DISRUPTIVE GRAVITATION THEORY
A mathematically consistent take on gravity

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Abstract

Viewing gravity as a spacetime bending force instead of just a spacetime curvature, we come to the conclusion of rest mass relativity since it yields equivalent equations as General Relativity. A close analysis of the Schwarzschild metric leads us naturally to the Vacuum Energy Invariance principle from which we derive the metric equation. Applying this theory to cosmology, we can explain the acceleration of the universe expansion in a way that doesn't require Dark Energy. This theory has the same predictive power as General Relativity for every local experimental tests of the latter since it's based on a slight modification of the Schwarzschild metric.

INTRODUCTION

This theory is a new take on gravity that deserves further investigations. It shows the mathematical consistency of seeing gravity as a spacetime bending force and provides sort of a framework for a consistent theory of gravity even for violations of the weak equivalence principle and non-newtonian gravitational potentials. Seeing gravity as a spacetime bending force has two main advantages: gravity can be easily quantized and explains the acceleration of the universe expansion with no need for Dark Energy.

For every classical test of General Relativity, this theory gives the same measurable results since it uses a slightly modified Schwarzschild metric. Some other tests are possible where General Relativity and this theory would give different results. Many such tests are presented in this paper and are a way to either falsify this theory or to ascertain its physical consistency.

We should keep in mind that the overall implications of it could lead to paradoxes as it was the case for Special Relativity and General Relativity whose paradoxes have sometimes been resolved decades after their publication, so only the mathematical consistency is of relevance in this paper.

In this paper, Greek letters range from 0 to 3 (representing spacetime) while Roman letters range from 1 to 3 (representing space), the metric signature is $(+---)$ and we use Einstein's summation convention. The Greek capital letter Φ is the gravitational potential.

I - A Light-Speed-Invariance-like Principle

Most tests of General Relativity are based on the Schwarzschild metric [1] below. Let's see if we can give a physical meaning to it.

$$ds^2 = (1 + 2\Phi/c^2)c^2dt^2 - (1 + 2\Phi/c^2)^{-1}dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)$$

Space and time being disjoint, we can define the space metric:

$$ds^2_{Space} = (1 + 2\Phi/c^2)^{-1}dr^2 + r^2(d\theta^2 + sin^2\theta d\psi^2)$$

The volume element of a riemannian manifold is the square root of the determinant of the metric in absolute value times the coordinate elements. For the Schwarzschild space metric it yields:

$$dV = \sqrt{(1 + 2\Phi/c^2)^{-1} \cdot r^2 \cdot r^2 sin^2\theta} \cdot drd\theta d\psi$$

In weak fields, neglecting second order terms, it comes:

$$dV = (1 - \Phi/c^2) \cdot r^2|sin\theta|drd\theta d\psi$$

Then, considering a hypothetical mass density of vacuum ρ in a volume element dV under Φ gravitational potential, the energy inside the volume element is:

$$(\rho c^2 + \rho\Phi) \cdot dV = \rho c^2(1 + \Phi/c^2) \cdot (1 - \Phi/c^2) \cdot r^2|sin\theta|drd\theta d\psi$$

Neglecting second order terms we have: $(1+\Phi/c^2)\cdot(1-\Phi/c^2) = 1-(\Phi/c^2)^2 = 1$

It yields:

$$(\rho c^2 + \rho\Phi) \cdot dV = \rho c^2 \cdot r^2|sin\theta|drd\theta d\psi$$

It doesn't depend on Φ in first order approximation. In other words, analogous to the invariance of the speed of light, we have the following principle:

"The energy of vacuum is invariant".

It seems like the same way speed of light invariance induces time dilation, vacuum energy invariance induces space dilation. It is therefore a strong incentive to search for a consistent theory of gravity as a spacetime bending force instead of just a spacetime curvature.

II - A Spacetime Bending Force

Describing gravity as a spacetime bending force has to produce the same tested predictions as General Relativity which are: Mercury's Orbit Precession, Time Dilation, Light Bending, Shapiro Delay, Lens-Thirring and geodetic effects.

We know Lens-Thirring and geodetic effets are both well discribed by Gravitoelectromagnetism [2] which is a theory of gravity in a flat spacetime analoguous to Maxwell's theory of electromagnetism. So including spacetime curvature in Gravitoelectromagnetism would still make those predictions.

Analogous to electromagnetism in General Relativity, we can consider gravity as some kind of gravitoelectromagnetism in a curved spacetime and see if it makes the same predictions as General Relativity. The lagrangian of an electrically charged body in General Relativity is:

$$L = -mc\sqrt{g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} - q\dot{x}^{\mu}A_{\mu}$$

where A_μ is the electromagnetic four-vector potential, m the rest mass and q the electric charge of the body. The idea is to consider a gravitational four-vector potential G_μ analogous to the electromagnetic four-vector potential A_μ and consider the following lagrangian:

$$L = -m_{inertial}c\sqrt{g_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu}} - m_{gravitational}\dot{x}^{\mu}G_{\mu}$$

where $m_{inertial}$ is the inertial mass of the body and $m_{gravitational}$ is its gravitational mass. We will see, that under the hypothesis of inertial mass relativity, this lagrangian gives equivalent results as General Relativity.

