On the study of a fundamental modular equation for an initial theoretical framework concerning the motivations of the mathematical connections that are obtained between various formulas of Ramanujan's mathematics and different parameters of Particle Physics and String Theory

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Abstract

In this research thesis, we have analyzed a fundamental modular equation for an initial theoretical framework concerning the motivations of the mathematical connections that are obtained between various formulas of Ramanujan's mathematics and different parameters of Particle Physics and String Theory.

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https://www.britannica.com/biography/Srinivasa-Ramanujan

Introduction

"In the work of Ramanujan, the number 24 appears repeatedly. This is an example of what mathematicians call magic numbers, which continually appear, where we least expect them, for reasons that no one understands. Miraculously, Ramanujan's function also appears in string theory. The number 24 appearing in Ramanujan's function is also the origin of the miraculous cancellations occurring in string theory. In string theory, each of the 24 modes in the Ramanujan function corresponds to a physical vibration of the string. Whenever the string executes its complex motions in space-time by splitting and recombining, a large number of highly sophisticated mathematical identities must be satisfied. These are precisely the mathematical identities discovered by Ramanujan. (Since physicists add two more dimensions when they count the total number of vibrations appearing in a relativistic theory, this means that space-time must have 24 + 2 = 26 space-time dimensions.)"

"When the Ramanujan function is generalized, the number 24 is replaced by the number 8. Thus the critical number for the superstring is 8 + 2, or 10. This is the origin of the tenth dimension. The string vibrates in ten dimensions because it requires these generalized Ramanujan functions in order to remain self-consistent. In other words, physicists have not the slightest understanding of why ten and 26 dimensions are singled out as the dimension of the string. It's as though there is some kind of deep numerology being manifested in these functions that no one understands. It is precisely these magic numbers appearing in the elliptic modular function that determines the dimension of space-time to be ten."

(Michio Kaku, Hyperspace: A Scientific Odyssey Through Parallel Universes, Time Warps, and the Tenth Dimension)

Now, from the paper "Modular equation and approximations to π" of Srinivasa Ramanujan, we analyze the following formula:

$$G_{65} = \left\{\left(\frac{1+\sqrt{5}}{2}\right)\left(\frac{3+\sqrt{13}}{2}\right)\right\}^{\frac{1}{4}} \sqrt{\left\{\sqrt{\left(\frac{9+\sqrt{65}}{8}\right)} + \sqrt{\left(\frac{1+\sqrt{65}}{8}\right)}\right\}}.$$

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2

Input:

$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)} \sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)} + \sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}$$

Exact result:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)} + \frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}}}{\sqrt{2}}$$

Decimal approximation:

2.415871946186809362816339075281469555671777147068661130889...

2.41587194618…

Alternate forms:

$$\frac{1}{2}\sqrt[4]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)} \sqrt{\sqrt{1+\sqrt{65}} + \sqrt{9+\sqrt{65}}}$$

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\sqrt{1-8i} + \sqrt{1+8i} + \sqrt{5} + \sqrt{13}}}{2\sqrt{2}}$$

$$\sqrt{\text{root of } x^8 - 8x^7 + 12x^6 + 8x^5 - 27x^4 + 8x^3 + 12x^2 - 8x + 1 \text{ near } x = 5.83644}$$

Minimal polynomial:

$$x^{16} - 8x^{14} + 12x^{12} + 8x^{10} - 27x^8 + 8x^6 + 12x^4 - 8x^2 + 1$$

From which we calculate, squaring the two terms (1+√5)/2 and (3+√13)/2 and putting 1/24 instead of 8 that multiplied the two terms 9+√65 and 1+√65, we obtain:

[((1+sqrt5)/2)^2 ((3+sqrt13)/2)^2]^1/4 (((sqrt((9+65^0.5)/24)+sqrt((1+65^0.5)/24))))^1/2

Input:

$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)^2}\sqrt{\sqrt{\frac{1}{24}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{24}\left(1+\sqrt{65}\right)}}$$

Exact result:

$$\frac{1}{2}\sqrt{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{6}\left(9+\sqrt{65}\right)}\right)}$$

Decimal approximation:

2.791002819266536925164482620044353502833296791440335845365...

2.791002819266…

Alternate forms:

$$\frac{1}{4}\sqrt{\left(\sqrt{\frac{1}{3}-\frac{8i}{3}}+\sqrt{\frac{1}{3}+\frac{8i}{3}}+\sqrt{\frac{5}{3}}+\sqrt{\frac{13}{3}}\right)\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}$$

$$\frac{\sqrt{(1+\sqrt{5})(3+\sqrt{13})\left(\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}\right)}}{2\times 2^{3/4}\sqrt[4]{3}}$$

$$\sqrt[4]{\text{root of } 6561x^8-393660x^7-278478x^6+466560x^5+1169883x^4+51840x^3-3438x^2-540x+1 \text{ near } x=60.6794}$$

Minimal polynomial:

$$6561x^{32}-393660x^{28}-278478x^{24}+466560x^{20}+1169883x^{16}+51840x^{12}-3438x^8-540x^4+1$$

Now, performing the square root of the previous expression, we obtain:

From

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2

multiplying by 1/3, the following terms within the roots, (1+√5)/2, (3+√13)/2, 9+√65 and 1+√65, we obtain:

[(1/3(((1+sqrt5)/2) ((3+sqrt13)/2)))]^1/4 (((sqrt(1/3(9+65^0.5)/8)+sqrt(1/3(1+65^0.5)/8))))^1/2

Input:

$$\sqrt[4]{\frac{1}{3}\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)\right)} \sqrt{\sqrt{\frac{1}{3}\left(\frac{1}{8}\left(9+\sqrt{65}\right)\right)}+\sqrt{\frac{1}{3}\left(\frac{1}{8}\left(1+\sqrt{65}\right)\right)}}$$

Exact result:

$$\frac{\sqrt[4]{\frac{1}{3}(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\frac{1}{2}\sqrt{\frac{1}{6}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{6}\left(9+\sqrt{65}\right)}}}{\sqrt{2}}$$

Decimal approximation:

1.394804318458619474625610830105273761964580280875442373809...

1.394804318458...

Alternate forms:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})\left(5+\sqrt{65}+\sqrt{74+10\sqrt{65}}\right)}}{2\sqrt{3}}$$

$$\frac{\sqrt[4]{\frac{1}{2}(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}}}{2\sqrt{3}}$$

$$\frac{\sqrt{\sqrt{\frac{1}{3}-\frac{8i}{3}}+\sqrt{\frac{1}{3}+\frac{8i}{3}}+\sqrt{\frac{5}{3}}+\sqrt{\frac{13}{3}}} \sqrt[4]{\frac{1}{3}(1+\sqrt{5})(3+\sqrt{13})}}{2\sqrt{2}}$$

Minimal polynomial:

$$6561x^{16}-17496x^{14}+8748x^{12}+1944x^{10}-2187x^8+216x^6+108x^4-24x^2+1$$

Or, equivalently:

[(1/3(((1+sqrt5)/2) ((3+sqrt13)/2)))]^1/4 (((sqrt(1/24(9+65^0.5))+sqrt(1/24(1+65^0.5)))))^1/2

Input:

$$\sqrt[4]{\frac{1}{3}\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)\right)} \sqrt{\sqrt{\frac{1}{24}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{24}\left(1+\sqrt{65}\right)}}$$

Exact result:

$$\frac{\sqrt[4]{\frac{1}{3}(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\frac{1}{2}\sqrt{\frac{1}{6}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{6}\left(9+\sqrt{65}\right)}}}{\sqrt{2}}$$

Decimal approximation:

1.394804318458619474625610830105273761964580280875442373809...

