Ingenious Proof of Fermat's Last Theorem

"5% of the people think; 10% of the people think that they think; and the other 85%would rather die than think."----Thomas Edison

"The simplest solution is usually the best solution"---Albert Einstein

Abstract

An ingenious proof of Fermat's Last theorem has been covered in this paper. Fermat's Last theorem states that if A, B, C, n are positive integers with A, B, and C being coprime, and $n > 2$, then the equation $A^n + B^n = C^n$ has no solutions. The principles applied in the proof are based on the same properties of the factored Beal equations. However, the proof is by contradiction. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. High school and college students can learn and prove this theorem for a class exam.

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Option 1

Preliminaries

Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Fermat''s Last theorem, one will be guided by the observational properties of the factored Beal equations.

Observation 1: $\boxed{2^3 + 2^3 = 2^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$2^3 + 2^3 = 2^4$$

$$2^3 + 2^3 = 2^3 \bullet 2$$

$$\underbrace{2^3}_{k}\underbrace{(1+1)}_{L} = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side..

Note above that the greatest common power of the prime factors on the left of the equation is the same as a power of the prime factor on the right side of the equation.

Note also, the following

The ratio $\frac{K}{M} = \frac{2^3}{2^3} = 1$.

If $\frac{K}{M} = 1$, then $K = M$

Similarly, $\frac{P}{L} = \frac{2}{1+1} = 1$.

If $\frac{P}{L} = 1$, then $P = L$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^y = F^z t^z$$

$$r = s = t = 2$$

$$x = 3 .y = 3, z = 4$$

$$(D = 1, E = 1, F = 1)$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x} F^z}_{P}$$

$$K = M,\ L = P$$

Observation 2: $\boxed{7^6 + 7^7 = 98^3}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$7^6 + 7^7 = 98^3$$
$$7^6 + 7^6 \bullet 7 = (49 \bullet 2)^3$$
$$7^6 + 7^6 \bullet 7 = 7^6 \bullet 2^3$$
$$7^6(1+7) = 7^6 \bullet 2^3$$
$$\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\dfrac{K}{M} = \dfrac{7^6}{7^6} = 1$.

Similarly, $\dfrac{P}{L} = \dfrac{2^3}{1+7} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 7$$
$$x = 6, y = 7, z = 3$$
$$(D = 1, E = 1, F = 14)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \quad L = P$$

Observation 3: $\boxed{3^3 + 6^3 = 3^5}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$3^3 + 6^3 = 3^5$$
$$3^3 + (3 \bullet 2)^3 = 3^5$$
$$3^3 + 3^3 \bullet 2^3 = 3^5$$
$$3^3(1 + 2^3) = 3^3 \bullet 3^2$$
$$\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\dfrac{K}{M} = \dfrac{3^3}{3^3} = 1$.

Similarly, $\dfrac{P}{L} = \dfrac{3^2}{1+8} = 1$.

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$$(Dr)^x + (Es)^y = (Ft)^z$$
$$D^x r^x + E^y s^y = F^z t^z$$
$$r = s = t = 3$$
$$x = 3, y = 3, z = 5$$
$$(D = 1, E = 2, F = 1)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K = M, \quad L = P$$

Observation 4: $\boxed{2^9+8^3=4^5}$

Identify the greatest common factor.of all three terms of the equation and factor it out on the left side.

$$2^9+8^3=4^5$$
$$2^9+([2^3])^3=([2^2])^5$$
$$2^9+2^9=2^{10}$$
$$2^9(1+1)=2^9\bullet 2$$
$$\underbrace{2^9}_{k}(\underbrace{1+1}_{L})=\underbrace{2^9}_{M}\bullet\underbrace{2}_{P}$$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M}=\frac{2^9}{2^9}=1.$

Similarly, $\frac{P}{L}=\frac{2}{1+1}=1.$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C.respectively, where D, E and F are positive integers, such that $A=Dr$, $B=Es$, $C=Ft$.

