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RESEARCH ARTICLE

BIPOLAR VAGUE A-IDEALS OF BCI-ALGEBRAS

*,[1]Cicily Flora, S., [1]Arockiarani, I. and [2]Jafari, S.

[1]Department of Mathematics, Nirmala College for Women, Coimbatore-18, India
[2]College of Vestsjaelland South, Herrestraede 11, 4200 slagelse, Denmark

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ABSTRACT

The notion of bipolar vague A-ideals of BCI-algebra is introduced and their properties are investigated. Characterizations of bipolar vague A-ideals are given. Relationship between bipolar vague ideal and the newly defined ideals are analyzed. Moreover, we studied some equivalent conditions for bipolar vague A-ideals. Finally, we prove a result in bipolar A-ideal using bipolar vague level cut.

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INTRODUCTION

Several authors from time to time have made a number of generalizations of Zadeh's fuzzy set theory (Zadeh, 1965). The notions of BCK/BCI-algebra were introduced by Iseki (1980) and were extensively investigated by many researchers. Liu and Meng (2004) introduced the notion of q-ideals and a-ideals in BCI-algebras. The study of fuzzy algebraic structure was started with the introduction of the concept of fuzzy sub-group in 1971 by Rosenfeld (1971). In 1993, Jun (1993, 1994) applied in BCI algebra. Of these, the notion of vague set theory was introduced by Gau and Buehrer (1993). Using this vague set, Biswas (2006) studied vague groups. Further Ramakrishnan and Eswarlal (2008, 2009) continued the study of vague algebra by studying the characterizations of cyclic groups in terms of vague groups, vague normal groups, vague normalizer, vague centralizer, vague ideals, normal vague ideals etc. Ganeshree Selvachandran (2012, 2016) introduced vague soft rings and vague soft ideals and studied some of their properties. Also Alhazaymeh (2012) introduced the concept of possibility vague soft set. Jun and Park (2007) studied vague ideals and vague deductive systems in subtraction algebras. In (Arsham Bourmand Saied, 2009), the concept of vague BCK/BCI-algebras is discussed. Lee (2004, 2000) introduced an extension of fuzzy sets named bipolar -valued fuzzy sets. Based on notion of bipolar-valued fuzzy sets, Jun and Song (2008) and Lee (2009) discussed subalgebra and ideals of BCH-algebras.The concept of bipolar vague fuzzy translation and bipolar-valued fuzzy S-extensions of a bipolar valued fuzzy subalgebra in BCK/BCI-algebra was introduced by Jun *et al*. (2009) In this paper, we apply the concept of bipolar vague A-ideals to BCI-algebras and investigate its properties. Also we discuss the relations among bipolar vague subalgebras, bipolar vague ideals and bipolar vague A-ideals.

Preliminaries

Definition 2.1 (Young Bae Jun *et al*., 2007) An algebra $(X;*,0)$ of type (2,0) is called a BCI-algebra if it satisfies the following conditions:

(i) $((x*y)*(x*z))*(z*y)=0$,
(ii) $(x*(x*y))*y=0$,

***Corresponding author: Cicily Flora, S.** Department of Mathematics, Nirmala College for Women, Coimbatore-18, India

(iii) $x*x=0$,

$(iv)\, x*y=0, y*x=0 \Rightarrow x=y$ for all $x,y,z \in X$

We can define a partial order $'\leq'$ on X by $x \leq y$ if and only if $x*y=0$. Any BCI-algebra X has the following properties:

(1) $x*0=x.$
(2) $(x*y)*z=(x*z)*y.$
(3) $x \leq y \Rightarrow x*z \leq y*z, z*y \leq z*x.$ for all $x,y,z \in X$

Definition 2.2(Zadeh, 1965): Let X be a nonempty set. A fuzzy set A drawn from X is defined as $A=\{<x:\mu_A(x)>: x \in X\}$, where $\mu_A : X \to [0,1]$ is the membership function of the fuzzy set A.

