The proton radius explained

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1. There is a precise CODATA value for the magnetic moment of a proton:

$$\mu = 1.41060679736\times10^{-26}\ \mathrm{J{\cdot}T^{-1}} \pm 0.00000000060\ \mathrm{J{\cdot}T^{-1}}$$

We also have a CODATA value for the proton mass:

$$m = 1.67262192369\times10^{-27}\ \mathrm{kg} \pm 0.00000000051\times10^{-27}\ \mathrm{kg}$$

These are *recommended* values, for use in elementary calculations, based on very precise *experiments*. In contrast, since the 2019 revision of SI units, the proton charge – which is, of course, nothing but the positive elementary charge[1], has been *defined* as:

$$q_e = 1.602176634\times10^{-19}\ \mathrm{C}$$

That's an exact value. As precise as the other *defined* values of Nature's constants, including but not limited to the speed of light (c), Planck's constant (h), and the fine-structure constant (α).[2] Hence, if we have three *given* constants (q_e, h, and c), what's the third *measured* value we should get out of them—apart from μ and m, that is? It's the proton radius which, recently, has been re-measured and is now supposed to be equal to[3]:

$$r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}\ \mathrm{fm}$$

[1] Stating that the charge of the proton is the positive or negative of the electron charge is stating the obvious, and then it is not. We have two very different elementary particles here, and the fact that they have the same (electric) charge – and not approximately but *exactly* – is, from an epistemological perspective, quite deep. It may, in fact, be the only reason why we can, perhaps, figure out how the Universe works—one day, that is.

[2] You will say the fine-structure constant has no exact value. You are right: there is still a uncertainty of 1.5×10^{-19} on it, which it shares with the electric and magnetic constants. I must assume that is just the uncertainty in the calculation because, since the 2019 revision of SI units, the fine-structure constant has now also been *defined* as:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}$$

The formula shows why any uncertainty in α is related to the same uncertainty in ε_0, and then ε_0 and μ_0 are, of course related through the $c^2 = 1/\varepsilon_0\mu_0$ relation. These two constants should, therefore, have the same standard error too. For a discussion of the physical meaning of the fine-structure constant as part of a larger realist interpretation of quantum physics, we may refer the reader to https://vixra.org/abs/1812.0273. I like to think of q_e, ħ and c as the *most fundamental* constants in Nature. In contrast, α, ε_0 and μ_0 are constants that can be directly *derived* from them using the relations above. As such, they are probably just another way of representing the same.

[3] We assume the reader is aware of the new measurement done by the PRad (proton radius) team using the Continuous Electron Beam Accelerator Facility (CEBAF) at Jefferson Lab. For an introductory discussion to the matter on hand here, we refer the reader to https://vixra.org/abs/2001.0513.

Hence, we have three constants and three variables. Now we need three equations, don't we? Let's start with the first one.

2. If we imagine this magnetic moment to be created by a circular current of the elementary charge, then it will be equal to the current times the area of the loop. The current itself will be the product of the charge (+q_e) and the frequency ($f = \omega/2\pi$). We, therefore, get the following easy formula:

$$\mu = I\pi a^2 = q_e f \pi a^2 = \frac{q_e \omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{q_e \omega}}$$

So far, so good. We now need to make an assumption about the frequency. This is where the *hocus-pocus* starts. The various crackpot theories we've entertained have one thing in common: we believe the Planck-Einstein relation ($E = h \cdot f = \hbar \cdot \omega$) reflects a fundamental *cycle*, and so we believe it also applies to this ring current idea. Hence, we write:

$$a = \sqrt{\frac{2\mu}{q_e \omega}} = \sqrt{\frac{2\mu\hbar}{q_e E}}$$

Of course, the question is: what energy should we use? For the electron we used the $E = mc^2$ formula – based on the assumption that all of the mass of the electron is the equivalent mass of the energy of the oscillation of the (elementary) pointlike electric charge – but a proton combines electric and strong charge. Hence, perhaps half of its energy (or mass) is to be explained by the (electric) current ring while the other is to be explained by the oscillation of the strong charge.[4] Hence, perhaps we should write: $E/2 = \hbar \cdot \omega$. Why half? I am not sure, but I am thinking of the energy equipartition theorem from kinetic gas theory here. However, you are right: perhaps we should generalize and write something like: $E/n = \omega \cdot \hbar$.

