The Discrete Nature of Mass in Gravitational Wave Events

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The mass of each precursor and remnant from eleven mergers observed by LIGO and Virgo during runs O1 and O2 is related through an inverse 5/2 power law to a sub-Planckian mass scale – in the range 1-10 TeV for the black holes – within one or more of three geometric sequences of mass scales that descend from Planck scale and are occupied by the particles. The three common ratios are 1/π, 2/π and 1/e. The results are in accord with those found for planetary bodies and provide evidence for the existence of a stringy theory of quantum gravity.

First, some results on planets and their moons [1] are reviewed. These results are directly relevant to our consideration of gravitational wave events. The (super-Planckian) masses *M* of planets and major moons of the solar system [2] are inputted into equation (1); Planck units are used, which explains the apparently unbalanced dimensions. The resulting value of *m* is a sub-Planckian mass scale.

$$2m^{-5} = M^2 \tag{1}$$

The sub-Planckian scales resulting from (1) coincide with mass levels and sub-levels of fractional level-number in Sequences 1, 2 and 3, which descend geometrically from Planck scale with common ratio π^{-1}, $(\pi/2)^{-1}$ and e^{-1}, respectively. Each mass level is related to the Planck Mass through an exponential factor, e.g. π^{-n_1} where n_1 is an integer or a half-integer, quarter-integer, eighth-integer etc. The sequences may derive from the geometry of a warped spacetime [3]. Note that π is the length in Planck units of the Planckian S^1/Z_2 interval. The occupation of levels of integer level-number in Sequences 1 and 3 is shown in Figure 1.

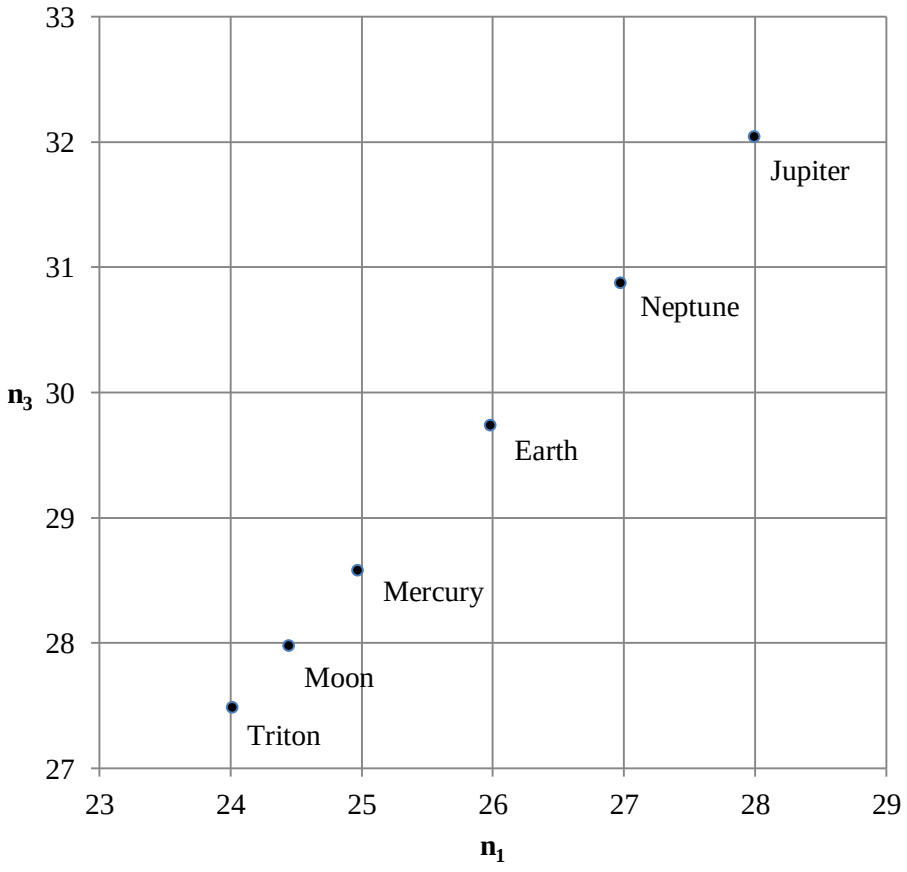

Figure 1: Sub-Planckian mass scales corresponding through (1) to the masses of selected planets and moons on the levels of Sequences 1 and 3. Data are taken from [3]. The mass scales are confined to a straight line as level numbers in the two sequences are in constant ratio.

