# on Λ-generalized continuous functions<sup>\*</sup> S.P. Missier, M.G. Rani, M. Caldas and S. Jafari

#### Abstract

In this paper, we introduce a new class of continuous functions as an application of  $\Lambda$ -generalized closed sets (namely  $\Lambda_g$ -closed set,  $\Lambda$ -g-closed set and  $g\Lambda$ -closed set) namely  $\Lambda$ -generalized continuous functions (namely  $\Lambda_g$ -continuous,  $\Lambda$ -g-continuous and  $g\Lambda$ -continuous) and study their properties in topological space.

### **1** Introduction and Preliminaries

Levine [7] introduced g-closed set. Maki [8] introduced the notion of  $\Lambda$ -sets in topological spaces. A subset A of a topological space  $(X, \tau)$  is called a  $\Lambda$ -set if it coincides with its kernel (the intersection of all open supersets of A). In [1], Arenas et al. introduced the notions of  $\lambda$ -open sets, and  $\lambda$ -closed sets and presented fundamental results for these sets. They also introduced [1]  $\lambda$ - continuity, which is weaker than continuity. Recently, M. Caldas, S. Jafari and T. Noiri [3] introduced  $\Lambda$ -generalized closed sets in topological space. The aim of this paper is to introduce a weak form of continuous functions called  $\Lambda$ -generalized continuous functions. Moreover, the relationships and properties of  $\Lambda$ -generalized continuous functions are obtained.

Throughout this paper, by  $(X, \tau)$  and  $(Y, \sigma)$  (or X and Y) we always mean topological spaces. Let A be a subset of X. We denote the interior, the closure and the complement of a set A by Int(A), Cl(A) and  $X \setminus A$  or  $A^c$ , respectively. A subset A of a space  $(X, \tau)$  is

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called  $\lambda$ -closed [1] if  $A = L \cap D$ , where L is a  $\Lambda$ -set and D is a closed set. The intersection of all  $\lambda$ -closed sets containing a subset A of X is called the  $\lambda$ -closure of A and is denoted by  $Cl_{\lambda}(A)$ . The complement of a  $\lambda$ -closed set is called  $\lambda$ -open. We denote the collection of all  $\lambda$ -open sets by  $\lambda O(X, \tau)$ .

Recall that a subset A of a topological space  $(X, \tau)$  is called generalized closed (briefly g-closed) [7] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is open in  $(X, \tau)$ . B is a g-open set of  $(X, \tau)$  if and only if  $B^c$  is g-closed.

**Definition 1** A subset A of a topological space  $(X, \tau)$  is called a  $\Lambda$ -generalized closed, briefly  $\Lambda_g$ -closed [3], (resp.  $\Lambda$ -g-closed,  $g\Lambda$ -closed) if  $Cl(A) \subseteq U$  (resp.  $Cl_{\lambda}(A) \subset U$ ,  $Cl_{\lambda}(A) \subset U$ ) whenever  $A \subset U$  and U is  $\lambda$ -open (resp. U is  $\lambda$ -open, U is open) in  $(X, \tau)$ .

**Remark 1.1** From the above definitions, we have the following.

- (1)  $\Lambda_g$ -closed sets and  $\lambda$ -closed sets are independent concepts.
- (2)  $\Lambda$ -g-closed sets and g-closed sets are independent concepts.
- (3)  $\lambda$ -closed sets and g-closed sets are also independent concepts.

From the above definitions and remark 1.1, we have the following diagram.

 $\begin{array}{cccc} closed & \Rightarrow & \Lambda_g\text{-}closed & \Rightarrow & g\text{-}closed \\ & \Downarrow & & \Downarrow & & \Downarrow \\ & \lambda\text{-}closed & \Rightarrow & \Lambda\text{-}g\text{-}closed & \Rightarrow & g\Lambda\text{-}closed \end{array}$ 

**Example 1.2** (i) Let  $X = \{a, b, c\}$  with a topology  $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ . Thus  $\lambda O(X, \tau) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ . Take  $A = \{a, c\}$ . Observe that A is a g-closed set but it is not  $\Lambda$ -g-closed.

(ii) Let  $X = \{a, b, c\}$  with a topology  $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ . Then,  $A = \{b\}$  is a  $\lambda$ -closed set but it is not g-closed.