Definition 2.3(Lee, 2000)**:** Let X be the universe. Then a bipolar valued fuzzy sets, A on X is defined by positive membership function μ_A^+, that is $\mu_A^+ : X \to [0,1]$, and a negative membership function μ_A^-, that is $\mu_A^- : X \to [-1,0]$. For the sake of simplicity, we shall use the symbol $A=\{\langle x, \mu_A^+(x), \mu_A^-(x)\rangle : x \in X\}$.

Definition 2.4 (Gau and Buehrer, 1993): A vague set A in the universe of discourse U is a pair $[t_A, f_A]$, where $t_A : U \to [0,1]$, $f_A : U \to [0,1]$ are the mappings (called truth membership function and false membership function respectively) where $t_A(x)$ is a lower bound of the grade of membership of x derived from the evidence for x and $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $t_A(x)+f_A(x) \leq 1$ for all $x \in U$.

Definition 2.5 (Gau and Buehrer, 1993): The interval $[t_A(x), 1-f_A(x)]$ is called the vague value of x in A, and it is denoted by $V_A(x)$. That is $V_A(x)=[t_A(x), 1-f_A(x)]$.

Definition 2.6 (Gau and Buehrer, 1993): A vague set A of U with $t_A(x)=1$ and $f_A(x)=0 \quad \forall x \in U$, is called the unit vague set of U. A vague set A of U with $t_A(x)=0$ and $f_A(x)=1 \ \forall x \in U$, is called the unit vague set of U

Definition 2.7 (Gau and Buehrer, 1993): Let Abe a non-empty set and the vague set A and B in the form $A=\{\langle x, t_A(x), 1-f_A(x)\rangle : x \in X\}$, $B=\{\langle x, t_B(x), 1-f_B(x)\rangle : x \in X\}$. Then

(i) $A \subseteq B$ if and only if $t_A(x) \leq t_B(x)$ and $1-f_A(x) \leq 1-f_B(x)$
$(ii)\, A \cup B = \max\{t_A(x), t_B(x)\}$ and $\max\{1-f_A(x), 1-f_B(x)\}$.
$(ii)\, A \cap B = \min\{t_A(x), t_B(x)\}$ and $\min\{1-f_A(x), 1-f_B(x)\}$.
(iv) $\overline{A} = \{\langle x, f_A(x), 1-t_A(x)\rangle : x \in X\}$.

Definition 2.8 (Cicily Flora and Arockiarani, 2016): Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form $A=\{\langle x, [t_A^+(x), 1-f_A^+(x)], [-1-f_A^-(x), t_A^-(x)]\rangle : x \in X\}$, where $[t_A^+, 1-f_A^+] : X \to [0,1]$ and $[-1-f_A^-, t_A^-] : X \to [-1,0]$ are the mapping such that $t_A^+ + f_A^+ \leq 1$ and $-1 \leq t_A^- + f_A^-$ The positive membership degree $[t_A^+(x), 1-f_A^+(x)]$ denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued set A and the negative membership degree $[-1-f_A^-(x), t_A^-(x)]$ denotes the satisfaction region of x to some implicit counter property of A. For a sake of simplicity, we shall use the notion of bipolar vague set $v_A^+ = [t_A^+, 1-f_A^+]$ and $v_A^- = [-1-f_A^-, t_A^-]$.

Definition 2.9 (Arockiarani and Cicily Flora, 2016): A bipolar vague set $A=(X; v_A^+, v_A^-)$ in X is called a bipolar vague subalgebra of X if it satisfies:

$$v_A^+(x*y) \geq \min\{v_A^+(x), v_A^+(y)\}$$
$$v_A^-(x*y) \leq \max\{v_A^-(x), v_A^-(y)\}$$

for all $x, y \in X$, that is

$t_A^+(x*y) \geq \min\{t_A^+(x), t_A^+(y)\}$

$1 - f_A^+(x*y) \geq \min\{1 - f_A^+(x), 1 - f_A^+(y)\}$

$t_A^-(x*y) \leq \max\{t_A^-(x), t_A^-(y)\}$

$1 - f_A^-(x*y) \leq \max\{1 - f_A^-(x), 1 - f_A^-(y)\}$

Definition 2.10 (Arockiarani and Cicily Flora, 2016): A bipolar vague set $A = (X; v_A^+, v_A^-)$ of a BCK algebra X is called a bipolar vague ideal of X if the following conditions are true:

