{\infty}\left(-0.917024(-1)^{k} q^{2 k}-\frac{2.18097 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(0.545242 i \pi-0.545242 \sqrt{0.992534})-\right. \\
& \left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t
\end{aligned}
$$

$-1+\exp \left(\left(\left(-1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-0.0864055 \wedge 2))-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}\left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:

$$
\begin{array}{r}
-1+\exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{array}
$$

## Result:

1.644380...
$1.644380 \ldots \approx \zeta(2)=\frac{\pi^{2}}{6}=1.644934 \ldots$

## Alternative representations:

$$
\begin{gathered}
-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
-1+\exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
\left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)
\end{gathered}
$$

$$
-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)((-1))=
$$

$$
-1+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right.
$$

$$
\left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)
$$

$$
-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)((-1))=
$$

$$
-1+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\right)
$$

## Series representations:

$$
\begin{aligned}
& -1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
& -1+\exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right) \text { for }
\end{aligned}
$$

$q=1.64564$

$$
\begin{aligned}
& -1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
& -1+\exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{gathered}
-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
-1+\exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}}+\right. \\
\left.\sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{2.18097 \sqrt{0.992534}_{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}^{2}}{}\right)\right) \text { for } q=1.64564
\end{gathered}
$$

$$
\begin{aligned}
& -1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)=-1+\exp ( \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
& \left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

$-26^{*} 1 / 10^{\wedge} 3-1+\exp \left(\left(\left(-1 / 2\left((()(1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.\right.$ $\left.\left.0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)$ )) ))
where 26 is the number of spacetime dimensions in bosonic string theory.

## Input interpretation:

$$
\begin{gathered}
-26 \times \frac{1}{10^{3}}-1+\exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{gathered}
$$

$\tanh (x)$ is the hyperbolic tangent function $\operatorname{coth}(x)$ is the hyperbolic cotangent function

## Result:

1.618380466924724004618081608251932169847324896621048284367...
$1.618380466 \ldots$. result that is a very good approximation to the value of the golden ratio 1,618033988749...

## Alternative representations:

$$
\begin{gathered}
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
-1+\exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
\left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)-\frac{26}{10^{3}}
\end{gathered}
$$

$$
\begin{array}{r}
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)((-1))= \\
-1+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right. \\
\left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)-\frac{26}{10^{3}}
\end{array}
$$

$$
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)((-1))=
$$

$$
-1+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\right)-\frac{26}{10^{3}}
$$

## Series representations:

$$
\begin{gathered}
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)= \\
-\frac{513}{500}+\exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right) \text { for }
\end{gathered}
$$

$q=1.64564$

$$
\begin{aligned}
& -\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
& -\frac{513}{500}+\exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{gathered}
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)= \\
-\frac{513}{500}+\exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}}+\right. \\
\sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{\left.\left.2.18097 \sqrt{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564}{} .\right.
\end{gathered}
$$

## Integral representation:

$$
\begin{gathered}
-\frac{26}{10^{3}}-1+\exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)=-\frac{513}{500}+\exp ( \\
\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
\left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{gathered}
$$

$\left(9^{\wedge} 3-1\right) * \exp \left(\left(\left(-1 / 2\left((()(1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.\right.$ $\left.\left.0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)$ )) )) ) $-144-55+$ golden ratio^2
where $9^{3}-1$ is a Ramanujan cube, while 144 and 5 are Fibonacci numbers

## Input interpretation:

$$
\begin{aligned}
&\left(9^{3}-1\right) \exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
&\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right)-144-55+\phi^{2}
\end{aligned}
$$

## Result:

1728.727...
1728.727...

This result is very near to the mass of candidate glueball $f_{0}(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the GrossZagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729

Alternative representations:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144-55+\phi^{2}= \\
& -199+\exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
& \left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)\left(-1+9^{3}\right)+\phi^{2}
\end{aligned}
$$

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144-55+\phi^{2}= \\
& -199+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right. \\
& \quad \sqrt{\left.\left.\frac{1.08641}{0.913595}\left(1+\frac{-1+e^{\sqrt{1-0.0864055^{2}}}}{}\right)\right)\right)\left(-1+9^{3}\right)+\phi^{2}} \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144-55+\phi^{2}= \\
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& -199+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right. \\
& \left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{\left.\left.1+e^{-\sqrt{1-0.0864055^{2}}}\right)\right)}{2}\right)\right)\left(\left(-1+9^{3}\right)+\phi^{2}\right.
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144-55+\phi^{2}= \\
& -199+\phi^{2}+728 \exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

$$
\begin{array}{r}
\left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\sqrt{\frac{1+0.0864055}{1-0.0864055}}\right.\right. \\
\left.\left.\operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144-55+\phi^{2}=-199+\phi^{2}+ \\
728 \exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{array}
$$

## for $q=1.64564$

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 144-55+\phi^{2}=-199+\phi^{2}+728 \exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}}+\right. \\
& \left.\sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{2.18097 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 144-55+\phi^{2}=-199+\phi^{2}+728 \exp \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
& \left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
& \left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

$\left(9^{\wedge} 3-1\right)^{*} \exp \left(\left(\left(-1 / 2\left((()(1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.\right.$
$\left.\left.0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$
$\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)$ )) )) ) $-144+$ golden ratio

## Input interpretation:

$$
\begin{aligned}
&\left(9^{3}-1\right) \exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
&\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-144+\phi
\end{aligned}
$$

$\tanh (x)$ is the hyperbolic tangent function $\operatorname{coth}(x)$ is the hyperbolic cotangent function $\phi$ is the golden ratio

## Result:

1782.727...
$1782.727 \ldots$ result in the range of the hypothetical mass of Gluino (gluino $=1785.16$ GeV).

## Alternative representations:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144+\phi= \\
& -144+\phi+\exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
& \left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)\left(-1+9^{3}\right)
\end{aligned}
$$

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144+\phi= \\
& -144+\phi+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right. \\
& \quad \sqrt{\left.\left.\frac{1.08641}{0.913595}\left(1+\frac{-1+e^{\sqrt{1-0.0864055^{2}}}}{}\right)\right)\right)\left(-1+9^{3}\right)} \\
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144+\phi= \\
& -144+\phi+\exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right. \\
& \quad \sqrt{\left.\frac{0.913595}{1.08641}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\left(-1+9^{3}\right)}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144+\phi= \\
& -144+\phi+728 \exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-144+\phi= \\
& -144+\phi+728 \exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 144+\phi=-144+\phi+728 \exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}}+\right. \\
& \sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{\left.\left.2.18097 \sqrt{0.992534}_{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}^{4}\right)\right) \text { for } q=1.64564}{}\right.
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \left(9^{3}-1\right) \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 144+\phi=-144+\phi+728 \exp \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
& \left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
& \left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

$55^{*} \exp \left(\left(\left(-1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2^{*} \tanh \left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-0.0864055 \wedge 2))\right)\right)\right)\right)\right)\right)-5-1 /$ golden ratio
where 55 and 5 are Fibonacci numbers

## Input interpretation:

$$
\begin{aligned}
& 55 \exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right)-5-\frac{1}{\phi}
\end{aligned}
$$

## Result:

139.8229..
139.8229.... result practically equal to the rest mass of Pion meson 139.57 MeV

## Alternative representations:

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-5-\frac{1}{\phi}= \\
& -5+55 \exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
& \left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)-\frac{1}{\phi}
\end{aligned}
$$

