Beal Conjecture Proved Finally

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

The author proves directly the original Beal conjecture that if $A^x + B^y = C^z$, where A, B, C, x, y, z are positive integers and $x, y, z > 2$, then A, B and C have a common prime factor. The principles applied in the proof are based the analytic observations of the factorization of sample numerical equations. Guided by the numerical examples, the factorization of the equations provided relationships between the prime factors involved. High school students can learn and prove this conjecture as a bonus question on a final class exam.

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Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. So also, to prove Beal Conjecture, one will be guided by the analytic observations of numerical examples of equations involved in Beal Conjecture.

Observation 1: $\boxed{2^3 + 2^3 = 2^4}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$2^3 + 2^3 = 2^4$$

$$2^3 + 2^3 = 2^3 \bullet 2$$

$$\underbrace{2^3}_{k}\underbrace{(1+1)}_{L} = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Also, $\dfrac{K}{M} = \dfrac{2}{1+1} = 1$

Corresponding relationship formula

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$r = s = t = 2$$

$$x = 3.y = 3, z = 4$$

$$(D = 1, E = 1, F = 1)$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M, \;\; P = L$$

Observation 2: $\boxed{7^6 + 7^7 = 98^3}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$7^6 + 7^7 = 98^3$$

$$7^6 + 7^6 \bullet 7 = (49 \bullet 2)^3$$

$$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$$

$$7^6(1+7) = 7^6 \bullet 2^3$$

$$\underbrace{7^6}_{K}\underbrace{(1+7)}_{L} = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Also, $\dfrac{K}{M} = \dfrac{2^3}{1+7} = 1$

Corresponding relationship formula

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$r = s = t = 7$$

$$x = 6, y = 7, z = 3$$

$$(D = 1, E = 1, F = 14)$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M, \;\; P = L$$

Observation 3: $\boxed{3^3+6^3=3^5}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$3^3+6^3=3^5$$
$$3^3+(3\bullet 2)^3=3^5$$
$$3^3+3^3\bullet 2^3=3^5$$
$$3^3(1+2^3)=3^3\bullet 3^2$$
$$\underbrace{3^3}_{K}(\underbrace{1+8}_{L})=\underbrace{3^3}_{M}\bullet\underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

$$\frac{K}{M}=\frac{3^2}{1+8}=1$$

Corresponding relationship formula

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^x=F^zt^z$$
$$r=s=t=3$$
$$x=3,y=3,z=5$$
$$(D=1,E=2,F=1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ \ P=\ L$$

Observation 4: $\boxed{2^9+8^3=4^5}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$2^9+8^3=4^5$$
$$2^9+([2^3])^3=([2^2])^5$$
$$2^9+2^9=2^{10}$$
$$2^9(1+1)=2^9\bullet 2$$
$$\underbrace{2^9}_{k}(\underbrace{1+1}_{L})=\underbrace{2^9}_{M}\bullet\underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side. $\frac{K}{M}=\frac{2}{1+1}=1$

Corresponding relationship formula

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^x=F^zt^z$$
$$r=s=t=2$$
$$x=9,y=3,z=5$$
$$(D=1,E=4,F=2)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ \ P=\ L$$

Observation 5: $\boxed{34^5+51^4=85^4}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$34^5+51^4=85^4$$
$$(17\bullet 2)^5+(17\bullet 3)^4=(17\bullet 5)^4$$
$$17^5\bullet 2^5+17^4\bullet 3^4=17^4\bullet 5^4$$
$$17^4(17\bullet 2^5+3^4)=17^4\bullet 5^4$$
$$\underbrace{17^4}_{k}(\underbrace{17\bullet 2^5+3^4}_{L})=\underbrace{17^4}_{M}\bullet\underbrace{5^4}_{P}$$

(Note: $17\bullet 2^5+3^4=17\bullet 32+81=625;$ $5^4=625$)

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side. $\frac{K}{M}=\frac{5^4}{17\bullet 2^5+3^4}=1$

Corresponding relationship formula

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^x=F^zt^z$$
$$r=s=t=17$$
$$x=5, y=4, z=4$$
$$(D=2, E=3, F=5)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ \ P=\ L$$

Observation 6: $\boxed{3^9+54^3=3^{11}}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$3^9+54^3=3^{11}$$
$$3^9+(3^3\bullet 2)^3=3^{11}$$
$$3^9+3^9\bullet 2^3=3^{11}$$
$$3^9(1+2^3)=3^9\bullet 3^2$$
$$\underbrace{3^9}_{k}(\underbrace{1+2^3}_{L})=\underbrace{3^9}_{M}\bullet\underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side. $\frac{K}{M}=\frac{3^2}{1+2^3}=1$

Corresponding relationship formula

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^x=F^zt^z$$
$$r=s=t=3$$
$$x=9, y=3, z=11$$
$$(D=1, E=18, F=1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ \ P=\ L$$

Observation 7: $\boxed{33^5 + 66^5 = 33^6}$

Identify the greatest common factor. of all three terms of the equation and factor it out on the left side.

$$33^5 + 66^5 = 33^6$$

$$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5 = (11 \bullet 3)^6$$

$$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$$

$$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$$

$$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

$$\frac{K}{M} = \frac{11 \bullet 3^6}{3^5 + 2^5 \bullet 3^5} = 1$$

Corresponding relationship formula

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$r = s = t = 11$$

$$x = 5, y = 5, z = 6$$

$$D = 3, E = 6, F = 3$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M, \quad P = L$$

Proof of Beal Conjecture

Given: $A^x + B^y = C^z$, A, B, C, x, y, z are positive integers and $x, y, z > 2$.
Required: To prove that A, B and C have a common prime factor.

Plan: Let r , s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.** The proof would be complete after showing that $r = s = t$

Proof: One would be guided by the analytic structure of the numerical examples covered. There are two main steps. In the first step, one will determine how r *and* t are related, and in the second step, one will determine how s *and* t are related.

Step 1: One will factor out r^x

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^x = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$

$$K = M, \ \ P = \ L$$

$$1.\ r^x = t^x; \quad 2.\ t^{z-x}F^z = D^x + E^y s^y r^{-x}$$

If $r^x = t^x$, then $r = t$. $\quad (\log r^x = \log t^x;\ x \log r = x \log t;\ \log r = \log t;\ r = t)$

Step 2: One will factor out s^y

$$(Es)^y + (Dr)^x + = (Ft)^z$$

$$E^y s^y + D^x r^x = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$

$$K = M, \ \ P = \ L$$

$$1.\ s^y = t^y; \ \ 2.\ t^{z-y}F^z = E^y + D^x r^x \bullet s^{-y}$$

If $s^y = t^y$,then $s = t$

Since it has been shown in Step 1 that $r = t$, and in Step 2 that, $s = t$; $r = s = t$.

Since $A = Dr$ **,** $B = Es$ **,** $C = Ft$ and $r = s = t$, A, B and C have a common prime factor, and the proof in complete.

Conclusion

Beal conjecture has been proved in this paper. The principles applied in the proof are based the analytic observations of the factorization of sample numerical equations. Since the main concern of this conjecture is a common prime factor, it was appropriate that factorization was the main tool in observing the structure of the factorization of the equations. The factorization of the equation revealed the relationships between the prime factors involved. The proof in this paper addresses the original Beal conjecture, and **not** the equivalent conjecture. High school students can learn and prove this conjecture as a bonus question on a final class exam.

PS: Other proofs of Beal Conjecture by the author are at viXra:1702.0331; viXra:1609.0383; viXra:1609.0157;

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