More examples of corresponding sub-Planckian mass scales coincident with mass levels and major sub-levels are shown in Figures 2 and 3.

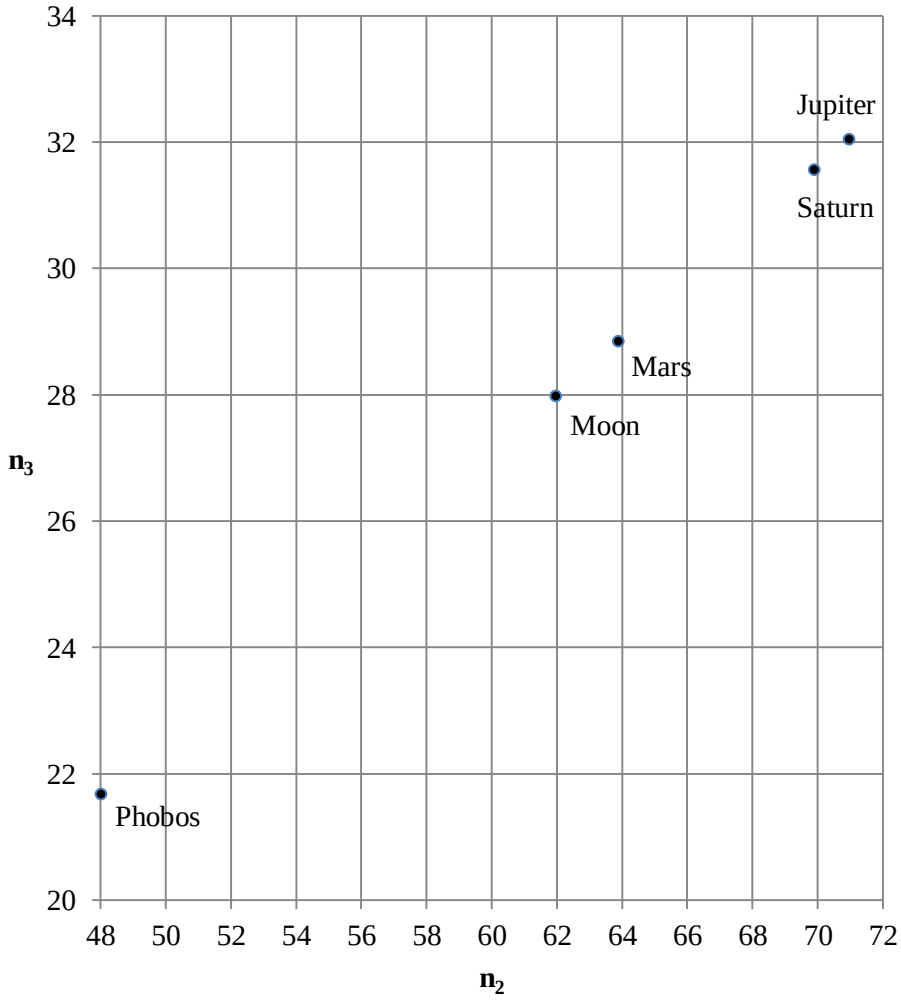

Figure 2: Sub-Planckian mass scales corresponding through (1) to the masses of planets and moons on the levels of Sequences 2 and 3.

