The gravitoelectric nuclear energy

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Abstract

In the present work we assume that in the atomic nucleus the gravitoelectric force $\left(F_{ge} = \frac{GKMm}{R^2}\right)$ acts as responsible for the stability of nucleus and for the nuclear size, and that the potential energy related to this force be given by the ratio $\frac{F_{ge}}{2\pi R}$ with R equal to the nuclear radius observed in the electron scattering experiments, obtaining surprising outcomes.

The new approach offers an occasion for discussing about the physics and chemistry foundations, in particular about the meaning of the gravitational potential energy and about the nature of the atomic nucleus, which perhaps should be reconsidered in deterministic terms, rather than probabilistic ones.

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- **The nuclear radius and the gravitoelectric force**

The nuclear force acting within the atomic nucleus as responsible for the nuclear stability and for the cohesion of nucleons is, according to the nuclear standard physics, the strong nuclear force.

Though, there are some evidences that seem to deny the existence of this force.

First of all, if the strong nuclear force were responsible for the cohesion of the nucleons, it should explain the nuclear size, which instead still remains without a theoretical motivation.

Moreover, in the reference [1] it has been remarked that the strong nuclear force is denied by the radius 7 fm of halo neutron in Be11.

In the mentioned reference we can read that the inexistence of the strong nuclear force, inferred from an impartial analysis of what really happens in the alpha decay of U238, seems to have been confirmed in 2009 by an experiment, reported in the reference [2], because, while the range of maximum actuation of the strong force is less than 3 fm, the experiment has detected that a halo neutron in Be11 is separated from the rest of the nucleus by a distance of 7 fm.

In the present work we show the proofs of the existence of the force of gravity even within the atomic nucleus, as responsible for the nuclear stability and for the cohesion of nucleons.

We know from Einstein's theory of relativity that the energy contained in the atomic nucleus is equal to $E = Mc^2$, where M is the mass of the nucleus.

We will seek to demonstrate that the above formula is incomplete, and that, once completed with the missing part, it will appear entirely nude to us, without that halo of mystery that previously prevented us from seeing more deeply, showing that perhaps it does not denote the mass-energy equivalence principle proposed by Einstein's special relativity, but something else.

As it's known, the gravitational potential energy of a mass body m subjected to the attractive force of gravity exerted by a mass body M is:

$$U = F_g * R \tag{1}$$

where F_g is the force of gravity $\frac{GMm}{R^2}$ and R is the distance between the centers of the two bodies (assumed as spherical).

Therefore the eq. (1) becomes (the value is considered as *positive* for the reasons explained below):

$$U = \frac{GMm}{R^2} * R$$

$$U = \frac{GMm}{R}$$

This is the gravitational potential energy of the mass body m, which is usually defined as the test body.

The reason of the direct proportionality between the gravitational potential energy and the distance, which we have seen in the equation (1), rather than the inverse proportionality, which instead appears in the equation of the force of gravity, is explained by the fact that in the first case we observe the phenomenon of gravitational attraction in terms of *potentiality* of the body subjected to a given gravitational force and located, for example, at a certain height above the Earth's surface, of affecting the surrounding reality, in particular by impacting the ground.

It is obvious that the higher up the body is located, the greater its gravitational potential will be, because the damage it will cause to the Earth's soil is the greater, the greater the height from which it begins to fall is (in this case, in fact, a body would reach the Earth's soil with the greater speed, the greater the distance from the Earth is).

It is a logical consequence of this new vision of the gravitational potential energy, although improvable as we'll see later, that it has to be mathematically expressed as positive, unlike the traditional definition in which it is negative, because we have considered the movement of the test body in the same direction and sense as those of the force of gravity, whereas, in the traditional standard physics, the mentioned energy is understood as the capacity of a body subjected to the gravitational force of doing work, in particular the work is considered what is needed to move the body, for instance, from the Earth up to infinite, in which case the sense of the motion of the body is opposite to that of the force, and consequently the potential energy is taken as negative.

We can say in this regard that to think of the gravitational potential energy in terms of *capacity of doing work* is misleading, because a body located at a certain distance, for example, from the Earth, doesn't have any other capacity than that of falling towards the planet, not at all that of moving *on its own* against the force of gravity.

The capacity of doing work is something different from the gravitational potential energy, because the work, like the term itself evokes, denotes something to do in order to prevail over some acting (for instance a persisting force) or resisting (for instance the inertia) entity, which obstacles the movement of the body.

In a nutshell, and more precisely than our previous definition, it seems more appropriate to consider the gravitational potential energy as the potentiality of the test body m, placed at distance R from the attractor body M, of accelerating towards the center of the aforementioned attractor, respecting a precise law $\left(g = \frac{GM}{R^2}\right)$, which is different not only from the work to be done by an extern cause in order to overcome gravity and move the test body up to a certain height above the planet's surface, but even from the work done by gravity itself to move the test body from its position to another specific point closer the planet.

In these two latter cases, in fact, we only determine the energy required to move a body from a point to another one, regardless the body's way of moving, whereas in the first case (gravitational potential energy) the way of moving of the body, in particular its way of accelerating, is taken as a fundamental element.

