Relationship between numbers with 3 prime factors and triangular numbers

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There is a sequence of numbers with four prime factors of the form:

a * b * c * d where a, b, c, d are prime where a = 2 and b = 3

They also have to fulfill that (a * d) - (b * c) = 1 or -1

For example if we build a number this way it would be like this.

2 * 3 * 19 * 29 we have that a * d = 58 and b * c = 57 so it is true that:

(a * d) - (b * c) = 1 or -1

That number exactly is:

3306 dividing by two is. 1653 which is the triangular number 57.

It seems that factor 2 in the formula is not necessary but it is necessary for the calculation of the largest factor.

It is also true that the triangular number is the product of 2 * by the last factor

Sometimes you have to multiply by 2 and subtract 1 or in other cases you just have to multiply by 2 the largest prime and they give us what triangular number it is. We will see that in the following sequence.

1122/2 = 3 × 11 × 17 triangular number 33
1482/2 = 3 × 13 × 19 triangular number 38
3306/2 = 3 × 19 × 29 triangular number 57
7482/2 = 3 × 29 × 43 triangular number 86

Now I am not going to include the factors but the sequence that follows is:

8742/2 triangular number 93
15006/2 triangle number 122
20022/2 triangular number 141
25122/2 triangular number 158
31506/2 triangular number 177
40602/2 triangular number 201

This sequence is not random the one created with the smallest primes that correspond to the previous equation. In this sequence you can see that even triangle numbers alternate with odd corner numbers.

In even triangular numbers its index is the largest prime factor by 2 and in odd numbers it is the largest factor by 2 -1

I tried it with numbers of the form $(a * b * c * d)$
where $a = 5$ and $b = 7$ that meet:
$(a * d) - (b * c) = 1$ or -1

But he did not find any factors that meet the above equation. So it would be a possible conjecture if they exist or not.