On the Ramanujan's mathematics and Quantum Theory of Fields: mathematical connections with ϕ, $\zeta(2)$, and some parameters of Particle Physics.

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Abstract

In this paper we have described and analyzed some Ramanujan equations and various formulas of Quantum Theory of Fields. Furthermore, we have obtained mathematical connections with ϕ, $\zeta(2)$, and some parameters of Particle Physics.

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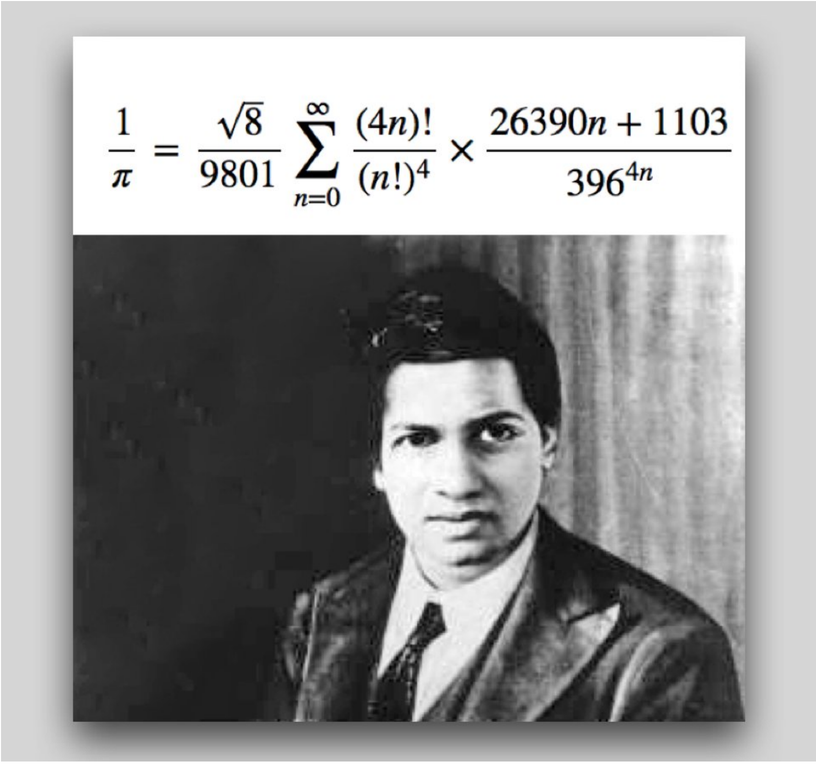


https://twitter.com/winkjs_org/status/973840788902866944

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From:

The Quantum Theory of Fields - Volume III - Supersymmetry
Steven Weinberg
University of Texas at Austin - Steven Weinberg 2000 - First published 2000
Printed in the United States of America

From (page 89)

To distinguish a chiral superfield $X(x,\theta)$ from the general superfield $S(x,\theta)$ of the previous section, we will use A, B, F, G, and ψ for its components, instead of C, M, N, Z, and ω. By using Eqs. (26.3.1), (26.3.2), and (26.3.8) in Eq. (26.2.10), we find the form of a general chiral superfield to be

$$\begin{aligned} X(x,\theta) = \; & A(x) - \left(\bar{\theta}\,\psi(x)\right) + \frac{1}{2}\left(\bar{\theta}\,\theta\right)F(x) - \frac{i}{2}\left(\bar{\theta}\,\gamma_5\,\theta\right)G(x) \\ & + \frac{i}{2}\left(\bar{\theta}\,\gamma_5\,\gamma_\mu\,\theta\right)\partial^\mu B(x) + \frac{1}{2}\left(\bar{\theta}\,\gamma_5\,\theta\right)\left(\bar{\theta}\gamma_5\,\not{\partial}\psi(x)\right) \\ & - \frac{1}{8}\left(\bar{\theta}\,\gamma_5\,\theta\right)^2 \Box A(x) \,. \end{aligned} \tag{26.3.9}$$

(We could just as well have taken $C = -B$, $\omega = \psi$, $M = -F$, $N = -G$, and $Z = A$. We make the identifications (26.3.8) because, as we see here, for a scalar superfield they are consistent with the usual convention that A and F are scalars while B and G are pseudoscalars.)

The chiral superfield (26.3.9) may be further decomposed, as

$$X(x,\theta) = \frac{1}{\sqrt{2}}\left[\Phi(x,\theta) + \tilde{\Phi}(x,\theta)\right], \qquad (26.3.10)$$

where

$$\Phi(x,\theta) = \phi(x) - \sqrt{2}\left(\bar{\theta}\psi_L(x)\right) + \mathscr{F}(x)\left(\bar{\theta}\left(\frac{1+\gamma_5}{2}\right)\theta\right) + \frac{1}{2}\left(\bar{\theta}\gamma_5\gamma_\mu\theta\right)\partial^\mu\phi(x)$$
$$-\frac{1}{\sqrt{2}}\left(\bar{\theta}\gamma_5\theta\right)\left(\bar{\theta}\,\not\partial\psi_L(x)\right) - \frac{1}{8}\left(\bar{\theta}\gamma_5\theta\right)^2\Box\phi(x), \qquad (26.3.11)$$

$$\tilde{\Phi}(x,\theta) = \tilde{\phi}(x) - \sqrt{2}\left(\bar{\theta}\psi_R(x)\right) + \tilde{\mathscr{F}}(x)\left(\bar{\theta}\left(\frac{1-\gamma_5}{2}\right)\theta\right) - \frac{1}{2}\left(\bar{\theta}\gamma_5\gamma_\mu\theta\right)\partial^\mu\tilde{\phi}(x)$$
$$+\frac{1}{\sqrt{2}}\left(\bar{\theta}\gamma_5\theta\right)\left(\bar{\theta}\,\not\partial\psi_R(x)\right) - \frac{1}{8}\left(\bar{\theta}\gamma_5\theta\right)^2\Box\tilde{\phi}(x), \qquad (26.3.12)$$

with component fields defined by

$$\phi \equiv \frac{A+iB}{\sqrt{2}}, \quad \psi_L \equiv \left(\frac{1+\gamma_5}{2}\right)\psi, \quad \mathscr{F} \equiv \frac{F-iG}{\sqrt{2}}, \qquad (26.3.13)$$

$$\tilde{\phi} \equiv \frac{A-iB}{\sqrt{2}}, \quad \psi_R \equiv \left(\frac{1-\gamma_5}{2}\right)\psi, \quad \tilde{\mathscr{F}} \equiv \frac{F+iG}{\sqrt{2}}. \qquad (26.3.14)$$

with A, B, and C running over the values 1,2, 3, *C, M, N,* V_a and D are real

for: A = 1, B = 2, F = 3, G = 5, γ_5 = 1, -1

$$C = A, \quad \omega = -i\gamma_5\psi, \quad M = G, \quad N = -F, \quad Z = B.$$

$\psi(x)$ is an arbitrary Majorana field, ψ = 8

$$\gamma_5 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix},$$

Input:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Determinant:

-1

Inverse:

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Characteristic polynomial:

$\lambda^2 - 1$

Eigenvalues:

$\lambda_1 = -1$

$\lambda_2 = 1$

Eigenvectors:

$v_1 = (0, 1)$

$v_2 = (1, 0)$

Condition number:

1

(We could just as well have taken $C = -B$, $\omega = \psi$, $M = -F$, $N = -G$, and $Z = A$. We make the identifications (26.3.8) because, as we see here, for a scalar superfield they are consistent with the usual convention that A and F are scalars while B and G are pseudoscalars.)

$$\phi \equiv \frac{A + iB}{\sqrt{2}}, \quad \psi_L \equiv \left(\frac{1+\gamma_5}{2}\right)\psi, \quad \mathscr{F} \equiv \frac{F - iG}{\sqrt{2}}, \qquad (26.3.13)$$

$$\tilde{\phi} \equiv \frac{A - iB}{\sqrt{2}}, \quad \psi_R \equiv \left(\frac{1-\gamma_5}{2}\right)\psi, \quad \tilde{\mathscr{F}} \equiv \frac{F + iG}{\sqrt{2}}. \qquad (26.3.14)$$

For: A = 1, B = 2, F = 3, G = 5, γ_5 = 1, -1; ψ = 8; θ = -i/(sqrt2)

(1+2i)/(sqrt2); (1+1)/2*8; (3-5i)/(sqrt2); (1-2i)/(sqrt2); (1+1)/2*8;

(3+5i)/(sqrt2)

we obtain:

Polar coordinates:

$r \approx 1.58114$ (radius), $\theta \approx 63.4349°$ (angle)

1.58114 (1-2i)/(sqrt2) ; (1+2i)/(sqrt2)

Polar coordinates:

$r \approx 4.12311$ (radius), $\theta \approx -59.0362°$ (angle)

4.12311 (3-5i)/(sqrt2) ; (3+5i)/(sqrt2)

((1+1)/2)*8 = 8

Input:

$$\frac{1+1}{2} \times 8$$

Result:

8

8

Now:

$$\Phi(x,\theta) = \phi(x) - \sqrt{2}\left(\bar{\theta}\psi_L(x)\right) + \mathscr{F}(x)\left(\bar{\theta}\left(\frac{1+\gamma_5}{2}\right)\theta\right) + \frac{1}{2}\left(\bar{\theta}\gamma_5\gamma_\mu\theta\right)\partial^\mu\phi(x) - \frac{1}{\sqrt{2}}\left(\bar{\theta}\gamma_5\theta\right)\left(\bar{\theta}\,\partial\!\!\!/\psi_L(x)\right) - \frac{1}{8}\left(\bar{\theta}\gamma_5\theta\right)^2\Box\phi(x)\,, \qquad (26.3.11)$$

1.58114-sqrt2(8*(-i/(sqrt2)))+4.12311((-i/(sqrt2))^2)+1/2((-i/(sqrt2))^2)*1.58114-1/(sqrt2)*((-i/(sqrt2))^2)*(((-i/(sqrt2)))*8)-1/8((-i/(sqrt2))^4)*1.58114

Input interpretation:

$$1.58114 - \sqrt{2}\left(8\left(-\frac{i}{\sqrt{2}}\right)\right) + 4.12311\left(-\frac{i}{\sqrt{2}}\right)^2 + \frac{1}{2}\left(-\frac{i}{\sqrt{2}}\right)^2 \times 1.58114 - \frac{1}{\sqrt{2}}\left(-\frac{i}{\sqrt{2}}\right)^2\left(-\frac{i}{\sqrt{2}} \times 8\right) + \frac{1}{8}\left(-\frac{i}{\sqrt{2}}\right)^4 \times (-1.58114)$$

i is the imaginary unit

Result:

$-0.925111... + 6\,i$

Polar coordinates:

$r = 6.0709$ (radius), $\theta = 98.7651°$ (angle)

6.0709

$$\tilde{\Phi}(x,\theta) = \tilde{\phi}(x) - \sqrt{2}\left(\bar{\theta}\psi_R(x)\right) + \tilde{\mathscr{F}}(x)\left(\bar{\theta}\left(\frac{1-\gamma_5}{2}\right)\theta\right) - \frac{1}{2}\left(\bar{\theta}\gamma_5\gamma_\mu\theta\right)\partial^\mu\tilde{\phi}(x)$$
$$+\frac{1}{\sqrt{2}}\left(\bar{\theta}\gamma_5\theta\right)\left(\bar{\theta}\,\partial\!\!\!/\psi_R(x)\right) - \frac{1}{8}\left(\bar{\theta}\gamma_5\theta\right)^2\Box\tilde{\phi}(x)\,, \qquad (26.3.12)$$

A = 1, B = 2, F = 3, G = 5, γ_5 = 1, -1; ψ = 8; θ = -i/(sqrt2)

1.58114-sqrt2(8*(-i/(sqrt2)))+4.12311((-i/(sqrt2))^2)-1/2((-i/(sqrt2))^2)*1.58114+1/(sqrt2)*((-i/(sqrt2))^2)*(((-i/(sqrt2)))*8)-1/8((((-i/(sqrt2))^4)*1.58114

Input interpretation:

$$1.58114 - \sqrt{2}\left(8\left(-\frac{i}{\sqrt{2}}\right)\right) + 4.12311\left(-\frac{i}{\sqrt{2}}\right)^2 + \frac{1}{2}\left(-\frac{i}{\sqrt{2}}\right)^2 \times (-1.58114) + \frac{1}{\sqrt{2}}\left(-\frac{i}{\sqrt{2}}\right)^2\left(-\frac{i}{\sqrt{2}} \times 8\right) + \frac{1}{8}\left(-\frac{i}{\sqrt{2}}\right)^4 \times (-1.58114)$$

i is the imaginary unit

Result:

$-0.134541\ldots + 10\,i$

Polar coordinates:

$r = 10.0009$ (radius), $\theta = 90.7708°$ (angle)

10.0009

Now, from

$$X(x,\theta) = \frac{1}{\sqrt{2}}\left[\Phi(x,\theta) + \tilde{\Phi}(x,\theta)\right],$$

We obtain:

1/(sqrt2) [6.0709+10.0009]

Input interpretation:

$$\frac{1}{\sqrt{2}}(6.0709 + 10.0009)$$

Result:

11.36447876587395450266549041887671278957802622334161702124...

11.3644787658...

That is about:

((1/(sqrt2))) [6+10]

16/(sqrt2)

Input:

$$\frac{16}{\sqrt{2}}$$

Result:

$$8\sqrt{2}$$

Decimal approximation:

11.31370849898476039041350979367758462855737500301558458541...

11.3137084...

From the two previous expression, we obtain:

1/7((1/(sqrt2))) [6.0709+10.0009]

Input interpretation:

$$\frac{1}{7}\times\frac{1}{\sqrt{2}}(6.0709+10.0009)$$

Result:

1.623496966553422071809355774125244684225432317620231003034...

1.62349696655...

and:

1/7((16/(sqrt2)))

((16/(7(sqrt2))))

Input:

$$\frac{16}{7\sqrt{2}}$$

Result:

$$\frac{8\sqrt{2}}{7}$$

Decimal approximation:

1.616244071283537198630501399096797804079625000430797797916...

1.616244071283...