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-5-\frac{1}{\phi}= \\
& -5+55 \exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right. \\
& \left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)-\frac{1}{\phi}
\end{aligned}
$$

$$
55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-5-\frac{1}{\phi}=
$$

$$
-5+55 \exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\right)-\frac{1}{\phi}
$$

## Series representations:

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-5-\frac{1}{\phi}= \\
& -5-\frac{1}{\phi}+55 \exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-5-\frac{1}{\phi}= \\
& -5-\frac{1}{\phi}+55 \exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

## for $q=1.64564$

$55 \exp \left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.$

$$
\begin{gathered}
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
5-\frac{1}{\phi}=-5-\frac{1}{\phi}+55 \exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}}+\right. \\
\sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{\left.\left.2.18097 \sqrt{0.992534}_{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564}{} .\right.
\end{gathered}
$$

## Integral representation:

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
&\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 5-\frac{1}{\phi}=-5-\frac{1}{\phi}+55 \exp \left(\int_{\frac{i \pi}{2}}^{\sqrt{0.092534}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
&\left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
&\left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

$55^{*} \exp \left(\left(\left(-1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2^{*} \tanh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2^{*} \operatorname{coth}\left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)-$
$21+1 /$ golden ratio
where 55 and 21 are Fibonacci numbers

## Input interpretation:

$$
\begin{aligned}
& 55 \exp \left(-\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-21+\frac{1}{\phi}
\end{aligned}
$$

$\tanh (x)$ is the hyperbolic tangent function $\operatorname{coth}(x)$ is the hyperbolic cotangent function $\phi$ is the golden ratio

## Result:

125.0590...
125.0590.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18 GeV

## Alternative representations:

$$
\begin{gathered}
55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-21+\frac{1}{\phi}= \\
-21+55 \exp \left(\frac { 1 } { 2 } \left(-\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{\frac{1.08641}{0.913595}}\left(1+\frac{1+e^{-\sqrt{1-0.0864055^{2}}}}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)+\right.\right. \\
\sqrt{2})+\frac{1}{\phi}
\end{gathered}
$$

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-21+\frac{1}{\phi}= \\
& -21+55 \exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.\right. \\
& \left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)+\frac{1}{\phi}
\end{aligned}
$$

$$
55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-21+\frac{1}{\phi}=
$$

$$
-21+55 \exp \left(\frac { 1 } { 2 } \left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\right)+\frac{1}{\phi}
$$

## Series representations:

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-21+\frac{1}{\phi}= \\
& -21+\frac{1}{\phi}+55 \exp \left(-0.0867299+\sum_{k=1}^{\infty}\left(-1.09048+0.917024(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

$55 \exp \left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.$

$$
\begin{gathered}
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)-21+\frac{1}{\phi}= \\
-21+\frac{1}{\phi}+55 \exp \left(-0.545242+\sum_{k=1}^{\infty}\left(-1.09048 q^{2 k}-\frac{1.83405 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{gathered}
$$

## for $q=1.64564$

$55 \exp \left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.$

$$
\begin{gathered}
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
21+\frac{1}{\phi}=-21+\frac{1}{\phi}+55 \exp \left(0.458512+\frac{1.09048}{\sqrt{0.992534}^{2}}+\right. \\
\sum_{k=1}^{\infty}\left(0.917024(-1)^{k} q^{2 k}+\frac{\left.\left.2.18097 \sqrt{0.992534}_{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564}{} .4 .\right.
\end{gathered}
$$

## Integral representation:

$$
\begin{aligned}
& 55 \exp \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)(-1)\right)- \\
& 21+\frac{1}{\phi}=-21+\frac{1}{\phi}+55 \exp \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
& \left(\operatorname{csch}^{2}(t)(-0.545242 i \pi+0.545242 \sqrt{0.992534})+\right. \\
& \left.\left.0.458512 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

From the formula (a), for $\gamma=0.0864055$, we obtain:
$\left(\left(\left(1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}\left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh \left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)$

## Input interpretation:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{aligned}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function $\tanh (x)$ is the hyperbolic tangent function

## Result:

0.7442060...
0.7442060...

## Alternative representations:

$$
\begin{gathered}
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
\frac{1}{2}\left(-\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right. \\
\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)
\end{gathered}
$$

$$
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.
$$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
$$

$$
\frac{1}{2}\left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)+\right.
$$

$$
\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)
$$

$$
\begin{gathered}
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
\frac{1}{2}\left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)-\right. \\
\left.\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& 0.0867299+\sum_{k=1}^{\infty}\left(-0.917024+1.09048(-1)^{k}\right) q^{2 k} \text { for } q=1.64564 \\
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& -0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}-\frac{\left.2.18097 \sqrt{0.992534}_{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}^{(1-95454}\right) \text { for } q=1.64564}{}\right.
\end{aligned}
$$

$$
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.
$$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
$$

$$
0.545242+\frac{0.917024}{\sqrt{0.992534}}+\sum_{k=1}^{\infty}\left(1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)
$$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})+\right. \\
& \left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t
\end{aligned}
$$

and:
$\left(\left(\left(1 / 2\left(\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}\left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-\right.\right.\right.\right.\right.\right.$ $\left.\left.\left.\left.\left.\left.((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 128$

## Input interpretation:

$$
\begin{aligned}
& \left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
& \left.\left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right) \wedge(1 / 128)
\end{aligned}
$$

## Result:

0.997694557208300922071449503706720027493605707487861199312...
$0.997694557208 \ldots$. result very near to the value of the following Rogers-Ramanujan continued fraction:
$\frac{\mathrm{e}^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^{5} \sqrt[4]{5^{3}}}-1}}-\varphi+1}=1-\frac{\mathrm{e}^{-\pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-2 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-3 \pi \sqrt{5}}}{1+\frac{\mathrm{e}^{-4 \pi \sqrt{5}}}{1+\ldots}}}} \approx 0.9991104684$
and to the dilaton value $\mathbf{0 . 9 8 9 1 1 7 3 5 2 2 4 3 = \boldsymbol { \phi }}$
$\log$ base $0.997694557(((1 / 2)((((1-0.0864055) /(1+0.0864055)) \wedge 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)\right)$ )) ) $-\mathrm{Pi}+1 /$ golden ratio

## Input interpretation:

$$
\begin{array}{r}
\log _{0.997694557}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-\pi+\frac{1}{\phi}
\end{array}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function
$\tanh (x)$ is the hyperbolic tangent function $\log _{b}(x)$ is the base- $b$ logarithm $\phi$ is the golden ratio

## Result:

125.476 .
125.476.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18 GeV

## Alternative representations:

$$
\begin{aligned}
& \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}= \\
& -\pi+\frac{1}{\phi}+\frac{\log \left(\frac{1}{2}\left(\operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right) \sqrt{\frac{0.913595}{1.08641}}-\sqrt{\frac{1.08641}{0.913595}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)}{\log (0.997695)}
\end{aligned}
$$

$$
\begin{gathered}
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}= \\
-\pi+\log _{0.997695}\left(\frac { 1 } { 2 } \left(-\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)+\frac{1}{\phi}
\end{gathered}
$$

$$
\log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}=
$$

$$
-\pi+\log _{0.997605}\left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)+\right.\right.
$$

$$
\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)+\frac{1}{\phi}
$$

## Series representations:

$$
\begin{aligned}
& \log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}= \\
& \frac{1}{\phi}-\pi+\log _{0.997695}\left(0.0867299+\sum_{k=1}^{\infty}\left(-0.917024+1.09048(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

