

On some Ramanujan's Approximations to π : mathematical connections with ϕ , $\zeta(2)$, and various parameters of Particle Physics.

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Abstract

In this paper we have described and analyzed some Ramanujan's Approximations to π . Furthermore, we have obtained various mathematical connections with ϕ , $\zeta(2)$, and several parameters of Particle Physics.

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$$\frac{1}{\pi} = \frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}}$$



https://twitter.com/winkjs_org/status/973840788902866944

$\text{sqrt}8/9801 \sum((4n)!)/((n!)^4)*(((26390n+1103)))/(((396)^(4n))), n = 0..infinity$

Input interpretation:

$$\frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}}$$

$n!$ is the factorial function

Result:

$$\frac{1}{\pi} \approx 0.31831$$

0.31831

$1+2((((\text{sqrt}8/9801 \sum((4n)!)/((n!)^4)*(((26390n+1103)))/(((396)^(4n))), n = 0..infinity))))-18/10^3$

Input interpretation:

$$1 + 2 \left(\frac{\sqrt{8}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} \right) - \frac{18}{10^3}$$

$n!$ is the factorial function

Result:

$$\frac{491}{500} + \frac{2}{\pi} \approx 1.61862$$

1.61862

Alternate form:

$$\frac{1000 + 491 \pi}{500 \pi}$$

From which:

$$[\frac{2(1/8*\sqrt{x})}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} * \frac{(26390*n+1103)}{(396)^{(4n)}}] = \frac{1}{\pi}$$

Input interpretation:

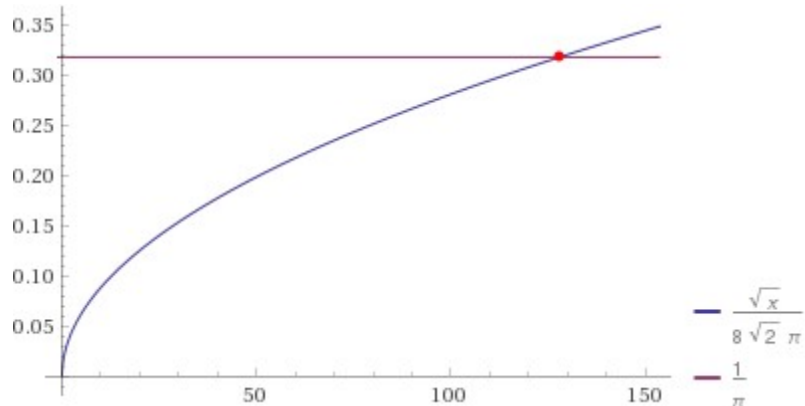
$$2 \times \frac{1}{8} \frac{\sqrt{x}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} = \frac{1}{\pi}$$

n! is the factorial function

Result:

$$\frac{\sqrt{x}}{8\sqrt{2}\pi} = \frac{1}{\pi}$$

Plot:



Alternate form assuming x is positive:

$$\sqrt{2} \sqrt{x} = 16$$

Solution:

$$x = 128$$

128

$$\left[\left(\frac{\sqrt{\sqrt{x/27}}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} \right) \right] = \frac{1}{\pi}$$

Input interpretation:

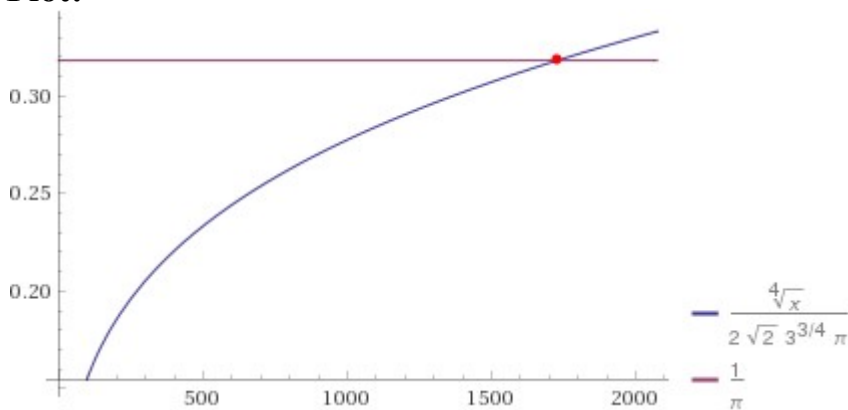
$$\frac{\sqrt{\sqrt{\frac{x}{27}}}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} = \frac{1}{\pi}$$

$n!$ is the factorial function

Result:

$$\frac{\sqrt[4]{x}}{2\sqrt{2} \cdot 3^{3/4} \pi} = \frac{1}{\pi}$$

Plot:



Alternate form assuming x is positive:

$$\sqrt{2} \sqrt[4]{3} \sqrt[4]{x} = 12$$

Solution:

$$x = 1728$$

1728

$$\left[\frac{\sqrt{\sqrt{\frac{x^{15}}{27}}}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} = \frac{1}{\pi} \right]$$