From:

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

We have:

$$64g_{22}^{-24} \quad = \quad 4096e^{-\pi\sqrt{22}} + \cdots$$

From the following equation:

$$\varphi \equiv e^{-i\alpha}\left(\frac{A+iB}{\sqrt{2}}\right). \qquad (26.4.19)$$

we obtain:

exp(-Pi*sqrt22) ((1+2i)/(sqrt2))

Input:

$$\exp\left(-\pi\sqrt{22}\right)\times\frac{1+2\,i}{\sqrt{2}}$$

i is the imaginary unit

Exact result:

$$\frac{(1+2i)\,e^{-\sqrt{22}\,\pi}}{\sqrt{2}}$$

Decimal approximation:

2.8183352319122236146662226044529672922947410469505830... × 10^-7 +
5.6366704638244472293324452089059345845894820939011 67... × 10^-7 i

Property:

$\frac{(1+2i)\,e^{-\sqrt{22}\,\pi}}{\sqrt{2}}$ is a transcendental number

Polar coordinates:

$r \approx 6.30199\times10^{-7}$ (radius), $\theta \approx 63.4349°$ (angle)

$6.30199*10^{-7}$

Alternate form:

$$\frac{e^{-\sqrt{22}\,\pi}}{\sqrt{2}} + i\sqrt{2}\,e^{-\sqrt{22}\,\pi}$$

Series representations:

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{\sqrt{2}} = \frac{(1+2i)\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{\sqrt{2}} = \frac{(1+2i)\exp\left(-\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(22-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(22-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{\sqrt{2}} = \left((1+2i)\exp\left(-\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(22-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(22-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right)\Bigg/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

From which:

1+((((exp(-Pi*sqrt22) ((1+2i)/(sqrt2))))))^1/30-Pi/10^3

Input:

$$1+\sqrt[30]{\exp\left(-\pi\sqrt{22}\right)\times\frac{1+2i}{\sqrt{2}}}-\frac{\pi}{10^3}$$

i is the imaginary unit

Exact result:

$$1+\frac{\sqrt[30]{1+2i}\; e^{-1/15\sqrt{11/2}\,\pi}}{\sqrt[60]{2}}-\frac{\pi}{1000}$$

Decimal approximation:

1.617756184812901332929946656929720788442790446237912472... +
0.02292461447580697514378576152082139278026912535370017310... i

Alternate forms:

$$1+\sqrt[60]{-\frac{3}{2}+2i}\; e^{-1/15\sqrt{11/2}\,\pi}-\frac{\pi}{1000}$$

$$\frac{1000-\pi}{1000}+\frac{\sqrt[30]{1+2i}\; e^{-1/15\sqrt{11/2}\,\pi}}{\sqrt[60]{2}}$$

$$\frac{-\sqrt[60]{2}\,(\pi-1000)+1000\sqrt[30]{1+2i}\,e^{-1/15\sqrt{11/2}\,\pi}}{1000\sqrt[60]{2}}$$

Series representations:

$$1+\sqrt[30]{\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{\sqrt{2}}}-\frac{\pi}{10^3}=\frac{1000-\pi+1000\sqrt[30]{\frac{(1+2i)\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)}{\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}}}{1000}$$

for (not ($z_0\in\mathbb{R}$ and $-\infty<z_0\leq 0$))

$$1+\sqrt[30]{\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{\sqrt{2}}}-\frac{\pi}{10^3}=\frac{1000-\pi+1000\sqrt[30]{\frac{(1+2i)\exp\left(-\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(22-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(22-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}}}{1000}$$

for ($x\in\mathbb{R}$ and $x<0$)

$$1+\sqrt[30]{\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{\sqrt{2}}}-\frac{\pi}{10^3}=\frac{1}{1000}\left(1000-\pi+1000\left(\left((1+2i)\exp\left(-\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(22-z_0)/(2\pi)\rfloor}z_0^{1/2(1+\lfloor\arg(22-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right)\Big/\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)\right)\hat{}\,(1/30)\right)$$

1.6177561848129013329+ 0.022924614475806975i

Input interpretation:

1.6177561848129013329 + 0.022924614475806975 i

i is the imaginary unit

Result:

1.617756184812901333... +
0.022924614475806975 i

Polar coordinates:

$r = 1.6179186047045934603$ (radius), $\theta = 0.81186260366559408°$ (angle)

1.6179186047045934603

But, we have that:

$$64 g_{22}^{-24} \quad = \quad 4096 e^{-\pi\sqrt{22}} + \cdots$$

Thence, we obtain also:

((((exp(-Pi*sqrt22) ((1+2i)/(sqrt2))))))1/4096

Input:

$$\left(\exp\left(-\pi\sqrt{22}\right)\times\frac{1+2\,i}{\sqrt{2}}\right)\times\frac{1}{4096}$$

i is the imaginary unit

Exact result:

$$\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}$$

Decimal approximation:

$6.88070124978570218424370753040275217845395763415669... \times 10^{-11}$ +
$1.37614024995714043684874150608055043569079152683133... \times 10^{-10}\,i$

Property:

$\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}$ is a transcendental number

Polar coordinates:

$r \approx 1.53857\times10^{-10}$ (radius), $\theta \approx 63.4349°$ (angle)

1.53857*10^{-10}

Series representations:

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}} = \frac{(1+2i)\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)}{4096\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}} = \frac{(1+2i)\exp\left(-\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(22-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(22-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)}{4096\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}} = \left((1+2i)\exp\left(-\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(22-z_0)/(2\pi)\rfloor} z_0^{1/2(1+\lfloor\arg(22-z_0)/(2\pi)\rfloor)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(22-z_0)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(2-z_0)/(2\pi)\rfloor}\right) \Bigg/ \left(4096\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)$$

From which:

3Pi-76*ln((((((exp(-Pi*sqrt22) ((1+2i)/(sqrt2))))))1/4096))

Input:

$$3\pi - 76\log\left(\left(\exp\left(-\pi\sqrt{22}\right)\times\frac{1+2i}{\sqrt{2}}\right)\times\frac{1}{4096}\right)$$

$\log(x)$ is the natural logarithm

i is the imaginary unit

Exact result:

$$3\pi - 76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)$$

Decimal approximation:

1726.644511514295261289208028614244095572728057478397404 93... − 84.143302552350878229296974973568815045323621050508881 1474... i

Alternate forms:

$$3\pi + 38\left(2\sqrt{22}\,\pi - 2\log\left(\frac{1}{4096}+\frac{i}{2048}\right)+\log(2)\right)$$

$$3\pi + 76\sqrt{22}\,\pi + 950\log(2) - 38\log(5) - 76\,i\tan^{-1}(2)$$

$$3\pi - 38\log\left(\frac{5\,e^{-2\sqrt{22}\,\pi}}{33554432}\right) - 76\,i\tan^{-1}(2)$$

$\tan^{-1}(x)$ is the inverse tangent function

Alternative representations:

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi - 76\log_e\left(\frac{(1+2i)\exp(-\pi\sqrt{22})}{4096\sqrt{2}}\right)$$

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi - 76\log(a)\log_a\left(\frac{(1+2i)\exp(-\pi\sqrt{22})}{4096\sqrt{2}}\right)$$

Series representations:

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) =$$
$$3\pi + 76\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{8192}\right)^k\left(-8192+(1+2i)\sqrt{2}\,e^{-\sqrt{22}\,\pi}\right)^k}{k}$$

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi - 152 i\pi \left\lfloor \frac{\arg\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}} - x\right)}{2\pi} \right\rfloor - 76\log(x) + 76\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi - 152 i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1+2i}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - 76\log(z_0) + 76\sum_{k=1}^{\infty} \frac{\left(-\frac{1}{8192}\right)^k \left((1+2i)\sqrt{2}\, e^{-\sqrt{22}\,\pi} - 8192 z_0\right)^k z_0^{-k}}{k}$$

Integral representations:

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi - 76\int_1^{\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}} \frac{1}{t}\,dt$$

$$3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right) = 3\pi + \frac{38 i}{\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)^{-s} \Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds \quad \text{for } -1 < \gamma < 0$$

1726.64451151429526 - 84.1433025523508i

Input interpretation:

1726.64451151429526 + i × (−84.1433025523508)

i is the imaginary unit

Result:

1726.644511514295… −
84.1433025523508… i

Polar coordinates:

$r = 1728.69354268096222$ (radius), $\theta = -2.78994610916412°$ (angle)

1728.69354268096222

and:

[3Pi-76*ln((((((exp(-Pi*sqrt22) ((1+2i)/(sqrt2))))))1/4096))]^1/15

Input:

$$\sqrt[15]{3\pi - 76\log\left(\left(\exp\left(-\pi\sqrt{22}\right)\times\frac{1+2i}{\sqrt{2}}\right)\times\frac{1}{4096}\right)}$$

log(x) is the natural logarithm

i is the imaginary unit

Exact result:

$$\sqrt[15]{3\pi - 76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)}$$

Decimal approximation:

1.643787141962770150129970034703442976140350500473884444... −
0.005336162267245953155274349993704432526608415170650366646... i

Alternate forms:

$$\sqrt[15]{3\pi + 38\left(2\sqrt{22}\,\pi - 2\log\left(\frac{1}{4096}+\frac{i}{2048}\right)+\log(2)\right)}$$

$$\sqrt[15]{3\pi + 76\sqrt{22}\,\pi + 950\log(2) - 38\log(5) - 76\,i\tan^{-1}(2)}$$

$$\sqrt[15]{\left(3+76\sqrt{22}\right)\pi - 38\left(-25\log(2) + 2\left(\frac{\log(5)}{2} + i\tan^{-1}(2)\right)\right)}$$

All 15th roots of 3 π - 76 log(((1/4096 + i/2048) e^(-sqrt(22) π))/sqrt(2)):

$$\sqrt[30]{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)^2+\left(3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)\right)^2}\ \exp\left(\frac{1}{15}\,i\tan^{-1}\left(\frac{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}{3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}\right)\right)\approx 1.64379-0.00534\,i$$

$$\sqrt[30]{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)^2+\left(3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)\right)^2}\ \exp\left(\frac{1}{15}\,i\left(2\pi+\tan^{-1}\left(\frac{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}{3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}\right)\right)\right)$$

$\approx 1.50384+0.6637\,i$ (principal root)

$$\sqrt[30]{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)^2+\left(3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)\right)^2}\ \exp\left(\frac{1}{15}\,i\left(4\pi+\tan^{-1}\left(\frac{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}{3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}\right)\right)\right)\approx 1.1039+1.2180\,i$$

$$\sqrt[30]{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)^2+\left(3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)\right)^2}\ \exp\left(\frac{1}{15}\,i\left(6\pi+\tan^{-1}\left(\frac{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}{3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}\right)\right)\right)\approx 0.5130+1.56169\,i$$

$$\sqrt[30]{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)^2+\left(3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)\right)^2}$$

$$\exp\left(\frac{1}{15}\,i\left(8\pi+\tan^{-1}\left(\frac{\operatorname{Im}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}{3\pi+\operatorname{Re}\left(-76\log\left(\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)\right)}\right)\right)\right)$$

$$\approx -0.16652+1.63534\,i$$

$\operatorname{Im}(z)$ is the imaginary part of z

$\operatorname{Re}(z)$ is the real part of z

Alternative representations:

$$\sqrt[15]{3\pi-76\log\left(\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{4096\sqrt{2}}\right)}=\sqrt[15]{3\pi-76\log_e\left(\frac{(1+2i)\exp\left(-\pi\sqrt{22}\right)}{4096\sqrt{2}}\right)}$$

$$\sqrt[15]{3\pi-76\log\left(\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{4096\sqrt{2}}\right)}=\sqrt[15]{3\pi-76\log(a)\log_a\left(\frac{(1+2i)\exp\left(-\pi\sqrt{22}\right)}{4096\sqrt{2}}\right)}$$

Series representations:

$$\sqrt[15]{3\pi-76\log\left(\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{4096\sqrt{2}}\right)}=\sqrt[15]{3\pi+76\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{8192}\right)^k\left(-8192+(1+2i)\sqrt{2}\,e^{-\sqrt{22}\,\pi}\right)^k}{k}}$$

$$\sqrt[15]{3\pi-76\log\left(\frac{\exp\left(-\pi\sqrt{22}\right)(1+2i)}{4096\sqrt{2}}\right)}=\left(3\pi-76\left(2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1+2i}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{8192}\right)^k\left((1+2i)\sqrt{2}\,e^{-\sqrt{22}\,\pi}-8192z_0\right)^k z_0^{-k}}{k}\right)\right)^{\wedge}(1/15)$$

$$\sqrt[15]{3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right)} = \left(3\pi - 76\left(\log(z_0) + \left\lfloor\frac{\arg\left((1+2i)\sqrt{2}\,e^{-\sqrt{22}\,\pi} - 8192\,z_0\right)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty}\frac{\left(-\frac{1}{8192}\right)^k\left((1+2i)\sqrt{2}\,e^{-\sqrt{22}\,\pi} - 8192\,z_0\right)^k z_0^{-k}}{k}\right)\right)\text{^}(1/15)$$

Integral representations:

$$\sqrt[15]{3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right)} = \sqrt[15]{3\pi - 76\int_1^{\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}}\frac{1}{t}\,dt}$$

$$\sqrt[15]{3\pi - 76\log\left(\frac{\exp(-\pi\sqrt{22})(1+2i)}{4096\sqrt{2}}\right)} = \sqrt[15]{3\pi + \frac{38\,i}{\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{\left(-1+\frac{\left(\frac{1}{4096}+\frac{i}{2048}\right)e^{-\sqrt{22}\,\pi}}{\sqrt{2}}\right)^{-s}\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds} \quad \text{for } -1 < \gamma < 0$$

1.64378714196277015 - 0.0053361622672459i

Input interpretation:

$1.64378714196277015 + i \times (-0.0053361622672459)$

i is the imaginary unit

Result:

1.643787141962770... –
0.0053361622672459 i

Polar coordinates:

$r = 1.64379580322796622$ (radius), $\theta = -0.18599640727761°$ (angle)

1.64379580322796622

From pag. (106)-(109)