From these important differences one can deduce not only that the gravitational potential energy cannot be other than positive, since the movement of the body caused by the force of gravity cannot have other sense than that of the force, but mostly that the gravitational potential energy can be mathematically expressed even in a different manner from the way of expressing the work, by not being work.

An example will help us to clarify the previous statements.

Let's take two planets, respectively of mass M and $\frac{M}{4}$.

Be A and B two test bodies of equal mass m and suppose that A is located at distance $2R$ from the planet M, and B at distance R from the planet $\frac{M}{4}$.

The two test bodies are subjected to the same force of gravity:

$$\frac{GMm}{(2R)^2} = \frac{G\frac{M}{4}m}{R^2}$$

$$\frac{GMm}{4R^2} = \frac{GMm}{4R^2}$$

Yet, even though both test bodies experience the same force of gravity, and even if the test body A will impact the planet M with a velocity which is greater than twice the velocity with which the body B will impact the planet $\frac{M}{4}$ (because the mass M is four times the mass $\frac{M}{4}$, and because the velocity of the falling objects increases exponentially according to the inverse square law $g = \frac{GM}{R^2}$), despite this, the body A hold a potentiality of accelerating towards the center of the planet M which is exactly twice that held by the body B of accelerating towards the center of the planet $\frac{M}{4}$, because the space available for accelerating is, in the first case, twice compared to the second, and because both falling bodies accelerate towards the respective attractors according to the same law: $g = \frac{GM}{R^2}$.

Notice in this regards that the potentiality held by a body of accelerating throughout a certain space according to a specific law, is different from the potential maximum acceleration of that body, because the first is an energy, the second is a prediction of the amount of the acceleration, that is an acceleration.

In essence, the reason why the gravitational potential energy of a falling object is directly proportional to the distance R, as we have written in the eq. (1), is that its possible maximum velocity increases as the distance increases, and that this increment happens according to a precise gravitational law.

The fact that in the macroscopic world the gravitational potential energy is mathematically expressed in the same way as the work (i.e. as the product of the force times distance) is only an accident that should not mislead and induce us to think that they are the same thing, consequently it is not consistent with the aforesaid nature of the potential energy to determine its value by means of the integral of the force of gravity over the distance (since gravity is not constant), even if the integral were taken as positive, because the aforesaid integral expresses the *work* done by the force of gravity to move the test body in the same sense of the force until it reaches the center of the attractor body, which is conceptually different from the *potentiality* stored in the test body of accelerating throughout a definite space toward the center of the attractor body according to a precise law.

In fact there could exist, in principle, cases in which the force of gravity acts in such a way that the test body m, even if at rest, does not fall towards the body M, but revolves around it, and that the revolution be the slower, the greater the distance from the central body is, and in this case the gravitational potential energy should be formally expressed in a different way from the product of the force of gravity times distance, in particular by means of the ratio of the force over the circular trajectory covered by the body $\left(\frac{F_g}{2\pi R}\right)$, as we will show below.

We are referring to the atomic nucleus, within which we can hypothesize that there exists an attractive-repulsive gravitational *zone* (notice that we have used the term zone only in a spatial sense, in order to not to generate confusion with space-temporal *field* used in the theory of relativity) produced by the nucleons themselves which would give rise to a pendulum, in particular a peculiar harmonic oscillator which would imply the complete revolution of the nucleons around the fixed point, rather than the oscillation around the equilibrium point, with the particularity that these latter would be considered as *self-orbitating* particles, namely which stay simultaneously both in the center of the nucleus and in orbit around it, and presenting the following features:

1) the center of the nucleus would be the fixed point (*fulcrum*) of the pendulum, which would be occupied by the nucleons at rest, among which the gravitational force would be only attractive, under the assumption that neutrons neutralize the electrostatic repulsion among protons;
2) the central core of such aggregated nucleons would simultaneously produce attractive and repulsive gravitational forces upon their orbitating *alter-ego*;
3) the attractive force experienced by the aforesaid orbitating alter-ego of nucleons would play the same role as that played by the tension of the wire in the Galilean pendulum;
4) the repulsive force experienced by the orbitating nucleons — equal in strength to the attractive force, but not aligned to it (having a non-radial direction) — would play the same role as that played by the force of gravity exerted by the Earth upon the Galilean pendulum, decomposing itself into two components, one radial component and one tangential component, the latter of which would represent the responsible for the movement of the pendulum (namely of the nucleons);
5) the letter g which appears in the formula of the period T of the harmonic oscillator $\left(T = 2\,\pi * \sqrt{\frac{l}{g}}\right)$ would be, in our model, the repulsive acceleration of gravity $g = \frac{GM}{l^2}$ experienced by the orbitating nucleons, which would play the same role as that played by the Earth's acceleration of gravity upon the pendulum, where the letter l, which denotes in the Galilean formula the length of the wire, in our model would represent the nuclear radius, which therefore will be replaced by

the letter R from now on.

In such a particular harmonic oscillator it should happen that, by increasing the distance from the center of the nucleus, the repulsive acceleration of gravity g should decrease — unlike the Galilean pendulum where, by increasing the length of wire, g increases, because the body approaches to the Earth's center — and consequently the formula of the potential energy should change.

