An explanation of the electron and its wavefunction

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26 March 2020
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Summary

This paper recaps our electron model – including our classical explanation of the anomalous magnetic moment – and slightly revises our interpretation of the elementary wavefunction that describes it. We also add some material from previous papers to combine all of the aspects of our electron model in one single paper.

Contents

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Introduction: basic concepts

The electromagnetic force **F** on a charge q is equal to **F** = q·(**E** + ***v***×**B**). This force is the sum of two (orthogonal) *component* vectors: q·**E** and q·***v***×**B**.

The velocity vector ***v*** in the equation shows *both* of these two component force vectors depend on our frame of reference. Hence, we should think of the separation of the electromagnetic force into an 'electric' (or electrostatic) and a 'magnetic' force *component* as being somewhat artificial: the *electromagnetic* force is (very) *real* – because it determines the *motion* of the charge – but our cutting-up of it in two separate components depends on our frame of reference and is, therefore, (very) *relative*.

At this point, we should probably also quickly note that both amateur as well as professional physicists often tend to neglect the magnetic force in their analysis because the *magnitude* of the magnetic field – and, therefore, of the force – is 1/*c* times that of the *electric* field or force. Hence, they often think of the magnetic force as a tiny – and, therefore, negligible – *fraction* of the electric force. That's a *huge* mistake, which becomes very obvious when using natural time and distance units so as to ensure Nature's constant is set to unity (*c* = 1). We will come back to this.

[…]

The remarks above sound rather trivial. Somewhat less easy to appreciate, perhaps, is that the concept of a force combines two different ideas. One is the idea of inertia: inertia is a measure of the resistance to a change of the state of motion. This measure is what we commonly refer to as *mass*. We think of it as a scalar (non-directional) quantity[1] and it is reflected in the relativistically correct expression of Newton's Law:

$$\mathbf{F} = \mathrm{m}_v \cdot \boldsymbol{a} = \frac{\mathrm{d}(\mathrm{m}_v \cdot \boldsymbol{v})}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$$

$$\mathrm{m}_v = \gamma \cdot \mathrm{m}_0 = \frac{1}{\sqrt{1 - v^2/c^2}} \cdot \mathrm{m}_0$$

[1] This may sound even more trivial than our introductory remarks but we may usefully remind ourselves that Albert Einstein, in his seminal 1905 article introducing the principle of relativity, did not hesitate to make a distinction between the "longitudinal" and "transverse" mass of a moving charge. We will come back to this because, at one point in the development of our argument, we will actually make use of that distinction ourselves.

The second idea is the idea of a charge: the electromagnetic force acts on an 'electric' charge.[2] Maxwell's equations describe how this happens—not approximately, but *exactly*.

Maxwell's equations also describe how – in the absence of a charge to *act* on – an electromagnetic *wave* propagates in space and, thereby, changes the *fields*. The latter idea – the idea of a *field*, traveling or static – is very deep—as fundamental as the idea of a charge, or the idea of a force itself. Indeed, the equations below – which combine the idea of inertia to motion and the electromagnetic force law – show that the electric and magnetic force are the product of (1) the electric and magnetic field respectively and (2) the charge (which is usually measured in terms of the historical Coulomb unit rather than in terms of the elementary charge[3]):

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\mathrm{m}_0 \cdot \boldsymbol{v}}{\sqrt{1 - v^2/c^2}}\right] = \boldsymbol{F} = \boldsymbol{F_E} + \boldsymbol{F_B}$$

$$= \mathrm{q}\boldsymbol{E} + \mathrm{q}\boldsymbol{v}\times\boldsymbol{B} = \mathrm{q}(\boldsymbol{E} + \boldsymbol{v}\times\boldsymbol{B})$$

[…]

As mentioned above, the *magnitude* of the magnetic field – and, therefore, of the force – is $1/c$ times that of the *electric* field or force. Because we use the historical *meter* and *second* unit for distance and time respectively, we think of the magnetic force being a *fraction* of the electric force only. That distorts our picture of what might be going at the smallest scale. Both the meter as well as the second are very large units at the sub-atomic scale. What we refer to as the clock speed[4] of an electron (which we denote as the cycle time T), for example, is expressed in units of 10^{-21} seconds. Very small, indeed. Especially when compared to the unit we use to describe the size of an electron[5], which is expressed in *pico*-meter (10^{-12} m): 10^{-12} is tiny too, but it is much larger than 10^{-21}.

[..]

We can combine the $\lambda_C = hc/\mathrm{E}$ and $\mathrm{T} = 1/f = h/\mathrm{E}$ equations to get the following fundamental *expression* for what – in our not-so-humble view – an electron actually *is*:

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[2] Other forces – not acting on a charge – may be thought of as non-elementary or *derived* forces. Think of friction or a contact force between macroscopic collections of matter. There is no such thing as a 'magnetic' charge—but that's because the term 'electric charge' was there, historically, and it hasn't changed: a linguistic purist looking at the language of physics would probably suggest renaming electric charge as electromagnetic charge.

[3] The elementary charge is the charge of the proton or the electron. They only differ by a plus or minus sign. We find it rather weird that no one seems to have attempted to introduce a system of natural units based on the $\mathrm{q_e} = 1$ equation. For an overview of systems of natural units, we refer the reader to the rather decent Wikipedia article on them (https://en.wikipedia.org/wiki/Natural_units#Systems_of_natural_units).

[4] We will elaborate this concept later but, as for now, you should think of it as the cycle time which – using the energy equivalent of the mass of an electron – is equal to: $\mathrm{T} = 1/f = h/\mathrm{E} \approx 8.1\times10^{-21}$ seconds (we use the energy equivalent of the mass of an electron here

[5] We may refer to both the Compton wavelength ($\lambda_C = hc/\mathrm{E}$) or, alternatively, the Compton radius ($r_C = \hbar c/\mathrm{E}$), which – in our not-so-humble opinion – both define the effective *space* of interference between an electron and the pointlike photons that scatter of it, elastically or non-elastically (we agree with the interpretation of Thomson scattering as the lower-limit of Compton scattering). The difference between the two measures is just a 2π factor. This 2π factor has a rather obvious meaning in our interpretation of what an electron actually *is*.

$$c = \frac{\lambda_C}{T} = \frac{\lambda_C}{T} \cdot \frac{2\pi}{2\pi} = a \cdot \omega \; (electron\ model)$$

The equation above may look trivial (the wavelength is the product of cycle time and velocity) but it is not: we are *not* talking the velocity of the electromagnetic wave here—read: the (absolute) velocity of a photon traveling through space. We think of *c* as a *tangential* velocity of a pointlike charge *zittering* around some center here—and the energy in this *Zitterbewegung* (which – following Hestenes and others – we will abbreviate as *zbw*) is the equivalent mass of the electron. The $c = a \cdot \omega$ is, therefore, nothing but the formula for the *tangential velocity* of the pointlike *zbw* charge.

To put it differently, the formula above sums up our *electron model*—reflecting the *structure* of what we think an electron actually *is*. The equation is, therefore, *very* different from the relation we get from the $E = h \cdot f$ Planck-Einstein relation and the more general $\lambda = c \cdot T$ relation for a photon.

The fundamental difference between the $\lambda_C = c \cdot T$ for an electron and the $\lambda = c \cdot T$ relation for a photon may be related to the fundamental difference between (charged) particles and light (photons): photons do not carry charge. They are, therefore, oscillating fields that travel at the speed of light. In contrast, a charge must have some tiny non-zero mass so as to make Newton's force law meaningful.[6] Hence, instead of a pointlike photon traveling through space, we have a charge – with a small but significant non-zero rest mass – in an orbital motion around a center. The tangential velocity *v* is, therefore, very near but not quite equal to *c*. The $c = \lambda_C/T = a \cdot \omega$ relation is, therefore, approximate only. We will come back to this.

As part of the *prolegomena* to this paper, we briefly want to walk over some basic oscillator math.