For some reason that will become clear in Section III, we define the gravitational mass as:

$$m_{gravitational} = \gamma^{-1}m_{inertial}$$

where γ is defined as $\gamma^{-1} = \sqrt{g_{\mu\nu}\dfrac{dx^{\mu}}{dx^{0}}\dfrac{dx^{\nu}}{dx^{0}}}$ similar to Lorentz factor.

We hypothesize that the rest mass is relative such that:

$$m_{inertial} = \alpha(\Phi)m_0$$

where m_0 is the rest mass, defined as the inertial mass if the gravitational potential is null: $\alpha(0) = 1$. The lagrangian becomes:

$$\boxed{L = -\alpha(\Phi) m_0 c \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - \gamma^{-1}\alpha(\Phi) m_0 \dot{x}^\mu G_\mu} \quad (1)$$

How the Gravitational four-vector potential G_μ is calculated is not of relevance in this paper since gravity is not postulated to be newtonian. It should then be subject to further investigations. It depends on the type of gravitational potential. If newtonian, it would be the exact analogous of electromagnetism in curved spacetime as we would just have to replace ϵ_0 by $-1/4\pi\mathcal{G}$ where $\mathcal{G}$ is Newton's constant.

In electromagnetism, the four-vector potential is of the form $A_\mu = (V/c, \vec{A})$ where V is the electrical potential and $\vec{A}$ is the potential vector . Even though G_μ remains to be calculated depending on the gravitational potential theory used (not necessarily newtonian), we now that, analogously to electromagnetism it is of the form $G_\mu = (\Phi/c, \vec{G})$ where Φ is the gravitational potential.

In electromagnetism, the magnetic field is derived as the curl of $\vec{A}$. Analogously, defining the gravitational tensor as:

$$F_{\mu\nu} = \partial_\mu G_\nu - \partial_\nu G_\mu = \begin{pmatrix} 0 & -\frac{1}{c}E^x_{\mathcal{G}} & -\frac{1}{c}E^y_{\mathcal{G}} & -\frac{1}{c}E^z_{\mathcal{G}} \\ \frac{1}{c}E^x_{\mathcal{G}} & 0 & B^z_{\mathcal{G}} & -B^y_{\mathcal{G}} \\ \frac{1}{c}E^y_{\mathcal{G}} & -B^z_{\mathcal{G}} & 0 & B^x_{\mathcal{G}} \\ \frac{1}{c}E^z_{\mathcal{G}} & B^y_{\mathcal{G}} & -B^x_{\mathcal{G}} & 0 \end{pmatrix}$$

provides a good description of Lens-Thirring and geodetic effects.

Another prediction of General Relativity is Gravitational Waves. It is not mentioned in the tests because it is in fact due to a gauge choice. Whereas viewing gravity as a spacetime bending force, gravitational waves would not be due to a gauge choice since G_μ is lorenzian by definition. Indeed, Lorentz gauge induces a wave equation of the potential.

III - First Order Non-Relativistic Dynamics

As we said in the previous section, we consider the following lagrangian:

$$L = -\alpha(\Phi) m_0 c \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - \gamma^{-1}\alpha(\Phi) m_0 \dot{x}^\mu G_\mu \quad [\text{i}]$$

Let's demonstrate that this lagrangian gives equivalent equations of motion as General Relativity for a certain choice of α when Lens-Thirring and geodetic effects can be neglected in case of non-relativistic speeds and in weak-fields.

Let's first simplify the lagrangian by neglecting second order terms. If Lens-Thirring and geodetic effects can be neglected, then cross-terms between space and time can be neglected. Parametrizing with the body's proper time, we have $c^2 = g_{00}(\dot{x}^0)^2 + g_{ij}\dot{x}^i\dot{x}^j$ which yields for non-relativistic fields:

$$\dot{x}^0 \cdot \sqrt{g_{00}} = c \cdot (1 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2) \qquad \text{[ii]}$$

Similarly, with $\gamma^{-1} = \sqrt{g_{\mu\nu}\frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0}}$, we have:

$$\gamma^{-1}/\sqrt{g_{00}} = \sqrt{g_{\mu\nu}/g_{00} \cdot \frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0}} = \sqrt{1 + g_{ij}/g_{00} \cdot \frac{dx^i}{dx^0}\frac{dx^j}{dx^0}} \qquad \text{[iii]}$$