If we admit, indeed, that the effect of the attractive-repulsive zone does not consist of making the bodies fall towards the central attractor-repulsor, but of making them move, even if initially at rest, around it at decreasing speed as the distance from the central body increases, according to the formulae of a pendulum in which g is inversely proportional to length of the wire squared (R^2), then it would follow that the equation of the gravitational potential energy would become as follows:

$$U = \frac{F_g}{2\,\pi R} \tag{2}$$

This time, differently from the eq. (1), the distance R is in the denominator, because, the greater is the distance, the lower the maximum linear velocity produced by the attractive-repulsive field will be.

In fact, the period T of the nuclear harmonic oscillator would increases if the nuclear radius R increases $\left(T = 2\,\pi * \sqrt{\frac{R}{g}}\right)$, and in this case not only the angular velocity of the pendulum, but even its linear velocity (more precisely the tangential velocity) decreases, because above we have assumed that in such a particular type of pendulum the repulsive acceleration of gravity g, as well as the attractive one, decreases with the increase of the nuclear radius squared ($g = \frac{GM}{R^2}$).

In detail, the formula of the tangential maximum velocity of the nuclear pendulum harmonic oscillator would be $v = \omega * R$, and, by knowing that the angular velocity of harmonic oscillator is $\omega = \sqrt{\frac{g}{R}}$, its tangential velocity will be $v = \sqrt{\frac{g}{R} * R^2}$ $= \sqrt{\frac{GM}{R^3} * R^2} = \sqrt{\frac{GM}{R}}$ which demonstrates that, in such a particular pendulum, the increase of the wire (the nuclear radius) implies the decrease of the possible maximum tangential velocity of the oscillating (better saying revolving) body of pendulum.

In essence, the reason why the gravitational potential energy of test body subjected to an attractive-repulsive gravitational zone, which is assumed to give rise to a pendulum harmonic oscillator generating a circular trajectory, is inversely proportional to the space covered ($2\pi R$), is that the possible maximum linear velocity caused by such a gravitational zone decreases as the distance increases, and that this reduction takes place according to the acceleration low of the harmonic oscillator $g = \omega^2 x$, where x is the projection of the position of the oscillating (better saying revolving) body onto the x-axis, consequently we can assert that the gravitational potential energy of nucleons can be defined as their potentiality of accelerating throughout a circumference of a certain radius R, according to the mentioned acceleration law, and that therefore this energy can be mathematically expressed as inversely proportional to the circular trajectory $\left(\frac{F_g}{2\pi R}\right)$ described by nucleons.

The term π appearing in the denominator of the eq. (2) is extremely important because from it one can deduce that it's not the case of an exclusively repulsive field, in which the potential energy should be inversely proportional to the rectilinear distance $\left(U = \frac{F_g}{R}\right)$, not to the circumference $\left(U = \frac{F_g}{2\pi R}\right)$.

But the equation (2) must still be modified if to be applied to the atomic nucleus.

Here, in fact, even if we admit that gravity acts, it would not be the only operating force, because it is not possible to neglect the electrostatic one.

Therefore I have supposed that in the nucleus the force of gravity and the electrostatic force were merged, giving rise to the *gravitoelectric* force F_{ge} (or, if one prefers, *electro-gravitational* force) having this magnitude:

$$F_{ge} = \frac{GKMm}{R^2} \tag{3}$$

where K is the Coulomb's constant and G is the gravitational constant (for now we leave out the dimensional analysis, faced in the next paragraph), so the eq. (2) becomes:

$$U = \frac{GKMm}{R^2} * \frac{1}{2\,\pi R} \tag{3a}$$

Consistently with our assumption that nucleons are self-orbitating particles, namely that stay simultaneously in the center of the nucleus and in orbit around it, we have to replace in the above equation m — which denotes the orbitating body, having a very small mass with respect to the central one — with M, i.e. with the total mass of the nucleons itself, so that the equation (3a) becomes:

$$U = \frac{GKMM}{R^2} * \frac{1}{2\,\pi R} \tag{4}$$

$$U = \frac{GKM^2}{2\,\pi R^3} \tag{5}$$

where R is the nuclear radius detected in the electron scattering experiments: for medium and heavy atoms, $R = 1.21 * \sqrt[3]{A}\,fm$, where A is the mass number (see references [3]).

Now, in order to mathematically demonstrate that this energy is operating within the atomic nucleus — deferring to later any further investigation concerning the dimensional analysis — we have to verify if the energy expressed in eq. (5) is equal to Mc^2, i.e. the total mass-energy of nucleons, so we can write:

$$\frac{GKM^2}{2\,\pi R^3} = Mc^2 \tag{6}$$

It's important to specify that M is taken as the mass of the nucleus, intended as the sum of the masses of the protons and of neutrons, without taking into account the binding energy (mass-defect), that therefore will not be subtracted from the mentioned sum.

