On various Ramanujan's equations (Hardy-Ramanujan number, taxicab numbers, etc) linked to some parameters of Standard Model Particles and String Theory: New possible mathematical connections. VI

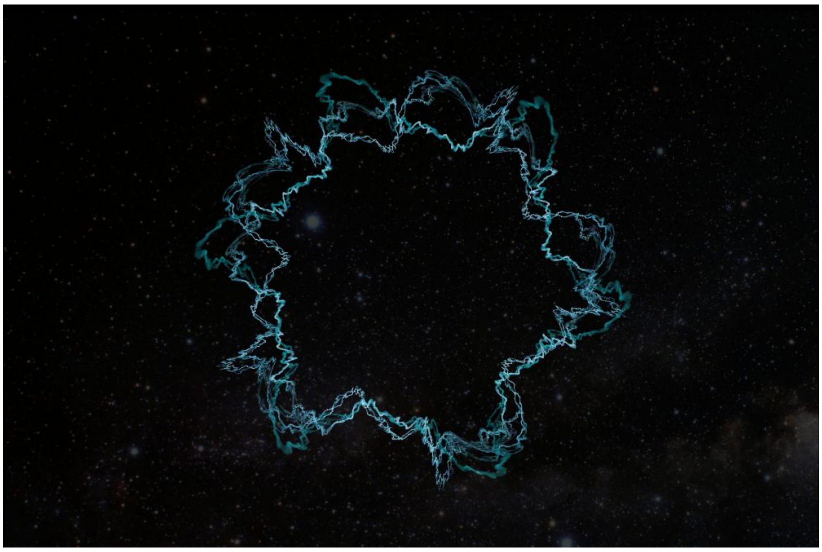
Michele Nardelli[1], Antonio Nardelli

Abstract

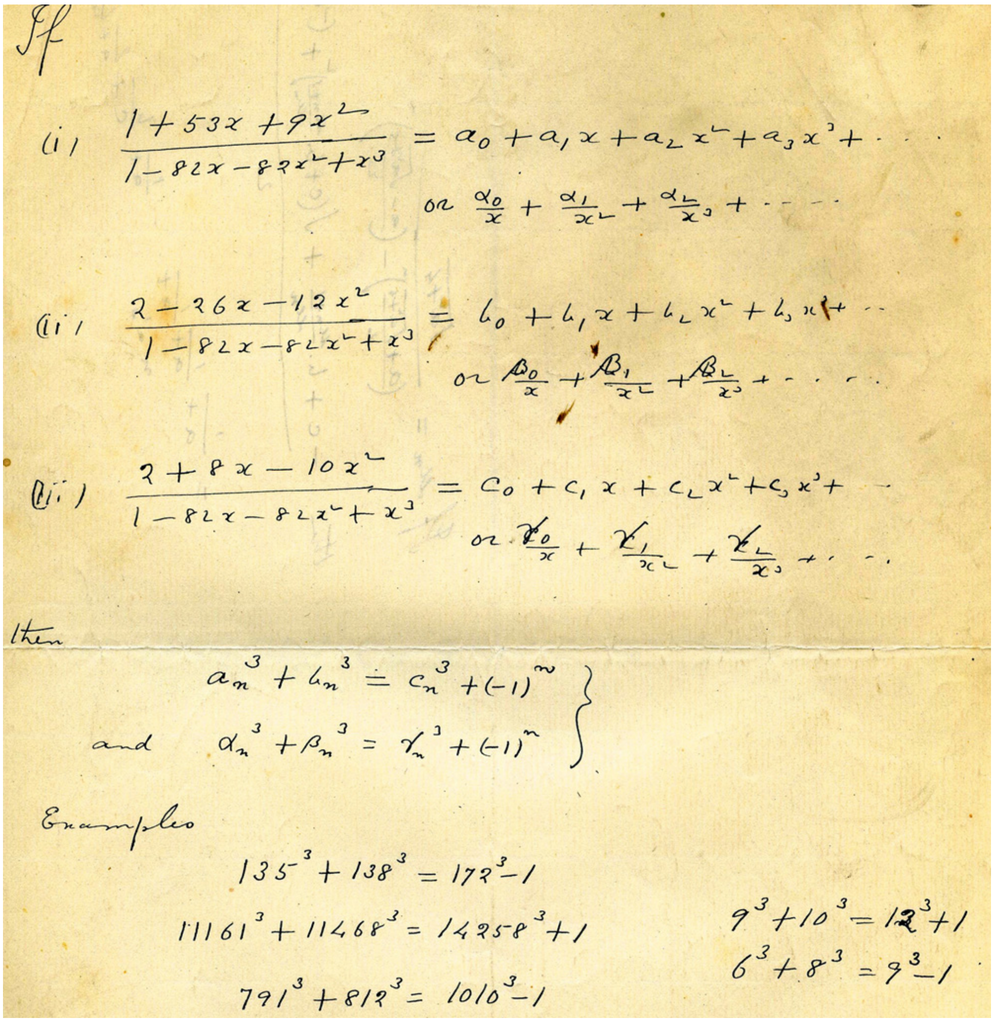
In this research thesis, we have described and deepened further Ramanujan equations (Hardy-Ramanujan number, taxicab numbers, etc) linked to some parameters of Standard Model Particles and String Theory. We have therefore obtained further possible mathematical connections.

[1] M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

https://www.britannica.com/biography/Srinivasa-Ramanujan



https://futurism.com/brane-science-complex-notions-of-superstring-theory



If

(i) $\frac{1+53x+9x^2}{1-82x-82x^2+x^3} = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \cdots$ or $\frac{\alpha_0}{x} + \frac{\alpha_1}{x^2} + \frac{\alpha_2}{x^3} + \cdots$

(ii) $\frac{2-26x-12x^2}{1-82x-82x^2+x^3} = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \cdots$ or $\frac{\beta_0}{x} + \frac{\beta_1}{x^2} + \frac{\beta_2}{x^3} + \cdots$

(iii) $\frac{2+8x-10x^2}{1-82x-82x^2+x^3} = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$ or $\frac{\gamma_0}{x} + \frac{\gamma_1}{x^2} + \frac{\gamma_2}{x^3} + \cdots$

then

$a_n^3 + b_n^3 = c_n^3 + (-1)^n$

and $\alpha_n^3 + \beta_n^3 = \gamma_n^3 + (-1)^n$

Examples

$135^3 + 138^3 = 172^3 - 1$

$11161^3 + 11468^3 = 14258^3 + 1$

$791^3 + 812^3 = 1010^3 - 1$

$9^3 + 10^3 = 12^3 + 1$

$6^3 + 8^3 = 9^3 - 1$

https://plus.maths.org/content/ramanujan

Ramanujan's manuscript

The representations of 1729 as the sum of two cubes appear in the bottom right corner. The equation expressing the near counter examples to Fermat's last theorem appears further up: $\alpha^3 + \beta^3 = \gamma^3 + (-1)^n$.

From Wikipedia

*The **taxicab number**, typically denoted Ta(n) or Taxicab(n), also called the nth **Hardy–Ramanujan number**, is defined as the smallest integer that can be expressed as a sum of two positive integer cubes in n distinct ways. The most famous taxicab number is 1729 = Ta(2) =* $1^3 + 12^3 = 9^3 + 10^3$.

From:

Integrable Scalar Cosmologies I. Foundations and links with String Theory
P. Fre , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013

Group III ($\gamma = \frac{2}{5}$) – **(Ramani potentials)** There are two scalar potentials in this class. The first potential is bounded from below and reads

$$\mathcal{V}_{IIIa}(\varphi) \;=\; \lambda\left[a\cosh^3\left(\frac{2\varphi}{5}\right) + b\sinh^2\left(\frac{2\varphi}{5}\right)\cosh\left(\frac{2\varphi}{5}\right) + c\sinh^3\left(\frac{2\varphi}{5}\right)\right], \quad (5.14)$$

The Liouville integrability of the system corresponding to eq. (5.14) is guaranteed by the existence of the additional conserved charge (see *e.g.* [15] and references therein)

$$\begin{aligned}\mathcal{Q}_{IIIa}(\xi,\eta) \;=\;& \dot\eta^4 + \frac{2}{\sqrt{3}}\dot\xi\dot\eta^3 - \frac{4\lambda}{25\sqrt{3}}\dot\xi^2\eta^3 + \frac{4\lambda}{25}(\eta^3 + \sqrt{3}\xi\eta^2)\dot\xi\dot\eta \\ &+ \frac{4\lambda}{25}\left(-2\sqrt{3}\xi^2\eta + \frac{1}{\sqrt{3}}\eta^3 - 2\xi\eta^2\right)\dot\eta^2 \\ &+ \frac{4\lambda^2}{25^2}\left(\frac{4}{\sqrt{3}}\xi^3\eta^3 - \frac{2}{\sqrt{3}}\xi\eta^5 - \xi^2\eta^4 + \frac{5}{9}\eta^6\right). \end{aligned} \quad (5.19)$$

From the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\dfrac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\cfrac{e^{-\pi\sqrt{5}}}{1+\cfrac{e^{-2\pi\sqrt{5}}}{1+\cfrac{e^{-3\pi\sqrt{5}}}{1+\cfrac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

We put for φ > 0 φ = 0.9991104684, and from

$$9^3 + 10^3 = 12^3 + 1$$

$$135^3 + 138^3 = 172^3 - 1$$

we have: a = 135, b = 138, c = 172, λ = 1, ξ = 9, η = 10, $\dot\xi = 12$, $\dot\eta = 1$

Thence, from the (5.14), we obtain:

φ = 0.9991104684

$$\mathcal{V}_{IIIa}(\varphi) \;=\; \lambda\left[a\cosh^3\left(\frac{2\varphi}{5}\right) \;+\; b\sinh^2\left(\frac{2\varphi}{5}\right)\cosh\left(\frac{2\varphi}{5}\right) \;+\; c\sinh^3\left(\frac{2\varphi}{5}\right)\right]$$

[135* cosh^3((2*0.9991104684)/5) + 138* sinh^2((2*0.9991104684)/5) cosh((2*0.9991104684)/5) + 172* sinh^3((2*0.9991104684)/5)]

Input interpretation:

$$135\cosh^3\left(\frac{2\times 0.9991104684}{5}\right)+138\sinh^2\left(\frac{2\times 0.9991104684}{5}\right)\cosh\left(\frac{2\times 0.9991104684}{5}\right)+172\sinh^3\left(\frac{2\times 0.9991104684}{5}\right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

207.5055085...

207.5055085…

Alternative representations:

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+172\sinh^3\left(\frac{2\times 0.99911}{5}\right)=135\cos^3\left(\frac{1.99822\,i}{5}\right)+138\cos\left(\frac{1.99822\,i}{5}\right)\left(\frac{1}{2}\left(-e^{-1.99822/5}+e^{1.99822/5}\right)\right)^2+172\left(\frac{1}{2}\left(-e^{-1.99822/5}+e^{1.99822/5}\right)\right)^3$$

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+172\sinh^3\left(\frac{2\times 0.99911}{5}\right)=135\cos^3\left(-\frac{1.99822\,i}{5}\right)+138\cos\left(-\frac{1.99822\,i}{5}\right)\left(i\cos\left(\frac{\pi}{2}+\frac{1.99822\,i}{5}\right)\right)^2+172\left(i\cos\left(\frac{\pi}{2}+\frac{1.99822\,i}{5}\right)\right)^3$$

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+ 138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+172\sinh^3\left(\frac{2\times 0.99911}{5}\right)= 135\cos^3\left(-\frac{1.99822\, i}{5}\right)+138\cos\left(-\frac{1.99822\, i}{5}\right)\left(\frac{1}{2}\left(-e^{-1.99822/5}+e^{1.99822/5}\right)\right)^2+ 172\left(\frac{1}{2}\left(-e^{-1.99822/5}+e^{1.99822/5}\right)\right)^3$$

Series representations:

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+ 172\sinh^3\left(\frac{2\times 0.99911}{5}\right)=1376\left(\sum_{k=0}^{\infty} I_{1+2k}(0.399644)\right)^3+ 552\left(\sum_{k=0}^{\infty} I_{1+2k}(0.399644)\right)^2\sum_{k=0}^{\infty}\frac{e^{-1.83436k}}{(2k)!}+135\left(\sum_{k=0}^{\infty}\frac{e^{-1.83436k}}{(2k)!}\right)^3$$

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+ 172\sinh^3\left(\frac{2\times 0.99911}{5}\right)=135\left(\sum_{k=0}^{\infty}\frac{e^{-1.83436k}}{(2k)!}\right)^3+ 138\left(\sum_{k=0}^{\infty}\frac{e^{-1.83436k}}{(2k)!}\right)\left(\sum_{k=0}^{\infty}\frac{0.399644^{1+2k}}{(1+2k)!}\right)^2+172\left(\sum_{k=0}^{\infty}\frac{0.399644^{1+2k}}{(1+2k)!}\right)^3$$

$$135\cosh^3\left(\frac{2\times 0.99911}{5}\right)+ 138\sinh^2\left(\frac{2\times 0.99911}{5}\right)\cosh\left(\frac{2\times 0.99911}{5}\right)+172\sinh^3\left(\frac{2\times 0.99911}{5}\right)= 172\left(\sum_{k=0}^{\infty}\frac{0.399644^{1+2k}}{(1+2k)!}\right)^3+138\, i\left(\sum_{k=0}^{\infty}\frac{0.399644^{1+2k}}{(1+2k)!}\right)^2\sum_{k=0}^{\infty}\frac{\left(0.399644-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}+ 135\, i^3\left(\sum_{k=0}^{\infty}\frac{\left(0.399644-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)^3$$

The Liouville integrability of the system corresponding to eq. (5.14) is guaranteed by the existence of the additional conserved charge (see *e.g.* [15] and references therein)

$$\begin{aligned}\mathcal{Q}_{IIIa}(\xi,\eta) &= \dot\eta^4+\frac{2}{\sqrt3}\dot\xi\,\dot\eta^3-\frac{4\lambda}{25\sqrt3}\xi^2\eta^3+\frac{4\lambda}{25}(\eta^3+\sqrt3\,\xi\,\eta^2)\,\dot\xi\,\dot\eta\\ &+\frac{4\lambda}{25}\Big(-2\sqrt3\,\xi^2\eta+\frac{1}{\sqrt3}\eta^3-2\,\xi\,\eta^2\Big)\,\dot\eta^2\\ &+\frac{4\lambda^2}{25^2}\Big(\frac{4}{\sqrt3}\xi^3\eta^3-\frac{2}{\sqrt3}\xi\,\eta^5-\xi^2\eta^4+\frac{5}{9}\eta^6\Big)\,. \end{aligned} \tag{5.19}$$

For: a = 135, b = 138, c = 172, $\lambda = 1$, $\xi = 9$, $\eta = 10$, $\dot{\xi} = 12$, $\dot{\eta} = 1$, we obtain:

1+(2*12)/(sqrt3)-(4*12^2*10^3)/(25sqrt3)+(((4/25(10^3+(sqrt3)*9*10^2)*12)))

Input interpretation:

$$1 + \frac{2\times 12}{\sqrt{3}} - \frac{4\times 12^2\times 10^3}{25\sqrt{3}} + \frac{4}{25}\left(10^3 + \sqrt{3}\times 9\times 10^2\right)\times 12$$

Result:

$$1 - 7672\sqrt{3} + \frac{48}{25}\left(1000 + 900\sqrt{3}\right)$$

Decimal approximation:

−8374.31000018940663272714105391090534910803442864890245316...

-8374.31…

Alternate form:

$$1921 - 5944\sqrt{3}$$

4/25(((-2sqrt3*9^2*10+(10^3)/(sqrt3)-2*9*10^2)))

Input interpretation:

$$\frac{4}{25}\left(-2\sqrt{3}\times 9^2\times 10 + \frac{10^3}{\sqrt{3}} - 2\times 9\times 10^2\right)$$

Result:

$$\frac{4}{25}\left(-1800 + \frac{1000}{\sqrt{3}} - 1620\sqrt{3}\right)$$

Decimal approximation:

−644.571526251512872160850286838008924607958841584430358629...

-644.57152625…

Alternate forms:

$$-288 - \frac{3088}{5\sqrt{3}}$$

$$-\frac{3088\sqrt{3}}{15}-288$$

$$\frac{1}{15}\left(-4320-3088\sqrt{3}\right)$$

4/(25)^2*(((((4*9^3*10^3)/(sqrt3)-(2*9*10^5)/(sqrt3)-9^2*10^4+(5*10^6)/9))))

Input interpretation:

$$\frac{4}{25^2}\left(\frac{4\times 9^3\times 10^3}{\sqrt{3}}-\frac{2\times 9\times 10^5}{\sqrt{3}}-9^2\times 10^4+\frac{5\times 10^6}{9}\right)$$

Result:

$$\frac{4}{625}\left(372000\sqrt{3}-\frac{2290000}{9}\right)$$

Decimal approximation:

2495.222118215538615985699805412736486772986303827309754830...

2495.2221182155…

Alternate forms:

$$\frac{11904\sqrt{3}}{5}-\frac{14656}{9}$$

$$\frac{64}{45}\left(1674\sqrt{3}-1145\right)$$

$$\frac{1}{45}\left(107136\sqrt{3}-73280\right)$$

From the algebraic sum of the three results, we obtain:

(-8374.3100001894-644.5715262515128+2495.222118215538)

Input interpretation:

-8374.3100001894 - 644.5715262515128 + 2495.222118215538

Result:

-6523.6594082253748

-6523.6594082253748

Note that:

-(((-8374.3100001894-644.5715262515128+2495.222118215538)+248))

We remember that the exceptional Lie group E_8 has dimension 248.

Input interpretation:

$$-((-8374.3100001894 - 644.5715262515128 + 2495.222118215538) + 248)$$

Result:

6275.6594082253748

6275.6594..... result practically equal to the rest mass of Charmed B meson 6275.6

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

for n = 232, and adding 34 and 7 that are Fibonacci and Lucas number respectively, we obtain:

(((sqrt(golden ratio) * exp(Pi*sqrt(232/15)) / (2*5^(1/4)*sqrt(232))))) + 34 +7

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{232}{15}}\right)}{2 \sqrt[4]{5} \sqrt{232}} + 34 + 7$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{2\sqrt{58/15}\,\pi} \sqrt{\frac{\phi}{58}}}{4 \sqrt[4]{5}} + 41$$

Decimal approximation:

6523.662817434028574054057424551281398144601158308922171541...

