

On some Ramanujan equations: mathematical connections with various formulas concerning some arguments of Cosmology and Black Holes/Wormholes Physics. IX

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Abstract

In this paper we have described several Ramanujan's formulas and obtained some mathematical connections with various equations concerning different sectors of Cosmology and Black Holes/Wormholes Physics.

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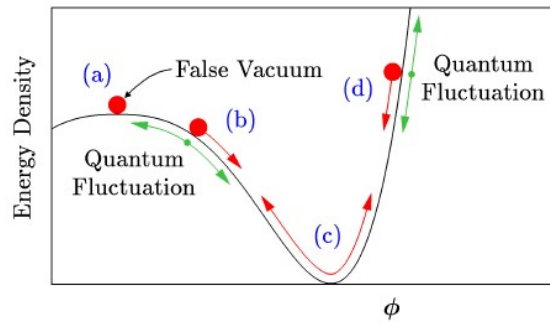
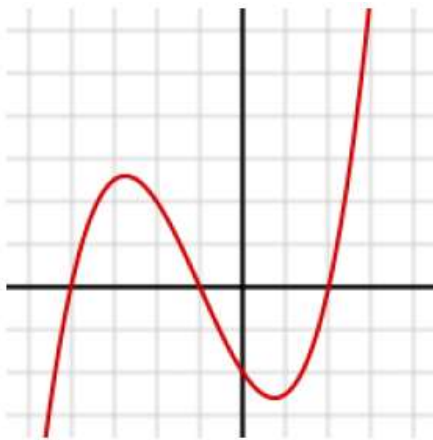
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"An equation for me has no meaning unless it expresses a thought of God."

~Srinivasa Ramanujan



<http://www.aicte-india.org/content/srinivasa-ramanujan>



From:

Traversable Wormholes with Exponential Shape Function in Modified Gravity and in General Relativity: A Comparative Study

Gauranga C. Samanta¹, Nisha Godani and Kazuharu Bamba - arXiv:1811.06834v2 [gr-qc] 23 Oct 2019

Tsujikawa [39] introduced a viable cosmological $f(R)$ model with the function $f(R)$ defined as

$$f(R) = R - \mu R_c \tanh \frac{R}{R_c}, \quad (31)$$

where μ and R_c are positive constants. The model is cosmologically viable and satisfies local gravity constraint. The cosmological viability of $f(R)$ models can be understood by quantities $m = R \frac{df_R(R)}{dR} / f_R(R)$ and $r = -R f_R(R) / f(R)$ by plotting corresponding curves in the (r, m) plane. It is considered that the cosmological evolution starts from radiation epoch with large and positive Ricci scalar R and approaches to de-Sitter attractor with $R = R_1$ in future. For m close to zero, the matter dominated point P_m lies on the line $m = -r - 1$. The viable saddle matter era exists if $m > 0$ and $-1 < \frac{dm}{dr} \leq 0$ at $r \approx -1$. If $f_R(R) > 0$ and $\frac{df_R(R)}{dR} > 0$ for $R \geq R_1$, then the model is stable and hence $m > 0$. The condition $-1 < \frac{dm}{dr} \leq 0$ is obtained, if $m(r)$ curves lies between $m = 0$ and $m = -r - 1$.

The effective equation of state is $w_{eff} = -1 - \frac{2\dot{H}}{3H^2}$. At point P_1 , $w_{eff} = -1$ which gives de-Sitter solution $\dot{H} = 0$. So this point is known as de-Sitter point. The de-Sitter point is stable, when $0 < m(r = -2) \leq 1$. If a $m(r)$ curve starting from P_m intersects with a line $r = -2$ for $0 < m \leq 1$, then $f(R)$ model is cosmologically viable.

Local gravity constraints on $f(R)$ model is given by $m(R_s) \ll \frac{1}{f_R(R)} (\frac{l}{R_s^{-1/2}})^2$, where $R_s \approx 8\pi G \rho_s$ is a curvature measured on the local structure, ρ_s is the energy density of the structure and l is the scale at which the gravity experiments are performed. Thus, in the region of high curvature, m should be a very small quantity.

For $R \gg R_c$, $f(R) \approx R - \mu R_c (1 - \exp(-\frac{2R}{R_c}))$. At the de-Sitter point,

$$\mu = \frac{x_1 \cosh^2(x_1)}{2 \sinh(x_1) \cosh(x_1) - x_1}, \quad (36)$$

where $x_1 = R_1/R_c$. For the stability of the model at de-Sitter point, $x_1 > 0.920$ and $\mu > 0.905$. For positive Ricci scalar R , $\frac{df_R(R)}{dR}$ is positive when $\mu > 0.905$. but $f_R(R)$ is positive only for $0.905 < \mu < 1$. Thus, for $0.905 < \mu < 1$, both $f_R(R)$ and $\frac{df_R(R)}{dR}$ are positive. Hence, the condition of stability is satisfied for this range of μ . For $\mu = 0.905$, the stability condition of de-Sitter point will not be satisfied.

Harko et al. [51] proposed an $f(R, T)$ model with the function $f(R, T) = R + 2\lambda T$, where λ is constant and T is the trace of energy-momentum tensor. In literature, various cosmological models are explored using this model [54, 56, 57, 59, 97].

In this section, the solutions of wormhole metric (1) and energy condition terms are derived and plotted with respect to above $f(R)$ and $f(R, T)$ models. The conclusions drawn are also analyzed for each set of plots.

A static and spherically symmetric wormhole structure is defined by the metric

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where the functions $b(r)$ and $e^{2\Phi(r)}$ are called as shape and redshift functions respectively. The radial coordinate r varies from r_0 to ∞ , where r_0 is called the radius of the throat. The angles θ and ϕ vary from 0 to π and 0 to 2π respectively. To avoid the presence of horizons and singularities, the redshift function should be finite and non-zero. The shape function should satisfy the following properties: (i) $\frac{b(r)}{r} < 1$ for $r > r_0$, (ii) $b(r_0) = r_0$ at $r = r_0$, (iii) $\frac{b(r)}{r} \rightarrow 0$ as $r \rightarrow \infty$, (iv) $\frac{b(r) - b'(r)r}{b(r)^2} > 0$ for $r > r_0$ and (v) $b'(r_0) \leq 1$. The condition (i) is necessary for the radial metric component to be negative. The shape function possesses minimum value equal to r_0 given by condition (ii). To obtain asymptotically flat space time as $r \rightarrow \infty$, the condition (iii) is required. Conditions (iv) and (v) are known as flaring out condition which are required to obtain traversable wormholes.

Schwarzschild metric

From Wikipedia

$$ds^2 = \left(1 - \frac{2Gm}{c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) - c^2 \left(1 - \frac{2Gm}{c^2 r}\right) dt^2$$

Note that:

$$\frac{2Gm}{c^2} = r_s$$

is the definition of the Schwarzschild radius for an object of mass m , so the Schwarzschild metric may be rewritten in the alternative form:

$$ds^2 = \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2) - c^2 \left(1 - \frac{r_s}{r}\right) dt^2$$

which shows that the metric becomes singular approaching the event horizon (that is, $r \rightarrow r_s$). The metric singularity is not a physical one (although there is a real physical singularity at $r = 0$), as can be shown by using a suitable coordinate transformation (e.g. the Kruskal–Szekeres coordinate system).

We place: $\mu = 0.9991104684$; $R_c = 5$; $r = -2$; $r_0 = -3$ and $b(r) =$ Schwarzschild radius

The wormhole solutions are

$$\begin{aligned}
\rho = & \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
& \times \left. \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 16\mu (r^2-2)^2 e^{2r_0} - 16\mu (r^2-2)^2 e^{r+r_0} \right) \right. \\
& \times \left. \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 6R_c^2 r^5 e^{2r+r_0} + 6R_c^2 r^4 e^{2r+r_0} - 2R_c^2 (r-1)r^4 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) \right. \\
& + 64R_c \mu r^2 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^2 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^5 e^{2r+r_0} \\
& \times \left. \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 16R_c \mu r^5 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^4 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \right. \\
& - 32R_c \mu r^4 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^3 e^{r+2r_0} \\
& \times \left. \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 256\mu r^4 e^{3r_0} - 256\mu r^4 e^{r+2r_0} - 1024\mu r^2 e^{3r_0} + 1024\mu r^2 e^{r+2r_0} \right. \\
& \left. - 1024\mu e^{r+2r_0} + 1024\mu e^{3r_0} \right] \tag{37}
\end{aligned}$$

$$\begin{aligned}
\rho = & \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
& \times \left. \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 16\mu (r^2-2)^2 e^{2r_0} - 16\mu (r^2-2)^2 e^{r+r_0} \right) \right.
\end{aligned}$$

Less the term $\frac{1}{16R_c^2 r^6}$, we obtain:

For: $\mu = 0.9991104684$; $R_c = 5$; $r = -2$; $r_0 = -3$

$$\begin{aligned}
& e^{(6)} \operatorname{sech}^4 \left(\frac{(2(-2-1)e^{(-1)})}{(5*(-2)^2)} \right) \left[(2*5^3*0.9991104684*e^{(-6)}*(-2)^6 \right. \\
& \sinh \left(\frac{(4(-2-1)e^{(-1)})}{(5*(-2)^2)} \right) \left. \right] + (5^3*0.9991104684*e^{(-6)}*(-2)^6) * \sinh \\
& \left(\frac{(8(-2-1)e^{(-1)})}{(5*(-2)^2)} \right) - 8*e^{(-3)} * \left((25*e^{(-4)} (-3-1)(-2)^4 + 16*0.9991104684*(4-2)^2*e^{(-6)} - 16*0.9991104684*4*e^{(-5)}) \right)
\end{aligned}$$

$$e^{(6)} \operatorname{sech}^4\left(\frac{(2(-2-1)*e^{(-1)})}{(5*(-2)^2)}\right) [(2*5^3*0.99911*e^{(-6)}*(-2)^6 \sinh\left(\frac{(4(-2-1)*e^{(-1)})}{(5*(-2)^2)}\right))]$$

Input:

$$e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{5(-2)^2}\right) \times \frac{2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{5(-2)^2}\right)}{e^6}$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

$\sinh(x)$ is the hyperbolic sine function

Result:

-3471.78...

-3471.78...

$$(5^3*0.99911*e^{(-6)}*(-2)^6) * \sinh\left(\frac{(8(-3)*e^{(-1)})}{(5*(-2)^2)}\right) - 8*e^{(-3)}*\left(\frac{(25*e^{(-4)}*(-2)^4 + 16*0.9991*4*e^{(-6)} - 16*0.9991*4*e^{(-5)})}{e^3}\right)$$

Input:

$$\frac{5^3 \times 0.99911 (-2)^6}{e^6} \sinh\left(\frac{8(-3)}{5(-2)^2}\right) - \frac{8 \left(\frac{25(-4(-2)^4)}{e^4} + \frac{16 \cdot 0.9991 \cdot 4}{e^6} - \frac{16 \cdot 0.9991 \cdot 4}{e^5} \right)}{e^3}$$

$\sinh(x)$ is the hyperbolic sine function

Result:

2.747422586626904859587937540748398602994826930158069946442...

2.7474225866...

$$e^{(6)} \operatorname{sech}^4\left(\frac{(2(-2-1)*e^{(-1)})}{(5*(-2)^2)}\right) [(2*5^3*0.99911*e^{(-6)}*(-2)^6 \sinh\left(\frac{(4(-2-1)*e^{(-1)})}{(5*(-2)^2)}\right))] + 2.7474225866269048595879$$

Input interpretation:

$$e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{5(-2)^2}\right) \times \frac{2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{5(-2)^2}\right)}{e^6} +$$

2.7474225866269048595879

$\operatorname{sech}(x)$ is the hyperbolic secant function

$\sinh(x)$ is the hyperbolic sine function

Result:

-3469.04...

-3469.04...

Alternative representations:

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \right) 2 \left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right)}{e^6} +$$

$$2.74742258662690485958790000 = 2.74742258662690485958790000 +$$

$$\frac{0.99911 (-2)^6 5^3 e^6 (e^{-12/(20e)} - e^{12/(20e)}) \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)} \right)^4}{e^6}$$

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \right) 2 \left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right)}{e^6} +$$

$$2.74742258662690485958790000 =$$

$$1.99822 i \cos\left(\frac{\pi}{2} + \frac{12i}{20e}\right) (-2)^6 5^3 e^6 \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)} \right)^4$$

$$2.74742258662690485958790000 - \frac{\phantom{1.99822 i \cos\left(\frac{\pi}{2} + \frac{12i}{20e}\right) (-2)^6 5^3 e^6 \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)} \right)^4}}{e^6}$$

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \right) 2 \left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right)}{e^6} +$$

$$2.74742258662690485958790000 = 2.74742258662690485958790000 +$$

$$\frac{0.99911 (-2)^6 5^3 e^6 (e^{-12/(20e)} - e^{12/(20e)}) \left(\frac{2e^{-6/(20e)}}{1+e^{-12/(20e)}} \right)^4}{e^6}$$

Series representations:

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \right) 2 \left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right)}{e^6} +$$

$$2.74742258662690485958790000 =$$

$$2.74742258662690485958790000 + 31\,971.5 \operatorname{sech}^4\left(-\frac{3}{10e}\right) \sum_{k=0}^{\infty} I_{1+2k}\left(-\frac{3}{5e}\right)$$

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)\right) 2\left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} +$$

$$2.74742258662690485958790000 =$$

$$511\,544. \left(5.37084 \times 10^{-6} + \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}\right)^4 \sum_{k=0}^{\infty} I_{1+2k}\left(-\frac{3}{5e}\right)\right) \text{ for } q = e^{-3/(10e)}$$

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)\right) 2\left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} +$$

$$2.74742258662690485958790000 = 255\,772.$$

$$\left(0.0000107417 + \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}\right)^4 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{3}\right)^{-1-2k} \left(-\frac{1}{e}\right)^{1+2k}}{(1+2k)!}\right) \text{ for } q = e^{-3/(10e)}$$

Integral representations:

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)\right) 2\left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} +$$

$$2.74742258662690485958790000 =$$

$$\frac{2.74742 \left(e \pi^4 - 55\,857.2 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^4 \int_0^1 \cosh\left(-\frac{3t}{5e}\right) dt\right)}{e \pi^4}$$

$$\frac{\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)\right) 2\left(5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} +$$

$$2.74742258662690485958790000 =$$

$$\frac{2.74742 \left(e i \pi^5 - 13\,964.3 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^4 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{9/(100e^2s)+s}}{s^{3/2}} ds\right) \sqrt{\pi}\right)}{e i \pi^5} \text{ for } \gamma > 0$$

$$\times \cosh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 6R_c^2 r^5 e^{2r+r_0} + 6R_c^2 r^4 e^{2r+r_0} - 2R_c^2 (r-1)r^4 e^{2r+r_0} \cosh\left(\frac{8(r-1)e^{r_0-r}}{R_c r^2}\right)$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$\cosh\left(\frac{4(-2-1)e^{-1}}{5(-2)^2}\right) - \left(\frac{6 \times 5^2 (-2)^5}{e^7} + \frac{6 \times 5^2 (-2)^4}{e^7} - \left(2 \times 5^2 \left(-\frac{3(-2)^4}{e^7}\right)\right)\right) \cosh\left(\frac{8(-2-1)}{5(-2)^2}\right) - \left(\frac{2 \times 5^2 (-3) (-2)^4 e^{-7}}{e^7}\right) * \cosh\left(\frac{8(-2-1)e^{-1}}{5(-2)^2}\right)$$

Input:

$$\cosh\left(\frac{4(-2-1)}{5(-2)^2}\right) - \frac{6 \times 5^2 (-2)^5}{e^7} + \frac{6 \times 5^2 (-2)^4}{e^7} - \left(2 \times 5^2 \left(-\frac{3(-2)^4}{e^7}\right)\right) \cosh\left(\frac{8(-2-1)}{5(-2)^2}\right)$$

cosh(x) is the hyperbolic cosine function

Exact result:

$$\frac{7200}{e^7} + \cosh\left(\frac{3}{5e}\right) + \frac{2400 \cosh\left(\frac{6}{5e}\right)}{e^7}$$

Decimal approximation:

9.995264267810911481773999157453520170969063323969109794779...

