$\log\left(\frac{1}{i}\right) = 0$

March 30, 2020
Yuji Masuda
(y_masuda0208@yahoo.co.jp)

$$\frac{f(x)}{g(x)} = \frac{e^{\frac{ix}{4}} - ie^{-\frac{ix}{4}}}{e^{\frac{ix}{4}} + ie^{-\frac{ix}{4}}} \Rightarrow \frac{f(x)g(x)}{g(x)^2} = \frac{e^{\frac{ix}{2}} + e^{-\frac{ix}{2}}}{g(x)^2}$$

$$\left(\frac{f(x)}{g(x)}\right)' = \frac{-1}{g(x)^2} \quad , \quad -16\big(f(x)\big)'\big(g(x)\big)' = e^{\frac{ix}{2}} + e^{-\frac{ix}{2}}$$

$$\therefore \frac{f(x)}{g(x)} = \left(\frac{f(x)}{g(x)}\right)' 16\big(f(x)\big)'\big(g(x)\big)' \quad \Rightarrow \quad \frac{\frac{f(x)}{g(x)}}{\left(\frac{f(x)}{g(x)}\right)'} = \frac{1}{16\big(f(x)\big)'\big(g(x)\big)'}$$

$$\therefore \log\left(\frac{\cos\left(\frac{x}{2}\right)}{i\left(\sin\left(\frac{x}{2}\right)+1\right)}\right) = \log\left(\frac{\cos\left(\frac{x}{2}\right)}{\left(\sin\left(\frac{x}{2}\right)+1\right)}\right) \quad \Rightarrow \quad \log\left(\frac{1}{i}\right) = 0$$

【Proof】

From Definition series,

$$\log\left(\frac{1}{i}\right) = 0 \quad \Rightarrow \quad e^0 = \frac{1}{i}$$

$$(-2)^5 = \frac{1}{2} \quad \Rightarrow \quad -32 = 3$$

That's all (proof end)