The metaphor of the two-dimensional oscillator

We said a force acts on a charge. A harmonic oscillator involves a different concept of force: it involves the idea of a *restoring* force—a force that wants to bring something back to a zero position. This idea is very different from the idea of a force acting on some charge: we could relate it to the idea of the random walk (as time goes by, the likelihood that an object moves much further out goes down), but that is not the topic of this paper.[7]

If the restoring force is proportional to the distance *x* from the zero position (*x* = 0), then we have what is, in physics, referred to as a harmonic oscillation. We write:

$$F = \frac{dp}{dt} = -kx$$

As you can see, k is the factor of proportionality and, as such, it may be said to *define* the force: for a simple mass-spring system, we will refer to it as the *stiffness* or – the opposite idea – the *elasticity* of the

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[6] When m_0 is equal to zero, and *v* is equal to *c*, we get a division of zero by zero: the mass $m_v = m_c = \gamma m_0$ is, therefore, undefined. We will come back to this in the next section(s).

[7] It is interesting that one of Einstein's seminal 1905 papers was on the Brownian motion, which is – essentially – a motion following the 'random walk' pattern in statistics. If our spacetime would, somehow, be a *physical* spacetime (as opposed to our mathematical Cartesian space and the idea of time as some universal clock), then we should probably relate the idea of a 'quantized' (physical) spacetime to the 'mathematical' (statistical) idea of a 'random walk'.

spring.[8] Also note that this equation is relativistically correct if we use the relativistically correct formula for the momentum, which is the one we introduced already: $p = m_v v = \gamma m_0 v$. Note that we don't use boldface here: F and p are scalars—*magnitudes* of what are actually vector quantities: **F** and **p**.

Let's move forward ! A harmonic oscillation will have an amplitude, which we will write as *a*: it is just the largest possible value of the variable *x*. If we know the amplitude, we can write the motion of our object as:

$$x = a \cdot \cos(\omega_0 \cdot t + \Delta)$$

The Δ here is a phase factor: it defines the $t = 0$ point. The ω_0 is the angular frequency of our oscillator. The subscript is there because we don't force the oscillation: ω_0 can thus be referred to as the natural or fundamental frequency. For a non-relativistic mass-spring system, one can show it is equal to:

$$\omega_0 = \sqrt{\frac{\mathrm{k}}{\mathrm{m}}}$$

This formula shows the fundamental frequency of a *non*-relativistic oscillator does *not* depend on the amplitude. We can – theoretically, at least[9] – imagine that the velocity might become very significant. In that case, we should use the relativistic mass concept and the formula above is no longer valid. One can show that the frequency – and, therefore, the *period* – of a relativistic oscillator will depend on the amplitude.[10] However, let us first have some fun with some non-relativistic oscillator. In fact, we want you to imagine an oscillation in two dimensions: up and down and sideways at the same time. The easiest *metaphor* to think of here is, perhaps, a V-2 engine with the pistons at a 90-degree angle, as illustrated below.

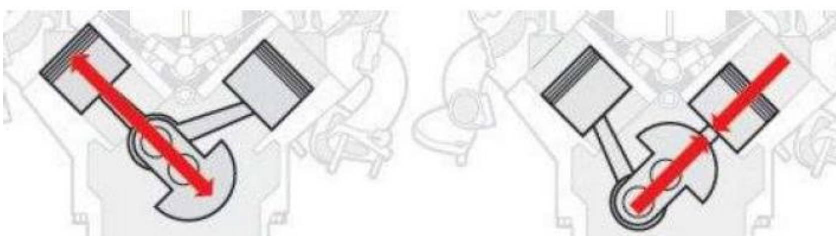


Figure 1: The V-2 metaphor[11]

[8] Strictly speaking, we should refer the term *elasticity* for the ability of some material to return to its original shape after some load or stress has been applied to it. However, at some point in this paper, we will want to vaguely discuss the idea of *elastic spacetime*. The only reason for including the word is, therefore, vanity. The concept of elastic spacetime sounds much fancier than the opposite concept: the *stiffness* of spacetime.

[9] Any actual spring would break long before the mass on it reaches relativistic speeds.

[10] We will let you *google* this: there are too many possible references here. We also refer to basic textbooks for other obvious formulas (e.g. potential or kinetic energy calculations) in this section.

[11] The illustration is from a January 2011 article in the *Car and Driver* magazine, titled: The Physics of Engine Cylinder-Bank Angles. See: https://www.caranddriver.com/features/a15126436/the-physics-of-engine-cylinder-bank-angles-feature/. The origin of this metaphor is, effectively, rather mundane: I was thinking about the relative efficiency of a Ducati versus a Harley-Davidson engines: the Ducati V-2 engine is more efficient because of the 90-degree angle between the pistons. The Harley-Davidson V-2 engine has a more characteristic sound – an irregular

The 90° angle makes it possible to perfectly balance the counterweight and the pistons, thereby ensuring smooth travel always. If we wouldn't have any friction or heat loss – not only from the cylinders but also from the internal motion of the gas – we would have a *perpetuum mobile* here. Indeed, with permanently closed valves, the air inside the cylinder compresses and decompresses as the pistons move up and down. It, therefore, provides a restoring force.

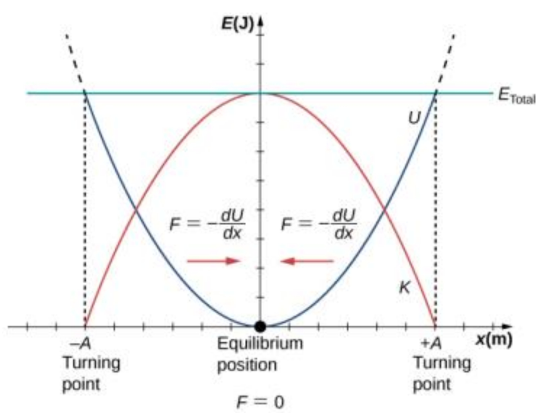
Hence, we have an oscillator here, and it will store potential energy—just like a spring. In fact, the motion of the pistons will also reflect that of a mass on a spring: it is described by a sinusoidal function, with the zero point at the center of each cylinder.[12] We can, therefore, think of the moving pistons as harmonic oscillators, just like mechanical springs. Despite the obvious shortcomings of this metaphorical thinking[13], the reader will – hopefully – appreciate the idea. Because of the 90° angle between the two oscillators, Δ would be 0 for one and –π/2 for the other. Hence, if the motion of one oscillator is given by $x = a\cdot\cos(\omega\cdot t)$, then the motion of the other is given by $y = a\cdot\cos(\omega\cdot t - \pi/2) = a\cdot\sin(\omega\cdot t)$. It is also easy to calculate the kinetic (T) and potential energy (U) of *one* oscillator and then sum them to get the *total* energy of *one* oscillator:

$$T = m\cdot v^2/2 = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot\sin^2(\omega\cdot t + \Delta)$$
$$U = k\cdot x^2/2 = (1/2)\cdot k\cdot a^2\cdot\cos^2(\omega\cdot t + \Delta)$$

$$E = T + U = (1/2)\cdot m\cdot\omega^2\cdot a^2\cdot[\sin^2(\omega\cdot t + \Delta) + \cos^2(\omega\cdot t + \Delta)] = m\cdot a^2\cdot\omega^2/2$$

These energy formulas are illustrated below.

Figure 2: Kinetic (K) and potential energy (U) of an oscillator[14]



sound, actually – because the cylinder bank angle is 45°. Riders love the Harley-Davidson sound, but speed records are/were set on Ducati motorbikes.

[12] The sideways motion of the rod connecting the piston to the crankshaft will result in not-so-perfect sinusoidal functions—but we hope the reader gets the idea.

[13] Apart from friction, we should think of heat – and, therefore, energy – loss as the gas is being compressed and then allowed to expand again.