Let's test now the eq. (6), considering the nucleus of bromum atom (^{79}Br), which contains 35 protons and 44 neutrons, whose radius — according to the empirical formula $R = 1.21151 * \sqrt[3]{A}\ fm$ — is 5.1983 $femtometers$:

$$\frac{(6.6743*10^{-11})*(8.9875*10^{9})*\{[(35*1.6726)+(44*1.6749)]*10^{-27}\}^2}{2*3.1415*(5.1983*10^{-15})^3} = [(35*1.6726)+(44*1.6749)]*10^{-27}*c^2$$

where c is the speed of light in vacuum: $299{,}792{,}458\ m/sec$

$$1.1884 * 10^{-8}\ joule = 1.1884 * 10^{-8}\ joule$$

$$\frac{E}{Mc^2} = \frac{1.1884 * 10^{-8}}{1.1884 * 10^{-8}} = 1$$

For summary reasons it's not worth reporting here the above calculation for all the atoms, since the empirical formula of the nuclear radius seen above $\left(R = 1.21151 * \sqrt[3]{A}\ fm\right)$ is applicable to every medium and heavy atom.

The only further atom that we can consider as a demonstration of the validity of the eq. (6) is the lead atom, the heaviest among the stable atoms.

The nucleus of the lead atom contains 82 protons and 126 neutrons, and its radius, according to the mentioned empirical formula $R = 1.21151 * \sqrt[3]{A}\ fm$, is $7.1781\ fm$, hence, applying the eq. (6), we obtain the following values:

$$\frac{(6.6743*10^{-11})*(8.9875*10^{9})*\{[(82*1.6726)+(126*1.6749)]*10^{-27}\}^2}{2*3.1415*(7.1781*10^{-15})^3} = [(82*1.6726)+(126*1.6749)]*10^{-27}*c^2$$

$$3.1295*10^{-8}\ joule = 3.1293*10^{-8}\ joule$$

$$\frac{E}{Mc^2} = \frac{3.1295*10^{-8}}{3.1293*10^{-8}} = 1.00005$$

The eq. (6) holds again.

Returning to our aim of demonstrating if there exists any other case in which the gravitational potential energy can be mathematically expressed in a different way from work (namely differently from the product of the force of gravity times distance), we think we have achieved the goal, because the expression $U = \frac{F_g}{2\,\pi R}$, which leads to the eq. (5), has a different configuration from the work, where instead the space appears in the numerator ($W = F * S$), and it is quite implausible to believe that the value expressed by the right-hand side of the eq. (5) not to express an energy, by being exactly equal to the value of another energy (mc^2), and, we ask, which kind of energy could it be if not the gravitoelectric potential energy, by having we used in the aforesaid equation the gravitational constant and the Coulomb's constant?

We reiterate that M which appears in the formulae (5) and (6) is taken as the total mass of nucleons, intended as the sum of the masses of protons and of neutrons, without taking into account the mass-defect detected in the nuclear reaction experiments and ascribed, by the nuclear standard physics, to the binding energy of nucleons, which therefore here has not been subtracted from the mentioned sum of the nucleonic masses, and, despite this, the equation (6) perfectly holds, and this seems to demonstrate that the mass-defect detected in the nuclear reactions is not the consequence of the mass-energy equivalence principle stated by Einstein's special theory of relativity, but most likely is the effect of the increase of the nuclear radius.

In other terms, from the relevant mathematical findings achieved in the eq. (6), it seems possible to infer that the mass of an unbound nucleon (proton or neutron) is not greater than that of a bound nucleon, but is exactly the same, and that the discrepancy detected in the nuclear reactions is not due to the mass defect of a bound nucleon with respect to a free nucleon, but is the consequence of the very probable increase of the nuclear radius occurring during the nuclear reactions which in turn implies, according to the eq. (5), the decrease of the nuclear potential energy (assuming that the velocity of nucleons, c, remains constant), which is very likely responsible for what is detected in the nuclear reactions and interpreted, perhaps mistakenly, as the mass-defect by the nuclear standard physics.

- **The gravitoelectric force seen in the light of the dimensional analysis**

According to the dimensional analysis, the force that in the eq. (3) we have supposed to be existing in the atomic nucleus should not be a force, because it scales as the square of a force over a charge or, in units of the International System, as $\left(\frac{N}{C}\right)^2$.

Firstly we can say that the dimensional analysis has empirical bases, hence it cannot reasonably represent the unique obstacle to the validity of a new theoretical equation, especially when, as in our case, the theoretical result is perfectly equal to the empirical one.

Anyway, even though we consider the dimensional analysis as a substantial issue, it is possible to overcome it simply by considering K as a dimensionless quantity precisely equal to the numerical value of the Coulomb's constant ($8.9875 * 10^9$), so that the new value of the gravitational constant, only operating at microscopic scale, would be:

$$\boldsymbol{G_k} = 6.6743 * 10^{-11} * 8.9875 * 10^9 \, \boldsymbol{Nm^2kg^{-2}}$$

Consequently the mathematical expression of the gravitoelectric force, seen in the eq. (3), would become:

$$F_{ge} = \frac{G_k Mm}{R^2}$$

We reiterate that G_k is the value of the gravitational constant only at microscopic scale, in particular at nuclear scale (at least until further studies will not confirm it even at atomic and molecular scale).

It's important to emphasize that the $8.9875 * 10^9$ parameter is a number with a precise physical meaning, because it is the numerical value of the Coulomb's constant, therefore it would not be possible to argue that the mentioned number be an ad hoc parameter deliberately chosen to fit the data, as well as it would be difficult to contest the appropriateness of the term "gravito*electric*" or "gravito*electrostatic*" force.

