# on $\Lambda$-generalized continuous functions* 

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#### Abstract

In this paper, we introduce a new class of continuous functions as an application of $\Lambda$-generalized closed sets (namely $\Lambda_{g}$-closed set, $\Lambda$ - $g$-closed set and $g \Lambda$-closed set) namely $\Lambda$-generalized continuous functions (namely $\Lambda_{g}$-continuous, $\Lambda$ - $g$-continuous and $g \Lambda$-continuous) and study their properties in topological space.


## 1 Introduction and Preliminaries

Levine [7] introduced g-closed set. Maki [8] introduced the notion of $\Lambda$-sets in topological spaces. A subset $A$ of a topological space $(X, \tau)$ is called a $\Lambda$-set if it coincides with its kernel (the intersection of all open supersets of A). In [1], Arenas et al. introduced the notions of $\lambda$-open sets, and $\lambda$-closed sets and presented fundamental results for these sets. They also introduced [1] $\lambda$ - continuity, which is weaker than continuity. Recently, M. Caldas, S. Jafari and T. Noiri [3] introduced $\Lambda$-generalized closed sets in topological space. The aim of this paper is to introduce a weak form of continuous functions called $\Lambda$-generalized continuous functions. Moreover, the relationships and properties of $\Lambda$-generalized continuous functions are obtained.

Throughout this paper, by $(X, \tau)$ and $(Y, \sigma)$ (or $X$ and $Y$ ) we always mean topological spaces. Let $A$ be a subset of $X$. We denote the interior, the closure and the complement of a set $A$ by $\operatorname{Int}(A), C l(A)$ and $X \backslash A$ or $A^{c}$, respectively. A subset $A$ of a space $(X, \tau)$ is

[^0]called $\lambda$-closed [1] if $A=L \cap D$, where $L$ is a $\Lambda$-set and $D$ is a closed set. The intersection of all $\lambda$-closed sets containing a subset $A$ of $X$ is called the $\lambda$-closure of $A$ and is denoted by $C l_{\lambda}(A)$. The complement of a $\lambda$-closed set is called $\lambda$-open. We denote the collection of all $\lambda$-open sets by $\lambda O(X, \tau)$.

Recall that a subset $A$ of a topological space ( $X, \tau$ ) is called generalized closed (briefly $g$-closed) [7] if $C l(A) \subset U$ whenever $A \subset U$ and $U$ is open in $(X, \tau) . B$ is a $g$-open set of $(X, \tau)$ if and only if $B^{c}$ is $g$-closed.

Definition 1 A subset $A$ of a topological space $(X, \tau)$ is called a $\Lambda$-generalized closed, briefly $\Lambda_{g}$-closed [3], (resp. $\Lambda$-g-closed, $g \Lambda$-closed) if $C l(A) \subseteq U\left(\right.$ resp. $\left.C l_{\lambda}(A) \subset U, C l_{\lambda}(A) \subset U\right)$ whenever $A \subset U$ and $U$ is $\lambda$-open (resp. $U$ is $\lambda$-open, $U$ is open) in $(X, \tau)$.

Remark 1.1 From the above definitions, we have the following.
(1) $\Lambda_{g}$-closed sets and $\lambda$-closed sets are independent concepts.
(2) $\Lambda$-g-closed sets and $g$-closed sets are independent concepts.
(3) $\lambda$-closed sets and $g$-closed sets are also independent concepts.