Input interpretation:

$$\frac{\sqrt{\sqrt{\frac{x^{15}}{27}}}}{9801} \sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}} = \frac{1}{\pi}$$

n! is the factorial function

Result:

$$\frac{\sqrt[4]{x^{15}}}{2\sqrt{2} 3^{3/4} \pi} = \frac{1}{\pi}$$

Alternate form assuming x is real:

$$\sqrt{2} \sqrt[4]{3} \sqrt[4]{x^{15}} = 12$$

Alternate form assuming x is positive:

$$\sqrt{2} \sqrt[4]{3} x^{15/4} = 12$$

Solutions:

$$x = -\sqrt[5]{-3} 2^{2/5}$$

$$x = (-2)^{2/5} \sqrt[5]{3}$$

$$x = 2^{2/5} \sqrt[5]{3}$$

$$x = -(-1)^{3/5} 2^{2/5} \sqrt[5]{3}$$

$$x = (-1)^{4/5} 2^{2/5} \sqrt[5]{3}$$

$$x = -\sqrt[5]{-6} \sqrt[5]{-1 - i\sqrt{3}}$$

$$x = \sqrt[5]{6} (-1 - i\sqrt{3})$$

$$x = (-1)^{2/5} \sqrt[5]{6} (-1 - i\sqrt{3})$$

$$x = -(-1)^{3/5} \sqrt[5]{6} (-1 - i\sqrt{3})$$

$$x = (-1)^{4/5} \sqrt[5]{6(-1 - i\sqrt{3})}$$

$$x = -\sqrt[5]{-6} \sqrt[5]{-1 + i\sqrt{3}}$$

$$x = \sqrt[5]{6(-1 + i\sqrt{3})}$$

$$x = (-1)^{2/5} \sqrt[5]{6(-1 + i\sqrt{3})}$$

$$x = -(-1)^{3/5} \sqrt[5]{6(-1 + i\sqrt{3})}$$

$$x = (-1)^{4/5} \sqrt[5]{6(-1 + i\sqrt{3})}$$

Real solution:

$$x = 2^{2/5} \sqrt[5]{3}$$

Complex solutions:

$$x = -\sqrt[5]{-3} 2^{2/5}$$

$$x = (-2)^{2/5} \sqrt[5]{3}$$

$$x = -(-1)^{3/5} 2^{2/5} \sqrt[5]{3}$$

$$x = (-1)^{4/5} 2^{2/5} \sqrt[5]{3}$$

$$x = -\sqrt[5]{6 + 6i\sqrt{3}}$$

Solutions:

$$x \approx -1.32982 - 0.966173 i$$

$$x \approx 0.507947 + 1.5633 i$$

$$x \approx 1.64375$$

$$x \approx 0.507947 - 1.5633 i$$

$$x \approx -1.32982 + 0.966173 i$$

$$x \approx -1.60783 - 0.341755 i$$

$$x \approx 1.50164 - 0.668574 i$$

$$x \approx 1.09988 + 1.22155 i$$

$$x \approx -0.171819 - 1.63475 i$$

$$x \approx -0.821876 + 1.42353 i$$

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$$x \approx 1.50164 + 0.668574 i$$

$$x \approx -0.171819 + 1.63475 i$$

$$x \approx 1.09988 - 1.22155 i$$

$$x \approx -1.60783 + 0.341755 i$$

Real solution:

$$x \approx 1.6438$$

1.6438

Now, we have that:

$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \binom{2n}{n}^3 \frac{42n + 5}{2^{12n+4}}$$

We have performed the following calculation:

$$\begin{aligned} & ((((-3 \cdot 17) / 7) \cdot ((-p(12, 6)^3 \cdot ((42 \cdot 6 + 5) / (2^{12 \cdot 6 + 4}))) - p(14, 7)^3 \\ & [((42 \cdot 7 + 5) / (2^{12 \cdot 7 + 4}))] - p(16, 8)^3 \cdot ((42 \cdot 8 + 5) / (2^{12 \cdot 8 + 4})))))) \end{aligned}$$

Input:

$$\begin{aligned} & \left(-\frac{1}{7} (3 \cdot 17)\right) \\ & \left(-\left(\frac{12!}{(12-6)!}\right)^3 \times \frac{42 \cdot 6 + 5}{2^{12 \cdot 6 + 4}} - \left(\frac{14!}{(14-7)!}\right)^3 \times \frac{42 \cdot 7 + 5}{2^{12 \cdot 7 + 4}} - \left(\frac{16!}{(16-8)!}\right)^3 \times \frac{42 \cdot 8 + 5}{2^{12 \cdot 8 + 4}}\right) \end{aligned}$$

n! is the factorial function

Exact result:

$$\frac{23995837177800922751625}{75557863725914323419136}$$

Decimal approximation:

0.317582260727297314190176661265858307908738211722265987191...

0.3175822607...

