

On the Ramanujan's mathematics (Rogers-Ramanujan continued fractions, Taxicab numbers and sixth order mock theta functions) applied to various parameters of Particle Physics: New possible mathematical connections.

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Abstract

In this research thesis, we have analyzed and deepened further Ramanujan expressions (Rogers-Ramanujan continued fractions, Taxicab numbers and sixth order mock theta functions) applied to various parameters of Particle Physics. We have therefore described new possible mathematical connections.

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<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

$$(i) \frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1x + a_2x^2 + a_3x^3 + \dots$$

$$\text{or } \frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \dots$$

$$(ii) \frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1x + b_2x^2 + b_3x^3 + \dots$$

$$\text{or } \frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \dots$$

$$(iii) \frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1x + c_2x^2 + c_3x^3 + \dots$$

$$\text{or } \frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \dots$$

then

$$\left. \begin{aligned} a_n^3 + b_n^3 &= c_n^3 + (-1)^n \\ \text{and } d_n^3 + \beta_n^3 &= \gamma_n^3 + (-1)^n \end{aligned} \right\}$$

Examples

$$135^3 + 138^3 = 172^3 - 1$$

$$11161^3 + 11468^3 = 14258^3 + 1$$

$$791^3 + 812^3 = 1010^3 - 1$$

$$9^3 + 10^3 = 12^3 + 1$$

$$6^3 + 8^3 = 9^3 - 1$$

<https://plus.maths.org/content/ramanujan>

Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$.

From Wikipedia

The **taxicab number**, typically denoted $Ta(n)$ or $Taxicab(n)$, also called the n th **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in n distinct ways. The most famous taxicab number is $1729 = Ta(2) = 1^3 + 12^3 = 9^3 + 10^3$.

Manuscript Book I of Srinivasa Ramanujan

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$$1 + 240 \left(\frac{1^2 y^3}{1-y^3} + \frac{2^2 y^6}{1-y^6} + \dots \right) = \left(1 + \frac{1 \cdot 2}{3!} x + \dots \right)^4 \left(1 - \frac{8}{9} x \right)$$

$$1 - 504 \left(\frac{1^2 y^3}{1-y^3} + \frac{2^2 y^6}{1-y^6} + \dots \right) = \left(1 + \frac{1 \cdot 2}{2!} x + \dots \right)^6 \left(1 - \frac{4}{3} x + \frac{8}{27} x^2 \right)$$

$$\left(1 + \frac{1 \cdot 2}{3!} x + \dots \right)^4 \left(1 - \frac{8}{9} x \right)$$

For $x = 2$, we obtain:

$$(1 + 2/9 * 2 + \dots)^4 (1 - 8/9 * 2)$$

Input interpretation:

$$\left(1 + \frac{2}{9} \times 2 + \dots\right)^4 \left(1 - \frac{8}{9} \times 2\right)$$

Results:

$$\frac{20\,925\,489\,375}{4\,294\,967\,296}$$

Intermediate result

$$\left(1 + \frac{2 \times 2}{9} + \dots\right) = \sum_{n=1}^{\infty} 9^{1-n} n^2 = \frac{405}{256}$$

Exact result:

$$\frac{20\,925\,489\,375}{4\,294\,967\,296} \text{ (irreducible)}$$

Decimal form:

-4.87209515995346009731292724609375

-4.8720951599...



$$(1 + 2/9 * 2 + \dots)^6 (1 - 4/3 * 2 + 8/27 * 2)$$

Input interpretation:

$$\left(1 + \frac{2}{9} \times 2 + \dots\right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2\right)$$

Results:

$$\frac{4\,739\,847\,545\,109\,375}{281\,474\,976\,710\,656}$$

Intermediate result

$$\left(1 + \frac{2 \times 2}{9} + \dots\right) = \sum_{n=1}^{\infty} 9^{1-n} n^2 = \frac{405}{256}$$

Exact result:

$$-\frac{4739847545109375}{281474976710656} \text{ (irreducible)}$$

Decimal form:

$$-16.839321208939050933395265019498765468597412109375$$

-16.8393212... result very near to the mass of the hypothetical light particle, the boson $m_x = 16.84$ MeV with minus sign

And:

$$-(((1+2/9*2+...)^6 (1-4/3*2+8/27*2)))-\pi$$

Input interpretation:

$$-\left(\left(1 + \frac{2}{9} \times 2 + \dots\right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2\right)\right) - \pi$$

Result:

$$\frac{4739847545109375}{281474976710656} - \pi$$

Alternate form:

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656}$$

Input:

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656}$$

Result:

$$\frac{4739847545109375}{281474976710656} - \pi$$

Decimal approximation:

$$13.69772855534925769493262163621926258440024270999989417902...$$

13.6977285...

In atomic physics, **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_{\infty} = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

Property:

$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656}$ is a transcendental number

Alternative representations:

$$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656} = \frac{4\,739\,847\,545\,109\,375 - 50\,665\,495\,807\,918\,080^\circ}{281\,474\,976\,710\,656}$$

$$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656} = \frac{4\,739\,847\,545\,109\,375 + 281\,474\,976\,710\,656 i \log(-1)}{281\,474\,976\,710\,656}$$

$$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656} = \frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \cos^{-1}(-1)}{281\,474\,976\,710\,656}$$

Series representations:

$$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656} = \frac{4\,739\,847\,545\,109\,375}{281\,474\,976\,710\,656} - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{4\,739\,847\,545\,109\,375 - 281\,474\,976\,710\,656 \pi}{281\,474\,976\,710\,656} = \frac{4\,739\,847\,545\,109\,375}{281\,474\,976\,710\,656} + \sum_{k=0}^{\infty} \frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}$$

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656} = \frac{4739847545109375}{281474976710656} - \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656} = \frac{4739847545109375}{281474976710656} - 4 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656} = \frac{4739847545109375}{281474976710656} - 2 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt$$

$$\frac{4739847545109375 - 281474976710656 \pi}{281474976710656} = \frac{4739847545109375}{281474976710656} - 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

The sum of the two results is:

$$(1+2/9*2+...) ^6 (1-4/3*2+8/27*2) + (1+2/9*2+...) ^4 (1-8/9*2)$$

Input interpretation:

$$\left(1 + \frac{2}{9} \times 2 + \dots\right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2\right) + \left(1 + \frac{2}{9} \times 2 + \dots\right)^4 \left(1 - \frac{8}{9} \times 2\right)$$

Result:

$$\frac{6111220416789375}{281474976710656}$$

Exact result:

$$\frac{6111220416789375}{281474976710656} \text{ (irreducible)}$$

Decimal form:

-21.711416368892511030708192265592515468597412109375
 -21.71141636...

Multiplying the two results, we obtain:

$$[(1+2/9*2+...) ^6 (1-4/3*2+8/27*2)] * [(1+2/9*2+...) ^4 (1-8/9*2)]$$

Input interpretation:

$$\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) \right)$$

Results:

$$\frac{99\,183\,629\,444\,306\,059\,775\,390\,625}{1\,208\,925\,819\,614\,629\,174\,706\,176}$$

Exact result:

$$\frac{99\,183\,629\,444\,306\,059\,775\,390\,625}{1\,208\,925\,819\,614\,629\,174\,706\,176} \text{ (irreducible)}$$

Decimal approximation:

82.04277535897359841720816931706633992464096361008074609344...
82.0427753...

We have that:

$$2[(1+2/9*2+...) ^6 (1-4/3*2+8/27*2)] * [(1+2/9*2+...) ^4 (1-8/9*2)] - 18 - 7$$

Input interpretation:

$$2 \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) \right) - 18 - 7$$

Results:

$$\frac{84\,072\,056\,699\,123\,195\,091\,563\,425}{604\,462\,909\,807\,314\,587\,353\,088}$$

Exact result:

$$\frac{84\,072\,056\,699\,123\,195\,091\,563\,425}{604\,462\,909\,807\,314\,587\,353\,088} \text{ (irreducible)}$$

Decimal approximation:

139.0855507179471968344163386341326798492819272201614921868...

139.08555071... result practically equal to the rest mass of Pion meson 139.57 MeV

$$2[(1+2/9*2+...) ^6 (1-4/3*2+8/27*2)] * [(1+2/9*2+...) ^4 (1-8/9*2)]-34-5$$

Input interpretation:

$$2\left(\left(1+\frac{2}{9}\times 2+\dots\right)^6\left(1-\frac{4}{3}\times 2+\frac{8}{27}\times 2\right)\right)\left(\left(1+\frac{2}{9}\times 2+\dots\right)^4\left(1-\frac{8}{9}\times 2\right)\right)-34-5$$

Result:

$$\frac{75\ 609\ 575\ 961\ 820\ 790\ 868\ 620\ 193}{604\ 462\ 909\ 807\ 314\ 587\ 353\ 088}$$

Exact result:

$$\frac{75\ 609\ 575\ 961\ 820\ 790\ 868\ 620\ 193}{604\ 462\ 909\ 807\ 314\ 587\ 353\ 088} \text{ (irreducible)}$$

Decimal approximation:

125.0855507179471968344163386341326798492819272201614921868...

125.08555071... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

We have that:

$$2[(1+2/9*2+...) ^6 (1-4/3*2+8/27*2)] * [(1+2/9*2+...) ^4 (1-8/9*2)]-18-7-2$$

Input interpretation:

$$2\left(\left(1+\frac{2}{9}\times 2+\dots\right)^6\left(1-\frac{4}{3}\times 2+\frac{8}{27}\times 2\right)\right)\left(\left(1+\frac{2}{9}\times 2+\dots\right)^4\left(1-\frac{8}{9}\times 2\right)\right)-18-7-2$$

Result:

$$\frac{82\ 863\ 130\ 879\ 508\ 565\ 916\ 857\ 249}{604\ 462\ 909\ 807\ 314\ 587\ 353\ 088}$$

Input:

$$\frac{82\ 863\ 130\ 879\ 508\ 565\ 916\ 857\ 249}{604\ 462\ 909\ 807\ 314\ 587\ 353\ 088}$$

Exact result:

$$\frac{82\,863\,130\,879\,508\,565\,916\,857\,249}{604\,462\,909\,807\,314\,587\,353\,088} \quad (\text{irreducible})$$

Decimal approximation:

137.0855507179471968344163386341326798492819272201614921868...

137.0855507179...

This result is very near to the inverse of fine-structure constant 137,035

$$27 * ((([(1+2/9*2+...) ^6 (1-4/3*2+8/27*2)] * [(1+2/9*2+...) ^4 (1-8/9*2)] - 18)))$$

Input interpretation:

$$27 \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) \right) - 18 \right)$$

Result:

$$\frac{2\,090\,420\,046\,663\,553\,835\,028\,345\,339}{1\,208\,925\,819\,614\,629\,174\,706\,176}$$

Exact result:

$$\frac{2\,090\,420\,046\,663\,553\,835\,028\,345\,339}{1\,208\,925\,819\,614\,629\,174\,706\,176} \quad (\text{irreducible})$$

Decimal approximation:

1729.154934692287157264620571560791177965306017472180144523...

1729.15493469...

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From Wikipedia:

“The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group $\mathbf{Z}/3\mathbf{Z}$, and its outer automorphism group is the cyclic

group $Z/2Z$. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories”.

$$\left(\left(\left(\left(27 \cdot \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots\right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2\right)\right) \cdot \left(1 + \frac{2}{9} \times 2 + \dots\right)^4 \left(1 - \frac{8}{9} \times 2\right) - 18\right)\right)\right)\right)\right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{27 \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) - 18 \right) \right)}$$

Result:

$$\frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}}$$

Input:

$$\frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}}$$

Decimal approximation:

1.643825048427037400581605187248848235343629361462611662293...

$$1.643825048427... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

Alternate form:

$$\frac{1}{64} \sqrt[3]{3} \sqrt[15]{8602551632360303847853273} 2^{2/3}$$

We have also:

$$\frac{1}{10^{27}} \left(\left(\left(\left(\left(\left(29 \cdot 10^3 + \left(\left(\left(\left(\left(27 \cdot \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \cdot \left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) - 18 \right) \right) \right) \right) \right) \right) \right) \right) \right)^{1/15} \right)$$

Input interpretation:

$$\frac{\frac{29}{10^3} + \sqrt[15]{27 \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) - 18 \right) \right)}}{10^{27}}$$

Result:

$$\frac{29}{1000} + \frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}}$$

1 000 000 000 000 000 000 000 000 000

Alternate forms:

$$\frac{29}{1000000000000000000000000000} + \frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{320000000000000000000000000 \sqrt[3]{2}}$$

$$\frac{232 + 125 \times 2^{2/3} \sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{800000000000000000000000000000}$$

$$\frac{116 \sqrt[3]{2} + 125 \sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{4000000000000000000000000000 \sqrt[3]{2}}$$

Input:

$$\frac{29}{1000} + \frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}}$$

1 000 000 000 000 000 000 000 000 000

Decimal approximation:

1.6728250484270374005816051872488482353436293614626116... × 10⁻²⁷

1.672825048... * 10⁻²⁷ result practically equal to the proton mass

Continued fraction:

$$597791144352066650561768933 + \frac{1}{10 + \frac{1}{2 + \frac{1}{3 + \frac{1}{1 + \frac{1}{20 + \frac{1}{1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2 + \frac{1}{\dots}}}}}}}}}}}}}}$$

$$-(18+7)/10^3 + ((((((27 * (((((1 + 2/9 * 2 + \dots)^6 (1 - 4/3 * 2 + 8/27 * 2)) * [(1 + 2/9 * 2 + \dots)^4 (1 - 8/9 * 2)] - 18))))))))))^{1/15}$$

Input interpretation:

$$-\frac{18+7}{10^3} + \sqrt[15]{27 \left(\left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^6 \left(1 - \frac{4}{3} \times 2 + \frac{8}{27} \times 2 \right) \right) \left(\left(1 + \frac{2}{9} \times 2 + \dots \right)^4 \left(1 - \frac{8}{9} \times 2 \right) \right) - 18 \right)}$$

Result:

$$\frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}} - \frac{1}{40}$$

Alternate forms:

$$\frac{1}{320} \left(5 \times 2^{2/3} \sqrt[3]{3} \sqrt[15]{8602551632360303847853273} - 8 \right)$$

$$\frac{5 \sqrt[3]{3} \sqrt[15]{8602551632360303847853273} - 4 \sqrt[3]{2}}{160 \sqrt[3]{2}}$$

$$\frac{1}{40} \left(\left(40 \sqrt[15]{\text{root of } 3518437208883200000x^5 + 439804651110400000x^4 + 21990232555520000x^3 + 549755813888000x^2 + 6871947673600x - 63794557088121149750620890438052371007 \text{ near } x = 7106.98} + 1 \right)^{(1/3) - 1} \right)$$

Input:

$$\frac{\sqrt[3]{3} \sqrt[15]{8602551632360303847853273}}{32 \sqrt[3]{2}} - \frac{1}{40}$$

Decimal approximation:

1.618825048427037400581605187248848235343629361462611662293...