From the above definitions and remark 1.1, we have the following diagram.

$$
\begin{array}{ccccc}
\text { closed } & \Rightarrow & \Lambda_{g} \text {-closed } & \Rightarrow & g \text {-closed } \\
\Downarrow & & \Downarrow & & \Downarrow \\
\lambda \text {-closed } & \Rightarrow & \Lambda \text {-g-closed } & \Rightarrow & g \Lambda \text {-closed }
\end{array}
$$

Example 1.2 (i) Let $X=\{a, b, c\}$ with a topology $\tau=\{\emptyset,\{a\},\{a, b\}, X\}$. Thus $\lambda O(X, \tau)=$ $\{\emptyset,\{a\},\{c\},\{a, b\},\{a, c\},\{b, c\}, X\}$. Take $A=\{a, c\}$. Observe that $A$ is a $g$-closed set but it is not $\Lambda$-g-closed.
(ii) Let $X=\{a, b, c\}$ with a topology $\tau=\{\emptyset,\{a\},\{b\},\{a, b\}, X\}$. Then, $A=\{b\}$ is a $\lambda$ closed set but it is not $g$-closed.
(iii) Let $X=\{a, b, c\}$ with a topology $\tau=\{\emptyset,\{a\}, X\}$. Then, $A=\{a, b\}$ is a $\Lambda_{g}$-closed set but it is not $\lambda$-closed.

Definition 2 A function $f:(X, \tau) \rightarrow(Y, \sigma)$ is called:
(1) $g$-continuous [7] if $f^{-1}(V)$ is $g$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.
(2) $\lambda$-continuous [1] if $f^{-1}(V)$ is $\lambda$-closed in $(X, \tau)$ for every closed set $V$ of $(Y, \sigma)$.

## $2 \Lambda$-generalized continuous functions

We introduce the following notions:

Definition 3 A function $f:(X, \tau) \rightarrow(Y, \sigma)$ is called:
(1) $\Lambda_{g}$-continuous if $f^{-1}(V)$ is $\Lambda_{g}$-closed in $X$, for every closed set in $Y$.
(2) $\Lambda$ - $g$-continuous if $f^{-1}(V)$ is $\Lambda$-g-closed in $X$, for every closed set in $Y$.
(3) $g \Lambda$-continuous if $f^{-1}(V)$ is $g \Lambda$-closed in $X$, for every closed set in $Y$.

Example 2.1 Let $X=\{a, b, c, d\}=Y, \tau=\{\phi, X,\{b\},\{b, c\},\{a, b\},\{a, b, c\}\}$ and $\sigma=$ $\{\phi, Y,\{a\},\{b, c\},\{a, b, c\},\{b, c, d\}\}$. Define the function $f:(X, \tau) \rightarrow(Y, \sigma)$ by $f(a)=b=$ $f(b), f(c)=c, f(d)=d$. Then $f$ is $\Lambda_{g}$-continuous, $\Lambda$ - $g$-continuous and $g \Lambda$-continuous.

Proposition 2.2 Every continuous function is $\Lambda_{g}$-continuous (resp. $\Lambda$ - $g$-continuous, $g \Lambda$ continuous).

Proof. By [3], every closed set is $\Lambda_{g}$-closed (resp $\Lambda$ - $g$-closed, $g \Lambda$-closed) and the proof follows.

Example 2.3 Let $X=Y=\{a, b, c, d\}, \tau=\{\phi, X,\{a\},\{b, c\},\{a, b, c\},\{b, c, d\}\}$ and $\sigma=$ $\{\phi, Y,\{b\},\{b, c\},\{a, b\},\{a, b, c\}\}$. Define the function $f:(X, \tau) \rightarrow(Y, \sigma)$ by $f(a)=f(b)=b$, $f(c)=c, f(d)=d$. Then $f$ is $\Lambda_{g}$-continuous, $\Lambda$ - $g$-continuous and $g \Lambda$-continuous but not continuous.

Proposition 2.4 Every $\Lambda_{g}$-continuous function is $g$-continuous.
Proof. It follows from the fact that every $\Lambda_{g}$-closed set is $g$-closed set [3].