1.39480431845…

Alternate forms:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})\left(5+\sqrt{65}+\sqrt{74+10\sqrt{65}}\right)}}{2\sqrt{3}}$$

$$\frac{\sqrt[4]{\frac{1}{2}(1+\sqrt{5})(3+\sqrt{13})} \sqrt{\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}}}{2\sqrt{3}}$$

$$\frac{\sqrt{\sqrt{\frac{1}{3}-\frac{8i}{3}}+\sqrt{\frac{1}{3}+\frac{8i}{3}}+\sqrt{\frac{5}{3}}+\sqrt{\frac{13}{3}}} \sqrt[4]{\frac{1}{3}(1+\sqrt{5})(3+\sqrt{13})}}{2\sqrt{2}}$$

Minimal polynomial:

$$6561x^{16}-17496x^{14}+8748x^{12}+1944x^{10}-2187x^{8}+216x^{6}+108x^{4}-24x^{2}+1$$

Note that:

1/10^27*sqrt((((((([((1+sqrt5)/2)^2 ((3+sqrt13)/2)^2]^1/4 (((sqrt((9+65^0.5)/24)+sqrt((1+65^0.5)/24))))^1/2))))))

Input:

$$\frac{1}{10^{27}}\sqrt{\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^2\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)^2}\ \sqrt{\sqrt{\frac{1}{24}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{24}\left(1+\sqrt{65}\right)}}}$$

Exact result:

$$\frac{\sqrt[4]{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\frac{1}{2}\sqrt{\frac{1}{6}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{6}\left(9+\sqrt{65}\right)}\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000\sqrt{2}}$$

Decimal approximation:

$1.6706294679750315373833269400701464529435819177968074\ldots \times 10^{-27}$

$1.6706294679\ldots*10^{-27}$ result very near to the value of the formula:

$$m_{p'} = 2 \times \frac{\eta}{R} m_P = 1.6714213 \times 10^{-27}\ \text{kg}$$

that is the holographic proton mass (N. Haramein) that we have obtained performing the square root of the expression and multiplying by 10^{-27} (this result is a sub-multiple of a "golden number" 1.6706294679...)

Alternate forms:

$$\frac{\sqrt[4]{\left(\sqrt{\frac{1}{3}-\frac{8i}{3}}+\sqrt{\frac{1}{3}+\frac{8i}{3}}+\sqrt{\frac{5}{3}}+\sqrt{\frac{13}{3}}\right)\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}}{2\,000\,000\,000\,000\,000\,000\,000\,000\,000}$$

$$\frac{\sqrt[4]{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\left(\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}\right)}}{1\,000\,000\,000\,000\,000\,000\,000\,000\,000 \times 2^{7/8}\sqrt[8]{3}}$$

Furthermore, we obtain:

a) [(1/3(((1+sqrt5)/2) ((3+sqrt13)/2)))]^1/4 (((sqrt(1/x(9+65^0.5))+sqrt(1/x(1+65^0.5)))))^1/2 =1.39480431845

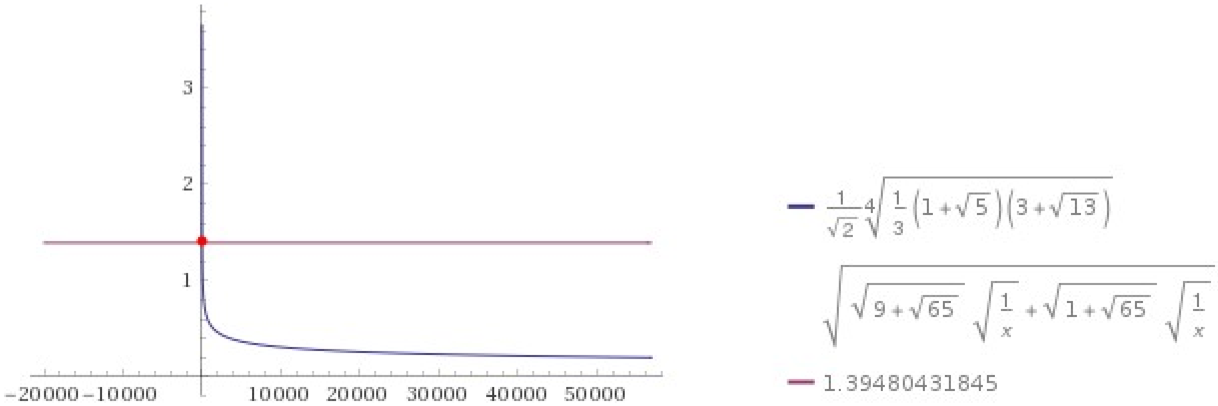
Input interpretation:

$$\sqrt[4]{\frac{1}{3}\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)\right)}\sqrt{\sqrt{\frac{1}{x}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{x}\left(1+\sqrt{65}\right)}} = 1.39480431845$$

Result:

$$\frac{\sqrt[4]{\frac{1}{3}\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\sqrt{\sqrt{9+\sqrt{65}}\sqrt{\frac{1}{x}}+\sqrt{1+\sqrt{65}}\sqrt{\frac{1}{x}}}}{\sqrt{2}} = 1.39480431845$$

Plot:



Solution:

$x = 24.0000000006$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

And

b) [(1/3(((x)) ((3+sqrt13)/2)))]^1/4
(((sqrt(1/24(9+65^0.5))+sqrt(1/24(1+65^0.5)))))^1/2 =1.39480431845

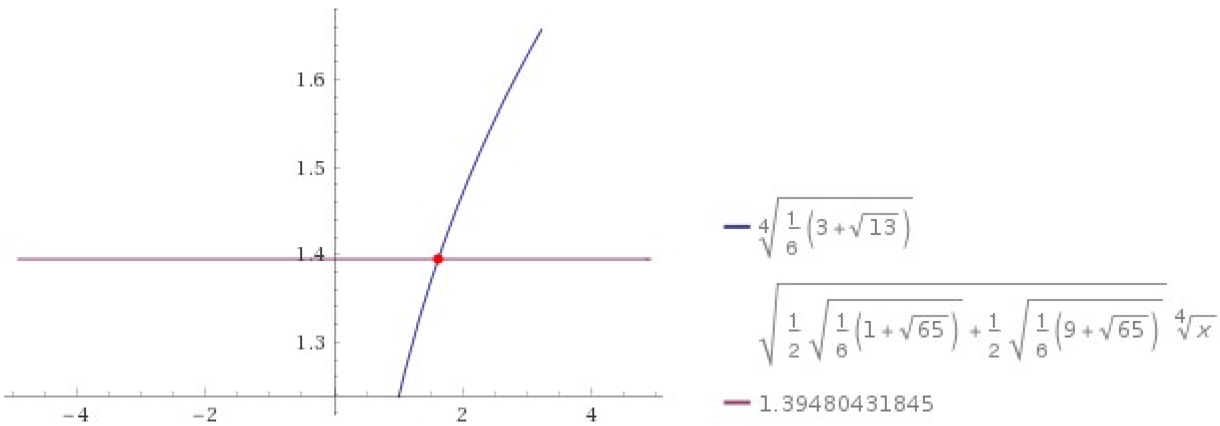
Input interpretation:

$$\sqrt[4]{\frac{1}{3}\left(x\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)\right)}\sqrt{\sqrt{\frac{1}{24}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{24}\left(1+\sqrt{65}\right)}} = 1.39480431845$$