The '$\mathscr{F}$-terms' and 'D-terms' from which we construct the Lagrangian density may be expressed as integrals over the superspace coordinates θ_α. The rules for integrals over fermionic parameters originally given by Berezin[4] are derived in Section 9.5. Briefly, because the square of any fermionic parameter vanishes, any function of a set of N fermionic parameters ξ_n may be expressed as

$$f(\xi) = \left(\prod_{n=1}^{N} \xi_n\right) c \; + \; \text{terms with fewer } \xi \text{ factors}\,, \qquad (26.6.1)$$

and its integral over the ξs is defined simply by

$$\int d^N\xi \, f(\xi) \equiv c\,. \qquad (26.6.2)$$

In the same way, using Eq. (26.3.11), we find that the spacetime integral of the $\mathscr{F}$-term of a general left-chiral superfield Φ (again, either elementary or composite) may be expressed as

$$\int d^4x \, [\Phi]_{\mathscr{F}} = \frac{1}{2}\int d^4x \int d^2\theta_L \, \Phi(x,\theta)\,. \qquad (26.6.6)$$

The representation of the action as an integral over superspace allows an easy derivation of the field equations in superfield form. Consider, for instance, the action for a set of left-chiral scalar superfields Φ_n (which includes as a special case the general renormalizable theory of left-chiral superfields Φ_n):

$$I = \frac{1}{2}\int d^4x \left[K(\Phi,\, \Phi^*)\right]_D + 2\,\mathrm{Re}\int d^4x \, [f(\Phi)]_{\mathscr{F}}\,, \qquad (26.6.9)$$

with K an arbitrary function of Φ_n and Φ_n^* without derivatives, and f an arbitrary function of Φ_n, also without derivatives. (The motivation superspace Feynman rules in Chapter 30. We write the Φ_n in terms of *potential superfields* $S_n(x,\theta)$ as

$$\Phi_n = \mathscr{D}_R^2 S_n , \quad (26.6.10)$$

from which it follows (using Eq. (26.A.21)) that

$$\Phi_n^* = -\mathscr{D}_L^2 S_n^* , \quad (26.6.11)$$

where $\mathscr{D}_R^2$ and $\mathscr{D}_L^2$ are abbreviations for $(\mathscr{D}_R^T\epsilon\mathscr{D}_R) = -(\bar{\mathscr{D}}_R\mathscr{D}_R)$ and $(\mathscr{D}_L^T\epsilon\mathscr{D}_L) = (\bar{\mathscr{D}}_L\mathscr{D}_L)$, respectively. To see that it is always possible to find an S_n (not necessarily local) that satisfies Eq. (26.6.10), note that for any left-chiral superfields Φ_n,

$$\mathscr{D}_R^2\mathscr{D}_L^2\Phi_n = -16\Box\Phi_n , \quad (26.6.12)$$

so that Eq. (26.6.10) is satisfied by the solution of

$$-16\Box S_n = \mathscr{D}_L^2\Phi_n . \quad (26.6.13)$$

$$\mathscr{D}_R^2 \frac{\delta K(\Phi, \Phi^*)}{\delta\Phi_n} = -4\frac{\partial f(\Phi)}{\partial\Phi_n} . \quad (26.6.15)$$

$$\mathscr{D}_L^2 \frac{\delta K(\Phi, \Phi^*)}{\delta\Phi_n^*} = 4\left(\frac{\partial f(\Phi)}{\partial\Phi_n}\right)^* . \quad (26.6.16)$$

$$\mathscr{D}_R^2(\theta_R^T\epsilon\theta_R) = -4,$$

We have that:

$\Phi_n = -16 / 4$; $\Phi_n = -4 \Box S_n$

$-4 \Box S_n = \Phi_n$

Thence:

$$\mathscr{D}_R^2\mathscr{D}_L^2\Phi_n = -16\Box\Phi_n$$

$64 \Box \Box S_n = 64 \Box 1/4 \Phi_n = 64 * \Phi_n / -4 S_n * 1/4 \Phi_n =$

$64 * -1/16 \Phi^2{}_n / S_n$ $= -64 / S_n$; -16 **$\Phi^2{}_n = -256$** ;

$\mathbf{\Phi^2{}_n = 16}$; $\mathbf{\Phi_n = -4}$; $\mathbf{\Box = 1}$;

$$\left(\mathscr{D}_R^2 \mathscr{D}_L^2 \Phi_n\right) = (-4)*(4)*(-4) = \mathbf{64}$$

That is the value for any left-chiral superfield $\mathbf{\Phi_n}$. We have also:

$$-16\Box S_n = \mathscr{D}_L^2 \Phi_n .$$

$$(-16) * 1 * 1 = 4 * (-4) ; \ -16 = -16$$

$$\left(\mathscr{D}_L^2 \Phi_n\right) = 4 * (-4) = -16$$

We note that:

$$\left(\mathscr{D}_R^2 \mathscr{D}_L^2 \Phi_n\right) \times \left(\mathscr{D}_L^2 \Phi_n\right) = -1024$$

And that -1024 / -16 = **64**

From:

Modular equations and approximations to π - *S. Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64 g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276 e^{-\pi\sqrt{22}} - \cdots, \\ 64 g_{22}^{-24} &= 4096 e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64 g_{22}^{-24} = 4096 e^{-\pi\sqrt{22}} + \cdots$$

and

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372 e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

that are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

We have that:

exp(Pi*sqrt22)-24+4372*exp(-Pi*sqrt22)+4096*exp(-Pi*sqrt22) = 64[(1+sqrt2)^12+(1-sqrt2)^12]

exp(Pi*sqrt22)-24+4372*exp(-Pi*sqrt22)+4096*exp(-Pi*sqrt22)

Input:

$$\exp\left(\pi\sqrt{22}\right) - 24 + 4372\exp\left(-\pi\sqrt{22}\right) + 4096\exp\left(-\pi\sqrt{22}\right)$$

Exact result:

$$-24 + 8468 e^{-\sqrt{22}\,\pi} + e^{\sqrt{22}\,\pi}$$

Decimal approximation:

$2.5089280016325388599005802278989666470854041530472866 4\ldots \times 10^6$

2508928.0016326

Property:

$-24 + 8468\, e^{-\sqrt{22}\,\pi} + e^{\sqrt{22}\,\pi}$ is a transcendental number

Alternate form:

$e^{-\sqrt{22}\,\pi}\left(8468 - 24\, e^{\sqrt{22}\,\pi} + e^{2\sqrt{22}\,\pi}\right)$

Series representations:

$$\exp\left(\pi\sqrt{22}\right) - 24 + 4372\exp\left(-\pi\sqrt{22}\right) + 4096\exp\left(-\pi\sqrt{22}\right) = -24 + 8468\exp\left(-\pi\sqrt{21}\sum_{k=0}^{\infty} 21^{-k}\binom{\frac{1}{2}}{k}\right) + \exp\left(\pi\sqrt{21}\sum_{k=0}^{\infty} 21^{-k}\binom{\frac{1}{2}}{k}\right)$$

$$\exp\left(\pi\sqrt{22}\right) - 24 + 4372\exp\left(-\pi\sqrt{22}\right) + 4096\exp\left(-\pi\sqrt{22}\right) = -24 + 8468\exp\left(-\pi\sqrt{21}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{21}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right) + \exp\left(\pi\sqrt{21}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{21}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)$$

$$\exp\left(\pi\sqrt{22}\right) - 24 + 4372\exp\left(-\pi\sqrt{22}\right) + 4096\exp\left(-\pi\sqrt{22}\right) = -24 + 8468\exp\left(-\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (22 - z_0)^k z_0^{-k}}{k!}\right) + \exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k (22 - z_0)^k z_0^{-k}}{k!}\right) \quad \text{for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

and:

64[(1+sqrt2)^12+(1-sqrt2)^12]

Input:

$64\left(\left(1+\sqrt{2}\right)^{12} + \left(1-\sqrt{2}\right)^{12}\right)$

Result:

2508928

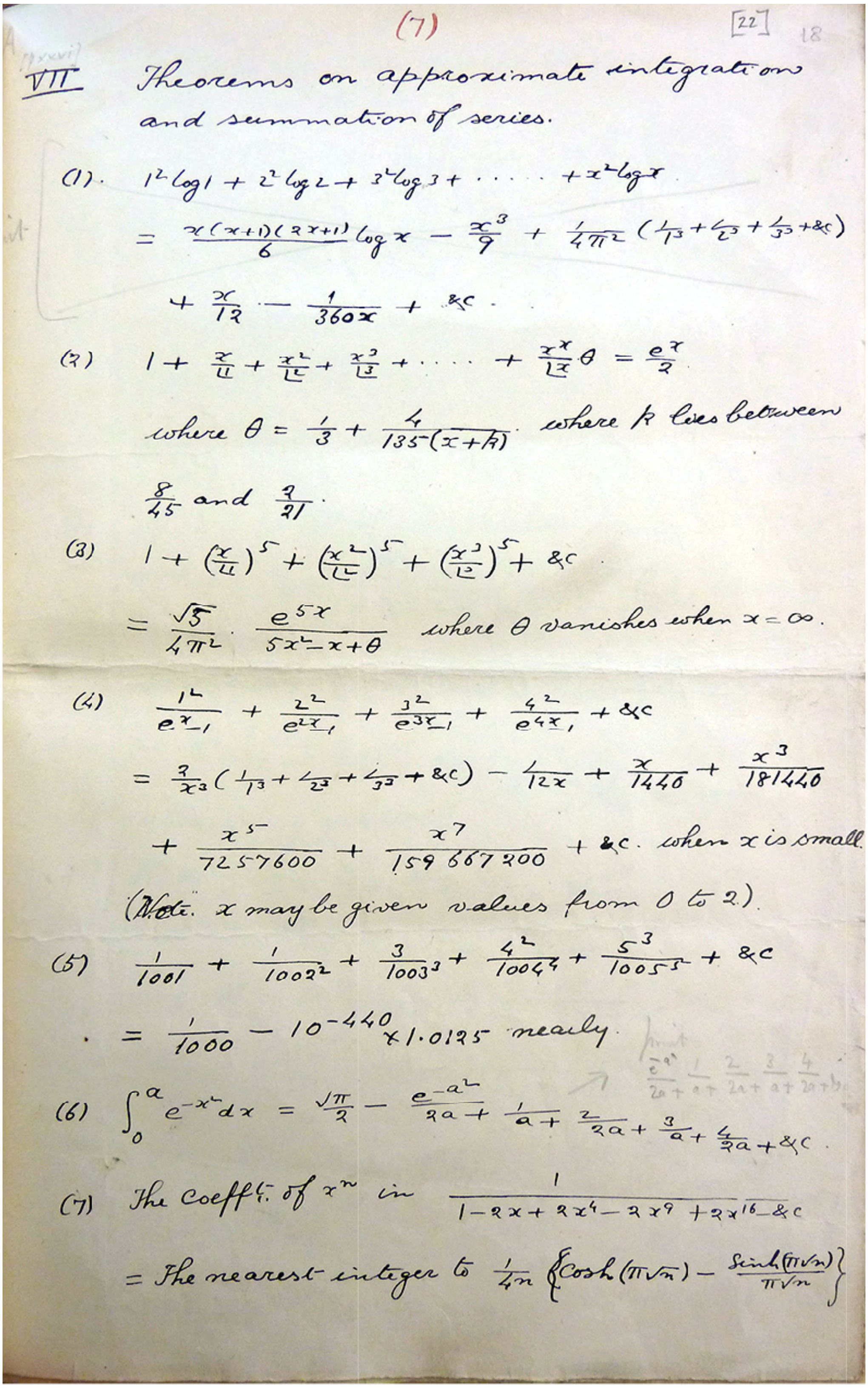
2508928

Note that:

2508928 / (64*2) = 19601; 19601*2 = 2508928; 2508928 / 39202 = 64

Now, we have that:

https://writings.stephenwolfram.com/2016/04/who-was-ramanujan/


(7) [22] 18

VII Theorems on approximate integration and summation of series.

(1). $1^2\log 1 + 2^2\log 2 + 3^2\log 3 + \cdots\cdots + x^2\log x$

$$= \frac{x(x+1)(2x+1)}{6}\log x - \frac{x^3}{9} + \frac{1}{4\pi^2}\left(\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\&c\right) + \frac{x}{12} - \frac{1}{360x} + \&c.$$

(2) $1 + \frac{x}{\lfloor 1} + \frac{x^2}{\lfloor 2} + \frac{x^3}{\lfloor 3} + \cdots\cdots + \frac{x^x}{\lfloor x}\theta = \frac{e^x}{2}$

where $\theta = \frac{1}{3} + \frac{4}{135(x+k)}$ where k lies between $\frac{8}{45}$ and $\frac{2}{21}$.

(3) $1 + \left(\frac{x}{\lfloor 1}\right)^5 + \left(\frac{x^2}{\lfloor 2}\right)^5 + \left(\frac{x^3}{\lfloor 3}\right)^5 + \&c$

$$= \frac{\sqrt{5}}{4\pi^2}\cdot\frac{e^{5x}}{5x^2 - x + \theta}$$ where θ vanishes when $x = \infty$.

(4) $\frac{1^2}{e^x - 1} + \frac{2^2}{e^{2x}-1} + \frac{3^2}{e^{3x}-1} + \frac{4^2}{e^{4x}-1} + \&c$

$$= \frac{2}{x^3}\left(\frac{1}{1^3}+\frac{1}{2^3}+\frac{1}{3^3}+\&c\right) - \frac{1}{12x} + \frac{x}{1440} + \frac{x^3}{181440} + \frac{x^5}{7257600} + \frac{x^7}{159667200} + \&c.$$ when x is small

(Note. x may be given values from 0 to 2).

(5) $\frac{1}{1001} + \frac{1}{1002^2} + \frac{3}{1003^3} + \frac{4^2}{1004^4} + \frac{5^3}{1005^5} + \&c$

$$= \frac{1}{1000} - 10^{-440}\times 1.0125 \text{ nearly}.$$

(6) $$\int_0^a e^{-x^2}dx = \frac{\sqrt{\pi}}{2} - \cfrac{e^{-a^2}}{2a + \cfrac{1}{a + \cfrac{2}{2a + \cfrac{3}{a + \cfrac{4}{2a + \&c}}}}}$$

(7) The coeff^t. of x^n in $\frac{1}{1 - 2x + 2x^4 - 2x^9 + 2x^{16} - \&c}$

$$= \text{The nearest integer to } \frac{1}{4n}\left\{\cosh(\pi\sqrt{n}) - \frac{\sinh(\pi\sqrt{n})}{\pi\sqrt{n}}\right\}$$

From (3), we have the following expression :

(sqrt5) / (4Pi^2) * ((exp(10))) / ((20-2+(Pi)))

Input:

$$\frac{\sqrt{5}}{4\pi^2} \times \frac{\exp(10)}{20-2+\pi}$$

Exact result:

$$\frac{\sqrt{5}\,e^{10}}{4\pi^2\,(18+\pi)}$$

Decimal approximation:

59.010919756724899356385867378965090454794490595851845158 81...