$\log _{0.997695}\left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.$

$$
\begin{array}{r}
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)-\pi+\frac{1}{\phi}=\frac{1}{\phi}-\pi+ \\
\log _{0.997605}\left(-0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}-\frac{2.18097 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{array}
$$

for $q=1.64564$

$$
\begin{aligned}
& \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}= \\
& \frac{1}{\phi}-\pi+\log _{0.997605}\left(0.545242+\frac{0.917024}{\sqrt{0.992534}}+\right. \\
& \left.\quad \sum_{k=1}^{\infty}\left(1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564
\end{aligned}
$$

## Integral representation:

$$
\begin{gathered}
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)-\pi+\frac{1}{\phi}= \\
\frac{1}{\phi}-\pi+\log _{0.997695}\left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
\left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})+\right. \\
\left.\left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{gathered}
$$

$\log$ base $0.997694557(((1 / 2)((((1-0.0864055) /(1+0.0864055)) \wedge 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)\right)$ )) ) $+11+1 /$ golden ratio
where 11 is a Lucas number

## Input interpretation:

$$
\begin{array}{r}
\log _{0.997694557}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right)+11+\frac{1}{\phi}
\end{array}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function $\tanh (x)$ is the hyperbolic tangent function $\log _{b}(x)$ is the base- $b$ logarithm $\phi$ is the golden ratio

## Result:

139.618...
139.618.... result practically equal to the rest mass of Pion meson 139.57 MeV

## Alternative representations:

$$
\begin{aligned}
& \log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)+11+\frac{1}{\phi}= \\
& 11+\frac{1}{\phi}+\frac{\log \left(\frac{1}{2}\left(\operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right) \sqrt{\frac{0.913595}{1.08641}}-\sqrt{\frac{1.08641}{0.913595}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)}{\log (0.997695)}
\end{aligned}
$$

$$
\begin{gathered}
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)+11+\frac{1}{\phi}= \\
11+\log _{0.997695}\left(\frac{1}{2}\left(-\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)+\right. \\
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)+\frac{1}{\phi}
\end{gathered}
$$

$$
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.
$$

$$
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+11+\frac{1}{\phi}=
$$

$$
11+\log _{0.997695}\left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)+\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)+\frac{1}{\phi}
$$

## Series representations:

$$
\begin{array}{r}
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+11+\frac{1}{\phi}= \\
11+\frac{1}{\phi}+\log _{0.997605}\left(0.0867299+\sum_{k=1}^{\infty}\left(-0.917024+1.09048(-1)^{k}\right) q^{2 k}\right)
\end{array}
$$

for $q=1.64564$
$\log _{0.997695}\left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right.$

$$
\begin{array}{r}
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+11+\frac{1}{\phi}=11+\frac{1}{\phi}+ \\
\log _{0.097695}\left(-0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}-\frac{2.18097 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{array}
$$

for $q=1.64564$

$$
\begin{gathered}
\log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+11+\frac{1}{\phi}= \\
11+\frac{1}{\phi}+\log _{0.907605}\left(0.545242+\frac{0.917024}{\sqrt{0.992534}}+\right. \\
\left.\sum_{k=1}^{\infty}\left(1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564
\end{gathered}
$$

## Integral representation:

$$
\begin{gathered}
\log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+11+\frac{1}{\phi}= \\
11+\frac{1}{\phi}+\log _{0.997695}\left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
\left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})+\right. \\
\left.\left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{gathered}
$$

27*1/2log base $0.997694557(((1 / 2)((((1-$
$0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}\left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-((1+0.0864055) /(1-$ $\left.\left.\left.\left.\left.\left.0.0864055))^{\wedge} 1 / 2 * \tanh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)$

From Wikipedia:
"The fundamental group of the complex form, compact real form, or any algebraic version of $E_{6}$ is the cyclic group $\boldsymbol{Z} / 3 \boldsymbol{Z}$, and its outer automorphism group is the cyclic group $\boldsymbol{Z} / 2 \boldsymbol{Z}$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, $E_{6}$ plays a role in some grand unified theories"'.

## Input interpretation:

$$
\begin{gathered}
27 \times \frac{1}{2} \log _{0.997694557}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{gathered}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function $\tanh (x)$ is the hyperbolic tangent function $\log _{b}(x)$ is the base- $b$ logarithm

## Result:

1728.00...
1728.00....

This result is very near to the mass of candidate glueball $\mathrm{f}_{0}(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the GrossZagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729

## Alternative representations:

$$
\begin{gathered}
\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
\end{gathered}
$$

$$
27 \log \left(\frac{1}{2}\left(\operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right) \sqrt{\frac{0.913595}{1.08641}}-\sqrt{\frac{1.08641}{0.913595}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)
$$

$2 \log (0.997695)$

$$
\begin{array}{r}
\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
\end{array}
$$

$$
\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(-\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right.
$$

$$
\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)
$$

$$
\begin{gathered}
\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)= \\
\left.\left.\sqrt{\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(-i\left(\operatorname { c o t } \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.913595}\right.\right.\right.\right.}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)
\end{gathered}
$$

## Series representations:

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)= \\
& \frac{27}{2} \log _{0.997695}\left(0.0867299+\sum_{k=1}^{\infty}\left(-0.917024+1.09048(-1)^{k}\right) q^{2 k}\right) \text { for } \\
& q=1.64564
\end{aligned}
$$

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)= \\
& \frac{27}{2} \log _{0.997695}\left(-0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}-\frac{2.18097 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{gathered}
\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)= \\
\frac{27}{2} \log _{0.997695}\left(0.545242+\frac{0.917024}{\sqrt{0.992534}}+\right. \\
\left.\sum_{k=1}^{\infty}\left(1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right) \text { for } q=1.64564
\end{gathered}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)=\frac{27}{2} \log _{0.997695}( \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})+\right. \\
& \left.\left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

$27 * 1 / 2 \log$ base $0.997694557(((1 / 2)((()(1-$
$0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-0.0864055 \wedge 2))-((1+0.0864055) /(1-$ $\left.\left.\left.\left.\left.\left.0.0864055))^{\wedge} 1 / 2 * \tanh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)+55-1 /$ golden ratio
where 55 is a Fibonacci number

## Input interpretation:

$27 \times \frac{1}{2} \log _{0.997694557}\left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)-\right.\right.$

$$
\left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right)+55-\frac{1}{\phi}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function $\tanh (x)$ is the hyperbolic tangent function $\log _{b}(x)$ is the base- $b$ logarithm $\phi$ is the golden ratio

## Result:

1782.38...
$1782.38 \ldots$ result in the range of the hypothetical mass of Gluino (gluino $=1785.16$ GeV ).