1.61882504842... result that is a very good approximation to the value of the golden ratio 1,618033988749...

Possible closed forms:

$$-e^{-20+1/e+22e+20/\pi+17\pi} \pi^{15-35e} \cos^9(e\pi) \cot^{18}(e\pi) \approx 1.61882504842703739339$$

$$\log\left(\frac{1}{26} \left(-48 - 359\sqrt{2} + 63e + 70e^2 - 167\pi + 53\pi^2\right)\right) \approx 1.61882504842703740083829$$

$$\frac{4237713901\pi}{8223971375} \approx 1.618825048427037400570834$$

$$\pi \sqrt{\text{root of } 184x^4 - 1293x^3 + 3393x^2 + 819x - 1159 \text{ near } x = 0.515288} \approx 1.618825048427037400596657$$

$$\sqrt{\text{root of } 117x^5 - 970x^4 + 1188x^3 - 710x^2 + 720x + 1016 \text{ near } x = 1.61883} \approx 1.6188250484270374005853454$$

$$\sqrt{\text{root of } 49467x^3 - 37358x^2 - 39835x - 47467 \text{ near } x = 1.61883} \approx 1.618825048427037400578687$$

$$\pi \sqrt{\text{root of } 56065x^3 + 49984x^2 + 34003x - 38464 \text{ near } x = 0.515288} \approx 1.6188250484270374005831219$$

$$\sqrt{\text{root of } 3645x^4 + 1262x^3 - 6332x^2 - 5283x - 5240 \text{ near } x = 1.61883} \approx 1.618825048427037400577277$$

$$\pi \sqrt{\text{root of } 1510x^5 - 1087x^4 + 884x^3 + 673x^2 - 351x - 97 \text{ near } x = 0.515288} \approx 1.6188250484270374005828611$$

$$\frac{1}{\sqrt{\text{root of } 47467x^3 + 39835x^2 + 37358x - 49467 \text{ near } x = 0.617732}} \approx 1.618825048427037400578687$$

$$\frac{1}{\sqrt{\text{root of } 5240x^4 + 5283x^3 + 6332x^2 - 1262x - 3645 \text{ near } x = 0.617732}} \approx 1.618825048427037400577277$$

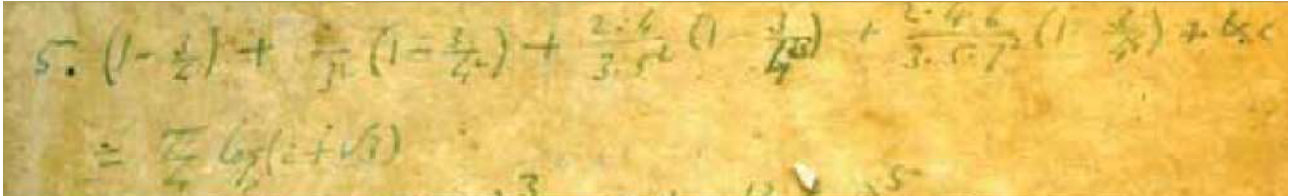
$$\frac{1}{4} \left(-93 - 16e - 7e^2 - 31\sqrt{1+e} + 38\sqrt{1+e^2} + \pi + 20\pi^2 - 13\sqrt{1+\pi} - 9\sqrt{1+\pi^2}\right) \approx 1.61882504842703740062238$$

$$\frac{-689 - 614e + 407e^2}{-100 - 343e + 194e^2} \approx 1.61882504842703739985$$

$$\frac{-334 + 664\pi - 201\pi^2}{-555 - 714\pi + 269\pi^2} \approx 1.61882504842703739675$$

$$\frac{4 + \sqrt{2} + 4\sqrt{3} + 8e - 4\pi - \pi^2 - \log(4)}{3 + 9\sqrt{2} - 7\sqrt{3} - e + \pi + \log\left(\frac{81}{8}\right)} \approx 1.618825048427037400565043$$

Now, we have that (page 156):



$$(1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256)+...$$

Input:

$$\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)$$

Exact result:

$$\frac{105269}{176400}$$

Decimal approximation:

0.596763038548752834467120181405895691609977324263038548752...

0.596763038...

$$0.989117352243 / (((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256)))$$

Input interpretation:

$$\frac{0.989117352243}{\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)}$$

Result:

1.657470869255575715547787097816071208047953338589708271190...

1.657470869... result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164,2696$ i.e. 1,65578...

We remember that the dilaton value **0.989117352243 = ϕ** in the above formula, is very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$8/((((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256))))$$

Input:

$$\frac{8}{\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)}$$

Exact result:

$$\frac{1411200}{105269}$$

Decimal approximation:

$$13.40565598609277185116226049454255288831469853423135016006\dots$$

$$13.40565598\dots$$

In atomic physics, **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

$$((((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256))))^{1/64}$$

Input:

$$\sqrt[64]{\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)}$$

Result:

$$\frac{\sqrt[64]{105269}}{\sqrt[16]{2} \sqrt[32]{105}}$$

Decimal approximation:

0.991966269843723145886293593462202407125435466185551937317...

0.991966269... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\phi^5 \sqrt[4]{5^3}} - 1}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the dilaton value **0.989117352243 = ϕ**

Alternate forms:

$$\frac{1}{210} \sqrt[64]{105269} 2^{15/16} \times 105^{31/32}$$

root of $176400x^{64} - 105269$ near $x = 0.991966$

$2 \log_{\text{base } 0.9919662698437}(\dots) - \pi + 1/\text{golden ratio}$

Input interpretation:

$$2 \log_{0.9919662698437} \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) - \pi + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

125.47644133...

125.47644133... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

Alternative representation:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) - \frac{1}{\phi} = -\pi + \frac{1}{\phi} + \frac{2 \log \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right)}{\log(0.99196626984370000)}$$

Series representations:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) - \frac{1}{\phi} = \frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{71131}{176400}\right)^k}{k}}{\log(0.99196626984370000)}$$

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) - \frac{1}{\phi} = \frac{1}{\phi} - 1.0000000000000000 \pi - 247.950358188420 \log\left(\frac{105269}{176400}\right) - 2 \log\left(\frac{105269}{176400}\right) \sum_{k=0}^{\infty} (-0.00803373015630000)^k G(k)$$

for $\left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

2log base 0.9919662698437((((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256))))+11+1/golden ratio

Input interpretation:

$$2 \log_{0.9919662698437} \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) + 11 + \frac{1}{\phi}$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

139.61803399...

139.61803399... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representation:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} + \frac{2 \log \left(1 - \frac{3}{4} + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)\right)}{\log(0.99196626984370000)}$$

Series representations:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{71131}{176400}\right)^k}{k}}{\log(0.99196626984370000)}$$

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) + 11 + \frac{1}{\phi} = 11 + \frac{1}{\phi} - 247.950358188420 \log\left(\frac{105269}{176400}\right) - 2 \log\left(\frac{105269}{176400}\right) \sum_{k=0}^{\infty} (-0.00803373015630000)^k G(k)$$

for $\left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

And:

$$2 \log \text{ base } 0.9919662698437 \left(\left(\left(\left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) \right) + 11 + \frac{1}{\phi} - 2 \right) \right)$$

Input interpretation:

$$2 \log_{0.9919662698437} \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) + 11 + \frac{1}{\phi} - 2$$

$\log_b(x)$ is the base- b logarithm

ϕ is the golden ratio

Result:

137.61803399...

137.61803399...

This result is very near to the inverse of fine-structure constant 137,035

Alternative representation:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{1}{75} \left(1 - \frac{3}{64}\right) + \frac{1}{735} \left(1 - \frac{3}{256}\right) \right) + 11 + \frac{1}{\phi} - 2 = 9 + \frac{1}{\phi} + \frac{2 \log \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right)}{\log(0.99196626984370000)}$$

Series representations:

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{1}{75} \left(1 - \frac{3}{64}\right) + \frac{1}{735} \left(1 - \frac{3}{256}\right) \right) + 11 + \frac{1}{\phi} - 2 = 9 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{71131}{176400}\right)^k}{k}}{\log(0.99196626984370000)}$$

$$2 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) + \frac{1}{75} \left(1 - \frac{3}{64}\right) + \frac{1}{735} \left(1 - \frac{3}{256}\right) \right) + 11 + \frac{1}{\phi} - 2 = 9 + \frac{1}{\phi} - 247.950358188420 \log\left(\frac{105269}{176400}\right) - 2 \log\left(\frac{105269}{176400}\right) \sum_{k=0}^{\infty} (-0.00803373015630000)^k G(k)$$

for $\left(G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

$$27 * \log \text{ base } 0.9919662698437 \left(\left(\left(\left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) \right) + 1 \right) \right)$$

Input interpretation:

$$27 \log_{0.9919662698437} \left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) + 1$$

$\log_b(x)$ is the base- b logarithm

Result:

1729.0000000...

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representation:

$$27 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) +$$

$$1 = 1 + \frac{27 \log \left(1 - \frac{3}{4} + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right)\right)}{\log(0.99196626984370000)}$$

Series representations:

$$27 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) +$$

$$1 = 1 - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-\frac{71131}{176400}\right)^k}{k}}{\log(0.99196626984370000)}$$

$$27 \log_{0.99196626984370000} \left(\left(1 - \frac{3}{4}\right) + \frac{1}{9} \left(1 - \frac{3}{16}\right) 2 + \frac{1}{75} \left(1 - \frac{3}{64}\right) 8 + \frac{1}{735} \left(1 - \frac{3}{256}\right) 48 \right) +$$

$$1 = 1.0000000000000000 - 3347.32983554368 \log\left(\frac{105269}{176400}\right) -$$

$$27.0000000000000000 \log\left(\frac{105269}{176400}\right) \sum_{k=0}^{\infty} (-0.00803373015630000)^k G(k)$$

for $\left(G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$

We have also:

$$((((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/(x+7)(1-3/256)))) = 0.59676303854875$$

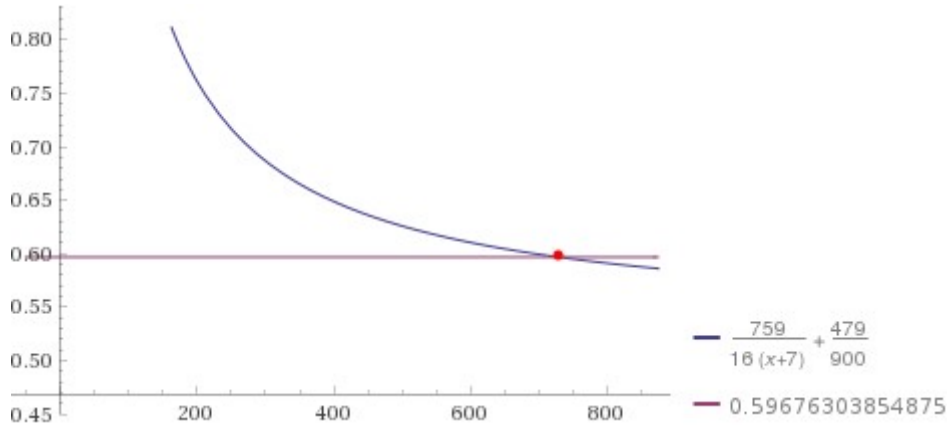
Input interpretation:

$$\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{x+7} \left(1 - \frac{3}{256}\right) = 0.59676303854875$$

Result:

$$\frac{759}{16(x+7)} + \frac{479}{900} = 0.59676303854875$$

Plot:



Alternate form assuming x is real:

$$\frac{735.00000}{1.000000000x + 7.00000000} = 1.00000000$$

Alternate form:

$$\frac{1916x + 184187}{3600(x + 7)} = 0.59676303854875$$

Alternate form assuming x is positive:

$$1.000000000000x = 728.000000000 \quad (\text{for } x \neq -7)$$

Solution:

$$x = 728$$

728 (Ramanujan taxicab number)

And:

$$(((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735((1-3/(2x-14)))))) = 0.59676303854875$$

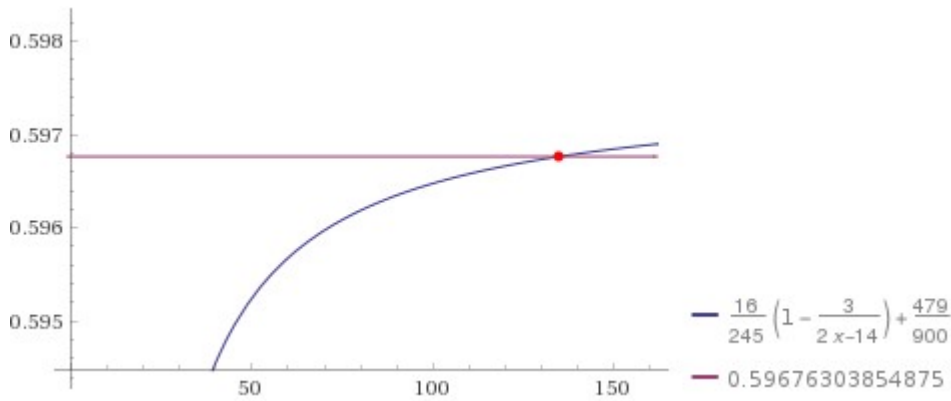
Input interpretation:

$$\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{2x-14}\right) = 0.59676303854875$$