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^y=F^zt^z$$
$$r=s=t=2$$
$$x=9, y=3, z=5$$
$$(D=1, E=4, F=2)$$
$$\underbrace{r^x}_{K}[\underbrace{D^x+E^ys^y\bullet r^{-x}}_{L}]=\underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$$
$$K=M,\ L=P$$

Observation 5: $\boxed{34^5+51^4=85^4}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$$34^5+51^4=85^4$$
$$(17\bullet 2)^5+(17\bullet 3)^4=(17\bullet 5)^4$$
$$17^5\bullet 2^5+17^4\bullet 3^4=17^4\bullet 5^4$$
$$17^4(17\bullet 2^5+3^4)=17^4\bullet 5^4$$
$$\underbrace{17^4}_{k}(\underbrace{17\bullet 2^5+3^4}_{L})=\underbrace{17^4}_{M}\bullet\underbrace{5^4}_{P}$$

(Note: $17\bullet 2^5+3^4=17\bullet 32+81=625$; $5^4=625$)

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M}=\frac{17^4}{17^4}=1.$

Similarly, $\frac{P}{L}=\frac{5^4}{17\bullet 2^5+3^4}=1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C.respectively, where D, E and F are positive integers, such that $A=Dr$, $B=Es$, $C=Ft$.

$$(Dr)^x+(Es)^y=(Ft)^z$$
$$D^xr^x+E^ys^y=F^zt^z$$
$$r=s=t=17$$
$$x=5, y=4, z=4$$
$$(D=2, E=3, F=5)$$
$$\underbrace{s^y}_{K}[\underbrace{E^y+D^xr^xs^{-y}}_{L}]=\underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$$
$$K=M,\ L=P$$

Note above that one factored out s^y .

One will apply the switch from r^x to s^y in the conjecture proof.

Observation 6: $\boxed{3^9 + 54^3 = 3^{11}}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$3^9 + 54^3 = 3^{11}$

$3^9 + (3^3 \bullet 2)^3 = 3^{11}$

$3^9 + 3^9 \bullet 2^3 = 3^{11}$

$3^9(1 + 2^3) = 3^9 \bullet 3^2$

$\underbrace{3^9}_{k}(\underbrace{1 + 2^3}_{L}) = \underbrace{3^9}_{M} \bullet \underbrace{3^2}_{P}$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side

Note the following The ratio $\frac{K}{M} = \frac{3^9}{3^9} = 1$.

Similarly, $\frac{P}{L} = \frac{3^2}{1 + 2^3} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$(Dr)^x + (Es)^y = (Ft)^z$

$D^x r^x + E^y s^y = F^z t^z$

$r = s = t = 3$

$x = 9, y = 3, z = 11$

$(D = 1, E = 18, F = 1)$

$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$

$K = M, \ L = P$

Observation 7: $\boxed{33^5 + 66^5 = 33^6}$

Identify the greatest common factor of all three terms of the equation and factor it out on the left side.

$33^5 + 66^5 = 33^6$

$(11 \bullet 3)^5 + (11 \bullet 2 \bullet 3)^5 = (11 \bullet 3)^6$

$11^5 \bullet 3^5 + 11^5 \bullet 2^5 \bullet 3^5 = 11^6 \bullet 3^6$

$11^5(3^5 + 2^5 \bullet 3^5) = 11^5 \bullet 11 \bullet 3^6$

$\underbrace{11^5}_{k}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$

Observe that the factor K on the left side equals the factor M on the right side of the equation, and the factor L on the left side of the equation equals the factor, P, on the right side.

Note the following

The ratio $\frac{K}{M} = \frac{11^5}{11^5} = 1$.

Similarly, $\frac{P}{L} = \frac{11 \bullet 3^6}{3^5 + 2^5 \bullet 3^5} = 1$

Corresponding relationship formula

Let r, s and t be prime factors of A, B and C .respectively, where D, E and F are positive integers, such that $A = Dr$, $B = Es$, $C = Ft$.