Example 2.5 The function $f$ in Example 2.3 with $\tau=\{\phi, X,\{b\},\{b, c\},\{a, b\},\{a, b, c\}\}$, $\sigma=\{\phi, Y,\{a\},\{a, c\},\{a, b\},\{a, b, c\}\}$ is $g$-continuous but not $\Lambda_{g}$-continuous since for the closed set $U=\{b, d\}$ in $(Y, \sigma), f^{-1}(U)=\{a, b, d\}$ which is not $\Lambda_{g}$-closed in $(X, \tau)$.

Proposition 2.6 Every $\lambda$-continuous function and $\Lambda_{g}$-continuous function are $\Lambda$ - $g$-continuous function.

Proof. By [3], every $\lambda$-closed set is $\Lambda$ - $g$-closed set and every $\Lambda_{g}$-closed set is $\Lambda$ - $g$-closed set, the proof follows.

Example 2.7 Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.3.
(i) Define a function $f:(X, \tau) \rightarrow(Y, \sigma)$ by $f(a)=a, f(c)=c, f(b)=d=f(d)$. Then $f$ is $\Lambda$-g-continuous but not $\lambda$-continuous since for the closed set $U=\{c, d\}$ in $(Y, \sigma)$, $f^{-1}(U)=\{b, c, d\}$ which is not $\lambda$-closed in $(X, \tau)$.
(ii) Define a function $f: X \rightarrow Y$ by $f(a)=b, f(b)=a, f(c)=d$ and $f(d)=c$. Then $f$ is $\Lambda$ -$g$-continuous but not $\Lambda_{g}$-continuous since for the closed set $U=\{d\}$ in $(Y, \sigma), f^{-1}(U)=\{c\}$ which is not $\Lambda_{g}$-closed in $(X, \tau)$.

Remark 2.8 (1) $\Lambda_{g}$-continuous and $\lambda$-continuous are independent.
(2) $\Lambda$-g-continuous and $g$-continuous are independent.
(3) $\lambda$-continuous and $g$-continuous are independent.

Example 2.9 (i) The function $f$ in Example 2.7(i) is $\Lambda_{g}$-continuous but not $\lambda$-continuous. (ii) Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.5. Then $f$ in Example 2.7(ii) is $\lambda$-continuous but not $\Lambda_{g}$-continuous.
(iii) $f$ is $\lambda$-continuous but not $g$-continuous.
(iv) $f$ is $\Lambda$-g-continuous but not $g$-continuous.
(v) Let $(X, \tau)$ and $(Y, \sigma)$ be as in Example 2.5 and the function $f$ be an identity function from $X$ to $Y$. Then $f$ is $g$-continuous but neither $\Lambda$ - $g$-continuous nor $\lambda$-continuous.

We get the following diagram:

```
continuous }=>\quad\mp@subsup{\Lambda}{g}{}\mathrm{ -continuous }=>g\mathrm{ -continuous
    \Downarrow
                                    \Downarrow
                                    \Downarrow
\lambda-continuous }=>|\Lambda\mathrm{ -g-continuous }=>g\\mathrm{ -continuous
```


## 3 Properties of $\Lambda$-generalized continuous functions

Theorem 3.1 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda_{g}$-continuous and $X$ is $T_{1}$ then $f$ is continuous.

Proof. Let $f$ be $\Lambda_{g}$-continuous and $X$ be $T_{1}$. Assume that $V$ is closed in $Y$. Hence $f^{-1}(V)$ is $\Lambda_{g}$-closed set in $X$. Since every $\Lambda_{g}$-closed is closed in a $T_{1}$ space $X$ [3], then $f^{-1}(V)$ is closed set in $X$. This shows that $f$ is continuous.

Corollary 3.2 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda_{g}$-continuous and $X$ is $T_{1}$ then $f$ is $\lambda$-continuous.

Theorem 3.3 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda$ - $g$-continuous and $X$ is $T_{0}$ then $f$ is $\lambda$-continuous.

