# The mass, radius, and magnetic moment of electrons and protons

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## Abstract

The electron-proton scattering experiment by the PRad (proton radius) team at Jefferson Lab measured the root mean square (rms) charge radius of the proton as $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm.[1] We offer a theoretical explanation of the new measurement based on a ring current model of a proton. This model further builds on older ring current and/or *Zitterbewegung* models for an electron and, hence, we will also highlight those results when relevant. We obtain a theoretical radius that is equal to four times the range parameter ($\hbar/m_p c$) in Yukawa's formula:

$$r_p = 4\hbar/m_p c \approx 0.841 \text{ fm}$$

The 1/4 factor stems from the energy equipartition theorem: using Wheeler's 'mass without mass' idea, we effectively assume half of the energy of a proton is explained by the electromagnetic, while the other half is attributed to the strong force, which we do not model but isolate from the analysis using the energy equipartition theorem.

As for the small difference between the theoretical and measured radius, we attribute this to the mathematical idealizations that underpin ring current models. While useful and necessary as a concept, we think pointlike electric charges with zero rest mass and/or zero dimension that, therefore, move at lightspeed, do not exist: they must have some (very) small dimension which explains the anomaly. We think mathematical idealization also explains the anomalous magnetic moment of an electron.

We think the calculations may offer a model of matter-particles in general.

[1] https://www.jlab.org/prad/collaboration.html

# The mass, radius, and magnetic moment of a proton and an electron

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**1.** The *recommended* CODATA values for the magnetic moment (mm) and the mass of a proton are the following:

$$\mu = 1.41060679736\times10^{-26}\ \mathrm{J{\cdot}T^{-1}} \pm 0.00000000060\ \mathrm{J{\cdot}T^{-1}}$$
$$\mathrm{m} = 1.67262192369\times10^{-27}\ \mathrm{kg} \pm 0.00000000051\times10^{-27}\ \mathrm{kg}$$

We also have the following defined (or *exact*) values for the elementary charge, the velocity of light and Planck's constant[2]:

$$q_e = 1.602176634\times10^{-19}\ \mathrm{C}$$
$$c = 299\ 792\ 458\ \mathrm{m/s}$$
$$h = 6.62607015\ \mathrm{J{\cdot}s}$$

From a mathematical point of view, we have a set of exact values (the physical constants) and a set of variables (mass, magnetic moment, radius, angular momentum of a proton, etcetera) that depend on them. The constants are related to the variables through a number of physical laws and theorems we accept to be valid.[3] The laws and theorems that we will use in this article are:

- The energy equipartition theorem
- The Planck-Einstein relation: $E = h{\cdot}f = \hbar{\cdot}\omega$
- The principle of relativity and the energy-mass equivalence relation: $E = m{\cdot}c^2$
- The force law, which states that a force acts upon a charge and changes its state of motion
- Maxwell's laws of electromagnetism

The system is completely determined – possible over-determined – and, hence, it is easy to derive the relations we seek: from the energy (or equivalent mass) of the particle (electron or proton), we can

[2] As part of the 2019 revision of SI units, *exact* numerical values were set for Planck's constant ($h$), the elementary electric charge ($q_e$), the Boltzmann constant ($k_B$), and Avogadro's constant ($N_A$). The fine-structure constant has now also been *defined* as:

$$\alpha = \frac{q_e^2}{4\pi\varepsilon_0\hbar c}$$

Its value still has an uncertainty of $1.5\times10^{-19}$ on it, which it shares with the electric and magnetic constants because of the $c^2 = 1/\varepsilon_0\mu_0$ relation.

[3] We adhere to a Popperian view here: we accept them to be valid because they have resisted falsification (Popper, 1959).

calculate all other observables using the three above-mentioned constants (Planck's quantum of action, the elementary charge and the speed of light).

**2.** We believe a realist interpretation of quantum mechanics is possible, which means the structure of the laws and theorems should not only reflect some structure in our mind, but in Nature as well. We, therefore, imagine the magnetic moment of a proton to be created by a circular current of the elementary charge. It is, therefore, equal to the current times the area of the loop:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \frac{\mathrm{q_e}\omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{\mathrm{q_e}\omega}}$$

The frequency is equal to the velocity of the charge ($v$) divided by the circumference of the loop ($2\pi a$). However, for a reason the reader will readily understand after reading this article, we prefer to use the Planck-Einstein relation for the frequency. We believe the Planck-Einstein relation ($\mathrm{E} = \mathrm{h}\cdot f = \hbar\cdot\omega$) reflects a fundamental *cycle* in Nature. It, therefore, makes sense to also apply it to the ring current idea of a proton.[4] Hence, we write:

$$a = \sqrt{\frac{2\mu}{\mathrm{q_e}\omega}} = \sqrt{\frac{2\mu\hbar}{\mathrm{q_e}\mathrm{E}}}$$

**3.** When applying this formula to an electron, we get the Compton radius of an electron ($a = \hbar/\mathrm{m}c$).[5] When applying the $a = \hbar/\mathrm{m}c$ radius formula to a proton, we get a value which is about 1/4 of the *measured* proton radius. We, therefore, need to consider using the same fraction of the proton energy to calculate the frequency:

$$\omega = \frac{1}{4}\frac{\mathrm{E}}{\hbar}$$

We should motivate the 1/4 factor, of course. We think the huge value of the proton mass and its tiny size – as compared to the mass and size of an electron – lend credibility to the assumption of another force (or another charge) inside of the proton.[6] Hence, the 1/4 factor combines (1) the energy

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[4] There is a long tradition of thinking of an electron in terms of a current ring. We may refer to Parson (1915), Schrödinger (1930) and, more recently, Hestenes (1990). It has been suggested it may also apply to protons (Consa, 2018) but, based on quick feedback from sympathetic researchers, we think this paper may be the first fully consistent theory in this regard. Alexander Burinskii, whose work on an integrated theory of the electron we admire greatly, drew our attention to earlier work of M.E. Shulman but Shulman's work seems to focus on leptons only (https://www.scirp.org/journal/paperinformation.aspx?paperid=78086). Giorgio Vassallo also sent useful references we will further examine over the coming months. We thank both for their quick feedback on our 'back-of-the-envelope' calculations.

[5] The reader who is not familiar with ring current and/or *Zitterbewegung* models of an electron may also not be familiar with the concept of a Compton *radius*. It is, of course, the reduced form of the Compton wavelength. We think of it as an actual (electric) charge radius (see Annex I and II).