$(Dr)^x + (Es)^y = (Ft)^z$

$D^x r^x + E^y s^y = F^z t^z$

$r = s = t = 11$

$x = 5, y = 5, z = 6$

$D = 3, E = 6, F = 3$

$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$

$K = M, \ L = P$

Surprise: The above properties will apply to $\boxed{6^2 + 8^2 = 10^2}$

$6^2 + 8^2 = 10^2$

$(2 \bullet 3)^2 + (2^3)^2 = (2 \bullet 5)^2$

$2^2 \bullet 3^2 + 2^6 = 2^2 \bullet 5^2$

$\underbrace{2^2}_{K}(\underbrace{3^2 + 2^4}_{L}) = \underbrace{2^2}_{M} \bullet \underbrace{5^2}_{P}$; $K = M, \ L = P$

Summary of Observations 1-7

The most important and useful observation in the above examples is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. This observation will be useful in proving Fermat's theorem.

1. $2^3 + 2^3 = 2^4$; $\underbrace{2^3}_{K}(\underbrace{1+1}_{L}) = \underbrace{2^3}_{M} \bullet \underbrace{2}_{P}$	**2.** $3^3 + 6^3 = 3^5$; $\underbrace{3^3}_{K}(\underbrace{1+8}_{L}) = \underbrace{3^3}_{M} \bullet \underbrace{3^2}_{P}$	**3.** $7^6 + 7^7 = 98^3$; $\underbrace{7^6}_{K}(\underbrace{1+7}_{L}) = \underbrace{7^6}_{M} \bullet \underbrace{2^3}_{P}$
4. $2^9 + 8^3 = 4^5$; $\underbrace{2^9}_{K}(\underbrace{1+1}_{L}) = \underbrace{2^9}_{M} \bullet \underbrace{2}_{P}$	**5.** $34^5 + 51^4 = 85^4$; $\underbrace{17^4}_{K}(\underbrace{17 \bullet 2^5 + 3^4}_{L}) = \underbrace{17^4}_{M} \bullet \underbrace{5^4}_{P}$	**6.** $3^9 + 54^3 = 3^{11}$; $\underbrace{3^9}_{K}(\underbrace{1+2^3}_{L}) = \underbrace{3^9}_{M} \bullet \underbrace{3^2}_{P}$
7. $33^5 + 66^5 = 33^6$; $\underbrace{11^5}_{K}(\underbrace{3^5 + 2^5 \bullet 3^5}_{L}) = \underbrace{11^5}_{M} \bullet \underbrace{11 \bullet 3^6}_{P}$	**Corresponding relationship formulas** $\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^{z'}}_{P}$ $K = M,\ L = P$ **or** $\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$; $K = M,\ L = P$	

Properties of the Factored Beal Equation

Let r, s and t be prime factors of A, B and C .respectively, such that $A = Dr$ **,** $B = Es$ **,** $C = Ft$ **.** where D, E and F are positive integers; and the equation becomes $(Dr)^x + (Es)^y = (Ft)^z$.

Step 1: Factor out r^x on the left side of the equation and on the right side of the equation, replace t^z by $t^x \bullet t^{z-x}$ (Note $t^x \bullet t^{z-x} = t^z$)

$$(Dr)^x + (Es)^y = (Ft)^z$$

$$D^x r^x + E^y s^y = F^z t^z$$

$$\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P};\ K = M,\ L = P$$

For the factorization $\underbrace{r^x}_{K}[\underbrace{D^x + E^y s^y \bullet r^{-x}}_{L}] = \underbrace{t^x}_{M}\underbrace{t^{z-x}F^z}_{P}$ with respect to r^x, $r^x = t^x$ $(K = M)$

Step 2: Factor out s^y on the left side of the equation and on the right side of the equation, replace t^z by $t^y \bullet t^{z-y}$ (Note $t^y \bullet t^{z-y} = t^z$)

$$(Es)^y + (Dr)^x + = (Ft)^z$$

$$E^y s^y + D^x r^x = F^z t^z$$

$$\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}\ ;K = M,\ L = P$$

For the factorization $\underbrace{s^y}_{K}[\underbrace{E^y + D^x r^x \bullet s^{-y}}_{L}] = \underbrace{t^y}_{M}\underbrace{t^{z-y}F^z}_{P}$ with respect to s^y, $s^y = t^y$ $(K = M)$

Option 2

Ingenious Proof of Fermat's Last Theorem

To prove Fermat's last theorem from the proof of the equivalent Beal conjecture, let $x, y, z = n > 2$. Then Fermat's Last theorem states that if A, B, C, n are positive integers and A, B, and C are coprime, with $n > 2$, then the equation $A^n + B^n = C^n$ has no solutions. The proof would be constructed by contradiction.