(iii) Let  $X = \{a, b, c\}$  with a topology  $\tau = \{\emptyset, \{a\}, X\}$ . Then,  $A = \{a, b\}$  is a  $\Lambda_g$ -closed set but it is not  $\lambda$ -closed.

**Definition 2** A function  $f : (X, \tau) \to (Y, \sigma)$  is called :

(1) g-continuous [7] if  $f^{-1}(V)$  is g-closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ .

(2)  $\lambda$ -continuous [1] if  $f^{-1}(V)$  is  $\lambda$ -closed in  $(X, \tau)$  for every closed set V of  $(Y, \sigma)$ .

### **2** Λ-generalized continuous functions

We introduce the following notions:

**Definition 3** A function  $f : (X, \tau) \to (Y, \sigma)$  is called:

(1)  $\Lambda_g$ -continuous if  $f^{-1}(V)$  is  $\Lambda_g$ -closed in X, for every closed set in Y.

(2)  $\Lambda$ -g-continuous if  $f^{-1}(V)$  is  $\Lambda$ -g-closed in X, for every closed set in Y.

(3)  $g\Lambda$ -continuous if  $f^{-1}(V)$  is  $g\Lambda$ -closed in X, for every closed set in Y.

**Example 2.1** Let  $X = \{a, b, c, d\} = Y$ ,  $\tau = \{\phi, X, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$  and  $\sigma = \{\phi, Y, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$ . Define the function  $f : (X, \tau) \to (Y, \sigma)$  by f(a) = b = f(b), f(c) = c, f(d) = d. Then f is  $\Lambda_g$ -continuous,  $\Lambda$ -g-continuous and  $g\Lambda$ -continuous.

**Proposition 2.2** Every continuous function is  $\Lambda_g$ -continuous (resp.  $\Lambda$ -g-continuous,  $g\Lambda$ -continuous).

*Proof.* By [3], every closed set is  $\Lambda_g$ -closed (resp  $\Lambda$ -g-closed,  $g\Lambda$ -closed) and the proof follows.

**Example 2.3** Let  $X = Y = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$  and  $\sigma = \{\phi, Y, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}\}$ . Define the function  $f : (X, \tau) \to (Y, \sigma)$  by f(a) = f(b) = b, f(c) = c, f(d) = d. Then f is  $\Lambda_g$ -continuous,  $\Lambda$ -g-continuous and  $g\Lambda$ -continuous but not continuous.

**Proposition 2.4** Every  $\Lambda_g$ -continuous function is g-continuous.

*Proof.* It follows from the fact that every  $\Lambda_g$ -closed set is g-closed set [3].

**Example 2.5** The function f in Example 2.3 with  $\tau = \{\phi, X, \{b\}, \{b, c\}, \{a, b\}, \{a, b, c\}\},$  $\sigma = \{\phi, Y, \{a\}, \{a, c\}, \{a, b\}, \{a, b, c\}\}$  is g-continuous but not  $\Lambda_g$ -continuous since for the closed set  $U = \{b, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(U) = \{a, b, d\}$  which is not  $\Lambda_g$ -closed in  $(X, \tau)$ .

**Proposition 2.6** Every  $\lambda$ -continuous function and  $\Lambda_g$ -continuous function are  $\Lambda$ -g-continuous function.

*Proof.* By [3], every  $\lambda$ -closed set is  $\Lambda$ -g-closed set and every  $\Lambda_g$ -closed set is  $\Lambda$ -g-closed set, the proof follows.

**Example 2.7** Let  $(X, \tau)$  and  $(Y, \sigma)$  be as in Example 2.3.

(i) Define a function  $f : (X, \tau) \to (Y, \sigma)$  by f(a) = a, f(c) = c, f(b) = d = f(d). Then f is  $\Lambda$ -g-continuous but not  $\lambda$ -continuous since for the closed set  $U = \{c, d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(U) = \{b, c, d\}$  which is not  $\lambda$ -closed in  $(X, \tau)$ .

(ii) Define a function  $f : X \to Y$  by f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then f is  $\Lambda$ g-continuous but not  $\Lambda_g$ -continuous since for the closed set  $U = \{d\}$  in  $(Y, \sigma)$ ,  $f^{-1}(U) = \{c\}$ which is not  $\Lambda_g$ -closed in  $(X, \tau)$ .

