

The Riemann Hypothesis is false.

Abstract In this paper I prove that the RH is false, that is there aren't zeros on the critical strip because I prove that the zeta function $\zeta(s)$ converges for part real of s greater than zero.

Theorem 0.1. *The zeta function $\zeta(s)$ for $s \in \mathbb{C}$ converges for part real of $s > 0$*

Proof. Let the sequence $a_n : \mathbb{N} \rightarrow \mathbb{R}$ defined by $a_n = (n)^{\frac{l}{m}}$ with l, m natural $0 < l < m$. The sequence is a Cauchy sequence in fact let $n, m \in \mathbb{N}$ $\lim_{n, m \rightarrow +\infty} |a_n - a_m| = 0$ we suppose that $m = n + k$ for a constant $k \in \mathbb{N}$ so

$$\lim_{n \rightarrow +\infty} a_{n+k} - a_n = \lim_{n \rightarrow +\infty} (n+k)^{\frac{l}{m}} - n^{\frac{l}{m}} = \lim_{n \rightarrow +\infty} \frac{l}{m} k \left(\frac{1}{n}\right)^{\frac{m-l}{m}} + o\left(\frac{1}{n}\right) = 0$$

a_n is a Cauchy sequence in \mathbb{R} so a_n converges to $a \in \mathbb{R}$ and a_n is an increasing sequence so $a > a_n \forall n \in \mathbb{N}$

$$a = \lim_{n \rightarrow +\infty} n^{\frac{l}{m}} = \lim_{x \rightarrow +\infty} x^{\frac{l}{m}} > x^{\frac{l}{m}} \quad \forall 0 < l < m \quad l, m \in \mathbb{N} \quad \text{and} \quad x \in \mathbb{R}$$

$\frac{m}{l} > 0$ so

$$\begin{aligned} \mathbf{c} &= \frac{m}{l} a > \frac{m}{l} x^{\frac{l}{m}} = \frac{m}{l} \int_0^x \frac{d}{dt} t^{\frac{l}{m}} dt = \frac{m}{l} \int_0^x \frac{l}{m} \left(\frac{1}{t}\right)^{\frac{m-l}{m}} dt \\ \mathbf{c} &\geq \int_0^{+\infty} \frac{1}{x} \frac{m-l}{m} dx \end{aligned}$$

$\frac{1}{x} \frac{m-l}{m}$ is a decreasing function so

$$\mathbf{c} \geq \int_0^{+\infty} \frac{1}{x} \frac{m-l}{m} > \sum_{j=1}^{+\infty} \frac{1}{j} \frac{m-l}{m}$$

For $l, m \in \mathbb{N}$ with $0 < l < m$ so $0 < \frac{m-l}{m}$ so for $s \in \mathbb{C}$ with part real of $s > 0$, $\zeta(s)$ converges. □