[14] You will find this diagram in many texts, but we took this one from the https://phys.libretexts.org/ site—which is a great hub for open-access textbooks.

We can now think of *two* oscillators in a 90-degree angle working together.[15] Think of the pistons of our V-2 engine metaphor turning the same crankshaft around. We can show it is a perpetuum mobile by doing some very simple math. To make it even easier, we will briefly assume that $k = m\cdot\omega^2$ and a are both equal to 1.[16] The motion of our first oscillator is given by the $\cos(\omega\cdot t) = \cos\theta$ function (so the phase varies with time only: $\theta = \omega\cdot t$). Its kinetic energy will be equal to $\sin^2\theta$. Hence, the (instantaneous) *change* in kinetic energy at any point in time will be equal to:

$$d(\sin^2\theta)/d\theta = 2\cdot\sin\theta\cdot d(\sin\theta)/d\theta = 2\cdot\sin\theta\cdot\cos\theta$$

Let us look at the second oscillator now. Just think of the second piston going up and down in the V-2 engine. Its motion is given by the $\sin\theta$ function, which is equal to $\cos(\theta-\pi/2)$. Hence, its kinetic energy is equal to $\sin^2(\theta-\pi/2)$, and how it *changes* – as a function of θ – will be equal to:

$$2\cdot\sin(\theta-\pi/2)\cdot\cos(\theta-\pi/2) = -2\cdot\cos\theta\cdot\sin\theta = -2\cdot\sin\theta\cdot\cos\theta$$

We have our *perpetuum mobile*! While transferring kinetic energy from one piston to the other, the crankshaft will rotate with a constant angular velocity: linear motion becomes circular motion, and vice versa, and the *total* energy that is stored in the system is $T + U = ma^2\omega^2$.

We have a great *metaphor* here. Somehow, in this beautiful interplay between linear and circular motion, energy is borrowed from one place and then returns to the other, cycle after cycle.[17]

The ring current model of an electron (1)

The ring current model of an electron uses the same equations. The motion of the pointlike *Zitterbewegung* charge is described by Euler's function. Indeed, the *origin* of both the force and momentum vectors in Figure 1 is the position vector $\boldsymbol{r}$, which we can write using Euler's function, which is nothing but the elementary wavefunction:

$$\boldsymbol{r} = a\cdot e^{i\theta} = x + i\cdot y = a\cdot\cos(\theta) + i\cdot a\cdot\sin(\theta) = a\cdot\cos(\omega t) + i\cdot a\cdot\sin(\omega t) = (x, y)$$

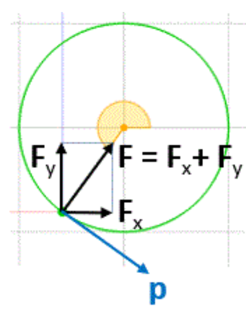


Figure 3: The ring current (or *Zitterbewegung*) model of an electron

[15] Working together—*somehow*, obviously. We have no idea as to how – in the actual quantum-mechanical world that we seem to be living in – this 'mechanism' might work in practice!

[16] Think of it as a normalization of units. There is no trick here. You can re-do the calculations for $a \neq 1$ and $k = m\cdot\omega^2 \neq 1$.

[17] The idea of 'borrowing energy from space' is very deep and fundamental. We've thought long and hard about it, but this metaphor is all we can offer—for the time being, that is.

The reader may not be familiar with the ring current or *Zitterbewegung* model. Oliver Consa (2018) offers a good overview of its history.[18] It was first proposed in 1915, by the British physicist and chemist Alfred Lauck Parson—but it got a lot more attention when Schrödinger stumbled upon it when exploring solutions to Dirac's wave equation for a free electron. Schrödinger shared the 1933 Nobel Prize for Physics with Paul Dirac for "the discovery of new productive forms of atomic theory", and it is worth quoting Dirac's summary of Schrödinger's discovery:

> "The variables give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

We know the wavefunction consist of a sine and a cosine: the cosine is the real component, and the sine is the imaginary component. *Could they both be real?* If so, each of the two oscillations should effectively account for *half* of the total energy of the electron.[19]

Of course, we cannot answer the question as to what is real or not.[20] What we do know is that the description of an electron in terms of a two-dimensional oscillation should be equivalent to the description of what keeps the pointlike *Zitterbewegung* charge going, and that is – as far as we know[21] – the *electromagnetic* force resulting from the current. We will come to that now.

The ring current model of an electron (2)

The idea of current rings – or, more generally, of perpetual currents – is all but outlandish: we can observe them, not at the nano- or picometer scale but at the macroscopic level! To be precise, when temperatures are low enough to cause superconducting materials to become superconducting, we can easily create a perpetual electric current—or a *persistent* current, I should say, as that is what it is usually referred to. The principle is illustrated and described below.

[18] See: Oliver Consa, Helical Solenoid Model of the Electron (http://www.ptep-online.com/2018/PP-53-06.PDF).

[19] Of course, the reader may want to know how we think of the motion of non-free electrons—electron *orbitals*, most notably. We refer our reader to our manuscript (https://vixra.org/abs/1901.0105), which is based on the notion of *layers* of motion.

[20] Only God can, we guess. Or – who knows? – perhaps it's just an irrelevant question. These may be two equivalent statements from a philosophical/epistemological point of view.

[21] Because of the relativity of electric and magnetic forces, we are – for the time being – not willing to accept our two-dimensional oscillation would be just some mathematical equivalent to a description in terms of electromagnetic fields and forces. We still feel there may be something more *real* – something more fundamental – to our description. We will come back to this.

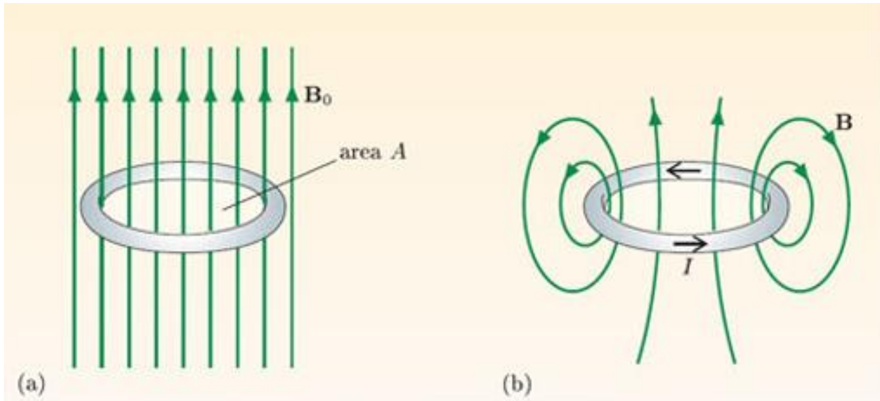


Figure 4: A perpetual current in a superconducting ring[22]

If we have some magnetic field – let us denote it by $\mathbf{B}_0$, as in the left-hand side (a) of the illustration above – going through a ring made of superconducting material, we can then cool the ring below the critical temperature and switch off the field. Lenz's law – which is nothing but a consequence of Faradays' law of induction – then tells us the *change* in the magnetic field will induce an electromotive force. Hence, we get an *induced* electric current, and its direction and magnitude will be such that the magnetic flux it generates will compensate for the flux change due to the change in the applied field. This gives rise to Hestenes' interpretation of the *zbw* model of an electron, which he summarizes as follows:

> "The electron is nature's most fundamental superconducting current loop. Electron spin designates the orientation of the loop in space. The electron loop is a superconducting LC circuit. The mass of the electron is the energy in the electron's electromagnetic field. Half of it is magnetic potential energy and half is kinetic."[23]