Alternative representations:

$$\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6 + 5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7 + 5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8 + 5)}{2^{12 \times 8+4}} \right) (-3 \times 17) =$$

$$-\frac{51}{7} \left(-\frac{257 \left(\frac{\Gamma(13)}{\Gamma(7)}\right)^3}{2^{76}} - \frac{299 \left(\frac{\Gamma(15)}{\Gamma(8)}\right)^3}{2^{88}} - \frac{341 \left(\frac{\Gamma(17)}{\Gamma(9)}\right)^3}{2^{100}} \right)$$

$$\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6 + 5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7 + 5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8 + 5)}{2^{12 \times 8+4}} \right) (-3 \times 17) =$$

$$-\frac{51}{7} \left(-\frac{257 \left(\frac{11!! \times 12!!}{5!! \times 6!!}\right)^3}{2^{76}} - \frac{299 \left(\frac{13!! \times 14!!}{6!! \times 7!!}\right)^3}{2^{88}} - \frac{341 \left(\frac{15!! \times 16!!}{7!! \times 8!!}\right)^3}{2^{100}} \right)$$

$$\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6 + 5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7 + 5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8 + 5)}{2^{12 \times 8+4}} \right) (-3 \times 17) =$$

$$-\frac{51}{7} \left(-\frac{257 \left(\frac{\Gamma(13,0)}{\Gamma(7,0)}\right)^3}{2^{76}} - \frac{299 \left(\frac{\Gamma(15,0)}{\Gamma(8,0)}\right)^3}{2^{88}} - \frac{341 \left(\frac{\Gamma(17,0)}{\Gamma(9,0)}\right)^3}{2^{100}} \right)$$

Series representation:

$$\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6 + 5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7 + 5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8 + 5)}{2^{12 \times 8+4}} \right) (-3 \times 17) =$$

$$\left(51 \left(4311744512 \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right. \right.$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(12-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 + \right.$$

$$1224704 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3$$

$$\left(\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 + 341 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3$$

$$\left. \left. \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(16-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right) \right) /$$

$$\left(8873554201597605810476922437632 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right.$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right)$$

for $(n_0 \geq 0$ or $n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow 6$ and $n_0 \rightarrow 7$ and $n_0 \rightarrow 8$ and $n_0 \rightarrow 12$ and $n_0 \rightarrow 14$ and $n_0 \rightarrow 16)$

$$1/((((((-3*17)/7(((p(12, 6)^3 [((42*6+5))/((2^(12*6+4))))]-p(14, 7)^3 [((42*7+5))/((2^(12*7+4))))] - p(16, 8)^3 [((42*8+5))/((2^(12*8+4))))])])])])])$$

Input:

$$\frac{1}{(-\frac{1}{7}(3 \times 17)) \left(-\left(\frac{12!}{(12-6)!}\right)^3 \times \frac{42 \times 6 + 5}{2^{12 \times 6 + 4}} - \left(\frac{14!}{(14-7)!}\right)^3 \times \frac{42 \times 7 + 5}{2^{12 \times 7 + 4}} - \left(\frac{16!}{(16-8)!}\right)^3 \times \frac{42 \times 8 + 5}{2^{12 \times 8 + 4}} \right)}$$

$n!$ is the factorial function

Exact result:

75557863725914323419136
 23995837177800922751625

Decimal approximation:

3.148790482534802602339462206749581465833907402874323394049...

3.14879048... $\approx \pi$

Mixed fraction:

$$3 \frac{3570352192511555164261}{23995837177800922751625}$$

Alternative representations:

$$\frac{1}{\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6+5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7+5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8+5)}{2^{12 \times 8+4}} \right) (-3 \times 17)}$$

$$= \frac{1}{\frac{51}{7} \left(-\frac{257 \left(\frac{\Gamma(13)}{\Gamma(7)}\right)^3}{2^{76}} - \frac{299 \left(\frac{\Gamma(15)}{\Gamma(8)}\right)^3}{2^{88}} - \frac{341 \left(\frac{\Gamma(17)}{\Gamma(9)}\right)^3}{2^{100}} \right)}$$

$$\frac{1}{\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6+5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7+5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8+5)}{2^{12 \times 8+4}} \right) (-3 \times 17)}$$

$$= \frac{1}{\frac{51}{7} \left(-\frac{257 \left(\frac{\Gamma(13,0)}{\Gamma(7,0)}\right)^3}{2^{76}} - \frac{299 \left(\frac{\Gamma(15,0)}{\Gamma(8,0)}\right)^3}{2^{88}} - \frac{341 \left(\frac{\Gamma(17,0)}{\Gamma(9,0)}\right)^3}{2^{100}} \right)}$$

$$\frac{1}{\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6+5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7+5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8+5)}{2^{12 \times 8+4}} \right) (-3 \times 17)}$$

$$= \frac{1}{\frac{51}{7} \left(-\frac{257 \left(\frac{(1)_{12}}{(1)_6}\right)^3}{2^{76}} - \frac{299 \left(\frac{(1)_{14}}{(1)_7}\right)^3}{2^{88}} - \frac{341 \left(\frac{(1)_{16}}{(1)_8}\right)^3}{2^{100}} \right)}$$

