I think that it is possible the quantization of the gravity using the quantum field theory (I use the Lewis Ryder book) and the classical theory of fields (I use the Landau-Lifsits book).

It is evident, from the Lifsits book, that the Euler-Lagrange equation for the gravitational field is the Einstein field equation obtained from the Lagrangian of the gravitational field (I write the vacuum solution):

$$G = \mathcal{L} = g^{ik}\left(\Gamma_{il}^m \Gamma_{km}^l - \Gamma_{ik}^l \Gamma_{lm}^m\right)$$

the generalized variables, used in the Euler-Lagrange equation are:

$$\mathcal{L}\left(g^{ik}\ ,\ \frac{\partial g^{lm}}{\partial x^r}\right)$$

as is evident from the variation of the action:

$$\delta S_g = -\frac{c^3}{16\pi k}\int\left\{\frac{\partial(G\sqrt{-g})}{\partial g^{ik}} - \frac{\partial}{\partial x^l}\frac{\partial(G\sqrt{-g})}{\partial\frac{\partial g^{ik}}{\partial x^l}}\right\}\delta g^{ik}\,d\Omega$$

so that it is simple to obtain the Hamiltonian from this action if the Christoffel symbols are expressed in the variables subject to variation (how I got it):

$$\Gamma_{kl}^i = -\frac{1}{2}\left(g_{pk}\frac{\partial g^{ip}}{\partial x^l} + g_{lp}\frac{\partial g^{ip}}{\partial x^k} - g^{ip}g_{kw}g_{lz}\frac{\partial g^{wz}}{\partial x^p}\right)$$

then, the Hamiltonian is

$$\mathcal{H} = \frac{\partial g^{ij}}{\partial x^0}\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ij}}{\partial x^0}} - \mathcal{L}$$

the Lagrangian is a quadratic function in the derivative of the metric tensor, so that the Hamiltonian is just a quadratic function of the metric tensor; there are no linear terms of the time derivative of the metric tensor:

$$\mathcal{H} = \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ik}}{\partial x^0}\partial\frac{\partial g^{ik}}{\partial x^0}}(\partial_0 g^{ik})^2 - \frac{1}{2}\frac{\partial\mathcal{L}}{\partial\frac{\partial g^{ij}}{\partial x^\mu}\partial\frac{\partial g^{lm}}{\partial x^\nu}}\partial_\mu g^{ik}\partial_\nu g^{lm}$$

where $\mu\in 1,2,3$ and so for each greek letter.

It is evident that the classical quantization is possible, if it is possible to write the time derivative of the metric tensor like a function of the generalized moment (I don't use the Heisenberg picture):

$$\hat{H} = \int\mathcal{H}\,d^3\mathbf{x}$$

$$i\hbar\frac{\partial\Psi\left(g^{ij},\partial_\mu g^{kl}\right)}{\partial t} = H\Psi\left(g^{ij},\partial_\mu g^{kl}\right)$$
$$\hat{\pi}_{rs}\Psi(g^{ik},\partial_\mu g^{lm}) = -i\hbar\frac{\partial}{\partial\partial_0 g^{rs}}\Psi(g^{ik},\partial_\mu g^{lm})$$
$$\hat{g}^{rs}\Psi(g^{ik},\partial_\mu g^{lm}) = g^{rs}\Psi(g^{ik},\partial_\mu g^{lm})$$

I write the Hamiltonian, and the momentum of the Lagrangian of the vacuum:

$$\frac{\partial^2\mathcal{L}}{\partial\frac{\partial g^{ab}}{\partial x^r}\partial\frac{\partial g^{cd}}{\partial x^s}} = g^{ik}\left(\frac{\partial\Gamma_{il}^m}{\partial\frac{\partial g^{ab}}{\partial x^r}}\frac{\partial\Gamma_{km}^l}{\partial\frac{\partial g^{cd}}{\partial x^s}} + \frac{\partial\Gamma_{il}^m}{\partial\frac{\partial g^{cd}}{\partial x^s}}\frac{\partial\Gamma_{km}^l}{\partial\frac{\partial g^{ab}}{\partial x^r}} - \frac{\partial\Gamma_{ik}^l}{\partial\frac{\partial g^{ab}}{\partial x^r}}\frac{\partial\Gamma_{lm}^m}{\partial\frac{\partial g^{cd}}{\partial x^s}} - \frac{\partial\Gamma_{ik}^l}{\partial\frac{\partial g^{cd}}{\partial x^s}}\frac{\partial\Gamma_{lm}^m}{\partial\frac{\partial g^{ab}}{\partial x^r}}\right) =$$
$$= \delta_a^s\delta_c^r g_{bd} + \delta_a^s\delta_d^r g_{bc} + \delta_b^s\delta_c^r g_{ad} + \delta_b^s\delta_d^r g_{ac} - g_{ac}g_{bd}g^{rs} - g_{ad}g_{bc}g^{rs} - \delta_a^s\delta_b^r g_{cd} - \delta_a^r\delta_b^s g_{cd} - \delta_c^s\delta_d^r g_{ab} - \delta_c^r\delta_d^s g_{ab} + 2g_{ab}g_{cd}g^{rs}$$

then:

$$\frac{\partial^2 \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^s}} = \delta_a^s \delta_c^0 g_{bd} + \delta_a^s \delta_d^0 g_{bc} + \delta_b^s \delta_c^0 g_{ad} + \delta_b^s \delta_d^0 g_{ac} - g_{ac} g_{bd} g^{0s} - g_{ad} g_{bc} g^{0s} - \delta_a^s \delta_b^0 g_{cd} - \delta_a^0 \delta_b^s g_{cd} - \delta_c^s \delta_d^0 g_{ab} - \delta_c^0 \delta_d^s g_{ab} + 2 g_{ab} g_{cd} g^{0s}$$

$$\frac{\partial \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^0}} = -2\delta_a^0 \delta_b^0 g_{cd} + \delta_a^0 \delta_c^0 g_{bd} + \delta_a^0 \delta_d^0 g_{bc} + \delta_b^0 \delta_c^0 g_{ad} + \delta_b^0 \delta_d^0 g_{ac} - 2\delta_c^0 \delta_d^0 g_{ab} + 2 g^{00} g_{ab} g_{cd} - g^{00} g_{ac} g_{bd} - g^{00} g_{ad} g_{bc}$$

$$\frac{\partial \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{ab}}{\partial x^0}} = -2\delta_a^0 \delta_b^0 g_{ab} + \delta_a^0 \delta_a^0 g_{bb} + \delta_b^0 \delta_b^0 g_{aa} + g^{00} g_{ab} g_{ab} - g^{00} g_{aa} g_{bb}$$

$$\frac{\partial \mathcal{L}}{\partial \frac{\partial g^{aa}}{\partial x^0} \partial \frac{\partial g^{aa}}{\partial x^0}} = 0$$

$$\pi_{ab} = \frac{\partial \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^0}} \frac{\partial g^{cd}}{\partial x^0} + \frac{\partial \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^\mu}} \frac{\partial g^{cd}}{\partial x^\mu}$$

$$\boxed{\pi_{ab} = \frac{\partial \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^l}} \partial_l g^{cd} = -\delta_a^0 \delta_b^0 g_{cd} \partial_0 g^{cd} + \delta_a^0 g_{bd} \partial_0 g^{d0} + \delta_b^0 g_{ad} \partial_0 g^{d0} - \delta_c^0 g_{ab} \partial_0 g^{c0} + g^{00} g_{ab} g_{cd} \partial_0 g^{cd} - g^{00} g_{ac} g_{bd} \partial_0 g^{cd} + F_{ab}}$$

the F_{ab} terms are:

$$F_{ab} = \frac{\partial^2 \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^\mu}} \partial_\mu g^{cd}$$