This sounds good but it raises an obvious question: where *exactly* is the energy? Our answer to this question is much more precise than Hestenes: we think half of the energy is in the motion of the *zbw* charge, and the other half is, effectively, in the electromagnetic field which perpetuates the motion. We, therefore, introduce the concept of the *effective mass* (or, what amounts to the same, its energy equivalent) of the *zbw* charge as it whizzes around its center of motion. We write it as m_γ and we will – in later section(s) of this paper – show why it is, effectively, equal to *half* of the electron mass[24]:

$$m_\gamma = m_e/2$$

It is a very interesting equation because the concept of effective mass also pops up very naturally in the quantum-mechanical analysis of the linear motion of electrons. Feynman, for example, gets the equation out of a quantum-mechanical analysis of how an electron could move along a line of atoms in a

[22] Source: Open University, Superconductivity, https://www.open.edu/openlearn/science-maths-technology/engineering-and-technology/engineering/superconductivity/content-section-2.2#. The reader who is interested in the detailed equations proving this fact will find them there.
[23] Email from Dr. David Hestenes to the author dated 17 March 2019.
[24] Using Einstein's mass-energy equivalence relation we can, of course, re-write everything in terms of *energies* rather than *masses*.

crystal lattice[25]. However, his interpretation of it is rather fuzzy: he treats it as just one of those weird quantum-mechanical (half-)'truths' (like the Uncertainty Principle or other 'derived laws' in quantum mechanics) and, therefore, limits his comments to a rather confusing connection to the non-relativistic kinetic energy formula (E = mv^2/2).

In contrast, we will show (or hope to show) – in our derivations in later section(s) – that the formula is actually *relativistically correct*.

However, before we go there, we should first do simpler stuff: we will re-visit the two-dimensional oscillator model but using relativistically correct equations this time around. We will do that now but – as we know we have been asking the reader for an extraordinary amount of patience and dedication here (we are already at (almost) 10 pages but have added little or nothing in terms of understanding what might actually "be the case"[26]), we will already mention the results of our analysis.

The ring current model implements Wheeler's idea of mass without mass: the rest mass of the electron is the *equivalent* mass of the energy of the *Zitterbewegung* of the pointlike charge.

Magic? No. We will get these results from rather plain calculations using the actual mass and the actual magnetic moment of the electron, as measured in zillions of experiments. Some more patience, *please!*

The relativistic oscillator

If the velocity of our mass on a spring – on *two* springs, really – becomes a sizable fraction of the speed of light, then we can no longer treat the mass as a constant factor: it will vary with velocity, and its variation is given by the Lorentz factor (γ). We already wrote the relativistically correct force equation for *one* oscillator:

$$F = dp/dt = F = -kx \text{ with } p = m_v v = \gamma m_0 v$$

The $m_v = \gamma m_0$ varies with speed because γ varies with speed:

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \beta^2}} = \frac{\mathrm{dt}}{\mathrm{d\tau}}$$

What's the dt/dτ here? We do not need this expression for what follows but we quickly wanted to remind the reader that we are, effectively, using relativistically correct equations and that we should, therefore, distinguish between the time in our reference frame (t) – aka as the *coordinate time* – and the time in the reference frame of the object itself (τ) – which is known as the *proper time*.

Let us now get on with the equation above. It involves a derivative and it is, therefore, a differential equation. It is a simple equation but, as simple as it is, we are not going to solve it—because we don't have to. We will just derive an energy conservation equation from it. We do so by multiplying both sides with v = dx/dt. I am skipping a few steps (we are not going to do *all* of the work for you) but you should be able to verify the following:

[25] See: Feynman's *Lectures*, Vol. III, Chapter 16: *The Dependence of Amplitudes on Position* (https://www.feynmanlectures.caltech.edu/III_16.html).
[26] We use one of Wittgenstein's expressions to refer to 'reality' here.

$$v\frac{\mathrm{d}(\gamma \mathrm{m}_0 v)}{\mathrm{dt}} = -\mathrm{kx}v \Leftrightarrow \frac{\mathrm{d}(\mathrm{m}c^2)}{\mathrm{dt}} = -\frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2\right] \Leftrightarrow$$

$$\frac{\mathrm{dE}}{\mathrm{dt}} = \frac{\mathrm{d}}{\mathrm{dt}}\left[\frac{1}{2}\mathrm{k}x^2 + \mathrm{m}c^2\right] = 0$$

So what are the energy concepts here? We recognize the potential energy: it is the same $\mathrm{k}x^2/2$ formula we got for the non-relativistic oscillator. No surprises: potential energy depends on position only, not on *velocity*, and there is nothing relative about position.

However, the $(½)\mathrm{m}_0v^2$ term that we would get when using the non-relativistic formulation of Newton's Law – which captures the kinetic energy – is now replaced by the $\mathrm{m}c^2 = \gamma\mathrm{m}_0c^2$ term. You should note this $\mathrm{m}c^2 = \gamma\mathrm{m}_0c^2$ is *not* a constant: it varies with time – just like $\mathrm{k}x^2/2$ – because of the use of the relativistic mass concept.

So how can we calculate the energy? The total energy is constant at any point, so we may equate x to 0 and calculate the energy there. At that point, the potential energy will be zero and, crossing the $x = 0$ point, our pointlike charge will also reach the speed of light. Using the $\mathrm{m}_\gamma = \mathrm{m}_c = \mathrm{m}_e/2$ equation which – admittedly – we still need to motivate, we write this:

$$\mathrm{E} = \frac{1}{2}\mathrm{k}\cdot 0^2 + \mathrm{m}_{x\,=\,0}\cdot c^2 = \mathrm{m}_\gamma \; = \frac{\mathrm{m_e}c^2}{2}$$

We can now add the energies in both oscillators so as to arrive at the *total* energy of the electron:

$$\mathrm{E} = \mathrm{m_e}c^2$$

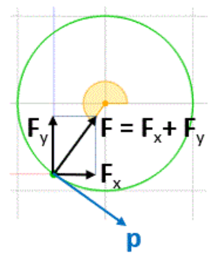
It is a wonderful result: we think it actually amounts to a rather elegant and intuitive common-sense explanation of Einstein's mass-energy equivalence relation. However, we will leave it to the reader to judge that statement. In terms of progress on this paper, I would say we are now – *finally!* – ready to get into the meat of the matter.

The ring current model of an electron (3)

We mentioned that we think of the ring current model as an implementation of Wheeler's idea of mass without mass: the rest mass of the electron is the *equivalent* mass of the energy of the *Zitterbewegung* of the pointlike charge.[27] We effectively think of the electron as consisting of a pointlike[28] charge which moves about some center at lightspeed.

However, unlike Hestenes' or other ring current model, we will incorporate the reality of the anomalous magnetic moment right from the start by assuming that the pointlike charge has some non-zero physical (spatial) dimension. This allows us to distinguish an *effective* radius (r) and an *effective* speed (v) which is *nearly* but *not quite* equal to the theoretical radius (a) and the theoretical speed (c). This should also address the following inconsistency or difficulty in *interpreting* the idea of a *zbw* charge moving about at the speed of light.

Let us copy Figure 3 once again:



The momentum (**p**) of the *zbw* charge is relativistic momentum, of course. Hence, its magnitude $|\mathbf{p}| = p$ is equal to:

$$p = mc = \gamma m_0 c$$

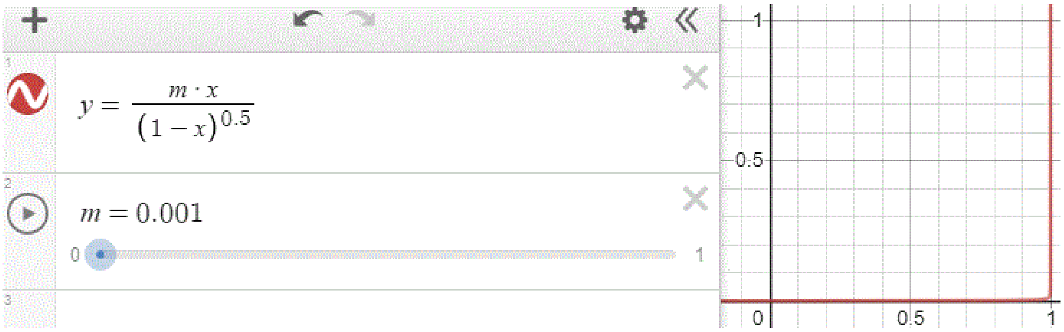
Now, it is easy to see that this formula becomes meaningless when the Lorentz factor (γ) goes to infinity as the velocity goes to c. To put it differently – but it amounts to saying the same – we should *not* assume that the pointlike *Zitterbewegung* charge has zero rest mass. We must, therefore, conclude that m_0 must be *close* to zero but not *exactly* zero. We will calculate the value for m_0 in the next section(s). Here, we only want to illustrate the problem by the following easy graph, which shows what happens with the $p = m_v v = \gamma m_0 v$ function for $m_0 = 0.001$ for the relative velocity $\beta = v/c$ ranging between 0 and 1.[29]

[27] We use the terms ring current model and *Zitterbewegung* model interchangeably.

