Shannon's Information and Heisenberg's Uncertainty

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Abstract: It is shown that the least possible amount of Shannon's information (a single bit) corresponds to the least possible value of the Heisenberg Uncertainty Principle.

As Richard Feynman noted nearly 60 years, physicists gave up even trying to understand quantum theory, long ago. But it is not that hard to arrive at a common-sense interpretation, if one actually tries.

Here are a few points to consider:
(1) In the 1920's the Stern Gerlach Experiment showed that some quantum observables, like spin, only took-on one of two values. This suggests that such an observable encodes only a single-bit-of-information. But at the time, "information" was not understood.

(2) In the late 1940's Claude Shannon published his Capacity Theorem specifying the maximum number of bits of information per second (C), that can be recovered by any set of measurements of a continuous signal:

$C = \Delta f \log_2(1+S/N)$, where Δf is the bandwidth and S/N is the signal-to-noise ratio.

(3) Multiplying Shannon's Capacity, by the duration (Δt) of the observations, yields the maximum number of bits of information that can be recovered, from a signal with a finite duration:
Max #bits = $\Delta t\ \Delta f \log_2(1+S/N)$

(4) The single-bit noted above in (1), thus suggests setting: $1 = (\Delta t\ \Delta f = 1)(\log_2(1+S/N) = 1)$, so $\Delta t\ \Delta f = 1$.

(5) Combining the following:
Distance traveled by a photon at light speed: $c\Delta t = \Delta x$ with the Uncertainty Principle: $\Delta x = h/\Delta p$, yields: $c\Delta t = \Delta x = h/\Delta p$

Combining the following:
The de Broglie relation: $\lambda = h/p$ with $\lambda = c/f$ from the relation between wavelength and frequency, yields: $h/p = c/f$, so $p = hf/c$, so $\Delta p = (h/c)\Delta f$

Combining these two results yields: $c\Delta t = \Delta x = h/\Delta p = h/((h/c)\Delta f) = c/\Delta f$

Hence: $c\Delta t = c/\Delta f$, so:
$\Delta t\Delta f = 1$, the same expression obtained in (4), for a single-bit of information.

In other words, the Heisenberg uncertainty principle corresponds to the definition of a single-bit of information, in the limiting case in which the minimum possible amount of information results in the minimum possible value for $\Delta x\Delta p$.

This fact provides a motivation for examining the behavior of classical objects, constructed such that they each encode only a single-bit of information: objects that have a severely limited duration, bandwidth and S/N, in accordance with Shannon's Capacity expression. For example, do such objects behave in the same manner as quantum entities, when subjected to Bell-tests? They do.