Proof of Riemann Hypothesis using the decomposition of $\zeta(z) = X(z) - Y(z)$

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Abstract:

The Riemann Zeta function or Euler–Riemann Zeta function, $\zeta(s)$, is a function of a complex variable z that analytically continues the sum of the Dirichlet series:

[1] $$\zeta(z) = \sum_{k=1}^{\infty} k^{-z}$$

The Riemann zeta function is a meromorphic function on the whole complex z-plane, which is holomorphic everywhere except for a simple pole at z = 1 with residue 1. One of the most important advance in the study of Prime numbers was the paper by Bernhard Riemann in November 1859 called "Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse" (On the number of primes less than a given quantity). In this paper, Riemann gave a formula for the number of primes less than x in terms the integral of 1/log(x), and also provided insights into the roots (zeros) of the zeta function, formulating a conjecture about the location of the zeros of $\zeta(z)$ in the critical line Re(z)=1/2.

[2] Riemann Hypothesis: All nontrivial zeros lie on the critical line, or Re(z) = 1/2.

In this paper, we use the decomposition of the Riemann Zeta function in the form:

[3] $\zeta(z) = X(z) - Y(z)$

To prove the Riemann Hypothesis.

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Nomenclature and conventions

a. $\zeta(z) = \sum_{k=1}^{\infty} k^{-z}$ is the Zeta function of Riemann

b. z*: any nontrivial solution of the Zeta function verifying that ζ(z*)=0. By default, a reference to zero of ζ(z) will mean a nontrivial zero of ζ(z).

c. ß(n) is the n^{th} zero of the Riemann function in the critical line x=1/2 in C. e.g. b1=14.134725…

d. α=Re(z) is the real part of z

e. ß=Im(z) is the imaginary part of z

f. If z=α+iß, Modulus(z)= $|z| = \sqrt{(\alpha^2+ß^2)}$

g. If z=α+iß, Absolute Square(z)= $|z|^2 = \alpha^2+ß^2$

1. A decomposition of $\zeta(z)$ for Re(z)>0, z≠1

From (2, Caceres 2020), one can write $\zeta(z)$ as the difference between the functions X(z) and Y(z):

[3] $\zeta(z) = X(z) - Y(z)$, where:

[4] $$X(z,n) = (\sum_{k=1}^{n} k^{-\alpha} (\cos(ß * \ln(k)) + \frac{1}{2} n^{-\alpha} \cos(ß \ln(n)) +$$
$$+ i * (\sum_{k=1}^{n} k^{-\alpha} (\sin(ß * \ln(k)) + \frac{1}{2} n^{-\alpha} \sin(ß \ln(n)))$$

and: $X(z) = \lim_{n\to\infty} X(z,n)$

[5] $$Y(z,n) = n^{(1-\alpha)} \frac{1}{[(1-\alpha)^2 + ß^2]} [((1-\alpha) * \cos(ß \ln(n)) + ß * \sin(ß \ln(n))) +$$
$$+ i \ (ß * \cos(ß \ln(n)) - (1-\alpha) * \sin(ß \ln(n)))]$$

and: $Y(z) = \lim_{n\to\infty} Y(z,n)$

2. Analysis of Absolute Square $|Y(z,n)|^2$

[6] $$|Y(z,n)|^2 = [(n^{(1-\alpha)} \frac{1}{[(1-\alpha)^2 + ß^2]} [(1-\alpha) * cos(ß * ln(n)) + ß * sin(ß * ln(n))])2$$
$$+ (n^{(1-\alpha)} \frac{1}{[(1-\alpha)^2 + ß^2]} [ß * cos(ß * ln(n)) - (1-\alpha) * sin(ß * ln(n))])2]$$

[7] $$|Y(z,n)|^2 = n^{2(1-\alpha)} * \frac{1}{[ß^2 + (1-\alpha)^2]}$$

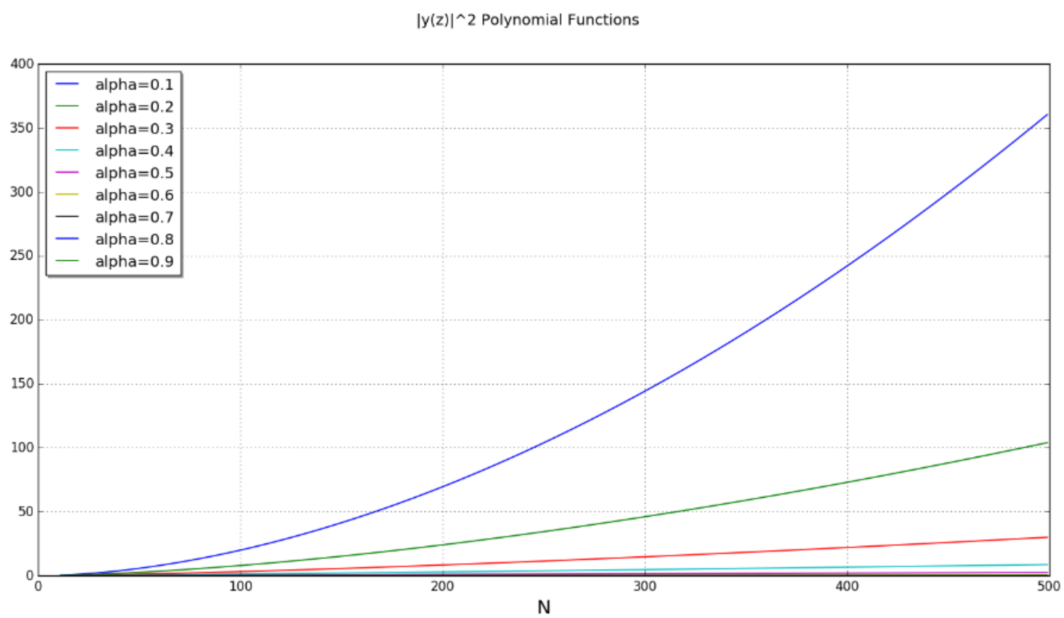

Figure 1: $|Y(z,n)|^2$ has a polynomial representation

2.1 $|Y(z,n)|^2$ is a straight line if and only if $\alpha = \frac{1}{2}$

The slope of $|Y(z,n)|^2$ with respect to n is given by:

[8] $slope(|Y(z,n)|^2) = d(|Y(z,n)|^2)/dn$

Which equals to:

[9] $slope(|Y(z,n)|^2) = 2(1-\alpha)\, n^{1-2\alpha} * \frac{1}{[ß^2+(1-\alpha)^2]}$

$|y(z)|^2$ can only be a line when the slope is constant, which can only happen if and only if:

$$(1-2\alpha) = 0$$

therefore:

$$|Y(z,n)|^2 \text{ is a straight line if and only if } \alpha = \frac{1}{2}$$

2.2 Summary for $|Y(z,n)|^2$ for $\alpha = \frac{1}{2}$:

- ⇨ the slope $|Y(z,n)|^2$ is constant if and only if $\alpha = \frac{1}{2}$
- ⇨ The slope for $|Y(z,n)|^2$ is $\frac{1}{[ß^2+\frac{1}{4}]}$
- ⇨ When α=1/2, $|Y(z,n)|^2 = \frac{n}{[ß^2+\frac{1}{4}]}$

3. Analysis of Absolute Square $|X(z,n)|^2$

[10] $$|X(z,n)|^2 = (\frac{1}{2}n^{-\alpha}\cos(ß\ln(n)) + \sum k^{-\alpha}\, cos(ß\, ln(n))^2 + (\frac{1}{2}n^{-\alpha}\sin(ß\ln(n)) + \sum k^{-\alpha}\, sin(ß\, ln(n))^2$$

[11] $$|X(z,n)|^2 = \frac{1}{4}n^{-2\alpha}(\cos^2(ß\ln(n)) + \sin^2(ß\ln(n))) + \sum k^{-\alpha}\, cos(ß\, ln(n))^2 + \sum k^{-\alpha}\, sin(ß\, ln(n))^2 + n^{-\alpha}[\cos(ß\ln(n)) * \sum k^{-\alpha}\, cos(ß\, ln(k))] + n^{-\alpha}[\sin(ß\ln(n)) * \sum k^{-\alpha}\, sin(ß\, ln(k))] =$$

[12] $$|X(z,n)|^2 = \frac{1}{4}n^{-2\alpha} + \sum_{k=1}^{n}\sum_{1}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right) + n^{-\alpha}[\cos(ß\ln(n)) * \sum k^{-\alpha}\, cos(ß\, ln(k))] + n^{-\alpha}[\sin(ß\ln(n)) * \sum k^{-\alpha}\, sin(ß\, ln(k))]$$

[13] $$|X(z,n)|^2 = \frac{1}{4}n^{-2a} + \sum_{k=1}^{n}\sum_{1}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right) +$$
$$n^{-a} * \sum k^{-\alpha}\left[cos\left(ß\, ln\left(\frac{k}{n}\right) + \cos(ß * \ln(kn))\right)\right] +$$
$$n^{-a} * \sum k^{-\alpha}\left[cos\left(ß\, ln\left(\frac{k}{n}\right) - \cos\,(ß * \ln(kn))\right)\right) =$$

[14] $$|X(z,n)|^2 = \frac{1}{4}n^{-2a} + \sum_{k=1}^{n}\sum_{1}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right)$$
$$+2\, n^{-a} * \sum k^{-\alpha}\cos\,(ß * \ln\left(\frac{k}{n}\right))$$

When $|X(z,n)|^2$ is represented graphically, one can observe that:

- $|X(z,n)|^2$ is a wave that converges when n → ∞ and α>1 (Fig. 2)
- $|X(z,n)|^2$ is a wave that does not converge when n → ∞ and α<1 (Fig. 3)
- $|X(z,n)|^2$ is a wave that collapses to a line when n → ∞ and α=1/2 and ß=Im($\zeta(z^*)$) (Fig. 4)
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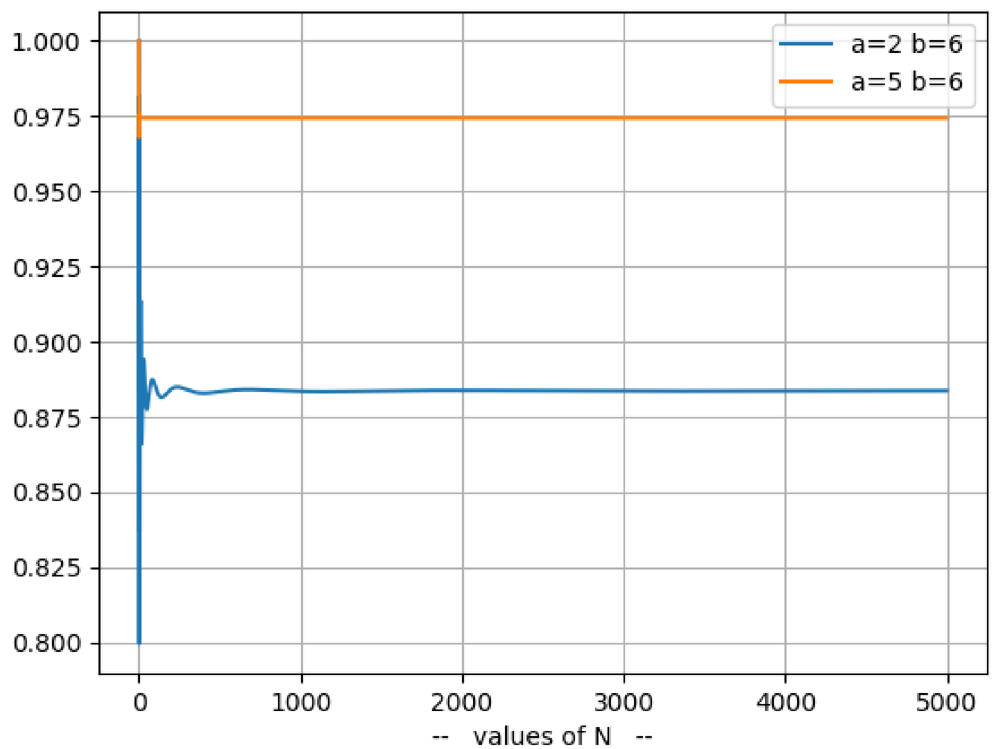