59.010919756…

Alternate forms:

$$\frac{e^{10}\sqrt{5}}{4\pi^2\,(18+\pi)}$$

$$\frac{\sqrt{5}\,e^{10}}{72\pi^2} - \frac{\sqrt{5}\,e^{10}}{1296\pi} + \frac{\sqrt{5}\,e^{10}}{1296\,(18+\pi)}$$

Series representations:

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)\left(4\pi^2\right)} = \frac{\exp(10)\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{4\pi^2\,(18+\pi)}$$

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)\left(4\pi^2\right)} = \frac{\exp(10)\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}{4\pi^2\,(18+\pi)}$$

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)\left(4\pi^2\right)} = \frac{\exp(10)\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\pi^2\,(18+\pi)\sqrt{\pi}}$$

From which:

(sqrt5) / (4Pi^2) * ((exp(10))) / ((20-2+(Pi)))+5

Input:

$$\frac{\sqrt{5}}{4\pi^2} \times \frac{\exp(10)}{20-2+\pi} + 5$$

Exact result:

$$5 + \frac{\sqrt{5}\, e^{10}}{4\pi^2 (18+\pi)}$$

Decimal approximation:

64.01091975672489935638586737896509045479449059585184515881...

64.0109197…. ≈ 64

Alternate forms:

$$\frac{e^{10}\sqrt{5}}{4\pi^2 (18+\pi)} + 5$$

$$\frac{\sqrt{5}\, e^{10} + 20\pi^2 (18+\pi)}{4\pi^2 (18+\pi)}$$

$$\frac{\sqrt{5}\, e^{10} + 360\pi^2 + 20\pi^3}{4\pi^2 (18+\pi)}$$

Series representations:

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)(4\pi^2)} + 5 = 5 + \frac{\exp(10)\sqrt{4}\sum_{k=0}^{\infty} 4^{-k}\binom{\frac{1}{2}}{k}}{4\pi^2 (18+\pi)}$$

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)(4\pi^2)} + 5 = 5 + \frac{\exp(10)\sqrt{4}\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}{4\pi^2 (18+\pi)}$$

$$\frac{\exp(10)\sqrt{5}}{(20-2+\pi)(4\pi^2)} + 5 = 5 + \frac{\exp(10)\sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 4^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}{8\pi^2 (18+\pi)\sqrt{\pi}}$$

From:

FIELD THEORY. A MODERN PRIMER - *Pierre Ramond* - (Florida U. and Caltech) - Jan 1, 1981 - 329 pages - Published in: *Front.Phys.* 51 (1981) 1-397, *Front.Phys.* 74 (1989) 1-329

We have that:

$$V(v) = -\frac{\mu^4}{32\pi^2}\frac{d}{ds}\left\{\frac{1}{(s-2)(s-1)}\left(\frac{m^2+\frac{\lambda}{2}v^2}{\mu^2}\right)^{2-s}\right\}\Bigg|_{s=0}$$
$$= \frac{1}{64\pi^2}\left[m^2+\frac{\lambda}{2}v^2\right]^2\left(-\frac{3}{2}+\ln\frac{m^2+\frac{\lambda}{2}v^2}{\mu^2}\right) . \qquad (3.5.21\text{-}3.5.22)$$

We place: m = 0.510998950 MeV; v = 2.189e+6 ; (parameters of an electron); μ = 2.

The value for the Compton wavelength of the <u>electron</u> is $2.4263102367(11)\times10^{-12}$ m , thence: $\lambda = 2.4263102367\times10^{-12}$. We obtain:

1/(64*Pi^2) [(((0.510998950^2+(2.426310e-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln(0.510998950^2+(2.426310e-12)1/2(2.189e+6)^2)/4)))

Input interpretation:

$$\frac{1}{64\pi^2}\left(0.510998950^2+2.426310\times10^{-12}\times\frac{1}{2}\left(2.189\times10^6\right)^2\right)^2$$
$$\left(-\frac{3}{2}+\frac{1}{4}\log\left(0.510998950^2+2.426310\times10^{-12}\times\frac{1}{2}\left(2.189\times10^6\right)^2\right)\right)$$

log(x) is the natural logarithm

Result:

-0.0612733...

-0.0612733…

Or, for m = 1/√2

1/(64*Pi^2) [((((1/(sqrt2))^2+(2.426310e-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln((1/(sqrt2))^2+(2.426310e-12)1/2(2.189e+6)^2)/4)))

Input interpretation:

$$\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2+2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2$$
$$\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

-0.0655790...

-0.0655790…

Or, for m = 1/√2 and λ = √6 × 10^{-12}

1/(64*Pi^2) [((((1/(sqrt2))^2+((sqrt6)*10^-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln((1/(sqrt2))^2+((sqrt6)*10^-12)1/2(2.189e+6)^2)/4)))

Input interpretation:

$$\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{\sqrt{6}}{10^{12}}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2$$
$$\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{\sqrt{6}}{10^{12}}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

-0.0665972...

-0.0665972…

Note that:

1-10(((1/(64*Pi^2) [(((0.510998950^2+(2.426310e-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln(0.510998950^2+(2.426310e-12)1/2(2.189e+6)^2)/4))))))+5/10^3

Input interpretation:

$$1 - 10\left(\frac{1}{64\pi^2}\left(0.510998950^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(0.510998950^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)\right)+\frac{5}{10^3}$$

$\log(x)$ is the natural logarithm

Result:

1.617733019588095810174418736284546640738267998923496151589...

1.617733019...

1-10((((1/(64*Pi^2) [((((1/(sqrt2))^2+(2.426310e-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln((1/(sqrt2))^2+(2.426310e-12)1/2(2.189e+6)^2)/4)))))))

Input interpretation:

$$1 - 10\left(\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)\right)$$

$\log(x)$ is the natural logarithm

Result:

1.655789818338243540800340589322645293439947652974754739030...

1.655789818338...

We note that, the result 1,655789818338... is practically equal to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164{,}2696$ i.e. 1,65578...

Indeed:

$$\sqrt[14]{\left(\sqrt{\frac{113+5\sqrt{505}}{8}}+\sqrt{\frac{105+5\sqrt{505}}{8}}\right)^3} = 1{,}65578\ldots \Rightarrow$$

$$\Rightarrow \left[1 - 10\left(\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2 + 2.426310\times 10^{-12}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)\right)\right] = 1.655789818338\ldots$$

or, for m = 1/√2; v = 2.189 and λ = 2.426310, μ = 2 we obtain:

1-10((((1/(64*Pi^2) [((((1/(sqrt2))^2+(2.426310)1/2(2.189)^2)))]^2 (((-3/2+ln((1/(sqrt2))^2+(2.426310)1/2(2.189)^2)/4)))))))

Input interpretation:

$$1-10\left(\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2+2.426310\times\frac{1}{2}\times 2.189^2\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+2.426310\times\frac{1}{2}\times 2.189^2\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1.655789818338243540800340589322645293439947652974754739030...

1.655789818338…. result equal to the previous

Alternative representations:

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=1-\frac{10\left(-\frac{3}{2}+\frac{1}{4}\log_e\left(1.21316\times 2.189^2+\left(\frac{1}{\sqrt{2}}\right)^2\right)\right)\left(1.21316\times 2.189^2+\left(\frac{1}{\sqrt{2}}\right)^2\right)^2}{64\pi^2}$$

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=1-\frac{10\left(-\frac{3}{2}+\frac{1}{4}\log(a)\log_a\left(1.21316\times 2.189^2+\left(\frac{1}{\sqrt{2}}\right)^2\right)\right)\left(1.21316\times 2.189^2+\left(\frac{1}{\sqrt{2}}\right)^2\right)^2}{64\pi^2}$$

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=1-\frac{10\left(-\frac{3}{2}-\frac{1}{4}\text{Li}_1\left(1-1.21316\times 2.189^2-\left(\frac{1}{\sqrt{2}}\right)^2\right)\right)\left(1.21316\times 2.189^2+\left(\frac{1}{\sqrt{2}}\right)^2\right)^2}{64\pi^2}$$

Series representations:

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=1-$$

$$\frac{1}{128\pi^2}\,5\left(-6+\log\left(5.8131+\frac{1}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}\right)\right)$$

$$\left(5.8131+\frac{1}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}\right)^2$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=$$

$$1-\frac{1}{32\pi^2}\,5\left(5.8131+\frac{1}{\exp^2\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}\right)^2$$

$$\left(-\frac{3}{2}+\frac{1}{4}\left(\log\left(4.8131+\frac{1}{\sqrt{2}^2}\right)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(4.8131+\frac{1}{\sqrt{2}^2}\right)^{-k}}{k}\right)\right)$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=$$

$$1-\frac{1}{32\pi^2}\,5\left(-\frac{3}{2}+\frac{1}{4}\log\left(5.8131+\frac{\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}\right)\right)$$

$$\left(5.8131+\frac{\left(\frac{1}{z_0}\right)^{-\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{-1-\lfloor\arg(2-z_0)/(2\pi)\rfloor}}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^2}\right)^2$$

Integral representations:

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=$$

$$1-\frac{1.32001\left(-6+\int_1^{5.8131+\frac{1}{\sqrt{2}^2}}\frac{1}{t}\,dt\right)\left(0.172025+\sqrt{2}^2\right)^2}{\pi^2\sqrt{2}^4}$$

$$1-\frac{10\left(\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{2.42631\times 2.189^2}{2}\right)\right)\right)}{64\pi^2}=1+$$

$$\frac{1}{i\pi^3\sqrt{2}^4}\,7.92003\left(i\pi-0.0833333\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(-s)^2\,\Gamma(1+s)\left(4.8131+\frac{1}{\sqrt{2}^2}\right)^{-s}}{\Gamma(1-s)}\,ds\right)\left(0.172025+\sqrt{2}^2\right)^2 \quad \text{for } -1<\gamma<0$$

1-10(((1/(64*Pi^2) [((((1/(sqrt2))^2+((sqrt6)*10^-12)1/2(2.189e+6)^2)))]^2 (((-3/2+ln((1/(sqrt2))^2+((sqrt6)*10^-12)1/2(2.189e+6)^2)/4))))))

Input interpretation:

$$1-10\left(\frac{1}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{\sqrt{6}}{10^{12}}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)^2\left(-\frac{3}{2}+\frac{1}{4}\log\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{\sqrt{6}}{10^{12}}\times\frac{1}{2}\left(2.189\times 10^6\right)^2\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1.665972368708553632214947030278573018425256385981698744852...

1.6659723687… result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

Now, we have:

$$V(\phi_{cl}) = \frac{1}{2}m^2\phi_{cl}^2(\bar{x}) + \frac{\lambda}{4!}\phi_{cl}^4(\bar{x}) + \frac{\hbar}{64\pi^2}\left(m^2 + \frac{\lambda}{2}\phi_{cl}^2\right)^2 \times \left[-\frac{3}{2} + \ln\frac{m^2 + \frac{\lambda}{2}\phi_{cl}^2}{\mu^2}\right] + \mathcal{O}(\hbar^2) \ . \tag{3.5.23}$$

For:

m = 1/√2; ϕ_{cl}^2 = 2.189 and λ = 2.426310, μ = 2 , we obtain:

1/2*(1/(sqrt2))^2*2.189^2+(2.426310*2.189^4)/4!+(((1.054571e-34)/(64Pi^2)))*((((1/(sqrt2))^2+1/2(2.426310*2.189^2))))^2*[-3/2+ln((((1/(sqrt2))^2+(2.426310)1/2(2.189)^2)/4))]+(1.054571e-34)^2

Input interpretation:

$$\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2 \times 2.189^2 + \frac{2.426310 \times 2.189^4}{4!} + \frac{1.054571\times10^{-34}}{64\,\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{2}\left(2.426310\times2.189^2\right)\right)^2 \left(-\frac{3}{2} + \log\left(\frac{1}{4}\left(\left(\frac{1}{\sqrt{2}}\right)^2 + 2.426310\times\frac{1}{2}\times2.189^2\right)\right)\right) + \left(1.054571\times10^{-34}\right)^2$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

3.519159811127093196249999999999999993055457635065936866243...

3.519159811127...

From which:

13/(9Pi) [1/2*(1/(sqrt2))^2*2.189^2+(2.426310*2.189^4)/4!+(((1.054571e-34)/(64Pi^2)))((((1/(sqrt2))^2+1/2(2.426310*2.189^2))))^2[-3/2+ln((((1/(sqrt2))^2+(2.426310)1/2(2.189)^2)/4))]+(1.054571e-34)^2]

Input interpretation:

$$\frac{13}{9\pi}\left(\frac{1}{2}\left(\frac{1}{\sqrt{2}}\right)^2\times 2.189^2+\frac{2.426310\times 2.189^4}{4!}+\frac{1.054571\times 10^{-34}}{64\pi^2}\left(\left(\frac{1}{\sqrt{2}}\right)^2+\frac{1}{2}\left(2.426310\times 2.189^2\right)\right)^2\left(-\frac{3}{2}+\log\left(\frac{1}{4}\left(\left(\frac{1}{\sqrt{2}}\right)^2+2.426310\times\frac{1}{2}\times 2.189^2\right)\right)\right)+\left(1.054571\times 10^{-34}\right)^2\right)$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.618042629583517672521255706910259951864433838493317445252...

1.618042629583…

We have this wonderful Ramanujan formula for the golden ratio, that is a fundamental constant of various fields of mathematics and physics

Input interpretation:

$$\sqrt[5]{\frac{1}{\left(\frac{1}{32}\left(-1+\sqrt{5}\right)^5+5\,e^{\left(-\sqrt{5}\,\pi\right)^5}\right)+\frac{1.6382898797095665677239458827012056245798314722584}{10^{7429}}}}$$

Result:

1.618033988749894848204586834365638117720309179805762862135...