## Alternative representations:

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+55-\frac{1}{\phi}=55-\frac{1}{\phi}+ \\
& \frac{27 \log \left(\frac{1}{2}\left(\operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right) \sqrt{\frac{0.913595}{1.08641}}-\sqrt{\frac{1.08641}{0.913595}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)}{2 \log (0.997695)}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)+55-\frac{1}{\phi}= \\
& 55+\frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(-\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right.\right. \\
& \left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)-\frac{1}{\phi} \\
& \frac{27}{2} \log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)+55-\frac{1}{\phi}= \\
& 55+\frac{27}{2} \log _{0.997605}\left(\frac { 1 } { 2 } \left(-i\left(\cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}\right)+\right.\right. \\
& \left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)-\frac{1}{\phi}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{27}{2} \log _{0.097605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+55-\frac{1}{\phi}= \\
& 55-\frac{1}{\phi}+\frac{27}{2} \log _{0.997695}\left(0.0867299+\sum_{k=1}^{\infty}\left(-0.917024+1.09048(-1)^{k}\right) q^{2 k}\right)
\end{aligned}
$$

## for $q=1.64564$

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997605}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+55-\frac{1}{\phi}=55-\frac{1}{\phi}+ \\
& \frac{27}{2} \log _{0.997695}\left(-0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}-\frac{2.18097 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)
\end{aligned}
$$

for $q=1.64564$

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+55-\frac{1}{\phi}= \\
& 55-\frac{1}{\phi}+\frac{27}{2} \log _{0.997605}\left(0.545242+\frac{0.917024}{\sqrt{0.992534}}+\right. \\
& \left.\sum_{k=1}^{\infty}\left(1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}_{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}}{}\right)\right) \text { for } q=1.64564
\end{aligned}
$$

## Integral representation:

$$
\begin{aligned}
& \frac{27}{2} \log _{0.997695}\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)-\right.\right. \\
& \left.\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)\right)+55-\frac{1}{\phi}= \\
& 55-\frac{1}{\phi}+\frac{27}{2} \log _{0.997695}\left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\right. \\
& \left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})+\right. \\
& \left.\left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)
\end{aligned}
$$

We have also, from the following calculation:
$\left(\left((1 / 2)\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.\right.$
$\left.\left.0.0864055^{\wedge} 2\right)\right)+((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$
$\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)$ )) )) ))/0.7442060

## Input interpretation:

$\frac{1}{0.7442060} \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)+\right.$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
$$

## Result:

$1.674982778577893204872023223387147912787255509665369170774 \ldots$
1.6749827785...

## Alternative representations:

$\frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2}=$

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2}= \\
& i \cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}+\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e \sqrt{1-0.0864055^{2}}}\right)
\end{aligned}
$$

$$
2 \times 0.744206
$$

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2}= \\
& \frac{-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)}{2 \times 0.744206}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2}= \\
& -1.34876+\sum_{k=1}^{\infty}\left(-1.23222-1.4653(-1)^{k}\right) q^{2 k} \text { for } q=1.64564 \\
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2}= \\
& -\frac{2}{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\left.{\sqrt{1-0.0864055^{2}}}_{2}^{2}\right)}{0.744206 \sqrt{0.992534}}\right)= \\
& \left.\frac{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534} 2}{2}\right) \text { for } q=1.64564 \\
& -\sum_{k=1}^{\infty}\left(-1.23222 q^{2 k}+\frac{\infty}{\infty}\left(-1.4653(-1)^{k} q^{2 k}+\frac{2.46444 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}}{ }^{2}\right)\right.
\end{aligned}
$$

## for $q=1.64564$

## Integral representation:

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{0.744206 \times 2} \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.616109 i \pi+0.616109 \sqrt{0.992534})-\right. \\
& \left.\quad 0.732649 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t
\end{aligned}
$$

$1 / 10^{\wedge} 27^{*}\left(\left((1 / 2)\left(\left(((1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.\right.$
$\left.\left.0.0864055^{\wedge} 2\right)\right)+((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$
$\left.\left.\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)$ )) $/ 0.7442060$

## Input interpretation:

$$
\begin{gathered}
\frac{1}{10^{27}} \times \frac{1}{0.7442060} \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)+\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{gathered}
$$

$\operatorname{coth}(x)$ is the hyperbolic cotangent function $\tanh (x)$ is the hyperbolic tangent function

## Result:

## $1.674983 \ldots \times 10^{-27}$

$1.674983 \ldots * 10^{-27}$ result practically equal to the neutron mass

## Alternative representations:

$\left.\frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right.}{2}\right)=$

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{(2 \times 0.744206) 10^{27}}= \\
& \frac{i \cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}+\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e \sqrt{1-0.0864055^{2}}}\right)}{2 \times 0.744206 \times 10^{27}}
\end{aligned}
$$

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{(2 \times 0.744206) 10^{27}}= \\
& \frac{-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)}{2 \times 0.744206 \times 10^{27}}
\end{aligned}
$$

## Series representations:

$$
\frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{-1.34876 \times 10^{-27}+\sum_{k=1}^{\infty}\left(-1.23222 \times 10^{-27}-1.4653 \times 10^{-27}(-1)^{k}\right) q^{2 k}}
$$

for $q=1.64564$

$$
\begin{aligned}
& \sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right) \\
& \text { ( } 2 \times 0.744206 \text { ) } 10^{27} \\
& -6.16109 \times 10^{-28}+\sum_{k=1}^{\infty}\left(-1.23222 \times 10^{-27} q^{2 k}+\frac{2.9306 \times 10^{-27} \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)
\end{aligned}
$$

## for $q=1.64564$

$$
\begin{aligned}
& \frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{(2 \times 0.744206) 10^{27}}= \\
& -7.32649 \times 10^{-28}+\frac{1.23222 \times 10^{-27}}{\sqrt{0.992534}}+ \\
& \quad \sum_{k=1}^{\infty}\left(-1.4653 \times 10^{-27}(-1)^{k} q^{2 k}+\frac{2.46444 \times 10^{-27} \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right) \text { for } q=1.64564
\end{aligned}
$$

## Integral representation:

$$
\begin{gathered}
\frac{\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)}{(2 \times 0.744206) 10^{27}} \\
\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.902534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}} \\
\left(\operatorname{csch}^{2}(t)\left(-6.16109 \times 10^{-28} i \pi+6.16109 \times 10^{-28} \sqrt{0.992534}\right)-\right. \\
\left.7.32649 \times 10^{-28} \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t
\end{gathered}
$$

From

$$
\begin{align*}
\dot{\mathcal{A}} & =\frac{1}{2}\left[\sqrt{\frac{1-\gamma}{1+\gamma}} \operatorname{coth}\left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)+\sqrt{\frac{1+\gamma}{1-\gamma}} \tanh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)\right]  \tag{2.12}\\
\varphi & =\varphi_{0}+\frac{1}{1+\gamma} \log \sinh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)-\frac{1}{1-\gamma} \log \cosh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right), \\
\mathcal{A} & =\frac{1}{1+\gamma} \log \sinh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right)+\frac{1}{1-\gamma} \log \cosh \left(\frac{\tau}{2} \sqrt{1-\gamma^{2}}\right), \tag{2.13}
\end{align*}
$$

We obtain from the first of (2.12);
$\left(\left((1 / 2)\left((()(1-0.0864055) /(1+0.0864055))^{\wedge} 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-\right.\right.\right.$ $\left.\left.0.0864055^{\wedge} 2\right)\right)+((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)$ )) ))

## Input interpretation:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)+\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{aligned}
$$

## Result:

1.246532...
1.246532...