It may also be possible an oscillation packs several units (h) of physical action, so we should – perhaps – write $E = n \cdot h \cdot f = n \cdot \hbar \cdot \omega$. Combining this and the energy equipartition theorem, it seems to make sense to write ω as a $1/n$ fraction of $E/\hbar$:

$$\omega = \frac{1}{n}\frac{E}{\hbar}$$

What value should we use for n? Based on some rather weird results obtained in a previous paper[5], we suggest assuming the motion of the pointlike elementary charge represents only 1/4 ($0.25 = 0.5 \times 0.5$) of the total energy of the proton. Where is the other half? We think it's in the electromagnetic field that's being generated—the fields that, according to *Zitterbewegung* theorists[6], keep this ring current going. For the electron, Hestenes sums this up as follows:

[4] One should think of some equivalent of the *Zitterbewegung* motion of the electric charge here. Perhaps it has the same *structure*, perhaps not.

[5] See Annex I of our first paper on the PRad experiment (https://vixra.org/abs/2001.0513).

[6] The *Zitterbewegung* model assumes an electron consists of a pointlike charge whizzing around some center. The *rest* mass of the pointlike charge is zero, which is why its velocity is equal to the speed of light. However, because of its motion, it acquires an *effective* mass – pretty much like a photon, which has mass because of its motion. One can show the effective mass of the pointlike charge – which is a relativistic mass concept – is half the rest mass of the electron: $m_\gamma = m_e/2$. The concept goes back to Alfred Lauck Parson (1915) and Erwin Schrödinger, who

> "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field. Half of it is magnetic potential energy and half is kinetic."[7]

So let us do an actual calculation – effectively inserting an *n* = 4 factor into our equation – and see what we get:

$$a = \sqrt{\frac{2\mu}{q_e\omega}} = \sqrt{\frac{4 \cdot 2\mu\hbar}{q_e E}} = 2 \cdot \sqrt{\frac{2\mu\hbar}{q_e mc^2}} \approx 2 \cdot 0.35146 \ldots \times 10^{-15} \approx 0.703 \text{ fm}$$

Does this make sense? Maybe. Maybe not. The range is OK, but the gap between 0.831 and 0.703 is puzzling. How can we *fix* this?[8] How can we relate the two values? It turns out that the insertion of a √2 factor does the trick. Check it:

$$a = \sqrt{\frac{\sqrt{2} \cdot 2\mu}{q_e\omega}} = 2 \cdot \sqrt{\frac{2\sqrt{2}\mu\hbar}{q_e E}} \approx 0.8359278 \text{ fm}$$

The difference between this calculated value (which used all of the precision of the CODATA values) and the PRad result is only about 0.005 fm[9], which is well within the statistical standard error of the measurement. In other words: it is a pretty good result.

3. Of course, the question is: how can we motivate the insertion of a √2 factor? We are, effectively, using some *new* value for the magnetic moment here, which we'll write as μ_L:

$$\mu_L = \sqrt{2} \cdot (1.41060679736 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}) \approx 1.995 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}$$

The subscript L in the μ_L notation stands for (orbital) angular momentum. We thought it was a good subscript to use because it reminds us of the (orbital) angular momentum one would effectively associate with the circular motion. More importantly, we suddenly remembered that a magnetic dipole will *precess* when placed in a magnetic field—which is, of course, what you do when measuring the magnetic moment of a proton.

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stumbled upon the idea about 15 years later (1930) while exploring solutions to Dirac's wave equation for free electrons. It's always worth quoting Dirac's summary of it: "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. *It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment.*" (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

[7] Email from Dr. David Hestenes to the author dated 17 March 2019.