Since $\frac{dx^0}{d\tau} = \dot{x}^0$ and for non-relativistic speeds $\dot{x}^0 \approx c$, neglecting second order terms it comes:

$$\gamma^{-1}/\sqrt{g_{00}} = \sqrt{1 + g_{ij}/g_{00} \cdot \dot{x}^i\dot{x}^j/(\dot{x}^0)^2} = 1 + 1/2 \cdot g_{ij}/g_{00} \cdot \dot{x}^i\dot{x}^j/c^2 \qquad \text{[iv]}$$

Since in weak-fields $1/g_{00} \approx 1 - 2\Phi/c^2$, neglecting second order terms yields:

$$\gamma^{-1}/\sqrt{g_{00}} = (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2) \qquad \text{[v]}$$

Multiplying [ii] and [v] we get:

$$\gamma^{-1}\dot{x}^0 = c \cdot (1 + 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - 1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2 - (1/2 \cdot g_{ij}\dot{x}^i\dot{x}^j/c^2)^2) \qquad \text{[vi]}$$

Neglecting second order terms again it comes: $\gamma^{-1}\dot{x}^0 = c$ [vii]

Lens-Thirring and Geodetic effects being neglected, we also have $G_0 = \Phi/c$ and $G_i = 0$ we get:

$$\dot{x}^\mu G_\mu = \dot{x}^0 G_0 = \dot{x}^0\Phi/c \qquad \text{[viii]}$$

Recasting [vii] yields: $\gamma^{-1}\dot{x}^\mu G_\mu = \gamma^{-1}\dot{x}^0\Phi/c = \Phi$ [ix]

Introducing Lorentz factor in the definition of the gravitational mass is convienient as it suppresses perturbative terms. Its physical meaning is quite intuitive though: the faster a body, the more massive it gets in term of relativistic mass, and the less the influence of a force on it. Taking this into account implies the introduction of Lorentz factor in the definition of the gravitational mass.

The lagrangian [i] becomes:

$$\boxed{L = -\alpha(\Phi) m_0 c \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} - \alpha(\Phi) m_0 \Phi} \ (2) \qquad \text{[x]}$$

For more clarity, let's also write: $\dot{s_0} = \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}$

We then have: $L = -\alpha(\Phi) m_0 c\dot{s_0} - \alpha(\Phi) m_0 \Phi$ [xi]

The Lagrangian's variables are x^μ and $\dot{x}^\mu$ but parametrizing with the body's proper time, we have $c^2 = g_{00}(\dot{x}^0)^2 + g_{ij}\dot{x}^i\dot{x}^j$ which shows that the variables are not independant. We then have to choose a set of independant variables. Since space and time are disjoint by hypothesis, it is really convenient to choose x^i and $\dot{x}^i$ as a set of independant variables.

The lagrangian equation is: $\frac{\partial L}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L}{\partial \dot{x}^i} = 0$ [xii]

Since Φ doesn't depend explicitly on $\dot{x}^i$, we have:

$$-\frac{\partial \alpha(\Phi) m_0 c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi) m_0 \Phi}{\partial x^i} + \frac{d}{d\tau}\frac{\partial \alpha(\Phi) m_0 c\dot{s_0}}{\partial \dot{x}^i} = 0 \qquad \text{[xiii]}$$

Leading to: $-\frac{\partial \alpha(\Phi) c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^i} + \frac{d}{d\tau}(\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial \dot{x}^i}) = 0$ [xiv]

It comes:

$$-\alpha(\Phi)\frac{\partial c\dot{s_0}}{\partial x^i} - \frac{\partial \alpha(\Phi)}{\partial x^i}c\dot{s_0} - \frac{\partial \alpha(\Phi)\Phi}{\partial x_i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} + \alpha(\Phi)\frac{d}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} = 0 \qquad \text{[xv]}$$

We see the lagrangian equation of General Relativity in the first and last terms of the equation [xv]. Let $L_0 = -m_0 c\dot{s_0}$, it comes:

$$-\frac{\partial \alpha(\Phi)}{\partial x^i}c\dot{s_0} - \frac{\partial \alpha(\Phi)\Phi}{\partial x^i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^i})/m_0 = 0 \qquad \text{[xvi]}$$

Parametrizing with the body's proper time, we have: $\dot{s_0} = c$. Thus:

$$-\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} + \frac{d\alpha(\Phi)}{d\tau}\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} + \alpha(\Phi)(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial \dot{x}^i})/m_0 = 0 \qquad \text{[xvii]}$$

Notations can be misleading. We cannot replace $\dot{s_0}$ by c in the expression $\frac{\partial c\dot{s_0}}{\partial \dot{x}^i}$ since it's a partial derivative. We have in fact:

$$\frac{\partial c\dot{s_0}}{\partial \dot{x}^i} = c \cdot \frac{\partial \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}}{\partial \dot{x}^i} = c \cdot \frac{2 \cdot g_{\mu i}\dot{x}^\mu}{2 \cdot \sqrt{g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu}} = c \cdot \frac{2 \cdot \dot{x}_i}{2 \cdot c} = \dot{x}_i \qquad \text{[xviii]}$$