$$\boxed{F_{ab} = (\delta_a^\mu \delta_c^0 g_{bd} + \delta_a^\mu \delta_d^0 g_{bc} + \delta_b^\mu \delta_c^0 g_{ad} + \delta_b^\mu \delta_d^0 g_{ac} - g_{ac} g_{bd} g^{0\mu} - g_{ad} g_{bc} g^{0\mu} - \delta_a^\mu \delta_b^0 g_{cd} - \delta_a^0 \delta_b^\mu g_{cd} - \delta_c^\mu \delta_d^0 g_{ab} - \delta_c^0 \delta_d^\mu g_{ab} + 2 g_{ab} g_{cd} g^{0s}) \partial_\mu g^{cd}}$$

it is important to obtain a minimum number of terms in the momemntum, so that it is more simple to obtain the time derivative of the metric tensor like a function of the generalized moments:

$$\frac{\partial g^{ab}}{\partial x^0} = P^{ab.cd} \pi_{cd}$$

The Hamiltonian is:

$$\mathcal{H} = \boxed{g^{00}(g_{ab} g_{ab} - g_{aa} g_{bb}) \frac{\partial g^{ab}}{\partial x^0} \frac{\partial g^{ab}}{\partial x^0}} - \frac{1}{2}(+\delta_a^\beta \delta_c^\alpha g_{bd} + \delta_a^\beta \delta_d^\alpha g_{bc} + \delta_b^\beta \delta_c^\alpha g_{ad} + \delta_b^\beta \delta_d^\alpha g_{ac} - g_{ac} g_{bd} g^{\alpha\beta} - g_{ad} g_{bc} g^{\alpha\beta} +$$

$$-\delta_a^\beta \delta_b^\alpha g_{cd} - \delta_a^\alpha \delta_b^\beta g_{cd} - \delta_c^\beta \delta_d^\alpha g_{ab} - \delta_c^\alpha \delta_d^\beta g_{ab} + 2 g_{ab} g_{cd} g^{\alpha\beta}) \frac{\partial g^{ab}}{\partial x^\alpha} \frac{\partial g^{cd}}{\partial x^\beta} =$$

$$= g^{00}(g_{ab} g_{ab} - g_{aa} g_{bb}) \partial_0 g^{ab} \partial_0 g^{ab} - 2 g_{cd} \partial_\beta g^{\alpha d} \partial_\alpha g^{\beta c} + 2 g_{cd} \partial_\beta g^{cd} \partial_\alpha g^{\alpha\beta} +$$

$$+ g_{ac} g_{bd} g^{\alpha\beta} \partial_\alpha g^{ab} \partial_\beta g^{cd} - g_{ab} g_{cd} g^{\alpha\beta} \partial_\alpha g^{ab} \partial_\beta g^{cd}$$

$$\pi_{ab} = 2 g_{bc} \partial_a g^{c0} + 2 g_{ac} \partial_b g^{c0} - 2 g_{ab} \partial_c g^{c0} - 2 g_{ac} g_{bd} g^{0s} \partial_s g^{cd} + g_{cd}(2 g_{ab} g^{0s} \partial_s g^{cd} - \delta_b^0 \partial_a g^{cd} - \delta_a^0 \partial_b g^{cd}$$

it is possible to write the time derivative of the metric tensor using a reduced number of terms:

$$\frac{\partial^2 \mathcal{L}}{\partial \frac{\partial g^{ab}}{\partial x^0} \partial \frac{\partial g^{cd}}{\partial x^0}} \frac{\partial g^{cd}}{\partial x^0} = \Theta_{ab} = -F_{ab} + \pi_{ab} = -\delta_a^0 \delta_b^0 g_{cd} \partial_0 g^{cd} + \delta_a^0 g_{bc} \partial_0 g^{c0} + \delta_b^0 g_{ac} \partial_0 g^{c0} - g_{ab} \partial_0 g^{00} + g^{00} g_{ab} g_{cd} \partial_0 g^{cd} - g^{00} g_{ac} g_{bd} \partial_0 g^{cd}$$