[8] We have no qualms about *fixing* calculations. If anything, this paper may inspire the reader to develop some kind of toolbox to toy with some models himself!

[9] 0.831 – 0.836 = 0.005. We showed seven digits so as to illustrate with the value we will get out of another radius calculation, which we will present in the next section(s).

This precession is usually referred to as the *Larmor* precession and, in the context of quantum mechanics, its analysis is somewhat weird. We should, of course, say a few words about it and we, therefore, refer to Richard Feynman's rather wonderful analysis of the difference between the magnitude of a classical vector and its quantum-mechanical equivalent in his discussion of the concept of angular momentum in quantum mechanics[10]. He there shows that, for spin-1/2 particles, we can relate the magnitude of the actual – or imagined?[11] – angular momentum of a precessing magnet (L) and L_z (the *measured* quantum value) as:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{1/2(1/2+1)}}{1/2} = 2 \cdot \sqrt{\frac{3}{4}} = \sqrt{3}$$

You'll say: a √3 factor is *not* a √2 factor. You are right: we only get the 'right' factor for $j = 1$, as shown below:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{1(1+1)}}{1} = \sqrt{2}$$

We cannot possibly believe a proton is a spin-one particle, can we? Probably not. However, the reader will have to admit the relation between the magnetic moment and the angular momentum and the *spin* number – we are talking about the *g*-factor here – depends on a *form factor*. So as to ensure the reader understands what we are talking about, we will briefly remind him of our electron model[12], in which we assumed the mass or energy of the electron is, somehow, spread over the circular disk. That allows us to use the 1/2 form factor for the moment of inertia (I), which gives us the correct spin number (j = ½):

$$\mathrm{L} = I \cdot \omega = \frac{ma^2}{2}\frac{c}{a} = \frac{mc}{2}\frac{\hbar}{mc} = \frac{\hbar}{2}$$

We can combine this formula with the formula for the magnetic moment, which is – once again – just the current times the area of the loop:

$$\mu = \mathrm{I} \cdot \pi a^2 = \mathrm{q_e}\frac{\mathrm{m}c^2}{h} \cdot \pi a^2 = \mathrm{q_e} c \frac{\pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

We now get the 'correct' g-factor for the spin of an electron[13]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

Another form factor would have resulted in a $j = 1$ spin number, and a different g-factor as well. We, therefore, do not attach too much importance to discussions on spin numbers and g-factors—and, yes,

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[10] See: https://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S7

[11] The difference between actual and imagined here depends on your interpretation of quantum mechanics. From what we presented so far, it should be obvious to the reader that we like to think this magnitude is something real. However, such metaphysical questions should not be the concern of the reader now.

[12] See our model of the electron as a harmonic electromagnetic oscillator: https://vixra.org/abs/1905.0521.

[13] To remind you of the *physicality* of what we are discussing here, we briefly switched to vector notation—which you can see from the **boldface** notation in the equation. We, therefore, had to insert a *minus* sign: in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions.

we do know that is an offense that's worse than presenting crackpot theories: it should probably be qualified as sacrilegious.[14] We will, therefore, come back to this discussion at some later stage. For the time being, however, we request the reader to just go along and accept our √2 factor may be justified because of precession.

4. Because of the apparent randomness of this √2 factor, we should, perhaps, try another approach. Do we have one? Yes. In fact, the approach below is far more *direct*—not invoking Planck-Einstein relation or energy equipartition, that is. In fact, it was what made us see this weird √2 factor in the very first place.[15] If the elementary charge is really pointlike and rotating around, then the frequency will be equal to its velocity (v) divided by the circumference of the loop ($2\pi a$). The quintessential assumption here is that the elementary charge itself has *zero* rest mass: it is just a *naked* charge, with no other properties but its charge.[16] Its velocity will, therefore, be equal to the speed of light.[17]