Result:

$$\frac{16}{245} \left(1 - \frac{3}{2x-14}\right) + \frac{479}{900} = 0.59676303854875$$

Plot:



Alternate forms:

$$\frac{26\,351}{44\,100} - \frac{24}{245(x-7)} = 0.59676303854875$$

$$\frac{26\,351x - 188\,777}{44\,100(x-7)} = 0.59676303854875$$

Alternate form assuming x is positive:

$$\frac{128.0000000000}{7.000000000000 - 1.000000000000x} = 1.000000000000$$

Expanded form:

$$\frac{26\,351}{44\,100} - \frac{48}{245(2x-14)} = 0.59676303854875$$

Solution:

$$x \approx 134.99999999995$$

≈ 135 (Ramanujan taxicab number)

$$(((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735((1-3/(2x-20)))))) = 0.59676303854875$$

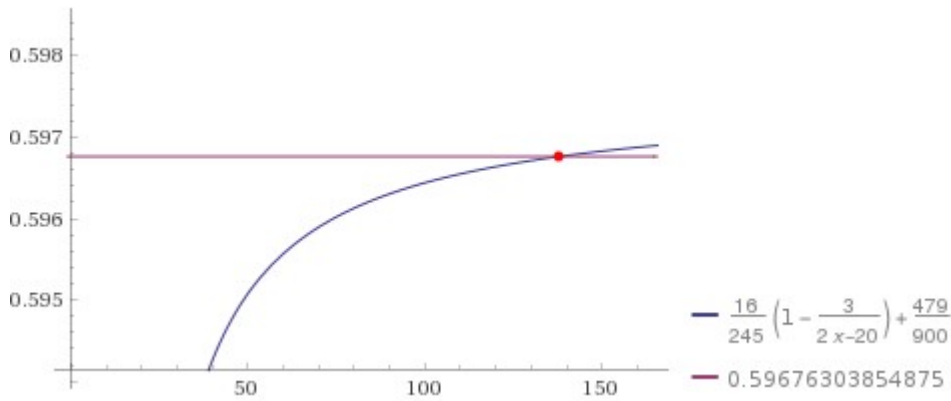
Input interpretation:

$$\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{2x-20}\right) = 0.59676303854875$$

Result:

$$\frac{16}{245} \left(1 - \frac{3}{2x-20}\right) + \frac{479}{900} = 0.59676303854875$$

Plot:



Alternate forms:

$$\frac{26\,351}{44\,100} - \frac{24}{245(x-10)} = 0.59676303854875$$

$$\frac{26\,351x - 267\,830}{44\,100(x-10)} = 0.59676303854875$$

$$\frac{8(2x-23)}{245(x-10)} + \frac{479}{900} = 0.59676303854875$$

Alternate form assuming x is positive:

$$-\frac{128.00000000}{10.000000000 - 1.0000000000x} = 1.0000000000$$

Expanded form:

$$\frac{26\,351}{44\,100} - \frac{48}{245(2x-20)} = 0.59676303854875$$

Solution:

$$x \approx 137.9999999995$$

≈ 138 (Ramanujan taxicab number)

From which:

$$(((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735((1-3/(2x-20-(\text{golden ratio}^2)))))) = 0.59676303854875$$

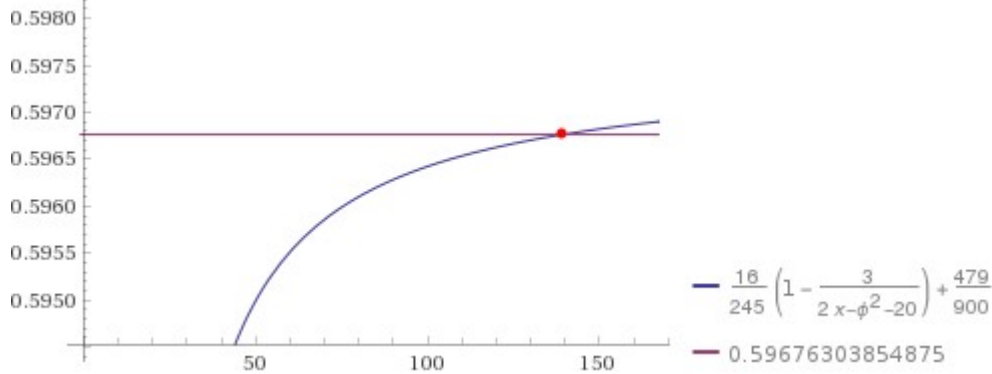
Input interpretation:

$$\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{2x-20-\phi^2}\right) = 0.59676303854875$$

ϕ is the golden ratio

Result:

$$\frac{16}{245} \left(1 - \frac{3}{2x - \phi^2 - 20} \right) + \frac{479}{900} = 0.59676303854875$$

Plot:**Alternate forms:**

$$\frac{16}{245} \left(\frac{3}{-2x + \phi^2 + 20} + 1 \right) + \frac{479}{900} = 0.59676303854875$$

$$\frac{-52702x + 26351\phi^2 + 535660}{44100(-2x + \phi^2 + 20)} = 0.59676303854875$$

$$\frac{16}{245} \left(1 - \frac{3}{2x + \frac{1}{2}(-43 - \sqrt{5})} \right) + \frac{479}{900} = 0.59676303854875$$

Alternate form assuming x is positive:

$$-\frac{128.00000}{11.30901699 - 1.00000000x} = 1.0000000$$

Expanded form:

$$\frac{26351}{44100} - \frac{48}{245(2x - \phi^2 - 20)} = 0.59676303854875$$

Solution:

$$x \approx 139.309016994$$

139.309016994 result practically equal to the rest mass of Pion meson 139.57 MeV

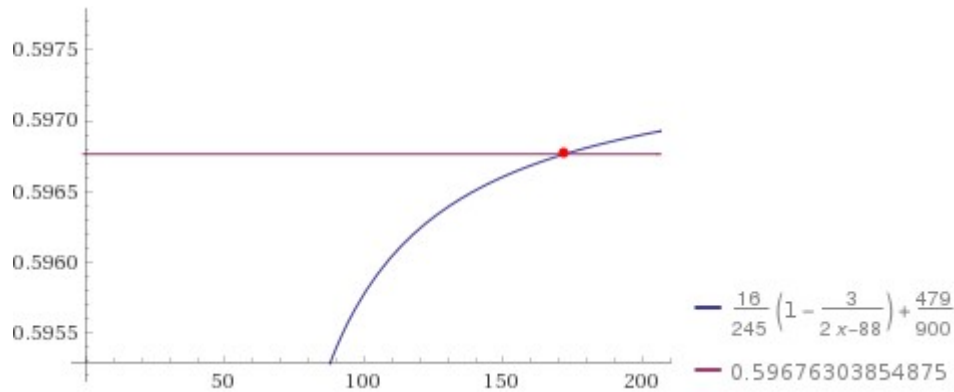
$$(((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735((1-3/(2x-88)))))) = 0.59676303854875$$

Input interpretation:

$$\left(1 - \frac{3}{4} \right) + \frac{2}{9} \left(1 - \frac{3}{16} \right) + \frac{8}{75} \left(1 - \frac{3}{64} \right) + \frac{48}{735} \left(1 - \frac{3}{2x - 88} \right) = 0.59676303854875$$

Result:

$$\frac{16}{245} \left(1 - \frac{3}{2x-88} \right) + \frac{479}{900} = 0.59676303854875$$

Plot:**Alternate forms:**

$$\frac{26\,351}{44\,100} - \frac{24}{245(x-44)} = 0.59676303854875$$

$$\frac{26\,351x - 1\,163\,764}{44\,100(x-44)} = 0.59676303854875$$

$$\frac{8(2x-91)}{245(x-44)} + \frac{479}{900} = 0.59676303854875$$

Alternate form assuming x is positive:

$$-\frac{128.00000000}{44.00000000 - 1.0000000000x} = 1.0000000000$$

Expanded form:

$$\frac{26\,351}{44\,100} - \frac{48}{245(2x-88)} = 0.59676303854875$$

Solution:

$$x \approx 172.0000000000$$

172 (Ramanujan taxicab number)

We have also:

The image shows a handwritten derivation on aged paper. The top line is a series expansion: $5 \cdot (1 - \frac{1}{2}) + \frac{5}{3} (1 - \frac{1}{4}) + \frac{2 \cdot 4}{3 \cdot 5^2} (1 - \frac{1}{4}) + \frac{2 \cdot 4 \cdot 6}{3 \cdot 5 \cdot 7} (1 - \frac{1}{5}) + \dots$. The bottom line shows the result: $= \frac{\pi}{4} \log(2 + \sqrt{3})$.

$$\frac{\pi}{4} \ln(2 + \sqrt{3})$$

Input:

$$\frac{\pi}{4} \log(2 + \sqrt{3})$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{1}{4} \pi \log(2 + \sqrt{3})$$

Decimal approximation:

1.034336313516517082033581770673406158619947276637927270391...

1.03433631351...

Alternative representations:

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log_e(2 + \sqrt{3})$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log(a) \log_a(2 + \sqrt{3})$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = -\frac{1}{4} \pi \text{Li}_1(-1 - \sqrt{3})$$

Series representations:

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{4} \pi \log(1 + \sqrt{3}) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{\left(\frac{-1}{1 + \sqrt{3}}\right)^k}{k}$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{2} i \pi^2 \left[\frac{\arg(2 + \sqrt{3} - x)}{2\pi} \right] + \frac{1}{4} \pi \log(x) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - x)^k x^{-k}}{k} \text{ for } x < 0$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{1}{2} i \pi^2 \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \frac{1}{4} \pi \log(z_0) - \frac{1}{4} \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - z_0)^k z_0^{-k}}{k}$$

Integral representations:

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = \frac{\pi}{4} \int_1^{2+\sqrt{3}} \frac{1}{t} dt$$

$$\frac{1}{4} \log(2 + \sqrt{3}) \pi = -\frac{i}{8} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + \sqrt{3})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

Thence:

$$(((1-3/4)+2/9(1-3/16)+8/75(1-3/64)+48/735(1-3/256)))x = \pi/4 \ln(2+\sqrt{3})$$

Input:

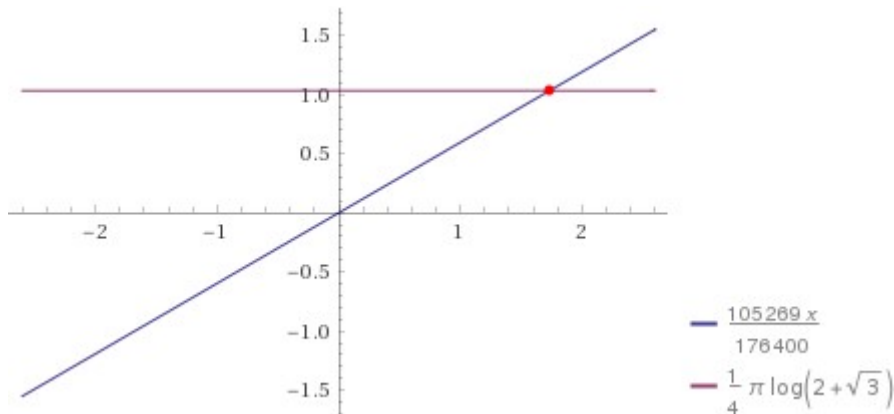
$$\left(\left(1 - \frac{3}{4}\right) + \frac{2}{9} \left(1 - \frac{3}{16}\right) + \frac{8}{75} \left(1 - \frac{3}{64}\right) + \frac{48}{735} \left(1 - \frac{3}{256}\right) \right) x = \frac{\pi}{4} \log(2 + \sqrt{3})$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{105269x}{176400} = \frac{1}{4} \pi \log(2 + \sqrt{3})$$

Plot:



Alternate form:

$$\frac{105\,269\,x}{176\,400} - \frac{1}{4} \pi \log(2 + \sqrt{3}) = 0$$

Solution:

$$x = \frac{44\,100 \pi \log(2 + \sqrt{3})}{105\,269}$$

Input:

$$\frac{44\,100 \pi \log(2 + \sqrt{3})}{105\,269}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

1.733244599115728403145501755947038979951920314612377532768...

1.733244599...