Hence: $\dfrac{d\alpha(\Phi)}{d\tau}\dfrac{\partial c\dot{s}_0}{\partial\dot{x}^i} = \dfrac{\partial\alpha(\Phi)}{\partial\Phi}\cdot\dfrac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\cdot\dot{x}_i$ [xix]

And calculating $(\dfrac{\partial L_0}{\partial x^i} - \dfrac{d}{d\tau}\dfrac{\partial L_0}{\partial\dot{x}^i})/m_0$ gives a known standard result of General Relativity [3] [4] [5] [6]:

$$(\frac{\partial L_0}{\partial x^i} - \frac{d}{d\tau}\frac{\partial L_0}{\partial\dot{x}^i})/m_0 = g_{\mu i}\ddot{x}^\mu + 1/2\cdot(-\partial^i g_{\mu\nu} + \partial^\mu g_{\nu i} + \partial^\nu g_{\mu i})\dot{x}^\mu\dot{x}^\nu \qquad \text{[xx]}$$

Thus, after multiplying [xiv] by g^{ik} (which is the inverse of the restriction of the metric to space), defining Christoffel symbols as:

$$\Gamma^k_{\mu\nu} = g^{ik}/2\cdot(-\partial^i g_{\mu\nu} + \partial^\mu g_{\nu i} + \partial^\nu g_{\mu i})\dot{x}^\mu\dot{x}^\nu$$

(as said in the introduction, Roman letters span from 1 to 3 whereas Greek letters span from 0 to 3) we get:

$$-g^{ik}\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} + \frac{\partial\alpha(\Phi)}{\partial\Phi}\frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^k + \alpha(\Phi)(\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu\dot{x}^\nu) = 0 \quad \text{[xxi]}$$

We see that, for it to give correct equations of motion in the newtonian limit, we necessarily have:

$$\frac{\partial(\alpha(\Phi)c^2 + \alpha(\Phi)\Phi)}{\partial x^i} = 0 \qquad \text{[xxii]}$$

That yields: $\alpha(\Phi) = (1 + \Phi/c^2)^{-1}$ [xxiii]

Then: $\dfrac{\partial\alpha(\Phi)}{\partial\Phi} = -1/c^2\cdot(1 + \Phi/c^2)^{-2}$ [xxiv]

Hence, recasting in [xxi] we get:

$$-(1 + \Phi/c^2)^{-2}\cdot\frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^k/c^2 + (1 + \Phi/c^2)^{-1}(\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu\dot{x}^\nu) = 0 \qquad \text{[xxv]}$$

After neglecting second order terms, that yields:

$$\boxed{\ddot{x}^k + \Gamma^k_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = \frac{\partial\Phi}{\partial x^\mu}\dot{x}^\mu\dot{x}^k/c^2}\ (3) \qquad \text{[xxvi]}$$

These equations of motion look like the geodesic equations of General Relativity. For weak-fields and low speeds, we trivially get the newtonian limit.

Hence, if the inertial mass is relative such that:

$$\boxed{m_{inertial} = (1 + \Phi/c^2)^{-1} m_0} \quad (4)$$

gravity described as a spacetime bending force instead of a spacetime curvature yields similar results. The small deviation from General Relativity induced by $\partial^\mu \Phi \dot{x}^\mu \dot{x}^k / c^2$ would be a test of the theory.

For more clarity, let's write: $m_0 \dfrac{\partial \Phi}{\partial x^\mu} \dot{x}^\mu \dot{x}^k / c^2 = -(\vec{F} \cdot \vec{v}) \cdot \vec{v} / c^2$

where $\vec{F}$ is the gravitational force and $\vec{v}$ the speed of the body. We can interpret it as an anomalous thrust unexpected from General Relativity. Such an anomaly is to be expected in the recently lauched Parker Solar Probe if solar wind and radiation pressure can be neglected so close to the Sun and would be a test of this theory.

In case of an orbital motion, we see that for a circular trajectory, this force is null. Thus, it can be neglected for low eccentricities yielding the same predictions of orbital precession as General Relativity.

However, it can be shown that the influence of this force over a revolution period is a resulting force parallel to the great axis and directed towards the aphelion of the trajectory that increases in magnitude with the eccentricity. Thus, it contributes to increasing the eccentricity of the trajectory over time. That might be the main reason why Mercury's eccentricity is high compared to other planets although tidal circularization would tend to make it null.

That is a good argument in favor of gravity as a spacetime bending force instead of just a spacetime curvature, even though it is not a test the theory in itself.

IV - Physical Implications

The hypothesis of inertial mass relativity yields equivalent results as General Relativity in weak fields and non-relativistic speeds, therefore it gives equivalent results for every experimental tests if the Schwarzschild metric is a solution of this theory. This hypothesis has physical implications and interpretations as we will see.