6523.662817434…

Property:

$$41 + \frac{e^{2\sqrt{58/15}\,\pi} \sqrt{\frac{\phi}{58}}}{4 \sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$41+\frac{1}{8}\sqrt{\frac{1}{145}\left(5+\sqrt{5}\right)}\; e^{2\sqrt{58/15}\,\pi}$$

$$41+\frac{\sqrt{\frac{1}{29}\left(1+\sqrt{5}\right)}\; e^{2\sqrt{58/15}\,\pi}}{8\sqrt[4]{5}}$$

$$\frac{47560+5^{3/4}\sqrt{29\left(1+\sqrt{5}\right)}\; e^{2\sqrt{58/15}\,\pi}}{1160}$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{232}{15}}\right)}{2\sqrt[4]{5}\sqrt{232}}+34+7=\left(410\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(232-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{232}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(232-z_0)^k z_0^{-k}}{k!}\right)\text{ for not }((z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{232}{15}}\right)}{2\sqrt[4]{5}\sqrt{232}}+34+7=\left(410\exp\left(i\pi\left\lfloor\frac{\arg(232-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(232-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{232}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{232}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(10\exp\left(i\pi\left\lfloor\frac{\arg(232-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(232-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{232}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{232}} + 34 + 7 = \left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(232-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(232-z_0)/(2\pi)\rfloor}\right.$$

$$\left(410\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(232-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(232-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (232-z_0)^k z_0^{-k}}{k!} + \right.$$

$$5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{232}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{232}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\right.$$

$$\left.\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{232}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$

$$\left.\left. z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!}\right)\right) \Big/$$

$$\left(10\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (232-z_0)^k z_0^{-k}}{k!}\right)$$

Now, we have that:

The Liouville integrability of the systems of eq. (5.6) is guaranteed by the existence of an additional conserved charge (see *e.g.* [15] and references therein) that in the three cases takes the form

$$\begin{aligned}
\mathcal{Q}^{(1)}_{Va}(\xi,\eta) &= \left[\dot\eta^2 - \frac{3}{2}\dot\xi^2 + \frac{9}{25}\lambda\left(\frac{9}{2}\xi^{\frac{4}{3}} + 3\xi^{-\frac{2}{3}}\eta^2\right)\right]\dot\eta - \frac{81}{25}\lambda\,\eta\,\xi^{\frac{1}{3}}\dot\xi\,, \\
\mathcal{Q}^{(2)}_{Va}(\xi,\eta) &= \left(\dot\eta^2 - 2\dot\xi^2 + \frac{36}{25}\lambda\,\xi^{-\frac{2}{3}}\eta^2\right)\dot\eta^2 - \frac{216}{25}\lambda\,\eta\,\xi^{\frac{1}{3}}\dot\eta\,\dot\xi - \frac{5832}{625}\lambda\,\eta^2\xi^{\frac{2}{3}}\,, \qquad (5.32) \\
\mathcal{Q}^{(3)}_{Va}(\xi,\eta) &= \left[\dot\eta^2 - 3\dot\xi^2 - \frac{54}{25}\lambda\left(3\xi^{\frac{4}{3}} - \xi^{-\frac{2}{3}}\eta^2\right)\right]\dot\eta^4 - \frac{648}{25}\lambda\,\eta\,\xi^{\frac{1}{3}}\dot\eta^3\dot\xi \\
&\quad - \left(\frac{9}{25}\lambda\right)^2 648\,\eta^2\xi^{\frac{2}{3}}\dot\eta^2 - \left(\frac{9}{25}\lambda\right)^3 648\,\eta^4\,.
\end{aligned}$$

For: a = 135, b = 138, c = 172, λ = 1, ξ = 9, η = 10, $\dot\xi = 12$, $\dot\eta = 1$, we obtain:

$$\mathcal{Q}^{(1)}_{Va}(\xi,\eta) = \left[\dot\eta^2 - \frac{3}{2}\dot\xi^2 + \frac{9}{25}\lambda\left(\frac{9}{2}\xi^{\frac{4}{3}} + 3\xi^{-\frac{2}{3}}\eta^2\right)\right]\dot\eta - \frac{81}{25}\lambda\,\eta\,\xi^{\frac{1}{3}}\dot\xi\,,$$

[1-3/2*12^2+9/25(9/2*9^(4/3)+3*9^(-2/3)*10^2)]-81/25*10*9^(1/3)*12

Input:

$$\left(1-\frac{3}{2}\times 12^2+\frac{9}{25}\left(\frac{9}{2}\times 9^{4/3}+3\times 9^{-2/3}\times 10^2\right)\right)-\frac{81}{25}\times 10\sqrt[3]{9}\times 12$$

Result:

$$-215 - \frac{1944 \times 3^{2/3}}{5} + \frac{9}{25}\left(\frac{100}{\sqrt[3]{3}} + \frac{81 \times 3^{2/3}}{2}\right)$$

Decimal approximation:

-968.4479623858607083650771829189132446392798503900030013 74...

-968.44796238…

Alternate forms:

$$-\frac{1}{50}\left(10\,750 + 18\,111 \times 3^{2/3}\right)$$

$$-215 - \frac{18\,111 \times 3^{2/3}}{50}$$

$$-\frac{18\,111\; 3^{2/3}}{50} - 215$$

Minimal polynomial:

$$125\,000\,x^3 + 80\,625\,000\,x^2 + 17\,334\,375\,000\,x + 54\,707\,325\,189\,679$$

$$\mathcal{Q}^{(2)}_{Va}(\xi,\eta) \;=\; \left(\dot\eta^2 \;-\; 2\dot\xi^2 \;+\; \frac{36}{25}\,\lambda\,\xi^{-\frac{2}{3}}\,\eta^2\right)\dot\eta^2 \;-\; \frac{216}{25}\,\lambda\,\eta\,\xi^{\frac{1}{3}}\,\dot\eta\,\dot\xi \;-\; \frac{5832}{625}\,\lambda\,\eta^2\,\xi^{\frac{2}{3}}$$

$\lambda = 1$, $\xi = 9$, $\eta = 10$, $\dot\xi = 12$, $\dot\eta = 1$,

(((1-2*12^2+36/25*9^(-2/3)*10^2)))-216/25*10*9^(1/3)*12-(5832/625)*10^2*9^(2/3)

Input:

$$\left(1 - 2 \times 12^2 + \frac{36}{25} \times 9^{-2/3} \times 10^2\right) - \frac{216}{25} \times 10\,\sqrt[3]{9} \times 12 - \frac{5832}{625} \times 10^2 \times 9^{2/3}$$

Result:

$$-287 - \frac{69\,984\,\sqrt[3]{3}}{25} - \frac{5104 \times 3^{2/3}}{5}$$

Decimal approximation:

-6447.7253237071304492481834279699369797342948049289679943 3...

-6447.725323…

Alternate forms:

$$-\frac{1}{25}\left(7175 + 69\,984\,\sqrt[3]{3} + 25\,520 \times 3^{2/3}\right)$$

$$\frac{1}{25}\left(-7175-69984\sqrt[3]{3}-25520\times 3^{2/3}\right)$$

Minimal polynomial:

$$15625x^3+13453125x^2-397987081125x+1062917307488087$$

$$\mathcal{Q}^{(3)}_{Va}(\xi,\eta) = \left[\dot\eta^2 - 3\dot\xi^2 - \frac{54}{25}\lambda\left(3\xi^{\frac{4}{3}} - \xi^{-\frac{2}{3}}\eta^2\right)\right]\dot\eta^4 - \frac{648}{25}\lambda\eta\xi^{\frac{1}{3}}\dot\eta^3\dot\xi - \left(\frac{9}{25}\lambda\right)^2 648\eta^2\xi^{\frac{2}{3}}\dot\eta^2 - \left(\frac{9}{25}\lambda\right)^3 648\eta^4 .$$

λ = 1, ξ = 9, η = 10, $\dot\xi = 12$, $\dot\eta = 1$,

(((1-3*12^2-54/25(3*9^(4/3)-9^(-2/3)*10^2))))-(648/25)*10*9^(1/3)*12-(9/25)^2*648*10^2*9^(2/3)-(9/25)^3*648*10^4

Input interpretation:

$$\left(1-3\times 12^2-\frac{54}{25}\left(3\times 9^{4/3}-9^{-2/3}\times 10^2\right)\right)-\frac{648}{25}\times 10\sqrt[3]{9}\times 12-\left(\frac{9}{25}\right)^2\times 648\times 10^2\times 9^{2/3}-\left(\frac{9}{25}\right)^3\times 648\times 10^4$$

Result:

$$-\frac{7569047}{25}-\frac{629856\sqrt[3]{3}}{25}-\frac{15552\times 3^{2/3}}{5}-\frac{54}{25}\left(27\times 3^{2/3}-\frac{100}{3\sqrt[3]{3}}\right)$$

Decimal approximation:

-345639.54301424950446926807309168339752837019123126208311 6...

-345639.54301424…

Alternate forms:

$$\frac{1}{25}\left(-7569047-629856\sqrt[3]{3}-78618\times 3^{2/3}\right)$$

$$-\frac{7569047}{25}-\frac{629856\sqrt[3]{3}}{25}-\frac{78618\times 3^{2/3}}{25}$$

$$-\frac{629856\sqrt[3]{3}}{25}-\frac{78618\,3^{2/3}}{25}-\frac{7569047}{25}$$

From the algebraic sum of the three results, we obtain:

(-968.4479623858607-6447.7253237071304-345639.5430142495)

Input interpretation:

$-968.4479623858607 - 6447.7253237071304 - 345\,639.5430142495$

Result:

-353055.7163003424911

-353055.7163003424911

(-(-968.4479623858607-6447.7253237071304-345639.5430142495))^1/2 – 47

Input interpretation:

$\sqrt{-(-968.4479623858607 - 6447.7253237071304 - 345\,639.5430142495)} - 47$

Result:

547.1849175974955...

547.1849175974955...result practically equal to the rest mass of Eta meson 547.862

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n))

for n = 421 and adding the algebraic sum of 7, 47, 322 and 2207, that are Lucas numbers, we obtain:

((sqrt(golden ratio) * exp(Pi*sqrt(421/15)) / (2*5^(1/4)*sqrt(421)))) +(2207+322-47+7)

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{421}{15}}\right)}{2\sqrt[4]{5}\sqrt{421}} + (2207 + 322 - 47 + 7)$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{421/15}\,\pi}\sqrt{\frac{\phi}{421}}}{2\sqrt[4]{5}} + 2489$$

Decimal approximation:

353055.9600836020986373407401357209343803893766975472609319...

353055.960083602…

Property:

$$2489 + \frac{e^{\sqrt{421/15}\,\pi}\sqrt{\frac{\phi}{421}}}{2\sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$2489 + \frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{4210}}\, e^{\sqrt{421/15}\,\pi}$$

$$2489 + \frac{\sqrt{\frac{1}{842}\left(1+\sqrt{5}\right)}\, e^{\sqrt{421/15}\,\pi}}{2\sqrt[4]{5}}$$

$$\frac{20\,957\,380 + 5^{3/4}\sqrt{842\left(1+\sqrt{5}\right)}\, e^{\sqrt{421/15}\,\pi}}{8420}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{421}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{421}} + (2207+322-47+7) = \left(24\,890\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (421-z_0)^k z_0^{-k}}{k!} + 5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{421}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right) \Big/ \left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(421-z_0)^k z_0^{-k}}{k!}\right) \quad \text{for not } ((z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{421}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{421}} + (2207+322-47+7) =$$

$$\left(24890 \exp\left(i\pi\left\lfloor\frac{\arg(421-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(421-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{421}{15}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{421}{15}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(\phi-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(10\exp\left(i\pi\left\lfloor\frac{\arg(421-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k\,(421-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\sqrt{\phi}\ \exp\left(\pi\sqrt{\frac{421}{15}}\right)}{2\sqrt[4]{5}\ \sqrt{421}} + (2207+322-47+7) =$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(421-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(421-z_0)/(2\pi)\rfloor}\left(24890\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(421-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(421-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(421-z_0)^k z_0^{-k}}{k!} + 5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{421}{15}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{421}{15}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{421}{15}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right) \Big/ \left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(421-z_0)^k z_0^{-k}}{k!}\right)$$

From:

Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials?
P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013

Now, we have that:

Sporadic Integrable Potentials

$$\mathcal{V}_{Ia}(\varphi) = \frac{\lambda}{4}\left[(a+b)\cosh\left(\frac{6}{5}\varphi\right) + (3a-b)\cosh\left(\frac{2}{5}\varphi\right)\right]$$

$$\mathcal{V}_{Ib}(\varphi) = \frac{\lambda}{4}\left[(a+b)\sinh\left(\frac{6}{5}\varphi\right) - (3a-b)\sinh\left(\frac{2}{5}\varphi\right)\right]$$

$$\mathcal{V}_{II}(\varphi) = \frac{\lambda}{8}\left[3a+3b-c+4(a-b)\cosh\left(\frac{2}{3}\varphi\right) + (a+b+c)\cosh\left(\frac{4}{3}\varphi\right)\right]$$

$$\mathcal{V}_{IIIa}(\varphi) = \frac{\lambda}{16}\left[\left(1-\frac{1}{3\sqrt{3}}\right)e^{-6\varphi/5} + \left(7+\frac{1}{\sqrt{3}}\right)e^{-2\varphi/5} + \left(7-\frac{1}{\sqrt{3}}\right)e^{2\varphi/5} + \left(1+\frac{1}{3\sqrt{3}}\right)e^{6\varphi/5}\right] .$$

$$\mathcal{V}_{IIIb}(\varphi) = \frac{\lambda}{16}\left[(2-18\sqrt{3})\,e^{-6\varphi/5} + (6+30\sqrt{3})\,e^{-2\varphi/5} + (6-30\sqrt{3})\,e^{2\varphi/5} + (2+18\sqrt{3})\,e^{6\varphi/5}\right]$$

$$9^3 + 10^3 = 12^3 + 1$$

$$\mathcal{V}_{Ia}(\varphi) = \frac{\lambda}{4}\left[(a+b)\cosh\left(\frac{6}{5}\varphi\right) + (3a-b)\cosh\left(\frac{2}{5}\varphi\right)\right]$$

$$\mathcal{V}_{Ib}(\varphi) = \frac{\lambda}{4}\left[(a+b)\sinh\left(\frac{6}{5}\varphi\right) - (3a-b)\sinh\left(\frac{2}{5}\varphi\right)\right]$$

$\varphi = 4$, $\lambda = 0.9991104$, $a = 9$, $b = 10$ and $c = 12$, we obtain:

0.9991104/4 [(9+10) cosh(24/5) + (3*9-10) cosh (8/5)] + 0.9991104/4 [(9+10) sinh(24/5) - (3*9-10) sinh (8/5)]

Input interpretation:

$$\frac{0.9991104}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)+\frac{0.9991104}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

577.5183...

577.5183…

Alternative representations:

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\frac{1}{4}\times 0.99911\left(-\frac{17}{2}\left(-e^{-8/5}+e^{8/5}\right)+\frac{19}{2}\left(-e^{-24/5}+e^{24/5}\right)\right)+\frac{1}{4}\times 0.99911\left(\frac{17}{2}\left(e^{-8/5}+e^{8/5}\right)+\frac{19}{2}\left(e^{-24/5}+e^{24/5}\right)\right)$$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\frac{1}{4}\times 0.99911\left(17\cos\left(\frac{8i}{5}\right)+19\cos\left(\frac{24i}{5}\right)\right)+\frac{1}{4}\times 0.99911\left(-\frac{17}{2}\left(-e^{-8/5}+e^{8/5}\right)+\frac{19}{2}\left(-e^{-24/5}+e^{24/5}\right)\right)$$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\frac{1}{4}\times 0.99911\left(17\cos\left(-\frac{8i}{5}\right)+19\cos\left(-\frac{24i}{5}\right)\right)+\frac{1}{4}\times 0.99911\left(-\frac{17}{2}\left(-e^{-8/5}+e^{8/5}\right)+\frac{19}{2}\left(-e^{-24/5}+e^{24/5}\right)\right)$$

Series representations:

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\sum_{k=0}^{\infty}\left(\frac{4.24622\left(\frac{64}{25}\right)^k}{(2k)!}+4.74577\left(\frac{\left(\frac{576}{25}\right)^k}{(2k)!}+\frac{\left(\frac{24}{5}\right)^{1+2k}}{(1+2k)!}\right)-\frac{4.24622\left(\frac{5}{8}\right)^{-1-2k}}{(1+2k)!}\right)$$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\sum_{k=0}^{\infty}\frac{-6.79395\left(\frac{64}{25}\right)^k+22.7797\left(\frac{576}{25}\right)^k+4.24622\,i\left(\frac{8}{5}-\frac{i\pi}{2}\right)^{1+2k}+4.74577\,i\left(\frac{24}{5}-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}$$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\sum_{k=0}^{\infty}\left(-8.49244\,I_{1+2k}\left(\frac{8}{5}\right)+\frac{4.24622\left(\frac{64}{25}\right)^k+4.74577\left(\frac{576}{25}\right)^k+9.49155\,I_{1+2k}\left(\frac{24}{5}\right)(2k)!}{(2k)!}\right)$$

Integral representations:

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=8.99199+\int_0^1\left(-6.79395\cosh\left(\frac{8t}{5}\right)+6.79395\sinh\left(\frac{8t}{5}\right)+22.7797\left(\cosh\left(\frac{24t}{5}\right)+\sinh\left(\frac{24t}{5}\right)\right)\right)dt$$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+\frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911=\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{16/(25s)+s}\left(-1.69849+2.12311\,s+e^{128/(25s)}(5.69493+2.37289\,s)\right)\sqrt{\pi}}{i\pi s^{3/2}}\,ds$$

for $\gamma > 0$

$$\frac{1}{4}\left((9+10)\cosh\left(\frac{24}{5}\right)+(3\times 9-10)\cosh\left(\frac{8}{5}\right)\right)0.99911+ \frac{1}{4}\left((9+10)\sinh\left(\frac{24}{5}\right)-(3\times 9-10)\sinh\left(\frac{8}{5}\right)\right)0.99911 = \int_0^1\left(-6.79395\cosh\left(\frac{8t}{5}\right)+22.7797\cosh\left(\frac{24t}{5}\right)\right)dt + \int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{16/(25s)+s}\left(2.12311+2.37289\,e^{128/(25s)}\right)\sqrt{\pi}}{i\pi\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\mathcal{V}_{II}(\varphi) \;=\; \frac{\lambda}{8}\left[3a+3b-c+4(a-b)\cosh\left(\tfrac{2}{3}\varphi\right)+(a+b+c)\cosh\left(\tfrac{4}{3}\varphi\right)\right]$$

φ = 4 , λ = 0.9991104, a = 9, b = 10 and c = 12, we obtain:

0.9991104/8 [(3*9+3*10-12+4(9-10) cosh(8/3) + (9+10+12) cosh (16/3)]

Input interpretation:

$$\frac{0.9991104}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)$$

cosh(x) is the hyperbolic cosine function

Result:

402.9692...