Figure 2: $|X(z,n)|^2$ *for α>1*

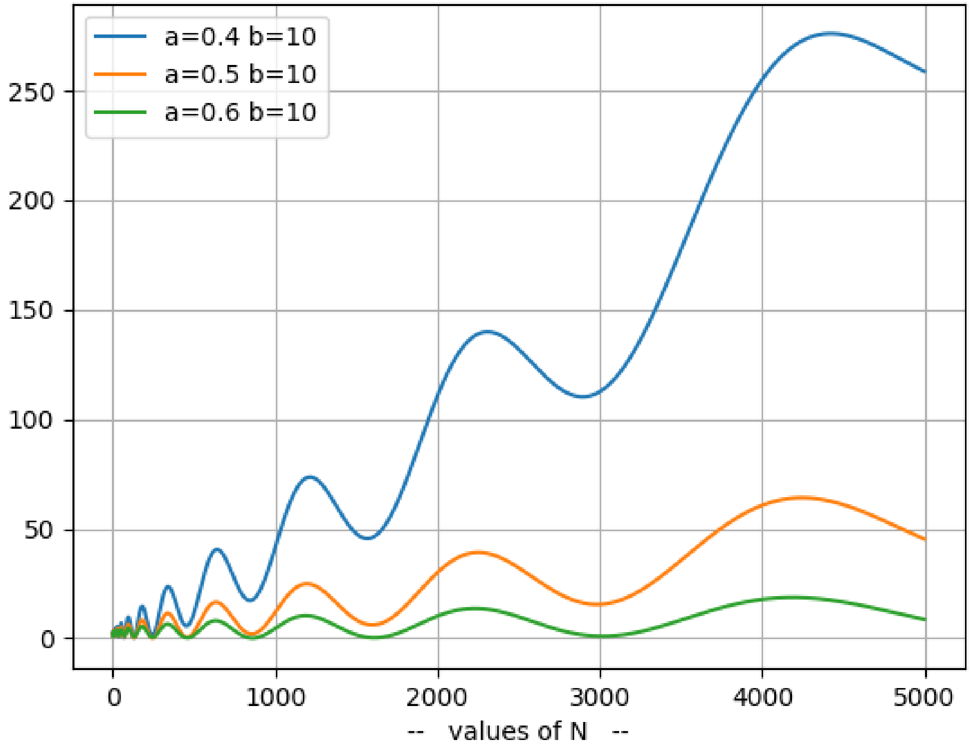


Figure 3: $|X(z,n)|^2$ *for* $\alpha<1$

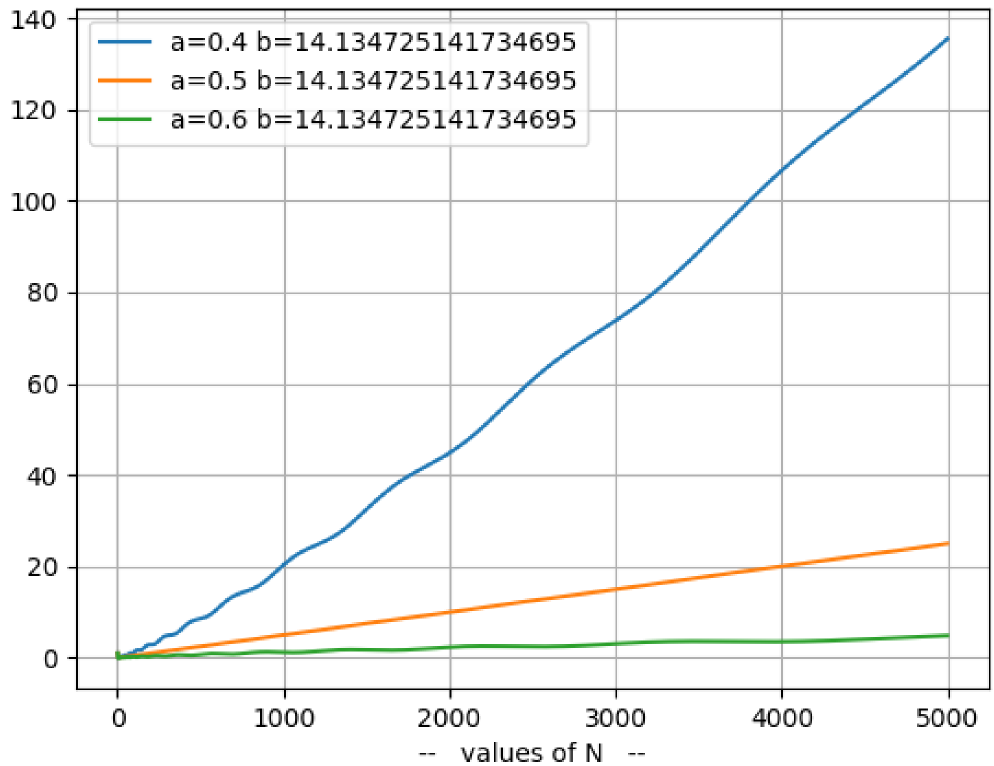


Figure 4: For a=0.5, b=b1, $|X(z,n)|^2$ *collapses to a line*

3.1. $|X(z,n)|^2$ converges when $n \to \infty$ and α>1 to $|\zeta(\alpha, ß)|^2$