Series representation:

$$\frac{1}{7} \left(-\frac{\left(\frac{12!}{(12-6)!}\right)^3 (42 \times 6+5)}{2^{12 \times 6+4}} - \frac{\left(\frac{14!}{(14-7)!}\right)^3 (42 \times 7+5)}{2^{12 \times 7+4}} - \frac{\left(\frac{16!}{(16-8)!}\right)^3 (42 \times 8+5)}{2^{12 \times 8+4}} \right) (-3 \times 17) =$$

$$\left(8873554201597605810476922437632 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right.$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right) /$$

$$\left(51 \left(4311744512 \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right.

$$\left. \left(\sum_{k=0}^{\infty} \frac{(12-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 + \right.$$

$$1224704 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(8-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3$$

$$\left. \left(\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 + 341 \left(\sum_{k=0}^{\infty} \frac{(6-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right.$$

$$\left. \left. \left(\sum_{k=0}^{\infty} \frac{(7-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \left(\sum_{k=0}^{\infty} \frac{(16-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} \right)^3 \right) \right)$$$$

for $(n_0 \geq 0$ or $n_0 \notin \mathbb{Z})$ and $n_0 \rightarrow 6$ and $n_0 \rightarrow 7$ and $n_0 \rightarrow 8$ and $n_0 \rightarrow 12$ and $n_0 \rightarrow 14$ and $n_0 \rightarrow 16$)

Now, from Ramanujan, we know this approximation to π :

$$\pi \approx 768 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}}}$$

$$\approx 3.141590463236763.$$

768sqrt(2-sqrt(2+sqrt(2+sqrt(2+sqrt(2+sqrt(2+sqrt(2+sqrt(2+sqrt(2+1))))))))))

Input:

$$768 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}}$$

Exact result:

$$\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \left(\frac{3840 \sqrt{30}}{\pi^2} - 961 \right) - \pi$$

Decimal approximation:

1.644597823976448432899606375852148231569257628478985413172...

[1.64459782397...](#)

Property:

$$\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \left(-961 + \frac{3840 \sqrt{30}}{\pi^2} \right) - \pi$$

is a transcendental number

Alternate forms:

$$-961 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} +$$

$$\sqrt{\frac{3840 \sqrt{30} \left(\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}}} \right)}{\pi^2}} - \pi$$

$$\frac{1}{\pi^2} \left(3840 \sqrt{30 \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}} \right}}} \right)} - \right.$$

$$\left. 961 \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}} \right}}} \right) \pi^2 - \pi^3$$

Series representations:

$$\left(\frac{768 (5 \sqrt{30})}{\pi^2} - 10^3 + 34 + 5 \right)$$

$$\begin{aligned}
 & \sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}} - \pi = \\
 & \left. \begin{aligned}
 & -\frac{1}{\pi^2} \left(\pi^3 + 961 \pi^2 \sqrt{z_0} \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2}\right)_k \right. \\
 & \left. \left(\left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} - z_0 \right)^k \right. \right. \\
 & \left. \left. 3840 \sqrt{z_0}^2 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (-1)^{k_1+k_2} \left(-\frac{1}{2}\right)_{k_1} \left(-\frac{1}{2}\right)_{k_2} (30 - z_0)^{k_1} \right. \right. \\
 & \left. \left. \left(\left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} - z_0 \right)^{k_2} \right. \right. \\
 & \left. \left. z_0^{-k_1-k_2} \right) \right) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
 \end{aligned}
 \right.
 \end{aligned}$$

$$\left(\frac{768(5\sqrt{30})}{\pi^2} - 10^3 + 34 + 5 \right)$$

$$\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}}} - \pi =$$

$$-\frac{1}{\pi^2} \pi^3 + 961 \pi^2 \left(\frac{1}{z_0} \right) \left[\left(\frac{1/2 \operatorname{arg} \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} \right) - z_0}{(2\pi)} \right) \right]$$

$$z_0 \left[\left(\frac{1/2 + 1/2 \operatorname{arg} \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} \right) - z_0}{(2\pi)} \right) \right] \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(-\frac{1}{2} \right)_k$$

$$\left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} - z_0 \right) z_0^{-k} -$$

$$3840 \left(\frac{1}{z_0} \right) \left[\left(\frac{1/2 [\operatorname{arg}(30 - z_0)/(2\pi)] + 1/2 \operatorname{arg} \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} \right) - z_0}{(2\pi)} \right) \right]$$

$$1 + 1/2 [\operatorname{arg}(30 - z_0)/(2\pi)] + 1/2 \operatorname{arg} \left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} - z_0 \right) / (2\pi)$$

$$z_0 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{1}{k_1! k_2!} (-1)^{k_1+k_2} \left(-\frac{1}{2} \right)_{k_1} \left(-\frac{1}{2} \right)_{k_2} (30 - z_0)^{k_1}$$

$$\left(2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{3}}}}}}}}} - z_0 \right) z_0^{-k_1 - k_2}$$

Now, from Ramanujan expression concerning π , we have that:

$$\frac{4}{\pi} = \sum_{n=0}^{\infty} \frac{(-1)^n (1123 + 21460n)(2n-1)!!(4n-1)!!}{882^{2n+1} 32^n (n!)^3}$$

we obtain:

$$\text{Sum } [(((-1)^n (1123+21460n)(2n-1)!! (4n-1)!!)) / (((882^{2n+1} 32^n (n!)^3)))] , n = 0..infinity$$

Infinite sum:

$$\sum_{n=0}^{\infty} \frac{(-1)^n ((21460n + 1123)(2n-1)!!)(4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} = \frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365} {1829677248} {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)$$

$n!!$ is the double factorial function

$n!$ is the factorial function

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

Decimal approximation:

1.273239544735162686151070106980114896275677165923651589981...