1.6180339887…

We have that:

$$\ln \frac{\lambda M^2}{2\mu^2} = -\frac{8}{3} \ , \tag{3.5.26}$$

as seen by differentiating (3.5.23), setting $m^2 = 0$ and using (3.5.25). Thus we can eliminate $2\mu^2/\lambda$ in favor of M^2 and express the result as

$$V(\phi_{cl}) = \frac{\lambda}{4!}\phi_{cl}^4 + \frac{\lambda^2\phi_{cl}^4}{256\pi^2}\left[\ln\frac{\phi_{cl}^2}{M^2} - \frac{25}{6}\right] \ , \tag{3.5.27}$$

in accordance with the result of S. Coleman and E. Weinberg, *Phys. Rev.* **D7**, 1888 (1973). This little exercise shows that we must carefully define the input parameters in the Lagrangian in order to handle the quantum corrections. The result (3.5.27) still seems to depend on one arbitrary scale M^2 but it really does not, because, given the normalization condition, if we change the scale from M^2 to M'^2 we have to change at the same time λ to λ', where

$$\lambda' = \lambda + \frac{3\lambda^2}{16\pi^2}\ln\frac{M'}{M} \tag{3.5.28}$$

using (3.5.25). We see that the potential

$$V(\phi_{cl}) = \frac{\lambda'}{4!}\phi_{cl}^4 + \frac{\lambda'^2\phi_{cl}^4}{256\pi^2}\left[\ln\frac{\phi_{cl}^2}{M'^2} - \frac{25}{6}\right] + O(\lambda^3) \tag{3.5.29}$$

For $m = 1/\sqrt{2}$; $\phi_{cl}^2 = 2.189$ and $\lambda = 2.426310$, $\mu = 2$, $M^2 = 2\mu^2/\lambda = 8/2.426310$

we obtain, from (3.5.27):

$$V(\phi_{cl}) = \frac{\lambda}{4!}\phi_{cl}^4 + \frac{\lambda^2\phi_{cl}^4}{256\pi^2}\left[\ln\frac{\phi_{cl}^2}{M^2} - \frac{25}{6}\right] \ ,$$

((2.426310*(2.189)^2))/4! + ((2.426310^2*(2.189)^2))/(256*Pi^2)*[ln((2.189)/(8/2.426310))-25/6]

nput interpretation:

$$\frac{2.426310\times 2.189^2}{4!} + \frac{2.426310^2\times 2.189^2}{256\,\pi^2}\left(\log\left(\frac{2.189}{\frac{8}{2.426310}}\right) - \frac{25}{6}\right)$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

0.433332413042619057720972968666320218383725566400251085454...

0.4333324130426…

Alternative representations:

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}=\frac{2.42631\times 2.189^2}{\Gamma(5)}+\frac{2.189^2\times 2.42631^2\left(\log(a)\log_a\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\,\pi^2}$$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}=\frac{2.42631\times 2.189^2}{3!!\times 4!!}+\frac{2.189^2\times 2.42631^2\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\,\pi^2}$$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}=\frac{2.42631\times 2.189^2}{(1)_4}+\frac{2.189^2\times 2.42631^2\left(\log_e\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\,\pi^2}$$

Series representations:

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}=-\frac{0.459127}{\pi^2}+\frac{0.11019\log(0.663899)}{\pi^2}+\frac{11.6262}{\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0\notin\mathbb{Z}$ or $n_0\geq 0)$ and $n_0\to 4)$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$\left(11.6262\left(\pi^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}\,(-0.336101)^{k_1}\,(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)\text{ for }((n_0\notin\mathbb{Z}\text{ or }n_0\geq 0)\text{ and }n_0\to 4)$$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$\left(11.6262\left(\pi^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.00947777\sum_{j_1=1}^{\infty}\sum_{j_2=0}^{\infty}\frac{\left(\operatorname{Res}_{s=-j_1}\frac{(-0.336101)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\right)(4-n_0)^{j_2}\,\Gamma^{(j_2)}(1+n_0)}{j_2!}\right)\right)/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)\text{ for }((n_0\notin\mathbb{Z}\text{ or }n_0\geq 0)\text{ and }n_0\to 4)$$

Integral representations:

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=-\frac{0.459127}{\pi^2}+\frac{11.6262}{\int_0^\infty e^{-t}\,t^4\,dt}+\frac{0.11019\log(0.663899)}{\pi^2}$$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=-\frac{0.459127}{\pi^2}+\frac{11.6262}{\int_0^1\log^4\left(\frac{1}{t}\right)dt}+\frac{0.11019\log(0.663899)}{\pi^2}$$

$$\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=-\frac{0.459127}{\pi^2}+\frac{0.11019\log(0.663899)}{\pi^2}+\frac{11.6262}{\int_1^\infty e^{-t}\,t^4\,dt+\sum_{k=0}^\infty\frac{(-1)^k}{(5+k)k!}}$$

From which, we obtain also:

4((((2.426310*(2.189)^2))/4! + ((2.426310^2*(2.189)^2))/(256*Pi^2)*[ln((2.189)/(8/2.426310))-25/6]))

Input interpretation:

$$4\left(\frac{2.426310\times 2.189^2}{4!}+\frac{2.426310^2\times 2.189^2}{256\,\pi^2}\left(\log\left(\frac{2.189}{\frac{8}{2.426310}}\right)-\frac{25}{6}\right)\right)$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.733329652170476230883891874665280873534902265601004341819...

1.7333296521704…

1.7333… ≈ √3

Possible closed form:

$\sqrt{3} \approx 1.7320508$

Alternative representations:

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}\right)=$$

$$4\left(\frac{2.42631\times 2.189^2}{\Gamma(5)}+\frac{2.189^2\times 2.42631^2\left(\log(a)\log_a\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)}{256\,\pi^2}\right)$$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}\right)=$$

$$4\left(\frac{2.42631\times 2.189^2}{3!!\times 4!!}+\frac{2.189^2\times 2.42631^2\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\,\pi^2}\right)$$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}\right)=$$

$$4\left(\frac{2.42631\times 2.189^2}{(1)_4}+\frac{2.189^2\times 2.42631^2\left(\log_e\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\,\pi^2}\right)$$

Series representations:

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}\right)=$$

$$-\frac{1.83651}{\pi^2}+\frac{0.440762\log(0.663899)}{\pi^2}+\frac{46.5048}{\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}}$$

for $((n_0\notin\mathbb{Z}$ or $n_0\geq 0)$ and $n_0\to 4)$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}\right)=$$

$$\left(46.5048\left(\pi^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}\,(-0.336101)^{k_1}\,(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)$$

for $((n_0\notin\mathbb{Z}$ or $n_0\geq 0)$ and $n_0\to 4)$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}\right)=$$

$$\left(46.5048\left(\pi^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.00947777\sum_{j_1=1}^{\infty}\sum_{j_2=0}^{\infty}\frac{\left(\operatorname{Res}_{s=-j_1}\frac{(-0.336101)^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\right)(4-n_0)^{j_2}\,\Gamma^{(j_2)}(1+n_0)}{j_2!}\right)\right)\Bigg/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)\text{ for }((n_0\notin\mathbb{Z}\text{ or }n_0\geq 0)\text{ and }n_0\to 4)$$

Integral representations:

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}\right)=$$

$$-\frac{1.83651}{\pi^2}+\frac{46.5048}{\int_0^\infty e^{-t}\,t^4\,dt}+\frac{0.440762\log(0.663899)}{\pi^2}$$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}\right)=$$

$$-\frac{1.83651}{\pi^2}+\frac{46.5048}{\int_0^1\log^4\left(\frac{1}{t}\right)dt}+\frac{0.440762\log(0.663899)}{\pi^2}$$

$$4\left(\frac{2.42631\times 2.189^2}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}\right)=$$

$$-\frac{1.83651}{\pi^2}+\frac{0.440762\log(0.663899)}{\pi^2}+\frac{46.5048}{\int_1^\infty e^{-t}\,t^4\,dt+\sum_{k=0}^{\infty}\frac{(-1)^k}{(5+k)k!}}$$

We remember that

$$A = l^2 \text{ x } \varphi$$

The AREA of a REGULAR POLYGON is given by the PRODUCT between the SQUARE of the SIDE size and the CONSTANT of the considered polygon. We define this constant φ

consequently

$$l = \sqrt{\frac{A}{\varphi}}$$

Now, we know that φ = 0.433 for an equilateral triangle. If we have a side of √3/2 , we obtain:

(sqrt(3/2))^2 * 0.433

Input:

$$\sqrt{\frac{3}{2}}^2 \times 0.433$$

Result:

0.6495

0.6495

Thence, in the previous formula, we obtain:

(sqrt(3/2))^2 * ((2.426310*(2.189)^2))/4! + ((2.426310^2*(2.189)^2))/(256*Pi^2)*[ln((2.189)/(8/2.426310))-25/6]

Input interpretation:

$$\sqrt{\frac{3}{2}}^2 \times \frac{2.426310 \times 2.189^2}{4!} + \frac{2.426310^2 \times 2.189^2}{256\,\pi^2}\left(\log\left(\frac{2.189}{\frac{8}{2.426310}}\right) - \frac{25}{6}\right)$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

0.6755449251157440577209729686663202183837255664002510854 54...

0.675544925115744…

Alternative representations:

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\pi^2} = \frac{2.189^2 \times 2.42631^2 \left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)}{256\pi^2} + \frac{2.42631 \times 2.189^2 \sqrt{\frac{3}{2}}^2}{(1)_4}$$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\pi^2} = \frac{2.189^2 \times 2.42631^2 \left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)}{256\pi^2} + \frac{2.42631 \times 2.189^2 \sqrt{\frac{3}{2}}^2}{3!! \times 4!!}$$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\pi^2} = \frac{2.189^2 \times 2.42631^2 \left(\log_e\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)}{256\pi^2} + \frac{2.42631 \times 2.189^2 \sqrt{\frac{3}{2}}^2}{(1)_4}$$

Series representations:

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\pi^2} = \left(11.6262\left(\pi^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^2 - 0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!} - 0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right) \Bigg/ \left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right) \text{ for } ((n_0 \notin \mathbb{Z} \text{ or } n_0 \geq 0) \text{ and } n_0 \to 4)$$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256 \pi^2} =$$

$$\left(11.6262 \left(\pi^2 \exp^2\left(i \pi \left\lfloor \frac{\arg\left(\frac{3}{2} - x\right)}{2 \pi} \right\rfloor\right) \sqrt{x}^2 \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{3}{2} - x\right)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right)^2 - 0.0394907 \sum_{k=0}^{\infty} \frac{(4 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - 0.00947777 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} (-0.336101)^{k_1} (4 - n_0)^{k_2} \Gamma^{(k_2)}(1 + n_0)}{k_2! \, k_1}\right)\right) / \left(\pi^2 \sum_{k=0}^{\infty} \frac{(4 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!}\right)$$

for $(x \in \mathbb{R}$ and $(n_0 \notin \mathbb{Z}$ or $n_0 \geq 0)$ and $x < 0$ and $n_0 \to 4)$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256 \pi^2} =$$

$$\left(11.6262 \left(\pi^2 \left(\frac{1}{z_0}\right)^{\lfloor \arg\left(\frac{3}{2} - z_0\right)/(2 \pi) \rfloor} z_0^{1 + \lfloor \arg\left(\frac{3}{2} - z_0\right)/(2 \pi) \rfloor} \left(\sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k \left(\frac{3}{2} - z_0\right)^k z_0^{-k}}{k!}\right)^2 - 0.0394907 \sum_{k=0}^{\infty} \frac{(4 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!} - 0.00947777 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1} (-0.336101)^{k_1} (4 - n_0)^{k_2} \Gamma^{(k_2)}(1 + n_0)}{k_2! \, k_1}\right)\right) / \left(\pi^2 \sum_{k=0}^{\infty} \frac{(4 - n_0)^k \Gamma^{(k)}(1 + n_0)}{k!}\right)$$

for $((n_0 \notin \mathbb{Z}$ or $n_0 \geq 0)$ and $n_0 \to 4)$

Integral representations:

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256 \pi^2} =$$

$$-\frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_0^{\infty} e^{-t} t^4 \, dt}$$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} = -\frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_0^1 \log^4\left(\frac{1}{t}\right) dt}$$

$$\frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} = -\frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_1^\infty e^{-t}\, t^4\, dt + \sum_{k=0}^\infty \frac{(-1)^k}{(5+k)k!}}$$

Adding the value of the following spectral index $n_s = 0.965 \pm 0.004$, consistent with the predictions of slow-roll, single-field, inflation, we obtain:

0.965+(sqrt(3/2))^2 * ((2.426310*(2.189)^2))/4! + ((2.426310^2*(2.189)^2))/(256*Pi^2)*[ln((2.189)/(8/2.426310))-25/6]

Input interpretation:

$$0.965 + \sqrt{\frac{3}{2}}^2 \times \frac{2.426310 \times 2.189^2}{4!} + \frac{2.426310^2 \times 2.189^2}{256\,\pi^2} \left(\log\left(\frac{2.189}{\frac{8}{2.426310}}\right) - \frac{25}{6}\right)$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.640544925115744057720972968666320218383725566400251085454...

1.640544925115744...