Result:

$$\sqrt[4]{\frac{1}{6}\left(3+\sqrt{13}\right)}\sqrt{\frac{1}{2}\sqrt{\frac{1}{6}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{6}\left(9+\sqrt{65}\right)}}\sqrt[4]{x} = 1.39480431845$$

Plot:



Alternate forms:

$$\frac{\sqrt[4]{\left(3+\sqrt{13}\right)\left(5+\sqrt{65}+\sqrt{74+10\sqrt{65}}\right)}\sqrt[4]{x}}{2^{3/4}\sqrt{3}} = 1.39480431845$$

$$\frac{\sqrt[4]{3+\sqrt{13}}\sqrt{\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}}\sqrt[4]{x}}{2\sqrt{3}} = 1.39480431845$$

$$\sqrt[4]{x}\ \sqrt[4]{\text{root of } 43046721x^8 - 71744535x^7 - 60052833x^6 - 17714700x^5 - 603612x^4 + 218700x^3 - 9153x^2 + 135x + 1 \text{ near } x = 2.33919} =$$

1.39480431845

Alternate form assuming x is positive:

$1.00000000000\sqrt[4]{x} = 1.12783848555$

Solution:

$x = 1.61803398871$

1.61803398871 result that is the value of the golden ratio

From the following Ramanujan expression:

$$64a - 4096be^{-\pi\sqrt{n}} + \cdots = e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots,$$

for n = 65, calculating the right hand side, we obtain:

e^(Pi*sqrt65)-24+276*e^(-Pi*sqrt65)

Input:

$e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}$

Exact result:

$-24 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}$

Decimal approximation:

$9.998936958782675181210795051435844795060562663559055 6\ldots \times 10^{10}$

$9.998936958\ldots*10^{10}$

Property:

$-24 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}$ is a transcendental number

Alternate form:

$e^{-\sqrt{65}\,\pi}\left(276 - 24\,e^{\sqrt{65}\,\pi} + e^{2\sqrt{65}\,\pi}\right)$

Series representations:

$$e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}} = e^{-\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}}\left(276 - 24\, e^{\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}} + e^{2\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}}\right)$$

$$e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}} = \exp\left(-\pi\sqrt{64}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{64}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(276 - 24\, e^{\pi\sqrt{64}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{64}\right)^k\left(-\frac{1}{2}\right)_k}{k!}} + \exp\left(2\pi\sqrt{64}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{64}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}} = \exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65-z_0)^k z_0^{-k}}{k!}\right)\left(276 - 24\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65-z_0)^k z_0^{-k}}{k!}\right) + \exp\left(2\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(65-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0))$

From this result, performing the ln, we obtain:

ln(((e^(Pi*sqrt65)-24+276*e^(-Pi*sqrt65))))

Input:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right)$$

$\log(x)$ is the natural logarithm

Exact result:

$$\log\left(-24 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}\right)$$

Decimal approximation:

25.328329713162086429474968922647242867615440600305372057 22...

25.328329713…

Alternate form:

$$\log\left(276 - 24\,e^{\sqrt{65}\,\pi} + e^{2\sqrt{65}\,\pi}\right) - \sqrt{65}\,\pi$$

Alternative representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = \log_e\left(-24 + 276\,e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = \log(a)\log_a\left(-24 + 276\,e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = -\text{Li}_1\left(25 - 276\,e^{-\pi\sqrt{65}} - e^{\pi\sqrt{65}}\right)$$

Series representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = \log\left(-25 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}\right) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{-25+276\,e^{-\sqrt{65}\,\pi}+e^{\sqrt{65}\,\pi}}\right)^k}{k}$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = 2\,i\,\pi \left\lfloor \frac{\arg\left(-24 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - x\right)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-24 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = 2\,i\,\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\,\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-24 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = \int_1^{-24+276\,e^{-\sqrt{65}\,\pi}+e^{\sqrt{65}\,\pi}} \frac{1}{t}\,dt$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) = -\frac{i}{2\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-25 + 276\,e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

From the result obtained, we subtract the value of the above expression:

$$\sqrt[4]{\frac{1}{3}\left(\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)\right)}\sqrt{\sqrt{\frac{1}{24}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{24}\left(1+\sqrt{65}\right)}}$$

that is equal to 1.394804318 and obtain:

ln(((e^(Pi*sqrt65)-24+276*e^(-Pi*sqrt65))))-1.394804318

Input interpretation:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) - 1.394804318$$

$\log(x)$ is the natural logarithm

Result:

23.933525395...

23.933525395… result very near to the black hole entropy 23.9078

(From: **Three-dimensional AdS gravity and extremal CFTs at c = 8m** - https://arxiv.org/abs/0708.3386)

Alternative representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) - 1.3948 = -1.3948 + \log_e\left(-24 + 276\,e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) - 1.3948 = -1.3948 + \log(a)\log_a\left(-24 + 276\,e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\,e^{-\pi\sqrt{65}}\right) - 1.3948 = -1.3948 - \text{Li}_1\left(25 - 276\,e^{-\pi\sqrt{65}} - e^{\pi\sqrt{65}}\right)$$

Series representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - 1.3948 =$$
$$-1.3948 + \log\left(-25 + 276\, e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-25 + 276\, e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)^{-k}}{k}$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - 1.3948 =$$
$$-1.3948 + \log\left(-24 + 276\, e^{-\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}} + e^{\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}}\right)$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - 1.3948 =$$
$$-1.3948 + \log\left(-24 + 276 \exp\left(-\pi\sqrt{64}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + e^{\pi\sqrt{64}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}\right)$$

Integral representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - 1.3948 = -1.3948 + \int_1^{-24+276\, e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}} \frac{1}{t}\, dt$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - 1.3948 = -1.3948 +$$
$$\frac{1}{2 i \pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-25 + 276\, e^{-\pi\sqrt{65}} + e^{\pi\sqrt{65}}\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

Or, subtracting $\pi^{1/4}$:

ln(((e^(Pi*sqrt65)-24+276*e^(-Pi*sqrt65))))-(Pi)^1/4

Input:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - \sqrt[4]{\pi}$$

log(x) is the natural logarithm

Exact result:

$$\log\left(-24+276\, e^{-\sqrt{65}\,\pi}+e^{\sqrt{65}\,\pi}\right)-\sqrt[4]{\pi}$$

Decimal approximation:

23.996994349361696716677434004696962014306074376487267798 76...