## Alternative representations:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \frac{1}{2}\left(\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right. \\
& \left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right) \\
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \frac{1}{2}\left(i \cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{1.08641}{0.913595}}+\right. \\
& \left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right) \\
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \frac{1}{2}\left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right. \\
& \left.\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& -1.00375+\sum_{k=1}^{\infty}\left(-0.917024-1.09048(-1)^{k}\right) q^{2 k} \text { for } q=1.64564
\end{aligned}
$$

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \quad-0.458512+\sum_{k=1}^{\infty}\left(-0.917024 q^{2 k}+\frac{2.18097 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right) \text { for } q=1.64564
\end{aligned}
$$

$$
\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.
$$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)=
$$

$$
-0.545242+\frac{0.917024}{\sqrt{0.992534}}+\sum_{k=1}^{\infty}\left(-1.09048(-1)^{k} q^{2 k}+\frac{1.83405 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)
$$

for $q=1.64564$

## Integral representation:

$$
\begin{aligned}
& \frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right. \\
& \left.\quad \sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)= \\
& \int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.002534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(-0.458512 i \pi+0.458512 \sqrt{0.992534})-\right. \\
& \left.0.545242 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t
\end{aligned}
$$

and from (2.13):
$1 /(1+0.0864055) \ln \sinh \left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)+1 /(1-0.0864055) \ln$ $\cosh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)$

## Input interpretation:

$$
\begin{aligned}
& \frac{1}{1+0.0864055} \log \left(\sinh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)+ \\
& \frac{1}{1-0.0864055} \log \left(\cosh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{aligned}
$$

$\sinh (x)$ is the hyperbolic sine function
$\log (x)$ is the natural logarithm
$\cosh (x)$ is the hyperbolic cosine function

## Result:

-0.4731811...
$-0.4731811 \ldots$

## Alternative representations:

$\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}+\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$
$\frac{\log (a) \log _{a}\left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{0.913595}+\frac{\log (a) \log _{a}\left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1.08641}$
$\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}+\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$

$$
\frac{\log _{e}\left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{0.913595}+\frac{\log _{e}\left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1.08641}
$$

$$
\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}+\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=
$$

$$
\frac{\log \left(\frac{1}{2}\left(e^{-1 / 2 \sqrt{1-0.0864055^{2}}}+e^{\sqrt{1-0.0864055^{2}} / 2}\right)\right)}{0.913595}+
$$

$$
\log \left(\frac{1}{2}\left(-e^{-1 / 2 \sqrt{1-0.0864055^{2}}}+e^{\sqrt{1-0.0864055^{2}} / 2}\right)\right)
$$

1.08641

## Series representation:



## Integral representations:


$d t$

$\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}+\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$
$1.09458\left(\log \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.092534}}{2}} \sinh (t) d t\right)+\right.$
$\left.0.840933 \log \left(\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \cosh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)\right)$

We have also that:
(((1/(()(((1/2)((()1-0.0864055)/(1+0.0864055))^1/2*coth(1/2*sqrt(1$\left.\left.0.0864055^{\wedge} 2\right)\right)+((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)$ )) -0.4731810709136926398312$\left.\left.\left.\left.\left.)\right)\right)\right)\right)\right)^{\wedge} 2$

## Input interpretation:

$$
\begin{gathered}
\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}}\right.\right.\right. \\
\left.\left.\left.\tanh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-0.4731810709136926398312\right)\right)^{2}
\end{gathered}
$$

## Result:

1.672039...
1.672039...
$1 / 10 \wedge 27(((1 /(()(((1 / 2)((() 1-0.0864055) /(1+0.0864055)) \wedge 1 / 2 * \operatorname{coth}(1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.0.0864055^{\wedge} 2\right)\right)+((1+0.0864055) /(1-0.0864055))^{\wedge} 1 / 2 * \tanh (1 / 2 * \operatorname{sqrt}(1-$ $\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.\left.0.0864055^{\wedge} 2\right)\right)\right)\right)\right)\right)\right)\right)-0.4731810709136926398312\right)\right)\right)\right)\right)\right)^{\wedge} 2$

## Input interpretation:

$$
\begin{array}{r}
\left.\frac{\frac{1}{10^{27}}}{\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)+\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{1}{2}\right.\right.\right.\right.}\right) \\
\left.\left.\left.\left.\sqrt{1-0.0864055^{2}}\right)\right)-0.4731810709136926398312\right)\right)^{2}
\end{array}
$$

## Result:

$1.6720394281601935876775696902945353434060445890216239 \ldots \times 10^{-27}$ $1.672039428 \ldots * 10^{-27}$ result practically equal to the proton mass

## Alternative representations:

$$
\begin{gathered}
\frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)- \\
0.47318107091369263983120000))^{2}= \\
\frac{1}{10^{27}}(1 /(-0.47318107091369263983120000+ \\
\frac{1}{2}\left(\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)+\right. \\
\left.\left.\left.\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)\right)^{2}
\end{gathered}
$$

$$
\begin{array}{r}
\frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)-
\end{array}
$$

$0.47318107091369263983120000)^{2}=$

$$
\frac{1}{10^{27}}\left(1 / \int-0.47318107091369263983120000+\right.
$$

$$
\frac{1}{2}\left(-i\left(\cot \left(-\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right) \sqrt{\frac{0.913595}{1.08641}}\right)+\right.
$$

$$
\left.\left.\left.\sqrt{\frac{1.08641}{0.913595}}\left(-1+\frac{2}{1+e^{-\sqrt{1-0.0864055^{2}}}}\right)\right)\right)\right)^{2}
$$

$$
\begin{aligned}
& \frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)- \\
& 0.47318107091369263983120000))^{2}=\frac{1}{10^{27}} \\
& \left(1 /\left(-0.47318107091369263983120000+\frac{1}{2}\left(i \cot \left(\frac{\pi}{2}+\frac{1}{2} i \sqrt{1-0.0864055^{2}}\right)\right.\right.\right. \\
& \left.\left.\left.\sqrt{\frac{1.08641}{0.913595}}+\sqrt{\frac{0.913595}{1.08641}}\left(1+\frac{2}{-1+e^{\sqrt{1-0.0864055^{2}}}}\right)\right)\right)\right)^{2}
\end{aligned}
$$

## Series representations:

$$
\begin{aligned}
& \frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
& \left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)- \\
& 0.47318107091369263983120000))^{2}= \\
& \frac{1.18916 \times 10^{-27}}{\left(1.61057+\sum_{k=1}^{\infty}\left(1+1.18916(-1)^{k}\right) q^{2 k}\right)^{2}} \\
& \text { for } \\
& q= \\
& 1.64564
\end{aligned}
$$

$$
\begin{gathered}
\frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)- \\
0.47318107091369263983120000))^{2}= \\
\left(1.016+\sum_{k=1}^{\infty}\left(q^{2 k}-\frac{2.37831 \sqrt{0.992534}}{(1-2 k)^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)^{2}
\end{gathered}
$$

> for
$q=$

### 1.64564

$\frac{1}{10^{27}}\left(1 /\left(\frac{1}{2}\left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right.$

$$
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)-
$$

$0.47318107091369263983120000))^{2}=$
$\left(4.75662 \times 10^{-27} \sqrt{0.992534}^{2}\right) /\left(2-2.22115 \sqrt{0.992534}+\sum_{k=1}^{\infty} \sqrt{0.992534}\right.$

$$
\left.\left(-2.37831(-1)^{k} q^{2 k}+\frac{4 \sqrt{0.992534}}{4 k^{2} \pi^{2}+\sqrt{0.992534}^{2}}\right)\right)^{2} \text { for } q=1.64564
$$