[6] We use a model explaining mass as the equivalent mass of energy here, i.e. Wheeler's idea of "mass without mass". Energy is force over a distance and, hence, we can distinguish between electromagnetic energy (and the equivalent mass) and some new strong energy or mass, which is defined in terms of some strong force and the related strong charge. Our interpretation of Wheeler's "mass without mass" theory is explained in a previous paper (https://vixra.org/abs/2001.0453).

equipartition theorem (half of the energy or mass of the electron is to be explained by the strong force) and (2) Hestenes' interpretation of Schrödinger's *Zitterbewegung* interpretation of an electron.[7] We can, finally, do an actual calculation now:

$$a = \sqrt{\frac{2\mu}{q_e\omega}} = \sqrt{\frac{4 \cdot 2\mu\hbar}{q_e E}} = 2 \cdot \sqrt{\frac{2\mu\hbar}{q_e mc^2}} \approx 2 \cdot 0.35146 \ldots \times 10^{-15} \approx 0.703 \text{ fm}$$

The gap between the 0.831 and 0.703 values suggests we are missing a √2 factor:

$$a = \sqrt{\frac{\sqrt{2} \cdot 2\mu}{q_e\omega}} = 2 \cdot \sqrt{\frac{2\sqrt{2}\mu\hbar}{q_e E}} \approx 0.8359278 \text{ fm}$$

The difference between this calculated value (which used all of the precision of the CODATA values) and the PRad result is only about 0.005 fm[8], which is well within the statistical standard error of the measurement. Hence, it is a good result.

**4.** We now need to motivate the insertion of the √2 factor. We think there is some *real* magnetic moment here, which we denote as $\mu_L$:

$$\mu_L = \sqrt{2} \cdot (1.41060679736 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}) \approx 1.995 \times 10^{-26} \text{ J} \cdot \text{T}^{-1}$$

The subscript L in the $\mu_L$ notation stands for (orbital) angular momentum. A magnetic dipole will *precess* when placed in a magnetic field—which is what is being done when measuring the magnetic moment of a proton. We refer to Feynman[9] for an easy and very meaningful explanation of the relation between the magnitude of the actual – or imagined?[10] – angular momentum of a precessing magnet ($L$) and $L_z$ (the *measured* quantum value) as:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{j(j+1)}}{j}$$

For $j$ = ½, we get:

$$= \frac{\sqrt{1/2(1/2+1)}}{1/2} = 2 \cdot \sqrt{\frac{3}{4}} = \sqrt{3}$$

[7] Hestenes summarizes his various papers as follows: "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field. Half of it is magnetic (potential) energy and half is kinetic." (email from Dr. David Hestenes to the author dated 17 March 2019)

[8] 0.831 – 0.836 = 0.005. We showed a result with seven digits to show the difference between this calculation and another value we will get out of another calculation (see Section 5).

[9] See: https://www.feynmanlectures.caltech.edu/II_34.html#Ch34-S7

[10] The difference between actual and imagined here depends on one's *interpretation* of quantum-mechanical laws. From what we present in this article, it should be obvious to the reader that we like to think this magnitude is something real. However, such metaphysical questions should not be the concern of the reader: he or she should just check our calculations so as to *verify* them. The *interpretation* of the results is a different matter.

We need a √2 factor. Hence, the spin number must be one:

$$\frac{L}{L_z} = \frac{\sqrt{j(j+1)} \cdot \hbar}{j \cdot \hbar} = \frac{\sqrt{1(1+1)}}{1} = \sqrt{2}$$

We know this assumption relates to the theoretical distinction between fermions and bosons. However, we will show the *j* = 1 assumption makes sense.

**5.** Because of the apparent randomness of this √2 factor, we must try the simpler approach to calculating the magnetic moment, which calculates the frequency from the $f = c/2\pi a$ formula:

$$\mu_L = I\pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a}\pi a^2 = \frac{q_e c}{2} a$$

$$\Leftrightarrow a = \frac{2\mu_L}{q_e c} = \sqrt{2} \cdot \frac{2\mu}{q_e c} = \sqrt{2} \cdot 0.587 \times 10^{-15} \approx 0.83065344 \ldots \mathrm{fm}$$

The result differs – slightly but significantly – from the result we obtained from using the Planck-Einstein relation for the frequency calculation (see Section 3). It is a *very* small difference. To be precise, it is, again, of the order of 0.005 fm. At the same time, this result is closer to the 0.831 PRad value: the difference is 0.000346656… fm only, which is less than 5% of the standard error of the PRad point estimate (0.007 fm).

**6.** In our calculations, we used the CODATA value for the magnetic moment of a proton in two different formulas for the radius, and we found the result is slightly different. While the two values do not differ significantly from the experimentally measured value for the proton radius – and, thereby, may be seen as a confirmation of the relevance of the PRad experiment – the two different values suggest we may think of some *unique* or *absolute* theoretical value for the magnetic moment. Indeed, because we have two equations for the radius *a* – and both of them involve $\mu_L$ – we can just equate them:

$$a = 2 \cdot \sqrt{\frac{2\mu_L \hbar}{q_e E}} = \frac{2\mu_L}{q_e c} \Leftrightarrow \sqrt{\frac{2\mu_L \hbar \cdot q_e^2 c^2}{q_e E \cdot \mu_L^2}} = 1$$

$$\Leftrightarrow \mu_L = \frac{2q_e}{m}\hbar \approx 2.02035 \times 10^{-26}\ \mathrm{J \cdot T^{-1}}$$

We get a value that is *almost* 2, but not quite. We think of this as a coincidence. We can now calculate an exact theoretical value for the proton radius:

$$\boxed{a = \frac{2\mu_L}{q_e c} = \frac{2}{q_e c} \cdot \frac{2q_e \hbar}{m} = 4 \cdot \frac{\hbar}{mc} \approx 4 \cdot (0.21 \ldots \mathrm{fm}) \approx 0.8413564 \ldots\ \mathrm{fm}}$$

This value is not within the 0.831 ± 0.007 fm interval, but it is well within the wider $r_p = 0.831 \pm 0.007_{stat} \pm 0.012_{syst}$ fm interval.[11]

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[11] We readily admit the insertion of the √2 factor needs further examination. We have a $\mu_L = 2q_e\hbar/m_p \approx 2.02\ldots$ J/T value for the magnetic proton which, we argue, differs from the CODATA value with a √2 factor because of precession. In contrast, the formula for the magnetic moment of an electron ($\mu_e = q_e\hbar/2m_e \approx 9.274$ J/T) gives us the

It may be noted that the $\hbar/m_pc$ factor is equal to the range parameter in Yukawa's formula for the nuclear potential.[12] As such, it is equivalent to the concept of the Compton radius for an electron.