(I) $v_A^+(0) \geq v_A^+(x)$ and $v_A^-(0) \leq v_A^-(x)$

(II) $v_A^+(x) \geq \min\{v_A^+(x*y), v_A^+(y)\}$ and $v_A^-(x) \leq \max\{v_A^-(x*y), v_A^-(y)\}$

3. Bipolar vague A-ideal

Definition 3.1: A bipolar vague set $A = (X; V_A^+, V_A^-)$ in X is called a *bipolar vague A-ideal* of X if it satisfies:

(i) $V_A^+(0) \geq V_A^+(x)$

(ii) $V_A^-(0) \leq V_A^-(x)$

(iii) $V_A^+(y*x) \geq \min\{V_A^+((x*z)*(0*y)), V_A^+(z)\}$

(iv) $V_A^-(y*x) \leq \max\{V_A^-((x*z)*(0*y)), V_A^-(z)\}$ for all $x, y, z \in X$.

Example 3.2: Consider a BCI-algebra X= {0, a, b, c} with the following Cayley table:

*	0	A	b	c
0	0	A	b	c
a	a	0	c	b
b	b	C	0	a
c	c	B	a	0

Define a bipolar vague set $A = (X; V_A^+, V_A^-)$ in X by

X	0	A	b	C
V_A^+	(0.6, 0.8)	(0.6, 0.8)	(0.3,0.4)	(0.3,0.4)
V_A^-	(-0.6,-0.4)	(-0.6,-0.4)	(-0.4,-0.3)	(-0.4,-0.3)

Then $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.3: If $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X , then $V_A^-(x) = V_A^-(0*x)$ and $V_A^+(x) = V_A^+(0*x)$ for all $x \in X$.

Proof: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague a-ideal of X . Taking $y = z = 0$ in Definition 3.1 and using Definition 2.1(iii) and (1), we get $V_A^-(0*x) \leq V_A^-(x)$ and $V_A^+(0*x) \geq V_A^+(x)$ ….*

Setting $x = z = 0$ in Definition 3.1 and using Definition 2.1(iii), (*) we have $V_A^-(y) = V_A^-(y*0) \leq V_A^-(0*(0*y)) \leq V_A^-(0*y)$ and $V_A^+(y) = V_A^+(y*0) \geq V_A^+(0*(0*y)) \geq V_A^+(0*y)$ for all $y \in X$.

Hence, $V_A^-(x) = V_A^-(0*x)$ and $V_A^+(x) = V_A^+(0*x)$ for all $x \in X$.

Theorem 3.4: Every bipolar vague A-ideal of X is both a bipolar vague subalgebra of X and a bipolar vague ideal of X .
Proof: $A = (X; V_A^+, V_A^-)$ be a bipolar vague A-ideal of X . By Definition 3.1 and Theorem 3.3, we have

$V_A^-(x) = V_A^-(0*x) \leq \max\{V_A^-((x*z)*(0*0)), V_A^-(z)\}$

$= \max\{V_A^-(x*z), V_A^-(z)\}$ and

$V_A^+(x) = V_A^+(0*x) \geq \min\{V_A^+((x*z)*(0*0)), V_A^+(z)\}$

$= \min\{V_A^+(x*z), V_A^+(z)\}$ for all $x, z \in X$.

Hence $A = (X; V_A^+, V_A^-)$ is a bipolar vague ideal of X. Now for any $x, y \in X$. We obtain

$V_A^-(x*y) \leq \max\{V_A^-((x*y)*x), V_A^-(x)\}$

$= \max\{V_A^-(0*y), V_A^-(x)\} = \max\{V_A^-(x), V_A^-(y)\}$ and

$V_A^+(x*y) \geq \min\{V_A^+((x*y)*x), V_A^+(x)\}$

$= \min\{V_A^+(0*y), V_A^+(x)\} = \min\{V_A^+(x), V_A^+(y)\}$.