[28] Pointlike does *not* necessarily imply that it has no spatial dimension whatsoever. On the contrary: in our previous papers, we do associate the *classical* electron radius with the *zbw* charge.

[29] We used the online desmos.com graphing tool to produce the graph.

Figure 5: $p = m_v v = \gamma m_0 v$ for $m_0 \rightarrow 0$



It is quite enlightening: p is (very close to) zero for *v*/*c* going from 0 to (very close to) 1 but then becomes undefined at *v*/*c* = 1 itself. The idea of the momentum of an object with zero rest mass is, therefore, not an easy one.[30]

The electron anomaly and the rest mass of the *zbw* charge

We do not think of the anomaly as an anomaly. We see an immediate perfectly rational explanation for it: we think the *zbw* charge has some very tiny (but non-zero) spatial dimension. As a result, we should distinguish between its *effective* and theoretical (tangential) velocity. The *effective* velocity – which we will denote as *v* – is *very near* but not exactly equal to *c*. Likewise, we should distinguish between an effective radius – which we will denote as *r* – versus its theoretical radius *a* = ħ/mc. Let us get through the logic here.

We should, first and foremost, note the crucial assumption here, which is that we think the accuracy of the Planck-Einstein relation is preserved, *always*! We, therefore, think we should not only distinguish between a theoretical and an actual (i.e. experimentally determined) magnetic moment but also between a theoretical and an actual radius of the ring current. To be precise, based on the *measured* value of the magnetic moment (i.e. the CODATA value), we can calculate the anomaly of the *radius* of the presumed ring current. Indeed, the frequency is, of course, the velocity of the charge divided by the circumference of the loop. Because we assume the velocity of our charge is equal to *c*, we get the following radius value:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e} c}{2} a \Leftrightarrow a = \frac{2\mu}{\mathrm{q_e} c} \approx 0.38666\ \mathrm{pm}$$

We should note that we get a value that is slightly different from the theoretical *a* = *c*/ω = ħ/m*c* radius which was equal to 0.38616… pm. We, therefore, have an anomaly, indeed! We can confirm this anomaly by re-doing this calculation using the Planck-Einstein relation to calculate the frequency:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \frac{\mathrm{q_e}\omega a^2}{2} \Leftrightarrow a = \sqrt{\frac{2\mu}{\mathrm{q_e}\omega}} = \sqrt{\frac{2\mu\hbar}{\mathrm{q_e E}}} = \sqrt{\frac{2\mu\hbar}{\mathrm{q_e m}c^2}} \approx 0.38638\ \mathrm{pm}$$

We again get a slightly different value—again a slightly *larger* value that the theoretical *a* = ħ/m*c* value. How can we explain this? Let us go through the calculations here.

[30] The reader will note the same can be said of a photon, but a photon does not carry charge!

1. Let us first find a theoretical value for the magnetic moment by equating the two formulas for the radius that we have presented so far. Both are based on a different *physical* concept of the frequency of the oscillation. While *different*, we can only have one radius, of course. We, therefore, get this:

$$\left.\begin{aligned} a &= \sqrt{\frac{2\mu\hbar}{\mathrm{q_e m}c^2}} \\ a &= \frac{2\mu}{\mathrm{q_e}c} \end{aligned}\right\} \Leftrightarrow \sqrt{\frac{2\mu\hbar \mathrm{q_e^2}c^2}{4\mu^2\mathrm{q_e m}c^2}} = \sqrt{\frac{\hbar \mathrm{q_e}}{2\mu \mathrm{m}}} = 1 \Leftrightarrow \mu = \frac{\mathrm{q_e}}{2\mathrm{m}}\hbar$$

So that confirms the *theoretical* value of the magnetic moment, which is equal to the above-mentioned μ_{CODATA} = 9.27401… J·T^{-1}.

2. Now, we know that a magnetic moment is generated by a current in a loop and, from experiment, we *know* that the *actual* magnetic moment is slightly higher than the above-mentioned value. We can, therefore, calculate the *effective* radius – using one of the two formulas above – from the *actual* magnetic moment. If you do this, you should get this:

$$r = \frac{2\mu}{\mathrm{q_e}c} \approx 0.3866 \ldots \mathrm{fm}$$

We effectively get a *larger* value than the Compton radius, which is equal to 0.38616 fm—more or less. We can now calculate an anomaly based on these two radii:

$$\frac{r-a}{a} \approx 0.00115965 \Leftrightarrow \frac{r}{a} = 1.00115965 \ldots$$

We get the same thing here: the anomaly of the *radius* is, once again, equal to about 99.85% of Schwinger's factor: α/2π = 0.00116141…

This allows us to guide the reader through the following calculations.[31]

3. Our assumption is that the anomaly is not an anomaly at all. We get it because of our mathematical idealizations: we do not *really* believe that pointlike charge are, effectively, pointlike and, therefore, dimensionless. In other words, we think the assumption that the electron is just a pointlike or dimensionless charge is non-sensical: when thinking of what might be going on at the smallest scale of Nature, we should abandon these mathematical idealizations: an object that has no physical dimension whatsoever does – quite simply – not exist.

We should, therefore, effectively distinguish the effective radius *r* and the effective velocity *v* from the theoretical values *a* and *c*. We can write this:

$$\frac{\mu_r}{\mu_a} = \frac{\frac{\mathrm{q_e}vr}{2}}{\frac{\mathrm{q_e}}{2\mathrm{m}}\hbar} = \frac{v \cdot r}{c \cdot a} = \frac{\omega \cdot r^2}{\omega \cdot a^2} = \frac{r^2}{a^2} \approx 1 + \frac{\alpha}{2\pi} \Leftrightarrow r = \sqrt{1 + \frac{\alpha}{2\pi}} \cdot a \approx 1.00058 \cdot \frac{\hbar}{\mathrm{m}c}$$

There is a crucial step here: we equated the anomaly to 1 + α/2π. Is that a good approximation? In a

[31] We realize this is a long text. However, we beg the reader to bear with us. We feel the view from the top warrants the climb—and more than a bit! Of course, this is just a mountaineer's opinion. 😊

first-order approximation, it is. In fact, the reader will probably have heard that Schwinger's $\alpha/2\pi$ factor explains about 99.85% of the anomaly, but it is actually better than. Check it: **the μ_r/μ_a ratio is about 99.99982445% of $1 + \alpha/2\pi$.**[32]

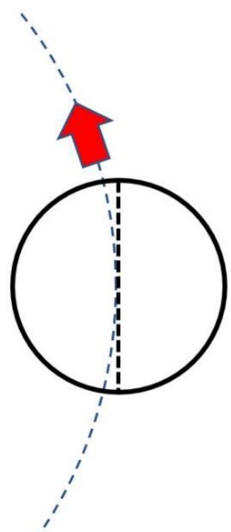
4. We can also calculate the effective velocity. We will use the fact that the *v*/*c* and *r*/*a* ratios must be the same, as we can see from the tangential velocity formula:

$$1 = \frac{\omega}{\omega} = \frac{v/r}{c/a} \Leftrightarrow \frac{v}{c} = \frac{r}{a}$$

We can, therefore, calculate the relative velocity as:

$$1 = \frac{\omega}{\omega} = \frac{v/r}{c/a} \Leftrightarrow v = \frac{r}{a} \cdot c = \frac{\sqrt{1 + \frac{\alpha}{2\pi}} \cdot a}{a} \cdot c = \sqrt{1 + \frac{\alpha}{2\pi}} \cdot c \approx 1.00058 \cdot c$$

Great ! We're done ! The only thing that's left to explain is… Well… How can the effective radius be *larger* than the theoretical one? And how can the effectively velocity be *larger* than *c*? Think of about the physicality of the situation here—as depicted below.