## Integral representation:

$$
\begin{gathered}
\frac{1}{10^{27}}\left(1 /\left(\frac { 1 } { 2 } \left(\sqrt{\frac{1-0.0864055}{1+0.0864055}} \operatorname{coth}\left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)+\right.\right.\right. \\
\left.\sqrt{\frac{1+0.0864055}{1-0.0864055}} \tanh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)- \\
0.47318107091369263983120000))^{2}=4.75662 \times 10^{-27} / \\
\left(1.03199+\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \frac{1}{i \pi-\sqrt{0.992534}}\left(\operatorname{csch}^{2}(t)(i \pi-\sqrt{0.992534})+\right.\right. \\
\left.\left.1.18916 \operatorname{sech}^{2}\left(\frac{(i \pi-2 t) \sqrt{0.992534}}{2 i \pi-2 \sqrt{0.992534}}\right) \sqrt{0.992534}\right) d t\right)^{2}
\end{gathered}
$$

We have also, from the second of (2.12):
$\mathrm{x}+1 /(1+0.0864055) \ln \sinh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-1 /(1-0.0864055) \ln$ $\cosh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)$

## Input interpretation:

$$
\begin{aligned}
& x+\frac{1}{1+0.0864055} \log \left(\sinh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)- \\
& \frac{1}{1-0.0864055} \log \left(\cosh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)
\end{aligned}
$$

$\sinh (x)$ is the hyperbolic sine function
$\log (x)$ is the natural logarithm
$\cosh (x)$ is the hyperbolic cosine function

## Result:

$x-0.734242$

## Geometric figure:

line
Alternate forms:
$x-0.734242$
$3.13809 \times 10^{-9}\left(3.18665 \times 10^{8} x-2.33977 \times 10^{8}\right)$
Root:
$x \approx 0.734242$
0.734242

## Properties as a real function:

Domain
$\mathbf{R}$ (all real numbers)

## Range

## R (all real numbers)

## Bijectivity

bijective from its domain to $\mathbb{R}$
Derivative:
$\frac{d}{d x}(x-0.734242)=1$

## Indefinite integral:

$$
\begin{aligned}
& \int\left(x+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}\right) d x= \\
& 0.5 x^{2}-0.734242 x+\text { constant }
\end{aligned}
$$

Definite integral after subtraction of diverging parts:

$$
\int_{0}^{\infty}((-0.734242+x)-(-0.734242+x)) d x=0
$$

$0.734242+1 /(1+0.0864055) \ln \sinh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)-1 /(1-0.0864055) \ln$ $\cosh \left(1 / 2 * \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)$

## Input interpretation:

```
\(0.734242+\frac{1}{1+0.0864055} \log \left(\sinh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-\)
    \(\frac{1}{1-0.0864055} \log \left(\cosh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\)
```

$\sinh (x)$ is the hyperbolic sine function
$\log (x)$ is the natural logarithm
$\cosh (x)$ is the hyperbolic cosine function

## Result:

$3.91796 \ldots \times 10^{-7}$
$3.91796 \ldots * 10^{-7}=\varphi$

## Alternative representations:

$0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$
$0.734242-\frac{\log (a) \log _{a}\left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{0.913595}+\frac{\log (a) \log _{a}\left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1.08641}$
$0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$
$0.734242-\frac{\log _{e}\left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{0.913595}+\frac{\log _{e}\left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1.08641}$


## Series representation:

$0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=0.734242+$
$\sum_{k=1}^{\infty} \frac{(-1)^{k}\left(1.09458\left(-1+\cosh \left(\frac{\sqrt{0.992534}}{2}\right)\right)^{k}-0.920467\left(-1+\sinh \left(\frac{\sqrt{0.992534}}{2}\right)\right)^{k}\right)}{k}$

## Integral representations:

$$
\begin{aligned}
& 0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}= \\
& 0.734242+\int_{1}^{\cosh \left(\frac{\sqrt{0.092534}}{2}\right)}\left(\left(0.174111 t-1.09458 \cosh \left(\frac{\sqrt{0.992534}}{2}\right)+\right.\right. \\
& \left.(1.09458-0.174111 t) \sinh \left(\frac{\sqrt{0.992534}}{2}\right)\right) / \\
& \left.\left(t\left(-t+\cosh \left(\frac{\sqrt{0.992534}}{2}\right)+(-1+t) \sinh \left(\frac{\sqrt{0.992534}}{2}\right)\right)\right)\right) d t \\
& 0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}= \\
& 0.920467\left(0.797685+\log \left(\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \cosh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)-\right. \\
& \left.1.18916 \log \left(1+\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \sinh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)\right)
\end{aligned}
$$

$0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1+0.0864055}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}=$ $-1.09458\left(-0.670799+\log \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.992534}}{2}} \sinh (t) d t\right)-\right.$ $\left.0.840933 \log \left(\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \cosh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)\right)$
$\left[1 /\left(\left(\left(0.734242+1 /(1+0.0864055) \ln \sinh \left(1 / 2^{*} \operatorname{sqrt}(1-0.0864055 \wedge 2)\right)-1 /(1-0.0864055)\right.\right.\right.\right.$ $\left.\left.\left.\ln \cosh \left(1 / 2^{*} \operatorname{sqrt}\left(1-0.0864055^{\wedge} 2\right)\right)\right)\right)\right]^{\wedge} 1 / 30-(11+5+1)^{*} 1 / 10^{\wedge} 3$
where 11 is a Lucas number and 5 is a Fibonacci number

## Input interpretation:

$$
\begin{array}{r}
\left(1 /\left(0.734242+\frac{1}{1+0.0864055} \log \left(\sinh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)-\frac{1}{1-0.0864055}\right.\right. \\
\left.\left.\log \left(\cosh \left(\frac{1}{2} \sqrt{1-0.0864055^{2}}\right)\right)\right)\right) \wedge(1 / 30)-(11+5+1) \times \frac{1}{10^{3}}
\end{array}
$$

$\sinh (x)$ is the hyperbolic sine function $\log (x)$ is the natural logarithm
$\cosh (x)$ is the hyperbolic cosine function

## Result:

1.618176597492330675952672454695510581717883715917510599300...
$1.61817659749 \ldots$ result that is a very good approximation to the value of the golden ratio 1,618033988749...

## Alternative representations:

$$
\sqrt{\left.\frac{1}{\left.\left.0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}{20}\right)\right)}-\sqrt\left[{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}\right)\right]{1+0.0864055}-\frac{11+5+1}{10^{3}}}=
$$



## Series representation:




## Integral representations:

$$
\left.\left.\left.\left.\begin{array}{l}
\sqrt[30]{\frac{1}{\left.0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}{2}\right)}} \begin{array}{l}
\left.\frac{1+0.0864055}{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}\right) \\
\frac{1}{1000}\left(-17+1000\left(1 /\left(0.734242-1.09458 \log \left(\int_{\frac{i \pi}{2}}^{\frac{\sqrt{0.902534}}{2}} \sinh (t) d t\right)+0.920467\right.\right.\right. \\
\log \left(\frac{\sqrt{0.992534}}{2}\right.
\end{array} \int_{0}^{1} \cosh \left(\frac{11+5+1}{10^{3}}=\right. \\
2
\end{array}\right) d t\right)\right) \wedge(1 / 30)\right) .
$$