402.9692…

Alternative representations:

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = \frac{1}{8}\times 0.99911\left(45-4\cos\left(\frac{8i}{3}\right)+31\cos\left(\frac{16i}{3}\right)\right)$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = \frac{1}{8}\times 0.99911\left(45-2\left(\frac{1}{e^{8/3}}+e^{8/3}\right)+\frac{31}{2}\left(\frac{1}{e^{16/3}}+e^{16/3}\right)\right)$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = \frac{1}{8}\times 0.99911\left(45-\frac{4}{\sec\left(\frac{8i}{3}\right)}+\frac{31}{\sec\left(\frac{16i}{3}\right)}\right)$$

Series representations:

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 5.62+\sum_{k=0}^{\infty}\frac{-0.499555\left(\frac{64}{9}\right)^k+3.87155\left(\frac{256}{9}\right)^k}{(2k)!}$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 5.62+\sum_{k=0}^{\infty}\frac{(-i)^k\cosh\left(\frac{i k\pi}{2}+z_0\right)\left(-0.499555\left(\frac{8}{3}-z_0\right)^k+3.87155\left(\frac{16}{3}-z_0\right)^k\right)}{k!}$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 5.62+\sum_{k=0}^{\infty}\frac{i\left(-0.499555\left(\frac{8}{3}-\frac{i\pi}{2}\right)^{1+2k}+3.87155\left(\frac{16}{3}-\frac{i\pi}{2}\right)^{1+2k}\right)}{(1+2k)!}$$

Integral representations:

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 8.99199+\int_0^1\left(-1.33215\sinh\left(\frac{8t}{3}\right)+20.6483\sinh\left(\frac{16t}{3}\right)\right)dt$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 5.62+\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{16/(9s)+s}\left(-0.249778+1.93578\,e^{16/(3s)}\right)\sqrt{\pi}}{i\pi\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{8}\left(3\times 9+3\times 10-12+4(9-10)\cosh\left(\frac{8}{3}\right)+(9+10+12)\cosh\left(\frac{16}{3}\right)\right)0.99911 = 5.62+\int_{\frac{i\pi}{2}}^{\frac{8}{3}}\frac{(7.99288-1.49867\,i\pi)\sinh(t)+(-123.89+11.6147\,i\pi)\sinh\left(\frac{-32t+i\pi(8+3t)}{-16+3i\pi}\right)}{-16+3i\pi}\,dt$$

$$\mathcal{V}_{IIIa}(\varphi) = \frac{\lambda}{16}\left[\left(1-\frac{1}{3\sqrt{3}}\right)e^{-6\varphi/5}+\left(7+\frac{1}{\sqrt{3}}\right)e^{-2\varphi/5} \right. \quad (5.17)$$

$$\left. +\left(7-\frac{1}{\sqrt{3}}\right)e^{2\varphi/5}+\left(1+\frac{1}{3\sqrt{3}}\right)e^{6\varphi/5}\right] . \quad (5.18)$$

From the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\frac{\sqrt{5}}{1+\sqrt[5]{\sqrt{\varphi^5\sqrt[4]{5^3}}-1}}-\varphi+1} = 1-\frac{e^{-\pi\sqrt{5}}}{1+\frac{e^{-2\pi\sqrt{5}}}{1+\frac{e^{-3\pi\sqrt{5}}}{1+\frac{e^{-4\pi\sqrt{5}}}{1+\dots}}}} \approx 0.9991104684$$

We put for φ > 0 φ = 4 and for λ > 0 λ = 0.9991104, an obtain:

0.9991104/16[(1-1/(3sqrt3))*e^(-24/5)+(7+1/(sqrt3))*e^(-8/5)+(7-1/(sqrt3))*e^(8/5)+(1+1/(3sqrt3))*e^(24/5)]

Input interpretation:

$$\frac{0.9991104}{16}\left(\left(1-\frac{1}{3\sqrt{3}}\right)e^{-24/5}+\left(7+\frac{1}{\sqrt{3}}\right)e^{-8/5}+\left(7-\frac{1}{\sqrt{3}}\right)e^{8/5}+\left(1+\frac{1}{3\sqrt{3}}\right)e^{24/5}\right)$$

Result:

11.13029...

11.13029…

$$\mathcal{V}_{IIIb}(\varphi) = \frac{\lambda}{16}\left[\left(2-18\sqrt{3}\right)e^{-6\varphi/5}+\left(6+30\sqrt{3}\right)e^{-2\varphi/5} \right. \quad (5.23)$$

$$\left. +\left(6-30\sqrt{3}\right)e^{2\varphi/5}+\left(2+18\sqrt{3}\right)e^{6\varphi/5}\right] . \quad (5.24)$$

We put for φ > 0 φ = 4 and for λ > 0 λ = 0.9991104, an obtain:

0.9991104/16[(2-18(sqrt3))*e^(-24/5)+(6+30(sqrt3))*e^(-8/5)+(6-30(sqrt3))*e^(8/5)+(2+18(sqrt3))*e^(24/5)]

Input interpretation:

$$\frac{0.9991104}{16}\left(\left(2-18\sqrt{3}\right)e^{-24/5}+\left(6+30\sqrt{3}\right)e^{-8/5}+\left(6-30\sqrt{3}\right)e^{8/5}+\left(2+18\sqrt{3}\right)e^{24/5}\right)$$

Result:

238.2350...

238.235…

The sum of all results is:

(577.5183+402.9692+11.13029+238.235)

Input interpretation:

$$577.5183+402.9692+11.13029+238.235$$

Result:

1229.85279

1229.85279

And:

(577.5183+402.9692+11.13029+238.235) + sqrt5

Input interpretation:

$$(577.5183+402.9692+11.13029+238.235)+\sqrt{5}$$

Result:

1232.089...

1232.089….result practically equal to the rest mass of Delta baryon 1232

(577.5183+402.9692+11.13029+238.235)^1/14-(47-3)*1/10^3

Input interpretation:

$$\sqrt[14]{577.5183+402.9692+11.13029+238.235} - (47-3)\times\frac{1}{10^3}$$

Result:

1.618278528025204045559644100015878071296110723145114435322...

1.618278528…. result that is a very good approximation to the value of the golden ratio 1,618033988749...

From:

$$V(\phi) = a \exp\left[2\sqrt{3}\,\gamma\,\phi\right] + b \exp\left[\sqrt{3}\,(\gamma+1)\,\phi\right] \qquad (5.1)$$

For γ = -7/6 , a = 9, b = 10 and $\phi = 1$, we obtain:

9 exp(2sqrt3 *(-7/6)) + 10 exp (sqrt3(-7/6+1))

Input:

$$9 \exp\left(2\sqrt{3}\left(-\frac{7}{6}\right)\right) + 10 \exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)$$

Exact result:

$$9\,e^{-7/\sqrt{3}} + 10\,e^{-1/(2\sqrt{3})}$$

Decimal approximation:

7.650703203987310823474781471574658664071712021641445010344...

7.650703203…

Alternate form:

$$e^{-7/\sqrt{3}}\left(9+10\,e^{13/(2\sqrt{3})}\right)$$

Series representations:

$$9 \exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right) + 10 \exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right) = 9 \exp\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right) + 10 \exp\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$9\exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)=$$

$$9\exp\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+10\exp\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$9\exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)=$$

$$9\exp\left(-\frac{7\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{6\sqrt{\pi}}\right)+$$

$$10\exp\left(-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{12\sqrt{\pi}}\right)$$

From which:

(((9 exp(2sqrt3 *(-7/6)) + 10 exp (sqrt3(-7/6+1)))))^3 + 47 + Pi

Input:

$$\left(9\exp\left(2\sqrt{3}\left(-\frac{7}{6}\right)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)\right)^3+47+\pi$$

Exact result:

$$47+\left(9e^{-7/\sqrt{3}}+10e^{-1/\left(2\sqrt{3}\right)}\right)^3+\pi$$

Decimal approximation:

497.9621887686594240085776966016809960720158990385768739293...

497.962188….result very near to the rest mass of Kaon meson 497.614

Alternate forms:

$$47+e^{-7\sqrt{3}}\left(9+10e^{13/\left(2\sqrt{3}\right)}\right)^3+\pi$$

$$47+2430e^{-29/\left(2\sqrt{3}\right)}+2700e^{-8/\sqrt{3}}+729e^{-7\sqrt{3}}+1000e^{-\sqrt{3}/2}+\pi$$

$$e^{-7\sqrt{3}}\left(729+2430e^{13/\left(2\sqrt{3}\right)}+2700e^{13/\sqrt{3}}+1000e^{\left(13\sqrt{3}\right)/2}+47e^{7\sqrt{3}}+e^{7\sqrt{3}}\pi\right)$$

Series representations:

$$\left(9\exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)\right)^3+47+\pi =$$
$$47+\pi+729\exp^3\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+$$
$$2430\exp^2\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\exp\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+$$
$$2700\exp\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\exp^2\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)+$$
$$1000\exp^3\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\left(9\exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)\right)^3+47+\pi =$$
$$47+\pi+729\exp^3\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+$$
$$2430\exp^2\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\exp\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+$$
$$2700\exp\left(-\frac{7}{3}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\exp^2\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+$$
$$1000\exp^3\left(-\frac{1}{6}\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left(9\exp\left(\frac{1}{6}\left(2\sqrt{3}\right)(-7)\right)+10\exp\left(\sqrt{3}\left(-\frac{7}{6}+1\right)\right)\right)^3+47+\pi=$$

$$47+\pi+729\exp^3\left(-\frac{7\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{6\sqrt{\pi}}\right)+$$

$$2430\exp^2\left(-\frac{7\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{6\sqrt{\pi}}\right)$$

$$\exp\left(-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{12\sqrt{\pi}}\right)+$$

$$2700\exp\left(-\frac{7\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{6\sqrt{\pi}}\right)$$

$$\exp^2\left(-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{12\sqrt{\pi}}\right)+$$

$$1000\exp^3\left(-\frac{\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{12\sqrt{\pi}}\right)$$

From:

Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials? - P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013

Now, we have that:

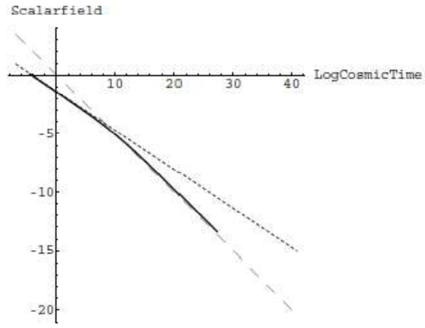


Figure 15: *Here we present the behavior of the scale factor and of the scalar field for the simplest of the hyperbolic type solutions ($\omega = \nu^2$) of the cosmological model based on the potential of eq.(5.19). The analytic form of the solution is given in eq.(5.62). For the plot we have chosen $\nu = \frac{1}{4}$. In the first graph, describing the scale factor, the solid line is the actual solution while the dashed curves are of the form $\alpha_{1,2} T_c^{\frac{1}{3}}$ with two different coefficient $\alpha_1 = \frac{3^{2/3}}{10^{1/3}}$ and $\alpha_2 = \frac{3}{5}^{1/3}$. The first curve is tangential to the solution at $T_c \to 0$ while the second is tangential to the solution at $T_c \to \infty$. The same style of presentation is adopted in the second picture. Here we plot the scalar field against the logarithm of the cosmic time. The two dashed straight lines represent the curves $-\frac{1}{3} \log [T_c]$, and $-\frac{1}{2} \log [T_c]$. The first is tangential to the solution at $T_c \to 0$, the second is tangential to the solution at $T_c \to \infty$.*

The simplest solution of the hyperbolic type is obtained for the choice $a = 0$, $c = 0$, $b = 1$, $\rho = 1$, since in this case the hypergeometric function disappears and we simply get:

$$\mathbf{a}(t,\nu) \equiv a\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; 0, 1, 1, \nu\right) = \frac{5^{2/3}e^{-\frac{2t\nu}{5}}\left(-1+e^{\frac{6t\nu}{5}}\right)}{\nu^{4/3}}$$

$$\exp\left[\mathbf{B}(t,\nu)\right] \equiv \exp\left[\mathcal{B}\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; 0, 1, 1, \nu\right)\right] = \frac{25\left(-1+e^{\frac{6t\nu}{5}}\right)^2}{\nu^4}$$

$$\mathbf{h}(t,\nu) \equiv \mathfrak{h}\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; a, b, \rho, \nu\right) = \log\left(\frac{1}{5\left(-1+e^{\frac{6t\nu}{5}}\right)}\right) + 2\log(\nu) \quad (5.62)$$

The shift in the parametric time variable $\tau \to t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}$ has been specifically arranged in such a way that $t = 0$ is a zero of the scale factor, namely corresponds to the Big Bang. Furthermore, in this case, which involves only elementary transcendental functions, the relation between parametric and cosmic time can be explicitly evaluated. We have:

$$T_c(t) \equiv \int_0^t dx \exp\left[\mathbf{B}(x,\nu)\right] = \frac{25t}{\nu^4} - \frac{125e^{\frac{6t\nu}{5}}}{3\nu^5} + \frac{125e^{\frac{12t\nu}{5}}}{12\nu^5} + \frac{125}{4\nu^5} \quad (5.63)$$

This corresponds to an equation of state of type 1.7 with $w = 1$. In view of eq.s(1.5) this means that at late times the predominant contribution to the energy density is the kinetic one, the potential energy being negligible. Such a conclusion can be matched with the information on the asymptotic behavior of the scalar field for late times. This latter can be worked in the following way. As $t \to \infty$ (for $\nu > 0$) we have:

$$T_c \overset{t\to\infty}{\simeq} \frac{125e^{\frac{12t\nu}{5}}}{12\nu^5} \quad (5.65)$$

We have that:

$$T_c \overset{t\to\infty}{\simeq} \frac{125e^{\frac{12t\nu}{5}}}{12\nu^5} \quad (5.65)$$

125*e^((12*0.25*x)/5) / ((12*0.25^5)) = y

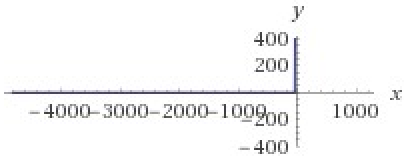
Input:

$$125 \times \frac{e^{1/5\,(12\times0.25\,x)}}{12\times0.25^5} = y$$

Result:

$$10\,666.7\,e^{0.6x} = y$$

Implicit plot:



Alternate form assuming x and y are real:

$10\,666.7\, e^{0.6 x} + 0 = y$

Real solution:

$y \approx 10\,666.7 \times 2.71828^{0.6 x}$

Solution:

$y = \frac{32\,000}{3}\, e^{(3 x)/5}$

Partial derivatives:

$\frac{\partial}{\partial x}\left(10\,666.7\, e^{0.6 x}\right) = 6400.\, e^{0.6 x}$

$\frac{\partial}{\partial y}\left(10\,666.7\, e^{0.6 x}\right) = 0$

Implicit derivatives:

$\frac{\partial x(y)}{\partial y} = \frac{26\,388\,279\,066\,624\, e^{-(1\,351\,079\,888\,211\,149\, x)/2\,251\,799\,813\,685\,248}}{168\,884\,986\,026\,393\,625}$

$\frac{\partial y(x)}{\partial x} = \frac{1\,351\,079\,888\,211\,149\, y}{2\,251\,799\,813\,685\,248}$

Limit:

$\lim_{x \to -\infty} 10\,666.7\, e^{0.6 x} = 0 \approx 0$

For

$y \approx 10\,666.7 \times 2.71828^{0.6 x}$

we obtain:

125*e^((12*0.25*x)/5) / ((12*0.25^5)) = 10666.7*2.71828^(0.6x)

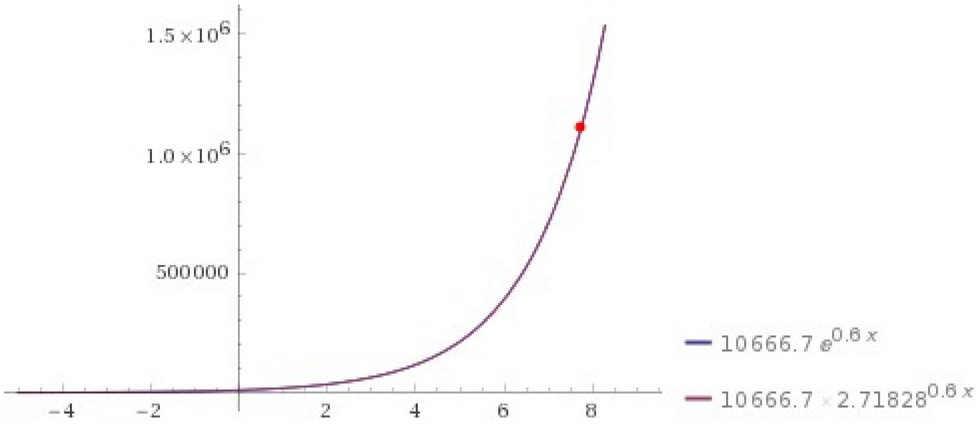
Input interpretation:

$125 \times \frac{e^{1/5\,(12 \times 0.25\, x)}}{12 \times 0.25^5} = 10\,666.7 \times 2.71828^{0.6 x}$

Result:

$10\,666.7\, e^{0.6 x} = 10\,666.7 \times 2.71828^{0.6 x}$

Plot:



Alternate forms:

$e^{0.6x} = 1. \times 2.71828^{0.6x}$

$10\,666.7\, e^{0.6x} = 10\,666.7\, e^{0.6x}$

Alternate form assuming x is positive:

$e^{0.6x} = 0.999997\, e^{0.6x}$

Alternate form assuming x is real:

$10\,666.7\, e^{0.6x} + 0 = 10\,666.7 \times 2.71828^{0.6x} + 0$

Real solution:

$x \approx 7.74296$

7.74296

For t = 7.74296 and ν = 1/4 = 0.25, from (5.62), we obtain:

$$\begin{aligned}
\mathbf{a}(t,\nu) &\equiv a\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; 0, 1, 1, \nu\right) = \frac{5^{2/3} e^{-\frac{2t\nu}{5}}\left(-1 + e^{\frac{6t\nu}{5}}\right)}{\nu^{4/3}} \\
\exp\left[\mathbf{B}(t,\nu)\right] &\equiv \exp\left[\mathcal{B}\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; 0, 1, 1, \nu\right)\right] = \frac{25\left(-1 + e^{\frac{6t\nu}{5}}\right)^2}{\nu^4} \\
\mathbf{h}(t,\nu) &\equiv \mathfrak{h}\left(t - \frac{5\log\left(\frac{\nu^2}{5}\right)}{6\nu}; a, b, \rho, \nu\right) = \log\left(\frac{1}{5\left(-1 + e^{\frac{6t\nu}{5}}\right)}\right) + 2\log(\nu) \quad (5.62)
\end{aligned}$$

((5^(2/3)*e^((-2*7.74296*0.25)/5)*(-1+e^((6*7.74296*0.25)/5)))) / (((0.25)^(4/3)))

Input interpretation:

$$\frac{5^{2/3} e^{1/5 (-2\times 7.74296\times 0.25)} \left(-1+e^{1/5 (6\times 7.74296\times 0.25)}\right)}{0.25^{4/3}}$$

Result:

78.7921...