The limit of $|X(z,n)|^2$outside the critical strip [0,1] can be calculated from [4]:

[15] $$\lim_{n\to\infty} |X(z,n)|^2 = \sum_{k=1}^{n} \sum_{j=1}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right)$$

As one can see in some examples in the following table where z=α+iß:

α	ß	$\lim_{n\to\infty} \lvert X(z,n)\rvert^2$	$\lvert\zeta(\alpha, ß)\rvert^2$
1.0	7	1.074711506185445	1.074756
1.0	10	1.4413521753699579	1.441430
2.5	7	1.0093487944300192	1.009349
2.5	10	1.0507402208589398	1.050740

Table 1

[16] $$\lim_{n\to\infty} |X(z,n)|^2 = |\zeta(z)|^2 = \zeta(\alpha + ßi) * \zeta(\alpha - ßi) \quad for\ \alpha > 1$$

And also, in the following figure 5:

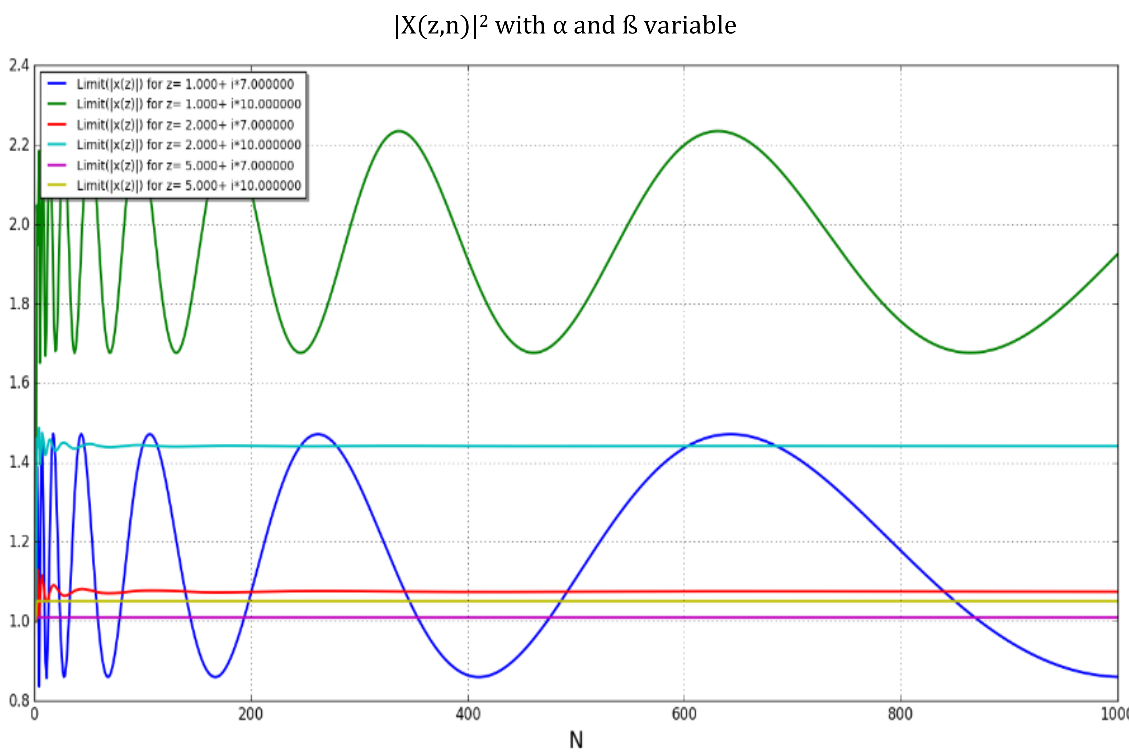


Figure 5. $|X(z,n)|^2$ converges when $n \to \infty$ and α>1

One can observe that the graphs for α=1 do not converge while graphs for α>1 they all converge. This observation can be used to prove that there are no zero values of ζ(z) for z with Re(z)=α>1.

3.2. $|X(z,n)|^2$ diverges when $n \to \infty$ for α≤1

$|X(z,n)|^2$ diverges when $n \to \infty$ for α<1 because:

[17] $|\cos\left(ß\,(\ln\left(\frac{k}{j}\right)\right)| < 1$

And:

[18] $\sum_{k=1}^{n}\sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha}\ \ diverges\ for\ \alpha < 1$

Therefore:

[19] $\lim_{n\to\infty} |X(z,n)|^2 \ = \sum_{k=1}^{n}\sum_{j=1}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\,(ln\left(\frac{k}{j}\right)\right)\ diverges\ for\ \alpha < 1$

3.3. $|X(z,n)|^2$ does not collapse to any polynomial function $|X(z,n)|^2 = \mathrm{C} * \mathrm{n}^{\mathrm{t}}$ for $\mathrm{t} > 1$, and C constant

One can prove it with a reduction to absurd.

Let's assume that $|X(z,n)|^2 = C * n^t\ for\ t > 1$ where C and t integers C>0 and t>0

If $|X(z,n)|^2 = C * n^t$ then:

[20] $\lim_{n\to\infty} |X(z,n)|^2/n^t = C$

But:

[21] $\lim_{n\to\infty} |X(z,n)|^2\,/n^t \ = \frac{1}{n^t} * \lim_{n\to\infty} \sum_{k=1}^{n} k^{-2\alpha} + \frac{1}{n^t} * \sum_{k=1}^{n}\sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * \cos\left(ß\,(\ln\left(\frac{k}{j}\right)\right)$

And:

[22] $\frac{1}{n^t} * \lim_{n\to\infty} \sum_{k=1}^{n} k^{-2\alpha} = 0\ \ for\ t > 1$

[23] $\frac{1}{n^t} * \sum_{k=1}^{n}\sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * \cos\left(ß\,(\ln\left(\frac{k}{j}\right)\right) = \ 0\ for\ t > 1$

So, C must be 0 which is an absurd.