**7.** We will now come back to the question of the spin number. Quantum-mechanical spin is expressed in units of $\hbar/2$ and, according to the Copenhagen interpretation of quantum mechanics, we should *not* try to think of it as a classical property—as something that has some *physical* meaning. We obviously disagree with this point of view. We think we can just use the classical $L = I \cdot \omega$ expression and substitute $I$ and ω for the angular mass and the angular frequency.[13] To calculate the angular mass, one must assume some form factor: a hoop, a disk, a sphere or a shell are associated with different form factors. Our electron model[14] assumes that the effective mass of the electron is spread over a circular disk. We can, therefore, calculate the angular momentum as:

$$\mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{\hbar}{mc} = \frac{\hbar}{2}$$

Hence, we may effectively refer to an electron as a spin-1/2 particle. However, we do not think of this property as some obscure 'intrinsic' property of an equally obscure 'pointlike' particle: we think of the electron as an actual disk-like structure with some *torque* on it. Its angular momentum is, therefore, *real*.[15] Likewise, we think of the magnetic moment as being equally real[16]:

$$\mu = \mathrm{I} \cdot \pi a^2 = \frac{\mathrm{q_e} c \cdot \pi a^2}{2\pi a} = \frac{\mathrm{q_e} c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \approx 9.274 \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$$

We think there is a confusion in regard to spin numbers and g-factors because we cannot directly measure the angular momentum: in real-life experiments, we measure the *magnetic moment*. Having

CODATA value (apart from the anomaly, of course) *without the need for any correction factor because of precession*. If an electron is some ring current as well, then it must precess as well. We looked on the NIST site, but could not find much in terms of methodology. We sent an email to the NIST Public Affairs section with a request to guide us to the necessary materials in this regard. Annex I offers a full discussion of the perceived issue.

[12] We calculated this range parameter in previous papers. See, for example, our *Metaphysics of Physics* paper (https://vixra.org/abs/2001.0453).

[13] The reader should not confuse the *I* and I symbols. The first (*I* in *italics*) stands for angular mass (expressed in $kg \cdot m^2$), while the second (I, normal type) is the symbol for current (expressed in C/s). We could have used different symbols, but we wanted to stick to the usual conventions. The reader will, of course, also not confuse the concepts of angular mass (*I*), also known as the moment of inertia, and angular momentum (L).

[14] See: https://vixra.org/abs/1905.0521.

[15] We will not engage in philosophical discussions here. We hope the reader understands what we want him/her to understand.

[16] The CODATA value for the magnetic moment includes the anomaly and is, therefore, slightly different from the theoretical value: $\mu_e \approx 9.285$ J/T. We think the difference between the theoretical and measured value is to be explained by a form factor: the circular point charge must have some (tiny) dimension and/or must have some (very tiny) non-zero rest mass. We believe the two letters of Gregory Breit to Gregory Breit to Isaac Rabi can easily be interpreted as Breit defending the idea that an intrinsic magnetic moment "of the order of $\alpha\mu_B$" is not anomalous at all. For more details on this conversation, see: Silvan S. Schweber, *QED and the Men Who Made It: Dyson, Feynman, Schwinger, and Tomonaga* , p. 222-223.

said that, it is true we can combine the two formulas to get the g-factor that is usually associated with the spin of an electron[17]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g} = 2$$

We should now apply these ideas to the proton. The idea of a current ring – and the idea of precession, of course – strongly suggests we should, once again, think of the proton as a disk-like structure. However, not all of the mass is in the electromagnetic oscillation: we think half of it remains to be explained by what is referred to as the strong force (or, what amounts to the same, the idea of a strong charge).[18] We will, therefore, use a 1/4 rather than a 1/2 factor in the angular mass formula. This yields the following result:

$$\mathrm{L_p} = I_\mathrm{p} \cdot \omega = \frac{\mathrm{m_p} r_\mathrm{p}^2}{4} \cdot \frac{c}{r_\mathrm{p}} = \frac{\mathrm{m_p} c}{4} \cdot r_\mathrm{p} = \frac{\mathrm{m_p} c}{4} \cdot \frac{4\hbar}{\mathrm{m_p} c} = \hbar$$

Hence, our 'spin number' is equal to one. Most academics will cry wolf here: we cannot possibly believe a proton is a spin-one particle, can we? We think we can. We think there is no need for the concept of a spin number and a g-factor in a *realist* interpretation of quantum mechanics. We think of the angular momentum and the magnetic moment as being *real* and, hence, whatever else is being calculated – be it a spin number or a g-factor – is not very relevant. Worse, we think it *confuses* rather than clarifies the analysis. We, therefore, think our calculation of $\mathrm{L_p}$ is consistent. We also think it is consistent with the use of the √2 factor – as opposed to a √3 factor – to calculate what we think of as a *real* magnetic moment of a proton ($\mu_\mathrm{p}$).

We should, of course, relate this to the usual conventions. We will, therefore, do some calculations involving a g-factor. Instead of the *Bohr magneton* $\mu_\mathrm{B} = \mathrm{q_e}\hbar/2\mathrm{m_e}$, we should use the *nuclear magneton* $\mu_\mathrm{N} = \mathrm{q_e}\hbar/2\mathrm{m_p}$. We get the following result:

$$\boldsymbol{\mu}_\mathbf{L} = \mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m_p}}\right)\mathbf{L} \Leftrightarrow \frac{2\mathrm{q_e}}{\mathrm{m_p}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\hbar \Leftrightarrow \mathrm{g} = 4$$

That is, of course, a strange number: the CODATA value is about 5.5857. However, this result depends on the use of a theoretical $\hbar/2$ value for the angular momentum. It also uses the CODATA value for the magnetic moment—as opposed to our $\mu_\mathrm{L}$ value, which is the CODATA value corrected for precession. Hence, the CODATA calculation of the g-factor is this:

$$\mu_\mathrm{p} = \mathrm{g_p}\frac{\mathrm{q_e}}{2\mathrm{m_p}}\frac{\hbar}{2} \Leftrightarrow \mathrm{g_p} = \frac{4\mu_\mathrm{p}\mathrm{m_p}}{\hbar\mathrm{q_e}} = 5.58569\ldots$$

We get a slightly different value when we insert our newly found theoretical value for the magnetic moment:

[17] We used vector notation (**boldface**) to draw attention, once again, to our *physical* interpretation of what might be going on: the *minus* sign (–) is there because, in the case of an electron, the magnetic moment and angular momentum vectors have opposite directions.

[18] See our paper on the idea of a strong force and/or a strong charge: https://vixra.org/abs/2001.0453.

$$\mu_p = g_p \frac{q_e}{2m_p}\frac{\hbar}{2} \Leftrightarrow g_p = \frac{4\mu_p m_p}{\hbar q_e} = \frac{4}{\sqrt{2}} \cdot \frac{\mu_L m_p}{\hbar q_e} = \frac{4}{\sqrt{2}} \cdot \frac{2q_e\hbar}{m_p} \cdot \frac{m_p}{\hbar q_e} = \frac{8}{\sqrt{2}} = 5.65685 \ldots$$

How can we explain the difference?

**8.** The difference of about 0.071 (about 1.2%) is not surprising: the difference is of the same order of magnitude as the difference between our theoretical value for the radius – which is based on the assumption of a pointlike charge – and the actually measured radius. We think this difference confirms both the theory as well as the PRad measurement. We anticipate theorists and experimenters to argue about the next digit of the anomalous magnetic moment of a proton in pretty much the same way as they have been arguing about the anomalous magnetic moment of an electron. We think both 'anomalies' are there because of the mathematical idealization in our assumptions: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever.[19]

Why not? We can only give a philosophical answer here: something that has no dimension whatsoever probably exists in our mind only. Something real – like a charge – *must* have *some* dimension.