78.7921…

Alternative representation:

$$\frac{5^{2/3} \left(e^{-(2\times 7.74296\times 0.25)/5} \left(-1+e^{(6\times 7.74296\times 0.25)/5}\right)\right)}{0.25^{4/3}} = \frac{5^{2/3} \left(\exp^{-\frac{2\times 7.74296\times 0.25}{5}}(z) \left(-1+\exp^{\frac{6\times 7.74296\times 0.25}{5}}(z)\right)\right)}{0.25^{4/3}} \text{ for } z=1$$

Series representations:

$$\frac{5^{2/3} \left(e^{-(2\times 7.74296\times 0.25)/5} \left(-1+e^{(6\times 7.74296\times 0.25)/5}\right)\right)}{0.25^{4/3}} = \frac{18.5664 \left(-1+\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{2.32289}\right)}{\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.774296}}$$

$$\frac{5^{2/3} \left(e^{-(2\times 7.74296\times 0.25)/5} \left(-1+e^{(6\times 7.74296\times 0.25)/5}\right)\right)}{0.25^{4/3}} = \frac{6.34679 \left(-5.00333+\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{2.32289}\right)}{\left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.774296}}$$

$$\frac{5^{2/3} \left(e^{-(2\times 7.74296\times 0.25)/5} \left(-1+e^{(6\times 7.74296\times 0.25)/5}\right)\right)}{0.25^{4/3}} = \frac{18.5664 \left(-1+\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{2.32289}\right)}{\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.774296}}$$

(((25(-1+ e^((6*7.74296*0.25)/5))^2))) / (0.25^4)

Input interpretation:

$$\frac{25\left(-1+e^{1/5\,(6\times 7.74296\times 0.25)}\right)^2}{0.25^4}$$

Result:

$5.42297\ldots \times 10^5$

5.44297...*10^5

ln(1/(5(-1+ e^((6*7.74296*0.25)/5))))+2 ln (0.25)

Input interpretation:

$$\log\left(\frac{1}{5\left(-1+e^{1/5\,(6\times 7.74296\times 0.25)}\right)}\right)+2\log(0.25)$$

log(x) is the natural logarithm

Result:

$-6.60178\ldots$

-6.60178

Alternative representations:

$$\log\left(\frac{1}{5\left(-1+e^{(6\,(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=2\log_e(0.25)+\log_e\left(\frac{1}{5\left(-1+e^{11.6144/5}\right)}\right)$$

$$\log\left(\frac{1}{5\left(-1+e^{(6\,(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=$$
$$2\log(a)\log_a(0.25)+\log(a)\log_a\left(\frac{1}{5\left(-1+e^{11.6144/5}\right)}\right)$$

$$\log\left(\frac{1}{5\left(-1+e^{(6\,(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=-2\,\mathrm{Li}_1(0.75)-\mathrm{Li}_1\left(1-\frac{1}{5\left(-1+e^{11.6144/5}\right)}\right)$$

Series representations:

$$\log\left(\frac{1}{5\left(-1+e^{(6(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left(2(-0.75)^k+\left(-1+\frac{1}{5\left(-1+e^{2.32289}\right)}\right)^k\right)}{k}$$

$$\log\left(\frac{1}{5\left(-1+e^{(6(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=4 i \pi\left\lfloor\frac{\arg(0.25-x)}{2\pi}\right\rfloor+2 i \pi\left\lfloor\frac{\arg\left(\frac{1}{5\left(-1+e^{2.32289}\right)}-x\right)}{2\pi}\right\rfloor+3\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left(2(0.25-x)^k+\left(\frac{1}{5\left(-1+e^{2.32289}\right)}-x\right)^k\right)x^{-k}}{k} \quad \text{for } x<0$$

$$\log\left(\frac{1}{5\left(-1+e^{(6(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=2\left\lfloor\frac{\arg(0.25-z_0)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+\left\lfloor\frac{\arg\left(\frac{1}{5\left(-1+e^{2.32289}\right)}-z_0\right)}{2\pi}\right\rfloor\log\left(\frac{1}{z_0}\right)+3\log(z_0)+2\left\lfloor\frac{\arg(0.25-z_0)}{2\pi}\right\rfloor\log(z_0)+\left\lfloor\frac{\arg\left(\frac{1}{5\left(-1+e^{2.32289}\right)}-z_0\right)}{2\pi}\right\rfloor\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^{1+k}\left(2(0.25-z_0)^k+\left(\frac{1}{5\left(-1+e^{2.32289}\right)}-z_0\right)^k\right)z_0^{-k}}{k}$$

Integral representation:

$$\log\left(\frac{1}{5\left(-1+e^{(6(7.74296\times 0.25))/5}\right)}\right)+2\log(0.25)=\int_1^{0.25}\frac{0.9-3.6t+e^{2.32289}(-0.5+3t)}{\left(0.45+e^{2.32289}(-0.25+t)-1.2t\right)t}\,dt$$

From the results: -6.60178, 5.44297...*10^5 and 78.7921

We obtain:

-(((5.44297e+5 / 78.7921 * 1/(-6.60178))))

Input interpretation:

$$-\left(\frac{5.44297\times 10^5}{78.7921}\left(-\frac{1}{6.60178}\right)\right)$$

Result:

1046.386715373374150816645021453991078634458092548842338177...

1046.3867153733...

From which:

-(((5.44297e+5 / 78.7921 * 1/(-6.60178)))) – 27

Input interpretation:

$$-\left(\frac{5.44297\times 10^5}{78.7921}\left(-\frac{1}{6.60178}\right)\right)-27$$

Result:

1019.386715373374150816645021453991078634458092548842338177...

1019.386715.... result practically equal to the rest mass of Phi meson 1019.461

With regard the number 27, we have that:

From Wikipedia:

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

And:

((-(((5.44297e+5 / 78.7921 * 1/(-6.60178))))))^1/14

Input interpretation:

$$\sqrt[14]{-\left(\frac{5.44297\times 10^5}{78.7921}\left(-\frac{1}{6.60178}\right)\right)}$$

Result:

1.643207...

1.643207.... ≈ $\zeta(2) = \frac{\pi^2}{6} = 1.644934$ ...

From the sum, we obtain:

(((5.44297e+5 +78.7921-6.60178))))

Input interpretation:

$5.44297\times 10^5 + 78.7921 - 6.60178$

Result:

544369.19032

544369.19032

From which, we obtain:

(((5.44297e+5 +78.7921-6.60178)))^1/2

Input interpretation:

$\sqrt{5.44297\times 10^5 + 78.7921 - 6.60178}$

Result:

737.814...

737.814...

(((5.44297e+5 +78.7921-6.60178)))^1/2 - Pi^2

Input interpretation:

$\sqrt{5.44297\times 10^5 + 78.7921 - 6.60178} - \pi^2$

Result:

727.944...

727.944... ≈ 728 (Ramanujan taxicab number)

10^3+(((((5.44297e+5 +78.7921-6.60178)))^1/2 - Pi^2))+1

Input interpretation:

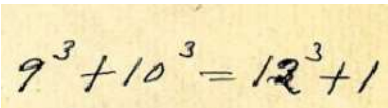
$$10^3+\left(\sqrt{5.44297\times 10^5+78.7921-6.60178}-\pi^2\right)+1$$

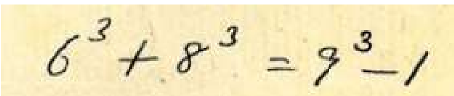
Result:

1728.944...

1728.944... ≈ 1729

This result is very near to the mass of candidate glueball $f_0(1710)$ meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)





Now, we have that:

$$\lambda_{\pm} = -\sqrt{3} \pm \sqrt{3 - 2\omega^2} \qquad (6.38)$$

In the integrable case $\omega = \sqrt{3}$, by means of the integrating transformation described in [1] we obtain the following general solution of eq.s (6.17) depending on three parameters, the scale λ

and the two angles ψ and θ, which applies to the case of the positive potential (upper choice in eq.(6.16)):

$$a_+(\tau) = \sqrt[3]{\left(\lambda \cos(\psi)\cosh\left(\sqrt{3}\tau\right) + \lambda \sinh\left(\sqrt{3}\tau\right)\right)^2 - \lambda^2 \cos^2\left(\theta - \sqrt{3}\tau\right)\sin^2(\psi)} \quad (6.18)$$

$$\phi_+(\tau) = \frac{1}{\sqrt{3}} \log\left[\frac{\cos(\psi)\cosh\left(\sqrt{3}\tau\right) - \cos\left(\theta - \sqrt{3}\tau\right)\sin(\psi) + \sinh\left(\sqrt{3}\tau\right)}{\cos(\psi)\cosh\left(\sqrt{3}\tau\right) + \cos\left(\theta - \sqrt{3}\tau\right)\sin(\psi) + \sinh\left(\sqrt{3}\tau\right)}\right] \quad (6.19)$$

analytic solution determined by the integrable cases. This time we use the solution eq.(6.19) of the integral model $\omega = \sqrt{3}$ characterized by parameters:

$$\lambda = 1 \quad ; \quad \theta = \pi \quad ; \quad \xi = \frac{\pi}{6} \qquad (6.54)$$

$\omega = \sqrt{3}$

-sqrt3+(sqrt(3-2*(sqrt3)^2))

Input:

$-\sqrt{3} + \sqrt{3 - 2\sqrt{3}^2}$

Result:

$(-1 + i)\sqrt{3}$

Decimal approximation:

$-1.732050807568877293527446341505872366942805253810380628 0\ldots + 1.7320508075688772935274463415058723669428052538103806280\ldots i$

Polar coordinates:

$r \approx 2.44949$ (radius), $\theta = 135°$ (angle)

$2.44949 = \lambda$

Alternate forms:

$-\sqrt{3} + i\sqrt{3}$

$(-1)^{3/4}\sqrt{6}$

For λ = 2.44949 , τ = 5 , θ = π , ψ = π/6 , we obtain:

$$a_+(\tau) = \sqrt[3]{\left(\lambda\cos(\psi)\cosh\left(\sqrt{3}\tau\right)+\lambda\sinh\left(\sqrt{3}\tau\right)\right)^2-\lambda^2\cos^2\left(\theta-\sqrt{3}\tau\right)\sin^2(\psi)}$$

(((((2.44949 cos (Pi/6) cosh (5*sqrt3) + 2.44949 sinh (5*sqrt3))^2 – 2.44949^2 cos^2(Pi-5*sqrt3) sin^2(Pi/6))))^1/3

Input interpretation:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\hat{}\,(1/3)$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

558.096...

558.096...

From which:

(((((2.44949 cos (Pi/6) cosh (5*sqrt3) + 2.44949 sinh (5*sqrt3))^2 – 2.44949^2 cos^2(Pi-5*sqrt3) sin^2(Pi/6))))^1/3 – 11

Input interpretation:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\hat{}\,(1/3)-11$$

cosh(x) is the hyperbolic cosine function

sinh(x) is the hyperbolic sine function

Result:

547.096...

547.096…. result very near to the rest mass of Eta meson 547.862

Addition formulas:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(2.44949\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-6.\sin^2\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(5\sqrt{3}\right)+\sin(\pi)\sin\left(5\sqrt{3}\right)\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(2.44949\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-6.\sin^2\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(-5\sqrt{3}\right)-\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(2.44949\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-6.\sin^2\left(\frac{\pi}{6}\right)\left(\cosh\left(-5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5i\sqrt{3}\right)\sin(\pi)\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(2.44949\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-6.\sin^2\left(\frac{\pi}{6}\right)\left(\cosh\left(5i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5i\sqrt{3}\right)\sin(\pi)\right)\right)^2\right)\text{^}(1/3)$$

Alternative representations:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+1.22475\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2-2.44949^2\cosh^2\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(\frac{-e^{-(i\pi)/6}+e^{(i\pi)/6}}{2i}\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(-2.44949^2\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\left(\frac{1}{2}\left(e^{-i\left(\pi-5\sqrt{3}\right)}+e^{i\left(\pi-5\sqrt{3}\right)}\right)\right)^2+\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+0.612373\left(e^{-(i\pi)/6}+e^{(i\pi)/6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(-2.44949^2\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2\left(\frac{1}{2}\left(e^{-i\left(\pi-5\sqrt{3}\right)}+e^{i\left(\pi-5\sqrt{3}\right)}\right)\right)^2+\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+0.612373\left(e^{-(i\pi)/6}+e^{(i\pi)/6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2\right)\text{^}(1/3)$$

Series representations:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(I_0\left(5\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k\pi^{2k}}{(2k)!}\right)^2-24.\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\pi-5\sqrt{3}\right)^{2k}}{(2k)!}\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=-11+\left(-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2+\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\sum_{k=0}^{\infty}\frac{25^k\sqrt{3}^{2k}}{(2k)!}\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=$$
$$-11+\left(-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2+\left(2.44949\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\sum_{k=0}^{\infty}\frac{25^k\sqrt{3}^{2k}}{(2k)!}+2.44949\sum_{k=0}^{\infty}\frac{5^{1+2k}\sqrt{3}^{1+2k}}{(1+2k)!}\right)^2\right)\text{^}(1/3)$$

Integral representations:

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=$$
$$-11+\left(-0.166667\pi^2\left(\int_0^1\cos\left(\frac{\pi t}{6}\right)dt\right)^2\left(\int_{\frac{\pi}{2}}^{\pi-5\sqrt{3}}\sin(t)\,dt\right)^2+\left(12.2475\sqrt{3}\int_0^1\cosh\left(5t\sqrt{3}\right)dt+\left(\int_{\frac{\pi}{2}}^{\frac{\pi}{6}}\sin(t)\,dt\right)\left(-2.44949-12.2475\sqrt{3}\int_0^1\sinh\left(5t\sqrt{3}\right)dt\right)\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=$$
$$-11+\left(-0.166667\pi^2\left(\int_0^1\cos\left(\frac{\pi t}{6}\right)dt\right)^2\left(-1+\pi-5\sqrt{3}\int_0^1\sin\left(t\left(\pi-5\sqrt{3}\right)\right)dt\right)^2+\left(\left(\int_{\frac{i\pi}{2}}^{5\sqrt{3}}\sinh(t)\,dt\right)\left(2.44949-0.408248\pi\int_0^1\sin\left(\frac{\pi t}{6}\right)dt\right)+12.2475\sqrt{3}\int_0^1\cosh\left(5t\sqrt{3}\right)dt\right)^2\right)\text{^}(1/3)$$

$$\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)-11=$$
$$-11+\left(-0.166667\pi^2\left(\int_0^1\cos\left(\frac{\pi t}{6}\right)dt\right)^2\left(\int_{\frac{\pi}{2}}^{\pi-5\sqrt{3}}\sin(t)\,dt\right)^2+\left(\int_0^1\int_0^1\sinh\left(\frac{1}{2}\left(i\pi+\left(-i\pi+10\sqrt{3}\right)t_1\right)\right)\sin\left(\frac{1}{6}\pi(3-2t_2)\right)dt_2\,dt_1-12.2475\sqrt{3}\int_0^1\cosh\left(5t\sqrt{3}\right)dt\right)^2\right)\text{^}(1/3)$$

Multiple-argument formulas:

$$\left(\left(2.44949 \cos\left(\frac{\pi}{6}\right) \cosh\left(5 \sqrt{3}\right) + 2.44949 \sinh\left(5 \sqrt{3}\right)\right)^2 - 2.44949^2 \cos^2\left(\pi - 5 \sqrt{3}\right) \sin^2\left(\frac{\pi}{6}\right)\right) \wedge (1/3) - 11 = -11 + \left(\left(2.44949 \left(-1 + 2 \cosh^2\left(\frac{5 \sqrt{3}}{2}\right)\right)\left(-1 + 2 \cos^2\left(\frac{\pi}{12}\right)\right) + 4.89898 \cosh\left(\frac{5 \sqrt{3}}{2}\right) \sinh\left(\frac{5 \sqrt{3}}{2}\right)\right)^2 - 24. \cos^2\left(\frac{\pi}{12}\right)\left(1 - 2 \cos^2\left(\frac{1}{2}\left(\pi - 5 \sqrt{3}\right)\right)\right)^2 \sin^2\left(\frac{\pi}{12}\right)\right) \wedge (1/3)$$

$$\left(\left(2.44949 \cos\left(\frac{\pi}{6}\right) \cosh\left(5 \sqrt{3}\right) + 2.44949 \sinh\left(5 \sqrt{3}\right)\right)^2 - 2.44949^2 \cos^2\left(\pi - 5 \sqrt{3}\right) \sin^2\left(\frac{\pi}{6}\right)\right) \wedge (1/3) - 11 = -11 + \left(\left(4.89898 \cosh\left(\frac{5 \sqrt{3}}{2}\right) \sinh\left(\frac{5 \sqrt{3}}{2}\right) + 2.44949 \left(-1 + 2 \cos^2\left(\frac{\pi}{12}\right)\right)\left(1 + 2 \sinh^2\left(\frac{5 \sqrt{3}}{2}\right)\right)\right)^2 - 24. \cos^2\left(\frac{\pi}{12}\right)\left(1 - 2 \cos^2\left(\frac{1}{2}\left(\pi - 5 \sqrt{3}\right)\right)\right)^2 \sin^2\left(\frac{\pi}{12}\right)\right) \wedge (1/3)$$

$$\left(\left(2.44949 \cos\left(\frac{\pi}{6}\right) \cosh\left(5 \sqrt{3}\right) + 2.44949 \sinh\left(5 \sqrt{3}\right)\right)^2 - 2.44949^2 \cos^2\left(\pi - 5 \sqrt{3}\right) \sin^2\left(\frac{\pi}{6}\right)\right) \wedge (1/3) - 11 = -11 + \left(\left(4.89898 \cosh\left(\frac{5 \sqrt{3}}{2}\right) \sinh\left(\frac{5 \sqrt{3}}{2}\right) + 2.44949 \left(-1 + 2 \cosh^2\left(\frac{5 \sqrt{3}}{2}\right)\right)\left(1 - 2 \sin^2\left(\frac{\pi}{12}\right)\right)\right)^2 - 24. \cos^2\left(\frac{\pi}{12}\right) \sin^2\left(\frac{\pi}{12}\right)\left(1 - 2 \sin^2\left(\frac{1}{2}\left(\pi - 5 \sqrt{3}\right)\right)\right)^2\right) \wedge (1/3)$$

1+1/(((((((2.44949 cos (Pi/6) cosh (5*sqrt3) + 2.44949 sinh (5*sqrt3))^2 – 2.44949^2 cos^2(Pi-5*sqrt3) sin^2(Pi/6))))^1/3)))

Input interpretation:

$$1 + 1 \Big/ \left(\left(\left(2.44949 \cos\left(\frac{\pi}{6}\right) \cosh\left(5 \sqrt{3}\right) + 2.44949 \sinh\left(5 \sqrt{3}\right)\right)^2 - 2.44949^2 \cos^2\left(\pi - 5 \sqrt{3}\right) \sin^2\left(\frac{\pi}{6}\right)\right) \wedge (1/3)\right)$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

Result:

1.00179181...