3.4. $|X(z,n)|^2$ collapses to a straight-line $|X(z,n)|^2 = Cn\ \ if\ Re(z) = 1/2$

The proposition says that the following limit exists only for Re(z) = 1/2

[24] $\lim_{n\to\infty} (|X(z,n)|^2/\,n) = S$

Using the expression:

[25] $$\lim_{n\to\infty} (|X(z,n)|^2 / n) = \lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} k^{-2\alpha} + \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right))$$

3.4.1. For α>1/2 , one can see that $\lim_{n\to\infty} (|x(z,n)|^2 / n) = 0$:

[26] $$\lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} k^{-2\alpha}) = 0$$ because 2α>1 and the series is convergent

[27] $$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right) < \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \sum_{j\neq k}^{n} (k^{-\alpha} * j^{-\alpha}) < \lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} k^{-2\alpha})$$

So:

[28] $$\lim_{n\to\infty} \left(\frac{1}{n} \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * \cos\left(ß\left(\ln\left(\frac{k}{j}\right)\right)\right)\right) = 0$$

3.4.2. For α<1/2, one can see that $\lim_{n\to\infty} (|X(z,n)|^2 / n) = \infty$ as:

[29] $$\lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} k^{-2\alpha}) < \lim_{n\to\infty} \frac{1}{n} \left(n * \frac{1}{n}\right) = \lim_{n\to\infty} \frac{1}{n} = 0$$

And:

[30] $$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-\alpha} * j^{-\alpha} * \cos\left(ß\left(\ln\left(\frac{k}{j}\right)\right)\right) > \lim_{n\to\infty} (\frac{1}{n} * n^2 * \frac{1}{n^{2\alpha}}) = \infty$$

Where the summations are replaced by the number of elements in the matrix (n x n) times the smallest value in each row (1/n) then 1>(2-1-2α)>0 when α<1/2

3.4.3. Limit for α=1/2.

When α=1/2, one can express $(|X(z,n)|^2 / n)$ as:

[31] $$\lim_{n\to\infty} (|X(z,n)|^2 / n) =$$

$$= \lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} k^{-1} + \sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * \cos\left(ß\left(\ln\left(\frac{k}{j}\right)\right)\right))$$

$$= \lim_{n\to\infty} \frac{1}{n} \left(\sum_{k=1}^{n} k^{-1}\right) + \lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right)) =$$

$$= 0 + \lim_{n\to\infty} \frac{1}{n} (\sum_{k=1}^{n} \sum_{j\neq k}^{n} k^{-1/2} * j^{-1/2} * cos\left(ß\left(ln\left(\frac{k}{j}\right)\right)\right)) =$$

$$= \lim_{n\to\infty} \frac{2n}{n} (\sum_{j=1}^{n-1} n^{-1/2} * j^{-1/2} * cos\left(ß\left(ln\left(\frac{n}{j}\right)\right)\right)) =$$

$$= \lim_{n\to\infty} 2(n^{-\frac{1}{2}} \sum_{j=1}^{n-1} * j^{-\frac{1}{2}} * cos\left(ß\,(ln\left(\frac{n}{j}\right)\right)) =$$

Using the integral approximation of the infinite series

$$= 2 * \lim_{n\to\infty} \frac{2 * \sqrt{n} * \cos\left(ß * ln\left(\frac{n}{n}\right)\right) - 2 * ß * \sin\,(ß * \ln\,(\frac{n}{n})}{4 * ß^2 + 1} * n^{-\frac{1}{2}}$$

$$= 2 * \frac{2 * \sqrt{n}}{4 * ß^2 + 1} n^{-\frac{1}{2}} = 2 * \frac{2}{4 * ß^2 + 1} = \frac{1}{ß^2 + 1/4}$$

So, if $\lim_{n\to\infty} (|X(z,n)|^2\,/n)$ exists will be equal to:

[32] $\lim_{n\to\infty} (|X(z,n)|^2\,/n) = \frac{1}{ß^2+1/4}$ if z=1/2+iß

And this limit can only exist when $|\mathrm{X(z,n)}|^2$ is monotonous which means that the curve will cross the x-axis only once.

[33] $$|X(z,n)|^2 = \left(\sum_{k=1}^{n} \sum_{j=k}^{n} k^{-\frac{1}{2}} * j^{-\frac{1}{2}} * cos\left(ß\,(ln\left(\frac{k}{j}\right)\right)\right)$$

$$= 2 * n^{-a} * \left(\sum_{j=1}^{n-1} j^{-a} * \cos\left(ß * (\ln\left(\frac{x}{j}\right))\right)\right)$$

4. Calculating the zeros of $|X(z,n)|^2$

Let's define the function $C_2(n,a,b) = |X(z,n)|^2$ in R (where z=a+bi) such that:

[34] $$C_2(n,a,b) = 2 * n^{-a} * \left(\sum_{j=1}^{n-1} j^{-a} * \cos\left(b * (\ln\left(\frac{n}{j}\right))\right)\right)$$

With the following wave representation for $C_2(n,a,b)$:

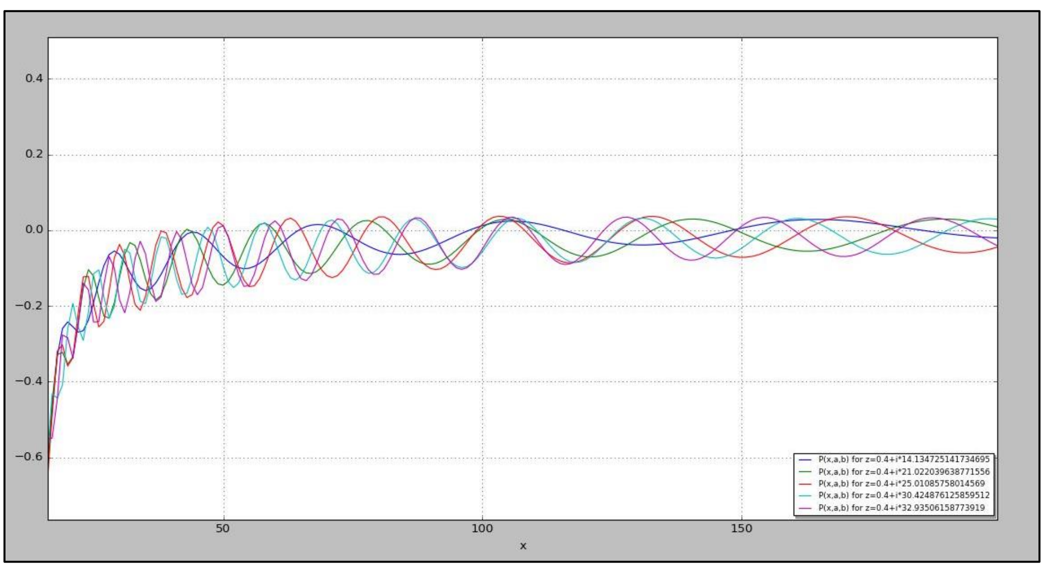


Figure 6. $C_2(x,a,b)$ for a=0.4 and variable b

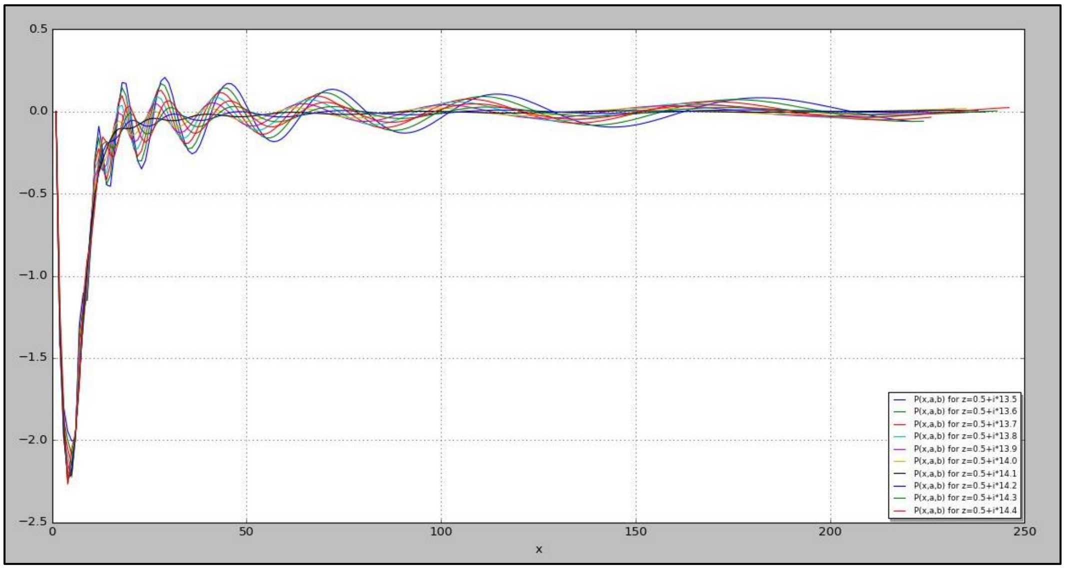


Figure 7. $C_2(n,a,b)$ *for a=0.5 and variable b*

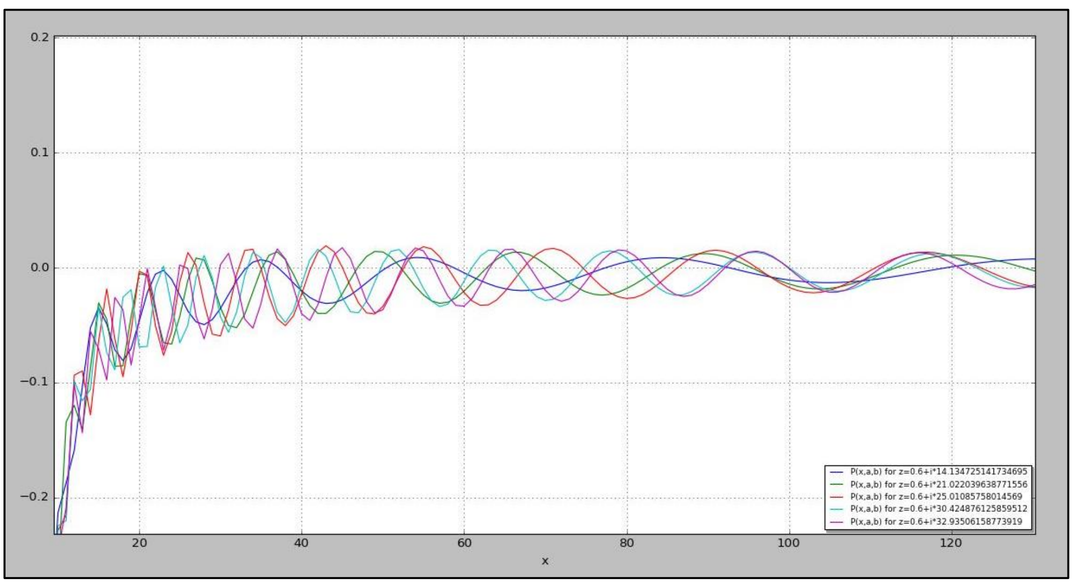


Figure 8. $C_2(n,a,b)$ *for a=0.6 and variable b*

As a wave, $C_2(n,a,b)$ can have one or more zeros. For $C_2(n,a,b)$ to have only one zero, it must cross the axis y=0 only once, which means that the wave collapses to a polynomial line. A numeric method has been created and coded to find the values of (n,a,b) such that $C_2(n,a,b)$=0. The following table shows an example of those calculated values, where x=n, a=Alfa, and b=Beta:

Alfa	Beta	Number of Zeros	Zero at X=
0.4	14.1	5	
0.4	14.2	5	
0.4	14.3	5	
0.4	14.4	5	
0.5	14.07	5	
0.5	14.08	5	
0.5	14.09	5	
0.5	14.1	4	
0.5	14.11	4	
0.5	14.12	3	
0.5	14.13	1	200
0.5	14.14	3	
0.5	20.97	11	
0.5	20.98	11	
0.5	20.99	11	
0.5	21	9	
0.5	21.01	5	
0.5	21.02	1	442
0.5	21.03	3	
0.5	24.96	16	
0.5	24.97	16	
0.5	24.98	15	
0.5	24.99	11	
0.5	25	7	
0.5	25.01	1	626
0.5	25.02	6	
0.5	25.03	10	

Table 2. Number of Zeros of $C_2(x,a,b)$ *for different values of a=Alfa, and b=Beta*

The calculations for $a \in (0,1)$ and $b \in [1, 100]$ only found single zeros for $C_2(x,a,b)$ for values of $a = 0.5$ as shown in the following table that summarizes the single zeros found in those intervals:

Values (x,a,b) | C2(x,a,b)=0 SINGLE		
x*	a*	b*
200.1000	0.5000	14.1368
442.2000	0.5000	21.0226
625.8000	0.5000	25.0110
926.0000	0.5000	30.4261
1085.0000	0.5000	32.9355
1413.0000	0.5000	37.5866
1674.6000	0.5000	40.9188
1877.5000	0.5000	43.3272
2304.8000	0.5000	48.0057

Table 3. List of first Zeros of $C_2(x,a,b)$

One can observe that:

[35] $if\ \ C_2(x,a,b) = 0 \rightarrow$

$a = 1/2$

$b = \ Im(z) \qquad with\ \zeta(z) = 0$

(a, b) are the Nontrivial Zeros of ζ(z) in the critical line.

$$x = b^2 + \frac{1}{4}$$

And the calculated values of $\lim_{x\to\infty} C_2(x,a,b)$ for the values of (a,b) from Table 3 are:

Values (x,a,b) | C2)x,a,b=0			Limit (C2(x,a,b))
x	a	b	when x->∞
200.1000	0.5000	14.1368	0.0050
442.2000	0.5000	21.0226	0.0023
625.8000	0.5000	25.0110	0.0016
926.0000	0.5000	30.4261	0.0011
1085.0000	0.5000	32.9355	0.0009
1413.0000	0.5000	37.5866	0.0007
1674.6000	0.5000	40.9188	0.0006
1877.5000	0.5000	43.3272	0.0005

Table 4. Limit of $C_2(x,a,b)$ for b in Table 10 and x->∞

Values (x,a,b) | C2(x,a,b)=0			Limit (C2(x,a,b))	
x	a	b	when x->∞	Known Zero
200.1000	0.5000	14.1368	0.0050	14.1347
442.2000	0.5000	21.0226	0.0023	21.0220
625.8000	0.5000	25.0110	0.0016	25.0109
926.0000	0.5000	30.4261	0.0011	30.4249
1085.0000	0.5000	32.9355	0.0009	32.9351
1413.0000	0.5000	37.5866	0.0007	37.5862
1674.6000	0.5000	40.9188	0.0006	40.9187
1877.5000	0.5000	43.3272	0.0005	43.3271
2304.8000	0.5000	48.0057	0.0004	48.0052
2477.7000	0.5000	49.7740	0.0004	49.7738

Table 5. Comparing "b" calculated with known zeros of ζ(z)

$|X(z,n)|^2 = \mathrm{C}(n,a,b)$ has the following special properties for all (a,b) such that ζ(a+bi)=0.

if $\quad S= \frac{1}{b^2+1/4}$

$C_2(n,a,b) = \ 0\ when\ x = \frac{1}{S},\ \ a = \frac{1}{2},\ \ b = Im(z^*)$ with z^* a nontrivial zero of $\zeta(z)$

$\lim_{x\to\infty} C_2\left(n,\frac{1}{2},b\right) = S$

Graphically:

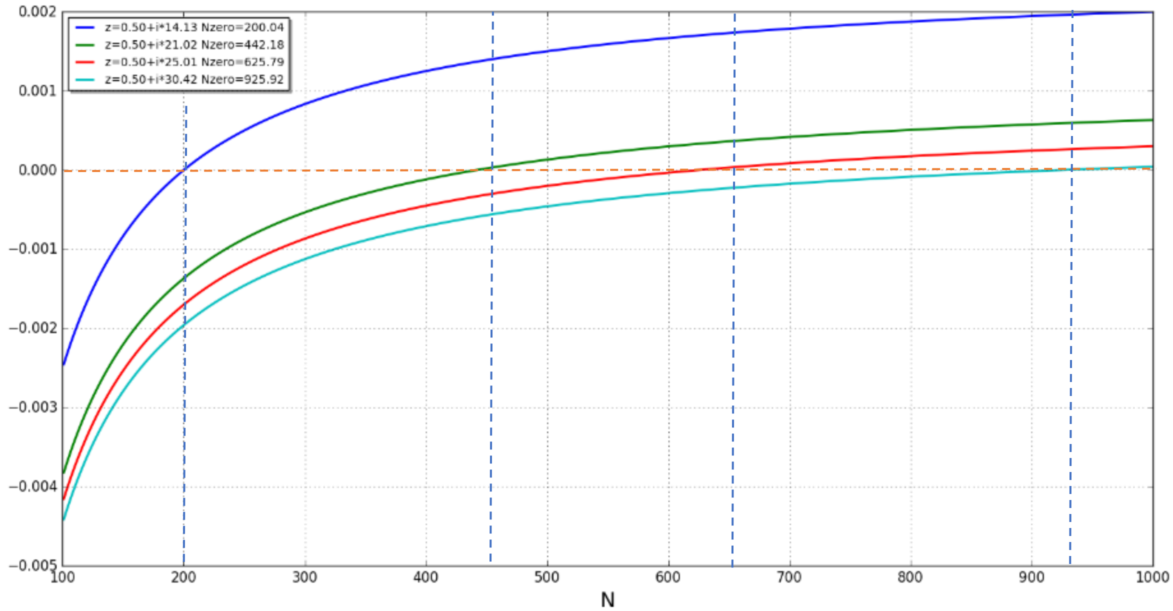


Figure 9. $C_2(n, 1/2, b)$ *such that ζ(1/2+b*i)=0*

The decomposition of *ζ(z) = X(z) – Y(z) for Re(z)≥0, z≠1,* has enabled us to study the zeros of ζ(z).