9.99526426781...

$$+ 64R_c\mu r^2 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 64R_c\mu r^2 e^{r+2r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 32R_c\mu r^5 e^{2r+r_0}$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$64 * ((5 * 0.9991104684 * (-2)^2 * e^{-7})) * \sinh\left(\frac{4(-2-1)e^{-1}}{5(-2)^2}\right) - 64 * ((5 * 0.9991104684 * (-2)^2 * e^{-8})) * \sinh\left(\frac{4(-2-1)e^{-1}}{5(-2)^2}\right) - 32 * ((5 * 0.9991104684 * (-2)^5 * e^{-7}))$$

Input interpretation:

$$64 \times \frac{5 \times 0.9991104684 (-2)^2}{e^7} \sinh\left(\frac{4(-2-1)}{5(-2)^2}\right) - 64 \times \frac{5 \times 0.9991104684 (-2)^2}{e^8} \sinh\left(\frac{4(-2-1)}{5(-2)^2}\right) - 32 \times \frac{5 \times 0.9991104684 (-2)^5}{e^7}$$

sinh(x) is the hyperbolic sine function

Result:

4.500646424...

4.500646424...

$$\times \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 16R_c\mu r^5 e^{r+2r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 32R_c\mu r^4 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right)$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$\begin{aligned} & \sinh\left(\frac{(4(-2-1)e^{-1})}{5(-2)^2}\right) + 16 * ((5 * 0.9991104684 * (-2)^5 * e^{-8})) * \\ & \sinh\left(\frac{(4(-2-1)e^{-1})}{5(-2)^2}\right) + 32 * ((5 * 0.9991104684 * (-2)^4 * e^{-7})) * \\ & \sinh\left(\frac{(4(-2-1)e^{-1})}{5(-2)^2}\right) \end{aligned}$$

Input interpretation:

$$\begin{aligned} & \sinh\left(\frac{e^{\frac{4(-2-1)}{5(-2)^2}}}{5(-2)^2}\right) + 16 \times \frac{5 \times 0.9991104684 (-2)^5}{e^8} \sinh\left(\frac{e^{\frac{4(-2-1)}{5(-2)^2}}}{5(-2)^2}\right) + \\ & 32 \times \frac{5 \times 0.9991104684 (-2)^4}{e^7} \sinh\left(\frac{e^{\frac{4(-2-1)}{5(-2)^2}}}{5(-2)^2}\right) \end{aligned}$$

$\sinh(x)$ is the hyperbolic sine function

Result:

-0.5505966964...

-0.5505966964...

$$- 32R_c\mu r^4 e^{r+2r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 64R_c\mu r^3 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 32R_c\mu r^3 e^{r+2r_0}$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$\begin{aligned} & -32 * ((5 * 0.9991104684 * (-2)^4 * e^{-8})) * \sinh\left(\frac{(4(-2-1)e^{-1})}{5(-2)^2}\right) + 64 * ((5 * 0.9991104684 * (-2)^3 * e^{-7})) * \\ & \sinh\left(\frac{(4(-2-1)e^{-1})}{5(-2)^2}\right) - 32 * ((5 * 0.9991104684 * (-2)^3 * e^{-8})) \end{aligned}$$

Input interpretation:

$$\begin{aligned} & -32 \times \frac{5 \times 0.9991104684 (-2)^4}{e^8} \sinh\left(\frac{e^{\frac{4(-2-1)}{5(-2)^2}}}{5(-2)^2}\right) + \\ & 64 \times \frac{5 \times 0.9991104684 (-2)^3}{e^7} \sinh\left(\frac{e^{\frac{4(-2-1)}{5(-2)^2}}}{5(-2)^2}\right) - 32 \times \frac{5 \times 0.9991104684 (-2)^3}{e^8} \end{aligned}$$

$\sinh(x)$ is the hyperbolic sine function

Result:

1.138943436...

1.138943436...

$$\times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 256\mu r^4 e^{3r_0} - 256\mu r^4 e^{r+2r_0} - 1024\mu r^2 e^{3r_0} + 1024\mu r^2 e^{r+2r_0} - 1024\mu e^{r+2r_0} + 1024\mu e^{3r_0} \Big]$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$\sinh(((4(-2-1)*e^{-1}))/((5*(-2)^2)))+256*((0.9991*(-2)^4*e^{-9}))-256*((0.9991*(-2)^4*e^{-8}))-1024*((0.9991*(-2)^2*e^{-9}))+1024*((0.9991*(-2)^2*e^{-8}))-1024*((0.9991*e^{-8}))+1024*((0.9991*e^{-9}))$$

Input:

$$\sinh\left(\frac{4(-2-1)}{5(-2)^2}\right) + 256 \times \frac{0.9991(-2)^4}{e^9} - 256 \times \frac{0.9991(-2)^4}{e^8} - 1024 \times \frac{0.9991(-2)^2}{e^9} + 1024 \times \frac{0.9991(-2)^2}{e^8} - 1024 \times \frac{0.9991}{e^8} + 1024 \times \frac{0.9991}{e^9}$$

sinh(x) is the hyperbolic sine function

Result:

-0.439471...

-0.43947320... (real precise result)

Thence, from the following equation

$$\begin{aligned}
\rho = & \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
& \times \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 16\mu (r^2-2)^2 e^{2r_0} - 16\mu (r^2-2)^2 e^{r+r_0} \right) \\
& \times \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 6R_c^2 r^5 e^{2r+r_0} + 6R_c^2 r^4 e^{2r+r_0} - 2R_c^2 (r-1)r^4 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) \\
& + 64R_c \mu r^2 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^2 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^5 e^{2r+r_0} \\
& \times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 16R_c \mu r^5 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^4 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
& - 32R_c \mu r^4 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^3 e^{r+2r_0} \\
& \times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 256\mu r^4 e^{3r_0} - 256\mu r^4 e^{r+2r_0} - 1024\mu r^2 e^{3r_0} + 1024\mu r^2 e^{r+2r_0} \\
& \left. - 1024\mu e^{r+2r_0} + 1024\mu e^{3r_0} \right] \tag{37}
\end{aligned}$$

we obtain:

$$\frac{1}{(16 \cdot 25 \cdot (-2)^6)} \cdot [(-3469.04) \cdot (9.99526426781) + (4.500646424) \cdot (-0.5505966964) - (1.138943436) \cdot (-0.43947320)]$$

Input interpretation:

$$\frac{1}{16 \times 25 \cdot (-2)^6} \cdot (-3469.04 \times 9.99526426781 + 4.500646424 \times (-0.5505966964) - 0.43947320 \times (-1.138943436))$$

Result:

$$-1.354529260216401693690375$$

$$\rho = -1.354529260216401693690375 \text{ (final result)}$$

Or:

$$\frac{1}{(25600)} \cdot ((([e^{(6)} \operatorname{sech}^4(((2(-2-1) \cdot e^{(-1)}) / (5 \cdot (-2)^2)))] \cdot (((2 \cdot 5^3 \cdot 0.99911 \cdot e^{(-6)} \cdot (-2)^6 \sinh \cdot (((4(-2-1) \cdot e^{(-1)}) / (5 \cdot (-2)^2)))))))] + 2.74742258] \cdot (9.99526) + (4.5) \cdot (-0.5506) - (1.13894) \cdot (-0.439473))$$

Input interpretation:

$$\frac{1}{25\,600} \left(\left(e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e} \right) \times \frac{2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right)}{e^6} + 2.74742258 \right) \times 9.99526 + 4.5 \times (-0.5506) - 0.439473 \times (-1.13894) \right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

$\sinh(x)$ is the hyperbolic sine function

Result:

-1.35453...

-1.35453...

Alternative representations:

$$\frac{1}{25\,600} \left(\left(\frac{e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e(5(-2)^2)} \right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right) \right)}{e^6} + 2.74742 \right) 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) =$$

$$\frac{-1.97717 + 9.99526 \left(2.74742 + \frac{0.99911 (-2)^6 5^3 e^6 \left(e^{-12/(20e)} - e^{12/(20e)} \right) \left(\frac{1}{\cos \left(\frac{6i}{20e} \right) \right)^4}{e^6} \right)}{25\,600}$$

$$\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)}{e^6} \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right) + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) =$$

$$\frac{-1.97717 + 9.99526 \left(2.74742 - \frac{1.99822 i \cos\left(\frac{\pi}{2} + \frac{12i}{20e}\right) (-2)^6 5^3 e^6 \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)}\right)^4}{e^6} \right)}{25600}$$

$$\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)}{e^6} \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right) + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) =$$

$$\frac{-1.97717 + 9.99526 \left(2.74742 + \frac{0.99911 (-2)^6 5^3 e^6 \left(e^{-12/(20e)} - e^{12/(20e)} \right) \left(\frac{2e^{-6/(20e)}}{1+e^{-12/(20e)}} \right)^4}{e^6} \right)}{25600}$$

Series representations:

$$\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right)}{e^6} \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right) \right) + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) =$$

$$0.00099547 + 12.483 \operatorname{sech}^4\left(-\frac{3}{10e}\right) \sum_{k=0}^{\infty} I_{1+2k}\left(-\frac{3}{5e}\right)$$

$$\frac{1}{25\,600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) = 199.727 \left(4.98415 \times 10^{-6} + \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \sum_{k=0}^{\infty} I_{1+2k} \left(-\frac{3}{5e} \right) \right) \text{ for } q = e^{-3/(10e)}$$

$$\frac{1}{25\,600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) = 99.8636 \left(9.96829 \times 10^{-6} + \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \sum_{k=0}^{\infty} \frac{\left(\frac{5}{3}\right)^{-1-2k} \left(-\frac{1}{e}\right)^{1+2k}}{(1+2k)!} \right) \text{ for } q = e^{-3/(10e)}$$

Integral representations:

$$\frac{1}{25\,600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) = \frac{0.00099547 \left(e \pi^4 - 60\,190.8 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^4 \int_0^1 \cosh\left(-\frac{3t}{5e}\right) dt \right)}{e \pi^4}$$

$$\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) = \frac{0.00099547 \left(e i \pi^5 - 15047.7 \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^4 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{R}^{9/(100e^2s)+s}}{s^{3/2}} ds \right) \sqrt{\pi} \right)}{e i \pi^5}$$

for $\gamma > 0$

From which:

$$\left[\frac{1}{25600} * \left(\left(\left(\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)*e^{-1}}{5*(-2)^2}\right) \right) * \left(\left(\left(2*5^3*0.99911*e^{-6} \right) * (-2)^6 \sinh\left(\frac{4(-2-1)e^{-1}}{5(-2)^2}\right) \right) \right) \right) + 2.74742258 \right) * (9.99526) + (4.5) * (-0.5506) - (1.13894) * (-0.439473) \right) \right]^{\sqrt{e}}$$

Input interpretation:

$$\left(\frac{1}{25600} \left(\left(e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{5(-2)^2}\right) \times \frac{2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{5(-2)^2}\right)}{e^6} + 2.74742258 \right) \times \left(9.99526 + 4.5 \times (-0.5506) - 0.439473 \times (-1.13894) \right) \right) \right)^{\sqrt{e}}$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

$\sinh(x)$ is the hyperbolic sine function

Result:

$$0.742826... - 1.47247... i$$

Polar coordinates:

$$r = 1.64923 \text{ (radius)}, \quad \theta = -63.2302^\circ \text{ (angle)}$$

$$1.64923... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$$

Alternative representations:

$$\left(\frac{1}{25\,600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. \left. \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{\sqrt{e}} = \right. \\ \left. \left(\frac{-1.97717 + 9.99526 \left(2.74742 + \frac{0.99911 (-2)^6 5^3 e^6 (e^{-12/(20e)} - e^{12/(20e)}) \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)}\right)^4}{e^6} \right)}{25\,600} \right)^{\sqrt{e}} \right)$$

$$\left(\frac{1}{25\,600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. \left. \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{\sqrt{e}} = \right. \\ \left. \left(\frac{-1.97717 + 9.99526 \left(2.74742 - \frac{1.99822 i \cos\left(\frac{\pi}{2} + \frac{12i}{20e}\right) (-2)^6 5^3 e^6 \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)}\right)^4}{e^6} \right)}{25\,600} \right)^{\sqrt{e}} \right)$$

$$\left(\frac{1}{25600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. \left. \left. 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) \right)^{\sqrt{e}} = \right. \\ \left. \left(\frac{1}{25600} \left(-1.97717 + 9.99526 \left(2.74742 + \frac{0.99911 (-2)^6 5^3 e^6 (e^{-12/(20e)} - e^{12/(20e)}) \left(\frac{2e^{-6/(20e)}}{1+e^{-12/(20e)}}\right)^4 \right)}{e^6} \right) \right) \right)^{\sqrt{e}} \right)$$