The magnetic moment can then, once again, be calculated as the product of the current (I) times the surface area of the loop (πa^2). However, we now use a much more straightforward formula for the calculation of the frequency in the current formula: $f = c/2\pi a$, i.e. the velocity of the charge divided by the circumference of the loop. We get the following result:

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[14] We are in the business of building a DIY kit for a proton model here, so we do not worry about conventional wisdom. However, as this is a footnote only, we may want to state some actual opinion here, which is that we *really* do not worry here. The distinction between spin-1 and spin-1/2 particles is supposed to be fundamental because of the supposedly quintessential distinction between bosons and fermions in physics: particles versus force carriers. We think this distinction is dogmatic. Worse, we think it is hampering a good understanding of what might actually be going on. Why? Several reasons, but the most obvious one is the following: the only boson which we have firm evidence of is a photon. For all other bosons, we only have indirect 'evidence': signals, traces, two- or three-jet events that may or may not corroborate the hypothesis of virtual particles being actually real (as opposed to intermediary mathematical constructs). Unfortunately, a real-life photon (not those imaginary virtual photons that are supposed to mediate the electromagnetic force) lacks an essential bosonic property: they have no zero-spin state. This is one of the things I never understood. All courses on quantum mechanics – think of Feynman's treatment of the difference between bosons and fermions here (http://www.feynmanlectures.caltech.edu/III_04.html) – devote plenty of space to the theoretical distinction between fermions and bosons but, when it comes to specifics, then the only boson we actually know (the photon) turns out to not be a typical boson because it cannot have zero spin. This observation actually led us to explore an alternative (read: non-mysterious) explanation of one-photon Mach-Zehnder interference (http://vixra.org/abs/1812.0455).

[15] See the first version(s) of our paper on the PRad experiment (https://vixra.org/abs/2001.0513), in which we just toyed around with various factors before taking them somewhat seriously ourselves. For the record, we were encouraged by Prof. Dr. Ashot Gasparian (JLAB's PRad team spokesman) himself after we had sent him these strange numbers, to which he replied: "Certainly your approach and the numbers are very interesting. […] We will distribute your email to our students and postdocs and see if they have any numerical comments on your observations." I hope this paper can serve as some additional inspiration to what – perhaps – might become an actual realist interpretation of what a proton might actually *be*.

[16] As such, some authors (e.g. Burinskii) have referred to it as a toroidal photon, or an electron photon. However, we don't like these terms because they are not only imprecise but also misleading: photons are not supposed to carry any charge. For the record, Prof. Dr. Alex Burinskii told us he (also) no longer uses these terms.

[17] The relativistically correct force formula tells us any force will give a charge with zero rest mass an infinite acceleration. We refer to Annex II of our previous paper (https://vixra.org/abs/2001.0513) for a more philosophical reflection on this rather weird mathematical fact.

$$\mu_{\mathrm{L}} = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e} c}{2} a$$

$$\Leftrightarrow a = \frac{2\mu_{\mathrm{L}}}{\mathrm{q_e} c} = \sqrt{2}\cdot\frac{2\mu}{\mathrm{q_e} c} = \sqrt{2}\cdot 0.587\times 10^{-15} \approx 0.83065344\ldots\,\mathrm{fm}$$

You should note this result differs from the result we obtained using the Planck-Einstein relation for the frequency calculation. It's a *very* small difference. To be precise, it is, again, of the order of 0.005 fm. At the same time, this result is even closer to the 0.831 PRad value: the difference is 0.000346656… fm only, which is less than 5% of the standard error of the PRad point estimate (0.007 fm).[18]

5. In our calculations, we used the CODATA value for the magnetic moment of a proton in two different formulas for the radius, and we found the result is slightly different. While the two values do not differ significantly from the experimentally measured value for the proton radius – and, thereby, may be seen as a confirmation of the relevance of the PRad experiment – we may wonder: can we find some *absolute* theoretical value for the magnetic moment?