$$\frac{\Theta_a^d}{2} = -\partial_0 g^{00} + g^{00} g_{cd} \partial_0 g^{cd} = (g^{00} g_{00} - 1) \partial_0 g^{00} + 2 g^{00} g_{\mu 0} \partial_0 g^{\mu 0} + g^{00} g_{\mu\nu} \partial_0 g^{\mu\nu}$$

$$\boxed{\Lambda_{ab} = \Theta_{ab} - \frac{g_{ab} \Theta_m^m}{2} = \delta_a^0 g_{bc} \partial_0 g^{c0} + \delta_b^0 g_{ac} \partial_0 g^{c0} - \delta_a^0 \delta_b^0 g_{cd} \partial_0 g^{cd} - g^{00} g_{ac} g_{bd} \partial_0 g^{cd}}$$

$$\boxed{\Lambda_{00} = 2 g_{c0} \partial_0 g^{c0} - (g_{cd} + g^{00} g_{c0} g_{d0}) \partial_0 g^{cd} = g_{00} (1 - g^{00} g_{00}) \partial_0 g^{00} + g^{00} g_{00} g_{\alpha 0} \partial_0 g^{\alpha 0} - (g_{\alpha\beta} + g^{00} g_{\alpha 0} g_{\beta 0}) \partial_0 g^{\alpha\beta}}$$

$$\Lambda_{\mu 0} = g_{\mu c} \partial_0 g^{c0} - g^{00} g_{\mu c} g_{d0} \partial_0 g^{cd} =$$
$$= g_{\mu 0} \partial_0 g^{00} + g_{\mu\alpha} \partial_0 g^{\alpha 0} - g^{00} g_{\mu 0} g_{00} \partial_0 g^{00} - g^{00} g_{\mu\alpha} g_{00} \partial_0 g^{\alpha 0} - g^{00} g_{\mu 0} g_{\alpha 0} \partial_0 g^{0\alpha} - g^{00} g_{\mu\alpha} g_{\beta 0} \partial_0 g^{\alpha\beta} =$$
$$= (g_{\mu 0} - g^{00} g_{\mu 0} g_{00}) \partial_0 g^{00} + (g_{\mu\alpha} - g^{00} g_{\mu\alpha} g_{00} - g^{00} g_{\mu 0} g_{\alpha 0}) \partial_0 g^{\alpha 0} - g^{00} g_{\mu\alpha} g_{\beta 0} \partial_0 g^{\alpha\beta}$$
$$\Lambda_{\mu\nu} = -g^{00} g_{\mu c} g_{\nu d} \partial_0 g^{cd} = g^{00} g_{\mu c} g^{cd} \partial_0 g_{\nu d} = g^{00} \partial_0 g_{\nu\mu}$$

$$\frac{\Lambda_{\mu\nu}}{g^{00}} = \partial_0 g_{\mu\nu}$$

$$g^{\alpha\nu} g^{\beta\mu} \partial_0 g_{\mu\nu} = \frac{\Lambda^{\alpha\beta}}{g^{00}}$$

$$\boxed{\partial_0 g^{\alpha\beta} = -\frac{\Lambda^{\alpha\beta}}{g^{00}} = -g^{\alpha\mu} g^{\beta\nu} \frac{\Lambda_{\mu\nu}}{g^{00}}}$$

the calculations are simpler if a new terms Υ_0^a is introduced to evaluate the remaining components of the metric tensor:

$$\Lambda_{a0} = (\delta_a^0 g_{c0} + g_{ac}) \partial_0 g^{c0} - (\delta_a^0 g_{cd} + g^{00} g_{ac} g_{d0}) \partial_0 g^{cd} =$$
$$= +(\delta_a^0 g_{00} + g_{a0}) \partial_0 g^{00} + (\delta_a^0 g_{\alpha 0} + g_{a\alpha}) \partial_0 g^{\alpha 0} - (\delta_a^0 g_{00} + g^{00} g_{a0} g_{00}) \partial_0 g^{00} - (\delta_a^0 g_{\alpha 0} + g^{00} g_{a\alpha} g_{00}) \partial_0 g^{\alpha 0} +$$
$$- (\delta_a^0 g_{\alpha 0} + g^{00} g_{a0} g_{\alpha 0}) \partial_0 g^{\alpha 0} - (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0}) \partial_0 g^{\alpha\beta} =$$
$$= g_{a0}(1 - g^{00} g_{00}) \partial_0 g^{00} + (g_{a\alpha} - \delta_a^0 g_{\alpha 0} - g^{00} g_{a\alpha} g_{00} - g^{00} g_{a0} g_{\alpha 0}) \partial_0 g^{\alpha 0} - (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0}) \partial_0 g^{\alpha\beta}$$
$$\Upsilon_{a0} = \Lambda_{a0} + (\delta_a^0 g_{\alpha\beta} + g^{00} g_{a\alpha} g_{\beta 0}) \partial_0 g^{\alpha\beta} = \Lambda_{a0} - \left(\frac{\delta_a^0 g_{\alpha\beta}}{g^{00}} + g_{a\alpha} g_{\beta 0} \right) g^{\alpha\mu} g^{\beta\nu} \Lambda_{\mu\nu} =$$
$$= g_{a0}(1 - g^{00} g_{00}) \partial_0 g^{00} + \left(g_{a\alpha} - \delta_a^0 g_{\alpha 0} - g^{00} g_{00} g_{a\alpha} - g^{00} g_{a0} g_{\alpha 0} \right) \partial_0 g^{\alpha 0}$$

$$\boxed{\Upsilon_{\kappa 0} = \Lambda_{\kappa 0} - g_{\kappa\alpha} g_{\beta 0} g^{\alpha\mu} g^{\beta\nu} \Lambda_{\mu\nu} = g_{\kappa 0} (1 - g^{00} g_{00}) \partial_0 g^{00} + [g_{\kappa\alpha}(1 - g^{00} g_{00}) - g^{00} g_{\kappa 0} g_{\alpha 0}] \partial_0 g^{\alpha 0}}$$

$$\boxed{\Upsilon_{00} = \Lambda_{00} - \left(\frac{g_{\alpha\beta}}{g^{00}} + g_{a\alpha} g_{\beta 0} \right) g^{\alpha\mu} g^{\beta\nu} \Lambda_{\mu\nu} = g_{00}(1 - g^{00} g_{00}) \partial_0 g^{00} - 2 g^{00} g_{00} g_{\alpha 0} \partial_0 g^{\alpha 0}}$$

then it is possible to obtain the $\partial_0 g^{a0}$ as a $\Upsilon(\Lambda(\Theta(\pi^{lm})))$ function:

$$\begin{pmatrix} \Upsilon_{10} - \frac{g_{10}}{g_{00}} \Upsilon_{00} \\ \Upsilon_{20} - \frac{g_{20}}{g_{00}} \Upsilon_{00} \\ \Upsilon_{30} - \frac{g_{30}}{g_{00}} \Upsilon_{00} \end{pmatrix} = \begin{pmatrix} g_{11}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{10} & g_{12}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{20} & g_{13}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{30} \\ g_{21}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{10} & g_{22}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{20} & g_{23}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{30} \\ g_{31}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{10} & g_{32}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{20} & g_{33}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{30} \end{pmatrix} \begin{pmatrix} \partial_0 g^{10} \\ \partial_0 g^{20} \\ \partial_0 g^{30} \end{pmatrix}$$

the determinant of the linear system is in general not null:

$$\begin{vmatrix} g_{11}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{10} & g_{12}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{20} & g_{13}(1 - g^{00} g_{00}) + g^{00} g_{10} g_{30} \\ g_{21}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{10} & g_{22}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{20} & g_{23}(1 - g^{00} g_{00}) + g^{00} g_{20} g_{30} \\ g_{31}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{10} & g_{32}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{20} & g_{33}(1 - g^{00} g_{00}) + g^{00} g_{30} g_{30} \end{vmatrix} = \gamma - g^{00}(1 - g^{00} g_{00})^2 g \geq 0$$

where $g = det(g_{ab})$ and $\gamma = det(g_{\alpha\beta})$, then it is possible to obtain the transformation between time derivatives of the metric tensor and generalized moments $\partial_0 g^{ab} = P^{ab.lm} \pi_{lm}$.