The mathematical identity demonstrated in the eq. (6) is so perfect that an underlying physical meaning must subsist beyond any reasonable doubt, by being unconvincing to claim that one deals with a mere accident.

But what is this underlying physical meaning of the eq. (6)?

In the next paragraph we'll try to discuss this issue.

- **Are nucleons self-orbitating particles?**

The result achieved above gives rise to a philosophical question.

How to interpret the eq. (5)?

In other words the fact that the energy expressed by the eq. (5) depends on the mass of nucleons squared could have a precise physical meaning, in particular could mean, as anticipated above, that the nucleons stay both in the center of the nucleus and, at *same time*, in orbit around it, because we have replaced in the eq. (5) the mass m — which denotes the orbitating body, having a very small mass with respect to the central one — with the mass M, that is the total mass of nucleons.

If we accept this assumption, there would be non-irrelevant consequences on the foundations of physics, because this would mean that the nucleons would not be everywhere, as the probabilistic vision of quantum mechanics proposes, but would be only in two specific points and conditions simultaneously (at rest and revolving), always having, even before the measurement, a precise position, trajectory and velocity in while they are orbitating about the center of the nucleus (occupied by their alter ego at rest).

In this weird scenario, one should accept not only the idea that the nucleons stay in two places at the same time, but also the fact that they are both at rest, in the center of nucleus, and revolving at same time around this point, with the further specification that, when they are moving, they would do at the speed of light at a distance equal to the nuclear radius.

In this framework, in fact, the right-hand side of the eq. (6) would be twice the kinetic energy of the nucleons ($2 * \frac{1}{2} Mc^2 = Mc^2$).

From the planetary orbits, indeed, we know that the orbit will be as stable as possible whether the gravitational potential energy will be equal to twice the kinetic energy of the planet.

In our solar system we have in particular that, for each planet, the following relation is operating:

$$U = 2 E_k \tag{7}$$

where U is the gravitational potential energy held by the planet in the point located at the distance R from the Sun, and E_k is the kinetic energy of the planet (equal to $E_k = \frac{1}{2}mv^2$, where m is the mass of the planet), both considered as constant, by approximating the planetary orbit to a circumference.

By knowing that U is equal to $m * g * R$, the eq. (7) becomes:

$$m * g * R = 2\left(\frac{1}{2}mv^2\right)$$

$$g * R = v^2$$

$$\frac{GM}{R^2} * R = v^2$$

$$\frac{GM}{R} = v^2$$

$$v = \sqrt{\frac{GM}{R}} \tag{8}$$

(where M is the mass of the Sun) which is notoriously the velocity required to have a circular orbit, namely the most stable orbit.

After all, from the eq. (6) it is possible to derive the theoretical formula of the speed of light c:

$$c = \sqrt{\frac{GKM}{2\,\pi R^3}}$$

which is not very different from the planetary orbital velocity seen in the eq. (8).

Furthermore in a recent research [4] it has been experimentally shown that the missing momentum of a knockout proton, in some collisions, can be up to 1,000 Mev/c, in contrast with the previous experiments, from which the value of the missing momentum turned out to be 250 Mev/c.

The value of 1,000 Mev/c is very high and could be well-justified by assuming that the nucleons move within the nucleus at the speed of light, or at a speed which is approaching it.

Moreover, in the mentioned research it has been shown that in the nucleus not only an attractive force exists, but also a repulsive force, and it is very likely that these two opposed forces are not aligned and this consequently gives life to the particular pendulum descripted in this work, which guarantees the dynamical equilibrium of nucleonic orbits.

In this regard it is important to remark the similarity of the nuclear model here proposed with the interatomic and intermolecular chemical bond, by being the balance between attractive and repulsive forces a fundamental feature of all these systems (see reference [8]).

- **Is the virial theorem always valid?**

The virial theorem (by R. Clausius, 1870) states, for a central potential $\langle \phi \rangle (\vec{R}) = \phi\ (R) \propto \pm\ R^{\pm b}$, that:

$$\langle E_K \rangle = \pm \frac{b}{2} \cdot \langle \phi \rangle \tag{9}$$

where $\langle \phi \rangle$ is the average over time of the potential energy, $\langle E_K \rangle$ is the average over time of the kinetic energy and b is the exponent of the radius as it appears in the formula of the potential energy.

Since the gravitational potential energy is inversely proportional to the distance $\left(U = \frac{GMm}{R}\right)$, then the exponent of the radius is $b = -1$ and the eq. (9) becomes:

$$\langle E_K \rangle = -\frac{1}{2} \cdot \langle \phi \rangle$$

Yet, in the light of the findings reached in eq. (6), whose left-hand side denotes quite indisputably the nuclear potential energy, the virial theorem [eq. (9)] doesn't hold.

Indeed, applying the eq. (9) and considering that the nuclear gravitoelectric potential energy is — as expressed in eq. (5), even though considered as negative like the traditional way of thinking — inversely proportional to R^3, the virial theorem would lead to:

$$\langle E_K \rangle = -\frac{3}{2} \cdot \langle \phi \rangle$$

$$\frac{1}{2}\ Mc^2 = -\frac{3}{2} \cdot \left(-\frac{GKM^2}{2\ \pi R^3}\right)$$

Multiplying both sides by 2:

$$Mc^2 = \frac{3GKM^2}{2\ \pi R^3}$$

which is not true.