If the *zbw* charge is effectively whizzing around at the speed of light, and we think of it as a charged sphere or shell, then its effective *center of charge* will *not* coincide with its center. Why not? Because the ratio between (1) the charge that is outside of the disk formed by the radius of its orbital motion and (2) the charge inside – note the triangular areas between the diameter line of the smaller circle (think of it as the *zbw* charge) and the larger circle (which represent its orbital) – is slightly *larger* than 1/2.

It all looks astonishingly simple, doesn't it? Too simple? We don't think so, but we will let you – the reader – judge, of course!

To conclude this section, we should add one more formula. It is an interesting one because it brings a very important *nuance* to the quantum-mechanical rule that angular momentum should come in units

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[32] Needless to say, for μ_r, we use the CODATA value.

of $\hbar$.

5. Indeed, our calculation shows the *actual* angular momentum of an electron must be slightly *larger* than $\hbar$:

$$1+\frac{\alpha}{2\pi}=\frac{v\cdot r}{c\cdot a}\Leftrightarrow v\cdot r=\left(1+\frac{\alpha}{2\pi}\right)\cdot c\cdot a=\left(1+\frac{\alpha}{2\pi}\right)\cdot c\cdot\frac{\hbar}{\mathrm{m}c}=\left(1+\frac{\alpha}{2\pi}\right)\frac{\hbar}{\mathrm{m}}$$

$$\Leftrightarrow \mathrm{L}=\mathrm{m}\cdot v\cdot r=\left(1+\frac{\alpha}{2\pi}\right)\cdot\hbar=\hbar+\frac{\alpha}{2\pi}\hbar$$

Unsurprisingly, the difference is, once again, given by Schwinger's α/2π factor.

The oscillator model of an electron

From Figure 3, it is obvious that we may usefully distinguish the components of the momentum vector **p** in the *x*- and *y*-direction respectively. We write:

$$\mathbf{p} = \mathbf{p_x} + \mathbf{p_y}$$

The *magnitude* of these vectors can then be written as $|\mathbf{p}| = \mathrm{p}$, $|\mathbf{p_x}| = \mathrm{p_x}$, and $|\mathbf{p_s}| = \mathrm{p_y}$ respectively.

This is easy enough. We will now do something very weird: we will briefly revive Einstein's idea that, perhaps, the concept of mass might be directional as well—just like the concepts of momentum and/or velocity. Indeed, we may usefully remind ourselves that Einstein actually used velocity-dependent concepts of mass in his seminal 1905 article introducing the principle of relativity, distinguishing between the "longitudinal" and "transverse" mass of a moving charge.[33] It is just a *temporary* assumption we feel we need to introduce here so as to make sure we keep track of directions here. Of course, we know the mass concept is scalar: the directional aspect is taken care of in the force law by writing both the force **F** and the acceleration ***a*** as vectors—using **bold type**, indeed.

However, as we will be calculating *magnitudes* hereunder, we want to make sure we do not make any mistakes. Hence, we will distinguish between an effective (relativistic) mass in the *x*- and *y*-directions as $\mathrm{m_x}$ and $\mathrm{m_y}$ respectively. If you think this does not make any sense, think again: it is quite intuitive. From our experience in daily life, we know it is much easier to change the *direction* of a massive object than its *velocity*.[34] We have the same situation here: the *zbw* charge whizzes around at near lightspeed, but changes direction all the time, which is why we need the distinction when separating out vectors into their components along this or that axis. Likewise, we will want – just to be on the safe side – also want to distinguish between a Lorentz factor that's applicable to the motion of the *zbw* charge along the x-

[33] See p. 21 of the English translation of Einstein's article on special relativity, which can be downloaded from: http://hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_1905_relativity.pdf. The distinction is related to the distinction between the electrostatic and magnetic forces, which is equally relative—in the sense that what is electromagnetic and what is magnetic depends on your reference frame!

[34] Think of all the movies involving some asteroid threatening to crash into our planet: the hero in his rocket will not try to stop it, but he will try to, somehow, change its direction (or, else, destroy it—or some combination of both).

direction as opposed to the one we should apply to the motion along the y-direction.[35] We can now write p_x and p_y as:

$$p_x = m_x v_x = \gamma_x m_0 v_x \text{ and } p_y = m_y v_y = \gamma_y m_0 v_y$$

The *origin* of both the force and momentum vectors is the position vector ***r***, which we can write using the elementary wavefunction, i.e. Euler's function:

$$\boldsymbol{r} = a\cdot e^{i\theta} = x + i\cdot y = a\cdot\cos(\theta) + i\cdot a\cdot\sin(\theta) = a\cdot\cos(\omega t) + i\cdot a\cdot\sin(\omega t) = (x, y)$$

We can also calculate the centripetal acceleration: it's equal to $a_c = v_t^2/a = a\cdot\omega^2$ (the reader should note that this formula is (also) relativistically correct). It might be useful to remind ourselves how we get this result. The position vector ***r*** has a horizontal and a vertical component: $x = a\cdot\cos(\omega t)$ and $y = a\cdot\sin(\omega t)$. We can now calculate the two components of the (tangential) velocity vector $\boldsymbol{v} = d\boldsymbol{r}/dt$ as $v_x = -a\cdot\omega\cdot\sin(\omega t)$ and $v_x\ y = -a\cdot\ \omega\cdot\cos(\omega t)$ and, in the next step, the components of the (centripetal) acceleration vector $\boldsymbol{a_c}$: $a_x = -a\cdot\omega^2\cdot\cos(\omega t)$ and $a_y = -a\cdot\omega^2\cdot\sin(\omega t)$. The magnitude of this vector is then calculated as follows:

$$a_c^2 = a_x^2 + a_y^2 = a^2\cdot\omega^4\cdot\cos^2(\omega t) + a^2\cdot\omega^4\cdot\sin^2(\omega t) = a^2\cdot\omega^4 \Leftrightarrow a_c = a\cdot\omega^2 = v_t^2/a$$

Now, Newton's force law tells us that the magnitude of the centripetal force $|\mathbf{F}| = F$ will be equal to:

$$F = m_\gamma\cdot a_c = m_\gamma\cdot a\cdot\omega^2$$

However, we again have this problem of determining what the mass of our pointlike charge actually is when we assume the m_0 in our $m_\gamma = \gamma m_0$ formula is zero ! Fortunately, we did find another way in the previous section ! However, we should continue to explore the basic geometry of the model here.