Series representations:

$$\left(\frac{1}{25600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. \left. \left. 9.99526 + 4.5(-0.5506) - 1.13894(-0.439473) \right) \right)^{\sqrt{e}} = \right. \\ \left(0.00099547 + 199.727 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \right. \\ \left. \sum_{k=0}^{\infty} I_{1+2k} \left(-\frac{3}{5e} \right) \right)^{\exp(i\pi [\arg(e-x)/(2\pi)]) \sqrt{x} \sum_{k=0}^{\infty} \left((-1)^k (e-x)^k x^{-k} \binom{-1}{k} / k! \right)}$$

for $(x \in \mathbb{R}$ and $x < 0$ and $q = e^{-3/(10e)})$

$$\left(\frac{1}{25600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right)^{\sqrt{e}} = \\ \left(0.00099547 + 99.8636 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \right. \\ \left. \sum_{k=0}^{\infty} \frac{\left(\frac{5}{3}\right)^{-1-2k} \left(-\frac{1}{e}\right)^{1+2k} \exp(i\pi [\operatorname{arg}(e-x)/(2\pi)]) \sqrt{x} \sum_{k=0}^{\infty} \left((-1)^k (e-x)^k x^{-k} \left(-\frac{1}{2}\right)_k\right) / k!}{(1+2k)!} \right) \\ \text{for } (x \in \mathbb{R} \text{ and } x < 0 \text{ and } q = e^{-3/(10e)})$$

$$\left(\frac{1}{25600} \left(\frac{e^6 \operatorname{sech}^4\left(\frac{2(-2-1)}{e(5(-2)^2)}\right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh\left(\frac{4(-2-1)}{e(5(-2)^2)}\right)\right)}{e^6} + 2.74742 \right) \right. \\ \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right)^{\sqrt{e}} = \\ \left(0.00099547 + 99.8636 i \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \right. \\ \left. \sum_{k=0}^{\infty} \frac{\left(-\frac{3}{5e} - \frac{i\pi}{2}\right)^{2k} \exp(i\pi [\operatorname{arg}(e-x)/(2\pi)]) \sqrt{x} \sum_{k=0}^{\infty} \left((-1)^k (e-x)^k x^{-k} \left(-\frac{1}{2}\right)_k\right) / k!}{(2k)!} \right) \\ \text{for } (x \in \mathbb{R} \text{ and } x < 0 \text{ and } q = e^{-3/(10e)})$$

and:

$$-[1/(25600) ((([e^6 \operatorname{sech}^4(((2(-2-1)*e^(-1))/(5*(-2)^2))(((2*5^3*0.99911*e^(-6)*(-2)^6 \sinh (((4(-2-1)e^(-1))/(5(-2)^2)))))))+2.74742258] (9.99526)+(4.5)(-0.5506)-(1.13894)(-0.439473)))]^(1.81147)$$

$$\begin{aligned}
& - \left(\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e(5(-2)^2)} \right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right) \right)}{e^6} + 2.74742 \right) \right. \right. \\
& \quad \left. \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{1.81147} = \\
& - \left(\frac{1}{25600} \left(-1.97717 + 9.99526 \left(2.74742 + \frac{0.99911 (-2)^6 5^3 e^6 (e^{-12/(20e)} - e^{12/(20e)}) \left(\frac{2e^{-6/(20e)}}{1+e^{-12/(20e)}} \right)^4 \right)}{e^6} \right) \right) \right)^{1.81147}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& - \left(\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e(5(-2)^2)} \right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right) \right)}{e^6} + 2.74742 \right) \right. \right. \\
& \quad \left. \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{1.81147} = \\
& -1.0342 \times 10^{-8} \left(25.484 + 319564 \cdot \operatorname{sech}^4 \left(-\frac{3}{10e} \right) \sum_{k=0}^{\infty} I_{1+2k} \left(-\frac{3}{5e} \right) \right)^{1.81147} \\
& - \left(\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e(5(-2)^2)} \right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right) \right)}{e^6} + 2.74742 \right) \right. \right. \\
& \quad \left. \left. 9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{1.81147} = \\
& -1.0342 \times 10^{-8} \left(25.484 + 5.11302 \times 10^6 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \sum_{k=0}^{\infty} I_{1+2k} \left(-\frac{3}{5e} \right) \right)^{1.81147} \\
& \text{for } q = e^{-3/(10e)}
\end{aligned}$$

$$\begin{aligned}
& - \left(\frac{1}{25600} \left(\left(\frac{e^6 \operatorname{sech}^4 \left(\frac{2(-2-1)}{e(5(-2)^2)} \right) \left(2 \times 5^3 \times 0.99911 (-2)^6 \sinh \left(\frac{4(-2-1)}{e(5(-2)^2)} \right) \right) \right)}{e^6} + 2.74742 \right) \right. \\
& \quad \left. \left. \left(9.99526 + 4.5 (-0.5506) - 1.13894 (-0.439473) \right) \right)^{1.81147} = \right. \\
& -1.0342 \times 10^{-8} \left(25.484 + 2.55651 \times 10^6 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^4 \right. \\
& \quad \left. \sum_{k=0}^{\infty} \frac{\left(\frac{5}{3} \right)^{-1-2k} \left(-\frac{1}{e} \right)^{1+2k}}{(1+2k)!} \right)^{1.81147} \quad \text{for } q = e^{-3/(10e)}
\end{aligned}$$

where 1.81147 is about equal to the following expression:

$$\sqrt{170/69}/\pi * \text{gamma}(1/4)$$

Input:

$$\frac{\sqrt{\frac{170}{69}}}{\pi} \Gamma\left(\frac{1}{4}\right)$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\sqrt{\frac{170}{69}}}{\pi} \Gamma\left(\frac{1}{4}\right)$$

Decimal approximation:

1.811469881748113503733095371306415938001933353811866987804...

1.8114698817...

Alternate forms:

$$\frac{4 \sqrt{\frac{170}{69}} \frac{1}{4}!}{\pi}$$

$$\frac{4 \sqrt{\frac{85(2+\sqrt{2})K\left(\frac{(-2-2\sqrt{2})^2}{(4+2\sqrt{2})^2}\right)}{6\varphi(4+2\sqrt{2})}}}{\pi^{3/4}}$$

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{G\left(1 + \frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi G\left(\frac{1}{4}\right)}$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{e^{-\log G(1/4) + \log G(1+1/4)} \sqrt{\frac{170}{6\varphi}}}{\pi}$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{\left(-1 + \frac{1}{4}\right)! \sqrt{\frac{170}{6\varphi}}}{\pi}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{4 \sqrt{\frac{170}{6\varphi}} \sum_{k=0}^{\infty} \frac{4^{-k} \Gamma^{(k)}(1)}{k!}}{\pi}$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{\sqrt{\frac{170}{6\varphi}}}{\pi \sum_{k=1}^{\infty} 4^{-k} c_k}$$

$$\text{for } \left(c_1 = 1 \text{ and } c_2 = 1 \text{ and } c_k = \frac{\gamma c_{-1+k} + \sum_{j=1}^{-2+k} (-1)^{1+j+k} c_j \zeta(-j+k)}{-1+k} \right)$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{\sqrt{\frac{170}{6\varphi}} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\pi} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{6\varphi}}}{\pi} = \frac{\sqrt{\frac{170}{6\varphi}}}{\sum_{k=0}^{\infty} \left(\frac{1}{4} - z_0\right)^k \sum_{j=0}^k \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}(-j+k)\pi + \pi z_0\right) \Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\zeta(s)$ is the Riemann zeta function

γ is the Euler-Mascheroni constant

\mathbb{Z} is the set of integers

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}}}{\pi} = \frac{\sqrt{\frac{170}{69}}}{\pi} \int_0^1 \frac{1}{\log^{3/4}\left(\frac{1}{t}\right)} dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}}}{\pi} = \frac{\sqrt{\frac{170}{69}}}{\pi} \int_0^\infty \frac{e^{-t}}{t^{3/4}} dt$$

$$\frac{\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}}}{\pi} = \frac{\sqrt{\frac{170}{69}}}{\pi} e^{\int_0^1 \frac{-\frac{3}{4} + \sqrt{x} - \frac{x}{4}}{(-1+x)\log(x)} dx}$$

Indeed, from the first result of the equation, we have:

$$1.3545273742037681^{\left(\left(\frac{\sqrt{170/69}}{\pi} * \Gamma(1/4)\right)\right)}$$

Input interpretation:

$$1.3545273742037681^{\sqrt{\frac{170}{69}} / \pi \Gamma(1/4)}$$

$\Gamma(x)$ is the gamma function

Result:

$$1.732724856977603\dots$$

$1.732724856977603\dots \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Alternative representations:

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \left(G\left(1+\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / (\pi G\left(\frac{1}{4}\right))$$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \left(e^{-\log G(1/4) + \log G(1+1/4)} \sqrt{\frac{170}{69}} \right) / \pi$$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi = 1.35452737420376810000 \left(\left(-1+\frac{1}{4}\right)! \sqrt{\frac{170}{69}} \right) / \pi$$

Series representations:

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$\exp \left(\frac{1.21381036864280163930 \sqrt{\frac{101}{69}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{69}{101}\right)^{k_1} 4^{-k_2} \binom{\frac{1}{2}}{k_1} \Gamma^{(k_2)}(1)}{k_2!}}{\pi} \right)$$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \left(\sqrt{\frac{101}{69}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{69}{101}\right)^{k_1} \binom{\frac{1}{2}}{k_1} \left(\frac{1-z_0}{4}\right)^{k_2} \Gamma^{(k_2)}(z_0)}{k_2!} \right) / \pi$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \left(4 \sqrt{\frac{101}{69}} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(-\frac{69}{101}\right)^{k_1} 4^{-k_2} \binom{-\frac{1}{2}}{k_1} \Gamma^{(k_2)}(1)}{k_1! k_2!} \right) / \pi$$

Integral representations:

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \sqrt{\frac{170}{69}} / \pi \int_0^{\infty} e^{-t} / t^{3/4} dt$$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \sqrt{\frac{170}{69}} / \pi \int_0^1 1 / \log^{3/4}\left(\frac{1}{t}\right) dt$$

$$1.35452737420376810000 \left(\Gamma\left(\frac{1}{4}\right) \sqrt{\frac{170}{69}} \right) / \pi =$$

$$1.35452737420376810000 \left(\csc\left(\frac{\pi}{8}\right) \sqrt{\frac{170}{69}} \right) / \pi \int_0^{\infty} \sin(t) / t^{3/4} dt$$

We observe that the result of the equation **-1.354529260216401693690375** is very near to the values of the following Ramanujan mock theta functions of 5th order:

$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \dots$$

$$\phi(-q) + \chi(q) = 2F(q).$$

We obtain:

$$1 + (0.449329^2) / (1 - 0.449329) + (0.449329)^8 / ((1 - 0.449329)(1 - 0.449329^3))$$

$$1 + \frac{0.449329^2}{1 - 0.449329} + \frac{0.449329^8}{(1 - 0.449329)(1 - 0.449329^3)}$$

$$1.369955709042580254965844050909072881396600348644448209935\dots$$

F(q) = 1.369955709...

$$2(((((((1 + (0.449329^2) / (1 - 0.449329) + (0.449329)^8 / ((1 - 0.449329)(1 - 0.449329^3))))))))))$$

$$2 \left(1 + \frac{0.449329^2}{1 - 0.449329} + \frac{0.449329^8}{(1 - 0.449329)(1 - 0.449329^3)} \right)$$

2.739911418085160509931688101818145762793200697288896419870...

$$2F(q) = 2.73991141808516...$$

Indeed: $F(q) = 1.369955709...$ is very near to the result 1.3545292602164

Thence, from the following equation

$$p_r = -\frac{1}{2}R_c\mu \tanh\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) + \frac{1}{R_c r^4} \left[\mu e^{r_0-2r} (R_c e^r r^3 - 8(r^2-2)(e^r - e^{r_0})) \right. \\ \left. \times \tanh\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) \operatorname{sech}^2\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) \right] - \frac{e^{r_0-r}}{r^2} \quad (38)$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$-1/2*5*0.99911 \tanh(\frac{2*(-3)*e^{-1}}{5*(-2)^2}) + 1/(5*(-2)^4) [0.99911 * e^{5*e^{-2}} * (-2)^3 - 8(2) * ((e^{-2} - e^{-3}))] \tanh(\frac{2*(-3)*e^{-1}}{5*(-2)^2}) \operatorname{sech}^2(\frac{2*(-3)*e^{-1}}{5*(-2)^2}) - 1/(4e)$$

Input:

$$-\frac{1}{2} \times 5 \times 0.99911 \tanh\left(\frac{2 \times (-3)}{5(-2)^2}\right) + \frac{1}{5(-2)^4} \left(0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2 \right) \left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh\left(\frac{2 \times (-3)}{5(-2)^2}\right) \operatorname{sech}^2\left(\frac{2 \times (-3)}{5(-2)^2}\right) \right) - \frac{1}{4e}$$

$\tanh(x)$ is the hyperbolic tangent function

$\operatorname{sech}(x)$ is the hyperbolic secant function

Result:

0.189333488469534537057381590771240422109365130020934778818...

$$p_r = 0.189333488469534537$$

Alternative representations:

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} = -\frac{1}{4e} - 2.49778 \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) + \frac{0.99911 e \left(-16 - \frac{40}{e^2}\right) \left(-\frac{1}{e^3} + \frac{1}{e^2}\right) \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)}\right)^2}{5(-2)^4}$$

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} = -\frac{1}{4e} - 2.49778 \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) + \frac{0.99911 e \left(-16 - \frac{40}{e^2}\right) \left(-\frac{1}{e^3} + \frac{1}{e^2}\right) \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) \left(\frac{2e^{-6/(20e)}}{1+e^{-12/(20e)}}\right)^2}{5(-2)^4}$$

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} = -\frac{1}{4e} - 2.49778 \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) + \frac{0.99911 e \left(-16 - \frac{40}{e^2}\right) \left(-\frac{1}{e^3} + \frac{1}{e^2}\right) \left(-1 + \frac{2}{1+e^{12/(20e)}}\right) \left(\frac{2}{e^{-6/(20e)} + e^{6/(20e)}}\right)^2}{5(-2)^4}$$

Series representations:

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} = 2.49778 - \frac{0.25}{e} + 4.99555 \sum_{k=1}^{\infty} (-1)^k q^{2k} - \frac{0.399644 (1-e)(5+2e^2) \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k}\right) \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k}\right)^2}{e^4} \text{ for } q = e^{-3/(10e)}$$

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} =$$

$$2.49778 - \frac{0.25}{e} + 4.99555 \sum_{k=1}^{\infty} (-1)^k q^{2k} + \frac{0.099911 (1-e)(5+2e^2) \left(\sum_{k=-\infty}^{\infty} \frac{1}{\left(-\frac{3}{10e} + i\left(\frac{1+k}{2}\right)\pi\right)^2}\right) \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k}\right)}{e^4} \text{ for } q = e^{-3/(10e)}$$

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} =$$

$$2.49778 - \frac{0.25}{e} + 4.99555 \sum_{k=1}^{\infty} (-1)^k q^{2k} - \frac{0.099911 (1-e)(5+2e^2) \pi^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k (1+2k)}{100e^2 + \left(\frac{1+k}{2}\right)^2 \pi^2}\right)^2 \left(1 + 2 \sum_{k=1}^{\infty} (-1)^k q^{2k}\right)}{e^4} \text{ for } q = e^{-3/(10e)}$$

Integral representation:

$$\frac{5}{2} (-1) 0.99911 \tanh\left(\frac{2(-3)}{(5(-2)^2)e}\right) + \frac{0.99911 e \left(\frac{5(-2)^3}{e^2} - 8 \times 2\right) \left(\frac{1}{e^2} - \frac{1}{e^3}\right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right)}{5(-2)^4} - \frac{1}{4e} =$$

$$-\frac{1}{e^4 \pi^2} 2.49778 \left(0.100089 e^3 \pi^2 + e^4 \pi^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt - 0.8 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt + 0.8 e \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt - 0.32 e^2 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt + 0.32 e^3 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt\right)^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt\right)$$

We have also that:

$$1/(\sqrt{3} e \log(2)) \approx 1/(0.189333488469534537)$$

Input interpretation:

$$\frac{1}{\sqrt{3} e \log(2)} \times \frac{1}{0.189333488469534537}$$

$\log(x)$ is the natural logarithm

Result:

1.61842281785993638...

1.61842281785993638.... result that is a very good approximation to the value of the golden ratio 1,618033988749

Alternative representations:

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{1}{0.1893334884695345370000 (e \log_e(2) \sqrt{3})}$$

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{1}{0.1893334884695345370000 (e \log(a) \log_a(2) \sqrt{3})}$$

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{1}{0.1893334884695345370000 (2 e \coth^{-1}(3) \sqrt{3})}$$

Series representations:

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{5.281685813130248237715}{e \log(2) \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}$$

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{5.281685813130248237715}{e \exp\left(i \pi \left\lfloor \frac{\arg(3-x)}{2 \pi} \right\rfloor\right) \log(2) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{5.281685813130248237715 \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(3-z_0)/(2 \pi) \rfloor} z_0^{-1/2 - 1/2 \lfloor \arg(3-z_0)/(2 \pi) \rfloor}}{e \log(2) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (3-z_0)^k z_0^{-k}}{k!}}$$

Integral representations:

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{5.281685813130248237715}{e \sqrt{3} \int_1^2 \frac{1}{t} dt}$$

$$\frac{1}{0.1893334884695345370000 (\sqrt{3} e \log(2))} = \frac{10.56337162626049647543 i \pi}{e \sqrt{3} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}$$

for $-1 < \gamma < 0$

From

$$\rho + p_t = \frac{1}{4R_c r^4} \left[e^{r_0 - 2r} \operatorname{sech}^2 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(16\mu (r^2 - 2) (e^r - e^{r_0}) \tanh \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \right. \right. \\ \left. \left. + R_c (2\mu - 1) e^r (r-2)r^2 - R_c e^r (r-2)r^2 \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \right) \right] \quad (41)$$

$$\mu = 0.9991104684; R_c = 5; r = -2; r_0 = -3$$

$$1/320 * e * \operatorname{sech}^2 \left(\frac{2(-3) * e^{(-1)}}{(5(-2)^2)} \right) 16 * 0.99911 * 2 \left((e^{(-2)} - e^{(-3)}) * \left(\frac{\tanh \left(\frac{2(-3) * e^{(-1)}}{(5(-2)^2)} \right)} \right) + 5(2 * 0.99911 - 1) * e^{(-2)} * (-16) - 5 * e^{(-2)} * (-16) * \cosh \left(\frac{4(-3) * e^{(-1)}}{(5(-2)^2)} \right) \right)$$

Input:

$$\frac{1}{320} e \operatorname{sech}^2\left(\frac{2 \times (-3)}{e}\right) (16 \times 0.99911 \times 2)$$

$$\left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh\left(\frac{2 \times (-3)}{e}\right) + \frac{5(2 \times 0.9911 - 1) \times (-16)}{e^2} - \frac{5 \times (-16) \cosh\left(\frac{4 \times (-3)}{5(-2)^2}\right)}{e^2} \right)$$

$\operatorname{sech}(x)$ is the hyperbolic secant function

$\tanh(x)$ is the hyperbolic tangent function

$\cosh(x)$ is the hyperbolic cosine function

Result:

0.120236...

$$\rho + p_t = 0.120236$$

Alternative representations:

$$\frac{1}{320} \left(e \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right) \right) (16 \times 0.99911 \times 2)$$

$$\left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh\left(\frac{4(-3)}{e(5(-2)^2)}\right)}{e^2} \right) =$$

$$\frac{1}{320} \times 31.9715 e$$

$$\left(-\frac{78.576}{e^2} + \frac{40(e^{-12/(20e)} + e^{12/(20e)})}{e^2} + \left(-\frac{1}{e^3} + \frac{1}{e^2}\right) \left(-1 + \frac{2}{1 + e^{12/(20e)}}\right) \right) \left(\frac{1}{\cos\left(\frac{6i}{20e}\right)} \right)^2$$

$$\frac{1}{320} \left(e \operatorname{sech}^2\left(\frac{2(-3)}{e(5(-2)^2)}\right) \right) (16 \times 0.99911 \times 2)$$

$$\left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh\left(\frac{2(-3)}{e(5(-2)^2)}\right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh\left(\frac{4(-3)}{e(5(-2)^2)}\right)}{e^2} \right) =$$

$$\frac{1}{320} \times 31.9715 e \left(-\frac{78.576}{e^2} + \frac{40(e^{-12/(20e)} + e^{12/(20e)})}{e^2} + \left(-\frac{1}{e^3} + \frac{1}{e^2}\right) \left(-1 + \frac{2}{1 + e^{12/(20e)}}\right) \right) \left(\frac{2e^{-6/(20e)}}{1 + e^{-12/(20e)}} \right)^2$$

$$\frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) =$$

$$\frac{1}{320} \times 31.9715 e \left(-\frac{78.576}{e^2} + \frac{80 \cos \left(\frac{12i}{20e} \right)}{e^2} + \left(-\frac{1}{e^3} + \frac{1}{e^2} \right) \left(-1 + \frac{2}{1 + e^{12/(20e)}} \right) \right) \left(\frac{2 e^{-6/(20e)}}{1 + e^{-12/(20e)}} \right)^2$$

Series representations:

$$\frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) =$$

$$\frac{1}{e^2} 31.9715 \left(0.0125 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 - 0.9947 e \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 + e I_0 \left(-\frac{3}{5e} \right) \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 - 0.025 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 \sum_{k=1}^{\infty} \left((-1)^k (-1+e) q^{2k} - 80 e I_{2k} \left(-\frac{3}{5e} \right) \right) \right) \text{ for } q = e^{-3/(10e)}$$

$$\frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) =$$

$$-\frac{1}{e^2} 0.799288 \left(-0.5 \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 + 39.788 e \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 - \left(\sum_{k=1}^{\infty} (-1)^k q^{2k} \right) \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 + e \left(\sum_{k=1}^{\infty} (-1)^k q^{2k} \right) \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 - 40 e \left(\sum_{k=1}^{\infty} (-1)^k q^{-1+2k} \right)^2 \sum_{k=0}^{\infty} \frac{\left(\frac{9}{25} \right)^k \left(-\frac{1}{e} \right)^{2k}}{(2k)!} \right) \text{ for } q = e^{-3/(10e)}$$

$$\begin{aligned}
& \frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \right. \\
& \quad \left. \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) = \\
& -\frac{1}{e^2} 7.99288 \left(0.0125 \sum_{k=-\infty}^{\infty} \frac{1}{\left(-\frac{3}{10e} + i \left(\frac{1}{2} + k \right) \pi \right)^2} - \right. \\
& \quad 0.9947 e \sum_{k=-\infty}^{\infty} \frac{1}{\left(-\frac{3}{10e} + i \left(\frac{1}{2} + k \right) \pi \right)^2} + e I_0 \left(-\frac{3}{5e} \right) \sum_{k=-\infty}^{\infty} \frac{1}{\left(-\frac{3}{10e} + i \left(\frac{1}{2} + k \right) \pi \right)^2} - \\
& \quad \left. 0.025 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=1}^{\infty} \frac{(-1)^{k_2} (-1+e) q^{2k_2} - 80 e I_{2k_2} \left(-\frac{3}{5e} \right)}{\left(-\frac{3}{10e} + i \pi \left(\frac{1}{2} + k_1 \right) \right)^2} \right) \text{ for } q = e^{-3/(10e)}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \\
& \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) = \\
& \frac{1}{e^2 \pi^2} 0.399644 \left(1.424 e \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 - \right. \\
& \quad \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt + e \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \\
& \quad \int_0^{-\frac{3}{10e}} \operatorname{sech}^2(t) dt - 48 \left(\int_0^{\infty} \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_0^1 \sinh \left(-\frac{3t}{5e} \right) dt \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \\
& \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) = \\
& \frac{1}{e^2 \pi^2} 0.399644 \left(-78.576 e \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 - \right. \\
& \left. \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_0^{-\frac{3}{10}e} \operatorname{sech}^2(t) dt + e \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \right. \\
& \left. \int_0^{-\frac{3}{10}e} \operatorname{sech}^2(t) dt + 80 e \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_{\frac{i\pi}{2}}^{-\frac{3}{5}e} \sinh(t) dt \right) \\
& \frac{1}{320} \left(e \operatorname{sech}^2 \left(\frac{2(-3)}{e(5(-2)^2)} \right) \right) (16 \times 0.99911 \times 2) \left(\left(\frac{1}{e^2} - \frac{1}{e^3} \right) \tanh \left(\frac{2(-3)}{e(5(-2)^2)} \right) + \right. \\
& \left. \frac{5(2 \times 0.9911 - 1)(-16)}{e^2} - \frac{5(-16) \cosh \left(\frac{4(-3)}{e(5(-2)^2)} \right)}{e^2} \right) = \\
& \frac{1}{e^2 i \pi^3} 0.399644 \left(-78.576 e i \pi \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 - \right. \\
& i \left(\pi \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_0^{-\frac{3}{10}e} \operatorname{sech}^2(t) dt \right) + \\
& e i \pi \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \int_0^{-\frac{3}{10}e} \operatorname{sech}^2(t) dt + \\
& \left. 40 e \left(\int_0^\infty \frac{t^{-(3i)/(5e\pi)}}{1+t^2} dt \right)^2 \left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\mathcal{A}^{\ominus/(100e^2s)+s}}{\sqrt{s}} ds \right) \sqrt{\pi} \right) \text{ for } \gamma > 0
\end{aligned}$$

We have that:

$$\rho = -1.354529260216401693690375$$

$$p_t = 0.189333488469534537$$

$$p_t = 1.474765260216401693690375$$

$$\rho + p_t = 0.120236$$

from:

$$\begin{aligned}
\rho + p_r = & \frac{1}{8R_c^2 r^6} \left[e^{r_0-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(4R_c^2 \mu e^{2r} r^5 + 4 \left(R_c^2 (\mu - 1) e^{2r} r^5 + 16\mu \right. \right. \right. \\
& \times (r^2 - 2)^2 e^{2r_0} - 16\mu (r^2 - 2)^2 e^{r+r_0} \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 3R_c^2 e^{2r} r^5 + R_c^2 e^{2r} r^5 \cosh \\
& \times \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 96R_c \mu e^{2r} r^2 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 96R_c \mu r^2 e^{r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
& + 16R_c \mu e^{2r} r^5 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 8R_c \mu r^5 e^{r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 16R_c \mu e^{2r} r^4 \sinh \\
& \times \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 16R_c \mu r^4 e^{r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu e^{2r} r^3 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
& + 16R_c \mu r^3 e^{r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 128\mu r^4 e^{2r_0} + 128\mu r^4 e^{r+r_0} + 512\mu r^2 \\
& \left. \times e^{2r_0} - 512\mu r^2 e^{r+r_0} + 512\mu e^{r+r_0} - 512\mu e^{2r_0} \right] \quad (40)
\end{aligned}$$

(-1.354529260216401693690375 + 0.189333488469534537)

Input interpretation:

-1.354529260216401693690375 + 0.189333488469534537

Result:

-1.165195771746867156690375

$\rho + p_r = -1.165195771746867156690375$

For to obtain p_t

$$\begin{aligned}
p_t = & \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(-2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - R_c^3 \mu e^{3r} r^6 \right. \right. \\
& \times \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) + 4R_c^2 \mu r^5 e^{2r+r_0} - 8R_c^2 \mu r^4 e^{2r+r_0} + 4e^{r_0} (R_c^2 e^{2r} r^4 (\mu(r-2) + r) \\
& + 32\mu (r^2 - 2)^2 e^{2r_0} - 32\mu (r^2 - 2)^2 e^{r+r_0} \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 3R_c^2 r^5 e^{2r+r_0} \\
& + R_c^2 r^5 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 128R_c \mu r^2 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 128R_c \mu r^2 e^{r+2r_0} \\
& \times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^5 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 16R_c \mu r^5 e^{r+2r_0} \\
& \times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^3 e^{r+2r_0} \\
& \times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 256\mu r^4 e^{3r_0} + 256\mu r^4 e^{r+2r_0} + 1024\mu r^2 e^{3r_0} - 1024\mu r^2 e^{r+2r_0} \\
& \left. + 1024\mu e^{r+2r_0} - 1024\mu e^{3r_0} \right] \quad (39)
\end{aligned}$$

From $\rho + p_t = 0.120236$

we calculate:

$$-1.354529260216401693690375 + x = 0.120236$$

$$0.120236 + 1.354529260216401693690375$$

Input interpretation:

$$0.120236 + 1.354529260216401693690375$$

Result:

$$1.474765260216401693690375$$

$$p_t = 1.474765260216401693690375$$

From the three principal result, we obtain:

$$\text{sqrt}(1.3545292602 - 0.1893334884 + 1.4747652602)$$

Input interpretation:

$$\sqrt{1.3545292602 - 0.1893334884 + 1.4747652602}$$

Result:

$$1.6247956893\dots$$

$$1.6247956893\dots$$

From which:

$$\text{sqrt}(1.3545292602 - 0.1893334884 + 1.4747652602) - 7/10^3$$

Input interpretation:

$$\sqrt{1.3545292602 - 0.1893334884 + 1.4747652602} - \frac{7}{10^3}$$

Result:

$$1.6177956893\dots$$

1.6177956893.... result that is a very good approximation to the value of the golden ratio 1,618033988749

and:

$$\sqrt{(1.3545292602 - 0.1893334884 + 1.4747652602) + (27 \times 4) / 10^3}$$

Input interpretation:

$$\sqrt{1.3545292602 - 0.1893334884 + 1.4747652602} + \frac{27 \times 4}{10^3}$$

Result:

1.7327956893...