23.99699434… ≈ 24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

Alternate form:

$$-\sqrt[4]{\pi}-\sqrt{65}\,\pi+\log\left(276-24\, e^{\sqrt{65}\,\pi}+e^{2\sqrt{65}\,\pi}\right)$$

Alternative representations:

$$\log\left(e^{\pi\sqrt{65}}-24+276\, e^{-\pi\sqrt{65}}\right)-\sqrt[4]{\pi}=\log_e\left(-24+276\, e^{-\pi\sqrt{65}}+e^{\pi\sqrt{65}}\right)-\sqrt[4]{\pi}$$

$$\log\left(e^{\pi\sqrt{65}}-24+276\, e^{-\pi\sqrt{65}}\right)-\sqrt[4]{\pi}=\log(a)\log_a\left(-24+276\, e^{-\pi\sqrt{65}}+e^{\pi\sqrt{65}}\right)-\sqrt[4]{\pi}$$

$$\log\left(e^{\pi\sqrt{65}}-24+276\, e^{-\pi\sqrt{65}}\right)-\sqrt[4]{\pi}=-\text{Li}_1\left(25-276\, e^{-\pi\sqrt{65}}-e^{\pi\sqrt{65}}\right)-\sqrt[4]{\pi}$$

Series representations:

$$\log\left(e^{\pi\sqrt{65}}-24+276\, e^{-\pi\sqrt{65}}\right)-\sqrt[4]{\pi}=$$

$$-\sqrt[4]{\pi}+\log\left(-25+276\, e^{-\sqrt{65}\,\pi}+e^{\sqrt{65}\,\pi}\right)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{-25+276\, e^{-\sqrt{65}\,\pi}+e^{\sqrt{65}\,\pi}}\right)^k}{k}$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - \sqrt[4]{\pi} = -\sqrt[4]{\pi} + 2\, i\, \pi \left\lfloor \frac{\arg\left(-24 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - x\right)}{2\,\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-24 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - \sqrt[4]{\pi} = -\sqrt[4]{\pi} + 2\, i\, \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\,\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(-24 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi} - z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - \sqrt[4]{\pi} = -\sqrt[4]{\pi} + \int_1^{-24+276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}} \frac{1}{t}\, dt$$

$$\log\left(e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}\right) - \sqrt[4]{\pi} = -\sqrt[4]{\pi} - \frac{i}{2\,\pi} \int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\left(-25 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}\right)^{-s} \Gamma(-s)^2\, \Gamma(1+s)}{\Gamma(1-s)}\, ds \quad \text{for } -1 < \gamma < 0$$

From the previous expression, we obtain:

a) ln(((x^(Pi*sqrt65)-24+276*x^(-Pi*sqrt65)))) = 25.328329713162

Input interpretation:

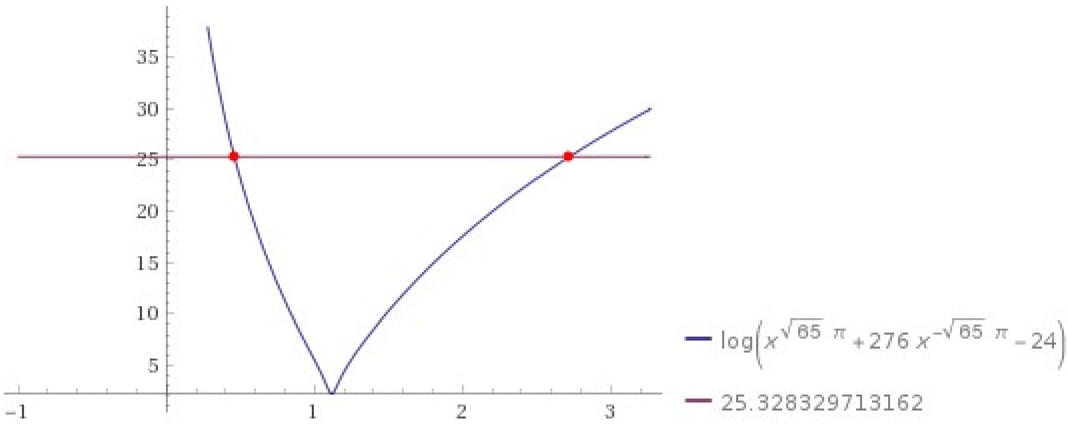
$$\log\left(x^{\pi\sqrt{65}} - 24 + 276\, x^{-\pi\sqrt{65}}\right) = 25.328329713162$$

log(x) is the natural logarithm

Result:

$$\log\left(x^{\sqrt{65}\,\pi} + 276\, x^{-\sqrt{65}\,\pi} - 24\right) = 25.328329713162$$

Plot:



Real solutions:

$x \approx 0.4592786176336052$

$x \approx 2.718281828459017$

2.718281828459017 = ***e***

Solutions:

$x \approx 2.718281828459045^{0.03948148224992763\,(6.283185307179586\,i\,n - 19.70792884768469)}$,
$-12.000000000000000 \leq n \leq 12.000000000000000$

$x \approx 2.718281828459045^{0.03948148224992763\,(6.283185307179586\,i\,n + 25.32832971340184)}$,
$-12.000000000000000 \leq n \leq 12.000000000000000$

And:

b) ln(((e^(x*sqrt65)-24+276*e^(-x*sqrt65)))) = 25.328329713162

Input interpretation:

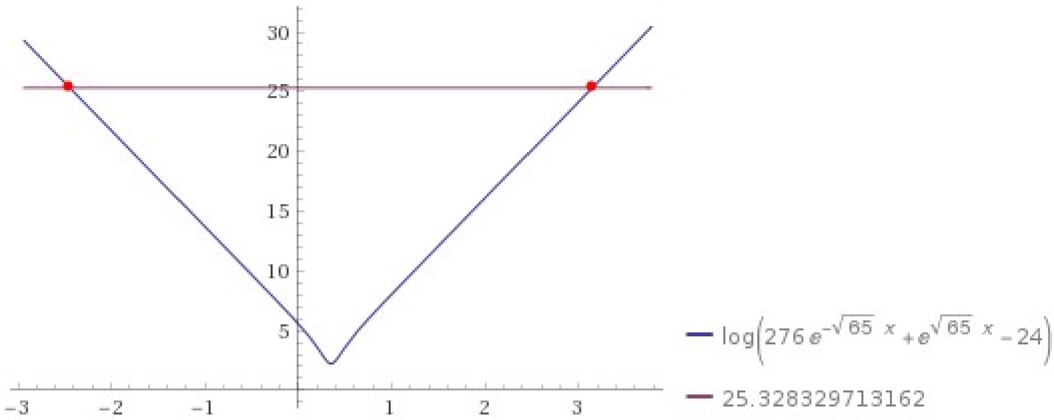
$$\log\left(e^{x\sqrt{65}} - 24 + 276\, e^{-x\sqrt{65}}\right) = 25.328329713162$$

log(x) is the natural logarithm

Result:

$$\log\left(276\, e^{-\sqrt{65}\, x} + e^{\sqrt{65}\, x} - 24\right) = 25.328329713162$$

Plot:



Real solutions:

$x \approx -2.444467723925576$

$x \approx 3.141592653589760$

3.141592653589760 = **π**

Solutions:

$x \approx 0.12403473458920846\,(6.283185307179586\, i\, n - 19.70792884768469)\,, \quad n \in \mathbb{Z}$