Mathematically, a natural physical interpretation arises. Indeed, we can give a physical meaning to $E_\Phi = m_{inertial} c^2$ thanks to inertial mass relativity:

$E_\Phi = m_0 c^2 / (1 + \Phi/c^2)$

Generalized to a Relativistic body, we have:

$E_\Phi = \gamma mc^2/(1+\Phi/c^2)$ where $\gamma = 1/\sqrt{1-v^2/c^2}$ is Lorentz factor [8].

Let's rewrite it as: $E_\Phi = \sqrt{m_0^2c^4 + p_0^2c^2}(1+\Phi/c^2)$

Or rather, for brevity : $\boxed{E_\Phi = E_0/(1+\Phi/c^2)}$ (5)

Applied to photons of energy $E_0 = h\nu_0$, with $E_\Phi = h\nu_\Phi$ we have:

$\nu_\Phi = \nu_0/(1+\Phi/c^2)$

That looks a lot like General Relativity's formula of gravitational redshift. Thus we define E_Φ as the Apparent Energy of the body.

Writing it as $E_\Phi = E_0\sqrt{g_{00}}$, it's as if the energy of a body could be redshifted. It's as if a body was also a wave which we know accurate since De Broglie's hypothesis of wave-particle duality.

Apparent Energy is nothing new. When a wave is Doppler-shifted for a moving observer, the shifted frequency is said to be apparent frequency. Analogously, the energy of a photon for a moving observer doesn't change, but since its frequency is Doppler-shifted, the change in energy is in fact Apparent Energy.

This physical meaning implies the time dilation factor be: $g_{00} = (1+\Phi/c^2)^2$

This provides another testable deviation from General Relativity. Indeed in General Relativity we have:

$g_{00,schwarzschild} = \sqrt{1+\Phi/c^2} \approx 1 + 2\Phi/c^2$

The second order difference is $(\Phi/c^2)^2$. It's really small but measurable so this theory is falsifiable.

V - Vacuum Energy Invariance

In Section I, we saw that the Schwarzschild metric could be interpreted in weak fields as the invariance of the energy of a vaccum with a virtual matter density. That was the incentive to consider gravity as a spacetime bending force instead of just a spacetime curvature.

Just as the Strong Equivalence principle is a postulate of General Relativity, Vaccum Energy Invariance (VEI) is the fundemental postulate of this theory. We will see that it yields the Schwarzschild metric in weak-fields and therefore provides the same predictions as General Relativity.

In Section VI and VII, we derive the metric thanks to this principle.

VI - Metric Derivation (part I)

We showed in Section II and III that gravity can be coherently described as a spacetime bending force if the rest mass is relative. We are left with how the metric can be derived such that the Schwarzschild metric is a particular case.

We naturally postulate that the metric $g_{\mu\nu}$ is of the form:

$$g = \begin{pmatrix} g_{00}(\Phi) & 0 \\ 0 & -g_s(\Phi) \end{pmatrix}$$

Indeed, in General Relativity, cross terms between space and time are responsible for Lens-Thirring and geodetic effects but since these are already accounted for by considering gravity as spacetime bending force, we can postulate that space and time curvature are disjoint.

We then consider that space and time are independently dilated by VEI.

Let's derive both $det(g_s)$ and g_{00} thanks to VEI principle.

At a given point in time t, in a volume element $dx_1 dx_2 dx_3$, under zero gravity (flat space) with vacuum energy density $\mathcal{E}_0$, we have:

$$dE_0 = \mathcal{E}_0 dx_1 dx_2 dx_3$$

and under Φ-gravity potential, we have:

$$dE_\Phi = \mathcal{E}_0 (1 + \Phi/c^2)\sqrt{det(g_s)} dx_1 dx_2 dx_3$$

Applying VEI, we have: $dE_0 = dE_\Phi$.

It comes:

$$\boxed{det(g_s) = (1 + \Phi/c^2)^{-2}} \quad (6)$$

Let's apply VEI in time domain to have a more rigorous way to find g_{00}.

The reasoning is a bit similar to the one for the derivation of the gravitational redshift. We reason in terms of observational events.

Let E_0 be the total vacuum energy and N be the number of observationnal events.

The total vacuum energy by time unit for an observer under a global 0-potential is:

$$P_0 = \frac{d(NE_0)}{dt}$$

The total vacuum energy by time unit for the same observer under a global Φ-potential is:

$$P_\Phi = \frac{d(NE_0(1+\Phi/c^2))}{d\tau}$$

Applying VEI, we have: $P_0 = P_\Phi$

It comes: $E_0 dN d\tau = E_0(1+\Phi/c^2) dN dt$

With $d\tau^2 = g_{00} dt^2$ it eventually comes:

$$\boxed{g_{00} = (1+\Phi/c^2)^2} \quad (7)$$

The equation of motion [xxvii] of Section III, for non-relativistic speeds becomes:

$$\ddot{x}^k + \Gamma^k_{00}\dot{x}^0\dot{x}^0 = 0$$

In weak-fields, standard result of linearized General Relativity yields :

$$\ddot{x}^k = -1/2 \cdot \partial_k h_{00} c^2$$

where $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$ is the perturbation of the metric.