The horizontal and vertical force component effectively behave like the restoring force that drives a (linear) harmonic oscillation: we just need to think of an oscillation along *two* (independent) dimensions here. This restoring force depends linearly on the (horizontal or vertical) displacement from the center, and the (linear) proportionality constant is usually written as k. In case of a mechanical spring, this constant will be the *stiffness* of the spring. We do not have a spring here so it is tempting to think it models some elasticity of space itself. However, we should probably not engage in such philosophical thought. Let us just write down the formulas:

$$F_x = dp_x/dt = -k\cdot x = -k\cdot a\cdot\cos(\omega t) = -F\cdot\cos(\omega t)$$

$$F_y = dp_y/dt = -k\cdot y = -k\cdot a\cdot\sin(\omega t) = -F\cdot\sin(\omega t)$$

Now, it is quite straightforward to show that the constant (k) can always be written as:

$$k = m\cdot\omega^2$$

We get that from the *solution* we find for ω when solving the differential equations $F_x = dp_x/dt = -k\cdot x$ and $F_y = dp_y/dt = F_y = dp_y/dt = -k\cdot y$ and assuming there is nothing particular about p and m. In other words, we assume there is nothing wrong with this $p = m\cdot v = \gamma m_0 v$ relation. So we just don't think about the weird behavior of that function. It's a bit like what Dirac did when he *defined* his rather (in)famous

[35] If you continue to think this sounds nonsensical, then just equate m_x and m_y (and γ_x and γ_y too). In other words, just drop the subscripts and continue reading: you should find it all makes sense as well!

Dirac function: the function doesn't make sense mathematically but it works – i.e. we get the right answers – when we use it.

So now we have the $k = m \cdot \omega^2$ equation and we know m is *not* the rest mass of our electron here. We referred to it as the *effective* mass of our pointlike charge as it's whizzing around at the speed of light. We need to remember mass is a measure of inertia and, hence, we can measure that inertia along the horizontal and vertical axis respectively. Hence, we can, effectively, write something like this: $m = m_\gamma = m_x = m_y$, in line with the distinction we made between p, p_x and p_y. Why m_γ? The notation is just a placeholder: we need to remind ourselves it is a relativistic mass concept and so I used γ (the symbol for the Lorentz factor) to remind ourselves of that. So let us write this:

$$k = m_\gamma \cdot \omega^2$$

From the equations for F_x and F_x, we know that $k \cdot a = F$, so $k = F/a$. Hence, the following equality must hold:

$$F/a = m_\gamma \cdot \omega^2 \Leftrightarrow F = m_\gamma \cdot a \cdot \omega^2 \Leftrightarrow F/a = m_\gamma \cdot a^2 \cdot \omega^2 = \Leftrightarrow F/a \cdot m_\gamma = a^2 \cdot \omega^2$$

We know the sum of the potential and kinetic energy in a linear oscillator adds up to $E = m \cdot a^2 \cdot \omega^2/2$. We have *two* independent linear oscillations here so we can just add their energies and the ½ factor vanishes.

Hence, if Einstein's mass-energy equivalence relation applies, we should all accept that **the total energy in this oscillation must be equal to $E = m \cdot c^2$**. The mass factor here is the *rest mass of our electron*, so it's *not* that weird relativistic m_γ concept. *However*, we did equate *c* to $a \cdot \omega^2$. Hence, we can now write the following:

$$E = m \cdot c^2 = m \cdot a^2 \cdot \omega^2 = m \cdot F/a \cdot m_\gamma$$

The force is, therefore, equal to:

$$F = (m_\gamma/m) \cdot (E/a)$$

What can we do with this result? What can we say about the m_γ/m ratio? Let us start yet another section in this paper.[36]

The effective mass of the *zbw* charge

We know m_γ is sort of undefined—but it shouldn't be zero and it shouldn't be infinity. It is also quite sensible to think m_γ should be smaller than m. It cannot be larger because than the energy of the oscillation would be larger than $E = mc^2$. What could it be? We think it is equal to 1/2, but can we *prove* that? In order to do so, we can try to calculate the angular momentum L. Common wisdom is that electrons are spin-1/2 particles, but can we *show* that?[37]

In a previous section, we took the ratio of the theoretical and actual magnetic moment so as to get the following formula for the anomaly:

[36] We apologize this seems to be becoming a small book rather than a paper or an article.

[37] Regular readers will remember we are extremely skeptical of the perceived wisdom in regard to spin-1/2 and spin-one particles. See: *Feynman's Worst Jokes and the Boson-Fermion Theory* (https://vixra.org/abs/2003.0012).

$$\frac{\mu_r}{\mu_a} = \frac{\frac{q_e vr}{2}}{\frac{q_e}{2m}\hbar} = \frac{v \cdot r}{c \cdot a}$$

We then equated the ratio above to 1 + α/2π. Why? Because we think a 99.99982445…% explanation is pretty good. So we get this:

$$1 + \frac{\alpha}{2\pi} = \frac{v \cdot r}{c \cdot a} \Leftrightarrow v \cdot r = \left(1 + \frac{\alpha}{2\pi}\right) \cdot c \cdot a = \left(1 + \frac{\alpha}{2\pi}\right) \cdot c \cdot \frac{\hbar}{mc} = \left(1 + \frac{\alpha}{2\pi}\right)\frac{\hbar}{m}$$

$$\Leftrightarrow L = m \cdot v \cdot r = \left(1 + \frac{\alpha}{2\pi}\right) \cdot \hbar = \hbar + \frac{\alpha}{2\pi}\hbar$$

This is surprising. Indeed, the reader should note we just derived a rather spectacular non-mainstream quantum-mechanical principle here:

> Mainstream quantum mechanics assumes angular momentum must come in units of ħ, and mainstream physicists think that is a direct implication of – or even an equivalent to – the Planck-Einstein law: E = *h·f* = ħ·ω. The calculation above brings some nuance to this statement: angular momentum does *not* come in *exact* units of ħ. There is an anomaly, and we think the anomaly is part and parcel of Nature.

The reader will or should also immediately note something else here: the formula above suggests our electron is a spin-one rather than a spin-1/2 particle ! This cannot be right, right?

The answer is: yes and no. If you push us, we'll say: more yes than no. We just need to wonder what mass concept we are using here: it is the *total* mass of the electron. To make sense of the formula, we should introduce a 1/2 factor. That is easy enough. All of what we wrote above is about orbital angular momentum. We should, therefore, relate it to the effective mass of the *zbw* charge that is spinning around. We can easily do that because the equipartition theorem tells us half of the energy of the electron would be in the electromagnetic field, while the other half will be kinetic energy related to the motion of the *zbw* charge. That kinetic energy is the energy equivalent of the effective mass of the *zbw* charge which must, therefore, be equal to 1/2 of the total energy of the electron.

Exactly equal? No. The same anomaly applies. We write:

$$L = m_\gamma \cdot v \cdot r = \frac{m_e}{2} \cdot v \cdot r = \frac{m_e}{2} \cdot \left(1 + \frac{\alpha}{2\pi}\right) \cdot c \cdot a = \frac{m_e}{2} \cdot \left(1 + \frac{\alpha}{2\pi}\right) \cdot \frac{\hbar}{m_e} = \frac{h}{2\pi + \alpha} \approx \frac{\hbar}{2}$$

In the latter part of the formula, we make abstraction of the anomaly, and so we get the simplified formula we wanted to find:

$$L_{orbital} \approx \frac{\hbar}{2}$$

We can now use the **L** = ***r*** × **p** formula. Indeed, the lever arm is the radius here, so we get:

1. L = ħ/2 ⇔ p = L/*a* = (ħ/2)/*a* = (ħ/2)·m*c*/ħ = m*c*/2
2. p = $m_\gamma c$

⇒ $m_\gamma c$ = m*c*/2 ⇔ m_γ = m/2

We found the grand result we expected to find: the *effective* mass of the pointlike charge – as it whizzes around the center of the two-dimensional oscillation that makes up our electron – is (about) half of the (rest) mass of the electron. To make mainstream physicists happy, we can plug this back into the more general formula for the angular momentum formula using the angular mass formula for a hoop: $\mathrm{I} = \mathrm{m}\cdot r^2$. We write[38]:

$$\mathrm{L} = \mathrm{I}\omega = \mathrm{m}_\gamma a^2 \frac{c}{a} = \frac{\mathrm{m_e} rc}{2} = \frac{\mathrm{m_e}\hbar c}{2\mathrm{m_e} c} = \frac{\hbar}{2}$$

Brilliant ! So we're done with this ! However, before we sign off, we should probably say a few words about the higher-order factors in the anomaly.