Therefore $A = (X; V_A^+, V_A^-)$ is a bipolar vague subalgebra of X.

The following example shows that the converse of the above need not be true.

Example 3.5: Let X= {0, a ,b} be a BCI-algebra with the following Cayley table:

*	0	a	b
0	0	b	a
a	a	0	b
b	b	a	0

Define a bipolar vague set $A = (X; V_A^+, V_A^-)$ in X by

X	0	a	B
V_A^+	(0.6,0.8)	(0.4,0.4)	(0.4,0.4)
V_A^-	(-0.8,-0.6)	(-0.7,-0.5)	(-0.7,-0.5)

Then $A = (X; V_A^+, V_A^-)$ is both a bipolar vague ideal and a bipolar vague subalgebra of X, but not a bipolar vague A-ideal of X.

Theorem 3.6: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague ideal of X. If the inequality $x*y \leq z$ holds in X. Then

$V_A^-(x) \leq \max\{V_A^-(y), V_A^-(z)\}$

$V_A^-(x) \geq \min\{V_A^+(y), V_A^+(z)\}$.

Proof: Let $x, y, z \in X$ be such that $x*y \leq z$. Then $(x*y)*z = 0$, and so

$V_A^-(x) \leq \max\{V_A^-(x*y), V_A^-(y)\} \leq \max\{\max\{V_A^-((x*y)*z), V_A^-(z)\}, V_A^-(y)\}$

$= \max\{\max\{V_A^-(0), V_A^-(z)\}, V_A^-(y)\} = \max\{V_A^-(y), V_A^-(z)\}$ and

$V_A^+(x) \geq \min\{V_A^+(x*y), V_A^+(y)\} \geq \min\{\min\{V_A^+((x*y)*z), V_A^+(z)\}, V_A^+(y)\}$

$= \min\{\min\{V_A^+(0), V_A^+(z)\}, V_A^+(y)\} = \min\{V_A^+(y), V_A^-(z)\}$.

Hence the proof.

Theorem 3.7: Let $A = (X; V_A^+, V_A^-)$ be a bipolar vague ideal of X. Then the following are equivalent.

(i) $A = (X; V_A^+, V_A^-)$ is a bipolar vague A-ideal of X.

(ii) $A = (X; V_A^+, V_A^-)$ satisfies the following conditions:

$V_A^-(y*(x*z)) \leq V_A^-((x*z)*(0*y))$

$V_A^+(y*(x*z)) \geq V_A^-((x*z)*(0*y))$ for all $x, y, z \in X$.

(iii) $A=(X;V_A^+,V_A^-)$ satisfies the following conditions:

$$V_A^-(y*x)\le V_A^-(x*(0*y))$$
$$V_A^+(y*x)\ge V_A^+(x*(0*y))$$ for all $x,y\in X$.

Proof:

$(i)\Rightarrow(ii)$ Assume that $A=(X;V_A^+,V_A^-)$ is a bipolar vague A-ideal of X . Using Definition 3.1, we get
$V_A^-(y*(x*z))\le \max\{V_A^-(((x*z)*0)*(0*y)),V_A^-(0)\}=V_A^-((x*z)*(0*y))$ and
$V_A^+(y*(x*z))\ge \min\{V_A^+(((x*z)*0)*(0*y)),V_A^+(0)\}=V_A^+((x*z)*(0*y))$.
$(ii)\Rightarrow(iii)$ taking z=0 in (ii) and using (1) induce (iii)
$(iii)\Rightarrow(i)$ Note that $(x*(0*y))*((x*z)*(0*y))\le x*(x*z)\le z$ for all $x,y,z\in X$. It follows from (iii) and Theorem 3.6 that $V_A^-(y*x)\le V_A^-(x*(0*y)\le \max\{V_A^-((x*z)*(0*y)),V_A^-(z)\}$ and $V_A^+(y*x)\ge V_A^+(x*(0*y)\ge \min\{V_A^+((x*z)*(0*y)),V_A^+(z)\}$. Hence $A=(X;V_A^+,V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.8: Assume that X is associative, i.e., X satisfies the following identity:

$((x*y)*z=x*(y*z)$. For all $x,y,z\in X$, then every bipolar vague ideal is a bipolar vague A-ideal of X .