1.27323954473...

Alternate forms:

$$\frac{2329623072} {1829677248} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365} {1829677248} {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)$$

$$\begin{aligned}
& \frac{2246 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{441 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} - \\
& \frac{1}{1829677248} 5365 \left(\frac{7318708992 \sqrt{\frac{2}{37}} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} + \right. \\
& \left. \frac{8297856 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} + \right. \\
& \left. \frac{12910202661888 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1\right)} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} + \right. \\
& \left. \frac{14637417984 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} - \right. \\
& \left. \left(\frac{25820405323776 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1\right)} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)}{E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)} \right) / \left(145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2} \right) - \\
& \left(\frac{29274835968 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)}{E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)} \right) / \left(\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2} \right) \right)
\end{aligned}$$

Series representations:

$$\frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{1829677248} =$$

$$\sum_{k=0}^{\infty} \frac{1}{1829677248 k!} \left(2329623072 {}_4\tilde{F}_3\left(1, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1-k, 1, 1; z_0\right) - \right.$$

$$\left. 5365 {}_4\tilde{F}_3\left(1, \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 1-k, 2, 2; z_0\right) \right)$$

$$\left(-\frac{1}{777924} - z_0\right)^k z_0^{-k} \text{ for (not } (z_0 \in \mathbb{R} \text{ and } 1 \leq z_0 < \infty))$$

$$\frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{1829677248} =$$

$$\sum_{k=0}^{\infty} \left(\frac{1123 (-1)^k 882^{-1-2k} \left(\frac{1}{4}\right)_k \left(\frac{1}{2}\right)_k \left(\frac{3}{4}\right)_k}{k! ((1)_k)^2} - \right.$$

$$\left. \frac{5365 (-1)^k 3^{-5-4k} \times 4^{-3-k} \times 49^{-3-2k} \left(\frac{5}{4}\right)_k \left(\frac{3}{2}\right)_k \left(\frac{7}{4}\right)_k}{k! ((2)_k)^2} \right)$$

$$\frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{1829677248} =$$

$$\frac{2329617707 \left(\prod_{k=1}^2 \Gamma(b_k)\right) \sum_{j=0}^{\infty} \text{Res}_{s=-j} \frac{777924^s \prod_{k=1}^3 \Gamma(-s+a_k)}{\prod_{k=1}^2 \Gamma(-s+b_k)}}{1829677248 \prod_{k=1}^3 \Gamma(a_k)}$$

$$\frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{1829677248} =$$

$$\sum_{k=0}^{\infty} \left(\left(-\frac{1}{777924} - x\right)^k \left(1164811536 \Gamma\left(\frac{5}{4}\right) \Gamma\left(\frac{7}{4}\right) \Gamma\left(\frac{1}{4} + k\right) \Gamma\left(\frac{1}{2} + k\right) \Gamma\left(\frac{3}{4} + k\right) \right.$$

$$\left. {}_3\tilde{F}_2\left(\frac{1}{2} + k, \frac{1}{4} + k, \frac{3}{4} + k; 1+k, 1+k; x\right) - 5365 \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4} + k\right) \right.$$

$$\left. \Gamma\left(\frac{3}{2} + k\right) \Gamma\left(\frac{7}{4} + k\right) {}_3\tilde{F}_2\left(\frac{3}{2} + k, \frac{5}{4} + k, \frac{7}{4} + k; 2+k, 2+k; x\right) \right) /$$

$$\left(914838624 \sqrt{\pi} k! \Gamma\left(\frac{1}{4}\right) \Gamma\left(\frac{3}{4}\right) \Gamma\left(\frac{5}{4}\right) \Gamma\left(\frac{7}{4}\right) \right) \text{ for } x > 1$$

and

$$4 / \left(\sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460n)(2n-1)!! (4n-1)!!)}{(882^{2n+1} \times 32^n (n!)^3)} \right),$$

Input interpretation:

$$\sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460n)(2n-1)!! (4n-1)!!)}{882^{2n+1} \times 32^n (n!)^3}$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$\frac{4}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \approx 3.14159$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

$$3.14159 \approx \pi$$

Alternate forms:

$$\frac{7318708992}{2329623072 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}$$

$$\frac{7318708992}{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) - 2329623072 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right)}$$