Given: A, B, C, n are positive integers and A, B, and C are coprime, with $n > 2$**.**

Required: To prove that the equation $A^n + B^n = C^n$ has no solutions.

Plan: Let r, s and t be prime factors of A, B and C.respectively, such that $A = Dr$, $B = Es$, $C = Ft$ where D, E and F are positive integers, and $\boxed{r \neq s \neq t}$ (A, B and C are **coprime**). The proof would be complete after showing that $r = s = t$, which would be a contradiction to the assumption that $r \neq s \neq t$.The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. Two main steps are involved in the proof. In the first step, one will determine how *r and t* are related, and in the second step, one will determine how *s and t* are related. The steps here are similar to those in the original Beal conjecture proof, except that the end of step 2 would conclude by contradiction.

Proof:

Step 1: One will factor out r^n

$(Dr)^n + (Es)^n = (Ft)^n \qquad (A = Dr,\ B = Es,\ C = Ft)$

$D^n r^n + E^n s^n = F^n t^n$

$$\underbrace{r^n}_{K}[\underbrace{D^n + E^n s^n r^{-n}}_{L}] = \underbrace{t^n}_{M}\underbrace{t^{n-n}F^n}_{P} = \underbrace{t^n}_{M}\underbrace{F^n}_{P}$$

$K = M,\ \ L = P$ (Properties of factored Beal equation)

From above, $r^n = t^n$;

If $r^n = t^n$, then $r = t$. $\quad (\log r^n = \log t^n;\ n\log r = n\log t;\ \log r = \log t;\ r = t)$

Step 2: One will factor out s^n

$(Es)^n + (Dr)^n + = (Ft)^n$

$E^n s^n + D^n r^n = F^n t^n$

$$\underbrace{s^n}_{K}[\underbrace{E^n + D^n r^n \bullet s^{-n}}_{L}] = \underbrace{t^n}_{M}\underbrace{t^{n-n}F^n}_{P} = \underbrace{t^n}_{M}\underbrace{F^n}_{P}$$

$K = M,\ \ L = P$ (Properties of factored Beal equation)

From above, $s^n = t^n$;

If $s^n = t^n$, then $s = t$. $\quad (\log s^n = \log t^n;\ n\log s = n\log t;\ \log s = \log t;\ s = t)$

Since it has been shown in Step 1 that $r = t$, and in Step 2 that, $s = t$; $r = s = t$**.**

This result. $\boxed{r = s = t,}$ is a contradiction to $\boxed{r \neq s \neq t}$. of the hypothesis, and therefore, the equation $A^n + B^n = C^n$ ($= (Dr)^n + (Es)^n = (Ft)^n$) is **not** true and has no solutions. The proof is complete.

Discussion

It is interesting that to prove Fermat's last theorem from the proof of the original Beal conjecture, all one has to do is to assume at the beginning of the proof that A, B and C do not have any common prime factor (part of the hypothesis) and then produce a contradictory result that A, B and C have a common prime factor. The principles upon which relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation are sound, since they are based on numerical sample problems research.

Conclusion

An ingenious proof of Fermat's Last Theorem has been covered in this paper. The principles applied in the proof are based on the properties of the factored Beal equations, and the proof is by contradiction. The main principle for obtaining relationships between the prime factors on the left side of the equation and the prime factor on the right side of the equation is that the greatest common power of the prime factors on the left side of the equation is the same as a power of the prime factor on the right side of the equation. High school and college students can learn and prove this theorem for a class exam.

PS: Other proofs of **Fermat's Last Theorem** by the author are at viXra:1605.0195; viXra:1609.0080; viXra:1609.0263; viXra:1609.0383;

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