Alternative representations:

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{44\,100 \pi \log_e(2 + \sqrt{3})}{105\,269}$$

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{44\,100 \pi \log(a) \log_a(2 + \sqrt{3})}{105\,269}$$

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = -\frac{44\,100 \pi \text{Li}_1(-1 - \sqrt{3})}{105\,269}$$

Series representations:

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{44\,100 \pi \log(1 + \sqrt{3})}{105\,269} - \frac{44\,100 \pi \sum_{k=1}^{\infty} \frac{\left(\frac{-1}{1+\sqrt{3}}\right)^k}{k}}{105\,269}$$

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{88\,200 i \pi^2 \left\lfloor \frac{\arg(2+\sqrt{3}-x)}{2\pi} \right\rfloor}{105\,269} + \frac{44\,100 \pi \log(x)}{105\,269} - \frac{44\,100 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2+\sqrt{3}-x)^k x^{-k}}{k}}{105\,269} \quad \text{for } x < 0$$

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{44\,100 \pi \left\lfloor \frac{\arg(2+\sqrt{3}-z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right)}{105\,269} + \frac{44\,100 \pi \log(z_0)}{105\,269} + \frac{44\,100 \pi \left\lfloor \frac{\arg(2+\sqrt{3}-z_0)}{2\pi} \right\rfloor \log(z_0)}{105\,269} - \frac{44\,100 \pi \sum_{k=1}^{\infty} \frac{(-1)^k (2+\sqrt{3}-z_0)^k z_0^{-k}}{k}}{105\,269}$$

Integral representations:

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = \frac{44\,100 \pi}{105\,269} \int_1^{2+\sqrt{3}} \frac{1}{t} dt$$

$$\frac{44\,100 (\pi \log(2 + \sqrt{3}))}{105\,269} = -\frac{22\,050 i}{105\,269} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + \sqrt{3})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

From

$$\frac{44\,100 \pi \log(2 + \sqrt{3})}{105\,269}$$

we obtain:

$$\left(\frac{44\,100 \pi \log(2 + \sqrt{3})}{105\,269}\right)^{12} + 47$$

Input:

$$\left(\frac{44\,100 \pi \log(2 + \sqrt{3})}{105\,269}\right)^{12} + 47$$

log(x) is the natural logarithm

Series representations:

$$\left(\frac{44\,100(\pi \log(2 + \sqrt{3}))}{105\,269}\right)^{12} + 47 =$$

$$47 + \left(54\,108\,198\,377\,272\,584\,130\,510\,593\,262\,881\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right)$$

$$\pi^{12} \left(\log(1 + \sqrt{3}) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{1+\sqrt{3}}\right)^k}{k} \right)^{12} /$$

1 851 850 693 974 386 435 142 694 354 411 325 235 870 722 703 448 137 570 ∙
038 161

$$\left(\frac{44\,100(\pi \log(2 + \sqrt{3}))}{105\,269}\right)^{12} + 47 = 47 +$$

$$\left(54\,108\,198\,377\,272\,584\,130\,510\,593\,262\,881\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right)$$

$$\pi^{12} \left(2i\pi \left[\frac{\arg(2 + \sqrt{3} - x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - x)^k x^{-k}}{k} \right)^{12} /$$

1 851 850 693 974 386 435 142 694 354 411 325 235 870 722 703 448 137 570 ∙
038 161 for $x < 0$

$$\left(\frac{44\,100(\pi \log(2 + \sqrt{3}))}{105\,269}\right)^{12} + 47 =$$

$$47 + \left(54\,108\,198\,377\,272\,584\,130\,510\,593\,262\,881\,000\,000\,000\,000\,000\,000\,000\,000\,000\,000\right)$$

$$\pi^{12} \left(\log(z_0) + \left[\frac{\arg(2 + \sqrt{3} - z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 + \sqrt{3} - z_0)^k z_0^{-k}}{k} \right)^{12} /$$

1 851 850 693 974 386 435 142 694 354 411 325 235 870 722 703 448 137 570 ∙
038 161

Integral representations:

$$\left(\frac{44\,100(\pi \log(2 + \sqrt{3}))}{105\,269}\right)^{12} + 47 =$$

$$47 + \left(54\,108\,198\,377\,272\,584\,130\,510\,593\,262\,881\,000\,000\,000\,000\,000\,000\,000\right)$$

$$\pi^{12} \left(\int_1^{2+\sqrt{3}} \frac{1}{t} dt\right)^{12} /$$

1 851 850 693 974 386 435 142 694 354 411 325 235 870 722 703 448 137 570 ∙
038 161

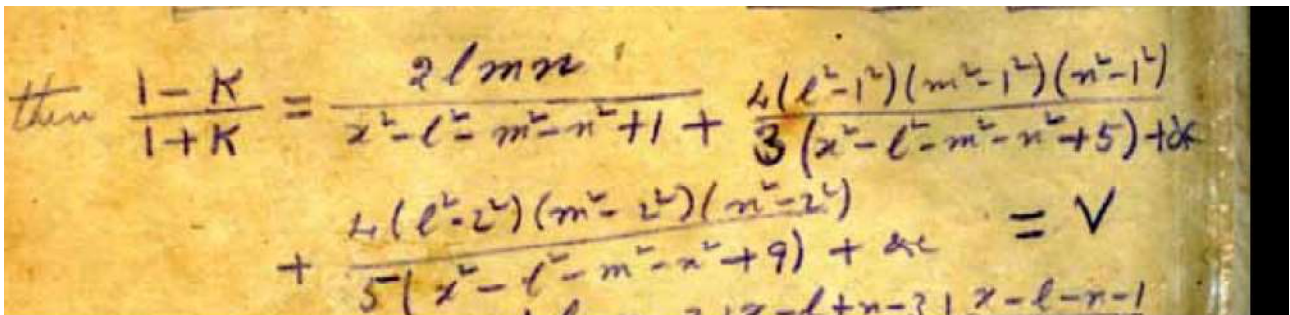
$$\left(\frac{44\,100(\pi \log(2 + \sqrt{3}))}{105\,269}\right)^{12} + 47 =$$

$$47 + \left(13\,210\,009\,369\,451\,314\,484\,987\,937\,808\,320\,556\,640\,625\,000\,000\,000\,000\right)$$

$$\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(1 + \sqrt{3})^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)^{12} /$$

1 851 850 693 974 386 435 142 694 354 411 325 235 870 722 703 448 137 570 ∙
038 161 for $-1 < \gamma < 0$

We have that (page 179)



For $x = 2, l = 8, m = 13$ and $n = 21$

$$(2 \cdot 8 \cdot 13 \cdot 21) / (4 - 8^2 - 13^2 - 21^2 + 1) + ((4(8^2 - 1)(13^2 - 1)(21^2 - 1))) / ((3(4 - 8^2 - 13^2 - 21^2 + 5))) + ((4(8^2 - 4)(13^2 - 4)(21^2 - 4))) / ((5(4 - 8^2 - 13^2 - 21^2 + 9)))$$

Input:

$$\frac{2 \times 8 \times 13 \times 21}{4 - 8^2 - 13^2 - 21^2 + 1} + \frac{4(8^2 - 1)(13^2 - 1)(21^2 - 1)}{3(4 - 8^2 - 13^2 - 21^2 + 5)} + \frac{4(8^2 - 4)(13^2 - 4)(21^2 - 4)}{5(4 - 8^2 - 13^2 - 21^2 + 9)}$$

Exact result:

$$\frac{40\,833\,183\,808}{2\,800\,657}$$

Decimal approximation:

-14579.8588716861793500596467186092406174694009298532451492...

-14579.8588716...

$$-\left(\frac{(2 \cdot 8 \cdot 13 \cdot 21) / (4 - 8^2 - 13^2 - 21^2 + 1) + ((4(8^2 - 1)(13^2 - 1)(21^2 - 1))) / ((3(4 - 8^2 - 13^2 - 21^2 + 5))) + ((4(8^2 - 4)(13^2 - 4)(21^2 - 4))) / ((5(4 - 8^2 - 13^2 - 21^2 + 9))))}{5}\right)^{1/2} - 5i$$

Input:

$$-\sqrt{\left(\frac{2 \times 8 \times 13 \times 21}{4 - 8^2 - 13^2 - 21^2 + 1} + \frac{4(8^2 - 1)(13^2 - 1)(21^2 - 1)}{3(4 - 8^2 - 13^2 - 21^2 + 5)} + \frac{4(8^2 - 4)(13^2 - 4)(21^2 - 4)}{5(4 - 8^2 - 13^2 - 21^2 + 9)}\right)} - 5i$$

i is the imaginary unit

Exact result:

$$-5i - 8i \sqrt{\frac{638\,018\,497}{2\,800\,657}}$$

Decimal approximation:

-125.747086390049923018103742363524139155344910227983688232... *i*

-125.74708639...*i* result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

Polar coordinates:

$r \approx 125.747$ (radius), $\theta = -90^\circ$ (angle)

Minimal polynomial:

$$7843\,679\,631\,649\,x^4 + 229\,111\,668\,109\,906\,162\,x^2 + 1\,661\,635\,815\,094\,475\,068\,689$$

Alternate forms:

$$i \left(-5 - 8 \sqrt{\frac{638\,018\,497}{2\,800\,657}} \right)$$

$$-i \left(5 + 8 \sqrt{\frac{638018497}{2800657}} \right) - \frac{i \left(14003285 + 8 \sqrt{1786870969752529} \right)}{2800657}$$

$$- \left(\frac{(2 \cdot 8 \cdot 13 \cdot 21) / (4 - 8^2 - 13^2 - 21^2 + 1) + ((4(8^2 - 1)(13^2 - 1)(21^2 - 1))) / ((3(4 - 8^2 - 13^2 - 21^2 + 5))) + ((4(8^2 - 4)(13^2 - 4)(21^2 - 4))) / ((5(4 - 8^2 - 13^2 - 21^2 + 9)))) \right)^{1/2} - 21i + 2i$$

Input:

$$- \sqrt{\left(\frac{2 \times 8 \times 13 \times 21}{4 - 8^2 - 13^2 - 21^2 + 1} + \frac{4(8^2 - 1)(13^2 - 1)(21^2 - 1)}{3(4 - 8^2 - 13^2 - 21^2 + 5)} + \frac{4(8^2 - 4)(13^2 - 4)(21^2 - 4)}{5(4 - 8^2 - 13^2 - 21^2 + 9)} \right) - 21i + 2i}$$

i is the imaginary unit

Exact result:

$$-19i - 8i \sqrt{\frac{638018497}{2800657}}$$

Decimal approximation:

$$-139.747086390049923018103742363524139155344910227983688232... i$$

-139.74708639...*i* result practically equal to the rest mass of Pion meson 139.57 MeV

Polar coordinates:

$$r \approx 139.747 \text{ (radius), } \theta = -90^\circ \text{ (angle)}$$

Minimal polynomial:

$$7843679631649 x^4 + 234382620822374290 x^2 + 1585803362300864650161$$

Alternate forms:

$$i \left(-19 - 8 \sqrt{\frac{638018497}{2800657}} \right)$$

$$-i \left(19 + 8 \sqrt{\frac{638018497}{2800657}} \right)$$

$$\frac{i \left(53212483 + 8 \sqrt{1786870969752529} \right)}{2800657}$$

$$27/2[-((((((2*8*13*21) / (4-8^2-13^2-21^2+1) + ((4(8^2-1)(13^2-1)(21^2-1))) / ((3(4-8^2-13^2-21^2+5))) + ((4(8^2-4)(13^2-4)(21^2-4))) / ((5(4-8^2-13^2-21^2+9)))))))))^{1/2} - 8i] + (11-2)i$$

Input:

$$\frac{27}{2} \left(-\sqrt{\left(\frac{2 \times 8 \times 13 \times 21}{4 - 8^2 - 13^2 - 21^2 + 1} + \frac{4(8^2 - 1)(13^2 - 1)(21^2 - 1)}{3(4 - 8^2 - 13^2 - 21^2 + 5)} + \frac{4(8^2 - 4)(13^2 - 4)(21^2 - 4)}{5(4 - 8^2 - 13^2 - 21^2 + 9)} \right)} - 8i \right) + (11 - 2)i$$

i is the imaginary unit

Exact result:

$$9i + \frac{27}{2} \left(-8i - 8i \sqrt{\frac{638018497}{2800657}} \right)$$

Decimal approximation:

$$-1729.08566626567396074440052190757587859715628807777979114... i$$

$$-1729.08566626... i$$

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the *j*-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Polar coordinates:

$$r \approx 1729.09 \text{ (radius)}, \quad \theta = -90^\circ \text{ (angle)}$$

Alternate forms:

$$-99i - 108i \sqrt{\frac{638018497}{2800657}}$$

$$i \left(-99 - 108 \sqrt{\frac{638018497}{2800657}} \right)$$

$$-9i \left(11 + 12 \sqrt{\frac{638018497}{2800657}} \right)$$

Minimal polynomial:

$$7843679631649x^4 + 41837877790526580210x^2 + 54973305261397849642082001$$

We have that (page 189):

Handwritten derivation on aged paper:

$$n \left\{ 1 + \frac{x^n}{\lfloor n} + \frac{x^{2n}}{\lfloor n} + \frac{x^{3n}}{\lfloor n} + \dots \right\}$$

$$= e^x + e^{x \cos \frac{2\pi}{n}} \cos(x \sin \frac{2\pi}{n}) + e^{x \cos \frac{4\pi}{n}} \cos(x \sin \frac{4\pi}{n}) + \dots$$

+ etc to n terms.

$$\frac{x^h}{\lfloor h} + \left(\frac{x^{h+2n}}{\lfloor h+2n} + \frac{x^{h-2n}}{\lfloor h-2n} \right) + \left(\frac{x^{h+4n}}{\lfloor h+4n} + \frac{x^{h-4n}}{\lfloor h-4n} \right) + \dots$$

$$= 1 + \left(\frac{x^n}{\lfloor n} + \frac{x^{-n}}{\lfloor -n} \right) + \left(\frac{x^{2n}}{\lfloor 2n} + \frac{x^{-2n}}{\lfloor -2n} \right) + \dots$$

$$\left\{ 6m^2 + (3m^3 - m) \right\}^3 + \left\{ 6m^2 - (3m^2 - m) \right\}^3$$

$$= \left\{ 6m^2 (3m^2 + 1) \right\}^2$$

For $m = 13$

$$((6 \cdot 13^2 \cdot (3 \cdot 13^2 + 1)))^2$$

Input:

$$(6 \times 13^2 (3 \times 13^2 + 1))^2$$

Result:

265 340 372 544

Scientific notation:

$2.65340372544 \times 10^{11}$

$2.65340372544 * 10^{11}$

$$\begin{aligned}
&= \{ m^7 - 3m^4(1+p) + m(3(1+p)^2 - 1) \}^3 \\
&+ \{ 2m^6 - 3m^3(1+2p) + (1+3p+3p^2) \}^3 \\
&+ \{ m^6 - (1+3p+3p^2) \}^3 \\
&= \{ m^7 - 3m^4p + m(3p^2 - 1) \}^3
\end{aligned}$$

For $m = 13$ and $p = 21$, we obtain:

$(13^7 - 3 * 13^4 * 21 + 13(3 * 21^2 - 1))^3$

Input:

$(13^7 - 3 \times 13^4 \times 21 + 13(3 \times 21^2 - 1))^3$

Result:

226 605 683 733 808 107 456 000

Scientific notation:

$2.26605683733808107456 \times 10^{23}$

$2.26605683733808107456 * 10^{23}$

From the ratio between the two results, and performing the 3th root, we obtain:

$$[(13^7 - 3 \cdot 13^4 \cdot 21 + 13(3 \cdot 21^2 - 1))^3 / ((6 \cdot 13^2 \cdot (3 \cdot 13^2 + 1)))^2]^{1/3} - 89$$

Input:

$$\sqrt[3]{\frac{(13^7 - 3 \times 13^4 \times 21 + 13(3 \times 21^2 - 1))^3}{(6 \times 13^2 (3 \times 13^2 + 1))^2}} - 89$$

Result:

$$\frac{390810 \sqrt[3]{\frac{3}{13}}}{127^{2/3}} - 89$$

Decimal approximation:

9398.5880926588896231860282480786451544773241000247249087038...