$$\begin{aligned}
& 4 / \left(\frac{2246 \sqrt{2} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right)^2}{441 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2} - \right. \\
& \frac{1}{1829677248} 5365 \left(\frac{7318708992 \sqrt{\frac{2}{37}} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2} + \right. \\
& \frac{8297856 \sqrt{2} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2} + \\
& \frac{12910202661888 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1 \right)} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2} + \\
& \frac{14637417984 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2} - \\
& \left. \left(25820405323776 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1 \right)} K \left(\frac{1}{2} - \right. \right. \right. \\
& \left. \left. \left. 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right) E \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right) \right) \right) / \\
& \left(145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2 \right) - \\
& \left(29274835968 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right) \right. \\
& \left. \left. E \left(\frac{1}{2} - 441 \sqrt{2 \left(-1 + \frac{145 \sqrt{37}}{882} \right)} \right) \right) \right) / \left(\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441}} \pi^2 \right) \right)
\end{aligned}$$

$$\left(\left(\sum_{n=0}^{\infty} \frac{((-1)^n ((1123+21460n)(2n-1)!! (4n-1)!!))}{882^{2n+1} \times 32^n (n!)^3} \right) / \left((882^{(2n+1)} 32^n (n!)^3) \right) \right)^2, n = 0..infinity$$

Input interpretation:

$$\left(\sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460 n) (2 n - 1)!! (4 n - 1)!!)}{882^{2n+1} \times 32^n (n!)^3} \right)^2$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$\left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right)^2 \approx 1.62114$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

$1.62114 \approx \text{golden ratio} = 1.61803398\dots$

Alternate forms:

$$\frac{\left(2329623072 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) \right)^2}{3347718831848853504}$$

$$\frac{1261129 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right)^2}{777924} - \frac{6024895 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{806887666368} + \frac{28783225 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)^2}{3347718831848853504}$$

$$\begin{aligned}
& \left(\frac{2246 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right)^2}{441 \sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2} - \right. \\
& \frac{1}{1829677248} 5365 \left(\frac{7318708992 \sqrt{\frac{2}{37}} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2} + \right. \\
& \frac{8297856 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2} + \\
& \left. \frac{12910202661888 \sqrt{\frac{1}{37}\left(\frac{145\sqrt{37}}{882} - 1\right)} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2} \right) \\
& + \\
& \frac{14637417984 \sqrt{\frac{145\sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2} - \\
& \left(25820405323776 \sqrt{\frac{1}{37}\left(\frac{145\sqrt{37}}{882} - 1\right)} \right. \\
& \left. K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right) \right. \\
& \left. E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right) \right) / \\
& \left(145 \sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2 \right) - \\
& \left(29274835968 \sqrt{\frac{145\sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right) \right. \\
& \left. E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145\sqrt{37}}{882}\right)}\right) \right) / \\
& \left. \left(\sqrt[4]{\frac{1555849}{777924} + \frac{145\sqrt{37}}{441}} \pi^2 \right) \right)^2
\end{aligned}$$

From the principal expression, we obtain also:

$$29+7+10^3 * (((1+1/2 \text{ Sum } [((-1)^n (1123+21460n)(2n-1)!! (4n-1)!!)] / ((882^{(2n+1)} 32^n (n!)^3))), n = 0..infinity)))$$

Input interpretation:

$$29 + 7 + 10^3 \left(1 + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460 n) (2 n - 1)!!) (4 n - 1)!!}{882^{2n+1} \times 32^n (n!)^3} \right)$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$1000 \left(\frac{1}{2} \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) + 1 \right) + 36 \approx 1672.62$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

1672.62 result practically equal to the rest mass of Omega baryon 1672.45

Alternate forms:

$$\frac{280750}{441} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{457419312} + 1036$$

$$\frac{1}{457419312} \left(291202884000 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) + 473886407232 \right)$$

$$\frac{1}{457419312} \left(291202884000 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 37 \left(18125 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) - 12807740736 \right) \right)$$

$$89+3+10^3*\left(\left(1+\frac{1}{2}\sum_{n=0}^{\infty}\frac{((-1)^n((1123+21460n)(2n-1)!!(4n-1)!!))}{882^{2n+1}\times 32^n(n!)^3}\right)\right)$$

Input interpretation:

$$89 + 3 + 10^3 \left(1 + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460 n) (2 n - 1)!!) (4 n - 1)!!}{882^{2n+1} \times 32^n (n!)^3} \right)$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$1000 \left(\frac{1}{2} \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) + 1 \right) + 92 \approx 1728.62$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

1728.62

Alternate forms:

$$\frac{280750}{441} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{457419312} + 1092$$

$$\frac{1}{457419312} \left(291202884000 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) + 499501888704 \right)$$

$$\left(\left((89+3+10^3 * \left(\sum_{n=0}^{\infty} \frac{((-1)^n (1123+21460n)(2n-1)!! (4n-1)!!)}{882^{2n+1} \times 32^n (n!)^3} \right) \right) / \left((882^{2n+1} 32^n (n!)^3) \right) \right)^{1/15}$$

Input interpretation:

$$\sqrt[15]{89 + 3 + 10^3 \left(1 + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460n)(2n-1)!!)(4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} \right)}$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$\left(1000 \left(\frac{1}{2} \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) + 1 \right) + 92 \right)^{(1/15)} \approx 1.64379$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