1.00179181…. result very near to the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{2\pi}{5}}}{\sqrt{\varphi\sqrt{5}-\varphi}} = 1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\ldots}}}} \approx 1.0018674362$$

Addition formulas:

$$1+1/(((2.44949\cos(\frac{\pi}{6})\cosh(5\sqrt{3})+2.44949\sinh(5\sqrt{3}))^2 - 2.44949^2\cos^2(\pi-5\sqrt{3})\sin^2(\frac{\pi}{6}))\verb|^|(1/3)) = 1+1/(((2.44949\cosh(5\sqrt{3})\cos(\frac{\pi}{6})+2.44949\sinh(5\sqrt{3}))^2 - 6.\sin^2(\frac{\pi}{6})(\cos(\pi)\cos(5\sqrt{3})+\sin(\pi)\sin(5\sqrt{3}))^2)\verb|^|(1/3))$$

$$1+1/(((2.44949\cos(\frac{\pi}{6})\cosh(5\sqrt{3})+2.44949\sinh(5\sqrt{3}))^2 - 2.44949^2\cos^2(\pi-5\sqrt{3})\sin^2(\frac{\pi}{6}))\verb|^|(1/3)) = 1+1/(((2.44949\cosh(5\sqrt{3})\cos(\frac{\pi}{6})+2.44949\sinh(5\sqrt{3}))^2 - 6.\sin^2(\frac{\pi}{6})(\cos(\pi)\cos(-5\sqrt{3})-\sin(\pi)\sin(-5\sqrt{3}))^2)\verb|^|(1/3))$$

$$1+1/(((2.44949\cos(\frac{\pi}{6})\cosh(5\sqrt{3})+2.44949\sinh(5\sqrt{3}))^2 - 2.44949^2\cos^2(\pi-5\sqrt{3})\sin^2(\frac{\pi}{6}))\verb|^|(1/3)) = 1+1/(((2.44949\cosh(5\sqrt{3})\cos(\frac{\pi}{6})+2.44949\sinh(5\sqrt{3}))^2 - 6.\sin^2(\frac{\pi}{6})(\cosh(-5i\sqrt{3})\cos(\pi)+i\sinh(-5i\sqrt{3})\sin(\pi))^2)\verb|^|(1/3))$$

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$
$$1+1\Big/\left(\left(\left(2.44949\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-6.\sin^2\left(\frac{\pi}{6}\right)\left(\cosh\left(5\,i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5\,i\sqrt{3}\right)\sin(\pi)\right)\right)^2\right)\wedge(1/3)\right)$$

Alternative representations:

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$
$$1+1\Big/\left(\left(\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+1.22475\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2-2.44949^2\cosh^2\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(\frac{-e^{-(i\pi)/6}+e^{(i\pi)/6}}{2\,i}\right)^2\right)\wedge(1/3)\right)$$

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$
$$1+1\Big/\left(\left(-2.44949^2\cos^2\left(\frac{\pi}{2}-\frac{\pi}{6}\right)\left(\frac{1}{2}\left(e^{-i\left(\pi-5\sqrt{3}\right)}+e^{i\left(\pi-5\sqrt{3}\right)}\right)\right)^2+\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+0.612373\left(e^{-(i\pi)/6}+e^{(i\pi)/6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2\right)\wedge(1/3)\right)$$

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$
$$1+1\Big/\left(\left(-2.44949^2\left(-\cos\left(\frac{\pi}{2}+\frac{\pi}{6}\right)\right)^2\left(\frac{1}{2}\left(e^{-i\left(\pi-5\sqrt{3}\right)}+e^{i\left(\pi-5\sqrt{3}\right)}\right)\right)^2+\left(1.22475\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+0.612373\left(e^{-(i\pi)/6}+e^{(i\pi)/6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)\right)^2\right)\wedge(1/3)\right)$$

Series representations:

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$

$$\left(1+\left(\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(I_0\left(5\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k\pi^{2k}}{(2k)!}\right)^2-24.\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\pi-5\sqrt{3}\right)^{2k}}{(2k)!}\right)^2\right)\wedge(1/3)\right)\Big/$$

$$\left(\left(\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(I_0\left(5\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(5\sqrt{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{36}\right)^k\pi^{2k}}{(2k)!}\right)^2-24.\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\pi-5\sqrt{3}\right)^{2k}}{(2k)!}\right)^2\right)\wedge(1/3)\right)$$

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$

$$\left(1+\left(-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2+\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\sum_{k=0}^{\infty}\frac{25^k\sqrt{3}^{2k}}{(2k)!}\right)^2\right)\wedge(1/3)\right)\Big/$$

$$\left(\left(-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2+\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\sum_{k=0}^{\infty}\frac{25^k\sqrt{3}^{2k}}{(2k)!}\right)^2\right)\wedge(1/3)\right)$$

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)\right)=$$

$$\left(1+\left(\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(I_0\left(5\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(5\sqrt{3}\right)\right)\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\right)^2-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)\text{^}(1/3)\right)\Big/$$

$$\left(\left(\left(4.89898\sum_{k=0}^{\infty}I_{1+2k}\left(5\sqrt{3}\right)+2.44949\left(I_0\left(5\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(5\sqrt{3}\right)\right)\left(J_0\left(\frac{\pi}{6}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{\pi}{6}\right)\right)\right)^2-24.\left(J_0\left(\pi-5\sqrt{3}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\pi-5\sqrt{3}\right)\right)^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{6}\right)\right)^2\right)\text{^}(1/3)\right)$$

Multiple-argument formulas:

$$1+1\Big/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\text{^}(1/3)\right)=$$

$$1+1\Big/\left(\left(\left(2.44949\left(-1+2\cosh^2\left(\frac{5\sqrt{3}}{2}\right)\right)\left(-1+2\cos^2\left(\frac{\pi}{12}\right)\right)+4.89898\cosh\left(\frac{5\sqrt{3}}{2}\right)\sinh\left(\frac{5\sqrt{3}}{2}\right)\right)^2-24.\cos^2\left(\frac{\pi}{12}\right)\left(1-2\cos^2\left(\frac{1}{2}\left(\pi-5\sqrt{3}\right)\right)\right)^2\sin^2\left(\frac{\pi}{12}\right)\right)\text{^}(1/3)\right)$$

$$1+1/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$

$$1+1/\left(\left(\left(\left(4.89898\cosh\left(\frac{5\sqrt{3}}{2}\right)\sinh\left(\frac{5\sqrt{3}}{2}\right)+2.44949\left(-1+2\cos^2\left(\frac{\pi}{12}\right)\right)\left(1+2\sinh^2\left(\frac{5\sqrt{3}}{2}\right)\right)\right)^2-24.\cos^2\left(\frac{\pi}{12}\right)\left(1-2\cos^2\left(\frac{1}{2}\left(\pi-5\sqrt{3}\right)\right)\right)^2\sin^2\left(\frac{\pi}{12}\right)\right)\wedge(1/3)\right)$$

$$1+1/\left(\left(\left(2.44949\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+2.44949\sinh\left(5\sqrt{3}\right)\right)^2-2.44949^2\cos^2\left(\pi-5\sqrt{3}\right)\sin^2\left(\frac{\pi}{6}\right)\right)\wedge(1/3)\right)=$$

$$1+1/\left(\left(\left(\left(4.89898\cosh\left(\frac{5\sqrt{3}}{2}\right)\sinh\left(\frac{5\sqrt{3}}{2}\right)+2.44949\left(-1+2\cosh^2\left(\frac{5\sqrt{3}}{2}\right)\right)\left(1-2\sin^2\left(\frac{\pi}{12}\right)\right)\right)^2-24.\cos^2\left(\frac{\pi}{12}\right)\sin^2\left(\frac{\pi}{12}\right)\left(1-2\sin^2\left(\frac{1}{2}\left(\pi-5\sqrt{3}\right)\right)\right)^2\right)\wedge(1/3)\right)$$

$$\phi_+(\tau) = \frac{1}{\sqrt{3}}\log\left[\frac{\cos(\psi)\cosh\left(\sqrt{3}\tau\right)-\cos\left(\theta-\sqrt{3}\tau\right)\sin(\psi)+\sinh\left(\sqrt{3}\tau\right)}{\cos(\psi)\cosh\left(\sqrt{3}\tau\right)+\cos\left(\theta-\sqrt{3}\tau\right)\sin(\psi)+\sinh\left(\sqrt{3}\tau\right)}\right]$$

For: $\tau = 5$, $\theta = \pi$, $\psi = \pi/6$, we obtain:

1/(sqrt3) ln [(((cos (Pi/6) cosh(5*sqrt3) – cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3)))) / (((cos (Pi/6) cosh(5*sqrt3) + cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3))))]

Input:

$$\frac{1}{\sqrt{3}}\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function log(x) is the natural logarithm

Exact result:

$$\frac{\log\left(\frac{\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})}{-\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})}\right)}{\sqrt{3}}$$

Decimal approximation:

-0.00007741323026660106173673189955194078598704989459953066...

-0.00007741323026….

Alternate forms:

$$-\frac{\log\left(\frac{-2+\sqrt{3}+2e^{10\sqrt{3}}+\sqrt{3}e^{10\sqrt{3}}-2e^{5\sqrt{3}}\cos(5\sqrt{3})}{-2+\sqrt{3}+2e^{10\sqrt{3}}+\sqrt{3}e^{10\sqrt{3}}+2e^{5\sqrt{3}}\cos(5\sqrt{3})}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\frac{1}{4}\left(e^{-5i\sqrt{3}}+e^{5i\sqrt{3}}\right)+\frac{1}{2}\left(e^{5\sqrt{3}}-e^{-5\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)}{\frac{1}{4}\left(-e^{-5i\sqrt{3}}-e^{5i\sqrt{3}}\right)+\frac{1}{2}\left(e^{5\sqrt{3}}-e^{-5\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})\right)}{\sqrt{3}}-\frac{\log\left(-\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})\right)}{\sqrt{3}}$$

Addition formulas:

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh(5\sqrt{3})-\cos(\pi-5\sqrt{3})\sin\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})}{\cos\left(\frac{\pi}{6}\right)\cosh(5\sqrt{3})+\cos(\pi-5\sqrt{3})\sin\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})}\right)}{\sqrt{3}}=$$

$$\frac{\log\left(\frac{\cosh(5\sqrt{3})\cos\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})-\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos(5\sqrt{3})+\sin(\pi)\sin(5\sqrt{3})\right)}{\cosh(5\sqrt{3})\cos\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos(5\sqrt{3})+\sin(\pi)\sin(5\sqrt{3})\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cos(\pi)\cos\left(-5\sqrt{3}\right)+\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(-5\sqrt{3}\right)-\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5\,i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5\,i\sqrt{3}\right)\sin(\pi)\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cosh\left(5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(5\,i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(5\,i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5\,i\sqrt{3}\right)\sin(\pi)\right)\right)}\right)}{\sqrt{3}}$$

Alternative representations:

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log_e\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log(a)\log_a\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = \frac{\log\left(\frac{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)-\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}\right)}{\sqrt{3}}$$

Series representations:

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = -\frac{\sum_{k=1}^{\infty}\frac{(-2)^k\left(-\frac{\cos\left(5\sqrt{3}\right)}{\cos\left(5\sqrt{3}\right)-\sqrt{3}\cosh\left(5\sqrt{3}\right)-2\sinh\left(5\sqrt{3}\right)}\right)^k}{k}}{\sqrt{3}}$$

$$\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}} = -\frac{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\frac{1}{2}\cos\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)}{-\frac{1}{2}\cos\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)}\right)^k}{k}}{\sqrt{3}}$$

sqrt[-1/((((1/(sqrt3) ln [(((cos (Pi/6) cosh(5*sqrt3) – cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3)))) / (((cos (Pi/6) cosh(5*sqrt3) + cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3))))])))))]+11+0.61803

Input:

$$\sqrt{-\frac{1}{\frac{1}{\sqrt{3}}\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}} + 11 + 0.61803$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

125.27404...

125.27404…. result very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV

Addition formulas:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803} =$$

$$11.618+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(5\sqrt{3}\right)+\sin(\pi)\sin\left(5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(5\sqrt{3}\right)+\sin(\pi)\sin\left(5\sqrt{3}\right)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803} =$$

$$11.618+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cos(\pi)\cos\left(-5\sqrt{3}\right)+\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(-5\sqrt{3}\right)-\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803} =$$

$$11.618+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5i\sqrt{3}\right)\sin(\pi)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803} =$$

$$11.618+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cosh\left(5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(5i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(5i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5i\sqrt{3}\right)\sin(\pi)\right)\right)}\right)}}$$

Alternative representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=$$
$$11.618+\sqrt{-\frac{1}{\frac{\log_e\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=$$
$$11.618+\sqrt{-\frac{1}{\frac{\log(a)\log_a\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=11.618+$$
$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)-\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}\right)}{\sqrt{3}}}}$$

Series representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803}=$$

$$11.618+\exp\left(i\pi\left\lfloor\frac{\arg\left(-x-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}\right)^k}{k!}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+11+0.61803}=$$

$$11.618+\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)/(2\pi)\right\rfloor}$$

$$z_0^{1/2\left(1+\left\lfloor\arg\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)/(2\pi)\right\rfloor\right)}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)^k z_0^{-k}}{k!}$$

Integral representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=11.618+\sqrt{-\frac{\sqrt{3}}{\log(1)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=11.618+\sqrt{-\frac{\sqrt{3}}{\log(1)}}$$

for $\gamma > 0$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+11+0.61803=$$

$$11.618+\sqrt{-\frac{\sqrt{3}}{\int_1^{\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}}\frac{1}{t}\,dt}}$$

sqrt[-1/((((1/(sqrt3) ln [(((cos (Pi/6) cosh(5*sqrt3) – cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3)))) / (((cos (Pi/6) cosh(5*sqrt3) + cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3))))])))))]+21+5

Input:

$$\sqrt{-\frac{1}{\frac{1}{\sqrt{3}}\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}}+21+5$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Exact result:

$$26+\sqrt[4]{3}\sqrt{-\frac{1}{\log\left(\frac{\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})}{-\frac{1}{2}\cos(5\sqrt{3})+\sinh(5\sqrt{3})+\frac{1}{2}\sqrt{3}\cosh(5\sqrt{3})}\right)}}$$

Decimal approximation:

139.6560098403096751305332842064453980061648767036868188465...

139.65600984…. result practically equal to the rest mass of Pion meson 139.57 MeV

Alternate forms:

$$26+\left(i\sqrt[4]{3}\right)/\left(\sqrt{\left(\log\left(\frac{1}{2}\cos\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)\right)-\log\left(-\frac{1}{2}\cos\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)\right)\right)}\right)$$

$$26+\frac{\sqrt[4]{3}}{\sqrt{\log\left(\frac{-2+\sqrt{3}+2e^{10\sqrt{3}}+\sqrt{3}\,e^{10\sqrt{3}}-2e^{5\sqrt{3}}\cos(5\sqrt{3})}{-2+\sqrt{3}+2e^{10\sqrt{3}}+\sqrt{3}\,e^{10\sqrt{3}}+2e^{5\sqrt{3}}\cos(5\sqrt{3})}\right)}}$$

$$26+\sqrt[4]{3}\sqrt{-\frac{1}{\log\left(\frac{\frac{1}{4}\left(e^{-5i\sqrt{3}}+e^{5i\sqrt{3}}\right)+\frac{1}{2}\left(e^{5\sqrt{3}}-e^{-5\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)}{\frac{1}{4}\left(-e^{-5i\sqrt{3}}-e^{5i\sqrt{3}}\right)+\frac{1}{2}\left(e^{5\sqrt{3}}-e^{-5\sqrt{3}}\right)+\frac{1}{4}\sqrt{3}\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)}\right)}}$$

Addition formulas:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh(5\sqrt{3})-\cos(\pi-5\sqrt{3})\sin\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})}{\cos\left(\frac{\pi}{6}\right)\cosh(5\sqrt{3})+\cos(\pi-5\sqrt{3})\sin\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})}\right)}{\sqrt{3}}}}+21+5=$$

$$26+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh(5\sqrt{3})\cos\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})-\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos(5\sqrt{3})+\sin(\pi)\sin(5\sqrt{3})\right)}{\cosh(5\sqrt{3})\cos\left(\frac{\pi}{6}\right)+\sinh(5\sqrt{3})+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos(5\sqrt{3})+\sin(\pi)\sin(5\sqrt{3})\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+21+5=$$

$$26+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cos(\pi)\cos\left(-5\sqrt{3}\right)+\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(-5\sqrt{3}\right)-\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+21+5=$$

$$26+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5i\sqrt{3}\right)\sin(\pi)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+21+5=$$

$$26+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cosh\left(5i\sqrt{3}\right)\cos(\pi)+i\sinh\left(5i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(5i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5i\sqrt{3}\right)\sin(\pi)\right)\right)}\right)}}$$

Alternative representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+21+5=$$

$$26+\sqrt{-\frac{1}{\frac{\log_e\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 21 + 5 =$$

$$26 + \sqrt{-\frac{1}{\frac{\log(a)\log_a\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 21 + 5 = 26 +$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)-\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}\right)}{\sqrt{3}}}}$$

Series representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 21 + 5 =$$

$$26 + \sqrt[4]{3}\sqrt{\frac{1}{\sum_{k=1}^{\infty}\frac{(-2)^k\left(-\frac{\cos\left(5\sqrt{3}\right)}{\cos\left(5\sqrt{3}\right)-\sqrt{3}\cosh\left(5\sqrt{3}\right)-2\sinh\left(5\sqrt{3}\right)}\right)^k}{k}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 21 + 5 =$$

$$26 + \sqrt[4]{3}\sqrt{\frac{1}{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{\frac{1}{2}\cos\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)}{-\frac{1}{2}\cos\left(5\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\cosh\left(5\sqrt{3}\right)+\sinh\left(5\sqrt{3}\right)}\right)^k}{k}}}$$

sqrt[-1/((((1/(sqrt3) ln [((((cos (Pi/6) cosh(5*sqrt3) – cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3)))) / (((cos (Pi/6) cosh(5*sqrt3) + cos (Pi-5*sqrt3) sin (Pi/6) + sinh (5*sqrt3))))]))))]+24-0.61803

Input:

$$\sqrt{-\frac{1}{\frac{1}{\sqrt{3}}\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}}+24-0.61803$$

$\cosh(x)$ is the hyperbolic cosine function

$\sinh(x)$ is the hyperbolic sine function

$\log(x)$ is the natural logarithm

Result:

137.03798...

137.03798….