Because we have two equations for the radius *a* – and both of them involve μ_L – we should probably see if anything would come out when we would *equate* them:

$$a = 2\cdot\sqrt{\frac{2\mu_{\mathrm{L}}\hbar}{\mathrm{q_e E}}} = \frac{2\mu_{\mathrm{L}}}{\mathrm{q_e} c} \Leftrightarrow \sqrt{\frac{2\mu_{\mathrm{L}}\hbar\cdot \mathrm{q_e^2} c^2}{\mathrm{q_e E}\cdot\mu_{\mathrm{L}}^2}} = 1$$

$$\Leftrightarrow \mu_{\mathrm{L}} = \frac{2\mathrm{q_e}}{\mathrm{m}}\hbar \approx 2.02035\times 10^{-26}\,\mathrm{J\cdot T^{-1}}$$

It is all perfect ! We even have a slight anomalous magnetic moment (we get a value that is *almost* 2, but not quite) which – we suspect – can be explained in very much the same way as the electron's anomalous magnetic moment.[19]

6. We can now calculate an exact theoretical value for the proton radius:

$$\boxed{a = \frac{2\mu_{\mathrm{L}}}{\mathrm{q_e} c} = \frac{2}{\mathrm{q_e} c}\cdot\frac{2\mathrm{q_e}\hbar}{\mathrm{m}} = 4\cdot\frac{\hbar}{\mathrm{m}c} \approx 4\cdot(0.21\ldots\,\mathrm{fm}) \approx 0.8413564\ldots\ \mathrm{fm}}$$

This value is not within the 0.831 ± 0.007 fm interval, but it is well within the wider $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm interval.

We may be accused of being a crackpot theorist, but we do think we just offered a fairly convincing theoretical proton model matching the PRad measurement.

7. We promised we would say a few words about spin numbers and g-factors. Let us do that now.

Quantum-mechanical spin is expressed – and, more importantly, also *measured* in real-life experiments (such as the Stern-Gerlach experiment, with which you should be familiar) – in units of ħ/2, and we are

[18] We invite the reader to check our calculations and inform us of any mistake they would find.
[19] See our paper on Consa's calculations, which also references our paper(s) on the topic: https://vixra.org/abs/2001.0264.

told that we should *not* try to think of it as a classical property—as something that has some physical meaning. It's just that weird number, right? We obviously don't think so. We think we can just use the classical L = *I*·ω expression and substitute *I* and ω for the angular mass (also known as the moment of inertia) and the angular frequency.[20] To calculate the angular mass, we need a form factor. A hoop, a disk, a sphere or a shell all have different form factors. Our electron model assumes that the effective mass of the electron is spread over a circular disk. We can, therefore, calculate the angular momentum as:

$$\mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{\hbar}{mc} = \frac{\hbar}{2}$$

So, yes, an electron is a spin-1/2 particle. However, we do not think of this some obscure 'intrinsic' property of an equally obscure 'pointlike' particle: we've got a disk-like structure and there's a *torque* on it, so we've got angular momentum. Likewise, we think of the magnetic moment as being equally real[21]:

$$\mu = \mathrm{I} \cdot \pi a^2 = \frac{\mathrm{q_e}c \cdot \pi a^2}{2\pi a} = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

In fact, as far as I know, we cannot directly measure the angular momentum: in real-life experiments, we measure the *magnetic moment*. It is, in that sense, more *real* than the angular momentum. The point is: we can combine the two formulas to get the g-factor that's associated with the spin of an electron[22]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