1.64379

Alternate forms:

$$\left(\frac{280750}{441} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{457419312} + 1092 \right)^{(1/15)}$$

$$\frac{1}{2^{4/15} \sqrt[3]{3} 7^{2/5}} \left(\left(291202884000 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 670625 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) + 499501888704 \right)^{(1/15)} \right)$$

$$-(21+5)1/10^3+(((89+3+10^3*(((1+1/2 \text{Sum} [((-1)^n (1123+21460n)(2n-1)!! (4n-1)!!) / (((882^{(2n+1)} 32^n (n!)^3))))], n = 0..infinity))))))^{1/15}$$

Input interpretation:

$$-(21+5) \times \frac{1}{10^3} + \sqrt[15]{89+3+10^3 \left(1 + \frac{1}{2} \sum_{n=0}^{\infty} \frac{(-1)^n ((1123+21460n)(2n-1)!!)(4n-1)!!}{882^{2n+1} \times 32^n (n!)^3}\right)}$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$\left(1000 \left(\frac{1}{2} \left(\frac{1123}{882} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{5365 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{1829677248}\right) + 1\right) + 92\right)^{(1/15)} - \frac{13}{500} \approx 1.61779$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

1.61779

Alternate forms:

$$\left(\frac{280750}{441} {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - \frac{670625 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right)}{457419312} + 1092\right)^{(1/15)} - \frac{13}{500}$$

$$\frac{1}{10500} \left(250 \times 2^{11/15} \times 3^{2/3} \times 7^{3/5} \left(291202884000 {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - 670625 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right) + 499501888704\right) - 273\right)^{(1/15)}$$

$$\frac{1}{500 \times 2^{4/15} \sqrt[3]{3} 7^{2/5}} \left(500 \left(291202884000 {}_3F_2\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924}\right) - 670625 {}_3F_2\left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924}\right) + 499501888704\right) - 13 \times 2^{4/15} \sqrt[3]{3} 7^{2/5}\right)^{(1/15)}$$

$$10^2 \left(\left(\sum_{n=0}^{\infty} \frac{(-1)^n (1123 + 21460n)(2n-1)!! (4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} \right) - 2 \right)$$

Input interpretation:

$$10^2 \sum_{n=0}^{\infty} \frac{(-1)^n (1123 + 21460n)(2n-1)!! (4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} - 2$$

$n!!$ is the double factorial function

$n!$ is the factorial function

Result:

$$100 \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) - 2 \approx 125.324$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

125.324

Alternate forms:

$$\frac{56150}{441} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{134125 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{457419312} - 2$$

$$\frac{1}{457419312} \left(58240576800 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - 134125 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) - 914838624 \right)$$

$$\begin{aligned}
& 100 \left(\frac{2246 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{441 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} - \right. \\
& \frac{1}{1829677248} 5365 \left(\frac{7318708992 \sqrt{\frac{2}{37}} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} + \right. \\
& \frac{8297856 \sqrt{2} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} + \\
& \frac{12910202661888 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1\right)} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} \\
& + \\
& \frac{14637417984 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right)^2}{\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2}} - \\
& \left(25820405323776 \sqrt{\frac{1}{37} \left(\frac{145 \sqrt{37}}{882} - 1\right)} \right. \\
& \left. K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right) \right. \\
& \left. E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right) \right) / \\
& \left(145 \sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2} \right) - \\
& \left(29274835968 \sqrt{\frac{145 \sqrt{37}}{882} - 1} K\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right) \right. \\
& \left. E\left(\frac{1}{2} - 441 \sqrt{2\left(-1 + \frac{145 \sqrt{37}}{882}\right)}\right) \right) / \\
& \left(\sqrt[4]{\frac{1555849}{777924} + \frac{145 \sqrt{37}}{441} \pi^2} \right) \right) - 2
\end{aligned}$$

$10^2 \left(\left(\sum_{n=0}^{\infty} \frac{((-1)^n (1123 + 21460n)(2n-1)!! (4n-1)!!)}{(882^{2n+1} \times 32^n (n!)^3)} \right) + 13 - \frac{1}{\phi} \right)$

Input interpretation:

$$10^2 \sum_{n=0}^{\infty} \frac{(-1)^n ((1123 + 21460 n) (2 n - 1)!!) (4 n - 1)!!}{882^{2n+1} \times 32^n (n!)^3} + 13 - \frac{1}{\phi}$$

$n!!$ is the double factorial function

$n!$ is the factorial function

ϕ is the golden ratio

Result:

$$100 \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) - \frac{1}{\phi} + 13 \approx 139.706$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

139.706

Alternate forms:

$$\frac{56150}{441} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{134125 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{457419312} - \frac{1}{\phi} + 13$$

$$- \frac{1}{457419312 \phi} \left(\phi \left(-58240576800 {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) + 134125 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right) - 5946451056 \right) + 457419312 \right)$$

$$100 \left(\frac{1123}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} \right) + \frac{1}{2} (27 - \sqrt{5})$$

$$\left(\sum_{n=0}^{\infty} \frac{(-1)^n \left(\left(x + 8 - \frac{1}{\phi} \right) + 21460n \right) (2n-1)!! (4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} \right) / \left((882^{2n+1}) 32^n (n!)^3 \right), n = 0..infinity) = 1.273239544735162686151$$