Proof: Let $A=(X;V_A^+,V_A^-)$ be a bipolar vague ideal of a associative BCI-algebra X . Since $0*x=x$ for all $x\in X$, it follows that $y*x=(0*y)*x=(0*x)*y=x*y=x*(0*y)$ for all $x,y,z\in X$. Therefore $V_A^-(y*x)=V_A^-(x*(0*y))$ and $V_A^+(y*x)=V_A^+(x*(0*y))$ using Theorem 3.7. We conclude that $A=(X;V_A^+,V_A^-)$ is a bipolar vague A-ideal of X .

Theorem 3.9: Let $A=(X;V_A^+,V_A^-)$ be a bipolar vague A-ideal of X . Then the set $\Omega=\{x\in X\,|\,V_A^-(x)=V_A^-(0),V_A^+(x)=V_A^+(0)\}$ is an A-ideal of X .

Proof: Obviously , $0\in\Omega$. Let $x,y,z\in X$ be such that $(x*z)*(0*y)\in\Omega$ and $z\in\Omega$. Then
$V_A^-(0)\le V_A^-(y*x)\le \max\{V_A^-((x*z)*(0*y)),V_A^-(z)\}=V_A^-(0)$ and
$V_A^+(0)\ge V_A^+(y*x)\ge \min\{V_A^+((x*z)*(0*y)),V_A^+(z)\}=V_A^+(0)$ by using Definition 3.1. It follows that $V_A^-(y*x)=V_A^-(0)$ and $V_A^+(y*x)=V_A^+(0)$. That is , $y*x\in\Omega$. Therefore Ω is an A-ideal of X .

Theorem 3.10: If $A=(X;V_A^+,V_A^-)$ and $B=(X;V_B^+,V_B^-)$ be two bipolar vague A-ideal of BCI-algebra X , then $A\cap B$ is a bipolar vague A-ideal of X .

Proof: $V_A^-(0)\le V_A^-(x)$ and $V_B^-(0)\le V_B^-(x)$ for all $x\in X$. $\max\{V_A^-(0),V_A^-(0)\}\le \max\{V_A^-(x),V_A^-(x)\}=V_{A\cap B}^-(0)\le V_{A\cap B}^-(x)$ for all $x\in X$. To verify second condition, $V_A^-(y*x)\le \max\{V_A^-((x*z)*(0*y)),V_A^-(z)\}$ and $V_B^-(y*x)\le \max\{V_B^-((x*z)*(0*y)),V_B^-(z)\}$.
$\max\{V_A^-(y*x),V_B^-(y*x)\}\le \max\{\max\{V_A^-((x*z)*(0*y)),V_A^-(z)\},\max\{V_B^-((x*z)*(0*y)),V_B^-(z)\}\}$
$V_{A\cap B}^-(y*x)\le \max\{\max\{V_A^-((x*z)*(0*y)),V_B^-((x*z)*(0*y))\},\max\{V_A^-(z),V_B^-(z)\}\}$
$V_{A\cap B}^-(y*x)\le \max\{V_{A\cap B}^-((x*z)*(0*y)),V_{A\cap B}^-(z)\}$. And $V_A^+(0)\ge V_A^-(x)$ and $V_B^+(0)\ge V_B^+(x)$ for all $x\in X$. $\min\{V_A^+(0),V_A^+(0)\}\ge \min\{V_A^+(x),V_A^+(x)\}=V_{A\cap B}^+(0)\ge V_{A\cap B}^+(x)$ for all $x\in X$. To verify second condition, $V_A^+(y*x)\ge \min\{V_A^+((x*z)*(0*y)),V_A^+(z)\}$ and $V_B^+(y*x)\ge \min\{V_B^+((x*z)*(0*y)),V_B^+(z)\}$. $\min\{V_A^+(y*x),V_B^+(y*x)\}$
$\ge \min\{\min\{V_A^+((x*z)*(0*y)),V_A^+(z)\},\min\{V_B^+((x*z)*(0*y)),V_B^+(z)\}\}$
$V_{A\cap B}^+(y*x)\ge \min\{\min\{V_A^+((x*z)*(0*y)),V_B^+((x*z)*(0*y))\},\min\{V_A^+(z),V_B^+(z)\}\}$
$V_{A\cap B}^+(y*x)\ge \min\{V_{A\cap B}^+((x*z)*(0*y)),V_{A\cap B}^+(z)\}$ for all $x,y,z\in X$. Therefore $A\cap B$ is a bipolar vague A-ideal of X .