$\sqrt[30]{\frac{1}{\left.0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}{1+0.0864055}\right)}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}}-\frac{11+5+1}{10^{3}}=$

$$
\frac{1}{1000}(-17+1000(1 / / 0.734242+0.920467
$$

$$
\log \left(\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \cosh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)-1.09458
$$

$$
\left.\left.\log \left(1+\frac{\sqrt{0.992534}}{2} \int_{0}^{1} \sinh \left(\frac{t \sqrt{0.992534}}{2}\right) d t\right)\right)\right) \wedge(1 / 30)
$$

$$
\sqrt[30]{\frac{1}{\left.0.734242+\frac{\log \left(\sinh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right.}{1+0.0864055}\right)}-\frac{\log \left(\cosh \left(\frac{\sqrt{1-0.0864055^{2}}}{2}\right)\right)}{1-0.0864055}}-\frac{11+5+1}{10^{3}}=
$$

$$
-\frac{17}{1000}+
$$

$$
\left(1 /\left(0.734242+\int_{1}^{\cosh \left(\frac{\sqrt{0.902534}}{2}\right)}\left(\left(0.174111 t-1.09458 \cosh \left(\frac{\sqrt{0.992534}}{2}\right)+\right.\right.\right.\right.
$$

$$
\left.(1.09458-0.174111 t) \sinh \left(\frac{\sqrt{0.992534}}{2}\right)\right) /
$$

$$
\left.\left(t\left(-t+\cosh \left(\frac{\sqrt{0.992534}}{2}\right)+(-1+t) \sinh \left(\frac{\sqrt{0.992534}}{2}\right)\right)\right)\right)
$$

$$
d t) \wedge(1 / 30)
$$

Now, we have that: (Aspects of SUSY Breaking in String Theory Augusto Sagnotti)

$$
V(\varphi)=V_{0}\left\{e^{2 \varphi}+\frac{1}{2} e^{2 \gamma \varphi}+a_{1} e^{-a_{2}\left(\varphi+a_{3}\right)^{2}}+\left[1-e^{-\frac{2}{3}(\varphi+\Delta)}\right]^{2}\right\}-v_{0}
$$

For $\Delta=0.351 \quad \varphi=3.91796 \ldots * 10^{-7}$
and we obtain:
$(((\exp (2 * 3.91796 \mathrm{e}-7)+1 / 2(\exp (2 * 0.0864055 * 3.91796 \mathrm{e}-7))+\exp (-2(3.91796 \mathrm{e}-$ $\left.\left.\left.\left.7+3)^{\wedge} 2\right)+(1-\exp (-2 / 3 *(3.91796 e-7+0.351)))^{\wedge} 2\right)\right)\right)-$ х

## Input interpretation:

$$
\begin{aligned}
& \left(\exp \left(2 \times 3.91796 \times 10^{-7}\right)+\frac{1}{2} \exp \left(2 \times 0.0864055 \times 3.91796 \times 10^{-7}\right)+\right. \\
& \left.\quad \exp \left(-2\left(3.91796 \times 10^{-7}+3\right)^{2}\right)+\left(1-\exp \left(-\frac{2}{3}\left(3.91796 \times 10^{-7}+0.351\right)\right)\right)^{2}\right)-x
\end{aligned}
$$

## Result:

$1.54353-x$
Plot:


Geometric figure:
line

## Alternate forms:

$1.54353-x$
$9.25774 \times 10^{-9}\left(1.66729 \times 10^{8}-1.08018 \times 10^{8} x\right)$

## Root:

$x \approx 1.54353$
1.54353

## Properties as a real function:

## Domain

$\mathbf{R}$ (all real numbers)

## Range

## $\mathbf{R}$ (all real numbers)

## Bijectivity

bijective from its domain to $\mathbb{R}$

Derivative:
$\frac{d}{d x}(1.54353-x)=-1$

## Indefinite integral:

$$
\begin{aligned}
& \int\left(\left(\exp \left(23.91796 \times 10^{-7}\right)+\frac{1}{2} \exp \left(2 \times 0.08640553 .91796 \times 10^{-7}\right)+\exp ( \right.\right. \\
& \left.\left.\quad-2\left(3.91796 \times 10^{-7}+3\right)^{2}\right)+\left(1-\exp \left(-\frac{2}{3}\left(3.91796 \times 10^{-7}+0.351\right)\right)\right)^{2}\right)- \\
& x) d x=1.54353 x-0.5 x^{2}+\text { constant }
\end{aligned}
$$

## Definite integral after subtraction of diverging parts:

$\int_{0}^{\infty}((1.54353-x)-(1.54353-x)) d x=0$
$\mathrm{x} *(((\exp (2 * 3.91796 \mathrm{e}-7)+1 / 2(\exp (2 * 0.0864055 * 3.91796 \mathrm{e}-7))+\exp (-2(3.91796 \mathrm{e}-$ $\left.\left.\left.\left.7+3)^{\wedge} 2\right)+(1-\exp (-2 / 3 *(3.91796 e-7+0.351)))^{\wedge} 2\right)\right)\right)-1.54353$

Input interpretation:

$$
\begin{aligned}
& x\left(\exp \left(2 \times 3.91796 \times 10^{-7}\right)+\right. \\
& \quad \frac{1}{2} \exp \left(2 \times 0.0864055 \times 3.91796 \times 10^{-7}\right)+\exp \left(-2\left(3.91796 \times 10^{-7}+3\right)^{2}\right)+ \\
& \left.\quad\left(1-\exp \left(-\frac{2}{3}\left(3.91796 \times 10^{-7}+0.351\right)\right)\right)^{2}\right)-1.54353
\end{aligned}
$$

## Result:

$1.54353 x-1.54353$
Plot:


## Geometric figure:

line
Alternate forms:
1.54353 ( $x-0.999999$ )
$9.25774 \times 10^{-14}\left(1.66729 \times 10^{13} x-1.66729 \times 10^{13}\right)$

## Root:

$x \approx 0.999999$
0.999999

## Properties as a real function:

Domain
$\mathbf{R}$ (all real numbers)

## Range

$\mathbf{R}$ (all real numbers)

## Bijectivity

bijective from its domain to $\mathbb{R}$

## Derivative:

$$
\frac{d}{d x}(1.54353 x-1.54353)=1.54353
$$

## Indefinite integral:

$$
\begin{gathered}
\int\left(x \left(\exp \left(23.91796 \times 10^{-7}\right)+\frac{1}{2} \exp \left(2 \times 0.08640553 .91796 \times 10^{-7}\right)+\right.\right. \\
\exp \left(-2\left(3.91796 \times 10^{-7}+3\right)^{2}\right)+ \\
\left.\left.\left(1-\exp \left(-\frac{2}{3}\left(3.91796 \times 10^{-7}+0.351\right)\right)\right)^{2}\right)-1.54353\right) \\
d x=0.771765 x^{2}-1.54353 x+\text { constant }
\end{gathered}
$$

Definite integral after subtraction of diverging parts:

$$
\int_{0}^{\infty}((-1.54353+1.54353 x)-(-1.54353+1.54353 x)) d x=0
$$

$0.999999^{*}(((\exp (2 * 3.91796 \mathrm{e}-7)+1 / 2(\exp (2 * 0.0864055 * 3.91796 \mathrm{e}-7))+\exp (-$ $\left.\left.\left.\left.2(3.91796 \mathrm{e}-7+3)^{\wedge} 2\right)+(1-\exp (-2 / 3 *(3.91796 e-7+0.351)))^{\wedge} 2\right)\right)\right)-1.54353$