9398.58809265... result practically equal to the rest mass of Bottom eta meson 9398

Alternate forms:

$$\frac{390810 \times 13^{2/3} \sqrt[3]{381} - 146939}{1651}$$

$$\frac{390810 (13^{2/3} \sqrt[3]{381})}{1651} - 89$$

$$\frac{390810 \sqrt[3]{\frac{3}{13}} - 89 \times 127^{2/3}}{127^{2/3}}$$

Minimal polynomial:

$$209677x^3 + 55983759x^2 + 4982554551x - 179067965689537987$$

$$[(13^7 - 3 \cdot 13^4 \cdot 21 + 13(3 \cdot 21^2 - 1))^3 / ((6 \cdot 13^2 \cdot (3 \cdot 13^2 + 1)))^2]^{1/4} + 47 + 11$$

Input:

$$\sqrt[4]{\frac{(13^7 - 3 \times 13^4 \times 21 + 13(3 \times 21^2 - 1))^3}{(6 \times 13^2 (3 \times 13^2 + 1))^2}} + 47 + 11$$

Result:

$$58 + \frac{3 \times 130270^{3/4}}{\sqrt[4]{13} \sqrt{127}}$$

Decimal approximation:

1019.317539098078624297925895746268266237750599677509359363...

1019.317539... result practically equal to the rest mass of Phi meson 1019.445

Alternate forms:

$$\frac{95\,758 + 3\sqrt{127} \, 1693510^{3/4}}{1651}$$

$$\frac{58\sqrt[4]{13} \sqrt{127} + 3 \times 130\,270^{3/4}}{\sqrt[4]{13} \sqrt{127}}$$

Minimal polynomial:

$$209\,677x^4 - 48\,645\,064x^3 + 4\,232\,120\,568x^2 - 163\,641\,995\,296x - 179\,065\,740\,696\,391\,208$$

From page 197

Handwritten mathematical derivation on aged paper:

$$y = e^{-\frac{2\pi}{\sqrt{3}}} \cdot \frac{1 + \frac{1-x}{3^2} + \dots}{1 + \frac{1-x}{3^2}x + \dots}$$

then

$$1 + 240 \left(\frac{1^3 y}{1-y} + \frac{2^3 y^2}{1-y^2} + \frac{3^3 y^3}{1-y^3} + \dots \right)$$

$$= \left\{ 1 + \frac{1-x}{3^2}x + \dots \right\}^4 (1 + 8x)$$

$$1 - 504 \left(\frac{1^5 y}{1-y} + \frac{2^5 y^2}{1-y^2} + \frac{3^5 y^3}{1-y^3} + \dots \right)$$

$$= \left\{ 1 + \frac{1-x}{3^2}x + \dots \right\}^6 (1 - 20x - 8x^2)$$

For $x = 2$, we obtain:

$$1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right)$$

Input interpretation:

$$1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right)$$

Result:

98.58439980392358353020126958645181230048708302484313015965...
 98.5843998039...

Note that: $98.5843998039 - 7 = 91.5843998039$ (Z boson)

And:

$$1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right) + 34 + 5$$

Input interpretation:

$$1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right) + 34 + 5$$

Result:

137.5843998039235835302012695864518123004870830248431301596...
 137.584399803...

This result is very near to the inverse of fine-structure constant 137,035

$$1 / \left(\frac{1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right) + 34 + 5}{1} \right)$$

Input interpretation:

$$\frac{1}{1 + 240 \left(\frac{0.141802}{1 - 0.141802} + \frac{8 \times 0.141802^2}{1 - 0.141802^2} + \frac{27 \times 0.141802^3}{1 - 0.141802^3} \right) + 34 + 5}$$

Result:

0.007268265889338729750838124283395111427326565914859944473...
 0.007268265...

This result is very near to the fine-structure constant

$$\exp[(-2\pi i)/\sqrt{3} * ((1+2/9(1-2)))/((1+2/9(2)))]$$

Input:

$$\exp\left(\frac{-2\pi}{\sqrt{3}} \times \frac{1 + \frac{2}{9}(1-2)}{1 + \frac{2}{9} \times 2}\right)$$

Exact result:

$$e^{-\frac{14\pi}{13\sqrt{3}}}$$

Decimal approximation:

0.141802165675737662311925226480247088194102889933489455592...

0.1418021656...

Property:

$e^{-\frac{14\pi}{13\sqrt{3}}}$ is a transcendental number

Series representations:

$$\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right) = e^{-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k}}}$$

$$\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right) = \exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \binom{-1}{k}}{k!}}\right)$$

$$\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right) = \exp\left(-\frac{28\pi\sqrt{\pi}}{13 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right)$$

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \text{ for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

$\text{Re}(z)$ is the real part of z

$\arg(z)$ is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

From the ratio between the two results, we obtain:

$$\frac{[1+240(((0.141802/(1-0.141802)+(8*0.141802^2)/(1-0.141802^2)+(27*0.141802^3)/(1-0.141802^3))))] / \exp[(-2\pi)/\sqrt{3}*((1+2/9(1-2)))/((1+2/9(2)))] + 34 - (89)*1/10^2-3*1/10$$

Input interpretation:

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{-2\pi}{\sqrt{3}} \times \frac{1+\frac{2}{9}(1-2)}{1+\frac{2}{9} \times 2}\right)} + 34 - 89 \times \frac{1}{10^2} - 3 \times \frac{1}{10}$$

Result:

728.035...

728.035... \approx 728 (Ramanujan taxicab number)

Series representations:

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1+\frac{2(1-2)}{9}\right)(-2\pi)}{\left(1+\frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 34 - \frac{89}{10^2} - \frac{3}{10} =$$

$$\frac{3281}{100} + \frac{98.5844}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{2} \binom{1}{k}}\right)}$$

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1+\frac{2(1-2)}{9}\right)(-2\pi)}{\left(1+\frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 34 - \frac{89}{10^2} - \frac{3}{10} =$$

$$\frac{3281}{100} + \frac{98.5844}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)}$$

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 34 - \frac{89}{10^2} - \frac{3}{10} =$$

$$\frac{3281}{100} + \frac{98.5844}{\exp\left(-\frac{28\pi\sqrt{\pi}}{13 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right)}$$

$$[1+240((((0.141802/(1-0.141802)+(8*0.141802^2)/(1-0.141802^2)+(27*0.141802^3)/(1-0.141802^3)))))] / \exp[(-2Pi)/sqrt3*((1+2/9(1-2)))/((1+2/9(2)))] + 89 - 2$$

Input interpretation:

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{-2\pi}{\sqrt{3}} \times \frac{1 + \frac{2}{9}(1-2)}{1 + \frac{2 \times 2}{9}}\right)} + 89 - 2$$

Result:

782.225...

782.225... result practically equal to the rest mass of Omega meson 782.65

Series representations:

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 89 - 2 =$$

$$87 + \frac{98.5844}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1}{k}}\right)}$$

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 89 - 2 =$$

$$87 + \frac{98.5844}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}\right)}$$

$$\frac{1 + 240 \left(\frac{0.141802}{1-0.141802} + \frac{8 \times 0.141802^2}{1-0.141802^2} + \frac{27 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right)} + 89 - 2 =$$

$$87 + \frac{98.5844}{\exp\left(-\frac{28\pi\sqrt{\pi}}{13 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right)}$$

And:

$$1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)$$

Input interpretation:

$$1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)$$

Result:

-763.436832032744557186622909007071485985957540611188681767...
-763.436832...

And again:

$$-1/(2e) \left(\frac{1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{2e} - 3 \right)$$

Input interpretation:

$$-\frac{1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{2e} - 3$$

Result:

137.426...

137.426...

This result is very near to the inverse of fine-structure constant 137,035

Alternative representation:

$$\frac{(1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))(-1)}{2e} - 3 =$$

$$\frac{(1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))(-1)}{2 \exp(z)} - 3 \text{ for } z = 1$$

Series representations:

$$\frac{(1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))(-1)}{2e} - 3 = -3 + 381.718 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}$$

$$\frac{(1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))(-1)}{2e} - 3 = -3 + \frac{381.718}{\sum_{k=0}^{\infty} \frac{1}{k!}}$$

$$\frac{(1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))(-1)}{2e} - 3 = -3 + \frac{763.437}{\sum_{k=0}^{\infty} \frac{1+k}{k!}}$$

1/(((((-1/(2e))(((1-504((((0.141802/(1-0.141802)+(32*0.141802^2)/(1-0.141802^2)+(243*0.141802^3)/(1-0.141802^3))))))))-3))))))

Input interpretation:

$$\frac{1}{\frac{1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{2e} - 3}$$

Result:

0.00727662...

0.00727662...

This result is very near to the fine-structure constant

Alternative representation:

$$\frac{1}{\frac{(1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))^{(-1)}}{2e} - 3} = \frac{1}{\frac{(1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))^{(-1)}}{2 \exp(z)} - 3} \quad \text{for } z = 1$$

Series representations:

$$\frac{1}{\frac{(1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))^{(-1)}}{2e} - 3} = \frac{1}{-3 + 381.718 \sum_{k=0}^{\infty} \frac{(-1)^k}{k!}}$$

$$\frac{1}{\frac{(1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))^{(-1)}}{2e} - 3} = \frac{1}{-3 + \frac{381.718}{\sum_{k=0}^{\infty} \frac{1}{k!}}}$$

$$\frac{1}{\frac{(1-504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right))^{(-1)}}{2e} - 3} = \frac{1}{-3 + \frac{763.437}{\sum_{k=0}^{\infty} \frac{1+k}{k!}}}$$

From the ratio, as previously, we obtain:

$$\frac{(((1-504((((0.141802/(1-0.141802)+(32*0.141802^2)/(1-0.141802^2)+(243*0.141802^3)/(1-0.141802^3)))))))) / \exp[(-2\pi)/\sqrt{3}*((1+2/9(1-2)))/((1+2/9(2)))] - 29 - 3$$

Input interpretation:

$$\frac{1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{-2\pi}{\sqrt{3}} \times \frac{1 + \frac{2}{9}(1-2)}{1 + \frac{2}{9} \times 2}\right)} - 29 - 3$$

Result:

-5415.82...

-5415.82... result practically equal to the rest mass of strange B meson 5415.4 with minus sign

Series representations:

$$\frac{1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(\frac{1 + \frac{2}{9}(1-2)}{1 + \frac{2}{9} \times 2}\right)(-2\pi)}{\sqrt{3}}\right)} - 29 - 3 =$$

$$-32 - \frac{763.437}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}\right)}$$

$$\frac{1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right)}{\exp\left(\frac{\left(\frac{1 + \frac{2}{9}(1-2)}{1 + \frac{2}{9} \times 2}\right)(-2\pi)}{\sqrt{3}}\right)} - 29 - 3 =$$

$$-32 - \frac{763.437}{\exp\left(-\frac{14\pi}{13\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(\frac{-1}{2}\right)^k \left(\frac{-1}{2}\right)_k}{k!}}\right)}$$

$$1 - 504 \left(\frac{0.141802}{1-0.141802} + \frac{32 \times 0.141802^2}{1-0.141802^2} + \frac{243 \times 0.141802^3}{1-0.141802^3} \right) - 29 - 3 =$$

$$\frac{\exp\left(\frac{\left(1 + \frac{2(1-2)}{9}\right)(-2\pi)}{\left(1 + \frac{2 \times 2}{9}\right)\sqrt{3}}\right)}{763.437}$$

$$- 32 - \frac{\exp\left(-\frac{28\pi\sqrt{\pi}}{13 \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}\right)}{}$$

Now, we have that (page 197):

Handwritten mathematical derivation on aged paper. The text reads: "If $y = e^{-\pi\sqrt{2}} \cdot \frac{1 + \frac{1.3}{4}(1-x) + 2x}{1 + \frac{1.3}{4}x + 4x^2}$ then $1 + 240 \left(\frac{1^4 y}{1-y} + \frac{2^4 y^2}{1-y^2} + \dots \right) = \left(1 + \frac{1.3}{4}x + 4x^2\right)^4 (1+3x)$, $1 - 504 \left(\frac{1^5 y}{1-y} + \frac{2^5 y^2}{1-y^2} + \dots \right) = \left(1 + \frac{1.3}{4}x + 4x^2\right)^6 (1-9x)$." The word "then" is written above the second equation.

For $x = 2$, we obtain:

$$e^{-(\pi \cdot \sqrt{2} \cdot (1 + 3/16(1-2)) / (1 + 3/16(2)))}$$

Input:

$$\exp\left(-\pi \left(\sqrt{2} \times \frac{1 + \frac{3}{16}(1-2)}{1 + \frac{3}{16} \times 2}\right)\right)$$

Exact result:

$$e^{-(13\pi)/(11\sqrt{2})}$$

Decimal approximation:

0.072415137641250910353698100217032990350306730559368789458...

0.07241513...

Property:

$e^{-(13\pi)/(11\sqrt{2})}$ is a transcendental number

Series representations:

$$e^{-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16}\right)}{1 + \frac{3 \times 2}{16}}} = \exp\left(-\frac{13}{22} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!}\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$e^{-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16}\right)}{1 + \frac{3 \times 2}{16}}} = \exp\left(-\frac{13}{22} \pi \exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$e^{-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16}\right)}{1 + \frac{3 \times 2}{16}}} = \exp\left(-\frac{13}{22} \pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2 (1 + \lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2 - z_0)^k z_0^{-k}}{k!}\right)$$

Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \text{ for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

$$1+240((((0.0724151/(1-0.0724151)+(8*0.0724151^2)/(1-0.0724151^2))))))$$

Input interpretation:

$$1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right)$$

Result:

29.85787806397533746306325940412262953738491441115058873744...
29.857878...

And:

$$1/\text{golden ratio}((((1+240((((0.0724151/(1-0.0724151)+(8*0.0724151^2)/(1-0.0724151^2))))))))))-(7/\text{sqrt}2)$$

Input interpretation:

$$\frac{1}{\phi} \left(1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - \frac{7}{\sqrt{2}}$$

Result:

13.5034...