In fact, if we again apply the above equation to the bromum atom ^{79}Br, it leads to:

$$\frac{Mc^2}{\frac{3GKM^2}{2\ \pi R^3}} = \frac{1.1884 * 10^{-8}}{3.5652 * 10^{-8}} \neq 1$$

At this point, the fact that the virial theorem doesn't hold for the nuclear gravitoelectric potential energy can be explained in two different ways.

The first is to assert that the eq. (5) doesn't contain the nuclear potential energy, and consequently that Mc^2 wouldn't represent twice the kinetic energy of nucleons, but would be, as the theory of relativity states, the total mass-energy of nucleons, more precisely the energy that the nucleons contains for the very fact of having a mass, even if they are at rest.

This interpretation, yet, doesn't allow to explain which would be the physical meaning of the perfect mathematical identity given by the eq. (6), which consequently should be ascribed, we repeat, only to the fortuity, nothing short of unrealistically.

The second possibility is to claim that the virial theorem, as formulated in eq. (9), is incorrect, and that the correct *law* would be:

$$\langle E_K \rangle = \frac{1}{2} \cdot \langle \phi \rangle \qquad (10)$$

where ϕ is the gravitational potential energy, taken as positive for the reasons stated above, and intended as the potentiality held by the body m, subjected to a gravitational attractive force or to two simultaneous gravitational attractive-repulsive forces, respectively exerted by an attractor or by an attractor-repulsor body M, of accelerating towards respectively the center of the attractor body (assumed spherical) or around the center of the attractor-repulsor central body (assumed again spherical), throughout a certain space and according to a precise law.

This interpretation is based on the fact that the virial theorem is an *ad hoc* solution, valid only in the case that the force of gravity be inversely proportional to the square of the distance and that the gravitational potential energy be proportional to R^{-1}.

Though, this is a fact that has never been explained logically, mathematically or geometrically, in essence scientifically, in particular nobody has never demonstrated the reason why the force of gravity can't be other than inversely proportional to the distance squared, and that the gravitational potential energy can't be other than proportional to R^{-1}.

Consequently one can argue, in principle, that, if the gravitational force were, for instance, inversely proportional to the fourth power of the distance, the theorem would fail, as we'll show shortly.

In fact, in the case that the force of gravity were $F = \frac{GMm}{R^4}$, the kinetic energy needed to have a stable orbit, applying the virial theorem, would turn out to be greater than the potential energy.

In particular, supposing that in the mentioned hypothesis the force of gravity be only attractive, then the gravitational potential energy, taken negative as traditionally done, would be:

$$U = -\frac{GMm}{R^4} \cdot R = -\frac{GMm}{R^3}$$

Consequently the exponent of the radius that would appear in the eq. (9) would be $b = -3$, so that the necessary condition to have a stable orbit would turn out to be:

$$\langle E_K \rangle = -\frac{3}{2} \cdot \langle \phi \rangle$$

$$\frac{1}{2} mv^2 = \frac{3}{2} \cdot \frac{GMm}{R^3}$$

but this is impossible because the kinetic energy of the mass body m would be greater than its potential energy ($E_K = 1.5 \cdot U$), and we know that in such a condition the orbit will be hyperbolic.

The same result would turn out in the case that the force of gravity were inversely proportional to the third power of the distance, in which case, applying the virial theorem, the most stable orbit would occur if the kinetic energy were equal to the potential energy, but it is well-known that in this case the orbiting body would reach the escape velocity, so the virial theorem would fail again.

The virial theorem, therefore, is implicitly based on two premises (namely the fact that the force of gravity can't be other than inversely proportional to the square of the distance, and that the gravitational potential energy can't be other than proportional to R^{-1}) which are not logically demonstrable, and this implies that it cannot be considered a *theorem* in the proper sense of the term, because a theorem is, by definition, a proposition which can be scientifically demonstrated, and this also holds for its logical premises.

Consequently one should admit that the eq. (9) would be replaced by the eq. (10), and that this latter would apply in any case, both when the object (body or particle) is subjected to only one attractive gravitational force, and when it is subjected to two gravitational forces (attractive and repulsive) at same time, regardless of the mathematical configuration of the potential energy (namely, regardless of the exponent of radius, *b*, appearing in the formula of the potential energy).

In other words, in this scenario one should admit that the eq. (10) be a fundamental principle of Nature, in the sense that it wouldn't have any mathematical derivation, but should be accepted as it is.

After all, there are some aspects of the force of gravity that are not entirely explainable, just think of the fact, we repeat, that it depends, without any apparent logical reason, on the inverse of the square — rather than on the inverse of the cube or of the fourth power — of the distance, or rather than simply on the inverse of the distance.

However the aim of this paper is not getting into the details of the debate between those who believe in the existence of the fundamental laws of Nature, and those who believe that the physical laws are created by humans to describe the reality and consequently that every natural law should be explainable in the light of the reason, but it’s undeniable that the answer to the question here proposed depends on the way of solving this dispute.

The only thing that I can say in this regard is that the deductive method doesn’t seem the best way of approaching the force of gravity, as it is shown by the paradoxical results of the virial theorem seen above.