Alternative representations:

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} = 0.965 + \frac{2.189^2 \times 2.42631^2 \left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)}{256\,\pi^2} + \frac{2.42631 \times 2.189^2 \sqrt{\frac{3}{2}}^2}{(1)_4}$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$0.965+\frac{2.189^2\times 2.42631^2\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\pi^2}+\frac{2.42631\times 2.189^2\sqrt{\frac{3}{2}}^{2}}{3!!\times 4!!}$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$0.965+\frac{2.189^2\times 2.42631^2\left(\log_e\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)}{256\pi^2}+\frac{2.42631\times 2.189^2\sqrt{\frac{3}{2}}^{2}}{(1)_4}$$

Series representations:

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$\left(11.6262\left(\pi^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+\right.\right.$$
$$0.0830022\,\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-$$
$$\left.\left.0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/$$
$$\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)\text{ for }((n_0\notin\mathbb{Z}\text{ or }n_0\geq 0)\text{ and }n_0\to 4)$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=\left(11.6262\left(\pi^2\exp^2\left(i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.0830022\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)$$

for ($x \in \mathbb{R}$ and ($n_0 \notin \mathbb{Z}$ or $n_0 \geq 0$) and $x < 0$ and $n_0 \to 4$)

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=\left(11.6262\left(\pi^2\left(\frac{1}{z_0}\right)^{\left\lfloor\arg\left(\frac{3}{2}-z_0\right)/(2\pi)\right\rfloor}z_0^{1+\left\lfloor\arg\left(\frac{3}{2}-z_0\right)/(2\pi)\right\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.0830022\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)$$

for (($n_0 \notin \mathbb{Z}$ or $n_0 \geq 0$) and $n_0 \to 4$)

Integral representations:

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}=$$

$$0.965-\frac{0.459127}{\pi^2}+\frac{0.11019\log(0.663899)}{\pi^2}+\frac{11.6262\sqrt{\frac{3}{2}}^2}{\int_0^\infty e^{-t}\,t^4\,dt}$$

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} =$$

$$0.965 - \frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_0^1 \log^4\left(\frac{1}{t}\right) dt}$$

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} =$$

$$0.965 - \frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_1^\infty e^{-t}\, t^4\, dt + \sum_{k=0}^\infty \frac{(-1)^k}{(5+k)k!}}$$

0.965+(sqrt(3/2))^2 * ((2.426310*(2.189)^2))/4! + ((2.426310^2*(2.189)^2))/(256*Pi^2)*[ln((2.189)/(8/2.426310))-25/6]-(18+4)1/10^3

Input interpretation:

$$0.965 + \sqrt{\frac{3}{2}}^2 \times \frac{2.426310 \times 2.189^2}{4!} + \frac{2.426310^2 \times 2.189^2}{256\,\pi^2}\left(\log\left(\frac{2.189}{\frac{8}{2.426310}}\right) - \frac{25}{6}\right) - (18+4) \times \frac{1}{10^3}$$

$n!$ is the factorial function

$\log(x)$ is the natural logarithm

Result:

1.618544925115744057720972968666320218383725566400251085454...

1.618544925115744...

Alternative representations:

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} - \frac{18+4}{10^3} =$$

$$0.965 - \frac{22}{10^3} + \frac{2.189^2 \times 2.42631^2 \left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right) - \frac{25}{6}\right)}{256\,\pi^2} + \frac{2.42631 \times 2.189^2 \sqrt{\frac{3}{2}}^2}{(1)_4}$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{18+4}{10^3}=$$

$$0.965-\frac{22}{10^3}+\frac{2.189^2\times 2.42631^2\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)}{256\,\pi^2}+\frac{2.42631\times 2.189^2\sqrt{\frac{3}{2}}^{2}}{3!!\times 4!!}$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{18+4}{10^3}=$$

$$0.965-\frac{22}{10^3}+\frac{2.189^2\times 2.42631^2\left(\log_e\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)}{256\,\pi^2}+\frac{2.42631\times 2.189^2\sqrt{\frac{3}{2}}^{2}}{(1)_4}$$

Series representations:

$$0.965+\frac{\sqrt{\frac{3}{2}}^{2}(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{18+4}{10^3}=$$

$$\left(11.6262\left(\pi^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+\right.\right.$$
$$0.0811099\,\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-$$
$$\left.\left.0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/$$
$$\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)\text{ for }((n_0\notin\mathbb{Z}\text{ or }n_0\geq 0)\text{ and }n_0\to 4)$$

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}-\frac{18+4}{10^3}=\left(11.6262\left(\pi^2\exp^2\left(i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.0811099\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)$$

for ($x \in \mathbb{R}$ and ($n_0 \notin \mathbb{Z}$ or $n_0 \geq 0$) and $x < 0$ and $n_0 \to 4$)

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}-\frac{18+4}{10^3}=\left(11.6262\left(\pi^2\left(\frac{1}{z_0}\right)^{\left\lfloor\arg\left(\frac{3}{2}-z_0\right)/(2\pi)\right\rfloor}z_0^{1+\left\lfloor\arg\left(\frac{3}{2}-z_0\right)/(2\pi)\right\rfloor}\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{3}{2}-z_0\right)^k z_0^{-k}}{k!}\right)^2-0.0394907\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}+0.0811099\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}-0.00947777\sum_{k_1=1}^{\infty}\sum_{k_2=0}^{\infty}\frac{(-1)^{k_1}(-0.336101)^{k_1}(4-n_0)^{k_2}\,\Gamma^{(k_2)}(1+n_0)}{k_2!\,k_1}\right)\right)\Big/\left(\pi^2\sum_{k=0}^{\infty}\frac{(4-n_0)^k\,\Gamma^{(k)}(1+n_0)}{k!}\right)$$

for (($n_0 \notin \mathbb{Z}$ or $n_0 \geq 0$) and $n_0 \to 4$)

Integral representations:

$$0.965+\frac{\sqrt{\frac{3}{2}}^2(2.42631\times 2.189^2)}{4!}+\frac{\left(\log\left(\frac{2.189}{\frac{8}{2.42631}}\right)-\frac{25}{6}\right)(2.42631^2\times 2.189^2)}{256\pi^2}-\frac{18+4}{10^3}=0.943-\frac{0.459127}{\pi^2}+\frac{0.11019\log(0.663899)}{\pi^2}+\frac{11.6262\sqrt{\frac{3}{2}}^2}{\int_0^\infty e^{-t}\,t^4\,dt}$$

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} - \frac{18+4}{10^3} =$$

$$0.943 - \frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_0^1 \log^4\left(\frac{1}{t}\right) dt}$$

$$0.965 + \frac{\sqrt{\frac{3}{2}}^2 (2.42631 \times 2.189^2)}{4!} + \frac{\left(\log\left(\frac{\frac{2.189}{8}}{2.42631}\right) - \frac{25}{6}\right)(2.42631^2 \times 2.189^2)}{256\,\pi^2} - \frac{18+4}{10^3} =$$

$$0.943 - \frac{0.459127}{\pi^2} + \frac{0.11019 \log(0.663899)}{\pi^2} + \frac{11.6262 \sqrt{\frac{3}{2}}^2}{\int_1^\infty e^{-t}\, t^4\, dt + \sum_{k=0}^\infty \frac{(-1)^k}{(5+k)k!}}$$

Now, we have that:

5.4 EVALUATION OF FERMION DETERMINANTS

$$V(\phi_0) = \frac{\lambda^2}{256\pi^2}\phi_0^4\left[-\frac{3}{2} + \ln\frac{\lambda\phi_0^2}{2\mu^2}\right] - \frac{f^4}{8\pi^2}\phi_0^4\left[-\frac{3}{2} + \ln\frac{f^2\phi_0^2}{\mu^2}\right] \quad , \qquad (5.4.30)$$

where the first term is the same as in the pure scalar case – it comes from the boson loops; the second term comes from the contribution of the closed fermion loops to the potential, and the relative minus sign comes from the closed fermion loop.

For: m = 1/√2; ϕ_0^2 = 2.189 and λ = 2.426310, μ = 2 , $M^2 = 2\mu^2/\lambda$ = 8/2.426310;

f = 1; we obtain:

(2.426310^2*2.189^2) / (256*Pi^2) [(-3/2+ln((2.426310*2.189)/8))]-2.189^2/(8*Pi^2) ((-3/2+ln((2.189)/4)))

Input interpretation:

$$\frac{2.426310^2 \times 2.189^2}{256\,\pi^2}\left(-\frac{3}{2} + \log\left(\frac{2.426310 \times 2.189}{8}\right)\right) - \frac{2.189^2}{8\,\pi^2}\left(-\frac{3}{2} + \log\left(\frac{2.189}{4}\right)\right)$$

log(x) is the natural logarithm

Result:

0.106297...

0.106297…

Series representations:

$$\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}=\frac{0.733162}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(0.598965\,(-0.45275)^k-0.11019\,(-0.336101)^k\right)}{k\,\pi^2}$$

$$\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}=\frac{0.733162}{\pi^2}-\frac{1.19793\,i\left\lfloor\frac{\arg(0.54725-x)}{2\pi}\right\rfloor}{\pi}+\frac{0.220381\,i\left\lfloor\frac{\arg(0.663899-x)}{2\pi}\right\rfloor}{\pi}-\frac{0.488775\log(x)}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(0.598965\,(0.54725-x)^k-0.11019\,(0.663899-x)^k\right)x^{-k}}{k\,\pi^2}\quad\text{for } x<0$$

$$\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}=\frac{0.733162}{\pi^2}-\frac{1.19793\,i\left\lfloor-\frac{-\pi+\arg\left(\frac{0.54725}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor}{\pi}+\frac{0.220381\,i\left\lfloor-\frac{-\pi+\arg\left(\frac{0.663899}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor}{\pi}-\frac{0.488775\log(z_0)}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(0.598965\,(0.54725-z_0)^k-0.11019\,(0.663899-z_0)^k\right)z_0^{-k}}{k\,\pi^2}$$

Integral representation:

$$\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}=\frac{0.733162}{\pi^2}+\int_1^{0.54725}\frac{-0.20788-0.488775\,t}{\pi^2\,t\,(0.347066+t)}\,dt$$

We know that α' is the Regge slope (string tension). The following result of the meson fits in the (J,M^2) plane concerning Ψ is 0.979

Ψ	3	$m_c = 1500$	0.979	-0.09

Adding this value to the previous expression that we have multiplied by 6, we obtain:

0.979+6*(((((2.426310^2*2.189^2) / (256*Pi^2) [(-3/2+ln((2.426310*2.189)/8))]-2.189^2/(8*Pi^2) ((-3/2+ln((2.189)/4)))))))

Input interpretation:

$$0.979 + 6\left(\frac{2.426310^2\times 2.189^2}{256\,\pi^2}\left(-\frac{3}{2}+\log\left(\frac{2.426310\times 2.189}{8}\right)\right)-\frac{2.189^2}{8\,\pi^2}\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)\right)$$

log(x) is the natural logarithm

Result:

1.616783009631382268098391915940444706725487577306398451725...

1.616783009361...

Alternative representations:

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$
$$0.979+6\left(-\frac{\left(-\frac{3}{2}+\log_e\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}+\frac{\left(-\frac{3}{2}+\log_e\left(\frac{5.31119}{8}\right)\right)2.189^2\times 2.42631^2}{256\,\pi^2}\right)$$

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$
$$0.979+6\left(-\frac{\left(-\frac{3}{2}+\log(a)\log_a\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}+\frac{\left(-\frac{3}{2}+\log(a)\log_a\left(\frac{5.31119}{8}\right)\right)2.189^2\times 2.42631^2}{256\,\pi^2}\right)$$

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$

$$0.979+6\left(-\frac{\left(-\frac{3}{2}-\operatorname{Li}_1\left(1-\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}+\frac{\left(-\frac{3}{2}-\operatorname{Li}_1\left(1-\frac{5.31119}{8}\right)\right)2.189^2\times 2.42631^2}{256\,\pi^2}\right)$$

Series representations:

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$

$$0.979+\frac{4.39897}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(3.59379\,(-0.45275)^k-0.661143\,(-0.336101)^k\right)}{k\,\pi^2}$$

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$

$$0.979+\frac{4.39897}{\pi^2}-\frac{7.18758\,i\left\lfloor\frac{\arg(0.54725-x)}{2\pi}\right\rfloor}{\pi}+\frac{1.32229\,i\left\lfloor\frac{\arg(0.663899-x)}{2\pi}\right\rfloor}{\pi}-\frac{2.93265\log(x)}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(3.59379\,(0.54725-x)^k-0.661143\,(0.663899-x)^k\right)x^{-k}}{k\,\pi^2}\quad\text{for } x<0$$

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$

$$0.979+\frac{4.39897}{\pi^2}-\frac{7.18758\,i\left\lfloor-\frac{-\pi+\arg\left(\frac{0.54725}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor}{\pi}+\frac{1.32229\,i\left\lfloor-\frac{-\pi+\arg\left(\frac{0.663899}{z_0}\right)+\arg(z_0)}{2\pi}\right\rfloor}{\pi}-\frac{2.93265\log(z_0)}{\pi^2}+\sum_{k=1}^{\infty}\frac{(-1)^k\left(3.59379\,(0.54725-z_0)^k-0.661143\,(0.663899-z_0)^k\right)z_0^{-k}}{k\,\pi^2}$$

Integral representation:

$$0.979+6\left(\frac{\left(-\frac{3}{2}+\log\left(\frac{2.42631\times 2.189}{8}\right)\right)(2.42631^2\times 2.189^2)}{256\,\pi^2}-\frac{\left(-\frac{3}{2}+\log\left(\frac{2.189}{4}\right)\right)2.189^2}{8\,\pi^2}\right)=$$

$$0.979+\frac{4.39897}{\pi^2}+\int_1^{0.54725}\frac{-1.24728-2.93265\,t}{\pi^2\,t\,(0.347066+t)}\,dt$$

Now, we have that:

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \frac{m^4}{\lambda}\left[\cos\frac{\sqrt{\lambda}\phi}{m} - 1\right] \quad , \qquad (1.6.5)$$

For: m = 64√2; $\phi_0^2 = \phi = 2.189$ and λ = 2.426310, μ = 2 , $M^2 = 2\mu^2/\lambda$ = 8/2.426310;

1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1]

Input interpretation:

$$\frac{1}{2}\times 2.189^2 + \frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times 2.189}{64\sqrt{2}}\right) - 1\right)$$

Result:

−19622.2...