**Remark 2.8** (1)  $\Lambda_q$ -continuous and  $\lambda$ -continuous are independent.

(2)  $\Lambda$ -g-continuous and g-continuous are independent.

(3)  $\lambda$ -continuous and g-continuous are independent.

**Example 2.9** (i) The function f in Example 2.7(i) is  $\Lambda_g$ -continuous but not  $\lambda$ -continuous. (ii) Let  $(X, \tau)$  and  $(Y, \sigma)$  be as in Example 2.5. Then f in Example 2.7(ii) is  $\lambda$ -continuous but not  $\Lambda_g$ -continuous.

(iii) f is  $\lambda$ -continuous but not g-continuous.

(iv) f is  $\Lambda$ -g-continuous but not g-continuous.

(v) Let  $(X, \tau)$  and  $(Y, \sigma)$  be as in Example 2.5 and the function f be an identity function from X to Y. Then f is g-continuous but neither  $\Lambda$ -g-continuous nor  $\lambda$ -continuous.

We get the following diagram:

 $\begin{array}{ccc} continuous & \Rightarrow & \Lambda_g\text{-}continuous & \Rightarrow & g\text{-}continuous \\ \\ \Downarrow & \qquad \end{array}$ 

 $\lambda$ -continuous  $\Rightarrow \Lambda$ -g-continuous  $\Rightarrow g\Lambda$ -continuous

## 3 Properties of $\Lambda$ -generalized continuous functions

**Theorem 3.1** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda_g$ -continuous and X is  $T_1$  then f is continuous.

Proof. Let f be  $\Lambda_g$ -continuous and X be  $T_1$ . Assume that V is closed in Y. Hence  $f^{-1}(V)$  is  $\Lambda_g$ -closed set in X. Since every  $\Lambda_g$ -closed is closed in a  $T_1$  space X [3], then  $f^{-1}(V)$  is closed set in X. This shows that f is continuous.

**Corollary 3.2** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda_g$ -continuous and X is  $T_1$  then f is  $\lambda$ -continuous.

**Theorem 3.3** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-continuous and X is  $T_0$  then f is  $\lambda$ -continuous.

Proof. Let f be  $\Lambda$ -g-continuous and X be  $T_0$ . Let V be closed in Y.  $f^{-1}(V)$  is  $\Lambda$ -g-closed in X. Since  $\Lambda$ -g-closed is  $\lambda$ -closed in a  $T_0$  space X [9], then  $f^{-1}(V)$  is  $\lambda$ -closed in X. This shows that f is  $\lambda$ -continuous.

**Definition 4** A function  $f : (X, \tau) \to (Y, \sigma)$  is said to be: (i)  $\Lambda_g$ -irresolute if  $f^{-1}(V)$  is  $\Lambda_g$ -closed in X for every  $\Lambda_g$ -closed set V in Y. (ii)  $\Lambda$ -g-irresolute if  $f^{-1}(V)$  is  $\Lambda$ -g-closed in X for every  $\Lambda$ -g-closed set V in Y. (iii)  $g\Lambda$ -irresolute if  $f^{-1}(V)$  is  $g\Lambda$ -closed in X for every  $g\Lambda$ -closed set V in Y.

Recall that a function  $f: (X, \tau) \to (Y, \sigma)$  is said to be  $\lambda$ -closed if f(F) is  $\lambda$ -closed in Y for every  $\lambda$ -closed set F of X.

**Lemma 3.4** [3]. A function  $f : (X, \tau) \to (Y, \sigma)$  is  $\lambda$ -closed if and only if for each subset B of Y and each  $U \in \lambda O(X, \tau)$  containing  $f^{-1}(B)$ , there exists  $V \in \lambda O(Y, \sigma)$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ .

**Theorem 3.5** Let  $f : (X, \tau) \to (Y, \sigma)$  be a continuous  $\lambda$ -closed function. Then f is  $\Lambda_g$ irresolute.