From what we write above, the reader will understand that we think some of the generalizations in quantum physics – most notably, the concept of bosons – are not necessary to understand Nature.

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[19] See our paper on Consa's calculations of the anomalous magnetic moment (https://vixra.org/abs/2001.0264), which also references our approach to the matter (https://vixra.org/abs/1906.0007).

## **Annex I**: Precession of electrons and protons

When inserting the CODATA value for the magnetic moment of an electron in the two formulas that we have used to calculate the theoretical magnetic moment of a proton, we get the electron's Compton radius. Calculating the frequency using the geometric formula ($f = c/2\pi a$), we get:

$$\mu_e = I\pi a^2 = q_e f \pi a^2 = q_e \frac{c}{2\pi a}\pi a^2 = \frac{q_e c}{2} a \Leftrightarrow a = \frac{2\mu_e}{q_e c} \approx 0.3866607 \dots \text{pm}$$

Calculating the frequency using the Planck-Einstein relation ($f = E/h$), we get the same value:

$$\mu_e = I\pi a^2 = q_e f \pi a^2 = \frac{q_e \omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu_e}{q_e \omega}} = \sqrt{\frac{2\mu_e \hbar}{q_e E}} \approx 0.3863831 \dots \text{pm}$$

There is, once again, a small difference between the two values and we can, therefore, equate the two formulas to calculate a theoretical value for the magnetic moment of an electron[20]:

$$a = \sqrt{\frac{2\mu_e \hbar}{q_e m c^2}} = \frac{2\mu_e}{q_e c} \Leftrightarrow \sqrt{\frac{\mu_e \hbar \cdot q_e^2 c^2}{2 q_e m c^2 \cdot \mu_e^2}} = 1$$

$$\Leftrightarrow \mu_e = \frac{q_e}{2m}\hbar \approx 9.274 \dots \times 10^{-24}\ \text{J} \cdot \text{T}^{-1}$$

The reader should note this differs slightly from the CODATA recommended value – which is equal to about 9.284 J·T$^{-1}$, which is based on experimental measurement. Again, we think this difference confirms both the theory as well as the measurement: we think the 'anomaly' is there because of the mathematical idealization in our assumptions: the pointlike charge may have zero rest mass (or some value *very* close to zero), but we should not assume it has no dimension whatsoever. We may also assume its velocity is, perhaps, *nearly* lightspeed but not quite. We, therefore, think it can be explained using classical physics.

We can now re-insert the theoretical magnetic moment in our formulas for the radius to calculate the theoretical radius of the electron:

$$a = \sqrt{\frac{2\mu_e \hbar}{q_e m c^2}} = \sqrt{\frac{2 q_e \hbar^2}{2 m q_e m c^2}} = \frac{2\mu_e}{q_e c} = \frac{2 q_e \hbar}{2 m q_e c} = \frac{\hbar}{mc} \approx 0.3861592 \dots \text{pm}$$

This is a nice result, but let us explore it some more. Do we assume precession and, if so, what factor should we use? The formulas do not suggest so. Our equality no longer holds. Indeed, if we would denote the so-called *real* magnetic moment as $\mu_J = \sqrt{n} \cdot \mu_e$[21] and then our equality becomes this:

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[20] We should add a *minus* sign because of the opposite direction of magnetic moment and angular momentum. However, here we are only calculating magnitudes.

[21] We use a *J* instead of *L* so as to avoid confusion with the $\mu_L$ symbol which we used for the angular momentum of a proton. The physical meaning of $\mu_L$ and $\mu_J$ is, therefore, exactly the same. Note that *n* is not necessarily a whole number. For example, when inserting *j* = 3/2 in the formula for the $L/L_z$ or $J/J_z$ ratio, we get $\sqrt{n} = \sqrt{15}/3$.

$$a = \sqrt{\frac{2\sqrt{n}\mu_e\hbar}{q_e mc^2}} = \frac{2\sqrt{n}\mu_e}{q_e c} \Leftrightarrow \frac{\sqrt{n}\mu_e\hbar \cdot q_e^2 c^2}{2q_e mc^2 \cdot n\mu_e^2} = \frac{\sqrt{n}q_e\hbar}{2m \cdot n\mu_e} = 1 \Leftrightarrow \frac{\sqrt{n}}{n} \cdot \frac{q_e\hbar}{2m} \cdot \frac{2m}{q_e\hbar} = 1 \Leftrightarrow \sqrt{n} = n$$

This equality only holds for *n* = 1. It is very puzzling: should we assume that the CODATA value for the magnetic moment of an electron has already been corrected to include the idea of precession?

While contemplating this possibility, we should also note we did not use a 1/2 factor in our $f = E/h$ formula. If we used such factor for our proton calculations – arguing half of the energy is kinetic and the other half is electromagnetic – then we should use such factor for our electron calculations as well. Let us see if this gets us anywhere. Substituting ω for $E/2\hbar$ instead of $E/\hbar$, we get this formula for the electron radius:

$$\mu = I\pi a^2 = qf\pi a^2 = \frac{q\omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{q\omega}} = \sqrt{\frac{4\mu\hbar}{qE}} = 2 \cdot \sqrt{\frac{\mu\hbar}{qE}}$$

The other way to calculate μ was like this:

$$\mu = I\pi a^2 = qf\pi a^2 = q\frac{c}{2\pi a}\pi a^2 = \frac{qc}{2}a \Leftrightarrow a = \frac{2\mu}{qc}$$

Equating both equations for *a* gives us this:

$$a = 2 \cdot \sqrt{\frac{\mu\hbar}{qmc^2}} = \frac{2\mu}{qc} \Leftrightarrow \sqrt{\frac{\mu\hbar}{qmc^2} \cdot \frac{q^2c^2}{\mu^2}} = 1 \Leftrightarrow \mu = \frac{q}{m}\hbar \approx 18.548 \times 10^{-24}\ \mathrm{J \cdot T^{-1}}$$

This is, unsurprisingly, *twice* the CODATA value for the magnetic moment. The corresponding radius is, unsurprisingly, *twice* the Compton radius:

$$a = 2 \cdot \sqrt{\frac{\mu\hbar}{qmc^2}} = 2 \cdot \sqrt{\frac{q\hbar^2}{qm^2c^2}} = 2\frac{\hbar}{mc} = \frac{2\mu}{qc} = \frac{2q\hbar}{qmc} = 2\frac{\hbar}{mc}$$

This is very weird, of course, even if the math here are very simple.[22] Let us quickly examine if this strange result respects conventional wisdom in regard to spin numbers and g-factors. The formula to be used depends, once again, on our assumption in regard to the form factor:

$$\boldsymbol{\mu} = -g\left(\frac{q_e}{2m}\right)\mathbf{L}$$

Do we think of the electron as a loop or a hoop, or do we think its mass is effectively spread out over a disk? The formulas below show that we only get the conventional g-factor (*g* = 2) if we assume, once