-19622.2…

Alternative representations:

$$\frac{2.189^2}{2} + \frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right) - 1\right)(64\sqrt{2})^4}{2.42631} = \frac{2.189^2}{2} + \frac{\left(-1 + \cosh\left(\frac{2.189\, i\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}$$

$$\frac{2.189^2}{2} + \frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right) - 1\right)(64\sqrt{2})^4}{2.42631} = \frac{2.189^2}{2} + \frac{\left(-1 + \cosh\left(-\frac{(2.189\, i)\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}$$

$$\frac{2.189^2}{2} + \frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right) - 1\right)(64\sqrt{2})^4}{2.42631} = \frac{2.189^2}{2} + \frac{\left(-1 + \frac{1}{2}\left(e^{-\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)} + e^{\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}\right)\right)(64\sqrt{2})^4}{2.42631}$$

Series representations:

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}=6.9147\times10^6$$
$$\left(3.46488\times10^{-7}-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.8645k}\left(\frac{1}{\sqrt{2}}\right)^{2k}}{(2k)!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}=6.9147\times10^6$$
$$\left(3.46488\times10^{-7}-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{0.0532769}{\sqrt{2}}-z_0\right)^k}{k!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}=-6.9147\times10^6$$
$$\left(-3.46488\times10^{-7}+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{0.0532769}{\sqrt{2}}\right)^{1+2k}}{(1+2k)!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

Integral representations:

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=$$
$$2.39586-368\,394.\ \sqrt{2}^3\int_0^1\sin\left(\frac{0.0532769\,t}{\sqrt{2}}\right)dt$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=$$
$$2.39586-6.9147\times10^6\sqrt{2}^4-6.9147\times10^6\sqrt{2}^4\int_{\frac{\pi}{2}}^{\frac{0.0532769}{\sqrt{2}}}\sin(t)\,dt$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=2.39586-6.9147\times10^6\sqrt{2}^4+$$
$$\frac{3.45735\times10^6\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-0.000709607/\left(s\sqrt{2}^2\right)}}{\sqrt{s}}\,ds\quad\text{for }\gamma>0$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=2.39586-6.9147\times10^6\sqrt{2}^4+$$
$$\frac{3.45735\times10^6\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{7.2508\,s}\,\Gamma(s)\left(\frac{1}{\sqrt{2}}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=$$
$$2.39586-1.38294\times10^7\sin^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=$$
$$2.39586+1.38294\times10^7\left(-1+\cos^2\left(\frac{0.0266384}{\sqrt{2}}\right)\right)\sqrt{2}^4$$

$$\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}=$$
$$2.39586+6.9147\times10^6\left(-1+\cos\left(\frac{0.017759}{\sqrt{2}}\right)\right)\left(1+2\cos\left(\frac{0.017759}{\sqrt{2}}\right)\right)^2\sqrt{2}^4$$

From which:

-1/11((((1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1])))))-55

Input interpretation:

$$-\frac{1}{11}\left(\frac{1}{2}\times 2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times 2.189}{64\sqrt{2}}\right)-1\right)\right)-55$$

Result:

1728.83...

1728.83…

Alternative representations:

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(\frac{2.189\, i\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(-\frac{(2.189\, i)\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\frac{1}{2}\left(e^{-\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}+e^{\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}\right)\right)(64\sqrt{2})^4}{2.42631}\right)$$

Series representations:

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\;2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=-628\,609.$$

$$\left(0.0000878412-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.8645k}\left(\frac{1}{\sqrt{2}}\right)^{2k}}{(2k)!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\;2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=-628\,609.$$

$$\left(0.0000878412-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{0.0532769}{\sqrt{2}}-z_0\right)^k}{k!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\;2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=628\,609.$$

$$\left(-0.0000878412+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{0.0532769}{\sqrt{2}}\right)^{1+2k}}{(1+2k)!}\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

Integral representations:

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55.2178+33490.4\sqrt{2}^3\int_0^1\sin\left(\frac{0.0532769\,t}{\sqrt{2}}\right)dt$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55.2178+628609.\sqrt{2}^4+628609.\sqrt{2}^4\int_{\frac{\pi}{2}}^{\frac{0.0532769}{\sqrt{2}}}\sin(t)\,dt$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=-55.2178+$$
$$628609.\sqrt{2}^4-\frac{314305.\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-0.000709607/\left(s\sqrt{2}^2\right)}}{\sqrt{s}}\,ds \quad \text{for } \gamma>0$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55.2178+628609.\sqrt{2}^4-$$
$$\frac{314305.\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{7.2508\,s}\,\Gamma(s)\left(\frac{1}{\sqrt{2}}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds \quad \text{for } 0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55.2178+1.25722\times10^6\sin^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55=$$
$$-55.2178+1.25722\times10^6\sqrt{2}^4-1.25722\times10^6\cos^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4$$

$$\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55 =$$

$$-55.2178 + 628\,609.\ \sqrt{2}^4 + 1.88583\times10^6 \cos\left(\frac{0.017759}{\sqrt{2}}\right)\sqrt{2}^4 -$$

$$2.51444\times10^6 \cos^3\left(\frac{0.017759}{\sqrt{2}}\right)\sqrt{2}^4$$

and:

((((-1/11(((((1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1])))))-55))))^1/15

Input interpretation:

$$\sqrt[15]{-\frac{1}{11}\left(\frac{1}{2}\times2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times2.189}{64\sqrt{2}}\right)-1\right)\right)-55}$$

Result:

1.643804696679158211483980921672874243681630633787502119696...

1.643804696679...

((((-1/11(((((1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1])))))-55))))^1/15-(21+5)1/10^3

Input interpretation:

$$\sqrt[15]{-\frac{1}{11}\left(\frac{1}{2}\times2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times2.189}{64\sqrt{2}}\right)-1\right)\right)-55}-(21+5)\times\frac{1}{10^3}$$

Result:

1.617804696679158211483980921672874243681630633787502119696...

1.617804696679…

Alternative representations:

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$
$$-\frac{26}{10^3}+\sqrt[15]{-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(\frac{2.189\,i\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$
$$-\frac{26}{10^3}+\sqrt[15]{-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(-\frac{(2.189\,i)\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$
$$-\frac{26}{10^3}+\sqrt[15]{-55-\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{\left(-1+\frac{1}{\sec\left(\frac{2.189\sqrt{2.42631}}{64\sqrt{2}}\right)}\right)(64\sqrt{2})^4}{2.42631}\right)}$$

Series representations:

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$
$$\frac{1}{500}\left(-13+500\left(-55.2178-628609.\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\left(-1+\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.8645k}\left(\frac{1}{\sqrt{2}}\right)^{2k}}{(2k)!}\right)\right)^\wedge(1/15)\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+500\left(-55.2178-628609.\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\left(-1+\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{0.0532769}{\sqrt{2}}-z_0\right)^k}{k!}\right)\right)^\wedge(1/15)\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$\frac{1}{500}\left(-13+500\left(-55.2178-628609.\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(-1+J_0\left(\frac{0.0532769}{\sqrt{2}}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{0.0532769}{\sqrt{2}}\right)\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)^\wedge(1/15)\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

Integral representations:

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{-55.2178+33490.4\sqrt{2}^3\int_0^1\sin\left(\frac{0.0532769\,t}{\sqrt{2}}\right)dt}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{-55.2178+628\,609.\sqrt{2}^4+628\,609.\sqrt{2}^4\int_{\frac{\pi}{2}}^{\frac{0.0532769}{\sqrt{2}}}\sin(t)\,dt}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\left(-55.2178+628\,609.\sqrt{2}^4-\frac{314\,305.\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-0.000709607/\left(s\sqrt{2}^2\right)}}{\sqrt{s}}\,ds\right)^{\wedge}(1/15)\quad\text{for }\gamma>0$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\left(-55.2178+628\,609.\sqrt{2}^4-\frac{314\,305.\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{7.2508\,s}\,\Gamma(s)\left(\frac{1}{\sqrt{2}}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\right)^{\wedge}(1/15)\quad\text{for }0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{-55.2178+1.25722\times10^6\sin^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{-55.2178+1.25722\times10^6\sqrt{2}^4-1.25722\times10^6\cos^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4}$$

$$\sqrt[15]{\frac{1}{11}\left(\frac{2.189^2}{2}+\frac{(64\sqrt{2})^4\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)}{2.42631}\right)(-1)-55} - \frac{21+5}{10^3} =$$

$$-\frac{13}{500}+\sqrt[15]{-55.2178+628609.\sqrt{2}^4-628609.\,T_{0.0532769}\left(\cos\left(\frac{1}{\sqrt{2}}\right)\right)\sqrt{2}^4}$$

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$:
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 279 , we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(279/15)) / (2*5^(1/4)*sqrt(279)) +123 -7

Input:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{279}{15}}\right)}{2\sqrt[4]{5}\sqrt{279}}+123-7$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{\sqrt{93/5}\,\pi}\sqrt{\frac{\phi}{31}}}{6\sqrt[4]{5}}+116$$

Decimal approximation:

19621.765125489367888371203513596437494619005720804345343 95...

19621.765125489…

Property:

$$116+\frac{e^{\sqrt{93/5}\,\pi}\sqrt{\frac{\phi}{31}}}{6\sqrt[4]{5}}$$ is a transcendental number

Alternate forms:

$$116+\frac{1}{6}\sqrt{\frac{1}{310}\left(5+\sqrt{5}\right)}\,e^{\sqrt{93/5}\,\pi}$$

$$116+\frac{\sqrt{\frac{1}{62}\left(1+\sqrt{5}\right)}\, e^{\sqrt{93/5}\,\pi}}{6\sqrt[4]{5}}$$

$$\frac{215\,760+5^{3/4}\sqrt{62\left(1+\sqrt{5}\right)}\, e^{\sqrt{93/5}\,\pi}}{1860}$$

Series representations:

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{279}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{279}}+123-7=\left(1160\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(279-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{93}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(279-z_0)^k z_0^{-k}}{k!}\right)\quad\text{for (not } (z_0\in\mathbb{R}\text{ and }-\infty<z_0\leq 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{279}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{279}}+123-7=\left(1160\exp\left(i\pi\left\lfloor\frac{\arg(279-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(279-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}+5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{93}{5}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{93}{5}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Bigg/\left(10\exp\left(i\pi\left\lfloor\frac{\arg(279-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(279-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\quad\text{for } (x\in\mathbb{R}\text{ and }x<0)$$

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{279}{15}}\right)}{2\sqrt[4]{5}\sqrt{279}}+123-7=$$

$$\left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(279-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(279-z_0)/(2\pi)\rfloor}\left(1160\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(279-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(279-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(279-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{93}{5}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{93}{5}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{93}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right)\Bigg/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(279-z_0)^k z_0^{-k}}{k!}\right)$$

Furthermore, we obtain also the following interesting expression:

-10((1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1]))+521+123+18

Input interpretation:

$$-10\left(\frac{1}{2}\times 2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times 2.189}{64\sqrt{2}}\right)-1\right)\right)+521+123+18$$

Result:

196883.7223574680154895292869871192649076656165029385963716...

196883.722357468…

196884 is a fundamental number of the following j-invariant

$$j(\tau)=q^{-1}+744+196884q+21493760q^2+864299970q^3+20245856256q^4+\cdots$$

(In mathematics, Felix Klein's j-invariant or j function, regarded as a function of a complex variable τ, is a modular function of weight zero for SL(2, Z) defined on the upper half plane of complex numbers. Several remarkable properties of j have to do with its q expansion (Fourier series expansion), written as a Laurent series in terms of $q = e^{2\pi i\tau}$ (the square of the nome), which begins:

$$j(\tau) = q^{-1} + 744 + 196884q + 21493760q^2 + 864299970q^3 + 20245856256q^4 + \cdots$$

Note that j has a simple pole at the cusp, so its q-expansion has no terms below q^{-1}.

All the Fourier coefficients are integers, which results in several almost integers, notably Ramanujan's constant:

$$e^{\pi\sqrt{163}} \approx 640320^3 + 744.$$

The asymptotic formula for the coefficient of q^n is given by

$$\frac{e^{4\pi\sqrt{n}}}{\sqrt{2}\,n^{3/4}},$$

as can be proved by the Hardy–Littlewood circle method)

Alternative representations:

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)+521+123+18 =$$

$$662-10\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(\frac{2.189\, i\sqrt{2.42631}}{64\sqrt{2}}\right)\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)+521+123+18 =$$

$$662-10\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(-\frac{(2.189\, i)\sqrt{2.42631}}{64\sqrt{2}}\right)\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)+521+123+18 =$$

$$662-10\left(\frac{2.189^2}{2}+\frac{\left(-1+\frac{1}{2}\left(e^{-\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}+e^{\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}\right)\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)$$

Series representations:

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=-6.9147\times10^7\left(-9.22731\times10^{-6}-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k e^{-5.8645k}\left(\frac{1}{\sqrt{2}}\right)^{2k}}{(2k)!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=-6.9147\times10^7\left(-9.22731\times10^{-6}-\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\left(\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right)+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{0.0532769}{\sqrt{2}}-z_0\right)^k}{k!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=6.9147\times10^7\left(9.22731\times10^{-6}+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4+\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{\pi}{2}+\frac{0.0532769}{\sqrt{2}}\right)^{1+2k}}{(1+2k)!}\right)\text{ for }(x\in\mathbb{R}\text{ and }x<0)$$

Integral representations:

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+3.68394\times10^6\sqrt{2}^3\int_0^1\sin\left(\frac{0.0532769\,t}{\sqrt{2}}\right)dt$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+6.9147\times10^7\sqrt{2}^4+6.9147\times10^7\sqrt{2}^4\int_{\frac{\pi}{2}}^{\frac{0.0532769}{\sqrt{2}}}\sin(t)\,dt$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+6.9147\times10^7\sqrt{2}^4-\frac{3.45735\times10^7\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-0.000709607/(s\sqrt{2}^2)}}{\sqrt{s}}\,ds\quad\text{for }\gamma>0$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+6.9147\times10^7\sqrt{2}^4-\frac{3.45735\times10^7\sqrt{2}^4\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{7.2508\,s}\,\Gamma(s)\left(\frac{1}{\sqrt{2}}\right)^{-2s}}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+1.38294\times10^8\sin^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$
$$638.041+1.38294\times10^8\sqrt{2}^4-1.38294\times10^8\cos^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4$$

$$-10\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)+521+123+18=$$

$$638.041+6.9147\times10^7\sqrt{2}^4+2.07441\times10^8\cos\left(\frac{0.017759}{\sqrt{2}}\right)\sqrt{2}^4-$$
$$2.76588\times10^8\cos^3\left(\frac{0.017759}{\sqrt{2}}\right)\sqrt{2}^4$$

and again:

(-Pi-1/golden ratio)((1/2*2.189^2+ (((64sqrt2))^4)/(2.426310) [cos((((2.426310)^0.5 * 2.189)/(64sqrt2)))-1]))-233-34-13

Input interpretation:

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{1}{2}\times2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times2.189}{64\sqrt{2}}\right)-1\right)\right)-233-34-13$$

ϕ is the golden ratio

Result:

73492.0...