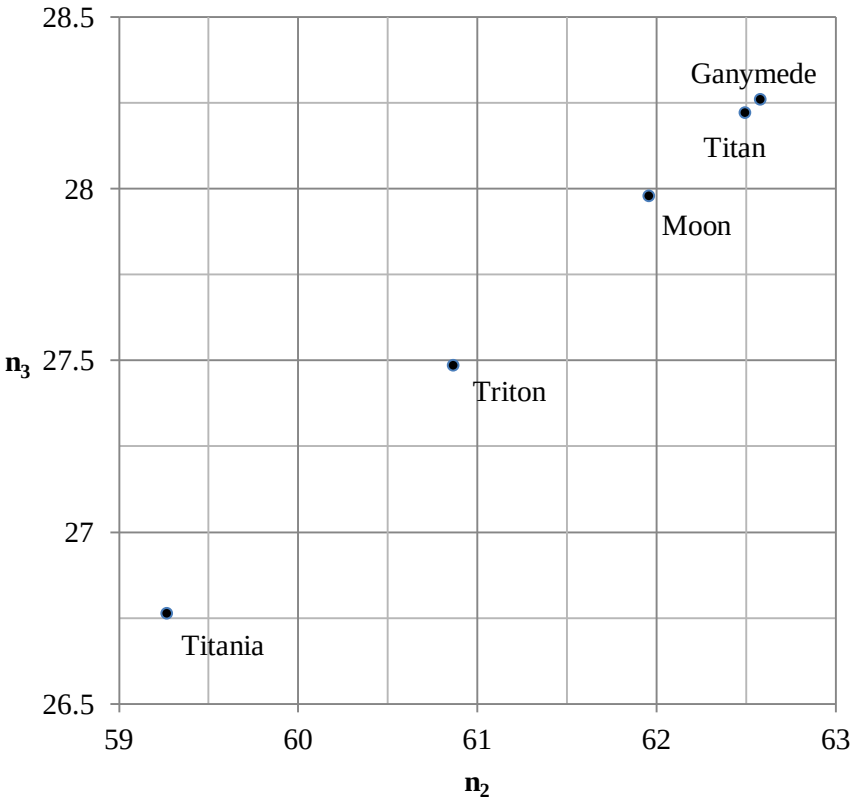

Figure 3: Sub-Planckian mass scales corresponding through (1) to the masses of the most massive moons of each of the five most massive planets of the solar system, on sub-levels in Sequences 2 and 3.

Particles and atoms occupy the levels and sub-levels of Sequences 1, 2 and 3. The lightest charged lepton (electron), lightest quark (up: mass of 2.16 MeV), lightest meson (π^0), lightest baryon (proton) and the highly stable nuclide ^{56}Fe are shown on the levels and sub-levels of Sequences 1 and 3 in Figure 4. The particle masses used are the evaluations of the Particle Data Group [4].

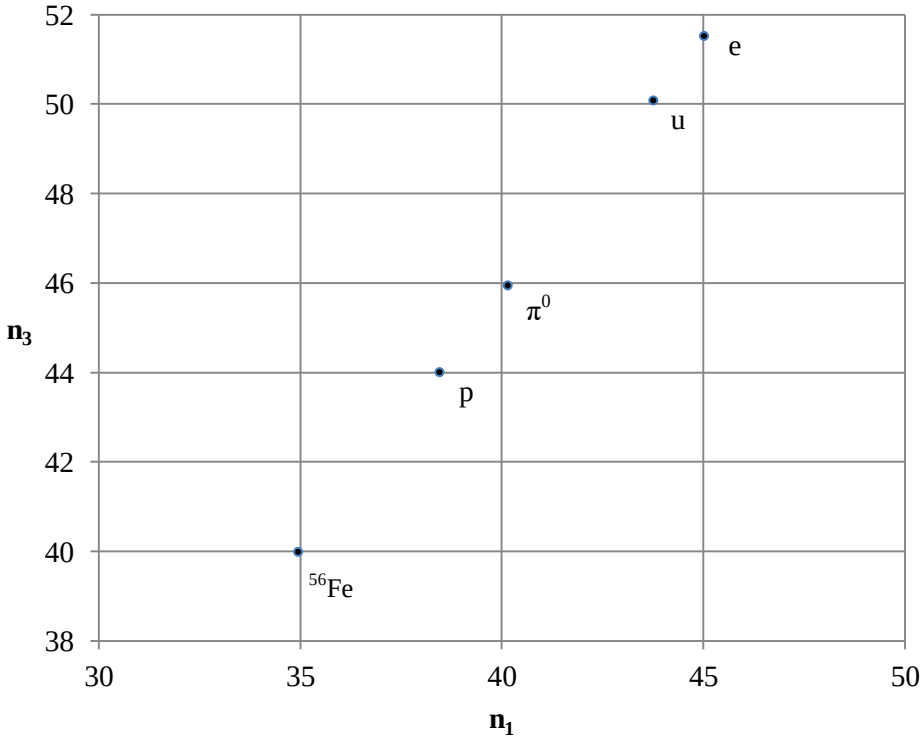

Figure 4: The electron, up quark, neutral pion, proton and the nuclide ^{56}Fe on the mass levels of Sequences 1 and 3.