The inductive method, on the contrary, by starting from the single cases in order to deduce, case by case, the existence of a general principle, seems to be more suitable to study the issues related to the force of gravity, which, as for every phenomenological entity, is not *a-priori* knowable in its every single aspect.

Obviously, the latter considerations would fail if we believe, as Einstein teaches, that the force of gravity is a geometrical entity, which would find its logical primary cause in the spacetime, in particular in its curvature, but we have already said that this is not entirely true, at least until the force of gravity will continue to receive no geometrical, logical, mathematical, scientific explanation with regard to the fact that it can’t be other than inversely proportional to the square of the distance and to the fact the gravitational potential energy can’t be other than proportional to R^{-1}.

- **Relative facts and absolute self-facts**

In the reference [5] the authors distinguish relative facts from stable facts, and conclude that the stable facts are only a subset of the more general category of relative facts.

According to this theory, called relational quantum mechanics (RMQ), relative facts are even those concerning the particles that are in two superimposed states, or even the particles that are demonstrated to be ubiquitous, which instead are stable according to quantum mechanics because they are ubiquitous as ubiquitous the decoherence is.

In essence, according to RQM, “*Schrodinger’s cat has no reason to feel superimposed*”, because this situation is similar as the man in Einstein’s elevator, which doesn’t feel that the elevator, in which he stays, is moving in the interagalactic space (where the absence of gravity is assumed) with uniform linear accelerated motion, but thinks that the elevator is coming up and that he, together with the lift, is subjected to the gravitational force.

No matter what the observer sees, the important thing is what the observed feels, what he perceives.

Consequently, if Schrodinger's cat doesn't feel any change after the measurement, then it means that, to cat, nothing has changed, in the sense that, after the measurement, it feels to be in a single state and doesn't perceive any difference with respect to the superimposition situation in which it was before the measurement.

If nothing has changed, it means that no wave function collapse has occurred.

A logical corollary of this fundamental conclusion is that a fact is absolute when the relationality is not possible, namely when observer and observed *coincide*.

In particular it is possible to arrive at the conclusion that no wave function collapse occurs even by assuming that the equation (5) expresses the potential energy of self-orbitating particles (nucleons).

In this framework, in fact, we have assumed that the nucleons revolve around themselves, but this means that the nucleons are observers and observed at same time.

In particular, the orbitating nucleons are revolving particles with respect to their central alter ego, but these latter are not different and separated particles from the orbitating ones: are the nucleons themselves.

Analogously, the central nucleons are at rest with respect to their orbitating alter-ego, but these latter are not different and separated particles from the central ones: are the nucleons themselves.

We can conclude, hence, that the nucleus constitutes a self-system, meaning that the nucleons are observers and observed at same time, and, in this case, the relationality isn't possible anymore.

In fact, claiming that every system is always relative to another one, and consequently that it cannot ever be absolute, holds until observer and observed are different and separated objects or systems, but obviously doesn't apply when observer and observed coincide.

In this particular case, we deal with systems (more precisely self-systems) originating absolute facts, because the relationality, as necessary requisite for a fact to be relative, lacks.

If the nucleons constitute a self-system originating only absolute facts, it means that their wave function cannot collapse, because absolute facts, by definition, cannot collapse, and this is the reason why we are able to see the proofs of this superimposition, as we'll see later.

Finding the evidences of superimposed states is fundamental to demonstrate that this phenomenon really occurs even before the measurement.

In other words, are we really sure that two entangled photons or electrons are really superimposed before measurement?

The question arises because, when we measure (namely observe) one photon entangled to another photon, both of them are never found superimposed, in the sense that the entangled photons manifest themselves in only one state (for instance showing only the spin "up" or only the spin "down"), even if opposed with respect to each other, but never in two states simultaneously.

But the fact that there is the absolute certainty that, when we measure a photon, the non-observed entangled photon has the opposite spin with respect to the observed photon doesn't necessary mean that the two photons were superimposed before measurement, and that, due to the measurement, they have collapsed in only one status, because we can also reasonably argue that the two photons were moving in that strange, entangled way even before the measurement, meaning that they were moving in such a way to have in every instant an opposite spin with respect to each other, namely changing their spin continuously and specularly, instant by instant, hence it's obvious that they show always opposite spin after measurement.

Moreover, to have the absolute certainty that the two photons were superimposed before the measurement we should observe them in this superimposed state.

Well, in this regard we can say that the nucleons represent a case in which this is possible.

Indeed we have seen above that the nuclear size is bigger than that resulting from the electron scattering experiments, in particular is twice the aforesaid dimension.

This can be well-explained, we repeat, by assuming that the nucleons are self-orbitating particles which are globally charged in while they are at rest and, at same time, electrically neutral in while they are in orbit.

In essence, the nucleons are in a double superimposed state, namely, they are both at rest and, at same time, in orbit, with the specification that, they are (positively) charged when they are at rest, and uncharged when in orbit; obviously all these considerations holds under the assumption that the electrostatic repulsion among the central protons be neutralized by the neutrons.

And this two superimpositions are both of them detectable in the experiments, descripted in the mentioned reference [6].