[22] It is especially weird because most *Zitterbewegung* theorists, including Hestenes (1990), Burinskii (2008, 2016) and Gauthier (2019), etcetera arrive at the conclusion that the radius of the oscillation must be equal to *half* the Compton radius. Oliver Consa (2018) is one of the few physicists who also equate the electron radius with the reduced Compton wavelength. As for the math, one should note that the current is inversely proportional to the radius ($f = c/2\pi a$) but that the surface of the loop ($\pi a^2$) is proportional to the *square* of the radius. The magnetic moment (μ)is the product of both. Hence, the radius (*a*) will be proportional to μ.

again, that the mass of the electron is spread out over a disk, which allows us to insert the necessary ½ factor [23]:

$$\boldsymbol{\mu} = -\mathrm{g}\left(\frac{\mathrm{q_e}}{2\mathrm{m}}\right)\mathbf{L} \Leftrightarrow \frac{\mathrm{q_e}}{\mathrm{m}}\hbar = \mathrm{g}\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar \Leftrightarrow \mathrm{g} = 2$$

$$\Leftrightarrow \mathrm{L} = I \cdot \omega = \frac{\mathrm{m}a^2}{2}\frac{c}{a} = \frac{\mathrm{m}c}{2} \cdot a = \frac{mc}{2} \cdot \frac{2\hbar}{mc} = \hbar$$

We are not sure how to make sense of the 1/2 factor and the thorny question of quantum-mechanical precession. Perhaps they are related to two different concepts of the radius: while we can calculate the radius of a loop of a pointlike charge, our model suggests the electromagnetic field will extend beyond the current ring. This may result in an *effective* charge radius which is larger than the Compton radius. It may also explain the 1/2 factor we used for the energy: if we do *not* include the energy of the magnetic field, then we get a radius that is only half the Compton radius. We welcome suggestions as to how to improve on this rather sloppy answer.

As for the methodology used to calculate the CODATA value of the magnetic moment of an electron, we have requested NIST to provide us with more details. This may or may not lead to future revisions of some of the remarks we presented in this paper.

[23] This allows us to insert a 1/2 factor in the formula for the angular mass.

## **Annex II**: The Compton radius: calculation and interpretation

## Introduction

The reader who is not familiar with ring current and/or *Zitterbewegung* models of an electron may also not be familiar with the concept of a Compton *radius*. It is, of course, the reduced form of the Compton wavelength: $r_C = \lambda_C/2\pi$. From what we wrote in the body of this paper, it is obvious we think of it as an actual radius—but a radius of what, *exactly*? We think of it as a distance, or a *scale*, within which a photon interacts with the electromagnetic field of the ring current electron or – let us drop the word again – the *Zitterbewegung* electron. As this is an annex, we have some more space to develop the idea. *Zitter* is German for shaking or trembling, and the *Zitterbewegung* refers to a presumed local oscillatory motion—which we believe is *real*. Erwin Schrödinger stumbled upon the idea when he was exploring *solutions* to Dirac's wave equation for free electrons, and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

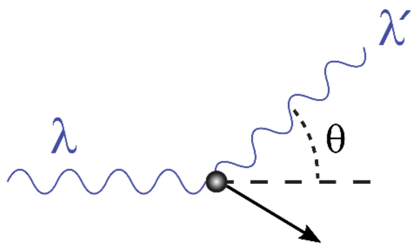
The reference to the 'law of scattering of light by an electron' is, of course, a reference to *Compton* scattering. For the convenience of the reader, we will quickly remind him or her how one gets the Compton wavelength from more standard (mainstream) calculations.[24] The reader should note that the calculations do *not* involve any quantum-mechanical weirdness: the uncertainty principle, for example, is *not* being invoked. In fact, we wanted to add this annex to illustrate how classical basic quantum-mechanical calculations can actually be.[25]

## Compton scattering: energy and angle formulas

Compton scattering is referred to as inelastic because the frequency of the incoming and outgoing photon are different. The situation is illustrated below.

[24] The presentation in this annex reflects standard analysis and can, therefore, be *googled* easily. However, we relied on Prof. Dr. Barton Zwiebach's introduction to quantum mechanics in MIT's edX course on quantum mechanics (8.01.1*x*). We took the illustration from the Wikipedia article on scattering. Finally, we found the notes that are part of Prof. Dr. Patrick LeClair's Physics 253 course (http://pleclair.ua.edu/PH253/Notes/compton.pdf) very useful, and we will also refer to them later.

[25] We repeat: the presentation in this annex is mainstream analysis. For a more speculative – but also more *intuitive* – theory of what might actually be going on, we refer to our own classical analysis of Compton scattering (https://vixra.org/abs/1912.0251).




We are not sure how to describe the interaction. We may, perhaps, think of an electron first absorbing the photon before re-emitting the new photon. Indeed, a photon is *defined* by its frequency, so we should think of the outgoing photon as a different photon. We will come back to these questions. What we do know is that the energy difference between the two photons is converted into some linear momentum. Hence, before the photon hits it, the electron is thought of as being stationary. Two classical laws govern the process: (1) energy conservation, and (2) momentum conservation.

**1.** The energy conservation law tells us that the total (relativistic) energy of the electron ($E = m_e c^2$) and the incoming photon must be equal to the total energy of the outgoing photon and the electron, which is now moving and, hence, includes the kinetic energy from its (linear) motion. We use a prime (‘) to designate variables measured after the interaction. Hence, $E_{e'}$ and $E_{\gamma'}$ are the energy of the moving electron (e’) and the outgoing photon (γ’) in the state after the event. We write:

$$\mathrm{E_e} + \mathrm{E_\gamma} = \mathrm{E_{e'}} + \mathrm{E_{\gamma'}}$$

We can now use (i) the mass-energy equivalence relation ($E = mc^2$), (ii) the Planck-Einstein relation for a photon ($E = h \cdot f$) and (iii) the relativistically correct relation ($E^2 - p^2c^2 = m^2c^4$) between energy and momentum for *any* particle – charged or non-charged, matter-particles or photons or whatever other distinction one would like to make[26] – to re-write this as[27]:

$$\mathrm{m_e}c^2 + hf = \sqrt{\mathrm{p_{e'}^2}c^2 + \mathrm{m_e^2}c^4} + hf' \Leftrightarrow \mathrm{p_{e'}^2}c^2 = (hf - hf' + \mathrm{m_e^2}c^4)^2 - \mathrm{m_e^2}c^4 \ (1)$$

**2.** This looks rather monstrous but things will fall into place soon enough because we will now derive another equation based on the momentum conservation law. Momentum is a vector, and so we have a vector equation here[28]:

$$\vec{\mathrm{p}}_\gamma = \vec{\mathrm{p}}_{\gamma'} + \vec{\mathrm{p}}_{e'} \Leftrightarrow \vec{\mathrm{p}}_{e'} = \vec{\mathrm{p}}_\gamma - \vec{\mathrm{p}}_{\gamma'}$$

---

[26] This is, once again, a standard textbook equation but – if the reader would require a reminder of how this formula comes out of special relativity theory – we may refer him to the online *Lectures* of Richard Feynman. Chapters 15 and 16 offer a concise but comprehensive overview of the basics of relativity theory and section 5 of Chapter 6 (https://www.feynmanlectures.caltech.edu/I_16.html#Ch16-S5) gives the reader the formula he needs here. It should be noted that we dropped the 0 subscript for the rest mass or energy: $m_0 = m$. The *prime* symbol (‘) takes care of everything here and so you should carefully distinguish between primed and non-primed variables.
[27] We realize we are cutting some corners. We trust the reader will be able to *google* the various steps in-between.
[28] We could have used **boldface** to denote vectors, but the calculations make the arrow notation more convenient here. So as to make sure our reader stays awake, we note that the objective of the step from the first to the second equation is to derive a formula for the (linear) momentum of the electron *after* the interaction. As mentioned, the linear momentum of the electron *before* the interaction is zero, because its (linear) velocity is zero: $\mathbf{p}_e = \mathbf{0}$.