The W, Z and H bosons occupy sub-levels in the mass sequences, as shown in Figure 5. The top quark is not included in this figure. Its mass (172.9 GeV) and that of the up quark (2.16 MeV) are related through the equation

$$m_{\mathrm{t}} = \left(\frac{\pi}{2}\right)^{25.00} m_{\mathrm{u}} \tag{2}$$

Powers of $(\pi/2)^{25}$ feature widely in the Planck Model [5]. For example, the Bohr radius a_0 is related to the Planck Length through the equation

$$a_0 = \left(\frac{\pi}{2}\right)^{125.00} l_{\mathrm{Planck}} \tag{3}$$

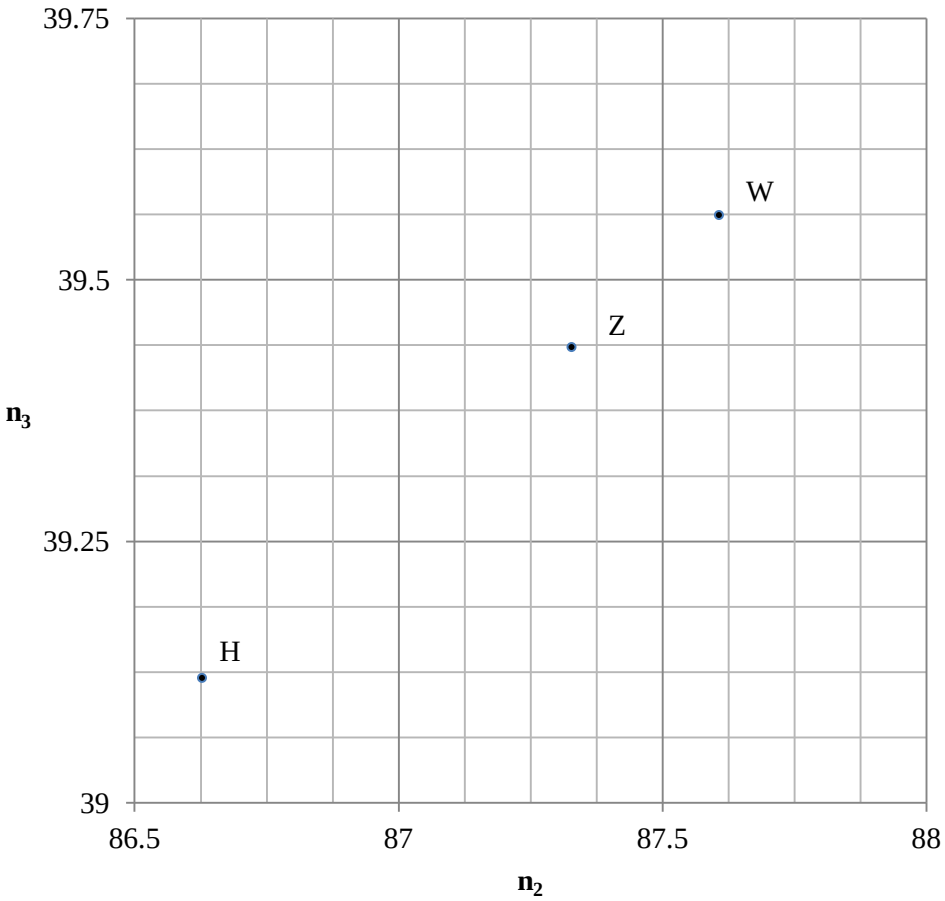

Figure 5: The W, Z and H bosons on sub-levels in Sequences 2 and 3.

We now turn to the black hole mergers observed by the LIGO and Virgo collaborations during runs O1 and O2 [6]. A sub-Planckian mass scale is calculated from (1) for the precursors and remnants of each of the ten confidently detected events, whose relevant parameters are included in Table 1. The sub-Planckian mass scales calculated for each event are plotted on the levels and sub-levels of Sequences 1, 2 and 3 in Figure 6.