1.7327956893... $\approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

We have also:

$$\left(\frac{1}{(-1.354529260216 + 0.1893334884695 + 1.474765260216)} \right)^4 + 18 - \phi$$

Input interpretation:

$$\left(\frac{1}{-1.354529260216 + 0.1893334884695 + 1.474765260216} \right)^4 + 18 - \phi$$

ϕ is the golden ratio

Result:

125.26680213...

125.26680213... result very near to the Higgs boson mass 125.18 GeV

Alternative representations:

$$\left(\frac{1}{-1.3545292602160000 + 0.18933348846950000 + 1.4747652602160000} \right)^4 + 18 - \phi = 18 + \left(\frac{1}{0.3095694884695000} \right)^4 - 2 \sin(54^\circ)$$

$$\left(\frac{1}{-1.3545292602160000 + 0.18933348846950000 + 1.4747652602160000} \right)^4 + 18 - \phi = 18 + 2 \cos(216^\circ) + \left(\frac{1}{0.3095694884695000} \right)^4$$

$$\left(\frac{1}{-1.3545292602160000 + 0.18933348846950000 + 1.4747652602160000} \right)^4 + 18 - \phi = 18 + \left(\frac{1}{0.3095694884695000} \right)^4 + 2 \sin(666^\circ)$$

$((1/(-1.354529260216 + 0.1893334884695 + 1.474765260216))))^4 + 29 + \text{golden ratio}$

Input interpretation:

$$\left(\frac{1}{-1.354529260216 + 0.1893334884695 + 1.474765260216} \right)^4 + 29 + \phi$$

ϕ is the golden ratio

Result:

139.502870107...

139.502870107.... result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$\left(\frac{1}{-1.3545292602160000 + 0.18933348846950000 + 1.4747652602160000} \right)^4 + 29 + \phi = 29 + \left(\frac{1}{0.3095694884695000} \right)^4 + 2 \sin(54^\circ)$$

$$\left(\frac{1}{-1.3545292602160000 + 0.18933348846950000 + 1.4747652602160000} \right)^4 + 29 + \phi = 29 - 2 \cos(216^\circ) + \left(\frac{1}{0.3095694884695000} \right)^4$$

$$\left(\frac{1}{-1.3545292602160000 + 0.1893334884695000 + 1.4747652602160000} \right)^4 + 29 + \phi = 29 + \left(\frac{1}{0.3095694884695000} \right)^4 - 2 \sin(666^\circ)$$

27*1/2((((1/(-1.35452926 + 0.189333488 + 1.47476526))))^4+18+golden ratio-1/2))+1

Input interpretation:

$$27 \times \frac{1}{2} \left(\left(\frac{1}{-1.35452926 + 0.189333488 + 1.47476526} \right)^4 + 18 + \phi - \frac{1}{2} \right) + 1$$

ϕ is the golden ratio

Result:

1729.039...

1729.039....

This result is very near to the mass of candidate glueball **f₀(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$\frac{27}{2} \left(\left(\frac{1}{-1.35453 + 0.189333 + 1.47477} \right)^4 + 18 + \phi - \frac{1}{2} \right) + 1 = 1 + \frac{27}{2} \left(\frac{35}{2} + \left(\frac{1}{0.309569} \right)^4 + 2 \sin(54^\circ) \right)$$

$$\frac{27}{2} \left(\left(\frac{1}{-1.35453 + 0.189333 + 1.47477} \right)^4 + 18 + \phi - \frac{1}{2} \right) + 1 = 1 + \frac{27}{2} \left(\frac{35}{2} - 2 \cos(216^\circ) + \left(\frac{1}{0.309569} \right)^4 \right)$$

$$\frac{27}{2} \left(\left(\frac{1}{-1.35453 + 0.189333 + 1.47477} \right)^4 + 18 + \phi - \frac{1}{2} \right) + 1 = 1 + \frac{27}{2} \left(\frac{35}{2} + \left(\frac{1}{0.309569} \right)^4 - 2 \sin(666^\circ) \right)$$

From the three results

$$\rho = -1.354529260216401693690375$$

$$p_r = 0.189333488469534537$$

$$p_t = 1.474765260216401693690375$$

we have also:

$$\begin{aligned} \rho + p_r + 2p_t &= -\frac{1}{8R_c^2 r^6} \left[\mu e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(R_c^3 e^{3r} r^6 \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8R_c^2 r^5 e^{2r+r_0} \right. \right. \\ &+ 8R_c^2 r^4 e^{2r+r_0} - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 8(r^2-2)^2 e^{2r_0} - 8(r^2-2)^2 e^{r+r_0} \right) \cosh \\ &\times \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 2R_c e^r r^2 \left(R_c^2 e^{2r} r^4 - 8(r^3-r^2-2r-2)e^{r+r_0} + 4(r^3-2r^2-2r \right. \\ &- 4)e^{2r_0} \left. \right) \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 128r^4 e^{3r_0} - 128r^4 e^{r+2r_0} - 512r^2 e^{3r_0} + 512r^2 e^{r+2r_0} \\ &\left. \left. - 512e^{r+2r_0} + 512e^{3r_0} \right) \right] \quad (42) \end{aligned}$$

$$\begin{aligned} &(-1.354529260216401693690375 + 0.189333488469534537 \\ &+ 2 * 1.474765260216401693690375) \end{aligned}$$

Input interpretation:

$$\begin{aligned} &-1.354529260216401693690375 + \\ &0.189333488469534537 + 2 \times 1.474765260216401693690375 \end{aligned}$$

Result:

$$1.784334748685936230690375$$

$$1.784334748685936230690375 = \rho + p_r + 2p_t$$

$$\begin{aligned}
\rho - |p_r| &= \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
&\times \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 16\mu (r^2-2)^2 e^{2r_0} - 16\mu (r^2-2)^2 e^{r+r_0} \right) \\
&\times \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 6R_c^2 r^5 e^{2r+r_0} + 6R_c^2 r^4 e^{2r+r_0} - 2R_c^2 (r-1)r^4 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&+ 64R_c \mu r^2 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^3 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^5 e^{2r+r_0} \\
&\times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 16R_c \mu r^5 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^4 e^{2r+r_0} \sinh \\
&\times \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^4 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&- 32R_c \mu r^3 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 256\mu r^4 e^{3r_0} - 256\mu r^4 e^{r+2r_0} - 1024\mu r^2 e^{3r_0} + 1024\mu \\
&\times \left. r^2 e^{r+2r_0} - 1024\mu e^{r+2r_0} + 1024\mu e^{3r_0} \right] - \left| -\frac{1}{2} R_c \mu \tanh \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) + \frac{1}{R_c r^4} \left[\mu e^{r_0-2r} \right. \right. \\
&\times \left. \left. \left(R_c e^r r^3 - 8(r^2-2)(e^r - e^{r_0}) \tanh \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \right) \operatorname{sech}^2 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \right] - \frac{e^{r_0-r}}{r^2} \right| \quad (43)
\end{aligned}$$

(-1.354529260216401693690375 - 0.189333488469534537)

Input interpretation:

-1.354529260216401693690375 - 0.189333488469534537

Result:

-1.543862748685936230690375

-1.543862748685936230690375 = $\rho - p_r$

From which:

1-1/(-1.354529260216401693690375 - 0.189333488469534537)

Input interpretation:

$1 - \frac{1}{-1.354529260216401693690375 - 0.189333488469534537}$

Result:

1.647725972306251467397924187034125922517015659438478454663...

1.6477259723.... $\approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

$$\begin{aligned}
\rho - |p_t| &= \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(2R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
&\times \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 8e^{r_0} \left(R_c^2 e^{2r} (r-1)r^4 + 16\mu (r^2-2)^2 e^{2r_0} - 16\mu (r^2-2)^2 e^{r+r_0} \right) \\
&\times \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 6R_c^2 r^5 e^{2r+r_0} + 6R_c^2 r^4 e^{2r+r_0} - 2R_c^2 (r-1)r^4 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&+ 64R_c \mu r^3 e^{2r-r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^2 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^5 e^{2r+r_0} \\
&\times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 16R_c \mu r^5 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^4 e^{2r+r_0} \sinh \\
&\times \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 32R_c \mu r^4 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&- 32R_c \mu r^3 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 256\mu r^4 e^{3r_0} - 256\mu r^4 e^{r+2r_0} - 1024\mu r^2 e^{3r_0} + 1024\mu r^2 e^{r+2r_0} \\
&- 1024\mu e^{r+2r_0} + 1024\mu e^{3r_0} \left. \right] - \left| \frac{1}{16R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4 \left(\frac{2(r-1)e^{r_0-r}}{R_c r^2} \right) \left(-2R_c^3 \mu e^{3r} r^6 \sinh \right. \right. \right. \\
&\times \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - R_c^3 \mu e^{3r} r^6 \sinh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) + 4R_c^2 \mu r^5 e^{2r+r_0} - 8R_c^2 \mu r^4 e^{2r+r_0} + 4e^{r_0} \left(R_c^2 \right. \\
&\times \left. e^{2r} r^4 (\mu(r-2) + r) + 32\mu (r^2-2)^2 e^{2r_0} - 32\mu (r^2-2)^2 e^{r+r_0} \right) \cosh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&+ 3R_c^2 r^5 e^{2r+r_0} + R_c^2 r^5 e^{2r+r_0} \cosh \left(\frac{8(r-1)e^{r_0-r}}{R_c r^2} \right) - 128R_c \mu r^2 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) \\
&+ 128R_c \mu r^2 e^{r+2r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^5 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 16R_c \mu r^5 e^{r+2r_0} \\
&\times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 64R_c \mu r^3 e^{2r+r_0} \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) + 32R_c \mu r^3 e^{r+2r_0} \\
&\times \sinh \left(\frac{4(r-1)e^{r_0-r}}{R_c r^2} \right) - 256\mu r^4 e^{3r_0} + 256\mu r^4 e^{r+2r_0} + 1024\mu r^2 e^{3r_0} - 1024\mu r^2 e^{r+2r_0} \\
&\left. \left. \left. + 1024\mu e^{r+2r_0} - 1024\mu e^{3r_0} \right) \right] \right| \quad (44)
\end{aligned}$$

(-1.354529260216401693690375 - 1.474765260216401693690375)

Input interpretation:

-1.354529260216401693690375 - 1.474765260216401693690375

Result:

-2.82929452043280338738075

-2.82929452043280338738075 = $\rho - p_t$

From which:

$$(-(-1.354529260216401693690375 - 1.474765260216401693690375))^{1/2} + 5/10^2$$

Input interpretation:

$$\sqrt{-(-1.354529260216401693690375 - 1.474765260216401693690375)} + \frac{5}{10^2}$$

Result:

1.732050689020043589298856...

1.732050689020043589298856... $\approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: [Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 \[gr-qc\] 28 Sep 2019](#))

1.7320506890

Possible closed forms:

$$\sqrt{3} \approx 1.73205080756$$

$$\frac{63727\pi}{115588} \approx 1.7320506889583$$

$$\frac{1}{90} (523\pi + 50) \approx 1.7320506888947$$

$$\begin{aligned}
\frac{p_r}{\rho} = & \left[-\frac{1}{2} R_c \mu \tanh\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) + \frac{1}{R_c r^4} \left[\mu e^{r_0-2r} (R_c e^r r^3 - 8(r^2-2)(e^r - e^{r_0})) \right. \right. \\
& \times \left. \left. \tanh\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) \operatorname{sech}^2\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) \right] - \frac{e^{r_0-r}}{r^2} \right] \\
& \div \frac{1}{16 R_c^2 r^6} \left[e^{-3r} \operatorname{sech}^4\left(\frac{2(r-1)e^{r_0-r}}{R_c r^2}\right) \left(2 R_c^3 \mu e^{3r} r^6 \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + R_c^3 \mu e^{3r} r^6 \right. \right. \\
& \times \left. \left. \sinh\left(\frac{8(r-1)e^{r_0-r}}{R_c r^2}\right) - 8 e^{r_0} \left(R_c^2 e^{2r} (r-1) r^4 + 16 \mu (r^2-2)^2 e^{2r_0} - 16 \mu (r^2-2)^2 e^{r+r_0} \right) \right. \right. \\
& \times \left. \left. \cosh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 6 R_c^2 r^5 e^{2r+r_0} + 6 R_c^2 r^4 e^{2r+r_0} - 2 R_c^2 (r-1) r^4 e^{2r+r_0} \cosh\left(\frac{8(r-1)e^{r_0-r}}{R_c r^2}\right) \right. \right. \\
& \left. \left. + 64 R_c \mu r^2 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 64 R_c \mu r^2 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 32 R_c \mu r^5 e^{2r+r_0} \right. \right. \\
& \times \left. \left. \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 16 R_c \mu r^5 e^{r+2r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 32 R_c \mu r^4 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) \right. \right. \\
& - \left. \left. 32 R_c \mu r^4 e^{r+2r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 64 R_c \mu r^3 e^{2r+r_0} \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) - 32 R_c \mu r^3 e^{r+2r_0} \right. \right. \\
& \times \left. \left. \sinh\left(\frac{4(r-1)e^{r_0-r}}{R_c r^2}\right) + 256 \mu r^4 e^{3r_0} - 256 \mu r^4 e^{r+2r_0} - 1024 \mu r^2 e^{3r_0} + 1024 \mu r^2 e^{r+2r_0} \right. \right. \\
& \left. \left. - 1024 \mu e^{r+2r_0} + 1024 \mu e^{3r_0} \right] \right] \tag{45}
\end{aligned}$$

(0.189333488469534537 / -1.354529260216401693690375)

Input interpretation:

$$\frac{0.189333488469534537}{-1.354529260216401693690375}$$

Result:

-0.13977807200657025709212291418517116652486406029253804873...