Definition 3.11: For a bipolar vague set $A=(X;V_A^+,V_A^-)$ in X and $(\alpha,\beta)\in[0,1]$ and $(\mu,\gamma)\in[-1,0]$, the positive (α,β) -cut and negative (μ,γ) -cut are denoted by $A_{(\alpha,\beta)}^+$ and $A_{(\mu,\gamma)}^-$, and are defined as follows:

$A^{+}_{(\alpha,\beta)} = \{x \in X / t^{+}_{A}(x) \geq \alpha \ and \ 1 - f^{+}_{A}(x) \geq \beta\}$ and $A^{-}_{(\mu,\gamma)} = \{x \in X / t^{-}_{A}(x) \leq \mu \ and \ -1 - f^{-}_{A}(x) \leq \gamma\}$, respectively with $\alpha + \beta \leq 1$ and $\mu + \gamma \geq -1$. The bipolar level-cut of $A = (X; V^{+}_{A}, V^{-}_{A})$ denoted by A_{cut} is denoted to be the set $A_{cut} = < A^{+}_{(\alpha,\beta)}, A^{-}_{(\mu,\gamma)} >$

Theorem 3.12: A bipolar vague set $A = (X; V^{+}_{A}, V^{-}_{A})$ in X is a bipolar vague A- ideal of X if and only if for all $(\alpha,\beta) \in [0,1]$ and $(\mu,\gamma) \in [-1,0]$, the non-empty positive (α,β) -cut and the non-empty negative (μ,γ) -cut are bipolar vague A- ideals of X .

Proof: Let $A = (X; V^{+}_{A}, V^{-}_{A})$ is a bipolar vague A-ideal of X and assume that $A^{+}_{(\alpha,\beta)}$ and $A^{-}_{(\mu,\gamma)}$ are non-empty for $(\alpha,\beta) \in [0,1]$ and $(\mu,\gamma) \in [-1,0]$. Obviously, $0 \in A^{+}_{(\alpha,\beta)} \cap A^{-}_{(\mu,\gamma)}$. Let $x, y, z \in X$ be such that $t^{-}_{A}((x*z)*(0*y)) \in A^{-}_{(\mu,\gamma)}$ and $t^{-}_{A}(z) \in A^{-}_{(\mu,\gamma)}$ also $-1 - f^{-}_{A}((x*z)*(0*y)) \in A^{-}_{(\mu,\gamma)}$ and $-1 - f^{-}_{A}(z) \in A^{-}_{(\mu,\gamma)}$. Then $t^{-}_{A}((x*z))*(0*y)) \leq \mu$ and $t^{-}_{A}(z) \leq \mu$ also $-1 - f^{-}_{A}((x*z))*(0*y)) \leq \gamma$ and $-1 - f^{-}_{A}(z) \leq \gamma$. It follows from Definition 3.1that $t^{-}_{A}(y*x) \leq \max\{t^{-}_{A}((x*z)*(0*y)), t^{-}_{A}(z)\} \leq \mu$ and $-1 - f^{-}_{A}(y*x) \leq \max\{t^{-}_{A}((x*z)*(0*y)), t^{-}_{A}(z)\} \leq \gamma$ so that $y*x \in A^{-}_{(\mu,\gamma)}$. Now assume that $t^{+}_{A}((x*z)*(0*y)) \in A^{+}_{(\alpha,\beta)}$ and $t^{+}_{A}(z) \in A^{+}_{(\alpha,\beta)}$ also $1 - f^{+}_{A}((x*z)*(0*y)) \in A^{+}_{(\alpha,\beta)}$ and $1 - f^{+}_{A}(z) \in A^{+}_{(\alpha,\beta)}$. Then $t^{+}_{A}((x*z))*(0*y)) \geq \alpha$ and $t^{+}_{A}(z) \geq \alpha$ also $1 - f^{+}_{A}((x*z))*(0*y)) \geq \beta$ and $1 - f^{+}_{A}(z) \geq \beta$. I t follows from Definition 3.1that $t^{+}_{A}(y*x) \geq \min\{t^{-}_{A}((x*z)*(0*y)), t^{-}_{A}(z)\} \geq \alpha$ and $1 - f^{+}_{A}(y*x) \geq \min\{t^{-}_{A}((x*z)*(0*y)), t^{-}_{A}(z)\} \geq \beta$ so that $y*x \in A^{+}_{(\alpha,\beta)}$. Therefore $A^{+}_{(\alpha,\beta)}$ and $A^{-}_{(\mu,\gamma)}$ are an A-ideal of X .