For reasons that will be obvious later – it is just the usual thing: ensuring we can combine two equations into one, as we did with our formulas for the radius – we square this equation and multiply with Einstein's constant $c^2$ to get this[29]:

$$\vec{p}_{e'}^{2} = \vec{p}_{\gamma}^{2} + \vec{p}_{\gamma'}^{2} - 2\vec{p}_{\gamma}\vec{p}_{\gamma'} \Leftrightarrow p_{e'}^{2}c^2 = p_{\gamma}^{2}c^2 + p_{\gamma'}^{2}c^2 - 2(p_{\gamma}c)(p_{\gamma'}c)\cdot\cos\theta$$

$$\Leftrightarrow p_{e'}^{2}c^2 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta \ (2)$$

**3.** We can now combine equations (1) and (2):

$$p_{e'}^{2}c^2 = (Eq.1) = (Eq.1) = (hf - hf' + m_e^2c^4)^2 - m_e^2c^4 = h^2f^2 + h^2f'^2 - 2(hf)(hf')\cdot\cos\theta$$

The reader will be able to do the horrible stuff of actually squaring the expression between the brackets and verifying only cross-products remain. We get:

$$(hf - hf')m_ec^2 = h(f - f')m_ec^2 = h^2ff'(1 - \cos\theta)$$

Multiplying both sides of the equation by the $1/hm_eff'$ constant yields the formula we were looking for:

$$\frac{(f - f')m_ec}{f\cdot f'} = \frac{fm_ec - f'^{m_e}c}{f\cdot f'} = \frac{h}{m_ec}(1 - \cos\theta)$$

$$\Leftrightarrow \frac{c}{f'} - \frac{c}{f} = \lambda' - \lambda = \Delta f = \frac{h}{m_ec}(1 - \cos\theta)$$

The formulas allow us also to calculate the angle in which the electron is going to recoil. It is equal to:

$$\cot\left(\frac{\theta}{2}\right) = \left(1 + \frac{E_\gamma}{E_e}\right)\tan\ \varphi$$

The $h/mc$ factor on the left-hand side of the right-hand side of the formula for the difference between the wavelengths is, effectively, a distance: about 2.426 *pico*meter ($10^{-12}$ m). The $1 - \cos\theta$ factor goes from 0 to 2 as θ goes from 0 to π. Hence, the maximum difference between the two wavelengths is about 4.85 pm. This corresponds, unsurprisingly, to *half* the (relativistic) energy of an electron.[30] Hence, a highly energetic photon could lose up to 255 keV.[31] That sounds enormous, but Compton scattering is usually done with highly energetic X- or gamma-rays.

Could we imagine that a photon loses all of its energy to the electron? No. We refer to Prof. Dr. Patrick LeClair's course notes on Compton scattering[32] for a very interesting and more detailed explanation of what actually happens to energies and frequencies, and what depends on what exactly. He shows that

[29] We do not want to sound disrespectful when referring to $c^2$ as Einstein's constant. It has a deep meaning, in fact. Einstein does not have any fundamental constant or unit named after him. Nor does Dirac. We think $c^2$ would be an appropriate 'Einstein constant'. Also, in light of Dirac's remarks on the nature of the strong force, we would suggest naming the unit of the strong charge after him. More to the point, note these steps – finally ! – incorporated the directional aspect we needed for the analysis. When everything is said and done, we don't only want some value for the Compton wavelength ($\lambda_C = h/mc$), but for the scattering angle (θ) as well! Note that we also use the rather obvious E = p$c$ relation for photons in the transformation of formulas here.

[30] The energy is inversely proportional to the wavelength: $E = h\cdot f = hc/\lambda$.

[31] The electron's rest energy is about 511 keV.

[32] See: http://pleclair.ua.edu/PH253/Notes/compton.pdf.

the electron's kinetic energy will always be a *fraction* of the incident photon's energy, and that fraction may approach but will never actually reach unity. In his words: "This means that there will always be some energy left over for a scattered photon. Put another way, it means that a stationary, free electron cannot absorb a photon! Scattering must occur. Absorption can only occur if the electron is bound to a nucleus."

Prof. Dr. LeClair also examines the scattering of photons from a proton and this is where he goes as far as mainstream physicists usually go when *interpreting* the actual meaning of the Compton wavelength or radius[33]:

> "The only difference is that the proton is heavier. We simply replace the electron mass in the Compton wavelength shift equation with the proton mass, and note that the maximum shift is at $\theta = \pi$. The maximum shift is $\Delta\lambda_{max} = 2h/m_p c \approx 2.64$ fm. Fantastically small. This is roughly the size attributed to a small atomic nucleus, *since the Compton wavelength sets the scale above which the nucleus can be localized in a particle-like sense*."[34] (my italics)

This is, effectively, what we like to hear from physicists. **The Compton radius has a *physical* meaning. It's the distance or scale within which we can, effectively, expect the photon to interfere with the electromagnetic field of the electron or proton current ring.**

Of course, we need a photon model to corroborate this: if a photon and an electron (or a proton) are going to interfere, we need to know what interferes with what, *exactly*. What is our photon model? We have elaborated that elsewhere and, hence, we will just copy the basics from one of our papers here.[35]

## The photon model

Angular momentum comes in units of $\hbar$. When analyzing the electron orbitals for the simplest of atoms (the one-proton hydrogen atom), this rule amounts to saying the electron orbitals are separated by a amount of *physical action* that is equal to $h = 2\pi\cdot\hbar$. Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of $h$. The photon that is emitted or absorbed will have to pack that somehow. It will also have to pack the related energy, which is given by the Rydberg formula:

$$\mathrm{E}_{n_2} - \mathrm{E}_{n_1} = -\frac{1}{{n_2}^2}\mathrm{E}_R + \frac{1}{{n_1}^2}\mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \mathrm{E}_R = \left(\frac{1}{{n_1}^2} - \frac{1}{{n_2}^2}\right)\cdot \frac{\alpha^2 \mathrm{m}c^2}{2}$$

To focus our thinking, let us consider the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ is equal 0.75. Hence, the photon energy should be equal to $(0.75)\cdot\mathrm{E}_R \approx 10.2$ eV. Now, if the total action is equal to $h$, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T} = h \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} \approx \frac{4.135\times10^{-15}\,\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}} \approx 0.4\times10^{-15}\ \mathrm{s}$$

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[33] As mentioned before, the concept of a Compton *radius* is usually not mentioned in physics textbooks. They only talk about the *reduced* value ($r_C = \lambda_C/2\pi$), pretty much like the difference between $\hbar$ and $h$, which – in our realist interpretation of quantum mechanics – is also *physical*.