## Input interpretation:

$$
\begin{aligned}
& 0.999999\left(\exp \left(2 \times 3.91796 \times 10^{-7}\right)+\right. \\
& \frac{1}{2} \exp \left(2 \times 0.0864055 \times 3.91796 \times 10^{-7}\right)+\exp \left(-2\left(3.91796 \times 10^{-7}+3\right)^{2}\right)+ \\
& \left.\left(1-\exp \left(-\frac{2}{3}\left(3.91796 \times 10^{-7}+0.351\right)\right)\right)^{2}\right)-1.54353
\end{aligned}
$$

## Result:

$-7.32737 \ldots \times 10^{-7}$
$-7.32737 \ldots * 10^{-7}=V(\varphi)$

We observe that:
$1+1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 / 3.91796 e-7\right)^{\wedge} 2\right)\right)^{\wedge} 1 / 3$

## Input interpretation:

$1+\frac{1}{\sqrt[3]{\left(-\frac{7.32737 \times 10^{-7}}{3.91796 \times 10^{-7}}\right)^{2}}}$

## Result:

1.65878...
1.65878 ... result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505} / G_{101 / 5}\right)^{3}=1164,2696$ i.e. $1,65578 \ldots$
$\left.\left(\left(\left(-1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 * 3.91796 \mathrm{e}-7\right)\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 4+21$
where 21 is a Fibonacci number

## Input interpretation:

$\sqrt[4]{\frac{-1}{-7.32737 \times 10^{-7} \times 3.91796 \times 10^{-7}}}+21$

## Result:

1387.15...
$1387.15 \ldots$ result practically equal to the rest mass of Sigma baryon 1387.2
$2\left(\left(\left(-1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 * 3.91796 \mathrm{e}-7\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 7+$ golden ratio

## Input interpretation:

$2 \sqrt[7]{\frac{-1}{-7.32737 \times 10^{-7} \times 3.91796 \times 10^{-7}}}+\phi$

## Result:

125.4244..
125.4244.... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $\mathrm{T}=0$ and to the Higgs boson mass 125.18 GeV
$2\left(\left(\left(-1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 * 3.91796 \mathrm{e}-7\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 7+13+$ golden ratio ${ }^{\wedge} 2$
where 13 is a Fibonacci number

## Input interpretation:

$2 \sqrt[7]{\frac{-1}{-7.32737 \times 10^{-7} \times 3.91796 \times 10^{-7}}}+13+\phi^{2}$

## Result:

139.4244..
139.4244.... result practically equal to the rest mass of Pion meson 139.57 MeV
$\left(\left(\left(-1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 * 3.91796 e-7\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 4+322+47-7$
where 322, 47 and 7 are Lucas numbers
Input interpretation:
$\sqrt[4]{\frac{-1}{-7.32737 \times 10^{-7} \times 3.91796 \times 10^{-7}}}+322+47-7$

## Result:

1728.15.
1728.15....

This result is very near to the mass of candidate glueball $\mathrm{f}_{0}(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the $j$-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the GrossZagier theorem. The number 1728 is one less than the Hardy-Ramanujan number 1729
$\left(\left(\left(-1 /\left(\left(\left(-7.32737 \times 10^{\wedge}-7 * 3.91796 e-7\right)\right)\right)\right)\right)\right)^{\wedge} 1 / 4+322+76+18$
where 322,76 and 18 are Lucas numbers

## Input interpretation:

$\sqrt[4]{\frac{-1}{-7.32737 \times 10^{-7} \times 3.91796 \times 10^{-7}}}+322+76+18$

## Result:

1782.15..
$1782.15 \ldots$ result in the range of the hypothetical mass of Gluino (gluino $=1785.16$ GeV ).

## Appendix

From:
Lectures on $\mathbf{N}=2$ String Theory
Doron Gepner
Joseph Henry Laboratories - Princeton University
Princeton, New Jersey 08544- April, 1989

Let us consider now the right moving vector representation. From table 2 the internal dimension for the vector fields is $\bar{\Delta}_{i}=\frac{1}{2}$ and the charge is $\bar{Q}_{i}= \pm 1$. Again, these fields obey $\bar{\Delta}=|\bar{Q}| / 2$, and are thus chiral or anti-chiral fields of the right moving $N=2$ algebra with charges $\pm 1$. Denote such a field with $\bar{Q}_{i}=1$ by $C=\hat{C} \exp (i \bar{\phi} / \sqrt{3})$, where $\hat{C}$ is a neutral field. The vertex operator for the massless fields in the vector representation of $S O(10)$ is $V_{\mu} \hat{C} \exp (i \bar{\phi} / \sqrt{3})$, where $V_{\mu}, \mu=1,2, \ldots 10$ represents the vector of $S O(10)$ at level one. ( $V_{\mu}$ can be taken to be 10 free Majorana fermions.) Acting on this field with the right moving supersymmetry generator $Q^{\dagger}$ we obtain a massless spinor field, $S_{\alpha} \hat{C} \exp (-i \bar{\phi} / 2 \sqrt{3})$, where $S_{\alpha}$ is the spin field of $S O(10)$. Acting once more with $Q^{\dagger}$ gives the massless singlet field $\hat{C} \exp (-2 i \bar{\phi} / \sqrt{3})$. Counting states we find $10+16+1=27$. Indeed these fields together give the 27 representation of $E_{6}$. It can be easily checked that the weights of these fields are the correct ones for the 27 of $E_{6}$ (as we did for the adjoint), and that this is precisely the vertex operator representation for the 27 of $E_{6}$ at level one.

Similarly, the right moving vector fields with $\bar{Q}_{i}=-1$ give the $\overline{27}$ representation of $E_{6}$ when acting with $Q$ twice, $\overline{27}=10+\overline{16}+1$. It can be further seen that these are all the possible fields in the right moving sector of the theory.

How are the right movers in the 27 and $\overline{27}$ representations of $E_{6}$ connected together with the right movers? The only possible fields in the right moving sector that these fields can multiply are the spinor and anti-spinor multiplets of $S O(2)$. We thus have four possibilities: space-time left fermions which are 27 $\left(Q_{i}=\bar{Q}_{i}=1\right)$, right fermions which are $27\left(-Q_{i}=\bar{Q}_{i}=1\right)$, left fermions which are $\overline{27}\left(Q_{i}=-\bar{Q}_{i}=1\right)$ and right fermions which are $\overline{27}\left(Q_{i}=\bar{Q}_{i}=-1\right)$. The last two are CPT conjugates of the first two $\left(Q_{i} \rightarrow-Q_{i}\right.$ along with $\left.\bar{Q}_{i} \rightarrow-\bar{Q}_{i}\right)$. We conclude that the matter content of the theory consists of a number of left
handed fermions in the 27 of $E_{6}$ and a number of left-handed fermions in the $\overline{27}$ of $E_{6}$. The 27 fields correspond to left and right chiral fields, $(c, c)$, whereas the $\overline{27}$ correspond to the fields which are left chiral and right anti-chiral, $(c, a)$. In general the number of 27 fields, $N_{27}$ would be different from the number of $\overline{27}, N_{27}$, giving rise to a net number of chiral generations in the theory, $N=$ $N_{27}-N_{27}$.

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We would like to thank Professor Augusto Sagnotti theoretical physicist at Scuola Normale Superiore (Pisa - Italy) for his very useful explanations and his availability and George E. Andrews Evan Pugh Professor of Mathematics at Pennsylvania State University for his great availability and kindness

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Scuola Normale Superiore and INFN -Pisa - OKC Colloquium
Stockholm, September 2018

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