$x \approx 0.12403473458920846\,(6.283185307179586\, i\, n + 25.32832971340184)\,, \quad n \in \mathbb{Z}$

$\mathbb{Z}$ is the set of integers

c) ln(((e^(Pi*sqrt65)-3x+276*e^(-Pi*sqrt65)))) = 25.328329713162

Input interpretation:

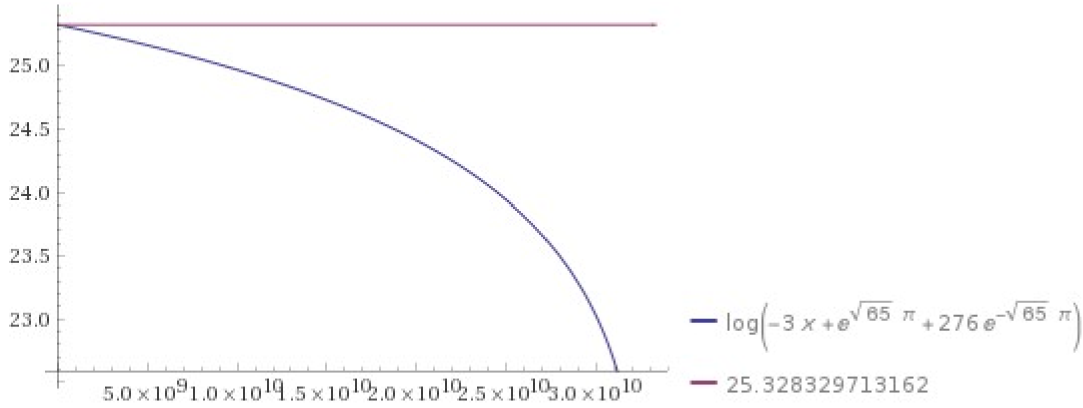
$\log\left(e^{\pi\sqrt{65}} - 3x + 276\, e^{-\pi\sqrt{65}}\right) = 25.328329713162$

log(x) is the natural logarithm

Result:

$\log\left(-3x + e^{\sqrt{65}\,\pi} + 276\, e^{-\sqrt{65}\,\pi}\right) = 25.328329713162$

Plot:



Alternate form assuming x is positive:

$$\log\left(-3 e^{\sqrt{65}\pi} x + e^{2\sqrt{65}\pi} + 276\right) = 50.656659426564$$

Solution:

$x = 8$

8

Note that when the Ramanujan function is generalized, 24 is replaced by 8 (8 + 2 = 10) for fermionic strings

Integer solution:

$x = 0$

From the first expression

$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\,\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}$$

we obtain:

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/x)+sqrt((1+65^0.5)/x))))^1/2 = 2.4158719461868

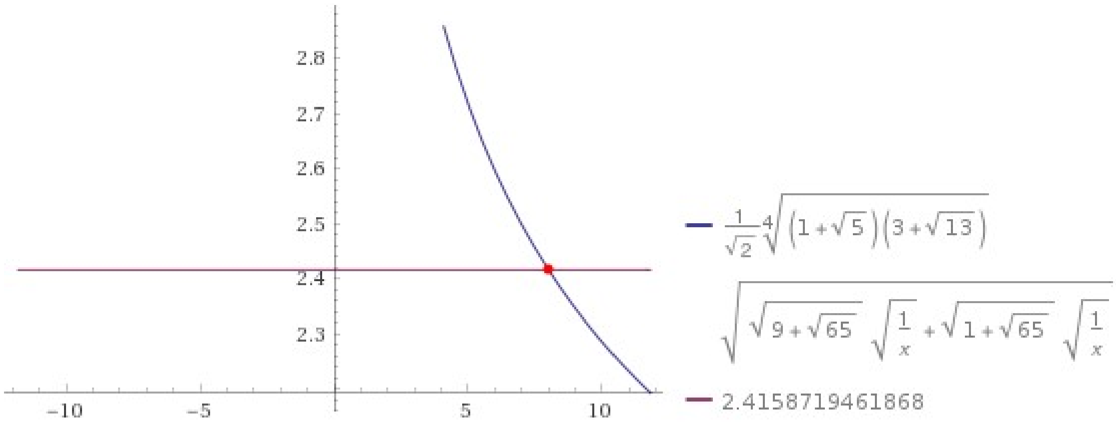
Input interpretation:

$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{9+\sqrt{65}}{x}}+\sqrt{\frac{1+\sqrt{65}}{x}}} = 2.4158719461868$$

Result:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})}\sqrt{\sqrt{9+\sqrt{65}}\sqrt{\frac{1}{x}}+\sqrt{1+\sqrt{65}}\sqrt{\frac{1}{x}}}}{\sqrt{2}} = 2.4158719461868$$

Plot:



Solution:

$x = 8$

Integer solution:

$x = 8$

8

Note that when the Ramanujan function is generalized, 24 is replaced by 8 (8 + 2 = 10) for fermionic strings and:

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/(x-2))+sqrt((1+65^0.5)/(x-2)))))^1/2 = 2.4158719461868

Input interpretation:

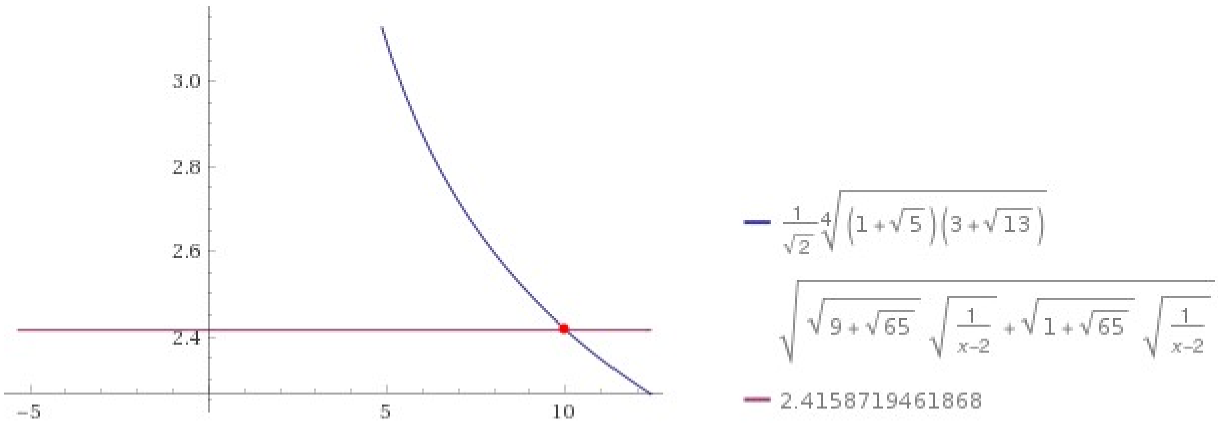
$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{9+\sqrt{65}}{x-2}}+\sqrt{\frac{1+\sqrt{65}}{x-2}}} = 2.4158719461868$$

Result:

$$\frac{\sqrt[4]{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\sqrt{\sqrt{9+\sqrt{65}}\sqrt{\frac{1}{x-2}}+\sqrt{1+\sqrt{65}}\sqrt{\frac{1}{x-2}}}}{\sqrt{2}} =$$

2.4158719461868

Plot:



Solution:

$x = 10$

Integer solution:

$x = 10$

10 (dimensions number in Superstring theory)

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/(x*1/3))+sqrt((1+65^0.5)/(x*1/3)))))^1/2 = 2.4158719461868

Input interpretation:

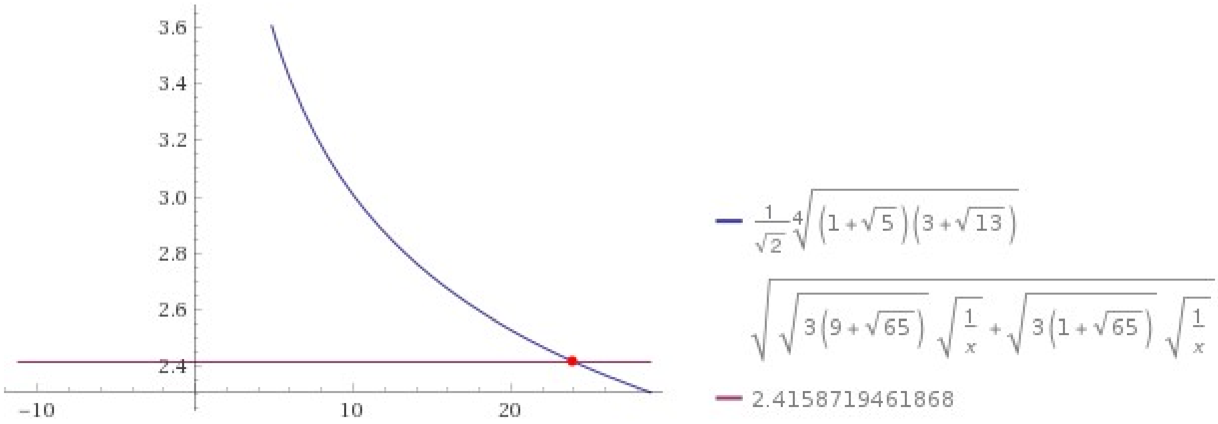
$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{9+\sqrt{65}}{x\times\frac{1}{3}}}+\sqrt{\frac{1+\sqrt{65}}{x\times\frac{1}{3}}}} = 2.4158719461868$$

Result:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})}\sqrt{\sqrt{3\left(9+\sqrt{65}\right)}\sqrt{\frac{1}{x}}+\sqrt{3\left(1+\sqrt{65}\right)}\sqrt{\frac{1}{x}}}}{\sqrt{2}} =$$

2.4158719461868

Plot:



Solution:

$x = 24$

Integer solution:

$x = 24$

24

This value is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

[((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 ((((sqrt((9+65^0.5)/((x*1/3)-2/3)))+sqrt((1+65^0.5)/((x*1/3)-2/3)))))^1/2 = 2.4158719461868

Input interpretation:

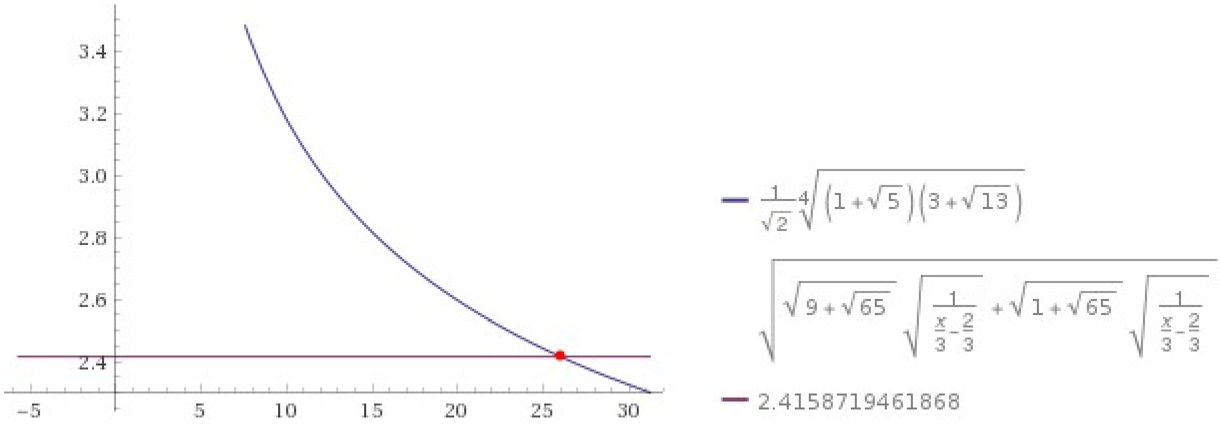
$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{9+\sqrt{65}}{x\times\frac{1}{3}-\frac{2}{3}}}+\sqrt{\frac{1+\sqrt{65}}{x\times\frac{1}{3}-\frac{2}{3}}}} = 2.4158719461868$$

Result:

$$\frac{\sqrt[4]{(1+\sqrt{5})(3+\sqrt{13})}\sqrt{\sqrt{9+\sqrt{65}}\sqrt{\frac{1}{\frac{x}{3}-\frac{2}{3}}}+\sqrt{1+\sqrt{65}}\sqrt{\frac{1}{\frac{x}{3}-\frac{2}{3}}}}}{\sqrt{2}} =$$

2.4158719461868

Plot:



Solution:

$x = 26$

26 (dimensions number in bosonic string theory)

From the previous Ramanujan expression

$$e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}$$

we have:

(((e^(Pi*sqrt65)-24+276*e^(-Pi*sqrt65))))^1/4 - 18 + Pi

Input:

$$\sqrt[4]{e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}} - 18 + \pi$$

Exact result:

$$-18 + \sqrt[4]{-24 + 276\, e^{-\sqrt{65}\,\pi} + e^{\sqrt{65}\,\pi}} + \pi$$

Decimal approximation:

547.4679724479696711078215509400894231688865989504543944248...

547.4679724479… result practically equal to the rest mass of Eta meson 547.862 that we have obtained performing the 4th root, subtracting 18, that is a Lucas number (linked to golden ratio), and adding π

Alternate forms:

$$-18 + e^{-(\sqrt{65}\,\pi)/4} \sqrt[4]{276 - 24\, e^{\sqrt{65}\,\pi} + e^{2\sqrt{65}\,\pi}} + \pi$$

$$e^{-(\sqrt{65}\,\pi)/4} \left(-18\, e^{(\sqrt{65}\,\pi)/4} + \sqrt[4]{276 - 24\, e^{\sqrt{65}\,\pi} + e^{2\sqrt{65}\,\pi}} + e^{(\sqrt{65}\,\pi)/4}\,\pi\right)$$

Series representations:

$$\sqrt[4]{e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}} - 18 + \pi = -18 + \sqrt[4]{-24 + 276\, e^{-\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}} + e^{\pi\sqrt{64}\sum_{k=0}^{\infty} 64^{-k}\binom{1/2}{k}}} + \pi$$

$$\sqrt[4]{e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}} - 18 + \pi = -18 + \sqrt[4]{-24 + 276 \exp\left(-\pi\sqrt{64}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + e^{\pi\sqrt{64}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}} + \pi$$

$$\sqrt[4]{e^{\pi\sqrt{65}} - 24 + 276\, e^{-\pi\sqrt{65}}} - 18 + \pi = -18 + \left(-24 + 276 \exp\left(-\frac{\pi\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 64^{-s}\,\Gamma\left(-\frac{1}{2} - s\right)\Gamma(s)}{2\sqrt{\pi}}\right) + \exp\left(\frac{\pi\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 64^{-s}\,\Gamma\left(-\frac{1}{2} - s\right)\Gamma(s)}{2\sqrt{\pi}}\right)\right)^{\wedge}(1/4) + \pi$$

From the initial formula

$$\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}$$

we have also:

((((([((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2)))))^6-64+golden ratio^2

Input:

$$\left(\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}\right)^6-64+\phi^2$$

ϕ is the golden ratio

Exact result:

$$\phi^2-64+\frac{1}{8}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^3$$

Decimal approximation:

137.43043220895391474004100060746134732806935785485351099962...