Proof. Let $f$ be $\Lambda$ - $g$-continuous and $X$ be $T_{0}$. Let $V$ be closed in $Y . f^{-1}(V)$ is $\Lambda$ - $g$-closed in $X$. Since $\Lambda$ - $g$-closed is $\lambda$-closed in a $T_{0}$ space $X[9]$, then $f^{-1}(V)$ is $\lambda$-closed in $X$. This shows that $f$ is $\lambda$-continuous.

Definition 4 A function $f:(X, \tau) \rightarrow(Y, \sigma)$ is said to be:
(i) $\Lambda_{g}$-irresolute if $f^{-1}(V)$ is $\Lambda_{g}$-closed in $X$ for every $\Lambda_{g}$-closed set $V$ in $Y$.
(ii) $\Lambda$-g-irresolute if $f^{-1}(V)$ is $\Lambda$-g-closed in $X$ for every $\Lambda$ - $g$-closed set $V$ in $Y$.
(iii) $g \Lambda$-irresolute if $f^{-1}(V)$ is $g \Lambda$-closed in $X$ for every $g \Lambda$-closed set $V$ in $Y$.

Recall that a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is said to be $\lambda$-closed if $f(F)$ is $\lambda$-closed in $Y$ for every $\lambda$-closed set $F$ of $X$.

Lemma 3.4 [3]. A function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\lambda$-closed if and only if for each subset $B$ of $Y$ and each $U \in \lambda O(X, \tau)$ containing $f^{-1}(B)$, there exists $V \in \lambda O(Y, \sigma)$ such that $B \subset V$ and $f^{-1}(V) \subset U$.

Theorem 3.5 Let $f:(X, \tau) \rightarrow(Y, \sigma)$ be a continuous $\lambda$-closed function. Then $f$ is $\Lambda_{g^{-}}$ irresolute.

Proof. Let $B$ be $\Lambda_{g}$-closed in $(Y, \sigma)$ and $U$ a $\lambda$-open set of $(X, \tau)$ containing $f^{-1}(B)$. Since $f$ is $\lambda$-closed, by Lemma 3.4 there exists a $\lambda$-open set $V$ of $(Y, \sigma)$ such that $B \subset V$ and $f^{-1}(V) \subset U$. Since $B$ is $\Lambda_{g^{-}}$-closed in $(Y, \sigma), C l(B) \subset V$ and hence $f^{-1}(B) \subset f^{-1}(C l(B)) \subset$ $f^{-1}(V) \subset U$. Since $f$ is continuous, $f^{-1}(C l(B))$ is closed and hence $C l\left(f^{-1}(B)\right) \subset U$. This shows that $f^{-1}(B)$ is $\Lambda_{g}$-closed in $(X, \tau)$. Therefore $f$ is $\Lambda_{g}$-irresolute.

Theorem 3.6 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda_{g}$-irresolute and $Y$ is $T_{1}$ then $f$ is $\Lambda_{g}$-continuous.

Proof. Let $f$ be $\Lambda_{g}$-irresolute and $Y$ be $T_{1}$. Suppose $V$ is $\Lambda_{g}$-closed in $Y$. Then $f^{-1}(V)$ is $\Lambda_{g}$-closed set in $X$. Since $Y$ is $T_{1}, V$ is closed in $Y$. Thus $f$ is $\Lambda_{g}$-continuous.

Theorem 3.7 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda$ - $g$-irresolute and $Y$ is $T_{0}$ then $f$ is -g-continuous.

Proof. Let $f$ be $\Lambda$ - $g$-irresolute, $Y$ a $T_{0}$ space and $V$ be $\Lambda-g$-closed in $Y$. Then $f^{-1}(V)$ is $\Lambda-g$-closed set in $X$. Since $Y$ is $T_{0}, V$ is closed in $Y$. Thus $f$ is $\Lambda-g$-continuous.

Theorem 3.8 If $f:(X, \tau) \rightarrow(Y, \sigma)$ is a $\lambda$-g-irresolute bijection and $f$ is $\lambda$-open, then $f$ is $\Lambda$-g-irresolute.