Conversely, suppose that the non-empty negative (μ,γ) -cut and the non-empty positive (α,β) -cut are A-ideals of X for every $(\alpha,\beta) \in [0,1]$ and $(\mu,\gamma) \in [-1,0]$. If $t^{-}_{A}(0) > t^{-}_{A}(a)$, $-1 - f^{-}_{A}(0) > -1 - f^{-}_{A}(a)$ or $t^{+}_{A}(0) < t^{+}_{A}(b)$, $1 - f^{+}_{A}(0) < 1 - f^{+}_{A}(b)$ for some $a, b \in X$, then $0 \notin A^{-}_{(t^{-}_{A}(a), -1 - f^{-}_{A}(a))}$ or $0 \notin A^{+}_{(t^{+}_{A}(a), 1 - f^{+}_{A}(a))}$. This is a contradiction. Thus $t^{-}_{A}(0) \leq t^{-}_{A}(x)$, $-1 - f^{-}_{A}(0) \leq -1 - f^{-}_{A}(x)$ and $t^{+}_{A}(0) \geq t^{+}_{A}(x)$, $1 - f^{+}_{A}(0) \geq 1 - f^{+}_{A}(x)$ for all $x \in X$. Assume that $t^{-}_{A}(b*a) > \max\{t^{-}_{A}((a*c)*(0*b)), t^{-}_{A}(c)\} = \mu$ and $-1 - f^{-}_{A}(b*a) > \max\{-1 - f^{-}_{A}((a*c)*(0*b)), -1 - f^{-}_{A}(c)\} = \gamma$ for some $a, b, c \in X$. Then $(a*c)*(0*b) \in A^{-}_{(\mu,\gamma)}$ and $c \in A^{-}_{(\mu,\gamma)}$ but $b*a \notin A^{-}_{(\mu,\gamma)}$. This is impossible, and thus $t^{-}_{A}(y*x) \leq \max\{t^{-}_{A}((x*z)*(0*y)), t^{-}_{A}(z)\}$ for all $x, y, z \in X$. If $t^{+}_{A}(b*a) < \min\{t^{+}_{A}((a*c)*(0*b)), t^{+}_{A}(c)\} = \alpha$ and $1 - f^{+}_{A}(b*a) < \min\{1 - f^{+}_{A}((a*c)*(0*b)), 1 - f^{+}_{A}(c)\} = \beta$ for some $a, b, c \in X$. Then $(a*c)*(0*b) \in A^{+}_{(\alpha,\beta)}$ and $c \in A^{+}_{(\alpha,\beta)}$ but $b*a \notin A^{+}_{(\alpha,\beta)}$. This is impossible and thus $t^{-}_{A}(y*x) \geq \min\{t^{+}_{A}((x*z)*(0*y)), t^{+}_{A}(z)\}$ for all $x, y, z \in X$. Consequently, $A = (X; V^{+}_{A}, V^{-}_{A})$ is a bipolar vague A-ideal of X .

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