13.5034...

In atomic physics, **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

Series representations:

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\phi} - \frac{7}{\sqrt{2}} = \frac{29.8579}{\phi} - \frac{7}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\phi} - \frac{7}{\sqrt{2}} = \frac{29.8579}{\phi} - \frac{7}{\exp\left(i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\phi} - \frac{7}{\sqrt{2}} = \frac{29.8579}{\phi} - \frac{7 \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} z_0^{1/2 (-1 - \lfloor \arg(2-z_0)/(2\pi) \rfloor)}}{\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}}$$

And:

$5(((1+240((((0.0724151/(1-0.0724151)+(8*0.0724151^2)/(1-0.0724151^2))))))))-11-1/\text{golden ratio}$

Input interpretation:

$$5 \left(1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - 11 - \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

137.671...

137.671...

This result is very near to the inverse of fine-structure constant 137,035

Alternative representations:

$$5 \left(1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - 11 - \frac{1}{\phi} =$$

$$-11 + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - \frac{1}{2 \sin(54^\circ)}$$

$$5 \left(1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - 11 - \frac{1}{\phi} =$$

$$-11 - \frac{1}{2 \cos(216^\circ)} + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right)$$

$$5 \left(1 + 240 \left(\frac{0.0724151}{1 - 0.0724151} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - 11 - \frac{1}{\phi} =$$

$$-11 + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1 - 0.0724151^2} \right) \right) - \frac{1}{2 \sin(666^\circ)}$$

$$1/((((5(((1+240(((0.0724151/(1-0.0724151)+(8*0.0724151^2)/(1-0.0724151^2)))))))-11-1/\text{golden ratio}))))))$$

Input interpretation:

$$\frac{1}{5 \left(1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - 11 - \frac{1}{\phi}}$$

ϕ is the golden ratio

Result:

0.00726368...

0.00726368...

This result is very near to the fine-structure constant

Alternative representations:

$$\frac{1}{5 \left(1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - 11 - \frac{1}{\phi}} = \frac{-11 + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - \frac{1}{2 \sin(54^\circ)}}{1}$$

$$\frac{1}{5 \left(1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - 11 - \frac{1}{\phi}} = \frac{-11 - \frac{1}{2 \cos(216^\circ)} + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right)}{1}$$

$$\frac{1}{5 \left(1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - 11 - \frac{1}{\phi}} = \frac{-11 + 5 \left(1 + 240 \left(\frac{0.0724151}{0.927585} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right) \right) - \frac{1}{2 \sin(666^\circ)}}{1}$$

$$1-504((((0.0724151/(1-0.0724151)+(32*0.0724151^2)/(1-0.0724151^2))))))$$

Input interpretation:

$$1-504\left(\frac{0.0724151}{1-0.0724151} + \frac{32 \times 0.0724151^2}{1-0.0724151^2}\right)$$

Result:

-123.366704417840306398250998061305278708446795115827880560...
-123.3667044...

Note that: $-123.3667044 + 29.85787 + 1.61803398 = -91.89079242$ (Z boson with minus sign)

$$1-504((((0.0724151/(1-0.0724151)+(32*0.0724151^2)/(1-0.0724151^2))))))-2$$

Input interpretation:

$$1-504\left(\frac{0.0724151}{1-0.0724151} + \frac{32 \times 0.0724151^2}{1-0.0724151^2}\right) - 2$$

Result:

-125.366704417840306398250998061305278708446795115827880560...
-125.3667044... result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV

$$1-504((((0.0724151/(1-0.0724151)+(32*0.0724151^2)/(1-0.0724151^2))))))-18+2$$

Input interpretation:

$$1-504\left(\frac{0.0724151}{1-0.0724151} + \frac{32 \times 0.0724151^2}{1-0.0724151^2}\right) - 18 + 2$$

Result:

-139.366704417840306398250998061305278708446795115827880560...
-139.3667044... result practically equal to the rest mass of Pion meson 139.57 MeV

$$\frac{(((1+240((((0.0724151/(1-0.0724151)+(8*0.0724151^2)/(1-0.0724151^2)))))))) / \exp(((\text{-Pi}*\text{sqrt}2*(1+3/16(1-2)))/(1+3/16(2)))) + 89 - 8$$

Input interpretation:

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\exp \left(-\pi \left(\sqrt{2} \times \frac{1 + \frac{3}{16}(1-2)}{1 + \frac{3}{16} \times 2} \right) \right)} + 89 - 8$$

Result:

493.315...

493.315... result practically equal to the rest mass of Kaon meson 493.677

Series representations:

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\exp \left(-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16} \right)}{1 + \frac{3 \times 2}{16}} \right)} + 89 - 8 =$$

$$81 + \frac{29.8579}{\exp \left(-\frac{13}{22} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (2-z_0)^k z_0^{-k}}{k!} \right)} \text{ for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\exp \left(-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16} \right)}{1 + \frac{3 \times 2}{16}} \right)} + 89 - 8 =$$

$$81 + \frac{29.8579}{\exp \left(-\frac{13}{22} \pi \exp \left(i \pi \left[\frac{\text{arg}(2-x)}{2\pi} \right] \right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2} \right)_k}{k!} \right)}$$

for (x ∈ ℝ and x < 0)

$$\frac{1 + 240 \left(\frac{0.0724151}{1-0.0724151} + \frac{8 \times 0.0724151^2}{1-0.0724151^2} \right)}{\exp \left(-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16} \right)}{1 + \frac{3 \times 2}{16}} \right)} + 89 - 8 =$$

$$81 + \frac{29.8579}{\exp \left(-\frac{13}{22} \pi \left(\frac{1}{z_0} \right)^{1/2 [\text{arg}(2-z_0)/(2\pi)]} z_0^{1/2 (1+[\text{arg}(2-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (2-z_0)^k z_0^{-k}}{k!} \right)}$$

$$-\left[\frac{1-504\left(\frac{0.0724151}{1-0.0724151}+\frac{32 \times 0.0724151^2}{1-0.0724151^2}\right)}{\exp\left(-\pi \sqrt{2} \times \frac{1+\frac{3}{16}(1-2)}{1+\frac{3}{16} \times 2}\right)}-34+8+\frac{3}{5}\right] /$$

Input interpretation:

$$-\left[\frac{1-504\left(\frac{0.0724151}{1-0.0724151}+\frac{32 \times 0.0724151^2}{1-0.0724151^2}\right)}{\exp\left(-\pi \left(\sqrt{2} \times \frac{1+\frac{3}{16}(1-2)}{1+\frac{3}{16} \times 2}\right)\right)}-34+8+\frac{3}{5}\right]$$

Result:

1729.00...

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Series representations:

$$-\left[\frac{1-504\left(\frac{0.0724151}{1-0.0724151}+\frac{32 \times 0.0724151^2}{1-0.0724151^2}\right)}{\exp\left(-\frac{\pi \sqrt{2} \left(1+\frac{3(1-2)}{16}\right)}{1+\frac{3 \times 2}{16}}\right)}-34+8+\frac{3}{5}\right] =$$

$$\frac{127}{5} + \frac{123.367}{\exp\left(-\frac{13}{22} \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$-\left[\frac{1-504\left(\frac{0.0724151}{1-0.0724151}+\frac{32 \times 0.0724151^2}{1-0.0724151^2}\right)}{\exp\left(-\frac{\pi \sqrt{2} \left(1+\frac{3(1-2)}{16}\right)}{1+\frac{3 \times 2}{16}}\right)}-34+8+\frac{3}{5}\right] =$$

$$\frac{127}{5} + \frac{123.367}{\exp\left(-\frac{13}{22} \pi \exp\left(i \pi \left[\frac{\text{arg}(2-x)}{2 \pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$- \left(\frac{1 - 504 \left(\frac{0.0724151}{1-0.0724151} + \frac{32 \cdot 0.0724151^2}{1-0.0724151^2} \right)}{\exp \left(-\frac{\pi \sqrt{2} \left(1 + \frac{3(1-2)}{16} \right)}{1 + \frac{3 \times 2}{16}} \right)} - 34 + 8 + \frac{3}{5} \right) =$$

$$\frac{127}{5} + \frac{123.367}{\exp \left(-\frac{13}{22} \pi \left(\frac{1}{z_0} \right)^{1/2} [\arg(2-z_0)/(2\pi)] \right) z_0^{1/2 (1+[\arg(2-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2} \right)_k (2-z_0)^k z_0^{-k}}{k!}$$

From:

Ramanujan's "Lost" Notebook VII: The Sixth Order Mock Theta Functions
GEORGE E. ANDREWS AND DEAN HICKERSON - ADVANCES IN
 MATHEMATICS 89, 60-105 (1991)

We have that:

For $x \neq 0$ and $|q| < 1$,

$$j(x, q) = (x, q/x, q; q)_{\infty} = \sum_n (-1)^n q^{\binom{n}{2}} x^n; \quad (0.4)$$

THEOREM 1.0. *If $|q| < 1$ and ω is a primitive cube root of unity, then*

$$j(\omega, q) = (1 - \omega) J_3, \quad (1.6)$$

$$j(-\omega, q) = (1 + \omega) \frac{J_{1,2} J_6}{J_3}, \quad (1.7)$$

and

$$j(q, \omega q^2) j(q, \omega^2 q^2) = \frac{J_6^3 J_{1,6}}{J_2 J_{3,18}}. \quad (1.8)$$

We have that:

$$\begin{aligned}
 & (\omega x, \omega^2 x, \omega q^6 x^{-1}, \omega^2 q^6 x^{-1}; q^6)_\infty \\
 &= \prod_{i \geq 0} (1 - \omega q^{6i} x)(1 - \omega^2 q^{6i} x)(1 - \omega q^{6i+6} x^{-1})(1 - \omega^2 q^{6i+6} x^{-1}) \\
 &= \prod_{i \geq 0} \frac{(1 - q^{18i} x^3)(1 - q^{18i+18} x^{-3})}{(1 - q^{6i} x)(1 - q^{6i+6} x^{-1})} = \frac{J_6 j(x^3, q^{18})}{J_{18} j(x, q^6)}.
 \end{aligned}$$

For $x = 2$, $q = 0.5$

$$\frac{(((1-0.5^{18i}) \cdot 2^3)(1-0.5^{(18i+18)} \cdot 2^{-3})))}{(((1-0.5^{6i}) \cdot 2)(1-0.5^{(6i+6)} \cdot 2^{-1}))}$$

Input:

$$\frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3}\right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6}\right)}$$

i is the imaginary unit

Result:

$$\begin{aligned}
 & -1.82071\dots - \\
 & 1.88047\dots i
 \end{aligned}$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 2.61747 \text{ (radius), } \theta = -134.075^\circ \text{ (angle)}$$

$$2.61747$$

$$\text{sqrt}(\frac{(((1-0.5^{18i}) \cdot 2^3)(1-0.5^{(18i+18)} \cdot 2^{-3})))}{(((1-0.5^{6i}) \cdot 2)(1-0.5^{(6i+6)} \cdot 2^{-1}))})$$

Input:

$$\sqrt{\frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3}\right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6}\right)}}$$

i is the imaginary unit

Result:

$$\begin{aligned}
 & 0.631172\dots - \\
 & 1.48966\dots i
 \end{aligned}$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.61786$ (radius), $\theta = -67.0375^\circ$ (angle)

1.61786 result that is a very good approximation to the value of the golden ratio
1,618033988749...

$$1 / \sqrt{\left(\frac{\left((1 - 0.5^{18i}) \times 2^3 \right) \left(1 - \frac{0.5^{18i+18}}{2^3} \right)}{\left((1 - 0.5^{6i}) \times 2 \right) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} \right)}$$

Input:

$$\frac{1}{\sqrt{\frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}}$$

i is the imaginary unit

Result:

0.241138... +
0.569123... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 0.6181$ (radius), $\theta = 67.0375^\circ$ (angle)

0.6181

$$1/10^{27} \left(\frac{\left((128)/10^3 + \sqrt{\left(\frac{\left((1 - 0.5^{18i}) \times 2^3 \right) \left(1 - \frac{0.5^{18i+18}}{2^3} \right)}{\left((1 - 0.5^{6i}) \times 2 \right) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} \right)}{\left((1 - 0.5^{6i}) \times 2 \right) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} \right) \right)$$

Input:

$$\frac{1}{10^{27}} \left(\frac{128}{10^3} + \sqrt{\frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}} \right)$$

i is the imaginary unit

Result:

$7.59172... \times 10^{-28}$ -
 $1.48966... \times 10^{-27} i$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.67196 \times 10^{-27}$ (radius), $\theta = -62.9954^\circ$ (angle)

1.67196×10^{-27} result practically equal to the proton mass

We have also:

$1/\text{golden ratio} - 5 * ((((((1 - 0.5^{18i}) * 2^3)(1 - 0.5^{18i+18}) * 2^{-3}))) / (((1 - 0.5^{6i}) * 2)(1 - 0.5^{6i+6}) * 2^{-1}))))))$

Input:

$$\frac{1}{\phi} - 5 \times \frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3}\right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6}\right)}$$

ϕ is the golden ratio

i is the imaginary unit

Result:

9.72161... +
9.40233... i

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 13.5246$ (radius), $\theta = 44.0435^\circ$ (angle)

13.5246

In atomic physics, **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_\infty = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

Alternative representations:

$$\frac{1}{\phi} - \frac{5 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} = \frac{5(1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)} + \frac{1}{2 \sin(54^\circ)}$$

$$\frac{1}{\phi} - \frac{5 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} = \frac{1}{2 \cos(216^\circ)} - \frac{5(1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)}$$

$$\frac{1}{\phi} - \frac{5 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)} = \frac{5(1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)} + \frac{1}{2 \sin(666^\circ)}$$

And:

$$8 + \text{golden ratio} + 55 \left(\frac{(((((1 - 0.5^{18i}) * 2^3) (1 - 0.5^{(18i+18)} * 2^{-3}))))}{(((1 - 0.5^{(6i)} * 2) (1 - 0.5^{(6i+6)} * 2^{-1}))))} \right)$$