Proof. Let $V$ be $\Lambda$ - $g$-closed and let $f^{-1}(V) \subset U$, where $U \in \lambda O(X, \tau)$. Clearly, $V \subseteq f(U)$. Since $f(U) \in \lambda O(X, \tau)$ and since $V$ is $\Lambda$ - $g$-closed in $Y$, then $C l_{\lambda}(V) \subset f(U)$ and thus $f^{-1}\left(C l_{\lambda}(V)\right) \subset U$. Since $f$ is $\lambda$-irresolute and $C l_{\lambda}(V)$ is a $\lambda$-closed set, then $f^{-1}\left(C l_{\lambda}(V)\right)$ is $\lambda$-closed in $X$. Thus $C l_{\lambda}\left(f^{-1}(V)\right) \subset C l_{\lambda}\left(f^{-1}\left(C l_{\lambda}(V)\right)\right)=f^{-1}\left(C l_{\lambda}(V)\right) \subset U$. Therefore, $C l_{\lambda}\left(f^{-1}(V)\right) \subseteq U$. So, $f^{-1}(V)$ is $\Lambda$ - $g$-closed and $f$ is a $\Lambda$ - $g$-irresolute bijection.

Definition 5 A topological space $(X, \tau)$ is called:
(1) a $T_{g} \Lambda$-space if every $g \Lambda$-closed is $g$-closed.
(2) a $T_{\Lambda_{g}}$-space if every $\Lambda$-g-closed is $\Lambda_{g}$-closed.

Recall that a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is said to be $g c$-irresolute [2] if $f^{-1}(V)$ is $g$-closed in $X$ for every $g$-closed set $V$ in $Y$. It is clear that a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $g c$-irresolute if and only if $f^{-1}(V)$ is $g$-open in $X$ for every $g$-open set $V$ in $Y$.

Theorem 3.9 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda_{g}$-irresolute and closed, then $f$ is gcirresolute.

Proof. It follows immediately from ([4], Proposition 2).
Theorem 3.10 If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $g \Lambda$-irresolute and $X$ is a $T_{g} \Lambda$-space, then $f$ is gc-irresolute.

Proof. Let $f$ be $g \Lambda$-irresolute and $V$ a $g$-closed set in $X$. Then $V$ is $g \Lambda$-closed in $Y$. Since $f$ is $g \Lambda$-irresolute, $f^{-1}(V)$ is $g \Lambda$-closed in $X$. But $X$ is a $T_{g} \Lambda$-space. Therefore $f^{-1}(V)$ is $g$-closed in $X$ and this implies that $f$ is $g c$-irresolute.

Remark 3.11 The condition that $X$ is a $T_{g} \Lambda$-space cannot be omitted in above theorem as shown in the following example.

Example 3.12 Let $X=\{a, b, c, d\}, \tau=\{\phi, X,\{a\},\{b, c\},\{a, b, c\},\{b, c, d\}\}$ and $\sigma=\{\phi, Y,\{b\},\{a, b\},\{b, c\},\{a, b, c\}\}$. Note that $(X, \tau)$ is not a $T_{g} \Lambda$-space. Let $f:(X, \tau) \rightarrow$ $(Y, \sigma)$ be the function defined as follows $f(a)=b, f(b)=a, f(c)=d$ and $f(d)=c$. Then $f$ is $g \Lambda$-irresolute but not gc-irresolute, since $f^{-1}(\{d\})=\{c\}$ is not $g$-closed in $(X, \tau)$.

Theorem 3.13 If a function function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda$-g-irresolute and $X$ is a $T_{\Lambda_{g}}$-space then $f$ is $\Lambda_{g}$-irresolute.