Event	$m_1/M_\odot$	$m_2/M_\odot$	$m_{final}/M_\odot$	$E_{rad}/M_\odot c^2$
GW150914	35.6 +4.7/-3.1	30.6 +3.0/-4.4	63.1 +3.4/-3.0	3.1 +0.4/-0.4
GW151012	23.2 +14.9/-5.5	13.6 +4.1/-4.8	35.6 +10.8/-3.8	1.6 +0.6/-0.5
GW151226	13.7 +8.8/-3.2	7.7 +2.2/-2.5	20.5 +6.4/-1.5	1.0 +0.1/-0.2
GW170104	30.8 +7.3/-5.6	20.0 +4.9/-4.6	48.9 +5.1/-4.0	2.2 +0.5/-0.5
GW170608	11.0 +5.5/-1.7	7.6 +1.4/-2.2	17.8 +3.4/-0.7	0.9 +0.0/-0.1
GW170729	50.2 +16.2/-10.2	34.0 +9.1/-10.1	79.5 +14.7/10.2	4.8 +1.7/-1.7
GW170809	35.0 +8.3/-5.9	23.8 +5.1/-5.2	56.3 +5.2/-3.8	2.7 +0.6/-0.6
GW170814	30.6 +5.6/-3.0	25.2 +2.8/-4.0	53.2 +3.2/-2.4	2.7 +0.4/-0.3
GW170817	1.46 +0.12/-0.10	1.27 +0.09/-0.09	≤ 2.8	≥ 0.04
GW170818	35.4 +7.5/-4.7	26.7 +4.3/-5.2	59.4 +4.9/-3.8	2.7 +0.5/-0.5
GW170823	39.5 +11.2/-6.7	29.0 +6.7/-7.8	65.4 +10.1/-7.4	3.3 +1.0/-0.9

Table 1: Mass parameters of gravitational wave detections. The median and 90% credible intervals are quoted. All mergers are of black holes except that of GW170817, which is of neutron stars.

We see from Figure 6 that the sub-Planckian scales corresponding through (1) to both precursor and remnant black hole masses are closely aligned with levels and sub-levels in Sequences 1, 2 and 3. As

usual in the Planck Model, mass levels of 'low order' (integer, half-integer, quarter integer level-number) and especially near-coincident such levels are preferred locations for the scales.