137.4304322…

This result is very near to the inverse of fine-structure constant 137,035 that we have obtained subtracting $64 = 8^2$ and adding the square of golden ratio

Alternate forms:

$$\phi^2-64+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3$$

$$\frac{1}{512}\left(512\phi^2-32768+\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3\right)$$

$$\phi^2-64+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{2\left(9+\sqrt{65}\right)}\right)^3$$

Minimal polynomial:

$$x^8+300x^7+21890x^6-2728960x^5-640057523x^4-53341132940x^3-2333619480060x^2-53575930102100x-511557630999769$$

(((((([((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2)))))^6-76+golden ratio^2

Input:

$$\left(\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}\right)^6-76+\phi^2$$

ϕ is the golden ratio

Exact result:

$$\phi^2-76+\frac{1}{8}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^3$$

Decimal approximation:

125.43043220895391474004100060746134732806935785485351099962...

125.4304322089… result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV, that we have obtained subtracting 76, that is a Lucas number, and adding the square of golden ratio

Alternate forms:

$$\phi^2-76+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3$$

$$\frac{1}{512}\left(512\phi^2-38912+\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3\right)$$

$$\phi^2-76+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{2\left(9+\sqrt{65}\right)}\right)^3$$

Minimal polynomial:

$$x^8+396x^7+51122x^6-148912x^5-736917203x^4-87005415452x^3-4845606479820x^2-137293944409652x-1596558217731721$$

We have also:

(((((([((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4
(((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2)))))^6-64+Pi+golden ratio

Input:

$$\left(\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}\right)^6-64+\pi+\phi$$

ϕ is the golden ratio

Exact result:

$$\phi-64+\frac{1}{8}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^3+\pi$$

Decimal approximation:

139.5720248625437079785036439907408502122665272542286168172...

139.57202486… result practically equal to the rest mass of Pion meson 139.57 MeV that we have obtained 64 = 8^2 and adding π and the golden ratio

Property:

$$-64+\frac{1}{8}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}\right)^3+\phi+\pi$$

is a transcendental number

Alternate forms:

$$\phi-64+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3+\pi$$

$$\phi-64+\frac{1}{512}\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{2\left(1+\sqrt{65}\right)}+\sqrt{2\left(9+\sqrt{65}\right)}\right)^3+\pi$$

$$\frac{1}{512}\left(512\phi-32768+\left(\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)\right)^{3/2}\left(\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}\right)^3+512\pi\right)$$

(((1/ sqrt(((([((1+sqrt5)/2) ((3+sqrt13)/2)]^1/4 (((sqrt((9+65^0.5)/8)+sqrt((1+65^0.5)/8))))^1/2))))))^1/496

Input:

$$\sqrt[496]{\frac{1}{\sqrt{\sqrt[4]{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(\frac{1}{2}\left(3+\sqrt{13}\right)\right)}\sqrt{\sqrt{\frac{1}{8}\left(9+\sqrt{65}\right)}+\sqrt{\frac{1}{8}\left(1+\sqrt{65}\right)}}}}}$$

Exact result:

$$\frac{\sqrt[1984]{\frac{2}{\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}}}}{\sqrt[3968]{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}}$$

Decimal approximation:

0.9991112215321754988038507004743031220098326538064352465 66...

0.9991112215… result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1}=1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\ldots}}}}\approx 0.9991104684$$

and to the dilaton value **0.989117352243** = $\boldsymbol{\phi}$ (see Observations for this value).

Alternate forms:

$$\frac{2^{5/3968}}{\sqrt[3968]{\left(1+\sqrt{5}\right)\left(3+\sqrt{13}\right)}\sqrt[1984]{\sqrt{1+\sqrt{65}}+\sqrt{9+\sqrt{65}}}}$$

$$\frac{2^{3/1984}\sqrt[1984]{\frac{1}{\sqrt{1-8i}+\sqrt{1+8i}+\sqrt{5}+\sqrt{13}}}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}}$$

All 496th roots of (2/(1/2 sqrt(1/2 (1 + sqrt(65))) + 1/2 sqrt(1/2 (9 + sqrt(65)))))^(1/4)/((1 + sqrt(5)) (3 + sqrt(13)))^(1/8):

$$\frac{\sqrt[1984]{2}\, e^{0}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}\ \sqrt[1984]{\frac{1}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{65}\right)}+\frac{1}{2}\sqrt{\frac{1}{2}\left(9+\sqrt{65}\right)}}} \approx 0.9991112$$

(real, principal root)

$$\frac{\sqrt[1984]{2}\, e^{(i\pi)/248}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}\ \sqrt[1984]{\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{65})}+\frac{1}{2}\sqrt{\frac{1}{2}(9+\sqrt{65})}}}$$

$\approx 0.9990311 + 0.012656\, i$

$$\frac{\sqrt[1984]{2}\, e^{(i\pi)/124}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}\ \sqrt[1984]{\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{65})}+\frac{1}{2}\sqrt{\frac{1}{2}(9+\sqrt{65})}}}$$

$\approx 0.9987906 + 0.025310\, i$

$$\frac{\sqrt[1984]{2}\, e^{(3i\pi)/248}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}\ \sqrt[1984]{\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{65})}+\frac{1}{2}\sqrt{\frac{1}{2}(9+\sqrt{65})}}}$$

$\approx 0.9983898 + 0.037960\, i$

$$\frac{\sqrt[1984]{2}\, e^{(i\pi)/62}}{\sqrt[3968]{(1+\sqrt{5})(3+\sqrt{13})}\ \sqrt[1984]{\frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{65})}+\frac{1}{2}\sqrt{\frac{1}{2}(9+\sqrt{65})}}}$$

$\approx 0.9978289 + 0.05060\, i$

Where 496 = 31*8*2. We note that 31 is a twin prime number (29-31) and 29 is a Lucas and prime number. Furthermore 496 is the dimension of the Lie group E_8 x E_8, fundamental in the heterotic string theory.