Input:

$$8 + \phi + 55 \times \frac{(1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}$$

ϕ is the golden ratio

i is the imaginary unit

Result:

$$-90.5213... - 103.426... i$$

(using the principal branch of the logarithm for complex exponentiation)

Result:

$$-0.00479177... + 0.00547487... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 0.00727567 \text{ (radius), } \theta = 131.193^\circ \text{ (angle)}$$

0.00727567

This result is very near to the fine-structure constant

Alternative representations:

$$\frac{1}{8 + \phi + \frac{55 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}} = \frac{1}{8 + \frac{55 (1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)} + 2 \sin(54^\circ)}$$

$$\frac{1}{8 + \phi + \frac{55 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}} = \frac{1}{8 - 2 \cos(216^\circ) + \frac{55 (1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)}}$$

$$\frac{1}{8 + \phi + \frac{55 \left((1 - 0.5^{18i} \times 2^3) \left(1 - \frac{0.5^{18i+18}}{2^3} \right) \right)}{(1 - 0.5^{6i} \times 2) \left(1 - \frac{1}{2} \times 0.5^{6i+6} \right)}} = \frac{1}{8 + \frac{55 (1 - 8 \times 0.5^{18i}) \left(1 - \frac{1}{8} \times 0.5^{18+18i} \right)}{(1 - 2 \times 0.5^{6i}) \left(1 - \frac{1}{2} \times 0.5^{6+6i} \right)} - 2 \sin(666^\circ)}$$

Now, we have that:

$$C_{n,k} = \frac{D_{n,k}}{P_k} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

$$\binom{5}{2} = 5! / (2!(5-2)!)$$

Input:

$$\frac{5!}{2!(5-2)!}$$

n! is the factorial function

Result:

10

10

From

$$\begin{aligned} & \sum_{k=0}^{n-1} \sum_s (-1)^{k+ns} q^{\binom{k+ns}{2}} x^{k+ns} \\ &= \sum_{k=0}^{n-1} (-1)^k q^{\binom{k}{2}} x^k \sum_s (-1)^{ns} q^{n^2 \binom{s}{2} + [\binom{n}{2} + kn] s} x^{ns} \\ &= \sum_{k=0}^{n-1} (-1)^k q^{\binom{k}{2}} x^k j((-1)^{n+1} q^{\binom{n}{2} + kn} x^n, q^{n^2}). \quad \blacksquare \end{aligned}$$

for k = 2, x = 2, n = 3 and s = 1, we obtain:

$$\sum_{k=0}^{n-1} \sum_s (-1)^{k+ns} q^{\binom{k+ns}{2}} x^{k+ns}$$

$$(-1)^5 * 0.5^{10} * 2^5$$

Input:

$$(-1)^5 \times 0.5^{10} \times 2^5$$

Result:

-0.03125

-0.03125

Rational form:

$$-\frac{1}{32}$$

And:

$$-2/(((-1)^5 * 0.5^{10} * 2^5))$$

Input:

$$\frac{2}{(-1)^5 \times 0.5^{10} \times 2^5}$$

Result:

64

64

From which:

$$-(3/\sqrt{2})^2 * 1/(((-1)^5 * 0.5^{10} * 2^5)) - 7 + 1/\text{golden ratio}$$

Input:

$$-\left(\frac{3}{\sqrt{2}}\right)^2 \times \frac{1}{(-1)^5 \times 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi}$$

ϕ is the golden ratio

Result:

137.618...

137.618...

This result is very near to the inverse of fine-structure constant 137,035

Series representations:

$$-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 \cdot 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi} = -7 + \frac{1}{\phi} + \frac{288}{\sqrt{z_0}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right)^2}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi} = -7 + \frac{1}{\phi} + \frac{288}{\exp^2\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right) \sqrt{x}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^2} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi} = -7 + \frac{1}{\phi} + \frac{288 \left(\frac{1}{z_0}\right)^{-[\arg(2-z_0)/(2\pi)]} z_0^{-1-[\arg(2-z_0)/(2\pi)]}}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)^2}$$

$$1/\left(\left(\left(-\left(\frac{3}{\sqrt{2}}\right)^2\right)^{2*1}/\left(\left(-1\right)^5 * 0.5^{10} * 2^5\right)\right)-7+1/\text{golden ratio}\right)$$

Input:

$$\frac{1}{-\left(\frac{3}{\sqrt{2}}\right)^2 \times \frac{1}{(-1)^5 \times 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi}}$$

ϕ is the golden ratio

Result:

0.00726649...

0.00726649...

This result is very near to the fine-structure constant

Series representations:

$$\frac{1}{-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi}} = \frac{1}{-7 + \frac{1}{\phi} + \frac{288}{\sqrt{z_0}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)^2}}$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

$$\frac{1}{-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi}} = \frac{1}{-7 + \frac{1}{\phi} + \frac{288}{\exp^2\left(i\pi\left[\frac{\arg(2-x)}{2\pi}\right]\right)\sqrt{x}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^2}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{1}{-\frac{\left(\frac{3}{\sqrt{2}}\right)^2}{(-1)^5 0.5^{10} \times 2^5} - 7 + \frac{1}{\phi}} = \frac{1}{-7 + \frac{1}{\phi} + \frac{288 \left(\frac{1}{z_0}\right)^{-\lceil \arg(2-z_0)/(2\pi) \rceil} z_0^{-1-\lceil \arg(2-z_0)/(2\pi) \rceil}}{\left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!}\right)^2}}$$

We have that:

$$\sum_{r=0}^{\infty} (-1)^r q^{\binom{r}{2}} x^r \sum_s (-1)^s q^{\binom{s}{2}} y^s = \sum_{r,s} (-1)^{r+s} q^{\binom{r}{2} + \binom{s}{2}} x^r y^s. \quad (1.17)$$

for $r = 2, x = 2, y = 3, n = 5$ and $s = 1,$
and

$$C_{n,k} = \frac{D_{n,k}}{P_k} = \frac{n!}{k!(n-k)!} = \binom{n}{k}$$

we obtain:

$$(-1)^3 * 0.5^{(((2!/(2!(2-2)!))+5*(1!/(2!(1-2)!))))} 2^2 * 3$$

Input:

$$(-1)^3 \times 0.5^{2!/(2!(2-2)!)+5 \times 1!/(2!(1-2)!)} (2^2 \times 3)$$

$n!$ is the factorial function

Result:

-6
-6

Alternative representations:

$$\left((-1)^3 0.5^{2!/(2!(2-2)!)+5 \times 1!/(2!(1-2)!)}\right) 2^2 \times 3 = -12 \times 0.5^{(5\Gamma(2))/(\Gamma(0)\Gamma(3))+\Gamma(3)/(\Gamma(1)\Gamma(3))}$$

$$\begin{aligned} &((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} 2^2 \times 3 = \\ &-12 \times 0.5^{(5 \times 0!! \times 1!!)/((-2)!! (-1)!! 1!! \times 2!!)+(1!! \times 2!!)/((-1)!! 0!! \times 1!! \times 2!!)} \end{aligned}$$

$$((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} 2^2 \times 3 = -12 \times 0.5^{(5(1)_1)/((1)_{-1} (1)_2)+(1)_2/((1)_0 (1)_2)}$$

Series representation:

$$((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} 2^2 \times 3 = -12 \times$$

0.:

$$5 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) / \left(\left(\sum_{k=0}^{\infty} \frac{(-1-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(2-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) + 1 / \left(\sum_{k=0}^{\infty} \frac{(-n_0)^k \Gamma(k)(1+n_0)}{k!} \right)$$

for $(n_0 \geq 0$ or $n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow -1$
and $n_0 \rightarrow 0$ and $n_0 \rightarrow 1$ and $n_0 \rightarrow 2$)

Integral representation:

$$((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} 2^2 \times 3 = -12 \times$$

0.:

$$5 \frac{1}{\left(\int_1^{\infty} e^{-t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+k)k!} \right) + \left(5 \int_1^{\infty} e^{-t} t dt \right) / \left(\left(\int_1^{\infty} \frac{e^{-t}}{t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right) \right) + \left(5 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2+k)k!} \right) / \left(\left(\int_1^{\infty} \frac{e^{-t}}{t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right) \right)}$$

-golden ratio^2 ((((-1)^3 * 0.5^(((2!/(2!(2-2)!))+5*(1!/(2!(1-2)!)))) 2^2*3)))-2

Input:

$$-\phi^2 \left((-1)^3 \times 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2$$

n! is the factorial function

φ is the golden ratio

Result:

13.7082...

13.7082...

In atomic physics, **Rydberg unit of energy**, symbol Ry, corresponds to the energy of the photon whose wavenumber is the Rydberg constant, i.e. the ionization energy of the hydrogen atom.

$$1 \text{ Ry} \equiv hcR_{\infty} = \frac{m_e e^4}{8\epsilon_0^2 h^2} = 13.605\,693\,009(84) \text{ eV} \approx 2.179 \times 10^{-18} \text{ J.}$$

Alternative representations:

$$-\phi^2 (-1)^3 \left(0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2 =$$

$$-2 + 12 \times 0.5^{(5 \Gamma(2))/(\Gamma(0) \Gamma(3))+\Gamma(3)/(\Gamma(1) \Gamma(3))} \phi^2$$

$$-\phi^2 (-1)^3 \left(0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2 =$$

$$-2 + 12 \times 0.5^{(5 \times 0!! \times 1!!)/((-2)! (-1)! 1!! \times 2!!)+(1!! \times 2!!)/((-1)! 0!! \times 1!! \times 2!!)} \phi^2$$

$$-\phi^2 (-1)^3 \left(0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2 =$$

$$-2 + 12 \times 0.5^{(5 (1)_1)/((1)_{-1} (1)_2)+(1)_2/((1)_0 (1)_2)} \phi^2$$

Series representation:

$$-\phi^2 (-1)^3 \left(0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2 = 2 \left(-1 + 6 \times$$

$$0. \left(\frac{5 \sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}}{\left(\sum_{k=0}^{\infty} \frac{(-1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(2-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + 1 \right) / \left(\sum_{k=0}^{\infty} \frac{(-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \phi^2$$

for $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow -1$ and $n_0 \rightarrow 0$ and $n_0 \rightarrow 1$ and $n_0 \rightarrow 2$)

Integral representation:

$$-\phi^2 (-1)^3 \left(0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)} (2^2 \times 3) \right) - 2 = 2 \left(-1 + 6 \times$$

$$0. \left(\frac{5 \int_1^{\infty} e^{-t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+k)k!}}{\left(\int_1^{\infty} e^{-t} t dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{k k!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right)} + 1 \right) / \left(\int_1^{\infty} e^{-t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{k k!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right) \phi^2$$

$$8 * (((-golden \text{ ratio}^2 (((-1)^3 * 0.5^{((2!/(2!(2-2)!))+5*(1!/(2!(1-2)!)))))$$

$$2^{2*3})))))) + 12 - 1/golden \text{ ratio}$$

Input:

$$8 \left(-\phi^2 \left((-1)^3 \times 0.5^{2!/(2!(2-2)!)+5 \times 1!/(2!(1-2)!)} (2^2 \times 3) \right) \right) + 12 - \frac{1}{\phi}$$

$n!$ is the factorial function

ϕ is the golden ratio

Result:

137.048...

137.048...

This result is very near to the inverse of fine-structure constant 137,035

Alternative representations:

$$8(-1)\left(\phi^2\left((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3)\right)\right) + 12 - \frac{1}{\phi} =$$

$$12 - \frac{1}{\phi} + 96 \times 0.5^{(5 \Gamma(2))/(\Gamma(0) \Gamma(3))+\Gamma(3)/(\Gamma(1) \Gamma(3))} \phi^2$$

$$8(-1)\left(\phi^2\left((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3)\right)\right) + 12 - \frac{1}{\phi} =$$

$$12 - \frac{1}{\phi} + 96 \times 0.5^{(5 \times 0!! \times 1!!)/((-2)!!(-1)!!1!! \times 2!!)+(1!! \times 2!!)/((-1)!!0!! \times 1!! \times 2!!)} \phi^2$$

$$8(-1)\left(\phi^2\left((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3)\right)\right) + 12 - \frac{1}{\phi} =$$

$$12 - \frac{1}{\phi} + 96 \times 0.5^{(5(1)_1)/((1)_{-1}(1)_2)+(1)_2/((1)_0(1)_2)} \phi^2$$

Series representation:

$$8(-1)\left(\phi^2\left((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3)\right)\right) + 12 - \frac{1}{\phi} = \frac{1}{\phi} \left(-1 + 12\phi + 96 \times$$

$$0. \cdot \left(5 \left(\sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) / \left(\left(\sum_{k=0}^{\infty} \frac{(-1-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(2-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) + 1 / \left(\sum_{k=0}^{\infty} \frac{(-n_0)^k \Gamma(k)(1+n_0)}{k!} \right) \right) \phi^3$$

for $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow -1$ and $n_0 \rightarrow 0$ and $n_0 \rightarrow 1$ and $n_0 \rightarrow 2$)

Integral representation:

$$8(-1)\left(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))\right) + 12 - \frac{1}{\phi} = \frac{1}{\phi} \left(-1 + 12\phi + 96 \times \right.$$

0.:

$$\left. 5 \int_1^\infty \frac{e^{-t}}{t} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(1+k)k!} \right) + \left(5 \int_1^\infty e^{-t} t dt \right) / \left(\left(\int_1^\infty \frac{e^{-t}}{t} dt + \sum_{k=0}^\infty \frac{(-1)^k}{k k!} \right) \left(\int_1^\infty e^{-t} t^2 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(3+k)k!} \right) \right) + \left(5 \sum_{k=0}^\infty \frac{(-1)^k}{(2+k)k!} \right) / \left(\left(\int_1^\infty \frac{e^{-t}}{t} dt + \sum_{k=0}^\infty \frac{(-1)^k}{k k!} \right) \left(\int_1^\infty e^{-t} t^2 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(3+k)k!} \right) \right) \phi^3 \right)$$

$$1/((((8*(((\text{-golden ratio}^2 ((((-1)^3 * 0.5^{((2!/(2!(2-2)!))+5*(1!/(2!(1-2)!))))(2^2*3)))))))+12-1/\text{golden ratio}))))))$$

Input:

$$\frac{1}{8(-\phi^2((-1)^3 \times 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3)) + 12 - \frac{1}{\phi}}$$

n! is the factorial function

φ is the golden ratio

Result:

0.00729673...