-0.139778072006... = p_r / ρ

From which:

$$-((21+2)/10^2)*1/ (0.189333488469534537 *1/ -1.354529260216401693690375)$$

Input interpretation:

$$-\frac{21+2}{10^2} \times \frac{1}{0.189333488469534537 \left(-\frac{1}{1.354529260216401693690375} \right)}$$

Result:

1.645465534745599212446526686268809747510858499904687499206...

1.6454655347.... ≈ ζ(2) = π²/6 = 1.644934 ...

From: **Manuscript Book 2 of Srinivasa Ramanujan**

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We have:

$$\frac{1}{8} \ln(5) + \frac{3}{10} \ln(2) + \frac{3}{4\sqrt{5}} \ln\left(\frac{\sqrt{5}+1}{2}\right) + \frac{1}{40} \sqrt{10-2\sqrt{5}} \ln\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \ln\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right)$$

Input:

$$\frac{1}{8} \log(5) + \frac{3}{10} \log(2) + \frac{3}{4\sqrt{5}} \log\left(\frac{1}{2}(\sqrt{5} + 1)\right) + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \log\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right)$$

log(x) is the natural logarithm

Exact result:

$$\frac{3 \log(2)}{10} + \frac{\log(5)}{8} + \frac{3 \log\left(\frac{1}{2}(1 + \sqrt{5})\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \log\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right)$$

Decimal approximation:

1.000301163885106499300594205362842555304287694349877380846...

1.00030116388510649....

Alternate forms:

$$\begin{aligned} & \frac{3 \log(2)}{10} + \frac{\log(5)}{8} + \frac{3 \log\left(\frac{1}{2}(1 + \sqrt{5})\right)}{4\sqrt{5}} + \\ & \frac{1}{20} \sqrt{\frac{1}{2}(5 - \sqrt{5})} \log\left(11 - 4\sqrt{5} + 2\sqrt{2(25 - 11\sqrt{5})}\right) + \\ & \frac{1}{20} \sqrt{\frac{1}{2}(5 + \sqrt{5})} \log\left(11 + 4\sqrt{5} + 2\sqrt{2(25 + 11\sqrt{5})}\right) \\ & \frac{3 \log(2)}{10} + \frac{\log(5)}{8} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \left(\log\left(4 + \sqrt{10 - 2\sqrt{5}}\right) - \log\left(4 - \sqrt{10 - 2\sqrt{5}}\right) \right) + \\ & \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \left(\log\left(4 + \sqrt{10 + 2\sqrt{5}}\right) - \log\left(4 - \sqrt{10 + 2\sqrt{5}}\right) \right) + \frac{3 \operatorname{csch}^{-1}(2)}{4\sqrt{5}} \\ & \frac{1}{40} \left(12 \log(2) + 5 \log(5) + 6\sqrt{5} \log\left(\frac{1}{2}(1 + \sqrt{5})\right) + \right. \\ & \left. \sqrt{2(5 - \sqrt{5})} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \sqrt{2(5 + \sqrt{5})} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \right) \end{aligned}$$

$\operatorname{csch}^{-1}(x)$ is the inverse hyperbolic cosecant function

Alternative representations:

$$\begin{aligned} & \frac{\log(5)}{8} + \frac{1}{10} \log(2) 3 + \frac{\log\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3}{4\sqrt{5}} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \\ & \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) = \frac{1}{8} \log(a) \log_a(5) + \\ & \frac{3}{10} \log(a) \log_a(2) + \frac{1}{40} \log(a) \log_a\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) \sqrt{10 - 2\sqrt{5}} + \\ & \frac{3 \log(a) \log_a\left(\frac{1}{2}(1 + \sqrt{5})\right)}{4\sqrt{5}} + \frac{1}{40} \log(a) \log_a\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \sqrt{10 + 2\sqrt{5}} \end{aligned}$$

$$\begin{aligned} & \frac{\log(5)}{8} + \frac{1}{10} \log(2) 3 + \frac{\log\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3}{4\sqrt{5}} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \\ & \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) = \\ & \frac{\log_e(5)}{8} + \frac{3 \log_e(2)}{10} + \frac{1}{40} \log_e\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) \sqrt{10 - 2\sqrt{5}} + \\ & \frac{3 \log_e\left(\frac{1}{2}(1 + \sqrt{5})\right)}{4\sqrt{5}} + \frac{1}{40} \log_e\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \sqrt{10 + 2\sqrt{5}} \end{aligned}$$

$$\begin{aligned} & \frac{\log(5)}{8} + \frac{1}{10} \log(2) 3 + \frac{\log\left(\frac{1}{2}(\sqrt{5} + 1)\right) 3}{4\sqrt{5}} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \\ & \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) = \\ & -\frac{\text{Li}_1(-4)}{8} - \frac{3 \text{Li}_1(-1)}{10} - \frac{1}{40} \text{Li}_1\left(1 - \frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) \sqrt{10 - 2\sqrt{5}} - \\ & \frac{3 \text{Li}_1\left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)}{4\sqrt{5}} - \frac{1}{40} \text{Li}_1\left(1 - \frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \sqrt{10 + 2\sqrt{5}} \end{aligned}$$

From which:

$$\begin{aligned} & \left[\frac{1}{8} \ln(5) + \frac{3}{10} \ln(2) + \frac{3}{4\sqrt{5}} \ln\left(\frac{\sqrt{5} + 1}{2}\right) + \frac{1}{40} (10 - 2\sqrt{5})^{1/2} \ln\left(\frac{4 + (10 - 2\sqrt{5})^{1/2}}{4 - (10 - 2\sqrt{5})^{1/2}}\right) + \frac{1}{40} (10 + 2\sqrt{5})^{1/2} \ln\left(\frac{4 + (10 + 2\sqrt{5})^{1/2}}{4 - (10 + 2\sqrt{5})^{1/2}}\right) \right]^{1008} \end{aligned}$$

Input:

$$\left(\frac{1}{8} \log(5) + \frac{3}{10} \log(2) + \frac{3}{4\sqrt{5}} \log\left(\frac{1}{2}(\sqrt{5} + 1)\right) + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \right)^{1008}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\left(\frac{3 \log(2)}{10} + \frac{\log(5)}{8} + \frac{3 \log\left(\frac{1}{2}(1 + \sqrt{5})\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10 - 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 - 2\sqrt{5}}}{4 - \sqrt{10 - 2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10 + 2\sqrt{5}} \log\left(\frac{4 + \sqrt{10 + 2\sqrt{5}}}{4 - \sqrt{10 + 2\sqrt{5}}}\right) \right)^{1008}$$

Decimal approximation:

1.354628832863576262036204429036450040536614991419128393060...

1.354628832863576.... ≈ ρ

We note that, from the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

$$\sqrt{\phi} * \exp(\pi * \sqrt{n/15}) / (2 * 5^{(1/4)} * \sqrt{n})$$

for n = 160, we obtain:

$$\sqrt{\phi} * \exp(\pi * \sqrt{160/15}) / (2 * 5^{(1/4)} * \sqrt{160}) + 47$$

Input:

$$\sqrt{\phi} \times \frac{\exp\left(\pi \sqrt{\frac{160}{15}}\right)}{2 \sqrt[4]{5} \sqrt{160}} + 47$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{4\sqrt{2/3} \pi \sqrt{\frac{\phi}{2}}}}{8 \times 5^{3/4}} + 47$$

Decimal approximation:

1007.939053937509320996194866539663903214764263075917474080...

1007.9390539...

Property:

$$47 + \frac{e^{4\sqrt{2/3} \pi \sqrt{\frac{\phi}{2}}}}{8 \times 5^{3/4}} \text{ is a transcendental number}$$

Alternate forms:

$$47 + \frac{1}{80} \sqrt{5 + \sqrt{5}} e^{4\sqrt{2/3} \pi}$$

$$47 + \frac{\sqrt{1 + \sqrt{5}} e^{4\sqrt{2/3} \pi}}{16 \times 5^{3/4}}$$

$$\frac{1}{80} \left(3760 + \sqrt[4]{5} \sqrt{1 + \sqrt{5}} e^{4\sqrt{2/3} \pi} \right)$$

Series representations:

$$\begin{aligned} \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right)}{2 \sqrt[4]{5} \sqrt{160}} + 47 = & \left(470 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (160 - z_0)^k z_0^{-k}}{k!} + \right. \\ & \left. 5^{3/4} \exp\left[\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{32}{3} - z_0\right)^k z_0^{-k}}{k!}\right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi - z_0)^k z_0^{-k}}{k!} \right) / \\ & \left(10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (160 - z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)) \end{aligned}$$

$$\begin{aligned} \frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right)}{2 \sqrt[4]{5} \sqrt{160}} + 47 = & \left(470 \exp\left(i \pi \left\lfloor \frac{\arg(160 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (160 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + \right. \\ & \left. 5^{3/4} \exp\left(i \pi \left\lfloor \frac{\arg(\phi - x)}{2 \pi} \right\rfloor\right) \exp\left[\pi \exp\left(i \pi \left\lfloor \frac{\arg\left(\frac{32}{3} - x\right)}{2 \pi} \right\rfloor\right) \sqrt{x}\right] \right. \\ & \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{32}{3} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \sum_{k=0}^{\infty} \frac{(-1)^k (\phi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \\ & \left(10 \exp\left(i \pi \left\lfloor \frac{\arg(160 - x)}{2 \pi} \right\rfloor\right) \sum_{k=0}^{\infty} \frac{(-1)^k (160 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \end{aligned}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right)}{2 \sqrt[4]{5} \sqrt{160}} + 47 = \left(\left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(160-z_0)/(2\pi) \rfloor} z_0^{-1/2 \lfloor \arg(160-z_0)/(2\pi) \rfloor} \right. \\ \left. \left(470 \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(160-z_0)/(2\pi) \rfloor} z_0^{1/2 \lfloor \arg(160-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (160-z_0)^k z_0^{-k}}{k!} + \right. \right. \\ \left. \left. 5^{3/4} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg\left(\frac{32}{3}-z_0\right)/(2\pi) \rfloor} z_0^{1/2 (1+\lfloor \arg\left(\frac{32}{3}-z_0\right)/(2\pi) \rfloor)} \right. \right. \right. \\ \left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{32}{3}-z_0\right)^k z_0^{-k}}{k!} \right) \left(\frac{1}{z_0}\right)^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \right. \\ \left. \left. z_0^{1/2 \lfloor \arg(\phi-z_0)/(2\pi) \rfloor} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) \right) / \\ \left(10 \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (160-z_0)^k z_0^{-k}}{k!} \right)$$

Thence, for the previous expression, we obtain:

$$1.0003011638851064993005942^{((((\sqrt{\text{golden ratio}} * \exp(\text{Pi} * \sqrt{160/15})) / (2 * 5^{(1/4)} * \sqrt{160}) + 47))))}$$

Input interpretation:

$$1.0003011638851064993005942^{\left(\frac{\sqrt{\phi} \times \exp\left(\pi \sqrt{\frac{160}{15}}\right)}{\left(2 \sqrt[4]{5} \sqrt{160}\right) + 47}\right)}$$

ϕ is the golden ratio

Result:

1.3546039729574088491492...

1.3546039729574... $\approx \rho$

Series representations:

$$1.00030116388510649930059420000 \left(\frac{\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right)}{\left(2 \sqrt[4]{5} \sqrt{160}\right) + 47} \right) = \\ 1.0142531933516966087006200788 \times \\ 1.0003011638851064993005942000 \cdot \\ \left(\exp\left(\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{32}{3}-z_0\right)^k z_0^{-k}}{k!}\right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} \right) / \left(2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (160-z_0)^k z_0^{-k}}{k!} \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))$$

$$1.00030116388510649930059420000 \left(\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right) \right) / (2 \sqrt[4]{5} \sqrt{160})^{+47} =$$

$$1.0142531933516966087006200788 \times$$

$$1.00030116388510649930059420000 \cdot$$

$$\int_0^{\infty} \exp\left(i\pi \left[\frac{\arg(\phi-x)}{2\pi} \right]\right) \exp\left(i\pi \left[\frac{\arg\left(\frac{32-x}{3}\right)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{32-x}{3}\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \left(\frac{(-1)^k (\phi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) / \left(2 \sqrt[4]{5} \exp\left(i\pi \left[\frac{\arg(160-x)}{2\pi} \right]\right) \sum_{k=0}^{\infty} \frac{(-1)^k (160-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$1.00030116388510649930059420000 \left(\sqrt{\phi} \exp\left(\pi \sqrt{\frac{160}{15}}\right) \right) / (2 \sqrt[4]{5} \sqrt{160})^{+47} =$$

$$1.0142531933516966087006200788 \times$$

$$1.00030116388510649930059420000 \cdot$$

$$\int_0^{\infty} \exp\left(\pi \left(\frac{1}{z_0}\right)^{1/2} \left[\frac{\arg\left(\frac{32-z_0}{3}\right)}{2\pi} \right] \right)^{1/2} \left(1 + \left[\frac{\arg(z_0)}{2\pi} \right] \right) \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)_k \left(\frac{32-z_0}{3}\right)^k z_0^{-k}}{k!} \left(\frac{1}{z_0} \right)^{-1/2} \left[\frac{\arg(160-z_0)}{2\pi} \right] + 1/2 \left[\frac{\arg(\phi-z_0)}{2\pi} \right] \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)_k (\phi-z_0)^k z_0^{-k}}{k!} / \left(2 \sqrt[4]{5} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{1}{2}\right)_k (160-z_0)^k z_0^{-k}}{k!} \right)$$

$n!$ is the factorial function

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

$\Gamma(x)$ is the gamma function

$\text{Re}(z)$ is the real part of z

$|z|$ is the absolute value of z

$$\left[\frac{1}{8} \ln(5) + \frac{3}{10} \ln(2) + \frac{3}{4\sqrt{5}} \ln\left(\frac{\sqrt{5}+1}{2}\right) + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \log\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right) \right]^{1598}$$