Proof. Let B be  $\Lambda_g$ -closed in  $(Y, \sigma)$  and U a  $\lambda$ -open set of  $(X, \tau)$  containing  $f^{-1}(B)$ . Since f is  $\lambda$ -closed, by Lemma 3.4 there exists a  $\lambda$ -open set V of  $(Y, \sigma)$  such that  $B \subset V$  and  $f^{-1}(V) \subset U$ . Since B is  $\Lambda_g$ -closed in  $(Y, \sigma)$ ,  $Cl(B) \subset V$  and hence  $f^{-1}(B) \subset f^{-1}(Cl(B)) \subset f^{-1}(V) \subset U$ . Since f is continuous,  $f^{-1}(Cl(B))$  is closed and hence  $Cl(f^{-1}(B)) \subset U$ . This shows that  $f^{-1}(B)$  is  $\Lambda_g$ -closed in  $(X, \tau)$ . Therefore f is  $\Lambda_g$ -irresolute.

**Theorem 3.6** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda_g$ -irresolute and Y is  $T_1$  then f is  $\Lambda_g$ -continuous.

*Proof.* Let f be  $\Lambda_g$ -irresolute and Y be  $T_1$ . Suppose V is  $\Lambda_g$ -closed in Y. Then  $f^{-1}(V)$  is  $\Lambda_g$ -closed set in X. Since Y is  $T_1$ , V is closed in Y. Thus f is  $\Lambda_g$ -continuous.

**Theorem 3.7** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-irresolute and Y is  $T_0$  then f is  $\Lambda$ -g-continuous.

*Proof.* Let f be  $\Lambda$ -g-irresolute, Y a  $T_0$  space and V be  $\Lambda$ -g-closed in Y. Then  $f^{-1}(V)$  is  $\Lambda$ -g-closed set in X. Since Y is  $T_0$ , V is closed in Y. Thus f is  $\Lambda$ -g-continuous.

**Theorem 3.8** If  $f : (X, \tau) \to (Y, \sigma)$  is a  $\lambda$ -g-irresolute bijection and f is  $\lambda$ -open, then f is  $\Lambda$ -g-irresolute.

Proof. Let V be  $\Lambda$ -g-closed and let  $f^{-1}(V) \subset U$ , where  $U \in \lambda O(X, \tau)$ . Clearly,  $V \subseteq f(U)$ . Since  $f(U) \in \lambda O(X, \tau)$  and since V is  $\Lambda$ -g-closed in Y, then  $Cl_{\lambda}(V) \subset f(U)$  and thus  $f^{-1}(Cl_{\lambda}(V)) \subset U$ . Since f is  $\lambda$ -irresolute and  $Cl_{\lambda}(V)$  is a  $\lambda$ -closed set, then  $f^{-1}(Cl_{\lambda}(V))$ is  $\lambda$ -closed in X. Thus  $Cl_{\lambda}(f^{-1}(V)) \subset Cl_{\lambda}(f^{-1}(Cl_{\lambda}(V))) = f^{-1}(Cl_{\lambda}(V)) \subset U$ . Therefore,  $Cl_{\lambda}(f^{-1}(V)) \subseteq U$ . So,  $f^{-1}(V)$  is  $\Lambda$ -g-closed and f is a  $\Lambda$ -g-irresolute bijection. **Definition 5** A topological space  $(X, \tau)$  is called:

(1) a  $T_g\Lambda$ -space if every  $g\Lambda$ -closed is g-closed.

(2) a  $T_{\Lambda_g}$ -space if every  $\Lambda$ -g-closed is  $\Lambda_g$ -closed.

Recall that a function  $f : (X, \tau) \to (Y, \sigma)$  is said to be *gc*-irresolute [2] if  $f^{-1}(V)$  is *g*-closed in X for every *g*-closed set V in Y. It is clear that a function  $f : (X, \tau) \to (Y, \sigma)$  is *gc*-irresolute if and only if  $f^{-1}(V)$  is *g*-open in X for every *g*-open set V in Y.

**Theorem 3.9** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda_g$ -irresolute and closed, then f is gc-irresolute.

*Proof.* It follows immediately from ([4], Proposition 2).

**Theorem 3.10** If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $g\Lambda$ -irresolute and X is a  $T_g\Lambda$ -space, then f is gc-irresolute.

*Proof.* Let f be  $g\Lambda$ -irresolute and V a g-closed set in X. Then V is  $g\Lambda$ -closed in Y. Since f is  $g\Lambda$ -irresolute,  $f^{-1}(V)$  is  $g\Lambda$ -closed in X. But X is a  $T_g\Lambda$ -space. Therefore  $f^{-1}(V)$  is g-closed in X and this implies that f is gc-irresolute.