0.00729673...

This result is very near to the fine-structure constant

Alternative representations:

$$\frac{1}{8(-1)\left(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))\right) + 12 - \frac{1}{\phi}} = \frac{1}{12 - \frac{1}{\phi} + 96 \times 0.5^{(5 \Gamma(2))/(\Gamma(0) \Gamma(3))+\Gamma(3)/(\Gamma(1) \Gamma(3))} \phi^2}$$

$$\frac{1}{8(-1)\left(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))\right) + 12 - \frac{1}{\phi}} = \frac{1}{12 - \frac{1}{\phi} + 96 \times 0.5^{(5 \times 0!! \times 1!!)/((-2)!!(-1)!! \times 2!!)+(1!! \times 2!!)/((-1)!! \times 1!! \times 2!!)} \phi^2}$$

$$\frac{1}{8(-1)(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))) + 12 - \frac{1}{\phi}} = \frac{1}{12 - \frac{1}{\phi} + 96 \times 0.5^{(5(1)_1)/((1)_{-1}(1)_2)+(1)_2/((1)_0(1)_2)} \phi^2}$$

Series representation:

$$\frac{1}{8(-1)(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))) + 12 - \frac{1}{\phi}} = \phi / \left(-1 + 12\phi + 96 \times \right.$$

$$\left. 0. \therefore \left(5 \sum_{k=0}^{\infty} \frac{(1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) / \left(\left(\sum_{k=0}^{\infty} \frac{(-1-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \sum_{k=0}^{\infty} \frac{(2-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) + 1 / \left(\sum_{k=0}^{\infty} \frac{(-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right) \right)$$

$$\left. \phi^3 \right)$$

for $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow -1$ and $n_0 \rightarrow 0$ and $n_0 \rightarrow 1$ and $n_0 \rightarrow 2$

Integral representation:

$$\frac{1}{8(-1)(\phi^2((-1)^3 0.5^{2!/(2!(2-2)!)+(5 \times 1!)/(2!(1-2)!)}(2^2 \times 3))) + 12 - \frac{1}{\phi}} = \phi / \left(-1 + 12\phi + 96 \times \right.$$

$$\left. 0. \therefore \left(5 \int_1^{\infty} e^{-t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+k)k!} \right) + \left(5 \int_1^{\infty} e^{-t} t dt \right) / \left(\left(\int_1^{\infty} \frac{e^{-t}}{t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{kk!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right) \right) + \left(5 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2+k)k!} \right) / \left(\left(\int_1^{\infty} \frac{e^{-t}}{t} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{kk!} \right) \left(\int_1^{\infty} e^{-t} t^2 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(3+k)k!} \right) \right) \right)$$

$$\left. \phi^3 \right)$$

Now, we have that:

THEOREM 2.3. *Let $a, b, c,$ and q be complex numbers with $a \neq 1, b \neq 0, c \neq 0, q \neq 0,$ and none of $a/b, a/c, qb,$ and qc of the form q^{-k} with $k \geq 0.$ For $n \geq 0,$ define*

We have:

$$j = 0 \text{ and } j = 1$$

$$j \geq 2.$$

For $j = 3$ and $n = 5$, we obtain:

$$A'_n(1, -1, -q^{-1}, q^2) = \frac{2(-1)^n q^{n^2+n-1}(1+q)}{1+q^{2n-1}} + q^{2n^2-n}(1-q^{2n}) \left[\frac{1-q}{1+q} + 2 \sum_{j=1}^{n-1} \frac{(-1)^j (1-q^2)}{q^{(j-1)^2} (1+q^{2j-1})(1+q^{2j+1})} \right].$$

$$(((2(-1)^5 * 0.5^{29} * (1+0.5)))) / (((1+0.5^9))) + 0.5^{45} * (1-0.5^{10}) * [(1-0.5)/(1+0.5) + 2 * (((-1)^3 * (1-0.5^2)) / ((0.5^4 * (1+0.5^5)(1+0.5^7)))]$$

Input:

$$\frac{2(-1)^5 \times 0.5^{29} (1+0.5)}{1+0.5^9} + 0.5^{45} (1-0.5^{10}) \left(\frac{1-0.5}{1+0.5} + 2 \times \frac{(-1)^3 (1-0.5^2)}{0.5^4 (1+0.5^5)(1+0.5^7)} \right)$$

Result:

$$-5.577689003577036611760292646743526397781859737309601... \times 10^{-9}$$

$$-5.577689003577... * 10^{-9}$$

From which, we obtain:

$$(((((-1/4096 * 1 / ((((((2(-1)^5 * 0.5^{29} * (1+0.5)))) / (((1+0.5^9)))) + 0.5^{45} * (1-0.5^{10}) * [(1-0.5)/(1+0.5) + 2 * (((-1)^3 * (1-0.5^2)) / ((0.5^4 * (1+0.5^5)(1+0.5^7))))])))])^{1/2} - 64 - 8$$

Input:

$$\sqrt{\frac{1}{\frac{2(-1)^5 \times 0.5^{29} (1+0.5)}{1+0.5^9} + 0.5^{45} (1-0.5^{10}) \left(\frac{1-0.5}{1+0.5} + 2 \times \frac{(-1)^3 (1-0.5^2)}{0.5^4 (1+0.5^5)(1+0.5^7)} \right)}} - 64 - 8$$

Result:

137.2150286628013351569251614075194737771599828547063597079...
137.2150286628...

This result is very near to the inverse of fine-structure constant 137,035

And:

$$1/((((((((((-1/4096*1/((((((2(-1)^5*0.5^29 * (1+0.5))))/(((1+0.5^9))))+0.5^45*(1-0.5^10)*[(1-0.5)/(1+0.5)+2*((-1)^3*(1-0.5^2))]/((0.5^4 * (1+0.5^5)(1+0.5^7))))))))))))))^(1/2 - 64-8))))))$$

Input:

$$\sqrt{\frac{1}{\frac{2(-1)^5 \times 0.5^{29} (1+0.5)}{1+0.5^9} + 0.5^{45} (1-0.5^{10}) \left(\frac{1-0.5}{1+0.5} + 2 \times \frac{(-1)^3 (1-0.5^2)}{0.5^4 (1+0.5^5)(1+0.5^7)} \right)}} - 64 - 8}$$

Result:

0.007287831440515506733181622962843694147290578023024028856...
0.00728783144...

This result is very near to the fine-structure constant

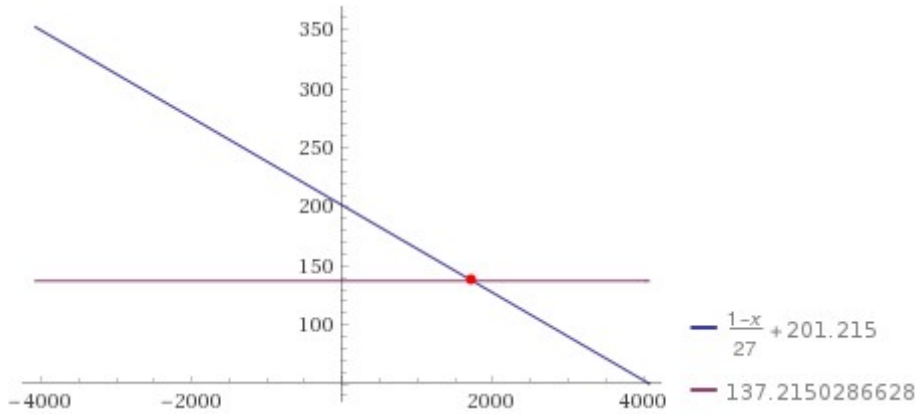
$$((((((-1/4096*1/((((((2(-1)^5*0.5^29 * (1+0.5))))/(((1+0.5^9))))+0.5^45*(1-0.5^10)*[(1-0.5)/(1+0.5)+2*((-1)^3*(1-0.5^2))]/((0.5^4 * (1+0.5^5)(1+0.5^7))))))))))))))^(1/2 - ((x-1)/27)-8 = 137.2150286628$$

Input interpretation:

$$\sqrt{\frac{1}{\frac{2(-1)^5 \times 0.5^{29} (1+0.5)}{1+0.5^9} + 0.5^{45} (1-0.5^{10}) \left(\frac{1-0.5}{1+0.5} + 2 \times \frac{(-1)^3 (1-0.5^2)}{0.5^4 (1+0.5^5)(1+0.5^7)} \right)}} - \frac{x-1}{27} - 8 = 137.2150286628$$

Result:

$$\frac{1-x}{27} + 201.215 = 137.2150286628$$

Plot:**Alternate forms:**

$$64.037 - \frac{x}{27} = 0$$

$$201.252 - 0.037037x = 137.2150286628$$

$$-0.037037(x - 5433.81) = 137.2150286628$$

Expanded form:

$$201.252 - \frac{x}{27} = 137.2150286628$$

Solution:

$$x \approx 1729.$$

1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Appendix

DILATON VALUE CALCULATIONS 0.989117352243

from:

Modular equations and approximations to π - Srinivasa Ramanujan
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

5. Since G_n and g_n can be expressed as roots of algebraical equations with rational coefficients, the same is true of G_n^{24} or g_n^{24} . So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \dots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \dots.$$

But we know that

$$\begin{aligned} 64e^{-\pi\sqrt{n}}g_n^{24} &= 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \dots, \\ 64g_n^{24} &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 64bg_n^{-24} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \\ 64a - 4096be^{-\pi\sqrt{n}} + \dots &= e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \dots, \end{aligned}$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \dots \quad (13)$$

Similarly, if

$$1 = aG_n^{-24} - bG_n^{-48} + \dots,$$

then

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \dots \quad (14)$$

From (13) and (14) we can find whether $e^{\pi\sqrt{n}}$ is very nearly an integer for given values of n , and ascertain also the number of 9's or 0's in the decimal part. But if G_n and g_n be simple quadratic surds we may work independently as follows. We have, for example,

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} (24 + 276e^{-\pi\sqrt{22}} + \dots), \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} (24 + 4372e^{-\pi\sqrt{22}} + \dots) = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Again

$$G_{37} = (6 + \sqrt{37})^{\frac{1}{4}},$$

$$\begin{aligned} 64G_{37}^{24} &= e^{\pi\sqrt{37}} (24 + 276e^{-\pi\sqrt{37}} + \dots), \\ 64G_{37}^{-24} &= 4096e^{-\pi\sqrt{37}} + \dots, \end{aligned}$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} (24 + 4372e^{-\pi\sqrt{37}} + \dots) = 64\{(6 + \sqrt{37})^6 + (6 - \sqrt{37})^6\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\dots$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5 + \sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} (24 + 4372e^{-\pi\sqrt{58}} + \dots) = 64 \left\{ \left(\frac{5 + \sqrt{29}}{2}\right)^{12} + \left(\frac{5 - \sqrt{29}}{2}\right)^{12} \right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24501257751.99999982\dots$$

From:

An Update on Brane Supersymmetry Breaking

J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Now, we have that:

From the following vacuum equations:

$$T e^{\gamma_E \phi} = - \frac{\beta_E^{(p)} h^2}{\gamma_E} e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

$$16 k' e^{-2C} = \frac{h^2 \left(p + 1 - \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}}{(7-p)}$$

$$(A')^2 = k e^{-2A} + \frac{h^2}{16(p+1)} \left(7 - p + \frac{2\beta_E^{(p)}}{\gamma_E} \right) e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

We have obtained, from the results almost equals of the equations, putting

$4096 e^{-\pi\sqrt{18}}$ instead of

$$e^{-2(8-p)C + 2\beta_E^{(p)} \phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p , C , β_E and ϕ correspond to the exponents of e (i.e. of exp).

Thence we obtain for $p = 5$ and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

Therefore with respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2 , while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

$\phi = -\pi\sqrt{18} + 6C$, for $C = 1$, we obtain:

$$\exp(-\pi\sqrt{18})$$

Input:

$$\exp(-\pi\sqrt{18})$$

Exact result:

$$e^{-3\sqrt{2}\pi}$$

Decimal approximation:

$$1.6272016226072509292942156739117979541838581136954016... \times 10^{-6}$$

$$1.6272016... * 10^{-6}$$

Now:

$$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$$

$$e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

$$\frac{1}{4096} e^{-6C+\phi} = 1.6272016... * 10^{-6}$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}} = 1.6272016... * 10^{-6}$$

$$\ln(e^{-\pi\sqrt{18}}) = -13.328648814475 = -\pi\sqrt{18}$$

$$(1.6272016 * 10^{-6}) * 1 / (0.000244140625)$$

Input interpretation:

$$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$$

Result:

$$0.0066650177536$$

$$0.006665017...$$

$$0.000244140625 e^{-6C+\phi} = e^{-\pi\sqrt{18}}$$

Dividing both sides by 0.000244140625, we obtain:

$$\frac{0.000244140625}{0.000244140625} e^{-6C+\phi} = \frac{1}{0.000244140625} e^{-\pi\sqrt{18}}$$

$$e^{-6C+\phi} = 0.0066650177536$$

$$(((\exp(-\pi\sqrt{18})))) * 1 / 0.000244140625$$

Input interpretation:

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625}$$

Result:

0.00666501785...

0.00666501785...

$$e^{-6C+\phi} = 0.0066650177536$$

$$\exp(-\pi\sqrt{18}) \times \frac{1}{0.000244140625} =$$

$$e^{-\pi\sqrt{18}} \times \frac{1}{0.000244140625}$$

$$= 0.00666501785...$$

$$\ln(0.00666501784619)$$

Input interpretation:

log(0.00666501784619)

Result:

-5.010882647757...

-5.010882647757...

Now:

$$-6C + \phi = -5.010882647757 \dots$$

For C = 1, we obtain:

$$\phi = -5.010882647757 + 6 = 0.989117352243 = \phi$$

Conclusions

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

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