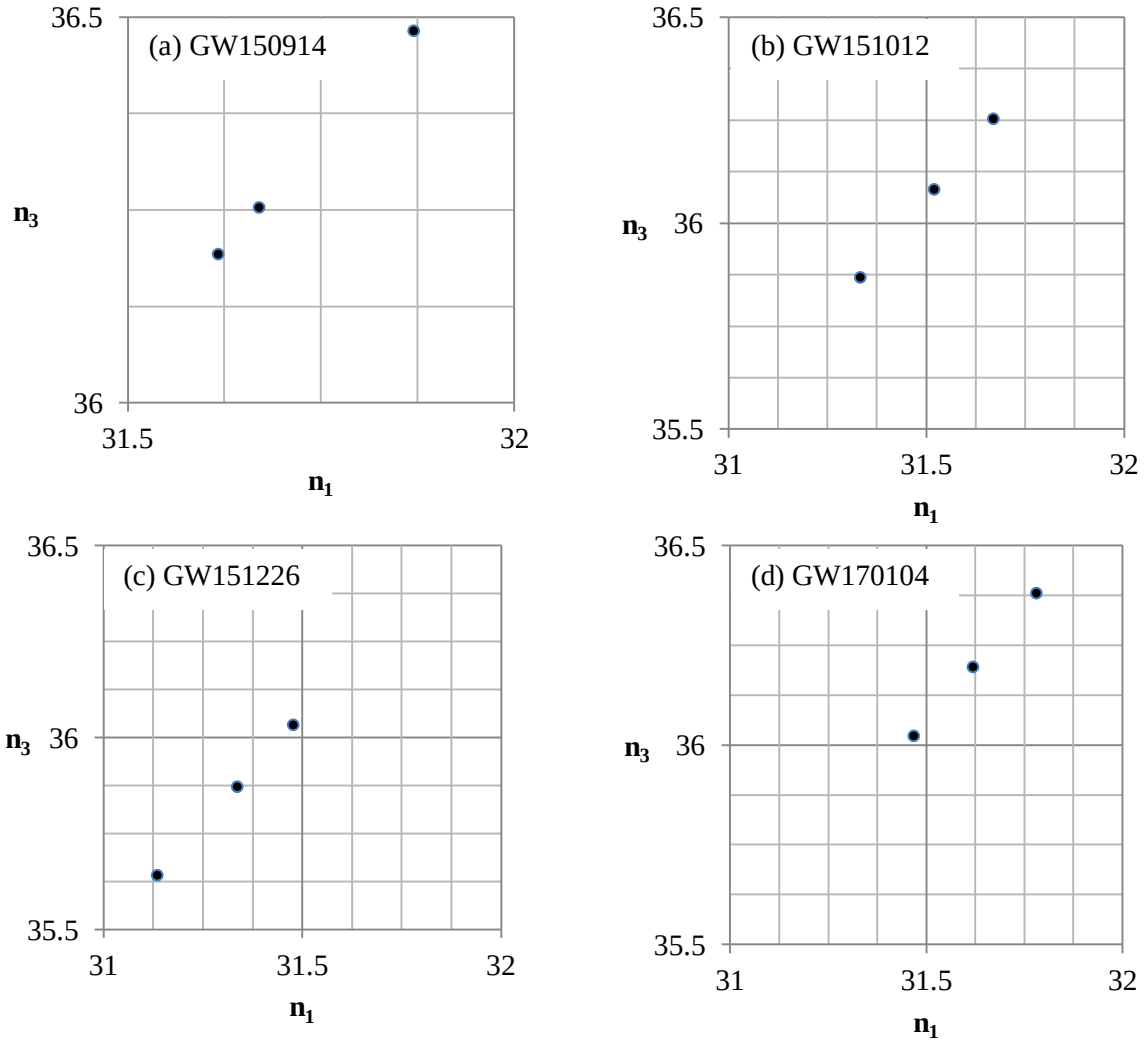

Figure 6: Sub-Planckian mass scales corresponding through (1) to precursor and remnant black hole masses from mergers producing gravitational wave events, on the mass levels of Sequences 1, 2 and 3. Mass scales corresponding to remnant black holes are of greater level-number than mass scales corresponding to precursors. Greater level-number means smaller sub-Planckian mass scale. Continued over.