**Remark 3.11** The condition that X is a  $T_g\Lambda$ -space cannot be omitted in above theorem as shown in the following example.

**Example 3.12** Let  $X = \{a, b, c, d\}, \tau = \{\phi, X, \{a\}, \{b, c\}, \{a, b, c\}, \{b, c, d\}\}$  and  $\sigma = \{\phi, Y, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}\}$ . Note that  $(X, \tau)$  is not a  $T_g\Lambda$ -space. Let  $f : (X, \tau) \rightarrow (Y, \sigma)$  be the function defined as follows f(a) = b, f(b) = a, f(c) = d and f(d) = c. Then fis  $g\Lambda$ -irresolute but not gc-irresolute, since  $f^{-1}(\{d\}) = \{c\}$  is not g-closed in  $(X, \tau)$ .

**Theorem 3.13** If a function function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-irresolute and X is a  $T_{\Lambda_q}$ -space then f is  $\Lambda_g$ -irresolute.

Proof. Let B be any  $\Lambda_g$ -closed set in Y. Then B is  $\Lambda$ -g-closed in Y. Since, f is  $\Lambda$ -g-irresolute, then  $f^{-1}(B)$  is  $\Lambda$ -g-closed in X. But X is  $T_{\Lambda_g}$ -space. Therefore  $f^{-1}(B)$  is  $\Lambda_g$ -closed in X which implies that f is  $\Lambda_g$ -irresolute.

**Remark 3.14** The condition that X is a  $T_{\Lambda_g}$ -space can not be omitted in Theorem 3.13 as it is shown in our next example.

**Example 3.15** Let f be as in Example 3.12. Then f is  $\Lambda$ -g-irresolute but not  $\Lambda_g$ -irresolute, where X is not  $T_{\Lambda_q}$ -space.  $f^{-1}(\{d\}) = \{c\}$  is not  $\Lambda_g$ -closed in  $(X, \tau)$ .

We recall that the space X is called a  $\lambda$ -space [1] if the set of all  $\lambda$ -open subsets form a topology on X. Clearly a space X is a  $\lambda$ - space if and only if the intersection of two  $\lambda$ -open sets is  $\lambda$ -open. An example of a  $\lambda$ -space is a  $T_{\frac{1}{2}}$ -space, where a space X is called  $T_{\frac{1}{2}}$  [5] if every singleton is open or closed.

**Theorem 3.16** If  $f_i : (X, \tau_i) \to (Y, \sigma_i) (i \in I)$  is a family of functions, where X is a  $\lambda$ -space and Y is any topological space, then every  $f_i$  is  $\Lambda$ -g-continuous.

*Proof.* It follows from ([9], Theorem 2.4).

**Theorem 3.17** (i) If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-continuous then  $f(Cl_{\lambda}(A)) \subset Cl_{\lambda}(f(A))$  for every A of X.

(ii) If a function  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-irresolute then for every subset A of X,  $f(Cl_{\Lambda-g}(A)) \subset Cl_{\lambda}(f(A))$  (where  $Cl_{\Lambda-g}(A)$  is the intersection of the smallest  $\Lambda$ -g-closed set containing A.)

*Proof.* (i) It follows from the fact that every  $\lambda$ -continuous is  $\Lambda_q$ -continuous.

(ii) If  $A \subset X$ , then consider  $Cl_{\lambda}(f(A))$  which is  $\lambda$ -closed in Y. Thus by Definition 4,  $f^{-1}Cl_{\lambda}(f(A))$  is  $\Lambda$ -g-closed in X. Furthermore,  $A \subset f^{-1}(f(A)) \subset f^{-1}(Cl_{\lambda}(f(A)))$ . Therefore  $Cl_{\Lambda-g}(A) \subset f^{-1}(Cl_{\lambda}(f(A)))$  and consequently,  $f(Cl_{\Lambda-g}(A)) \subset f(f^{-1}(Cl_{\lambda}(f(A)))) \subset Cl_{\lambda}(f(A))$ .

**Theorem 3.18** If a map  $f : X \to Y$  is  $\Lambda_g$ -irresolute, then it is  $\Lambda_g$ -continuous but not conversely.

*Proof.* Since every closed set is  $\Lambda_g$ -closed, it is proved that f is  $\Lambda_g$ -continuous. The converse need not be true as it is seen from the following example.