This result is very near to the inverse of fine-structure constant 137,035

Addition formulas:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=$$

$$23.382+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(5\sqrt{3}\right)+\sin(\pi)\sin\left(5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(5\sqrt{3}\right)+\sin(\pi)\sin\left(5\sqrt{3}\right)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=$$

$$23.382+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cos(\pi)\cos\left(-5\sqrt{3}\right)+\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cos(\pi)\cos\left(-5\sqrt{3}\right)-\sin(\pi)\sin\left(-5\sqrt{3}\right)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+24-0.61803} =$$

$$23.382+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5\,i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(-5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(-5\,i\sqrt{3}\right)\sin(\pi)\right)}\right)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+24-0.61803} =$$

$$23.382+\sqrt{-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(-\cosh\left(5\,i\sqrt{3}\right)\cos(\pi)+i\sinh\left(5\,i\sqrt{3}\right)\sin(\pi)\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\sin\left(\frac{\pi}{6}\right)\left(\cosh\left(5\,i\sqrt{3}\right)\cos(\pi)-i\left(\sinh\left(5\,i\sqrt{3}\right)\sin(\pi)\right)\right)}\right)}}$$

Alternative representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+24-0.61803} =$$

$$23.382+\sqrt{-\frac{1}{\frac{\log_e\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}+24-0.61803} =$$

$$23.382+\sqrt{-\frac{1}{\frac{\log(a)\log_a\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}{\sqrt{3}}}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 24 - 0.61803 = 23.382 +$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)-\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}{\frac{1}{2}\left(-e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{1}{2}\cosh\left(-\frac{i\pi}{6}\right)\left(e^{-5\sqrt{3}}+e^{5\sqrt{3}}\right)+\frac{\cosh\left(-i\left(\pi-5\sqrt{3}\right)\right)\left(-e^{-(i\pi)/6}+e^{(i\pi)/6}\right)}{2i}}\right)}{\sqrt{3}}}}$$

Series representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}} + 24 - 0.61803 =$$

$$23.382 + \exp\left(i\pi\left\lfloor\frac{\arg\left(-x-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}\right)}{2\pi}\right\rfloor\right)\sqrt{x}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k x^{-k}\left(-\frac{1}{2}\right)_k\left(-x-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}\right)^k}{k!}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=$$

$$23.382+\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)/(2\pi)\right\rfloor}$$

$$z_0^{1/2\left(1+\left\lfloor\arg\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)/(2\pi)\right\rfloor\right)}$$

$$\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-\frac{\sqrt{3}}{\log\left(\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}\right)}-z_0\right)^k z_0^{-k}}{k!}$$

Integral representations:

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=23.382+\sqrt{-\frac{\sqrt{3}}{\log(1)}}$$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=23.382+\sqrt{-\frac{\sqrt{3}}{\log(1)}}$$

for $\gamma > 0$

$$\sqrt{-\frac{1}{\frac{\log\left(\frac{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}{\cos\left(\frac{\pi}{6}\right)\cosh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)}\right)}{\sqrt{3}}}}+24-0.61803=$$

$$23.382+\sqrt{-\frac{\sqrt{3}}{\int_1^{\frac{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)-\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}{\cosh\left(5\sqrt{3}\right)\cos\left(\frac{\pi}{6}\right)+\sinh\left(5\sqrt{3}\right)+\cos\left(\pi-5\sqrt{3}\right)\sin\left(\frac{\pi}{6}\right)}}\frac{1}{t}\,dt}}$$

Now, we have that:

$$
\begin{aligned}
V \;=\; & \frac{1}{64}e^{-h_1-3h_2-3h_3}\lambda_1^2 - \frac{1}{32}e^{-3h_2}\lambda_1\lambda_4 + \frac{1}{64}e^{h_1-3h_2+3h_3}\lambda_4^2 + \frac{1}{32}e^{-2h_2+h_3}\lambda_4\lambda_5 \\
& -\frac{1}{192}e^{\;\;h_1\;\;h_2\;\;h_3}\lambda_5^2 - \frac{1}{32}e^{\;\;2h_2\;\;h_3}\lambda_1\lambda_8 + \frac{1}{32}e^{\;\;h_2}\lambda_5\lambda_8 - \frac{1}{192}e^{h_1\;\;h_2\,|\,h_3}\lambda_8^2 \qquad (7.12)
\end{aligned}
$$

For h_1, h_2 and h_3 = 1 and λ = 2, we obtain:

((1/64 * e^(-7)))*4 – ((1/32 * e^(-3)))*4+ ((1/64 * e))*4 + ((1/32 * e^(-1)))*4 – ((1/192 * e^(-3)))*4 – ((1/32 * e^(-3)))*4 + ((1/32*e^(-1)))*4 – ((1/192 * e))*4

Input:

$$\frac{\frac{1}{64}}{e^7}\times 4-\frac{\frac{1}{32}}{e^3}\times 4+\left(\frac{1}{64}e\right)\times 4+\frac{\frac{1}{32}}{e}\times 4-\frac{\frac{1}{192}}{e^3}\times 4-\frac{\frac{1}{32}}{e^3}\times 4+\frac{\frac{1}{32}}{e}\times 4-\left(\frac{1}{192}e\right)\times 4$$

Result:

$$\frac{1}{16e^7}-\frac{13}{48e^3}+\frac{1}{4e}+\frac{e}{24}$$

Decimal approximation:

0.1918045980852048045783212122801100032610740465607070733 99...

0.1918045980852…

Property:

$\frac{1}{16e^7}-\frac{13}{48e^3}+\frac{1}{4e}+\frac{e}{24}$ is a transcendental number

Alternate form:

$$\frac{3-13e^4+12e^6+2e^8}{48e^7}$$

Alternative representation:

$$\frac{4}{64e^7}-\frac{4}{32e^3}+\frac{4e}{64}+\frac{4}{32e}-\frac{4}{192e^3}-\frac{4}{32e^3}+\frac{4}{32e}-\frac{4e}{192}=$$
$$\frac{4}{64\exp^7(z)}-\frac{4}{32\exp^3(z)}+\frac{4\exp(z)}{64}+\frac{4}{32\exp(z)}-$$
$$\frac{4}{192\exp^3(z)}-\frac{4}{32\exp^3(z)}+\frac{4}{32\exp(z)}-\frac{4\exp(z)}{192} \text{ for } z=1$$

Series representations:

$$\frac{4}{64 e^7} - \frac{4}{32 e^3} + \frac{4 e}{64} + \frac{4}{32 e} - \frac{4}{192 e^3} - \frac{4}{32 e^3} + \frac{4}{32 e} - \frac{4 e}{192} = \frac{e}{24} + \sum_{k=0}^{\infty} \frac{3 (-7)^k - 13 (-3)^k + 12 (-1)^k}{48 k!}$$

$$\frac{4}{64 e^7} - \frac{4}{32 e^3} + \frac{4 e}{64} + \frac{4}{32 e} - \frac{4}{192 e^3} - \frac{4}{32 e^3} + \frac{4}{32 e} - \frac{4 e}{192} = \frac{3 - 13 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^4 + 12 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^6 + 2 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^8}{48 \left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^7}$$

$$\frac{4}{64 e^7} - \frac{4}{32 e^3} + \frac{4 e}{64} + \frac{4}{32 e} - \frac{4}{192 e^3} - \frac{4}{32 e^3} + \frac{4}{32 e} - \frac{4 e}{192} = \frac{384 - 104 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^4 + 24 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^6 + \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^8}{48 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^7}$$

From which:

sqrt((((1/2*1/((((((1/64 * e^(-7)))*4 – ((1/32 * e^(-3)))*4+ ((1/64 * e))*4 + ((1/32 * e^(-1)))*4 – ((1/192 * e^(-3)))*4 – ((1/32 * e^(-3)))*4 + ((1/32*e^(-1)))*4 – ((1/192 * e))*4)))))))

Input:

$$\sqrt{\frac{1}{2} \times \frac{1}{\frac{1}{\frac{64}{e^7}} \times 4 - \frac{1}{\frac{32}{e^3}} \times 4 + \left(\frac{1}{64} e\right) \times 4 + \frac{1}{\frac{32}{e}} \times 4 - \frac{1}{\frac{192}{e^3}} \times 4 - \frac{1}{\frac{32}{e^3}} \times 4 + \frac{1}{\frac{32}{e}} \times 4 - \left(\frac{1}{192} e\right) \times 4}}$$

Exact result:

$$\frac{1}{\sqrt{2 \left(\frac{1}{16 e^7} - \frac{13}{48 e^3} + \frac{1}{4 e} + \frac{e}{24}\right)}}$$

Decimal approximation:

1.614564856167353921101495127353026358078820089280400290911...

1.614564856167…. result that is near to the value of the golden ratio 1,618033988749...

Property:

$$\frac{1}{\sqrt{2\left(\frac{1}{16 e^7}-\frac{13}{48 e^3}+\frac{1}{4 e}+\frac{e}{24}\right)}} \text{ is a transcendental number}$$

Alternate form:

$$2 e^{7/2} \sqrt{\frac{6}{3-13 e^4+12 e^6+2 e^8}}$$

All 2nd roots of 1/(2 (1/(16 e^7) - 13/(48 e^3) + 1/(4 e) + e/24)):

$$\frac{e^0}{\sqrt{2\left(\frac{1}{16 e^7}-\frac{13}{48 e^3}+\frac{1}{4 e}+\frac{e}{24}\right)}} \approx 1.6146 \text{ (real, principal root)}$$

$$\frac{e^{i \pi}}{\sqrt{2\left(\frac{1}{16 e^7}-\frac{13}{48 e^3}+\frac{1}{4 e}+\frac{e}{24}\right)}} \approx -1.6146 \text{ (real root)}$$

Series representations:

$$\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e 4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e 4}{192}\right) 2}} = \sqrt{-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}} \sum_{k=0}^{\infty}\left(-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}\right)^{-k} \binom{\frac{1}{2}}{k}$$

$$\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e 4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e 4}{192}\right) 2}} = \sqrt{-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}} \sum_{k=0}^{\infty} \frac{(-1)^k\left(-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}\right)^{-k}\left(-\frac{1}{2}\right)_k}{k!}$$

$$\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e\,4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e\,4}{192}\right)2}} =$$

$$\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}-z_0\right)^k z_0^{-k}}{k!}$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

1 / sqrt((((1/2*1/(((((1/64 * e^(-7)))*4 – ((1/32 * e^(-3)))*4+ ((1/64 * e))*4 + ((1/32 * e^(-1)))*4 – ((1/192 * e^(-3)))*4 – ((1/32 * e^(-3)))*4 + ((1/32*e^(-1)))*4 – ((1/192 * e))*4)))))))

Input:

$$\frac{1}{\sqrt{\frac{1}{2}\times\frac{1}{\frac{\frac{1}{64}}{e^7}\times 4-\frac{\frac{1}{32}}{e^3}\times 4+\left(\frac{1}{64}e\right)\times 4+\frac{\frac{1}{32}}{e}\times 4-\frac{\frac{1}{192}}{e^3}\times 4-\frac{\frac{1}{32}}{e^3}\times 4+\frac{\frac{1}{32}}{e}\times 4-\left(\frac{1}{192}e\right)\times 4}}}$$

Exact result:

$$\sqrt{2\left(\frac{1}{16 e^7}-\frac{13}{48 e^3}+\frac{1}{4 e}+\frac{e}{24}\right)}$$

Decimal approximation:

0.619361926639351645218721354141111885476807764049412329292...

0.619361926639935…. result very near to the conjugate of the golden ratio

Property:

$\sqrt{2\left(\frac{1}{16 e^7}-\frac{13}{48 e^3}+\frac{1}{4 e}+\frac{e}{24}\right)}$ is a transcendental number

Alternate form:

$$\frac{\sqrt{\frac{1}{6}\left(3-13 e^4+12 e^6+2 e^8\right)}}{2 e^{7/2}}$$

Series representations:

$$\frac{1}{\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e 4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e 4}{192}\right)^2}}} = \frac{1}{\sqrt{-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}} \sum_{k=0}^{\infty} \left(-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}\right)^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{1}{\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e 4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e 4}{192}\right)^2}}} = \frac{1}{\sqrt{-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-1+\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}\right)^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

$$\frac{1}{\sqrt{\frac{1}{\left(\frac{4}{64 e^7}-\frac{4}{32 e^3}+\frac{e 4}{64}+\frac{4}{32 e}-\frac{4}{192 e^3}-\frac{4}{32 e^3}+\frac{4}{32 e}-\frac{e 4}{192}\right)^2}}} = \frac{1}{\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{24 e^7}{3-13 e^4+12 e^6+2 e^8}-z_0\right)^k z_0^{-k}}{k!}} \quad \text{for not } ((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

We have that:

$$\{\phi_1, \phi_2, \phi_3\} = \left\{\frac{1}{24}\left(-73+\sqrt{6481}\right) h_1 - h_2 + h_3, h_2 + h_3, -\frac{1}{24}\left(73+\sqrt{6481}\right) h_1 - h_2 + h_3\right\} \tag{7.32}$$

We note that:

(((1/24(-73+sqrt6484)1-1+1,1+1,-1/24(73+sqrt6481)1-1+1)))

Input:

$$\left\{\frac{1}{24}\left(-73+\sqrt{6484}\right)\times 1-1+1,\ 1+1,\ -\frac{1}{24}\left(73+\sqrt{6481}\right)\times 1-1+1\right\}$$

Result:

$$\left\{\frac{1}{24}\left(2\sqrt{1621}-73\right),\ 2,\ \frac{1}{24}\left(-73-\sqrt{6481}\right)\right\}$$

Alternate forms:

$$\left\{\frac{1}{24}\left(2\sqrt{1621}-73\right), 2, -\frac{1}{24}\left(73+\sqrt{6481}\right)\right\}$$

$$\left\{\frac{\sqrt{1621}}{12}-\frac{73}{24}, 2, -\frac{73}{24}-\frac{\sqrt{6481}}{24}\right\}$$

Decimal approximation:

$\{0.31347, 2, -6.39603\}$

Total:

$$\frac{1}{24}\left(-73+2\sqrt{1621}\right)+2+\frac{1}{24}\left(-73-\sqrt{6481}\right)\approx -4.08256$$

Vector length:

$$\sqrt{4+\frac{1}{576}\left(2\sqrt{1621}-73\right)^2+\frac{1}{576}\left(73+\sqrt{6481}\right)^2}\approx 6.70876$$

Normalized vector:

$$\left(\frac{-73+2\sqrt{1621}}{24\sqrt{4+\frac{1}{576}\left(-73+2\sqrt{1621}\right)^2+\frac{1}{576}\left(73+\sqrt{6481}\right)^2}}, \frac{2}{\sqrt{4+\frac{1}{576}\left(-73+2\sqrt{1621}\right)^2+\frac{1}{576}\left(73+\sqrt{6481}\right)^2}}, \frac{-73-\sqrt{6481}}{24\sqrt{4+\frac{1}{576}\left(-73+2\sqrt{1621}\right)^2+\frac{1}{576}\left(73+\sqrt{6481}\right)^2}}\right)$$

Spherical coordinates (radial, polar, azimuthal):

$r\approx 6.70876, \quad \theta\approx 17.5632^\circ, \quad \phi\approx 81.0922^\circ$

6.70876

(1729+812-138+9)/10^4(((1/24(-73+sqrt6484)1-1+1,1+1,-1/24(73+sqrt6481)1-1+1)))

Input:

$$\frac{(1729+812-138+9)\left(\frac{1}{24}\left(-73+\sqrt{6484}\right)-1+1,\ 1+1,\ -\frac{1}{24}\left(73+\sqrt{6481}\right)-1+1\right)}{10^4}$$

Result:

$$\left\{\frac{201\left(2\sqrt{1621}-73\right)}{20000}, \frac{603}{1250}, \frac{201\left(-73-\sqrt{6481}\right)}{20000}\right\}$$

Alternate forms:

$$\left\{\frac{201\left(2\sqrt{1621}-73\right)}{20000}, \frac{603}{1250}, -\frac{201\left(73+\sqrt{6481}\right)}{20000}\right\}$$

$$\left\{\frac{201\sqrt{1621}}{10000}-\frac{14673}{20000}, \frac{603}{1250}, -\frac{14673}{20000}-\frac{201\sqrt{6481}}{20000}\right\}$$

$$\left\{\frac{402\sqrt{1621}-14673}{20000}, \frac{603}{1250}, \frac{-14673-201\sqrt{6481}}{20000}\right\}$$

Decimal approximation:

{0.075609, 0.4824, −1.54272}

Total:

$$\frac{201\left(-73+2\sqrt{1621}\right)}{20000}+\frac{603}{1250}+\frac{201\left(-73-\sqrt{6481}\right)}{20000} \approx -0.984713$$

Vector length:

$$\sqrt{\frac{363609}{1562500}+\frac{40401\left(2\sqrt{1621}-73\right)^2}{400000000}+\frac{40401\left(73+\sqrt{6481}\right)^2}{400000000}} \approx 1.61815$$

Normalized vector:

$$\left(\frac{201\left(-73+2\sqrt{1621}\right)}{20000\sqrt{\frac{363609}{1562500}+\frac{40401\left(-73+2\sqrt{1621}\right)^2}{400000000}+\frac{40401\left(73+\sqrt{6481}\right)^2}{400000000}}},\right.$$

$$\frac{603}{1250\sqrt{\frac{363609}{1562500}+\frac{40401\left(-73+2\sqrt{1621}\right)^2}{400000000}+\frac{40401\left(73+\sqrt{6481}\right)^2}{400000000}}},$$

$$\left.\frac{201\left(-73-\sqrt{6481}\right)}{20000\sqrt{\frac{363609}{1562500}+\frac{40401\left(-73+2\sqrt{1621}\right)^2}{400000000}+\frac{40401\left(73+\sqrt{6481}\right)^2}{400000000}}}\right)$$

Spherical coordinates (radial, polar, azimuthal):

$r \approx 1.61815$, $\theta \approx 17.5632°$, $\phi \approx 81.0922°$

1.61815 result that is a very good approximation to the value of the golden ratio 1,618033988749...

$9^3+10^3=12^3+1$ $\quad 9=\sqrt[3]{12^3+1-10^3}$; $9^3+10^3=1729$

$791^3+812^3=1010^3-1$ $\quad 812=\sqrt[3]{1010^3-1-791^3}$

$135^3+138^3=172^3-1$ $\quad 138=\sqrt[3]{172^3-1-135^3}$

Now, we have:

$$\begin{aligned} V_{dil}(\vec{\omega}) \;=\; & \frac{5}{32}-\frac{1}{64}e^{-3\omega_1}-\frac{e^{3\omega_1}}{64}+\frac{1}{32}e^{-\sqrt{3}\omega_2}+\frac{1}{32}e^{\sqrt{3}\omega_2}+\frac{1}{64}e^{-3\omega_1-\sqrt{3}\omega_2} \\ & +\frac{1}{64}e^{3\omega_1-\sqrt{3}\omega_2}+\frac{1}{64}e^{\sqrt{3}\omega_2-3\omega_1}+\frac{1}{64}e^{3\omega_1+\sqrt{3}\omega_2} \end{aligned} \qquad (7.53)$$

for ω = √3:

5/32-1/64*e^(-3sqrt3)-(e^(3sqrt3))/64+1/32*e^(-3)+1/32*e^3+1/64*e^(-3sqrt3-3)+1/64*e^(3sqrt3-3)+1/64*e^(3-3sqrt3)+1/64*e^(3sqrt3+3)

Input:

$$\frac{5}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{\frac{1}{32}}{e^3} + \frac{1}{32} e^3 + \frac{1}{64} e^{-3\sqrt{3}-3} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}+3}$$

Exact result:

$$\frac{5}{32} + \frac{1}{32 e^3} + \frac{e^3}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{1}{64} e^{-3-3\sqrt{3}} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3+3\sqrt{3}}$$

Decimal approximation:

54.77748803289322565065208644102577002773938033997914335476...