It is simple to obtain the Hamiltonian like a π_{ab} function:

$\Theta_{ab} = -F_{ab} + \pi_{ab}$
$\Theta_m^m = -F_m^m + g^{am}\pi_{am}$
$\Lambda_{ab} = \Theta_{ab} - \frac{g_{ab}\Theta_m^m}{2} = -F_{ab} + \pi_{ab} - \frac{g_{ab}\Theta_m^m}{2} = -F_{ab} + \pi_{ab} + g_{ab}\frac{F_m^m}{2} - g_{ab}\frac{g^{ms}\pi_{ms}}{2}$
$\Upsilon_{\kappa 0} = \Lambda^{\kappa 0} - g_{\kappa\alpha}g_{\beta 0}g^{\alpha\mu}g^{\beta\nu}\Lambda_{\mu\nu}$
$\partial_0 g^{\alpha\beta} = -g^{\alpha\mu}g^{\beta\nu}\frac{\Lambda_{\mu\nu}}{g^{00}} = -\frac{g^{\alpha\mu}g^{\beta\nu}}{g^{00}}\left[-F_{\mu\nu} + g_{\mu\nu}\frac{F_m^m}{2}\right] - \frac{g^{\alpha\mu}g^{\beta\nu}}{g^{00}}\left[\pi_{\mu\nu} - g_{\mu\nu}\frac{g^{ms}\pi_{ms}}{2}\right] = G_{\mu\nu} - \frac{g^{\alpha\mu}g^{\beta\nu}}{g^{00}}\left[\pi_{\mu\nu} - g_{\mu\nu}\frac{g^{ms}\pi_{ms}}{2}\right]$

$\mathcal{H} = 2g^{00}(g_{\alpha 0}g_{\alpha 0} - g_{\alpha\alpha}g_{00})\partial_0 g^{\alpha 0}\partial_0 g^{\alpha 0} + 2g^{00}(g_{\alpha\beta}g_{\alpha\beta} - g_{\alpha\alpha}g_{\alpha\beta})\partial_0 g^{\alpha\beta}\partial_0 g^{\alpha\beta} +$
$-2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} + 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta} + g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} - g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} =$

$= \boxed{\begin{array}{l} 2g^{00}(g_{\alpha 0}g_{\alpha 0} - g_{\alpha\alpha}g_{00})\left(P^{\alpha 0.ab}\pi_{ab}\right)^2 + \\ 2g^{00}(g_{\alpha\beta}g_{\alpha\beta} - g_{\alpha\alpha}g_{\beta\beta})\left\{G_{\alpha\beta} - \frac{g^{\alpha\mu}g^{\beta\nu}}{g^{00}}\left[\pi_{\mu\nu} - g_{\mu\nu}\frac{g^{ms}\pi_{ms}}{2}\right]\right\}^2 + \\ -2g_{cd}\partial_\beta g^{\alpha d}\partial_\alpha g^{\beta c} + 2g_{cd}\partial_\beta g^{cd}\partial_\alpha g^{\alpha\beta} + g_{ac}g_{bd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} - g_{ab}g_{cd}g^{\alpha\beta}\partial_\alpha g^{ab}\partial_\beta g^{cd} \end{array}}$

this is the Hamiltonian of the gravitational field, that it is quantizable (even if not uniquely), and it is similar to the Wheeler-De Witt equation.

I will try to simplify the description of the time derivative $\partial_0 g^{\alpha 0}$, in addition to giving an explicit description, and some other simplifications, but the complexity of the Hamiltonian should not be significantly reduced.