**Example 3.19** Let  $X = Y = \{a, b, c, d\}, \sigma = \{\phi, X, \{b\}, \{d\}, \{b, d\}\}, \tau = \{\phi, Y, \{a\}, \{b\}, \{a, b\}, \}.$ Define a function  $f : (X, \tau) \to (Y, \sigma)$  by f(a) = d = f(d), f(b) = b and f(c) = c. Then f is  $\Lambda_q$ -continuous but not  $\Lambda_q$ -irresolute.

**Theorem 3.20** Let  $(X, \tau)$  and  $(Z, \eta)$  be topological spaces and  $(Y, \sigma)$  be a  $T_1$  space. The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is  $\Lambda_g$ -continuous function where  $f : (X, \tau) \to (Y, \sigma)$  and  $g : (Y, \sigma) \to (Z, \eta)$  are  $\Lambda_g$ -continuous.

Proof. Let F be any closed set in Z. Since g is  $\Lambda_g$ -continuous,  $g^{-1}(F)$  is  $\Lambda_g$ -closed in Y. But Y is a  $T_1$ -space and so  $g^{-1}(F)$  is closed in Y. Since f is  $\Lambda_g$ -continuous,  $f^{-1}(g^{-1}(F))$  is  $\Lambda_g$ -closed in X. Hence,  $g \circ f$  is  $\Lambda_g$ -continuous.

**Theorem 3.21** Let  $(X, \tau)$  and  $(Z, \eta)$  be topological spaces and  $(Y, \sigma)$  be a  $T_1$  space. (1) The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is  $\lambda$ -continuous function where  $f : (X, \tau) \to (Y, \sigma)$ is  $\lambda$ -continuous and  $g : (Y, \sigma) \to (Z, \eta)$  is  $\Lambda_g$ -continuous.

(2) The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is g-continuous function where  $f : (X, \tau) \to (Y, \sigma)$ is g-continuous and  $g : (Y, \sigma) \to (Z, \eta)$  is  $\Lambda_g$ -continuous.

(3) The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is  $\Lambda$ -g-continuous function where  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda$ -g-continuous and  $g : (Y, \sigma) \to (Z, \eta)$  is  $\Lambda_g$ -continuous.

*Proof.* Similar to the proof of Theorem 3.20.

**Theorem 3.22** Let  $(X, \tau)$  and  $(Z, \eta)$  be any topological spaces and  $(Y, \sigma)$  be a  $T_0$  space. The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is  $\lambda$ -continuous function where  $f : (X, \tau) \to (Y, \sigma)$  is  $\lambda$ -irresolute and  $g : (Y, \sigma) \to (Z, \eta)$  is  $\Lambda$ -g-continuous.

Proof. Let V be any closed set in Z. Since g is  $\Lambda$ -g-continuous,  $g^{-1}(V)$  is  $\Lambda$ -g-closed in Y. But Y is a  $T_0$ -space and so  $g^{-1}(V)$  is  $\lambda$ -closed in Y. Since f is  $\lambda$ -irresolute,  $f^{-1}(g^{-1}(V))$  is  $\lambda$ -closed in X. Hence,  $g \circ f$  is  $\lambda$ -continuous.

**Theorem 3.23** Let  $(X, \tau)$  and  $(Z, \eta)$  be topological spaces and  $(Y, \sigma)$  be a  $T_g\Lambda$  space. The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is g-continuous function where  $f : (X, \tau) \to (Y, \sigma)$  is g-c-irresolute and  $g : (Y, \sigma) \to (Z, \eta)$  is g $\Lambda$ -continuous.

*Proof.* This follows from the definitions.

**Theorem 3.24** Let  $(X, \tau)$  and  $(Z, \eta)$  be topological spaces and  $(Y, \sigma)$  be a  $T_{\Lambda_g}$  space. The composition  $g \circ f : (X, \tau) \to (Z, \eta)$  is  $\Lambda_g$ -continuous function, where  $f : (X, \tau) \to (Y, \sigma)$  is  $\Lambda_g$ -irresolute and  $g : (Y, \sigma) \to (Z, \eta)$  is  $\Lambda$ -g-continuous.

*Proof.* This follows from definitions.

Recall that a space X is called locally indiscrete if and only if every open set is closed if and only if every  $\lambda$ -open set of X is open in X.

Finally, we get the following diagram:

where  $S_1$  is a locally indiscrete space.

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