Proof. Let $B$ be any $\Lambda_{g}$-closed set in $Y$. Then $B$ is $\Lambda$ - $g$-closed in $Y$. Since, $f$ is $\Lambda$ -$g$-irresolute, then $f^{-1}(B)$ is $\Lambda$ - $g$-closed in $X$. But $X$ is $T_{\Lambda_{g}}$-space. Therefore, $f^{-1}(B)$ is $\Lambda_{g}$-closed in $X$ which implies that $f$ is $\Lambda_{g}$-irresolute.

Remark 3.14 The condition that $X$ is a $T_{\Lambda_{g}}$-space can not be omitted in Theorem 3.13 as it is shown in our next example.

Example 3.15 Let $f$ be as in Example 3.12. Then $f$ is $\Lambda$ - $g$-irresolute but not $\Lambda_{g}$-irresolute, where $X$ is not $T_{\Lambda_{g}}$-space. $f^{-1}(\{d\})=\{c\}$ is not $\Lambda_{g}$-closed in $(X, \tau)$.

We recall that the space $X$ is called a $\lambda$-space [1] if the set of all $\lambda$-open subsets form a topology on $X$. Clearly a space $X$ is a $\lambda$ - space if and only if the intersection of two $\lambda$-open sets is $\lambda$-open. An example of a $\lambda$-space is a $T_{\frac{1}{2}}$-space, where a space $X$ is called $T_{\frac{1}{2}}$ [5] if every singleton is open or closed .

Theorem 3.16 If $f_{i}:\left(X, \tau_{i}\right) \rightarrow\left(Y, \sigma_{i}\right)(i \in I)$ is a family of functions, where $X$ is a $\lambda$-space and $Y$ is any topological space, then every $f_{i}$ is $\Lambda$ - $g$-continuous.

Proof. It follows from ([9], Theorem 2.4).
Theorem 3.17 (i) If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda$-g-continuous then $f\left(C l_{\lambda}(A)\right) \subset$ $C l_{\lambda}(f(A))$ for every $A$ of $X$.
(ii) If a function $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda$ - $g$-irresolute then for every subset $A$ of $X$, $f\left(C l_{\Lambda-g}(A)\right) \subset C l_{\lambda}(f(A))$ (where $C l_{\Lambda-g}(A)$ is the intersection of the smallest $\Lambda$ - $g$-closed set containing $A$.)

Proof. (i) It follows from the fact that every $\lambda$-continuous is $\Lambda_{g}$-continuous.
(ii) If $A \subset X$, then consider $C l_{\lambda}(f(A))$ which is $\lambda$-closed in $Y$. Thus by Definition 4, $f^{-1} C l_{\lambda}(f(A))$ is $\Lambda$ - $g$-closed in $X$. Furthermore, $A \subset f^{-1}(f(A)) \subset f^{-1}\left(C l_{\lambda}(f(A))\right)$. Therefore $C l_{\Lambda-g}(A) \subset f^{-1}\left(C l_{\lambda}(f(A))\right)$ and consequently, $f\left(C l_{\Lambda-g}(A)\right) \subset f\left(f^{-1}\left(C l_{\lambda}(f(A))\right)\right) \subset$ $C l_{\lambda}(f(A))$.

Theorem 3.18 If a map $f: X \rightarrow Y$ is $\Lambda_{g}$-irresolute, then it is $\Lambda_{g}$-continuous but not conversely.

Proof. Since every closed set is $\Lambda_{g}$-closed, it is proved that $f$ is $\Lambda_{g}$-continuous. The converse need not be true as it is seen from the following example.

Example 3.19 Let $X=Y=\{a, b, c, d\}, \sigma=\{\phi, X,\{b\},\{d\},\{b, d\}\}, \tau=\{\phi, Y,\{a\},\{b\},\{a, b\}$,$\} .$
Define a function $f:(X, \tau) \rightarrow(Y, \sigma)$ by $f(a)=d=f(d), f(b)=b$ and $f(c)=c$. Then $f$ is $\Lambda_{g}$-continuous but not $\Lambda_{g}$-irresolute.