So what do we get when we use our new formulas for the proton? We cannot be sure because, so far, we did not advance any form factor: the idea of a current ring – and the idea of precession, of course – strongly suggests we should, once again, think of the proton as a disk-like structure. However, not all of the mass is in the electromagnetic oscillation: we think half of it remains to be explained by some mysterious strong charge. We will, therefore, use a 1/4 rather than a 1/2 factor in the angular mass formula. Let us see where it leads us:

$$\mathrm{L_p} = I_\mathrm{p} \cdot \omega = \frac{\mathrm{m_p}r_\mathrm{p}^2}{4} \cdot \frac{c}{r_\mathrm{p}} = \frac{\mathrm{m_p}c}{4} \cdot r_\mathrm{p} = \frac{\mathrm{m_p}c}{4} \cdot \frac{4\hbar}{\mathrm{m_p}c} = \hbar$$

Our 'spin number' is equal to one. Academics will cry wolf here: we cannot possibly believe a proton is a spin-one particle, can we? We can. We think there is no need for the concept of a spin number and a g-factor in our *realist* interpretation of quantum mechanics. We think of the angular momentum and the magnetic moment as being *real* and, hence, whatever else you want to calculate – be it a spin number or a g-factor – is not very relevant. Worse, we think it *confuses* rather than clarifies the analysis.

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[20] Please do *not* confuse *I* and I. The first (*I* in *italics*) stands for angular mass (expressed in kg·m^2), while the second (I, normal type) is the symbol for current (expressed in C/s). We could have used different symbols, but we wanted to stick to the usual conventions. The reader should also not confuse the concepts of angular mass (*I*) and angular momentum (L), but that is quite clear from what follows.

[21] We use the Compton radius of an electron as the radius of the current ring. We have explained the theoretical justification elsewhere.

[22] The *minus* sign is there because, in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions.

Having said that, our calculation of L_p is wonderfully consistent with the use of the √2 factor – as opposed to a √3 factor – to calculate what we think is the *real* magnetic moment of a proton (μ_p).

Let us, therefore, continue to follow convention and calculate some g-factor. Instead of the *Bohr magneton* $\mu_B = q_e\hbar/2m_e$, we should now use the *nuclear magneton* $\mu_N = q_e\hbar/2m_p$. We get the following result:

$$\boldsymbol{\mu}_{\mathbf{L}} = \mathrm{g}\left(\frac{q_e}{2m_p}\right)\mathbf{L} \Leftrightarrow \frac{2q_e}{m_p}\hbar = \mathrm{g}\frac{q_e}{2m_p}\hbar \Leftrightarrow \mathrm{g} = 4$$

That is, of course, a strange number: the CODATA value is about 5.5857. How do they calculate this? Because of conventional wisdom, they use the $\hbar/2$ value for L and, obviously, they also use the CODATA value for the magnetic moment—as opposed to our μ_L value, which is the CODATA value corrected for precession. Hence, the CODATA calculation is this:

$$\mu_p = g_p\frac{q_e}{2m_p}\frac{\hbar}{2} \Leftrightarrow g_p = \frac{4\mu_p m_p}{\hbar q_e} = 5.58569 \ldots$$

Easy. What do we get when we insert our newly found theoretical value for the magnetic moment? Let's see:

$$\mu_p = g_p\frac{q_e}{2m_p}\frac{\hbar}{2} \Leftrightarrow g_p = \frac{4\mu_p m_p}{\hbar q_e} = \frac{4}{\sqrt{2}}\cdot\frac{\mu_L m_p}{\hbar q_e} = \frac{4}{\sqrt{2}}\cdot\frac{2q_e\hbar}{m_p}\cdot\frac{m_p}{\hbar q_e} = \frac{8}{\sqrt{2}} = 5.65685 \ldots$$

A difference of about 0.071, or 1.2%. This is not surprising: the difference is of the same order of magnitude as the difference between our theoretical value for the radius – which is based on the assumption of a pointlike charge – and the actually measured radius, which is slightly different because we do think there is an anomaly. We think this anomaly can be explained in the same way as the anomalous magnetic moment of an electron: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever.

Why not? I can only give a philosophical answer here: something that has no dimension whatsoever probably exists in our mind only. Something real – like a charge – *must* have *some* dimension.

Mystery solved. The Emperor has no clothes.

Jean Louis Van Belle, 1 February 2020