[34] See: http://pleclair.ua.edu/PH253/Notes/compton.pdf, p. 10

[35] See: https://vixra.org/abs/2001.0345.

This corresponds to a wave train with a length of $(3\times10^8\ \text{m/s})\cdot(0.4\times10^{-15}\ \text{s}) = 122$ nm. That is the size of a large molecule and it is, therefore, much more reasonable than the length of the wave trains we get when thinking of transients using the supposed Q of an atomic oscillator.[36] In fact, this length is the wavelength of the light ($\lambda = c/f = c\cdot\text{T} = \text{h}\cdot c/\text{E}$) that we would associate with this photon energy.[37]

Let us quickly insert another calculation, which you may find interesting—or not. If we think of an electromagnetic oscillation – as a beam or, what we are trying to do here, as some *quantum* – then its energy is going to be proportional to (a) the square of the amplitude of the oscillation – and we are *not* thinking of a quantum-mechanical amplitude here: we are talking the amplitude of a *physical* wave here – and (b) the square of the frequency. Hence, if we write the amplitude as $a$ and the frequency as $\omega$, then the energy should be equal to $\text{E} = \text{k}\cdot a^2\cdot\omega^2$. The k is just a proportionality factor.

However, relativity theory tells us the energy will have some equivalent mass, which is given by Einstein's mass-equivalence relation: $\text{E} = \text{m}\cdot c^2$. Hence, the energy will also be proportional to this equivalent mass. It is, therefore, very tempting to equate k and m. We can only do this, of course, if $c^2$ is equal to $a^2\cdot\omega^2$ or – what amounts to the same – if $c = a\cdot\omega$. You will recognize this as a tangential velocity formula, and so you should wonder: the tangential velocity of *what*? The $a$ in the $\text{E} = \text{k}\cdot a^2\cdot\omega^2$ formula that we started off with is an amplitude: why would we suddenly think of it as a radius now? I can*not* give you a very convincing answer to that question but – intuitively – we will probably want to think of our photon as having a circular polarization. Why? Because it is a boson and it, therefore, has angular momentum. To be precise, its angular momentum is $+\hbar$ or $-\hbar$. There is no zero-spin state.[38] Hence, if we think of this classically, then we will associate it with circular polarization.

We are now ready for the punch line, as Dr. Zwiebach would say. If the energy E in the Planck-Einstein relation ($\text{E} = \hbar\cdot\omega$) and the energy E in the energy equation for an oscillator ($\text{E} = \text{m}\cdot a^2\cdot\omega^2$) are the same –

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[36] In one of his famous *Lectures* (I-32-3), Feynman thinks about a sodium atom, which emits and absorbs sodium light, of course. Based on various assumptions – assumption that make sense in the context of the blackbody radiation model but *not* in the context of the Bohr model – he gets a Q of about $5\times10^7$. Now, the frequency of sodium light is about 500 THz ($500\times10^{12}$ oscillations per second). Hence, the *decay time* of the radiation is of the order of $10^{-8}$ seconds. So that means that, after $5\times10^7$ oscillations, the amplitude will have died by a factor $1/e \approx 0.37$. That seems to be very short, but it still makes for 5 million oscillations and, because the wavelength of sodium light is about 600 nm ($600\times10^{-9}$ meter), we get a wave train with a considerable length: $(5\times10^6)\cdot(600\times10^{-9}\ \text{meter}) = 3$ meter. *Surely you're joking, Mr. Feynman!* A photon with a length of 3 meter – or longer? While one might argue that relativity theory saves us here (relativistic length contraction should cause this length to reduce to zero as the wave train zips by at the speed of light), this just doesn't feel right – especially when one takes a closer look at the assumptions behind.

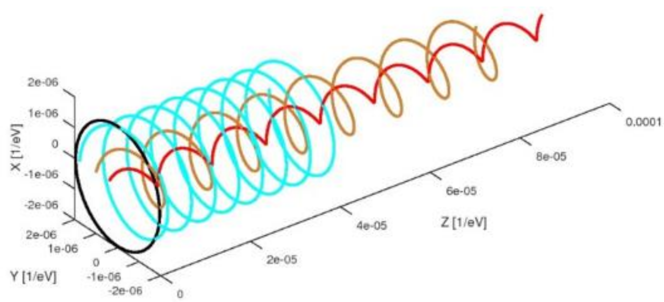
[37] This is short-wave ultraviolet light (UV-C). It is the light that is used to purify water, food or even air. It kills or inactivate microorganisms by destroying nucleic acids and disrupting their DNA. It is, therefore, harmful. The ozone layer of our atmosphere blocks most of it.

[38] This is one of the things in mainstream quantum mechanics that bothers me. All courses in quantum mechanics spend like two or three chapters on why bosons and fermions are different (spin-one versus spin-1/2) and, when it comes to the specifics, then the only boson we actually know (the photon) turns out to *not* be a typical boson because it can't have zero spin. Feynman gives some haywire explanation for this is section 4 of *Lecture* III-17. I will let you look it up (Feynman's *Lectures* are online) but, as far as I am concerned, I think it's really one of those things which makes me think of Prof. Dr. Ralston's criticism of his own profession: "*Quantum mechanics is the only subject in physics where teachers traditionally present haywire axioms they don't really believe, and regularly violate in research*." (John P. Ralston, *How To Understand Quantum Mechanics*, 2017, p. 1-10)

and they should be because we are talking about something that has some energy – then we get the following formula for the amplitude or radius $a$:

$$\mathrm{E} = \hbar \cdot \omega = \mathrm{m} \cdot a^2 \cdot \omega^2 \Leftrightarrow \hbar = \mathrm{m} \cdot a^2 \cdot \omega \Leftrightarrow a = \sqrt{\frac{\hbar}{\mathrm{m} \cdot \omega}} = \sqrt{\frac{\hbar}{\frac{\mathrm{E}}{c^2} \cdot \frac{\mathrm{E}}{\hbar}}} = \sqrt{\frac{\hbar^2}{\mathrm{m}^2 \cdot c^2}} = \frac{\hbar}{\mathrm{m} \cdot c}$$

This is the formula for the Compton radius of an electron ! How can we explain this? What relation could there possibly be between our *Zitterbewegung* model of an electron and the quantum of light? We do not want to confuse the reader too much but things become somewhat more obvious when staring at the illustration below (**Error! Reference source not found.**). We think of the *Zitterbewegung* of a free electron as a circular oscillation of a pointlike *charge* (with *zero* rest mass) moving about some center at the speed of light. However, as the electron starts moving along some *linear* trajectory at a relativistic velocity (i.e. a velocity that is a *substantial* fraction of *c*), then the *radius* of the oscillation will have to diminish – because the tangential velocity remains what it is: *c*. The geometry of the situation shows the circumference – so that's the Compton *wavelength* $\lambda_C = 2\pi \cdot a = 2\pi\hbar/\mathrm{m}c$ – becomes a wavelength in this process.




Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Let us quickly calculate it for our 10.2 eV photon. We should, of course, express the energy in SI units (10.2 eV ≈ $1.634 \times 10^{-18}$ *J*) to get what we should get:

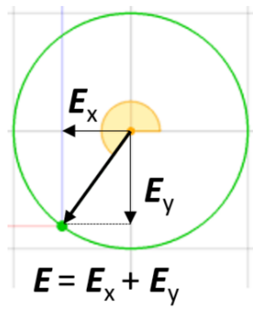
$$a = \frac{\hbar}{\mathrm{m} \cdot c} = \frac{\hbar}{\mathrm{E}/c} = \frac{(1.0545718 \times 10^{-18}\, J \cdot s) \cdot (3 \times 10^8\, m/s)}{1.634 \times 10^{-18}\, J} \approx 19.4 \times 10^{-9}\ \mathrm{m}$$

We have a *linear* structure here. The integrity of the photon is given by this wavelength, and we think it interferes with the disk-like structure of the electromagnetic field of the electron's ring current.

Of course, the example that was given was of a low-energy photon. The Compton radius of an electron is equal to about $386 \times 10^{-15}$ m, so that's about 50,000 times *smaller* than this 'Compton radius' of a photon. Unsurprisingly, that's the ratio between the electron's (rest) energy (about $8.187 \times 10^{-14}$ *J*) and the photon energy (about $1.634 \times 10^{-18}$ *J*). If we calculate the Compton radius for highly energetic photons, we get very different results. For example, the X-ray photons in the original Compton scattering experiment had an energy of about 17 keV = 17,000 eV and modern-day experiments will use gamma rays with even higher energies. One experiment, for example, uses a cesium-137 source emitting photons with an energy that is equal to 0.662 MeV = 662,000 eV. We can see these high photon energies can easily bridge the gap with the rest energy of the electron they are targeting.

I hope we were able to make the case: the Compton radius is, effectively, some kind of effective radius of interference.

As we may (or may not) have the attention of the reader, let us quickly further explore this *one-cycle photon model*. We can use the elementary wavefunction to represent the rotating field vector or, remembering the F = $q_e$E equation, the force field.




It is a delightfully simple model: the photon is just one single cycle traveling through space and time, which packs one unit of angular momentum ($\hbar$) or – which amounts to the same, one unit of physical action ($h$). This gives us an equally delightful interpretation of the Planck-Einstein relation ($f$ = 1/T = E/$h$) and we can, of course, do what we did for the electron, which is to express $h$ in two alternative ways: (1) the product of some momentum over a distance and (2) the product of energy over some time. We find, of course, that the distance and time correspond to the wavelength and the cycle time:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda \Leftrightarrow \lambda = \frac{hc}{\mathrm{E}}$$

$$h = \mathrm{E} \cdot \mathrm{T} \Leftrightarrow \mathrm{T} = \frac{h}{\mathrm{E}} = \frac{1}{f}$$

Needless to say, the E = m$c^2$ mass-energy equivalence relation can be written as p = m$c$ = E/$c$ for the photon. The two equations are, therefore, wonderfully consistent:

$$h = \mathrm{p} \cdot \lambda = \frac{\mathrm{E}}{c} \cdot \lambda = \frac{\mathrm{E}}{f} = \mathrm{E} \cdot \mathrm{T}$$

Let us now try something more adventurous: let us try to calculate the strength of the electric field. How can we do that? Energy is some force over a distance and, hence, the force must equal the ratio of the energy and the distance. What distance should we use? The force will vary over the cycle and, hence, this distance is a distance that we must be able to relate to this fundamental cycle. Is it the Compton *radius* ($a$) or the wavelength (λ)? They differ by a factor 2π only, so let us just try the radius and see if we get some kind of sensible result:

$$\mathrm{F} = \frac{\mathrm{E}}{a} = \frac{2\pi \cdot \mathrm{E}}{\lambda} = \frac{2\pi \cdot h \cdot f}{\lambda} = \frac{2\pi \cdot h \cdot c}{\lambda^2}$$

Does this look weird? Not really. We get the E·λ = h·$c$ equation from *de Broglie*'s $h$ = p·λ = m·$c$·λ = E·λ/$c$ equation and the equation above respects that equation:

$$\frac{\mathrm{E}}{a} = \frac{2\pi \cdot h \cdot c}{\lambda^2} \Leftrightarrow \mathrm{E} \cdot \lambda = \frac{2\pi \cdot a \cdot h \cdot c}{\lambda} = h \cdot c$$

Let's try the next logical step. The electric field – which we will write as *E*[39] – is the force per unit charge which, we should remind the reader, is the *coulomb* – *not* the electron charge. Why? Because we use SI units. We, therefore, get a delightfully simple formula for the strength of the electric field vector for a photon[40]:

$$E = \frac{\frac{2\pi hc}{\lambda^2}}{1} = \frac{2\pi hc}{\lambda^2} = \frac{2\pi \mathrm{E}}{\lambda} = \frac{\mathrm{E}}{a}$$

The electric field is the ratio of the energy and the Compton radius. Does this make sense? What about units? We divided by 1 *coulomb* and the physical dimension is, therefore, equal to [*E*] = [E/*a*] *per coulomb*. A *joule* is a newton·meter and [E/*a*] is, therefore, equal to N·m/m = N. We're fine. Let us calculate its value for our 10.2 eV photon (using SI units once again, of course):

$$E \approx \frac{1.634 \times 10^{-18}\,J}{19.4 \times 10^{-9}\,m \cdot C} \approx 84 \times 10^{-12}\,\frac{\mathrm{N}}{\mathrm{C}}$$

We hope the reader can now, finally, connect the dots and *imagine* how a photon and an electron *actually* interfere. The core of the story is an interference which – temporarily – creates an unstable wavicle which does *not* respect the integrity of Planck's quantum of action (E = h·*f*). The equilibrium situation is re-established as the electron – now moving – emits a new photon. Both the electron and the photon respect the integrity of Planck's quantum of action again and they are, therefore, *stable*.

END

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[39] The *E* and E symbols should not be confused. *E* is the magnitude of the electric field vector and E is the energy of the photon. We hope the italics (*E*) – and the context of the formula, of course! – will be sufficient to help the reader distinguish the electric field vector (*E*) from the energy (E). We do not needlessly want to multiply the number of symbols we are using here.

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