Theorem 3.20 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_{1}$ space. The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $\Lambda_{g}$-continuous function where $f:(X, \tau) \rightarrow(Y, \sigma)$ and $g:(Y, \sigma) \rightarrow(Z, \eta)$ are $\Lambda_{g}$-continuous.

Proof. Let $F$ be any closed set in $Z$. Since $g$ is $\Lambda_{g}$-continuous, $g^{-1}(F)$ is $\Lambda_{g}$-closed in $Y$. But $Y$ is a $T_{1}$-space and so $g^{-1}(F)$ is closed in $Y$. Since $f$ is $\Lambda_{g}$-continuous, $f^{-1}\left(g^{-1}(F)\right)$ is $\Lambda_{g}$-closed in $X$. Hence, $g \circ f$ is $\Lambda_{g}$-continuous.

Theorem 3.21 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_{1}$ space.
(1) The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $\lambda$-continuous function where $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\lambda$-continuous and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $\Lambda_{g}$-continuous.
(2) The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $g$-continuous function where $f:(X, \tau) \rightarrow(Y, \sigma)$ is $g$-continuous and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $\Lambda_{g}$-continuous.
(3) The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $\Lambda$-g-continuous function where $f:(X, \tau) \rightarrow$ $(Y, \sigma)$ is $\Lambda$-g-continuous and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $\Lambda_{g}$-continuous.

Proof. Similar to the proof of Theorem 3.20.
Theorem 3.22 Let $(X, \tau)$ and $(Z, \eta)$ be any topological spaces and $(Y, \sigma)$ be a $T_{0}$ space. The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $\lambda$-continuous function where $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\lambda$-irresolute and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $\Lambda$ - $g$-continuous.

Proof. Let $V$ be any closed set in $Z$. Since $g$ is $\Lambda$ - $g$-continuous, $g^{-1}(V)$ is $\Lambda-g$-closed in $Y$. But $Y$ is a $T_{0}$-space and so $g^{-1}(V)$ is $\lambda$-closed in $Y$. Since $f$ is $\lambda$-irresolute, $f^{-1}\left(g^{-1}(V)\right)$ is $\lambda$-closed in $X$. Hence, $g \circ f$ is $\lambda$-continuous.

Theorem 3.23 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_{g} \Lambda$ space. The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $g$-continuous function where $f:(X, \tau) \rightarrow(Y, \sigma)$ is $g c$-irresolute and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $g \Lambda$-continuous.

Proof. This follows from the definitions.

Theorem 3.24 Let $(X, \tau)$ and $(Z, \eta)$ be topological spaces and $(Y, \sigma)$ be a $T_{\Lambda_{g}}$ space. The composition $g \circ f:(X, \tau) \rightarrow(Z, \eta)$ is $\Lambda_{g}$-continuous function, where $f:(X, \tau) \rightarrow(Y, \sigma)$ is $\Lambda_{g}$-irresolute and $g:(Y, \sigma) \rightarrow(Z, \eta)$ is $\Lambda$ - $g$-continuous.

Proof. This follows from definitions.

Recall that a space $X$ is called locally indiscrete if and only if every open set is closed if and only if every $\lambda$-open set of $X$ is open in $X$.

Finally, we get the following diagram:

$$
\text { continuous } \Rightarrow \Lambda_{g} \text {-continuous } \Rightarrow g \text {-continuous }
$$

$$
\begin{array}{ccc}
S_{1} \Uparrow & T_{\Lambda_{g}} \Uparrow & T_{g} \Uparrow \\
\lambda \text {-continuous } \Rightarrow
\end{array} \Rightarrow \begin{gathered}
\text { - } g \text {-continuous }
\end{gathered} \Rightarrow \begin{aligned}
& g \Lambda \text {-continuous }
\end{aligned}
$$

where $S_{1}$ is a locally indiscrete space.

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