From VEI we have $h_{00} = 2\Phi/c^2$ which yields Newton's law [9].

VII - Metric Derivation (part 2)

We still don't fully know g_s. Any g_s formula predicting a correct Light Deflection and reproducing the Schwarzschild metric for the Sun's mass distribution works to account for every experimental tests.

Considering gravity as a spacetime bending force would give us a space metric g_s different from General Relativity. It doesn't change anything to the newtonian limit since in that case only g_{00} is relevant for the equations of motion. The idea is to aggregate the contributions of every mass of the distribution to the space deformation. In case of a compact spherical distribution, far from the sphere, space dilation would be purely radial just as in the Schwarzshild metric whereas it wouldn't be the case close to the mass distribution. This provides another test.

Space deformations induced by a single punctual mass must be radial for trivial physical reasons. Then in a local orthonormal basis $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ where $\vec{e_r}$ is radial, space metric is $-g_{s,ruv}$ of the form:

$$g_{s,ruv} = \begin{pmatrix} \beta^{-2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = I + (\beta^{-2} - 1) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Applying VAEI yields: $\beta = 1 + \Phi/c^2$.

Let M^T be the change of basis orthonormal matrix from $(\vec{e_r}, \vec{e_u}, \vec{e_v})$ to $(\vec{e_1}, \vec{e_2}, \vec{e_3})$. So with $\vec{e_r} = r_i \vec{e_i}$, $\vec{e_u} = u_i \vec{e_i}$ and $\vec{e_v} = v_i \vec{e_i}$, changing coordinates we have:

$$g_s = M^T g_{s,ruv} M \text{ with } M^T = \begin{pmatrix} r_1 & u_1 & v_1 \\ r_2 & u_2 & v_2 \\ r_3 & u_3 & v_3 \end{pmatrix}$$

Since $M^T M = I$, it comes: $g_s = I + (\beta^{-2} - 1) M^T \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} M$

Eventually:

$$g_s = I + (\beta^{-2} - 1) \begin{pmatrix} r_1^2 & r_1 r_2 & r_1 r_3 \\ r_2 r_1 & r_2^2 & r_2 r_3 \\ r_3 r_1 & r_3 r_2 & r_3^2 \end{pmatrix} \text{ or } g_{s,ij} = \delta_{ij} + (\beta^{-2} - 1) r_i r_j$$

In weak-fields, this is equivalent to the Schwarzschild metric written in cartesian coordinates. This doesn't depend on the choice of $\vec{e_u}$ and $\vec{e_v}$. For a mass distribution, the unit vector pointing from a massive point towards a local point in space is the same as the radial vector $\vec{e_r}$ so we can aggregate their influence thanks to the above formula.

Indeed, for an infinitely small potential $d\Phi$, we have $\beta^{-2} - 1 = -2d\Phi/c^2$ and the metric becomes, when integrating over every infinitely small potential:

$g_{s,ij} = \delta_{ij} + \lambda \cdot \int -2r_i r_j d\Phi/c^2$ with λ such that $det(g_s) = (1 + \Phi/c^2)^{-2}$

Space being curved there might not be a unique choice of r_i. Therefore we introduce the potential angular distribution $\phi(\vec{\sigma})$, where $\vec{\sigma}$ is the observed direction. Leading to the following metric equation:

$$\boxed{g_{s,ij} = \delta_{ij} + \lambda \cdot \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma} \quad (8)$$

With: $\boxed{B_{ij} = \int -2\phi(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma}$ (9)

We have: $g_{s,ij} = \delta_{ij} + \lambda B_{ij}$

In fact, for any 3x3 matricial function f such that $f(P^{-1}MP) = P^{-1}f(M)P$ and $f(M) = I + M$ if M is small, $g_s = f(\lambda B)$ would also be valid. For physical reasons, rather than summing the infinitely small pertubations, we should multiply the metrics induced by each infinitely small perturbations. That would yield:

$$\boxed{g_s = e^{\lambda B}} \quad (10)$$

Deriving λ is then straightforward since B being symmetric, it is diagonal in a certain basis, and $e^{\lambda B}$ would be a diagonal matrix in such a basis whose determinant is the exponential of the sum of its eigenvalues. The sum of the eigenvalues being the trace of λB, we have:

$det(e^{\lambda B}) = e^{Tr(\lambda B)}$

Applying VEI principle we then have : $e^{\lambda Tr(B)} = (1 + \Phi/c^2)^{-2}$

Hence : $\boxed{\lambda = -2 \cdot ln(1 + \Phi/c^2)/Tr(B)}$ (11)

So in the weak-fields limit we have: $\boxed{g_{s,ij} = \delta_{ij} - 2\Phi/c^2 \cdot B_{ij}/B_{kk}}$ (12)