But in order to justify the cited experiments in the light of the gravitoelectric force and gravitoelectric energy proposed in this paper, it's necessary to modify the eq. (5) as follows:

$$U = \frac{4GKM^2}{\pi R^3} \tag{11}$$

So the eq. (6) becomes:

$$\frac{4GKM^2}{\pi R^3} = Mc^2$$

In this way we obtain a nuclear radius $\left(R = \sqrt[3]{\frac{4GKM}{\pi c^2}}\right)$ which is exactly twice the radius observed in the electron scattering experiment, and therefore we manage to explain the real, total size of the nucleus resulting from both the electron scattering phenomenon and the absorption phenomenon descripted in the reference [6], provided that we assume that the orbitating alter ego of nucleons to be electrically neutral and that the repulsive force among the central protons is neutralized by the neutrons, but accepting the eq. (11) implies to accept the *containing-energy* concept as defined in the reference [7], where it is clarified the reason of the adjunct of 4 in the numerator and the lack of 2 in the denominator of eq. (5).

But why can we detect only superimposed states concerning nucleons and not even those concerning photons, or in general, entangled particles?

This question has two possible answers.

The first is to think that the wave function of nucleons, as we have already said, cannot collapse because it involves objects which originate only absolute facts.

The second is to think that the wave function doesn't physically exist, in the sense that it is only a mathematical artifice and, consequently, the superimposed states which are not detected, but only supposed, have to be considered inexistent until they are experimentally demonstrated.

After all, "entangled" doesn't mean superimposed, but just means "united", "linked" to each other, in the sense that, by measuring only one particle, also the other is immediately affected.

As regards the feature of particles' ubiquity, which is shown in the double slits experiment, again doesn't mean that these particles are superimposed, because being everywhere doesn't mean being simultaneously in two superimposed, *opposed* states.

Being superimposed means being in two contrary states in the same instant, namely two states which contradict one another, for instance at rest and in movement, charged and uncharged, dead and alive, but if a particle moves towards two slits, and passes simultaneously in these two slits, it doesn't mean that the particle was superimposed, but only that, in while it was moving towards the slits, it was not concentrated in only one point, but was everywhere, yet this is a different situation from the superimposition paradox, and can be also explained by resorting to the pilot wave concept of De Broglie-Bohm.

Anyway the aim of the present paper is seeking to give a response only to the superimposition paradox in microscopic mechanics, and how to understand when it occurs, so we don't go here in the details of the debate concerning the possible interpretations of double slits experiment, which, we repeat, denotes weirdness, not paradoxicalness.

The only thing that we can say in concluding this study is that considering the nucleons as objects originating absolute facts can represent a useful tool to conceptually motivate not only the fact that they remain superimposed even after the measurement as well as to elucidate the experiments reported in reference [6], but even to justify some other absolute facts.

In particular, if we accept the existence of self-systems, then we should accept even that the facts they produce can't be other than absolute, for instance the constancy of the speed of light, which is independent from any observer.

The endorsement of the idea that the photons can produce absolute facts could be supported by arguing that they are in a certain way related to protons, in particular if we think about the possibility that their mass could be equal to the proton mass squared, as it is better shown again in the reference [7].

- **Conclusions**

Through a new vision of the gravitational potential energy, intended as the potentiality stored in a body m, subjected to one gravitational attractive force, or to two gravitational attractive-repulsive forces simultaneously acting upon it, respectively exerted by an attractor, or by an attractor-repulsor body M, of accelerating respectively towards the center of the attractor body (assumed spherical) or around the center of the attractor-repulsor central body (assumed again spherical), throughout a given space and according to a precise law, and assuming that nucleons are self-orbitating particles, namely revolving around themselves, and supposing that the orbiting alter-ego of nucleons are subjected to two simultaneous gravitational forces, one attractive and one repulsive, both generated by the nuclear central core made up of the nucleons at-rest, and hypothesizing that this attractive-repulsive *zone* would give rise to a pendulum harmonic oscillator which would cause the movement of the nucleons like a body hanging by a pendulum, it has been possible to demonstrate that the force of gravity acts in the atomic nucleus, even if in its electric variant $\left(F_{ge} = \frac{GKMm}{R^2}\right)$.

Even though some physical aspects of the just described nuclear model still remain not very clear, for example the weird fact that the aforesaid harmonic oscillator does not oscillate around the equilibrium point, but revolve around the fixed point (the nuclear center), as well as the odd fact that the gravitoelectric repulsive force would have a non-radial direction, and even though, consequently, it is not absolutely certain whether the gravitoelectric force be one, only attractive, or two, attractive and repulsive at same time, nevertheless it seems quite indisputable that the gravitoelectric force acts within the atomic nucleus as responsible for its dimensions and then for the cohesion of nucleons, by being the numerical value of the gravitoelectric potential energy of nucleons, found in our research, exactly equal to the numerical value of mc^2, which could lead one to conclude that in the atomic nucleus a typical law of the macroscopic world be operating, namely the fact that the gravitational potential energy of the orbitating body has to be twice its kinetic energy in order to obtain a stable orbit, but, in this case, one should admit that the expression mc^2 not to express the mass-energy equivalence principle, as the special theory of relativity states, but twice the kinetic energy of nucleons, and then the foundations of chemical physics regarding the atomic nucleus, as well as those concerning the theory of relativity itself, could be questioned.

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