54.7774880328…

Alternate forms:

$$\frac{1 + 5 e^3 + e^6 + (1 - e^3 + e^6)\cosh(3\sqrt{3})}{32 e^3}$$

$$\frac{1}{64} e^{-3-3\sqrt{3}} \left(1 - e^3 + e^6\right) + \frac{1}{64} e^{3\sqrt{3}-3} \left(1 - e^3 + e^6\right) + \frac{1 + 5 e^3 + e^6}{32 e^3}$$

$$\frac{2 + 10 e^3 + 2 e^6 - e^{3-3\sqrt{3}} + e^{6-3\sqrt{3}} - e^{3+3\sqrt{3}} + e^{6+3\sqrt{3}} + 2\cosh(3\sqrt{3})}{64 e^3}$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$\frac{5}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{1}{e^3\, 32} + \frac{e^3}{32} + \frac{1}{64} e^{-3\sqrt{3}-3} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}+3} = \frac{1}{64} e^{-3-3\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} \left(1 - e^3 + e^6 + 2 e^{3\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} + e^{6\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} + 10 e^{3+3\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} + 2 e^{6+3\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} - e^{3+6\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}} + e^{6+6\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{1/2}{k}}\right)$$

$$\frac{5}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{1}{e^3\, 32} + \frac{e^3}{32} + \frac{1}{64} e^{-3\sqrt{3}-3} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}+3} = \frac{1}{64} \exp\left(-3 - 3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) \left(1 - e^3 + e^6 + 2 e^{3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + e^{6\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + 10 \exp\left(3 + 3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + 2 \exp\left(6 + 3\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) - \exp\left(3 + 6\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(6 + 6\sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{5}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{1}{e^3\, 32} + \frac{e^3}{32} + \frac{1}{64} e^{-3\sqrt{3}-3} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}+3} = \frac{1}{64} \exp\left(-3 - 3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) \left(1 - e^3 + e^6 + 2 \exp\left(3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) + \exp\left(6\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) + 10 \exp\left(3 + 3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) + 2 \exp\left(6 + 3\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) - \exp\left(3 + 6\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right) + \exp\left(6 + 6\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3 - z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

1/34*((((5/32-1/64*e^(-3sqrt3)-(e^(3sqrt3))/64+1/32*e^(-3)+1/32*e^3+1/64*e^(-3sqrt3-3)+1/64*e^(3sqrt3-3)+1/64*e^(3-3sqrt3)+1/64*e^(3sqrt3+3)))))

Input:

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{\frac{1}{32}}{e^3}+\frac{1}{32}e^3+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)$$

Exact result:

$$\frac{1}{34}\left(\frac{5}{32}+\frac{1}{32e^3}+\frac{e^3}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{64}e^{-3-3\sqrt{3}}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3+3\sqrt{3}}\right)$$

Decimal approximation:

1.611102589202741930901531954147816765521746480587621863375...

1.611102589… result near to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{1+5e^3+e^6+(1-e^3+e^6)\cosh(3\sqrt{3})}{1088e^3}$$

$$\frac{2+10e^3+2e^6-e^{3-3\sqrt{3}}+e^{6-3\sqrt{3}}-e^{3+3\sqrt{3}}+e^{6+3\sqrt{3}}+2\cosh(3\sqrt{3})}{2176e^3}$$

$$\frac{e^{-3-3\sqrt{3}}\left(1-e^3+e^6+2e^{3\sqrt{3}}+e^{6\sqrt{3}}+10e^{3+3\sqrt{3}}+2e^{6+3\sqrt{3}}-e^{3+6\sqrt{3}}+e^{6+6\sqrt{3}}\right)}{2176}$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)=\frac{1}{2176}e^{-3-3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}\left(1-e^3+e^6+2e^{3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+e^{6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+10e^{3+3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+2e^{6+3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}-e^{3+6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+e^{6+6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}\right)$$

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)=\frac{1}{2176}\exp\left(-3-3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\left(1-e^3+e^6+2e^{3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+e^{6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+10\exp\left(3+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+2\exp\left(6+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)-\exp\left(3+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\exp\left(6+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)=$$

$$\frac{1}{2176}\exp\left(-3-3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\left(1-e^3+e^6+2\exp\left(3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+\exp\left(6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+10\exp\left(3+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+2\exp\left(6+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)-\exp\left(3+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+\exp\left(6+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$

1/[1/34*((((5/32-1/64*e^(-3sqrt3)-(e^(3sqrt3))/64+1/32*e^(-3)+1/32*e^3+1/64*e^(-3sqrt3-3)+1/64*e^(3sqrt3-3)+1/64*e^(3-3sqrt3)+1/64*e^(3sqrt3+3)))))]

Input:

$$1\bigg/\left(\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{\frac{1}{32}}{e^3}+\frac{1}{32}e^3+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)\right)$$

Exact result:

$$34\bigg/\left(\frac{5}{32}+\frac{1}{32e^3}+\frac{e^3}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{64}e^{-3-3\sqrt{3}}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3+3\sqrt{3}}\right)$$

Decimal approximation:

0.6206929383031110722096181912483405807579366453335192299 16...

0.620692938… result very near to the conjugate of the golden ratio

Alternate forms:

$$\frac{1088 e^3}{1+5 e^3+e^6+(1-e^3+e^6)\cosh(3\sqrt{3})}$$

$$\frac{2176 e^3}{2+10 e^3+2 e^6-e^{3-3\sqrt{3}}+e^{6-3\sqrt{3}}-e^{3+3\sqrt{3}}+e^{6+3\sqrt{3}}+2\cosh(3\sqrt{3})}$$

$$\frac{2176 e^{3+3\sqrt{3}}}{1-e^3+e^6+2 e^{3\sqrt{3}}+e^{6\sqrt{3}}+10 e^{3+3\sqrt{3}}+2 e^{6+3\sqrt{3}}-e^{3+6\sqrt{3}}+e^{6+6\sqrt{3}}}$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$1\Big/\left(\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32 e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)\right)=\left(2176 e^{3+3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}\right)\Big/\left(1-e^3+e^6+2 e^{3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+e^{6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+10 e^{3+3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+2 e^{6+3\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}-e^{3+6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}+e^{6+6\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{1/2}{k}}\right)$$

$$1\Big/\left(\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32 e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)\right)=\left(2176\exp\left(3+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\Big/\left(1-e^3+e^6+2 e^{3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+e^{6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+10\exp\left(3+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+2\exp\left(6+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)-\exp\left(3+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+\exp\left(6+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$1/\left(\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)\right)=$$

$$\left(2176\exp\left(3+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)\Bigg/$$
$$\left(1-e^3+e^6+2\exp\left(3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+\right.$$
$$\exp\left(6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$
$$10\exp\left(3+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$
$$2\exp\left(6+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)-$$
$$\exp\left(3+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$
$$\left.\exp\left(6+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0))$

We have also:

1/34*((((5/32-1/64*e^(-3sqrt3)-(e^(3sqrt3))/64+1/32*e^(-3)+1/32*e^3+1/64*e^(-3sqrt3-3)+1/64*e^(3sqrt3-3)+1/64*e^(3-3sqrt3)+1/64*e^(3sqrt3+3)))))+7/10^3

Input:

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{\frac{1}{32}}{e^3}+\frac{1}{32}e^3+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)+\frac{7}{10^3}$$

Exact result:

$$\frac{7}{1000}+\frac{1}{34}\left(\frac{5}{32}+\frac{1}{32e^3}+\frac{e^3}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{64}e^{-3-3\sqrt{3}}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3+3\sqrt{3}}\right)$$

Decimal approximation:

1.618102589202741930901531954147816765521746480587621863375...

1.6181025892… result that is a very good approximation to the value of the golden ratio 1,618033988749...

Alternate forms:

$$\frac{1577 + 250 \cosh(3) + 125 (2 \cosh(3) - 1) \cosh(3 \sqrt{3})}{136000}$$

$$\frac{e^{-3-3\sqrt{3}} (1 - e^3 + e^6)}{2176} + \frac{e^{3\sqrt{3}-3} (1 - e^3 + e^6)}{2176} + \frac{125 + 1577 e^3 + 125 e^6}{136000 e^3}$$

$$\frac{1}{272000 e^3} \left(250 + 3154 e^3 + 250 e^6 + 125 e^{-3\sqrt{3}} + 125 e^{3\sqrt{3}} - 125 e^{3-3\sqrt{3}} + 125 e^{6-3\sqrt{3}} - 125 e^{3+3\sqrt{3}} + 125 e^{6+3\sqrt{3}}\right)$$

cosh(x) is the hyperbolic cosine function

Series representations:

$$\frac{1}{34} \left(\frac{5}{32} - \frac{1}{64} e^{-3\sqrt{3}} - \frac{e^{3\sqrt{3}}}{64} + \frac{1}{32 e^3} + \frac{e^3}{32} + \frac{1}{64} e^{-3\sqrt{3}-3} + \frac{1}{64} e^{3\sqrt{3}-3} + \frac{1}{64} e^{3-3\sqrt{3}} + \frac{1}{64} e^{3\sqrt{3}+3}\right) + \frac{7}{10^3} =$$

$$\frac{1}{272000} e^{-3-3\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} \left(125 - 125 e^3 + 125 e^6 + 250 e^{3\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 125 e^{6\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 3154 e^{3+3\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 250 e^{6+3\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} - 125 e^{3+6\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}} + 125 e^{6+6\sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{1/2}{k}}\right)$$

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)+$$

$$\frac{7}{10^3}=\frac{1}{272\,000}\exp\left(-3-3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\left(125-125e^3+125e^6+250e^{3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+125e^{6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}+\right.$$

$$3154\exp\left(3+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+250\exp\left(6+3\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)-$$

$$\left.125\exp\left(3+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)+125\exp\left(6+6\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\right)$$

$$\frac{1}{34}\left(\frac{5}{32}-\frac{1}{64}e^{-3\sqrt{3}}-\frac{e^{3\sqrt{3}}}{64}+\frac{1}{32e^3}+\frac{e^3}{32}+\frac{1}{64}e^{-3\sqrt{3}-3}+\frac{1}{64}e^{3\sqrt{3}-3}+\frac{1}{64}e^{3-3\sqrt{3}}+\frac{1}{64}e^{3\sqrt{3}+3}\right)+\frac{7}{10^3}=$$

$$\frac{1}{272\,000}\exp\left(-3-3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)$$

$$\left(125-125e^3+125e^6+250\exp\left(3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+\right.$$

$$125\exp\left(6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$

$$3154\exp\left(3+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$

$$250\exp\left(6+3\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)-$$

$$125\exp\left(3+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)+$$

$$\left.125\exp\left(6+6\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for not $((z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0))$

from which, we obtain:

((x+42) + 250 cosh(3) + 125 (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000 = 1.61810258920274

Input interpretation:

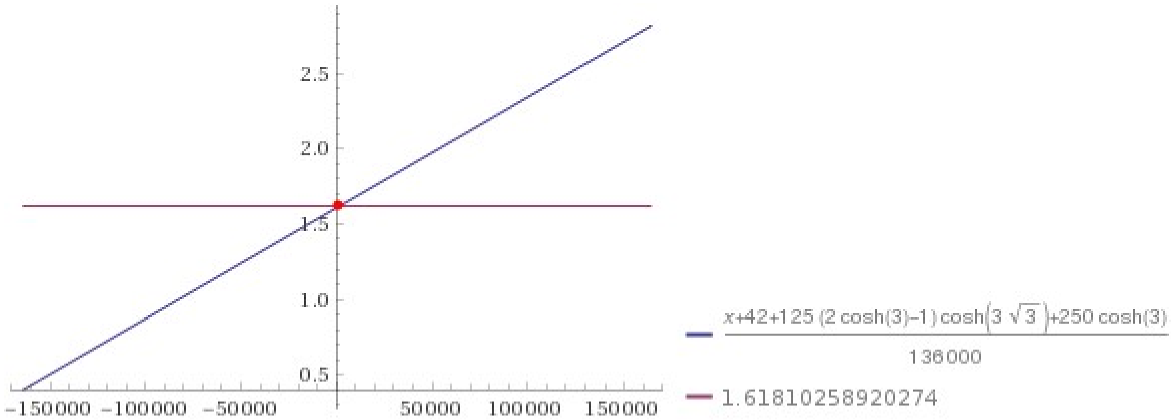
$$\frac{(x+42)+250\cosh(3)+125(-1+2\cosh(3))\cosh(3\sqrt{3})}{136\,000} = 1.61810258920274$$

cosh(x) is the hyperbolic cosine function

Result:

$$\frac{x+42+125(2\cosh(3)-1)\cosh(3\sqrt{3})+250\cosh(3)}{136\,000} = 1.61810258920274$$

Plot:



Alternate forms:

$$\frac{x}{136\,000} - 0.01128676470588 = 0$$

$$\frac{x+42+250\cosh(3)\cosh(3\sqrt{3})-125\cosh(3\sqrt{3})+250\cosh(3)}{136\,000} = 1.61810258920274$$

$$\frac{x+\frac{125}{2}\left(-1+\frac{1}{e^3}+e^3\right)\left(e^{-3\sqrt{3}}+e^{3\sqrt{3}}\right)+125\left(\frac{1}{e^3}+e^3\right)+42}{136\,000} = 1.61810258920274$$

Expanded form:

$$\frac{x}{136\,000}+\frac{21}{68\,000}+\frac{1}{544}\cosh(3)\cosh\left(3\sqrt{3}\right)-\frac{\cosh(3\sqrt{3})}{1088}+\frac{\cosh(3)}{544} = 1.61810258920274$$

Alternate form assuming x>0:

$$\frac{x + 42 + 250 \cosh(3) (1 + \cosh(3 \sqrt{3})) - 125 \cosh(3 \sqrt{3})}{136\,000} = 1.61810258920274$$

Solution:

$x \approx 1535.000000000$

1535 result equal to the rest mass of Xi baryon

(1577 + 250 cosh(3) + x (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000 = 1.61810258920274

Input interpretation:

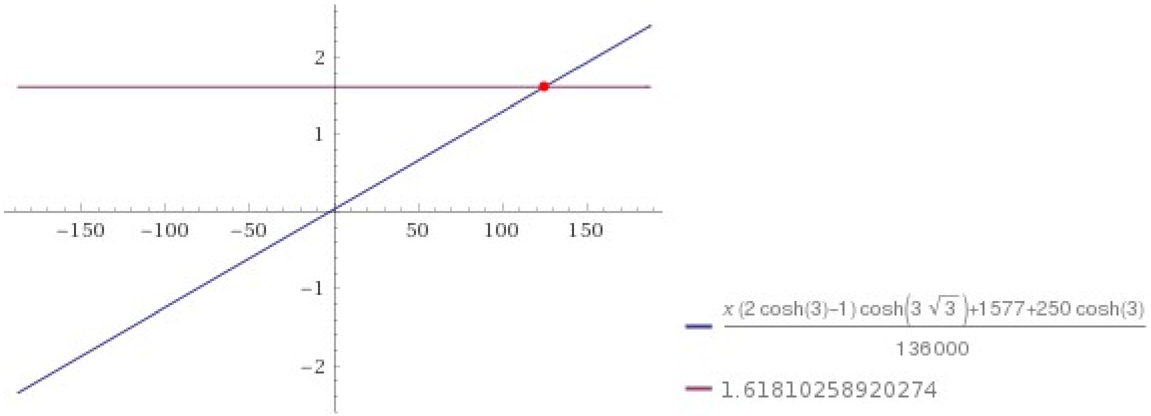
$$\frac{1577 + 250 \cosh(3) + x (-1 + 2 \cosh(3)) \cosh(3 \sqrt{3})}{136\,000} = 1.61810258920274$$

cosh(x) is the hyperbolic cosine function

Result:

$$\frac{x (2 \cosh(3) - 1) \cosh(3 \sqrt{3}) + 1577 + 250 \cosh(3)}{136\,000} = 1.61810258920274$$

Plot:



Alternate forms:

$$\frac{x \cosh(3) \cosh(3 \sqrt{3})}{68\,000} - \frac{x \cosh(3 \sqrt{3})}{136\,000} - 1.58800026935756 = 0$$

$$\frac{2 x \cosh(3) \cosh(3 \sqrt{3}) - x \cosh(3 \sqrt{3}) + 1577 + 250 \cosh(3)}{136\,000} = 1.61810258920274$$

$$\frac{\frac{1}{2}\left(-1 + \frac{1}{e^3} + e^3\right)\left(e^{-3\sqrt{3}} + e^{3\sqrt{3}}\right) x + 125 \left(\frac{1}{e^3} + e^3\right) + 1577}{136\,000} = 1.61810258920274$$

Expanded form:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68\,000} - \frac{x\cosh(3\sqrt{3})}{136\,000} + \frac{1577}{136\,000} + \frac{\cosh(3)}{544} = 1.61810258920274$$

Solution:

$x \approx 125.000000000000$

125 result practically equal to the Higgs boson mass 125.18 GeV

(1577 + 250 cosh(3) + (x-13) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000 = 1.61810258920274

Input interpretation:

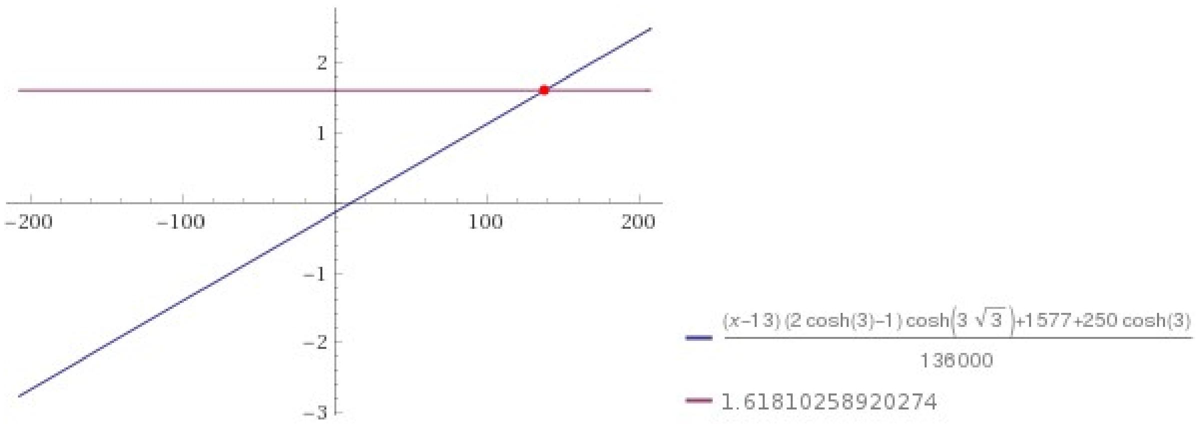
$$\frac{1577 + 250\cosh(3) + (x-13)(-1+2\cosh(3))\cosh(3\sqrt{3})}{136\,000} = 1.61810258920274$$

cosh(x) is the hyperbolic cosine function

Result:

$$\frac{(x-13)(2\cosh(3)-1)\cosh(3\sqrt{3}) + 1577 + 250\cosh(3)}{136\,000} = 1.61810258920274$$

Plot:



Alternate forms:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68\,000} - \frac{x\cosh(3\sqrt{3})}{136\,000} - 1.75315229737075 = 0$$

$$\frac{\frac{1}{2}\left(-1+\frac{1}{e^3}+e^3\right)\left(e^{-3\sqrt{3}}+e^{3\sqrt{3}}\right)(x-13) + 125\left(\frac{1}{e^3}+e^3\right) + 1577}{136\,000} = 1.61810258920274$$

$$\frac{1}{136\,000}\left(2\,x\cosh(3)\cosh\left(3\sqrt{3}\right)-x\cosh\left(3\sqrt{3}\right)+1577-26\cosh(3)\cosh\left(3\sqrt{3}\right)+13\cosh\left(3\sqrt{3}\right)+250\cosh(3)\right)=1.61810258920274$$

Expanded form:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68\,000}-\frac{x\cosh(3\sqrt{3})}{136\,000}+\frac{1577}{136\,000}-\frac{13\cosh(3)\cosh(3\sqrt{3})}{68\,000}+\frac{13\cosh(3\sqrt{3})}{136\,000}+\frac{\cosh(3)}{544}=1.61810258920274$$

Solution:

$x\approx 138.000000000000$

138 (Ramanujan taxicab number)

(1577 + 250 cosh(3) + (x-7-3) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000 = 1.61810258920274

Input interpretation:

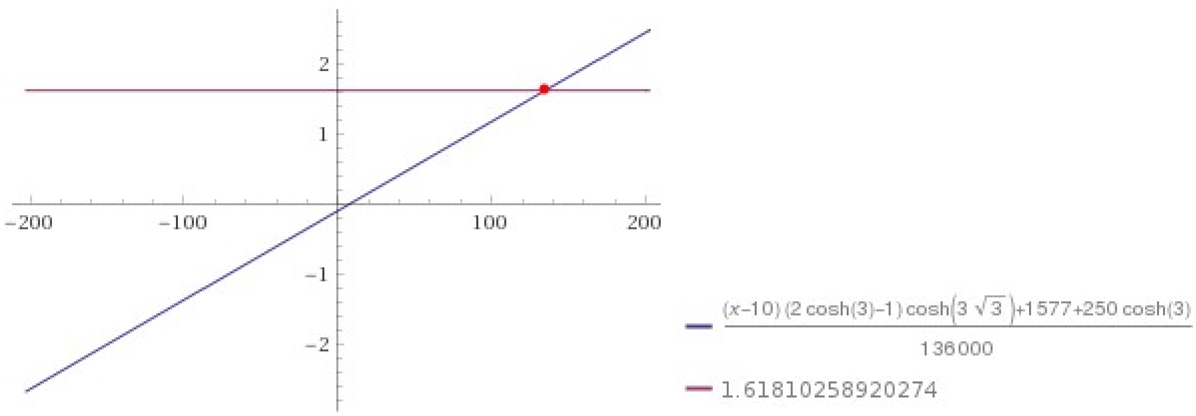
$$\frac{1577+250\cosh(3)+(x-7-3)(-1+2\cosh(3))\cosh(3\sqrt{3})}{136\,000}=1.61810258920274$$

cosh(x) is the hyperbolic cosine function

Result:

$$\frac{(x-10)(2\cosh(3)-1)\cosh(3\sqrt{3})+1577+250\cosh(3)}{136\,000}=1.61810258920274$$

Plot:



Alternate forms:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68000} - \frac{x\cosh(3\sqrt{3})}{136000} - 1.71504029090617 = 0$$

$$\frac{\frac{1}{2}\left(-1+\frac{1}{e^3}+e^3\right)\left(e^{-3\sqrt{3}}+e^{3\sqrt{3}}\right)(x-10)+125\left(\frac{1}{e^3}+e^3\right)+1577}{136000} = 1.61810258920274$$

$$\frac{1}{136000}\left(2x\cosh(3)\cosh\left(3\sqrt{3}\right)-x\cosh\left(3\sqrt{3}\right)+1577-20\cosh(3)\cosh\left(3\sqrt{3}\right)+10\cosh\left(3\sqrt{3}\right)+250\cosh(3)\right) = 1.61810258920274$$

Expanded form:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68000} - \frac{x\cosh(3\sqrt{3})}{136000} + \frac{1577}{136000} - \frac{\cosh(3)\cosh(3\sqrt{3})}{6800} + \frac{\cosh(3\sqrt{3})}{13600} + \frac{\cosh(3)}{544} = 1.61810258920274$$

Solution:

$x \approx 135.000000000000$

135 (Ramanujan taxicab number)

(1577 + 250 cosh(3) + (x-47) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000 = 1.61810258920274

Input interpretation:

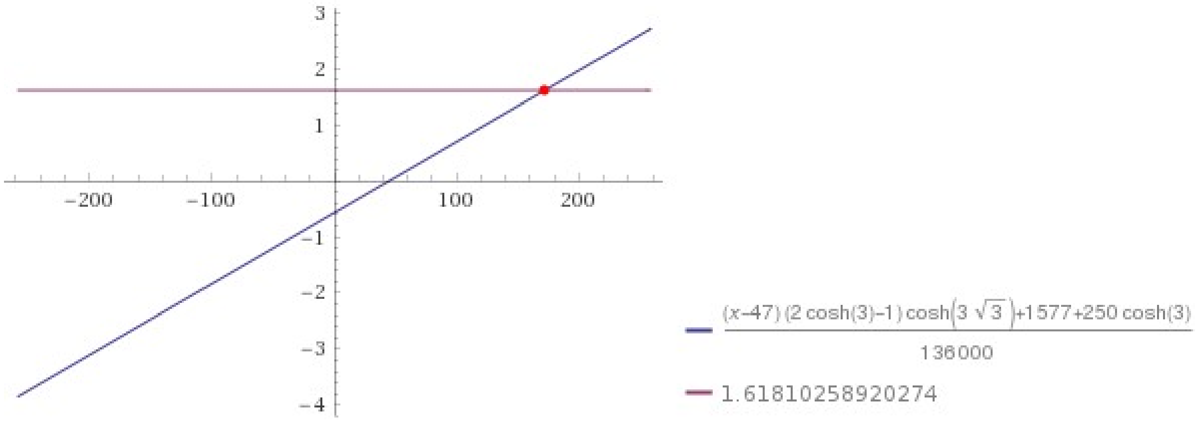
$$\frac{1577+250\cosh(3)+(x-47)(-1+2\cosh(3))\cosh(3\sqrt{3})}{136000} = 1.61810258920274$$

cosh(x) is the hyperbolic cosine function

Result:

$$\frac{(x-47)(2\cosh(3)-1)\cosh(3\sqrt{3})+1577+250\cosh(3)}{136000} = 1.61810258920274$$

Plot:



Alternate forms:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68000}-\frac{x\cosh(3\sqrt{3})}{136000}-2.18508837063600=0$$

$$\frac{\frac{1}{2}\left(-1+\frac{1}{e^3}+e^3\right)\left(e^{-3\sqrt{3}}+e^{3\sqrt{3}}\right)(x-47)+125\left(\frac{1}{e^3}+e^3\right)+1577}{136000}=1.61810258920274$$

$$\frac{1}{136000}\left(2x\cosh(3)\cosh\left(3\sqrt{3}\right)-x\cosh\left(3\sqrt{3}\right)+1577-94\cosh(3)\cosh\left(3\sqrt{3}\right)+47\cosh\left(3\sqrt{3}\right)+250\cosh(3)\right)=1.61810258920274$$

Expanded form:

$$\frac{x\cosh(3)\cosh(3\sqrt{3})}{68000}-\frac{x\cosh(3\sqrt{3})}{136000}+\frac{1577}{136000}-\frac{47\cosh(3)\cosh(3\sqrt{3})}{68000}+\frac{47\cosh(3\sqrt{3})}{136000}+\frac{\cosh(3)}{544}=1.61810258920274$$

Solution:

$x \approx 172.000000000000$

172 (Ramanujan taxicab number)

Where 138, 135 and 172 are Ramanujan's taxicab numbers

$$135^3+138^3=172^3-1$$

Indeed, we have:

(((1577 + 250 cosh(3) + (x-13) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3 + (((1577 + 250 cosh(3) + (x-7-3) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3 = 172^3-1

Input:

$$\left(\frac{1577 + 250\cosh(3) + (x-13)(-1+2\cosh(3))\cosh(3\sqrt{3})}{136\,000}\right)^3 + \left(\frac{1577 + 250\cosh(3) + (x-7-3)(-1+2\cosh(3))\cosh(3\sqrt{3})}{136\,000}\right)^3 = 172^3 - 1$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\frac{((x-13)(2\cosh(3)-1)\cosh(3\sqrt{3}) + 1577 + 250\cosh(3))^3}{2\,515\,456\,000\,000\,000} + \frac{((x-10)(2\cosh(3)-1)\cosh(3\sqrt{3}) + 1577 + 250\cosh(3))^3}{2\,515\,456\,000\,000\,000} = 5\,088\,447$$

Real solution:

$x =$

$$\text{root of } x^3 (-2 \cosh^3(3\sqrt{3}) + 12 \cosh(3) \cosh^3(3\sqrt{3}) + 16 \cosh^3(3) \cosh^3(3\sqrt{3}) - 24 \cosh^2(3) \cosh^3(3\sqrt{3})) + x^2 (69 \cosh^3(3\sqrt{3}) - 414 \cosh(3) \cosh^3(3\sqrt{3}) - 552 \cosh^3(3) \cosh^3(3\sqrt{3}) + 9462 \cosh^2(3\sqrt{3}) - 36\,348 \cosh(3) \cosh^2(3\sqrt{3}) + 31\,848 \cosh^2(3) \cosh^2(3\sqrt{3}) + 6000 \cosh^3(3) \cosh^2(3\sqrt{3}) + 828 \cosh^2(3) \cosh^3(3\sqrt{3})) + x (750\,000 \cosh^3(3) \cosh(3\sqrt{3}) - 807 \cosh^3(3\sqrt{3}) + 4842 \cosh(3) \cosh^3(3\sqrt{3}) + 6456 \cosh^3(3) \cosh^3(3\sqrt{3}) + 9\,087\,000 \cosh^2(3) \cosh(3\sqrt{3}) - 217\,626 \cosh^2(3\sqrt{3}) + 836\,004 \cosh(3) \cosh^2(3\sqrt{3}) - 732\,504 \cosh^2(3) \cosh^2(3\sqrt{3}) - 138\,000 \cosh^3(3) \cosh^2(3\sqrt{3}) - 9684 \cosh^2(3) \cosh^3(3\sqrt{3}) - 14\,921\,574 \cosh(3\sqrt{3}) + 25\,112\,148 \cosh(3) \cosh(3\sqrt{3})) - 12\,799\,764\,536\,824\,156\,225\,934 - 25\,576 \cosh^3(3) \cosh^3(3\sqrt{3}) - 19\,182 \cosh(3) \cosh^3(3\sqrt{3}) + 3197 \cosh^3(3\sqrt{3}) - 8\,625\,000 \cosh^3(3) \cosh(3\sqrt{3}) + 31\,250\,000 \cosh^3(3) + 4\,283\,556 \cosh^2(3) \cosh^2(3\sqrt{3}) - 4\,888\,806 \cosh(3) \cosh^2(3\sqrt{3}) + 1\,272\,639 \cosh^2(3\sqrt{3}) - 104\,500\,500 \cosh^2(3) \cosh(3\sqrt{3}) + 591\,375\,000 \cosh^2(3) + 38\,364 \cosh^2(3) \cosh^3(3\sqrt{3}) + 807\,000 \cosh^3(3) \cosh^2(3\sqrt{3}) - 288\,789\,702 \cosh(3) \cosh(3\sqrt{3}) + 171\,598\,101 \cosh(3\sqrt{3}) + 3\,730\,393\,500 \cosh(3) \text{ near } x = 10\,755.1$$

Complex solutions:

$x \approx 10\,755.0731577168$

$x \approx -5363.84085090725 - 9306.25970458265\,i$

$x \approx -5363.84085090725 + 9306.25970458265\,i$

(((1577 + 250 cosh(3) + (10755.0731577168-13) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3 + (((1577 + 250 cosh(3) + (10755.0731577168-7-3) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3 = 172^3-1

Input interpretation:

$$\left(\frac{1577 + 250\cosh(3) + (10\,755.0731577168 - 13)(-1 + 2\cosh(3))\cosh\left(3\sqrt{3}\right)}{136\,000}\right)^3 + \left(\frac{1}{136\,000}\left(1577 + 250\cosh(3) + (10\,755.0731577168 - 7 - 3)(-1 + 2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3 = 172^3 - 1$$

cosh(x) is the hyperbolic cosine function

Result:

True

(((1577 + 250 cosh(3) + (10755.0731577168-13) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3

Input interpretation:

$$\left(\frac{1577 + 250\cosh(3) + (10\,755.0731577168 - 13)(-1 + 2\cosh(3))\cosh\left(3\sqrt{3}\right)}{136\,000}\right)^3$$

cosh(x) is the hyperbolic cosine function

Result:

$2.54315807414543... \times 10^6$

$2.54315807…*10^6$

(((1577 + 250 cosh(3) + (10755.0731577168-7-3) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3

Input interpretation:

$$\left(\frac{1577 + 250\cosh(3) + (10\,755.0731577168 - 7 - 3)(-1 + 2\cosh(3))\cosh\left(3\sqrt{3}\right)}{136\,000}\right)^3$$

cosh(x) is the hyperbolic cosine function

Result:

$2.54528892585461... \times 10^6$

$2.54528892...*10^6$

2.54528892585461 × 10^6 + 2.54315807414543 × 10^6

Input interpretation:

$2.54528892585461 \times 10^6 + 2.54315807414543 \times 10^6$

Result:

$5.08844700000004 \times 10^6$

$5.088447...*10^6$

Indeed:

(((1577 + 250 cosh(3) + (10755.0731577168-13) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3 + (((1577 + 250 cosh(3) + (10755.0731577168-7-3) (-1 + 2 cosh(3)) cosh(3 sqrt(3)))/136000))^3

Input interpretation:

$$\left(\frac{1577 + 250 \cosh(3) + (10\,755.0731577168 - 13)(-1 + 2\cosh(3))\cosh(3\sqrt{3})}{136\,000}\right)^3 + \left(\frac{1577 + 250 \cosh(3) + (10\,755.0731577168 - 7 - 3)(-1 + 2\cosh(3))\cosh(3\sqrt{3})}{136\,000}\right)^3$$

cosh(x) is the hyperbolic cosine function

Result:

$5.08844700000004\ldots \times 10^6$

$5.088447\ldots*10^6$

Alternative representations:

$$\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=\left(\frac{1}{136000}\left(1577+250\cos(-3i)+10742.07315771680000(-1+2\cos(-3i))\cos\left(-3i\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136000}\left(1577+250\cos(-3i)+10745.07315771680000(-1+2\cos(-3i))\cos\left(-3i\sqrt{3}\right)\right)\right)^3$$

$$\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=\left(\frac{1577+250\cos(3i)+10742.07315771680000(-1+2\cos(3i))\cos\left(3i\sqrt{3}\right)}{136000}\right)^3+\left(\frac{1577+250\cos(3i)+10745.07315771680000(-1+2\cos(3i))\cos\left(3i\sqrt{3}\right)}{136000}\right)^3$$

$$\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136000}\left(1577+250\cosh(3)+(10755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=\left(\frac{1577+\frac{250}{\sec(3i)}+\frac{10742.07315771680000\left(-1+\frac{2}{\sec(3i)}\right)}{\sec\left(3i\sqrt{3}\right)}}{136000}\right)^3+\left(\frac{1577+\frac{250}{\sec(3i)}+\frac{10745.07315771680000\left(-1+\frac{2}{\sec(3i)}\right)}{\sec\left(3i\sqrt{3}\right)}}{136000}\right)^3$$

Series representations:

$$\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=$$

$$\left(\left(1577+250\sum_{k=0}^{\infty}\frac{9^k}{(2k)!}+10\,742.07315771680000\left(-1+2\sum_{k=0}^{\infty}\frac{9^k}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{9^k\sqrt{3}^{2k}}{(2k)!}\right)^3+\left(1577+250\sum_{k=0}^{\infty}\frac{9^k}{(2k)!}+10\,745.07315771680000\left(-1+2\sum_{k=0}^{\infty}\frac{9^k}{(2k)!}\right)\sum_{k=0}^{\infty}\frac{9^k\sqrt{3}^{2k}}{(2k)!}\right)^3\right)\Big/2\,515\,456\,000\,000\,000$$

$$\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=$$

$$\left(1577+250\left(I_0(3)+2\sum_{k=1}^{\infty}I_{2k}(3)\right)+10\,742.07315771680000\left(-1+2\left(I_0(3)+2\sum_{k=1}^{\infty}I_{2k}(3)\right)\right)\left(I_0\left(3\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(3\sqrt{3}\right)\right)\right)^3\Big/2\,515\,456\,000\,000\,000+\left(1577+250\left(I_0(3)+2\sum_{k=1}^{\infty}I_{2k}(3)\right)+10\,745.07315771680000\left(-1+2\left(I_0(3)+2\sum_{k=1}^{\infty}I_{2k}(3)\right)\right)\left(I_0\left(3\sqrt{3}\right)+2\sum_{k=1}^{\infty}I_{2k}\left(3\sqrt{3}\right)\right)\right)^3\Big/2\,515\,456\,000\,000\,000$$

$$\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3+\left(\frac{1}{136\,000}\left(1577+250\cosh(3)+(10\,755.07315771680000-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)\right)\right)^3=$$

$$\left(\left(1577+250\,i\sum_{k=0}^{\infty}\frac{\left(3-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}+10\,742.07315771680000\,i\left(-1+2\,i\sum_{k=0}^{\infty}\frac{\left(3-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+3\sqrt{3}\right)^{1+2k}}{(1+2k)!}\right)^3+\left(1577+250\,i\sum_{k=0}^{\infty}\frac{\left(3-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}+10\,745.07315771680000\,i\left(-1+2\,i\sum_{k=0}^{\infty}\frac{\left(3-\frac{i\pi}{2}\right)^{1+2k}}{(1+2k)!}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{i\pi}{2}+3\sqrt{3}\right)^{1+2k}}{(1+2k)!}\right)^3\right)\Big/2\,515\,456\,000\,000\,000$$

172^3-1

Input:

172^3-1

Result:

5 088 447

5088447

In conclusion:

$$\left(\frac{1577+250\cosh(3)+(10\,755.0731577168-13)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)}{136\,000}\right)^3+\left(\frac{1577+250\cosh(3)+(10\,755.0731577168-7-3)(-1+2\cosh(3))\cosh\left(3\sqrt{3}\right)}{136\,000}\right)^3$$

$$=\ 5.08844700000004\ldots\times10^6$$

$$172^3-1=5\,088\,447$$

Conclusions

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

References

Integrable Scalar Cosmologies I. Foundations and links with String Theory
P. Fre , A. Sagnotti and A.S. Sorin - arXiv:1307.1910v3 [hep-th] 16 Oct 2013

Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials?
P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013

Integrable Scalar Cosmologies II. Can they fit into Gauged Extended Supergavity or be encoded in N=1 superpotentials? - P. Fre, A.S. Sorin and M. Trigiante - arXiv:1310.5340v1 [hep-th] 20 Oct 2013