Applying it to a punctual mass, space deformation being radial, in spherical coordinates we trivially obtain a modified Schwarzschild metric:

$$\boxed{ds^2 = (1+\Phi/c^2)^2 c^2 dt^2 - (1+\Phi/c^2)^{-2} dr^2 - r^2(d\theta^2 + sin^2\theta d\psi^2)} \quad (13)$$

So this predicts Mercury's Orbital Precession and Light Deflection by the Sun since its mass is concentrated in its core. But in case of a homogenous spherical mass distribution like the Earth, the radial dilation would be smaller than the one predicted by the Schwarzschild metric because the deformation is fairly distributed according to the influence of every part of the mass distribution. This could be measured through interferometry and provide another test of the theory.

VIII - Summary

The formalism could be enhanced but this is not necessary to show the mathematical consistency of this theory. G_μ being Lorentzian, $G_0 = \Phi/c$ depends on the referential frame. So space dilation through VAEI would be relative. There is a paradox there that we won't adress and suppose that a better formalism would erase it. In last resort, General Covariance could be dropped.

The theory can be summarized by the equations below:

$$\Phi_0 = \Phi$$

$$L = -\alpha m_0 c\sqrt{g_{\mu\nu}\dot{x^\mu}\dot{x^\nu}} - \gamma^{-1}\alpha m_0 \dot{x^\mu} G_\mu$$

$$\alpha = (1+\Phi_0/c^2)^{-1}$$

$$\gamma^{-1} = \sqrt{g_{\mu\nu}\frac{dx^\mu}{dx^0}\frac{dx^\nu}{dx^0}}$$

$$g = \begin{pmatrix} (1+\Phi_0/c^2)^2 & 0 \\ 0 & -e^{\lambda B} \end{pmatrix}$$

$$B_{ij} = \int -2\phi_0(\vec{\sigma})/c^2 \cdot r_i(\vec{\sigma}) r_j(\vec{\sigma}) d\sigma$$

$$\lambda = -2 \cdot ln(1+\Phi/c^2)/Tr(B)$$

This can be easily adapted to any violation of Weak Equivalence principle by separating vacuum gravitational potential from the bodies' gravitational potential: $\Phi_0 \neq \Phi$

IX - Universe Expansion

The Cosmological Redshift can as well be interpreted as due to an expanding universe if we postulate that the universe is homogeneous and isotropic and has a beginning. This wouldn't be an extension of General Relativity but an alternative coherent model providing the same observational results. Indeed, if gravity is a force, gravitational potential propagates at the speed of light. The older the universe, the more propagated the gravitational potential and the greater space dilation would be.

Let's see how global vacuum gravitational potential evolves in a homogeneous and isotropic universe from its creation. The potential is induced by the mass in a cT radius sphere where T is the age of the universe. The gravitation potential is:

$\Phi = \int_0^{cT} \phi(r)\rho \cdot 4\pi r^2 dr$

Taking space dilation into account and conservation of matter, we have:

$\rho = \rho_0 \cdot (1 + \Phi/c^2)^{-1}$

And with the variable $t = r/c$ we have:

$\Phi = 4\pi\rho_0 c^3 \cdot \int_0^{T} \phi(ct)(1 + \Phi/c^2)^{-1} t^2 dt$

Hence the following gravitational potential differential equation:

$d\Phi/dT = 4\pi\rho_0 c^3 \cdot \phi(cT)(1 + \Phi/c^2)^{-1} T^2$

Seperating variables, we get:

$\Phi + \Phi^2/2c^2 = 4\pi\rho_0 c^3 \cdot \int_0^{T} \phi(ct) t^2 dt$

Hence the solution:

$$\boxed{1 + \Phi/c^2 = \sqrt{1 + 8c\pi\rho_0 \cdot \int_0^{T} \phi(ct) t^2 dt}} \quad (14)$$

The age T is the time elapsed from the point of view of an observer in a null gravitational potential, as if he was shielded from gravity.

Since the universe is homogeneous, the scale factor is $a = (1 + \Phi/c^2)^{-1/3}$ so recasting the solution yields:

$$\boxed{a(T) = (1 + 8c\pi\rho_0 \cdot \int_0^T \phi(ct)t^2 dt)^{-1/6}} \quad (15)$$

To be able to compare this model with Friedmann-Lemaitre-Robertson-Walker models, we need to express the dilation factor with a time equivalent to comoving observers. The time T_c of a comoving observer satisfies:

$$dT_c = g_{00} dT = (1 + \Phi(T)/c^2)dT$$

It comes: $$\boxed{T_c = \int_0^T (1 + 8c\pi\rho_0 \cdot \int_0^t \phi(c\tau)\tau^2 d\tau)^{1/2} dt} \quad (16)$$