The higher-order factors in the explanation of the anomaly

The μ_r/μ_a ratio is about 99.99982445% of 1 + α/2π.[39] So that is *very* good. It is actually *much* better than what is usually said about Schwinger's α/2π factor explaining about 99.85% explanation of the *measured* anomaly. However, *very* good is, perhaps not good enough. How can we explain the n^{th}-order factors (n > 1) that follow the α/2π factor in the expression below:

$$\frac{\mu_a - \mu}{\mu} = \frac{a_\mu - a}{a} = \frac{\alpha}{2\pi} + \cdots$$

We used the CODATA value for μ, of course, so perhaps there is a small error in the CODATA value? Possibly, but *not* likely: this is one of the most precise measurements in the history of physics, so we should not think there has been any fudging here.[40]

We have not any detailed calculations here, but we think we have an logical explanation. As mentioned earlier, the $\mu = \mathrm{I}\pi r^2 = \mathrm{q_e} f \pi r^2 = \mathrm{q_e}\frac{v}{2\pi r}\pi r^2 = \frac{1}{2}\mathrm{q_e} rv$ tells us that the moment is proportional to the radius of the loop, and the factor of proportionality is $\mathrm{q_e} v/2$. Hence, electric charge that is closer to the theoretical $a = \hbar/\mathrm{m}c$ radius will make a proportionally larger contribution to the magnetic moment. We should, therefore, *not* assume that the *zbw* charge has no spatial dimension whatsoever. On the contrary: the higher-order factors tell us *the zbw should have some radius of its own*.

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[38] The reader should not confuse the two symbols: I is angular mass or rotational inertia, while *I* denotes electric current. Note that we use the theoretical values for radius and velocity, i.e. the Compton radius and the speed of light.

[39] Needless to say, for μ_r, we use the CODATA value.

[40] In case you doubt fudging ever happens in physics, think again: Oliver Consa documents some very interesting cases in his article on the *Rotten State of QED* (https://vixra.org/abs/2002.0011).

Let us illustrate this point by thinking about the *physicality* of what we are modeling here. From the formula for the magnetic moment[41] – and from our calculations above – it is easy to see that we can also write the anomaly as an anomaly between an actual and a theoretical radius of the electron[42]:

$$\frac{a_\mu - a}{a} = \frac{\alpha}{2\pi} + \cdots \Leftrightarrow a_\mu - a = (\alpha + \cdots) \cdot \frac{a}{2\pi}$$

This is a very interesting equation. *A priori*, one might have expected that the *difference* between the $a = \hbar/\mathrm{m}c$ Compton radius and the *actual* radius r would be of the order of $\alpha \cdot a$. Why? Because $\alpha \cdot a$ is the classical electron radius, which explains elastic scattering. We, therefore, think it is, in effect, the actual radius of the *zbw* charge inside of the electron. But we have a 1/2π factor here, and it is rather obvious that we cannot explain it away. This 1/2π factor is equal to about 0.16.

So what can we say about this? Nothing much. We should note that we calculated the *difference* between what we think of the *real* radius of the ring current and its theoretical radius $a = \hbar/\mathrm{m}c$. We did *not* directly calculate a radius of the *zbw* charge ! We need other assumptions and/or other formulas for that. Do we have these?

We do. Richard Feynman gets the following interesting formula when calculating the electromagnetic mass or energy of a sphere of charge with radius a[43]:

$$\mathrm{U} = \frac{1}{2}\frac{\mathrm{e}^2}{a} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r_\mathrm{e}} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{\mathrm{m}c}{\alpha\hbar} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0\hbar\mathrm{m}c^2}{\mathrm{q_e^2}\hbar} = \frac{1}{2}\mathrm{m}c^2$$

In fact, Feynman does not write it like this, but we inserted and used the $\alpha = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0\hbar c}$ and $a = r_\mathrm{e} = \alpha\frac{\hbar}{\mathrm{m}c}$ identities above. The point is: we get only half of the (rest) energy or (rest) mass of the electron out of this assembly. Feynman was puzzled by that ½ factor: where is the other half? He should not have been puzzled by it: he is assembling the *zbw* charge here—*not* the electron as a whole. Hence, the missing mass is in the *Zitterbewegung* or orbital/circular motion of the *zbw* charge. We can now *derive* the classical electron radius from the formula above:

$$\mathrm{U} = \frac{1}{2}\frac{\mathrm{e}^2}{r_\mathrm{e}} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r_\mathrm{e}} \Leftrightarrow r_\mathrm{e} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0\mathrm{U}} = \alpha\frac{\hbar c}{2\mathrm{m}_\gamma c^2} = \alpha\frac{\hbar}{\mathrm{m_e}c} = \alpha r_\mathrm{C}$$

[41] The magnetic moment is the product of the current times the area of the loop. However, writing that all out shows that the magnetic moment is simply *inversely* proportional to the radius of the current loop. Indeed, we calculate the magnetic moment as:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e}f\pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e}c}{2}a$$

[42] We re-write the n^{th}-order factors (n > 1) here: we simply multiply them by 2π as we bring the 1/2π factor out of the brackets.

[43] See: https://www.feynmanlectures.caltech.edu/II_28.html. The basic idea is to 'assemble' the elementary charge by bringing infinitesimally small charge *fractions* together.

Conclusions

The formulas above are all very nice, but do they fully solve the enigma of the electron?[44]

Maybe, but – let us be honest – probably not. We did gloss over some rather important details here. Feynman was assembling a thin spherical shell of charge here—as opposed to a uniformly charged *sphere* of charge, in which case the coefficient becomes 3/5 instead of 1/2.[45] So is our *zbw* charge a thin spherical shell of charge or a uniformly charged *sphere* of charge? Our honest answer is: we don't know. The formulas suggest the former—and that makes sense, instinctively: negative charges repel each other, so they are always on the outside of a conductor. At the same time: such reasoning amounts to admitting we don't fully understand what's going on here. Consa's idea of some *fractal* structure does not appeal to us, but we have to admit it might make sense.

We still have a long way to go. Perhaps we should just accept that we cannot not push our classical ideas too far. Indeed, there are a few other – more important – things that don't make sense here. First, one should note that Feynman did not include the energy we associated with the spin of the *zbw* charge in this energy calculation. He only calculated *potential* energy when assembling the elementary charge by bringing infinitesimally small charges together. This undermines the logic of the derivation above.

So what can we say? Not all that much, for the time being. We don't think we managed to *fully* solve all of the quantum-mechanical mysteries. However, we are positive, and so we do think that we have a perfectly consistent realist interpretation of quantum mechanics here.

To be precise, we think we have a theory here which explains all of the mysterious *intrinsic* properties of an electron (its mass, its radius for elastic as well as inelastic scattering, and its magnetic moment) using common-sense physics. We, therefore, hope that we have managed to convince the reader that the assumption that the electron is just a dimensionless charge is non-sensical. When thinking of what might be going on at the smallest scale of Nature, we should abandon our mathematical idealizations: an object that has no physical dimension whatsoever does – quite simply – not exist. Pointlike and zero-dimension are not the same: the pointlike *zbw* charge has some (tiny) dimension.

In light of the title of this section, we should probably conclude here. We would love to add a few more thoughts on the applicability of this oscillator model to the proton. We will do so in a separate paper, however. This paper has become way too long. 😊

For more philosophical considerations, we also refer our reader to the Metaphysics page of our new physics blog, which we keep more up to date nowadays than our papers.[46]

Jean Louis Van Belle, 25 March 2020

[44] The language we use here refers to a book by Malcolm Mac Gregor: *The Enigmatic Electron, A Doorway to Particle Masses* (https://www.amazon.com/Enigmatic-Electron-Doorway-Particle-Masses/dp/1886838100)
[45] See: https://www.feynmanlectures.caltech.edu/II_08.html.
[46] See: https://ideez.org/philosophy/.