73492

Alternative representations:

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13=$$
$$-280+\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(\frac{2.189\,i\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13=$$
$$-280+\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(-1+\cosh\left(-\frac{(2.189\,i)\sqrt{2.42631}}{64\sqrt{2}}\right)\right)(64\sqrt{2})^4}{2.42631}\right)$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13=$$

$$-280+\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(-1+\frac{1}{2}\left(e^{-\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}+e^{\left(2.189\, i\sqrt{2.42631}\right)/\left(64\sqrt{2}\right)}\right)\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)$$

Series representations:

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13=$$

$$-280+\left(-\frac{1}{\phi}-\pi\right)\left(2.39586+6.9147\times10^6\left(\frac{1}{z_0}\right)^{2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(-1+\sum_{k=0}^{\infty}\frac{(-1)^k\,0.0532769^{2k}\left(\frac{1}{\sqrt{2}}\right)^{2k}}{(2k)!}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^4\right)$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13=$$

$$-280+\left(-\frac{1}{\phi}-\pi\right)\left(2.39586+6.9147\times10^6\left(\frac{1}{z_0}\right)^{2\lfloor\arg(2-z_0)/(2\pi)\rfloor}z_0^{2(1+\lfloor\arg(2-z_0)/(2\pi)\rfloor)}\left(-1+\sum_{k=0}^{\infty}\frac{\cos\left(\frac{k\pi}{2}+z_0\right)\left(\frac{0.0532769}{\sqrt{2}}-z_0\right)^k}{k!}\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(2-z_0)^k z_0^{-k}}{k!}\right)^4\right)$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13 =$$

$$-280-\frac{1}{\phi}(1+\phi\pi)\left(2.39586+6.9147\times10^6\exp^4\left(i\pi\left\lfloor\frac{\arg(2-x)}{2\pi}\right\rfloor\right)\sqrt{x}^4\left(-1+J_0\left(\frac{0.0532769}{\sqrt{2}}\right)+2\sum_{k=1}^{\infty}(-1)^k J_{2k}\left(\frac{0.0532769}{\sqrt{2}}\right)\right)\left(\sum_{k=0}^{\infty}\frac{(-1)^k(2-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^4\right) \text{ for } (x\in\mathbb{R} \text{ and } x<0)$$

Integral representations:

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13 =$$

$$-280+\frac{368394.\,(1+\phi\pi)\left(-6.50353\times10^{-6}+\sqrt{2}^3\int_0^1\sin\left(\frac{0.0532769\,t}{\sqrt{2}}\right)dt\right)}{\phi}$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13 =$$

$$-280-\frac{(1+\phi\pi)\left(2.39586-6.9147\times10^6\left(1+\int_{\frac{\pi}{2}}^{\frac{0.0532769}{\sqrt{2}}}\sin(t)\,dt\right)\sqrt{2}^4\right)}{\phi}$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)\left(64\sqrt{2}\right)^4}{2.42631}\right)-233-34-13 =$$

$$-280+\left(-\frac{1}{\phi}-\pi\right)\left(2.39586+6.9147\times10^6\sqrt{2}^4\left(-1+\frac{\sqrt{\pi}}{2i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{s-0.000709607/\left(s\sqrt{2}^2\right)}}{\sqrt{s}}\,ds\right)\right) \text{ for } \gamma>0$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13 =$$

$$-280+\left(-\frac{1}{\phi}-\pi\right)\left(2.39586+6.9147\times10^6\sqrt{2}^4\left(-1+\frac{\sqrt{\pi}}{2\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{0.0266384^{-2\,s}\,\Gamma(s)\left(\frac{1}{\sqrt{2}}\right)^{-2\,s}}{\Gamma\left(\frac{1}{2}-s\right)}\,d\,s\right)\right) \text{ for } 0<\gamma<\frac{1}{2}$$

Multiple-argument formulas:

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13 =$$

$$-280+\frac{1.38294\times10^7\,(1+\phi\,\pi)\left(-1.73244\times10^{-7}+\sin^2\left(\frac{0.0266384}{\sqrt{2}}\right)\sqrt{2}^4\right)}{\phi}$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13 =$$

$$-280-\frac{(1+\phi\,\pi)\left(2.39586+1.38294\times10^7\left(-1+\cos^2\left(\frac{0.0266384}{\sqrt{2}}\right)\right)\sqrt{2}^4\right)}{\phi}$$

$$\left(-\pi-\frac{1}{\phi}\right)\left(\frac{2.189^2}{2}+\frac{\left(\cos\left(\frac{\sqrt{2.42631}\ 2.189}{64\sqrt{2}}\right)-1\right)(64\sqrt{2})^4}{2.42631}\right)-233-34-13 = -280-$$

$$\frac{(1+\phi\,\pi)\left(2.39586+6.9147\times10^6\left(-1+\cos\left(\frac{0.017759}{\sqrt{2}}\right)\right)\left(1+2\cos\left(\frac{0.017759}{\sqrt{2}}\right)\right)^2\sqrt{2}^4\right)}{\phi}$$

We have the following mathematical connection:

$$\left(\left(-\pi-\frac{1}{\phi}\right)\left(\frac{1}{2}\times2.189^2+\frac{(64\sqrt{2})^4}{2.426310}\left(\cos\left(\frac{\sqrt{2.426310}\times2.189}{64\sqrt{2}}\right)-1\right)\right)-233-34-13\right) = 73492 \Rightarrow$$

$$\Rightarrow -3927 + 2\left(\sqrt[13]{\frac{N \exp\left[\int d\hat{\sigma}\left(-\frac{1}{4u^2}\mathrm{P}_i D\mathrm{P}_i\right)\right]|Bp\rangle_{\mathrm{NS}}}{\int [d\mathrm{X}^\mu] \exp\left\{\int d\hat{\sigma}\left(-\frac{1}{4v^2} D\mathrm{X}^\mu D^2 \mathrm{X}^\mu\right)\right\}|\mathrm{X}^\mu, \mathrm{X}^i = 0\rangle_{\mathrm{NS}}} +}\right) =$$

$$-3927 + 2\sqrt[13]{2.2983717437 \times 10^{59} + 2.0823329825883 \times 10^{59}}$$

$$= 73490.8437525.... \Rightarrow$$

$$\Rightarrow \left(A(r) \times \frac{1}{B(r)}\left(-\frac{1}{\phi(r)}\right) \times \frac{1}{e^{\Lambda(r)}}\right) \Rightarrow$$

$$\Rightarrow \left(-0.000029211892 \times \frac{1}{0.0003644621}\left(-\frac{1}{0.0005946833}\right) \times \frac{1}{0.00183393}\right) =$$

$$= 73491.78832548118710549159572042220548025195726563413398700\ldots$$

$$= 73491.7883254... \Rightarrow$$

$$\left(\begin{gathered} I_{21} \leqslant \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right)\left|\sum_{\lambda \leqslant P^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda)\lambda^{-i(T+t)}\right|^2 dt \leqslant \\ \leqslant H\left\{\left(\frac{4}{\varepsilon_2 \log T}\right)^{2r} (\log T)(\log X)^{-2\beta} + (\varepsilon_2^{-2r}(\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r})\, T^{-\varepsilon_1}\right\} \end{gathered}\right)/$$

$$/(26 \times 4)^2 - 24 = \left(\frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24}\right) = 73493.30662\ldots$$

Mathematical connections with the boundary state corresponding to the NSNS-sector of N Dp-branes in the limit of u → ∞, with the ratio concerning the general asymptotically flat solution of the equations of motion of the p-brane and with the Karatsuba's equation concerning the zeros of a special type of function connected with Dirichlet series.

Appendix

The OEIS Foundation is supported by donations from users of the OEIS and by a grant from the Simons Foundation.

THE ON-LINE ENCYCLOPEDIA OF INTEGER SEQUENCES ®

founded in 1964 by N. J. A. Sloane

A053261 Coefficients of the '5th order' mock theta function psi_1(q). 17

1, 1, 1, 1, 1, 1, 2, 1, 1, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 6, 6, 6, 7, 7, 7, 8, 8, 9, 10, 10, 10, 11, 11, 12, 13, 13, 14, 15, 16, 16, 17, 18, 19, 20, 20, 21, 23, 24, 25, 26, 27, 28, 30, 31, 32, 34, 35, 37, 39, 40, 41, 44, 45, 47, 50, 51, 53, 56, 58, 60, 63, 65 (list; graph; refs; listen; history; text; internal format)

OFFSET	0,7
COMMENTS	Number of partitions of n such that each part occurs at most twice and if k occurs as a part then all smaller positive integers occur. Strictly unimodal compositions with rising range 1, 2, 3, .., m where m is the largest part and distinct parts in the falling range (this follows trivially from the comment above). [Joerg Arndt, Mar 26 2014]
REFERENCES	Srinivasa Ramanujan, Collected Papers, Chelsea, New York, 1962, pp. 354-355 Srinivasa Ramanujan, The Lost Notebook and Other Unpublished Papers, Narosa Publishing House, New Delhi, 1988, pp. 19, 21, 22
LINKS	Vaclav Kotesovec, Table of n, a(n) for n = 0..10000 (terms 0..1000 from Alois P. Heinz)

Observations

Figs.

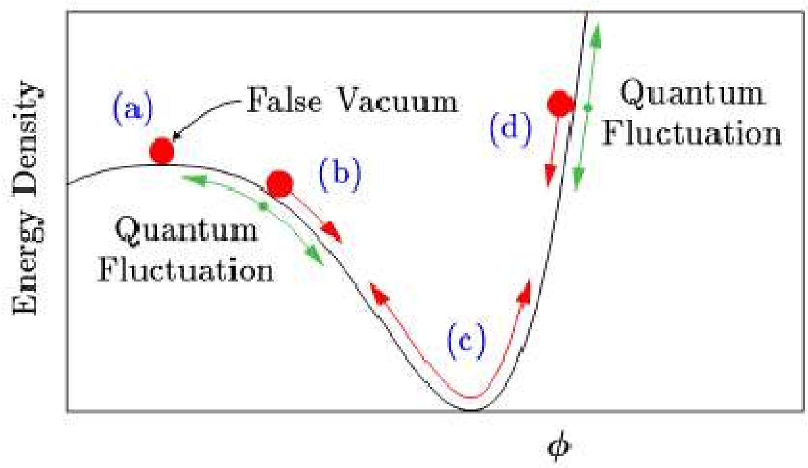
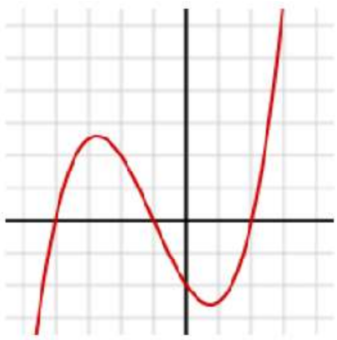


FIG. 1: In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field, ϕ. Red arrows show the classical motion of ϕ. When ϕ is near region (a), the energy density will remain nearly constant, $\rho \cong \rho_f$, even as the universe expands. Moreover, cosmic expansion acts like a frictional drag, slowing the motion of ϕ: Even near regions (b) and (d), ϕ behaves more like a marble moving in a bowl of molasses, slowly creeping down the side of its potential, rather than like a marble sliding down the inside of a polished bowl. During this period of "slow roll," ρ remains nearly constant. Only after ϕ has slid most of the way down its potential will it begin to oscillate around its minimum, as in region (c), ending inflation.



Graph of a cubic function with 3 real roots (where the curve crosses the horizontal axis at $y = 0$). The case shown has two critical points. Here the function is $f(x) = (x^3 + 3x^2 - 6x - 8)/4$.

The ratio between M_0 and q

$$M_0 = \sqrt{3q^2 - \Sigma^2},$$

$$q = \frac{(3\sqrt{3})\,M_s}{2}.$$

i.e. the gravitating mass M_0 and the Wheelerian mass q of the wormhole, is equal to:

$$\frac{\sqrt{3\,(2.17049\times 10^{37})^2 - 0.001^2}}{\frac{1}{2}\,((3\sqrt{3})\,(4.2\times 10^6\times 1.9891\times 10^{30}))}$$

1.732050787905194420703947625671018160083566548802082460520...

1.7320507879

$1.7320507879 \approx \sqrt{3}$ that is the ratio between the gravitating mass M_0 and the Wheelerian mass q of the wormhole

We note that:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right)$$

i is the imaginary unit

$i\sqrt{3}$

1.732050807568877293527446341505872366942805253810380628055... i

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

1.73205

This result is very near to the ratio between M_0 and q, that is equal to 1.7320507879 $\approx \sqrt{3}$

With regard $\sqrt{3}$, we note that is a fundamental value of the formula structure that we need to calculate a Cubic Equation

We have that the previous result

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) = i\sqrt{3} =$$

$= 1.7320508075688772935274463415058723669428052538103806280 55\ldots i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

can be related with:

$$u^2(-u)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)+v^2(-v)\left(\frac{1}{2}\pm\frac{i\sqrt{3}}{2}\right)=q$$

Considering:

$$(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q$$

$= i\sqrt{3} = 1.7320508075688772935274463415058723669428052538103806280 55\ldots i$

$r \approx 1.73205$ (radius), $\theta = 90°$ (angle)

Thence:

$$\left(-\frac{1}{2}+\frac{i}{2}\sqrt{3}\right)-\left(-\frac{1}{2}-\frac{i}{2}\sqrt{3}\right) \Rightarrow$$

$$\Rightarrow(-1)\left(\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)-(-1)\left(\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)=q=1.73205\approx\sqrt{3}$$

We observe how the graph above, concerning the cubic function, is very similar to the graph that represent the scalar field (in red). It is possible to hypothesize that cubic functions and cubic equations, with their roots, are connected to the scalar field.

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= \qquad\qquad 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a *factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

References

The Quantum Theory of Fields - Volume III - Supersymmetry
Steven Weinberg
University of Texas at Austin - Steven Weinberg 2000 - First published 2000
Printed in the United States of America

Modular equations and approximations to π – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

FIELD THEORY. A MODERN PRIMER - *Pierre Ramond* - (Florida U. and Caltech) - Jan 1, 1981 - 329 pages - Published in: *Front.Phys.* 51 (1981) 1-397, *Front.Phys.* 74 (1989) 1-329