Intuitively, the dilation factor has a positive acceleration because it is a division by a quantity that seems to near zero. In fact, the above equations shows that the absolute time T can have a finite limit value when the comoving time T_c tends to infinity. That depends on the gravitational potential theory used. Let's do the calculation for a newtonian potential $\phi(r) = -\mathcal{G}/r$. We have:

$$T_c = \int_0^T (1 - 4\pi\mathcal{G}\rho_0 t^2)^{1/2} dt$$

And: $a(T) = (1 - 4\pi\mathcal{G}\rho_0 T^2)^{-1/6}$

From this simple newtonian model, we see the scalar factor has a positive acceleration. The potential is not necessarily newtonian, but we see that an accelerating expanding universe would be more expected than a non-accelerating universe, especially for non-newtonian potentials such that $\mathcal{G}/r \cdot \phi(r)^{-1} = o(1)$. This model doesn't require Dark Energy to explain such acceleration.

X - Intrinsic Redshift Non-Standart Cosmology

From Halton Arp observations [12], it seems like the redshift is not only due to expansion. It seems like there is what we can call an Intrinsic Redshift. This can be understood through mass relativity. Indeed, hydrogen atom spectrum absorption lines is dependent on electron and proton mass which are relative. An hydrogen photon frequency emitted or absorbed in Φ_2 gravitational potential as seen by an observer shielded from gravity is proportional to the inertial reduced mass of a proton and an electron $\mu = \mu_0/(1 + \Phi_2/c^2)$ since hydrogen energy states are given by E_I/n^2 with $E_I = \mu e^4/8\epsilon_0^2 h^2$ and is then stretched through expansion. Such frequency observed in Φ_1 gravitational potential is of the form:

$$\nu_2 = \nu_0/(1 + \Phi_2/c^2) \cdot (1 + \Phi_1/c^2)^{-1}$$

Same reasonning for a photon emitted or absorbed in Φ_1 and observed in Φ_1 gives the observed frequency:

$$\nu_1 = \nu_0/(1+\Phi_1/c^2) \cdot (1+\Phi_1/c^2)^{-1}$$

Hence the redshift: $\nu_2/\nu_1 = (1+\Phi_1/c^2)/(1+\Phi_2/c^2)$

At a cosmological scale, a photon observed from a far away galaxy at a distance D is emitted or absorbed under the gravitational potential $\Phi_0(T-D/c)$ at time $T-D/c$ plus intrinsic potential Φ_{Gal} of the galaxy, hence the redshift observed in $\Phi_0(T)+\Phi_{local}$ gravitational potential taking into account space dilation with scale factor a:

$$\nu_{Gal}/\nu_{local} = \frac{1+\Phi_{local}/c^2+\Phi_0(T)/c^2}{1+\Phi_{Gal}/c^2+\Phi_0(T-D/c)/c^2} \cdot \frac{a(T-D/c)}{a(T)}$$

Since $a(T) = (1+\Phi_0(T)/c^2)^{-1/3}$, it comes:

$$\boxed{\nu_{Gal}/\nu_{local} = \frac{1+\Phi_{local}/(c^2+\Phi_0(T))}{1+\Phi_{Gal}/(c^2+\Phi_0(T-D/c))} \cdot \left(\frac{a(T-D/c)}{a(T)}\right)^4} \quad (17)$$

So we have this way an intrinsic redshift taken into account that explains Halton Arp's observations.

In case of no apparent intrinsic redshift, the redshift formula becomes:

$$\boxed{\nu_{Gal}/\nu_{local} = \left(\frac{a(T-D/c)}{a(T)}\right)^4} \quad (18)$$

It means that the Cosmological Redshift is not only due to the expansion of the universe but is mostly due to mass relativity.

CONCLUSION

Vacuum Apparent Energy Invariance is a principle analogous to speed of light invariance that is the fundamental postulate of this theory, just like Strong Equivalence principle is the fundamental postulate of General Relativity. From it we can derive the Schwarzschild metric in weak-fields and equations of space dilation similar to linearized General Relativity in weak-fields. We demonstrated that in every experimental tests done so far, viewing gravity as a spacetime bending force gives equivalent results as viewing gravity as a spacetime curvature. The only new concept introduced is rest mass relativity which is physically acceptable since the concept of rest mass and relative mass already exist. It also takes into account any possible violations of Weak Equivalence Principle and non-newtonian potentials which is an open door to further studies. It provides a model of an accelerating expanding universe witout Dark Energy and an explanation of Halton Arp observations which has never been done before.

Above all, more than being mathematically consistent, it is testable. Predicted deviations from General Relativity are described in Section III, IV and VII and could ascertain its physical consistency.

Gravity as a spacetime bending force instead of a spacetime curvature is trivially quantizable as a force in a curved spacetime analogous to electromagnetism (see V. Fock, Z. Phys. 57, 261 (1929) and H. Tetrode, Z. Phys. 50, 336 (1928)). It is then a possible alternative to General Relativity that is worth investigating further even though it could very well be ruled out as many other theories.

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