Input:

$$\left(\frac{1}{8} \log(5) + \frac{3}{10} \log(2) + \frac{3}{4\sqrt{5}} \log\left(\frac{1}{2}(\sqrt{5}+1)\right) + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \log\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right) \right)^{1598}$$

$\log(x)$ is the natural logarithm

Exact result:

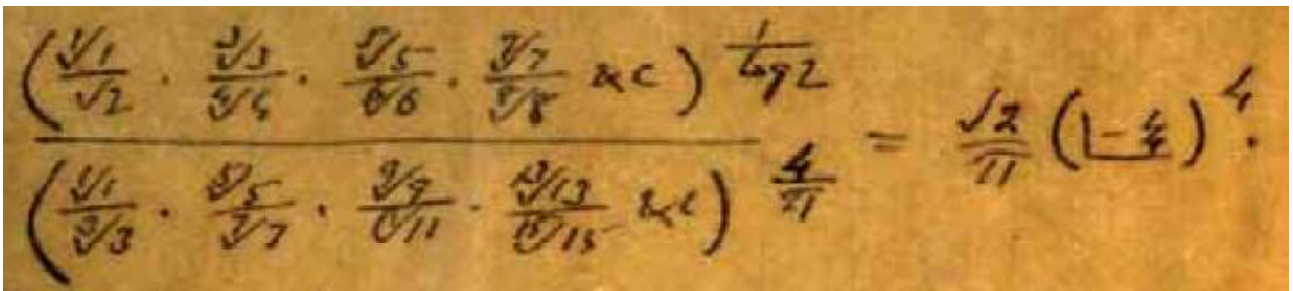
$$\left(\frac{3 \log(2)}{10} + \frac{\log(5)}{8} + \frac{3 \log\left(\frac{1}{2}(1+\sqrt{5})\right)}{4\sqrt{5}} + \frac{1}{40} \sqrt{10-2\sqrt{5}} \log\left(\frac{4+\sqrt{10-2\sqrt{5}}}{4-\sqrt{10-2\sqrt{5}}}\right) + \frac{1}{40} \sqrt{10+2\sqrt{5}} \log\left(\frac{4+\sqrt{10+2\sqrt{5}}}{4-\sqrt{10+2\sqrt{5}}}\right) \right)^{1598}$$

Decimal approximation:

1.617994523502815698633408228493231601606909823739008997699...

1.6179945235.... result that is a very good approximation to the value of the golden ratio 1,618033988749...

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$$\frac{((1)^{1/2} / (2)^{1/2} * (3)^{1/3} / (4)^{1/4} * (5)^{1/5} / (6)^{1/6} * (7)^{1/7} / (8)^{1/8})^{1/(\ln(2))}}{((1)^{1/2} / (3)^{1/3} * (5)^{1/5} / (7)^{1/7} * (9)^{1/9} / (11)^{1/11} * (13)^{1/13} / (15)^{1/15})^{(4/\pi)}}$$

Input:

$$\frac{\log(2) \sqrt{\frac{\sqrt{1}}{\sqrt{2}} \times \frac{\sqrt[3]{3}}{\sqrt[4]{4}} \times \frac{\sqrt[5]{5}}{\sqrt[6]{6}} \times \frac{\sqrt[7]{7}}{\sqrt[8]{8}}}}{\left(\frac{\sqrt{1}}{\sqrt[3]{3}} \times \frac{\sqrt[5]{5}}{\sqrt[7]{7}} \times \frac{\sqrt[9]{9}}{\sqrt[11]{11}} \times \frac{\sqrt[13]{13}}{\sqrt[15]{15}}\right)^{4/\pi}}$$

log(x) is the natural logarithm

Exact result:

$$\frac{1}{e} 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \times 2^{-13/(24 \log(2))} \times 3^{32/(45\pi)+1/(6 \log(2))} \times 5^{1/(5 \log(2))-8/(15\pi)} \times 7^{4/(7\pi)+1/(7 \log(2))}$$

Decimal approximation:

0.945219325331322556131536505652913783767455009206975077031...

0.94521932533... result very near to the spectral index n_s , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}} \approx 0.9568666373$$

Alternative representations:

$$\frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15})^2 \sqrt[7]{7}}\right)^{4/\pi}} = \frac{\log_e(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15})^2 \sqrt[7]{7}}\right)^{4/\pi}} = \frac{2 \coth^{-1}(3) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15})^2 \sqrt[7]{7}}\right)^{4/\pi}} = \frac{\log(a) \log_a(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

Series representations:

$$\frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15})^2 \sqrt[7]{7}}\right)^{4/\pi}} = \frac{1}{e} 2^{-13} \left(24 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times$$

$$3^{32/(45\pi)+1} \left(6 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times$$

$$5^{-8/(15\pi)+1} \left(5 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times$$

$$7^{4/(7\pi)+1} \left(7 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \text{ for } x < 0$$

$$\begin{aligned}
& \frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15})^7 \sqrt[7]{7}} \right)^{4/\pi}} = \frac{1}{e} \\
& 2^{-13} / \left(24 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 3^{32/(45\pi)+1} / \left(6 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 5^{-8/(15\pi)+1} / \left(5 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 7^{4/(7\pi)+1} / \left(7 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \\
& \frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15})^7 \sqrt[7]{7}} \right)^{4/\pi}} = \frac{1}{e} 2^{-13} / \left(24 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 3^{32/(45\pi)+1} / \left(6 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 5^{-8/(15\pi)+1} / \left(5 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \\
& 7^{4/(7\pi)+1} / \left(7 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)}
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15})^7 \sqrt[7]{7}} \right)^{4/\pi}} = \frac{1}{e} 2^{-13} / \left(24 \int_1^2 \frac{1}{t} dt \right) \times 3^{32/(45\pi)+1} / \left(6 \int_1^2 \frac{1}{t} dt \right) \times \\
& 5^{-8/(15\pi)+1} / \left(5 \int_1^2 \frac{1}{t} dt \right) \times 7^{4/(7\pi)+1} / \left(7 \int_1^2 \frac{1}{t} dt \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)}
\end{aligned}$$

$$\frac{\log(2) \sqrt{\frac{(\sqrt{1} \sqrt[5]{5} \sqrt[7]{7})^3 \sqrt[3]{3}}{(\sqrt{2} \sqrt[6]{6} \sqrt[8]{8})^4 \sqrt[4]{4}}}}{\left(\frac{(\sqrt{1} \sqrt[9]{9} \sqrt[13]{13})^5 \sqrt[5]{5}}{(\sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15})^7 \sqrt[7]{7}}\right)^{4/\pi}} = \frac{1}{e} 2^{-13i\pi} \left(12 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times$$

$$3^{32/(45\pi)+i\pi} \left(3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 5^{-8/(15\pi)+2i\pi} \left(5 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times$$

$$7^{4/(7\pi)+2i\pi} \left(7 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \text{ for } -1 < \gamma < 0$$

$$\sqrt{2} \left(\frac{1^{1/2}}{2^{1/2}} \cdot \frac{3^{1/3}}{4^{1/4}} \cdot \frac{5^{1/5}}{6^{1/6}} \cdot \frac{7^{1/7}}{8^{1/8}} \right)^{1/\ln(2)} \div \left(\frac{1^{1/2}}{3^{1/3}} \cdot \frac{5^{1/5}}{7^{1/7}} \cdot \frac{9^{1/9}}{11^{1/11}} \cdot \frac{13^{1/13}}{15^{1/15}} \right)^{4/\pi}$$

Input:

$$\sqrt{2} \times \frac{\log(2) \sqrt{\frac{\sqrt{1}}{\sqrt{2}} \times \frac{\sqrt[3]{3}}{\sqrt[4]{4}} \times \frac{\sqrt[5]{5}}{\sqrt[6]{6}} \times \frac{\sqrt[7]{7}}{\sqrt[8]{8}}}}{\left(\frac{\sqrt{1}}{\sqrt[3]{3}} \times \frac{\sqrt[5]{5}}{\sqrt[7]{7}} \times \frac{\sqrt[9]{9}}{\sqrt[11]{11}} \times \frac{\sqrt[13]{13}}{\sqrt[15]{15}}\right)^{4/\pi}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{\sqrt{2} 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \times 3^{32/(45\pi)+1/(6\log(2))} \times 5^{1/(5\log(2))-8/(15\pi)} \times 7^{4/(7\pi)+1/(7\log(2))}}{e^{37/24}}$$

Decimal approximation:

1.336741989300703152590514262199038178317406264039080945238...

1.3367419893... result very near to the value of ρ

Alternative representations:

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{\log_e(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{2 \coth^{-1}(3) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{\log(a) \log_a(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

Series representations:

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} \sqrt{2} 3^{32/(45\pi)+1} \left/ \left(6 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times \right.$$

$$5^{-8/(15\pi)+1} \left/ \left(5 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times \right.$$

$$7^{4/(7\pi)+1} \left/ \left(7 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \text{ for } x < 0$$

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}}$$

$$\sqrt{2} 3^{32/(45\pi)+1} \left/ \left(6 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right.$$

$$5^{-8/(15\pi)+1} \left/ \left(5 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right.$$

$$7^{4/(7\pi)+1} \left/ \left(7 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right.$$

$$11^{4/(11\pi)} \times 13^{-4/(13\pi)}$$

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3}}{4} \frac{\sqrt{1}}{\sqrt{2}} \frac{5\sqrt{5}}{\sqrt{6}} \frac{7\sqrt{7}}{\sqrt{8}}}}{\left(\frac{5\sqrt{5}}{7\sqrt{7}} \frac{\sqrt{1}}{\sqrt{3}} \frac{9\sqrt{9}}{11\sqrt{11}} \frac{13\sqrt{13}}{15\sqrt{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} \sqrt{2} 3^{32/(45\pi)+1} \left(6 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$5^{-8/(15\pi)+1} \left(5 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$7^{4/(7\pi)+1} \left(7 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)}$$

Integral representations:

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3}}{4} \frac{\sqrt{1}}{\sqrt{2}} \frac{5\sqrt{5}}{\sqrt{6}} \frac{7\sqrt{7}}{\sqrt{8}}}}{\left(\frac{5\sqrt{5}}{7\sqrt{7}} \frac{\sqrt{1}}{\sqrt{3}} \frac{9\sqrt{9}}{11\sqrt{11}} \frac{13\sqrt{13}}{15\sqrt{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} \sqrt{2} 3^{32/(45\pi)+1} \left(6 \int_1^2 \frac{1}{t} dt \right) \times$$

$$5^{-8/(15\pi)+1} \left(5 \int_1^2 \frac{1}{t} dt \right) \times 7^{4/(7\pi)+1} \left(7 \int_1^2 \frac{1}{t} dt \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)}$$

$$\frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3}}{4} \frac{\sqrt{1}}{\sqrt{2}} \frac{5\sqrt{5}}{\sqrt{6}} \frac{7\sqrt{7}}{\sqrt{8}}}}{\left(\frac{5\sqrt{5}}{7\sqrt{7}} \frac{\sqrt{1}}{\sqrt{3}} \frac{9\sqrt{9}}{11\sqrt{11}} \frac{13\sqrt{13}}{15\sqrt{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} \sqrt{2}$$

$$3^{32/(45\pi)+(i\pi)} \left(3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times 5^{-8/(15\pi)+(2i\pi)} \left(5 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times$$

$$7^{4/(7\pi)+(2i\pi)} \left(7 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \text{ for } -1 < \gamma < 0$$

$$(55+2)/10^3 + \sqrt{\pi} \left(\frac{1}{\sqrt{2}} \times \frac{3\sqrt{3}}{4} \times \frac{5\sqrt{5}}{\sqrt{6}} \times \frac{7\sqrt{7}}{\sqrt{8}} \right)^{\log(2)}$$

$$\frac{55+2}{10^3} + \sqrt{\pi} \times \frac{\sqrt{\frac{3\sqrt{3}}{4} \frac{\sqrt{1}}{\sqrt{2}} \frac{5\sqrt{5}}{\sqrt{6}} \frac{7\sqrt{7}}{\sqrt{8}}}}{\left(\frac{\sqrt{1}}{3\sqrt{3}} \times \frac{5\sqrt{5}}{7\sqrt{7}} \times \frac{9\sqrt{9}}{11\sqrt{11}} \times \frac{13\sqrt{13}}{15\sqrt{15}}\right)^{4/\pi}}$$

Input:

$$\frac{55+2}{10^3} + \sqrt{\pi} \times \frac{\sqrt{\frac{3\sqrt{3}}{4} \frac{\sqrt{1}}{\sqrt{2}} \frac{5\sqrt{5}}{\sqrt{6}} \frac{7\sqrt{7}}{\sqrt{8}}}}{\left(\frac{\sqrt{1}}{3\sqrt{3}} \times \frac{5\sqrt{5}}{7\sqrt{7}} \times \frac{9\sqrt{9}}{11\sqrt{11}} \times \frac{13\sqrt{13}}{15\sqrt{15}}\right)^{4/\pi}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{57}{1000} + \frac{1}{e} 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \sqrt{\pi} 2^{-13/(24 \log(2))} \times 3^{32/(45\pi)+1/(6 \log(2))} \times 5^{1/(5 \log(2))-8/(15\pi)} \times 7^{4/(7\pi)+1/(7 \log(2))}$$

Decimal approximation:

1.732357633133816438606656312983222832907779525581040188821...