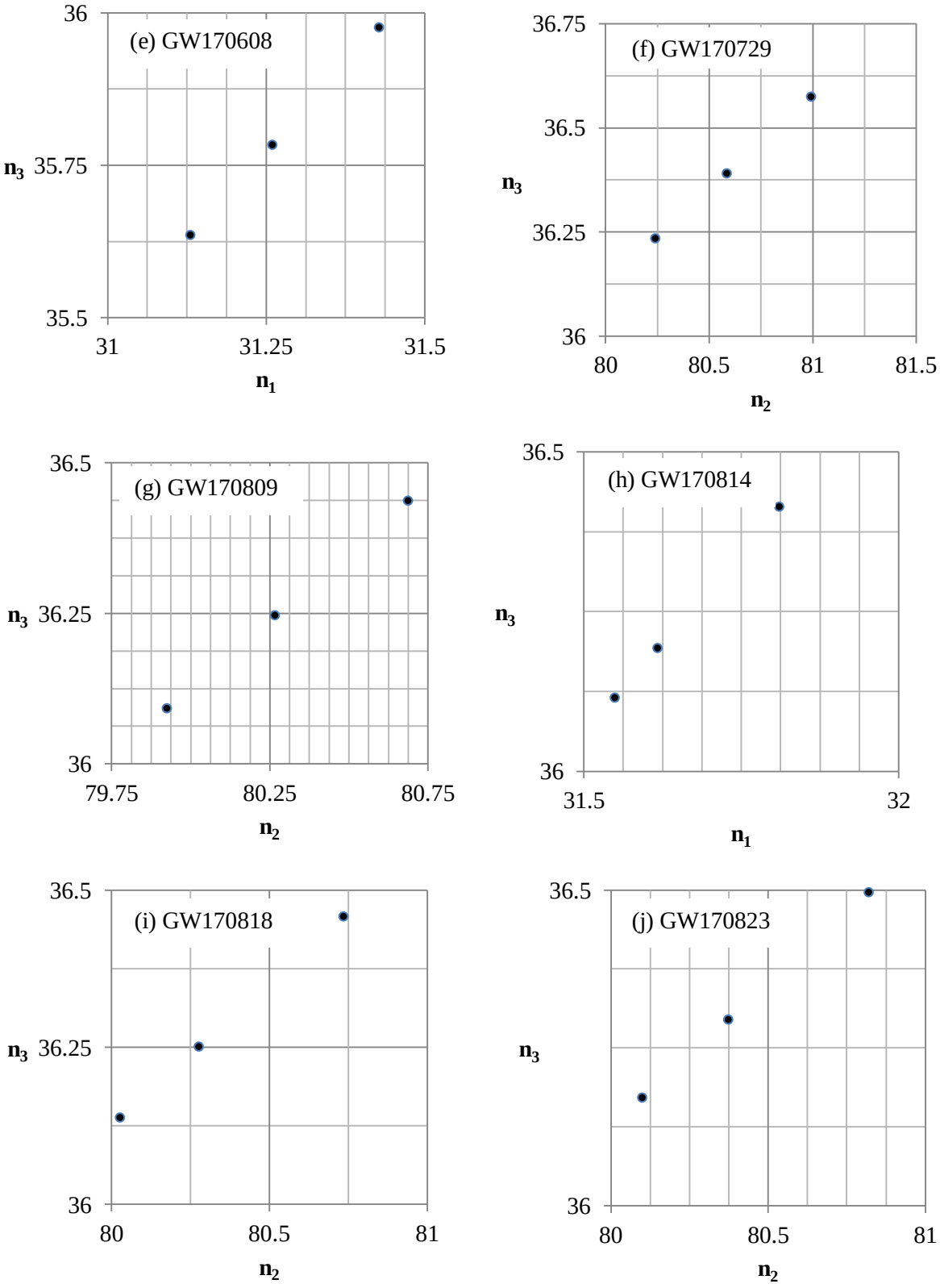

Figure 6 continued: Sub-Planckian mass scales corresponding through (1) to precursor and remnant black hole masses from mergers producing gravitational wave events, on the mass levels of Sequences 1, 2 and 3.

The sub-Planckian mass scales calculated from the masses of the neutron star precursors of GW170817, whose relevant parameters are included in Table 1, are plotted on the levels and sub-levels of Sequences 1 and 3 in Figure 7. We see that the mass scales are closely aligned with a mass level and a half-level that are nearly coincident.

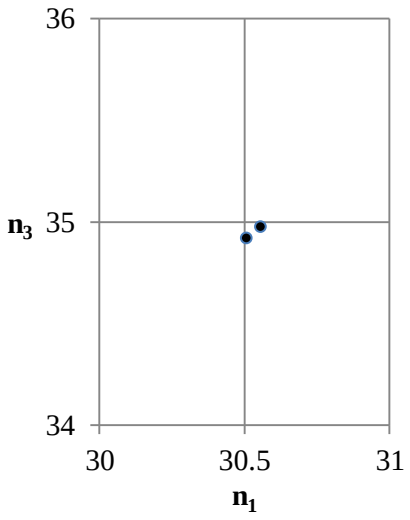

Figure 7: Sub-Planckian mass scales corresponding through (1) to the masses of the precursor neutron stars of gravitational wave event GW170817, on the levels of Sequences 1 and 3.

We have seen that the masses of planets, moons, black holes and neutron stars correspond through (1) to discrete sub-Planckian mass scales within the sequences of mass levels occupied by the particles. The large scales of astrophysics assume discrete values that are codified through (1).

The sub-Planckian mass scales corresponding to astrophysical scales are related to Planck scale through exponential factors, e.g. π^{-n_1}, where n_1 is an integer or a half-integer, quarter-integer, eighth-integer etc. The exponents n_1, n_2 and n_3 may be the winding numbers of intersecting D-branes that wrap cycles of Planckian compact spaces and whose intersections are the domains of the particles [3]. Fractional winding number, such as we see, is a feature of string theory [7]. The natural mass scale on an intersection sets the mass of a particle and, through (1), the mass of an astrophysical body.

References

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