Observations

DILATON VALUE CALCULATIONS

from:

Modular equations and approximations to π - *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have that:

5. Since G_n and g_n can be expressed as roots of algebraical equations with rational coefficients, the same is true of G_n^{24} or g_n^{24}. So let us suppose that

$$1 = ag_n^{-24} - bg_n^{-48} + \cdots,$$

or

$$g_n^{24} = a - bg_n^{-24} + \cdots.$$

But we know that

$$64e^{-\pi\sqrt{n}}g_n^{24} = 1 - 24e^{-\pi\sqrt{n}} + 276e^{-2\pi\sqrt{n}} - \cdots,$$
$$64g_n^{24} = e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots,$$
$$64a - 64bg_n^{-24} + \cdots = e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots,$$
$$64a - 4096be^{-\pi\sqrt{n}} + \cdots = e^{\pi\sqrt{n}} - 24 + 276e^{-\pi\sqrt{n}} - \cdots,$$

that is

$$e^{\pi\sqrt{n}} = (64a + 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \qquad (13)$$

Similarly, if

$$1 = aG_n^{-24} - bG_n^{-48} + \cdots,$$

then

$$e^{\pi\sqrt{n}} = (64a - 24) - (4096b + 276)e^{-\pi\sqrt{n}} + \cdots \qquad (14)$$

From (13) and (14) we can find whether $e^{\pi\sqrt{n}}$ is very nearly an integer for given values of n, and ascertain also the number of 9's or 0's in the decimal part. But if G_n and g_n be simple quadratic surds we may work independently as follows. We have, for example,

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$
\begin{aligned}
64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\
64g_{22}^{-24} &= \qquad\qquad\quad 4096e^{-\pi\sqrt{22}} + \cdots,
\end{aligned}
$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Again

$$G_{37} = (6+\sqrt{37})^{\frac{1}{4}},$$

$$
\begin{aligned}
64G_{37}^{24} &= e^{\pi\sqrt{37}} + 24 + 276e^{-\pi\sqrt{37}} + \cdots, \\
64G_{37}^{-24} &= \qquad\qquad\quad 4096e^{-\pi\sqrt{37}} - \cdots,
\end{aligned}
$$

so that

$$64(G_{37}^{24} + G_{37}^{-24}) = e^{\pi\sqrt{37}} + 24 + 4372e^{-\pi\sqrt{37}} - \cdots = 64\{(6+\sqrt{37})^{6} + (6-\sqrt{37})^{6}\}.$$

Hence

$$e^{\pi\sqrt{37}} = 199148647.999978\ldots.$$

Similarly, from

$$g_{58} = \sqrt{\left(\frac{5+\sqrt{29}}{2}\right)},$$

we obtain

$$64(g_{58}^{24} + g_{58}^{-24}) = e^{\pi\sqrt{58}} - 24 + 4372e^{-\pi\sqrt{58}} + \cdots = 64\left\{\left(\frac{5+\sqrt{29}}{2}\right)^{12} + \left(\frac{5-\sqrt{29}}{2}\right)^{12}\right\}.$$

Hence

$$e^{\pi\sqrt{58}} = 24591257751.99999982\ldots.$$

From:

An Update on Brane Supersymmetry Breaking
J. Mourad and A. Sagnotti - arXiv:1711.11494v1 [hep-th] 30 Nov 2017

Now, we have that:

From the following vacuum equations:

$$T\, e^{\gamma_E \phi} \;=\; -\,\frac{\beta_E^{(p)}\, h^2}{\gamma_E}\; e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

$$16\, k'\, e^{\;\;2C} \;=\; \frac{h^2 \left(p \,+\, 1 \,-\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right) e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}}{(7 \,-\, p)}$$

$$(A')^2 \;-\; k\, e^{-2A} \;+\; \frac{h^2}{16(p\,+\,1)}\;\left(7 \,-\, p \,+\, \frac{2\,\beta_E^{(p)}}{\gamma_E}\right)\; e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

We have obtained, from the results almost equals of the equations, putting

$4096\, e^{-\pi\sqrt{18}}$ instead of

$$e^{-2\,(8-p)\,C + 2\,\beta_E^{(p)}\,\phi}$$

a new possible mathematical connection between the two exponentials. Thence, also the values concerning p, C, β_E and ϕ correspond to the exponents of *e* (i.e. of exp). Thence we obtain for p = 5 and $\beta_E = 1/2$:

$$e^{-6C+\phi} = 4096 e^{-\pi\sqrt{18}}$$

Therefore with respect to the exponential of the vacuum equation, the Ramanujan's exponential has a coefficient of 4096 which is equal to 64^2, while $-6C+\phi$ is equal to $-\pi\sqrt{18}$. From this it follows that it is possible to establish mathematically, the dilaton value.

phi = -Pi*sqrt(18) + 6C, for C = 1, we obtain:

exp((-Pi*sqrt(18))

Input:

$\exp\left(-\pi\sqrt{18}\right)$

Exact result:

$e^{-3\sqrt{2}\,\pi}$

Decimal approximation:

$1.6272016226072509292942156739117979541838581136954016\ldots \times 10^{-6}$

$1.6272016\ldots * 10^{-6}$

Now:

$e^{-6C+\phi} = 4096e^{-\pi\sqrt{18}}$

$e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^{-6}

$\frac{1}{4096}e^{-6C+\phi}$ = 1.6272016… * 10^{-6}

0.000244140625 $e^{-6C+\phi} = e^{-\pi\sqrt{18}}$ = 1.6272016… * 10^{-6}

$\ln\left(e^{-\pi\sqrt{18}}\right) = -13.328648814475 = -\pi\sqrt{18}$

(1.6272016* 10^-6) *1/ (0.000244140625)

Input interpretation:

$\frac{1.6272016}{10^6} \times \frac{1}{0.000244140625}$

Result:

0.0066650177536

0.006665017...

0.000244140625 $e^{-6C+\phi} = e^{-\pi\sqrt{18}}$

Dividing both sides by 0.000244140625, we obtain:

$\frac{0.000244140625}{0.000244140625}e^{-6C+\phi} = \frac{1}{0.000244140625}e^{-\pi\sqrt{18}}$

$e^{-6C+\phi}$ = 0.0066650177536

((((exp((-Pi*sqrt(18)))))))*1/0.000244140625

Input interpretation:

$\exp\left(-\pi\sqrt{18}\right) \times \frac{1}{0.000244140625}$

Result:

0.00666501785...

0.00666501785…

$e^{-6C+\phi} = 0.0066650177536$

$\exp\left(-\pi\sqrt{18}\right)\times\frac{1}{0.000244140625} =$

$e^{-\pi\sqrt{18}}\times\frac{1}{0.000244140625}$

= 0.00666501785…

ln(0.00666501784619)

Input interpretation:

log(0.00666501784619)

Result:

-5.010882647757...

-5.010882647757…

Now:

$-6C + \phi = -5.010882647757 \ldots$

For C = 1, we obtain:

$\phi = -5.010882647757 + 6 = \mathbf{0.989117352243} = \boldsymbol{\phi}$

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} \quad - \quad 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(**Modular equations and approximations to π -** *S. Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

"For this reason Ramanujan elaborated a theory of reality around Zero (representing absolute Reality) and Infinity (the manifold manifestations of that reality): their mathematical product represented all the numbers, each of which corresponded to individual acts of creation. For him, "the numbers and their mathematical ratios let us understand how everything was in harmony in the universe"." - (https://www.cittanuova.it/ramanujanhardy-e-il-piacere-di-scoprire/?ms=006&se=007)

CONCLUSIONS

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the final results of the analyzed expressions.

References

Modular equations and approximations to π - *S. Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372