$1.732357633133... \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

(see: Can massless wormholes mimic a Schwarzschild black hole in the strong field lensing? - arXiv:1909.13052v1 [gr-qc] 28 Sep 2019)

Alternate forms:

$$\frac{57}{1000} + \frac{11^{4/(11\pi)} \times 13^{-4/(13\pi)} \sqrt{\pi} 3^{32/(45\pi)+1/\log(64)} \times 5^{1/\log(32)-8/(15\pi)} \times 7^{1/7(4/\pi+1/\log(2))}}{e^{37/24}}$$

$$\frac{1}{8 e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(57 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{2^{13/(24 \log(2))}} + 8 \times 11^{4/(11\pi)} \sqrt{\pi} 3^{32/(45\pi)+1/(6 \log(2))} \times 5^{3+1/(5 \log(2))} \times 7^{4/(7\pi)+1/(7 \log(2))} \right)$$

Alternative representations:

$$\frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt{6} \sqrt{8}}}}{\left(\frac{5\sqrt{5} \sqrt{1} \sqrt{9} \sqrt{13}}{\sqrt{7} \sqrt{3} \sqrt{11} \sqrt{15}}\right)^{4/\pi}} = \frac{57}{10^3} + \frac{\log_e(2) \sqrt{\frac{\sqrt{1} \sqrt{3} \sqrt{5} \sqrt{7}}{\sqrt{2} \sqrt{4} \sqrt{6} \sqrt{8}}} \sqrt{\pi}}{\left(\frac{\sqrt{1} \sqrt{5} \sqrt{9} \sqrt{13}}{\sqrt{3} \sqrt{7} \sqrt{11} \sqrt{15}}\right)^{4/\pi}}$$

$$\frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt{6} \sqrt{8}}}}{\left(\frac{5\sqrt{5} \sqrt{1} \sqrt{9} \sqrt{13}}{\sqrt{7} \sqrt{3} \sqrt{11} \sqrt{15}}\right)^{4/\pi}} = \frac{57}{10^3} + \frac{2 \coth^{-1}(3) \sqrt{\frac{\sqrt{1} \sqrt{3} \sqrt{5} \sqrt{7}}{\sqrt{2} \sqrt{4} \sqrt{6} \sqrt{8}}} \sqrt{\pi}}{\left(\frac{\sqrt{1} \sqrt{5} \sqrt{9} \sqrt{13}}{\sqrt{3} \sqrt{7} \sqrt{11} \sqrt{15}}\right)^{4/\pi}}$$

$$\frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{57}{10^3} + \frac{\log(a) \log_a(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{\pi}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

Series representations:

$$\begin{aligned} \frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \\ \frac{1}{e} 2^{-3-13} / \left(24 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \\ \left(57 \times 2^{13} / \left(24 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e + \right. \\ \left. 8 \times 3^{32/(45\pi)+1} / \left(6 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times \right. \\ \left. 5^{3+1} / \left(5 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times \right. \\ \left. 7^{4/(7\pi)+1} / \left(7 \left(2i\pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} \sqrt{\pi} \right) \text{ for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e} \\ 2^{-3-13} / \left(24 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times 5^{-3-8/(15\pi)} \times \\ 13^{-4/(13\pi)} \left(57 \times 2^{13} / \left(24 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right. \\ \left. 5^{8/(15\pi)} \times 13^{4/(13\pi)} e + \right. \\ \left. 8 \times 3^{32/(45\pi)+1} / \left(6 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right. \\ \left. 5^{3+1} / \left(5 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right. \\ \left. 7^{4/(7\pi)+1} / \left(7 \left(\log(z_0) + \left\lfloor \frac{\arg(2-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times \right. \\ \left. 11^{4/(11\pi)} \sqrt{\pi} \right) \end{aligned}$$

$$\begin{aligned}
& \frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4 \sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \\
& \frac{1}{e} \left[2^{-3-13} \left(24 \left[2i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right] \right) \times 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \right. \\
& \left. \left(57 \times 2^{13} \left[24 \left[2i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right] \right) \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e + \right. \right. \\
& \left. \left. 8 \times 3^{32/(45\pi)+1} \left(6 \left[2i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right] \right) \times \right. \right. \\
& \left. \left. 5^{3+1} \left(5 \left[2i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right] \right) \times \right. \right. \\
& \left. \left. 7^{4/(7\pi)+1} \left(7 \left[2i\pi \left| \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right| + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right] \right) \times 11^{4/(11\pi)} \sqrt{\pi} \right) \right]
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4 \sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e} \\
& 2^{-3-13} \left(24 \int_1^2 \frac{1}{t} dt \right) \times 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(57 \times 2^{13} \left(24 \int_1^2 \frac{1}{t} dt \right) \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e + \right. \\
& \left. 8 \times 3^{32/(45\pi)+1} \left(6 \int_1^2 \frac{1}{t} dt \right) \times 5^{3+1} \left(5 \int_1^2 \frac{1}{t} dt \right) \times 7^{4/(7\pi)+1} \left(7 \int_1^2 \frac{1}{t} dt \right) \times 11^{4/(11\pi)} \sqrt{\pi} \right)
\end{aligned}$$

$$\frac{55+2}{10^3} + \frac{\sqrt{\pi} \log(2) \sqrt{\frac{\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e} 2^{-3-(13i\pi)} \left(12 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 5^{-3-8/(15\pi)} \times$$

$$13^{-4/(13\pi)} \left(57 \times 2^{(13i\pi)} \left(12 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e + \right.$$

$$8 \times 3^{32/(45\pi)+(i\pi)} \left(3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 5^{3+(2i\pi)} \left(5 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times$$

$$\left. 7^{4/(7\pi)+(2i\pi)} \left(7 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right) \times 11^{4/(11\pi)} \sqrt{\pi}\right) \text{ for } -1 < \gamma < 0$$

$$(89+47)/10^3 + \sqrt{2} \left(\frac{((1)^{1/2} / (2)^{1/2} (3)^{1/3} / (4)^{1/4} (5)^{1/5} / (6)^{1/6} (7)^{1/7} / (8)^{1/8})^{1/(\ln(2))}}{((1)^{1/2} / (3)^{1/3} (5)^{1/5} / (7)^{1/7} (9)^{1/9} / (11)^{1/11} (13)^{1/13} / (15)^{1/15})^{4/\pi}} \right)$$

Input:

$$\frac{89+47}{10^3} + \sqrt{2} \times \frac{\log(2) \sqrt{\frac{\sqrt{1} \times \sqrt[3]{3} \times \sqrt[5]{5} \times \sqrt[7]{7}}{\sqrt{2} \times \sqrt[4]{4} \times \sqrt[6]{6} \times \sqrt[8]{8}}}}{\left(\frac{\sqrt{1} \times \sqrt[5]{5} \times \sqrt[9]{9} \times \sqrt[13]{13}}{\sqrt[3]{3} \times \sqrt[7]{7} \times \sqrt[11]{11} \times \sqrt[15]{15}}\right)^{4/\pi}}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{17}{125} + \frac{\sqrt{2} 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \times 3^{32/(45\pi)+1/(6\log(2))} \times 5^{1/(5\log(2))-8/(15\pi)} \times 7^{4/(7\pi)+1/(7\log(2))}}{e^{37/24}}$$

Decimal approximation:

1.472741989300703152590514262199038178317406264039080945238...

1.4727419893.... $\approx p_t$

Alternate forms:

$$\frac{1}{125 e^{37/24}} \left(17 e^{37/24} + \sqrt{2} 11^{4/(11\pi)} \times 13^{-4/(13\pi)} \times 3^{32/(45\pi)+1/(6\log(2))} \times 5^{3-8/(15\pi)+1/(5\log(2))} \times 7^{4/(7\pi)+1/(7\log(2))} \right)$$

$$\frac{1}{e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} + \sqrt{2} 11^{4/(11\pi)} \times 3^{32/(45\pi)+1/(6\log(2))} \times 5^{3+1/(5\log(2))} \times 7^{4/(7\pi)+1/(7\log(2))} \right)$$

Alternative representations:

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{136}{10^3} + \frac{\log_e(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{136}{10^3} + \frac{2 \coth^{-1}(3) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{136}{10^3} + \frac{\log(a) \log_a(2) \sqrt{\frac{\sqrt{1} \sqrt[3]{3} \sqrt[5]{5} \sqrt[7]{7}}{\sqrt{2} \sqrt[4]{4} \sqrt[6]{6} \sqrt[8]{8}}} \sqrt{2}}{\left(\frac{\sqrt{1} \sqrt[5]{5} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[3]{3} \sqrt[7]{7} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}}$$

Series representations:

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(\sqrt{2} 3^{32/(45\pi)+1} / \left(6 \left(2i\pi \left[\frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 5^{3+1} / \left(5 \left(2i\pi \left[\frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 7^{4/(7\pi)+1} / \left(7 \left(2i\pi \left[\frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right) \times 11^{4/(11\pi)} + 17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} \right) \text{ for } x < 0$$

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt{6} \sqrt{8}}}}{\left(\frac{5\sqrt{5} \sqrt{1} \sqrt{9} \sqrt{13}}{7\sqrt{7} \sqrt{3} \sqrt{11} \sqrt{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)}$$

$$\left(\sqrt{2} \ 3^{32/(45\pi)+1} / \left(6 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \right) \times$$

$$5^{3+1} / \left(5 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$7^{4/(7\pi)+1} / \left(7 \left(\log(z_0) + \left[\frac{\arg(2-z_0)}{2\pi} \right] \left(\log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$\left. 11^{4/(11\pi)} + 17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} \right)$$

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt{3} \sqrt{1} \sqrt{5} \sqrt{7}}{4\sqrt{4} \sqrt{2} \sqrt{6} \sqrt{8}}}}{\left(\frac{5\sqrt{5} \sqrt{1} \sqrt{9} \sqrt{13}}{7\sqrt{7} \sqrt{3} \sqrt{11} \sqrt{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)}$$

$$\left(\sqrt{2} \ 3^{32/(45\pi)+1} / \left(6 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \right) \times$$

$$5^{3+1} / \left(5 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$7^{4/(7\pi)+1} / \left(7 \left(2i\pi \left[\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right) \times$$

$$\left. 11^{4/(11\pi)} + 17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} \right)$$

Integral representations:

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{5\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{7\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} = \frac{1}{e^{37/24}}$$

$$5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(\sqrt{2} 3^{32/(45\pi)+1} \left/ \left(6 \int_1^2 \frac{1}{t} dt \right) \times 5^{3+1} \left/ \left(5 \int_1^2 \frac{1}{t} dt \right) \times \right. \right.$$

$$\left. \left. 7^{4/(7\pi)+1} \left/ \left(7 \int_1^2 \frac{1}{t} dt \right) \times 11^{4/(11\pi)} + 17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} \right) \right)$$

$$\frac{89+47}{10^3} + \frac{\sqrt{2} \log(2) \sqrt{\frac{3\sqrt[3]{3} \sqrt{1} \sqrt[5]{5} \sqrt[7]{7}}{4\sqrt[4]{4} \sqrt{2} \sqrt[6]{6} \sqrt[8]{8}}}}{\left(\frac{5\sqrt[5]{5} \sqrt{1} \sqrt[9]{9} \sqrt[13]{13}}{7\sqrt[7]{7} \sqrt[3]{3} \sqrt[11]{11} \sqrt[15]{15}}\right)^{4/\pi}} =$$

$$\frac{1}{e^{37/24}} 5^{-3-8/(15\pi)} \times 13^{-4/(13\pi)} \left(\sqrt{2} 3^{32/(45\pi)+(i\pi)} \left/ \left(3 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times \right.$$

$$5^{3+(2i\pi)} \left/ \left(5 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times 7^{4/(7\pi)+(2i\pi)} \left/ \left(7 \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \times \right.$$

$$\left. \left. 11^{4/(11\pi)} + 17 \times 5^{8/(15\pi)} \times 13^{4/(13\pi)} e^{37/24} \right) \text{ for } -1 < \gamma < 0$$

Observations

Figs.

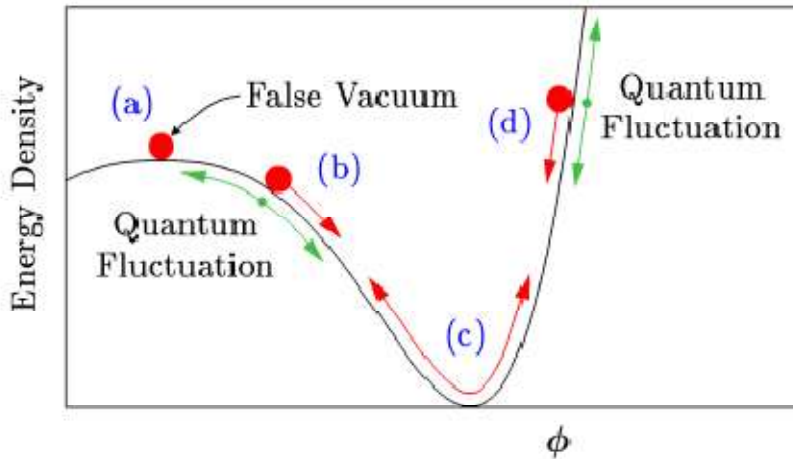
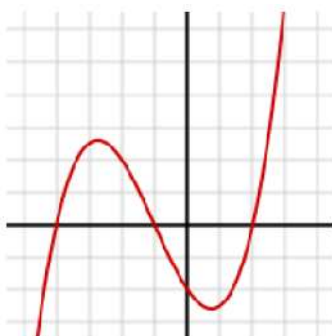


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ . Red arrows show the classical motion of ϕ . When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ : Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of “slow roll,” ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3}) M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3(2.17049 \times 10^{37})^2 - 0.001^2}}{\frac{1}{2}((3\sqrt{3})(4.2 \times 10^6 \times 1.9891 \times 10^{30}))}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2} + \frac{i}{2} \sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2} \sqrt{3}\right)$$

$$i\sqrt{3}$$

i is the imaginary unit

1.732050807568877293527446341505872366942805253810380628055... i

$r \approx 1.73205$ (radius), $\theta = 90^\circ$ (angle)

1.73205

This result is very near to the ratio between M_0 and q , that is equal to $1.7320507879 \approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$$= 1.732050807568877293527446341505872366942805253810380628055... i$$

$r \approx 1.73205$ (radius), $\theta = 90^\circ$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) + v^2(-v)\left(\frac{1}{2} \pm \frac{i\sqrt{3}}{2}\right) = q$$

Considering:

$$(-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q$$

$$= i\sqrt{3} = 1.732050807568877293527446341505872366942805253810380628055... i$$

$r \approx 1.73205$ (radius), $\theta = 90^\circ$ (angle)

Thence:

$$\left(-\frac{1}{2} + \frac{i}{2}\sqrt{3}\right) - \left(-\frac{1}{2} - \frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow (-1)\left(\frac{1}{2} - \frac{i\sqrt{3}}{2}\right) - (-1)\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = q = 1.73205 \approx \sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns,

such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from

0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ...
(sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is ϕ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of ϕ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

Traversable Wormholes with Exponential Shape Function in Modified Gravity and in General Relativity: A Comparative Study

Gauranga C. Samanta¹, Nisha Godani and Kazuharu Bamba - arXiv:1811.06834v2
[gr-qc] 23 Oct 2019

Manuscript Book 2 of Srinivasa Ramanujan