For Re(z)≥0, z≠1, the representation of X(z) and Y(z) only coincide, making ζ(z)=0, when X(z) and Y(z) are both a straight line with slope 1/(ß² + ¼), which happens only when Re(z)=1/2.

5. Theorem. For Re(z)≥0, z≠1, if z* is a nontrivial zero of ζ(z), then Re(z*)=1/2

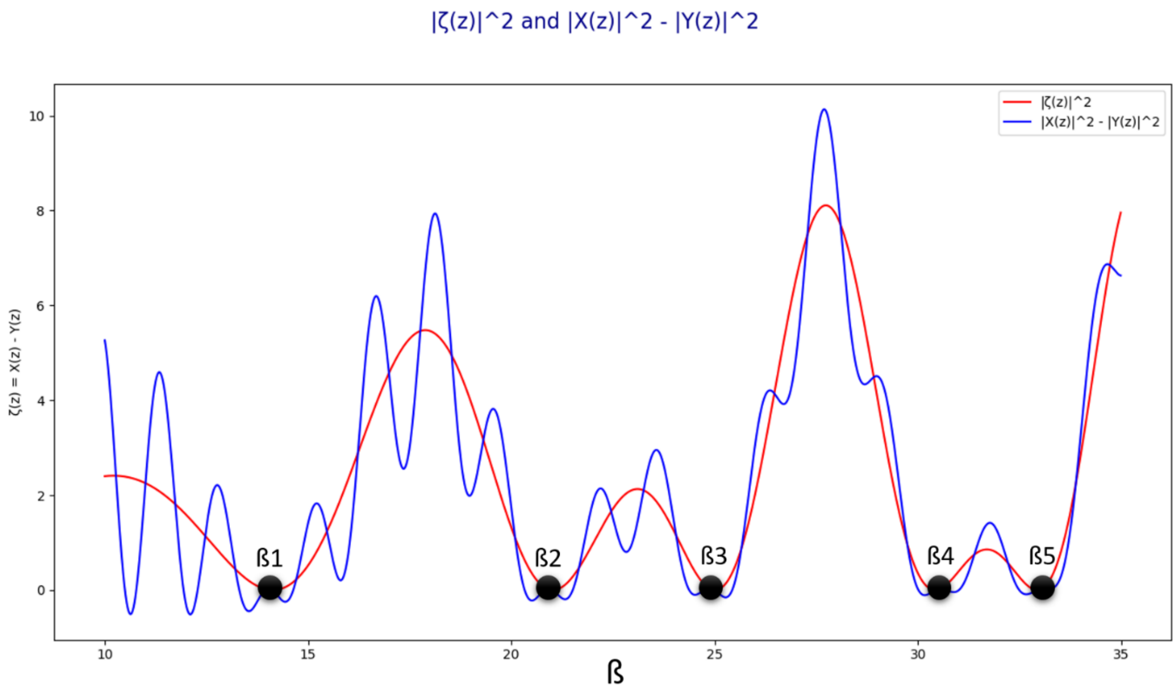
Proof:

- From [3], [4], [5] ζ(z) $= X(z) - Y(z)$ for Re(z)>0, z≠1
- From [7] is always a polynomial line.
- From [9]|Y(z, n)|² is only straight line if and only if Re(z)= ½

  $|Y(z^*)|^2 = \lim_{n\to\infty} |\mathrm{Y}(\mathrm{z}^*, \mathrm{n})|^2$ tends to a straight line with slope $\frac{1}{[ß^{*2}+1/4]}$
- From [32] |X(z, n)|² is a wave function that has only one polynomial representation in the form of a straight line if and only if Re(z)= ½ and for certain values of Im(z)=ß* that we calculated. This values of ß* coincide with the imaginary parts of the nontrivial zeros of Riemann Zeta z*, so:

  $|X(z^*)|^2 = \lim_{n\to\infty} |\mathrm{X}(\mathrm{z}^*, \mathrm{n})|^2$ tends to a straight line with slope $\frac{1}{[ß^{*2}+1/4]}$
- If z=z* is a zero of ζ(z) then there exists an N such that for any n>N then |X(z*)|² -|Y(z*)|² <∈ arbitrarily small

- From[9] and [32] Of all possible representations of $|X(z^*)|^2$ and $|Y(z^*)|^2$ at any z* nontrivial zero of ζ(z), the only one in common for both functions is a representation as a straight line with slope $\frac{1}{[ß^{*2}+1/4]}$ when Re(z)=1/2.
- Therefore, all z* nontrivial solution of ζ(z) must have Re(z*) = ½



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REFERENCES

(1) B. Riemann, "Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse", Monat. der Königl. Preuss. Akad. der Wissen. zu Berlin aus der Jahre 1859 (1860), 671–680; also, Gesammelte math. Werke und wissensch. Nachlass, 2. Aufl. 1892, 145–155.

(2) Caceres, Pedro. "ζ(z) = X(z) – Y(z) A decomposition of the Riemann Zeta function for $Re(z) > 0, z \neq 1$", 2020. VIXRA: https://vixra.org/abs/2003.0189

(3) Kopp, Konrad. "Theory and Application of Infinite Series." Blackie & Son Limited, London, 1928

(4) Sondow, Jonathan and Weisstein, Eric W. "Riemann Zeta Function." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/RiemannZetaFunction.html