Input interpretation:

$$\sum_{n=0}^{\infty} \frac{(-1)^n \left(\left(x + 8 - \frac{1}{\phi} \right) + 21460n \right) (2n-1)!! (4n-1)!!}{882^{2n+1} \times 32^n (n!)^3} = 1.273239544735162686151$$

$n!!$ is the double factorial function

$n!$ is the factorial function

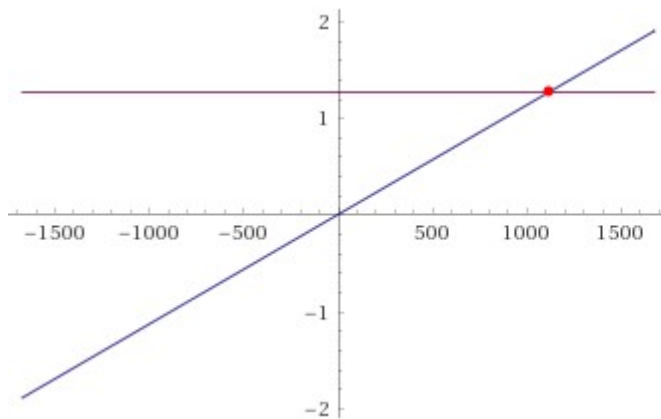
ϕ is the golden ratio

Result:

$$\frac{\frac{1}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} + \frac{(3+4\sqrt{5}) {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right)}{441(1+\sqrt{5})}}{441(1+\sqrt{5})} = 1.273239544735162686151$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

Plot:



$$\begin{aligned} & \frac{1}{882} {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right) - \\ & \frac{5365 {}_3F_2 \left(\frac{5}{4}, \frac{3}{2}, \frac{7}{4}; 2, 2; -\frac{1}{777924} \right)}{1829677248} + \\ & \frac{(3+4\sqrt{5}) {}_3F_2 \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}; 1, 1; -\frac{1}{777924} \right)}{441(1+\sqrt{5})} \\ & \text{--- } 1.273239544735162686151 \end{aligned}$$

Solution:

$$x \approx 1115.618033988749894848$$

1115.618033988749894848 result practically equal to the rest mass of Lambda baryon 1115.683

Now, we have also the following Ramanujan formula:

$$\begin{aligned} \frac{1}{\pi} &= 12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13\,591\,409 + 545\,140\,134n)}{(n!)^3 (3n)! (640\,320^3)^{n+1/2}} \\ &= \frac{163 \cdot 8 \cdot 27 \cdot 7 \cdot 11 \cdot 19 \cdot 127}{640\,320^{3/2}} \sum_{n=0}^{\infty} \left(\frac{13\,591\,409}{163 \cdot 2 \cdot 9 \cdot 7 \cdot 11 \cdot 19 \cdot 127} + n \right) \frac{(6n)!}{(3n)! (n!)^3} \frac{(-1)^n}{640\,320^{3n}} \end{aligned}$$

12 * sum (((-1)^n (6n)! (13591409+545140134n))) /(((n!)^3 (3n)! (640320^3)^(n+1/2))), n = 0..infinity

Input interpretation:

$$12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13\,591\,409 + 545\,140\,134n)}{(n!)^3 (3n)! (640\,320^3)^{n+1/2}}$$

$n!$ is the factorial function

Result:

$$\begin{aligned} &\left(1\,651\,969\,144\,908\,540\,723\,200 \, {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000} \right) - \right. \\ &\quad \left. 30\,285\,563 \, {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000} \right) \right) / \\ &\quad \left(51\,885\,171\,624\,116\,224\,000 \sqrt{10\,005} \right) \approx 0.31831 \end{aligned}$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

$$0.31831 \approx 1 / \pi$$

Alternate form:

$$\frac{13\,591\,409 \, {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000} \right) - 426\,880 \sqrt{10\,005}}{30\,285\,563 \, {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000} \right) - 51\,885\,171\,624\,116\,224\,000 \sqrt{10\,005}}$$

$$\frac{1}{6} * [1 / (((12 * \sum_{n=0}^{\infty} ((-1)^n (6n)! (13591409 + 545140134n))) / (((n!)^3 (3n)! (640320^3)^{(n+1/2)})))]^2$$

Input interpretation:

$$\frac{1}{6} \left(\frac{1}{12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13591409 + 545140134n)}{(n!)^3 (3n)! (640320^3)^{n+1/2}}} \right)^2$$

$n!$ is the factorial function

Result:

$$\frac{4489028449968712332417573157580308480000000}{\left(1651969144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - 30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2} \approx 1.64493$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

$$1.64493 = \zeta(2)$$

Alternate form:

$$\frac{4489028449968712332417573157580308480000000}{\left(30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) - 1651969144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) \right)^2}$$

$$89 - 5 + 10^3 * \frac{1}{6} * [1 / (((12 * \sum_{n=0}^{\infty} ((-1)^n (6n)! (13591409 + 545140134n))) / (((n!)^3 (3n)! (640320^3)^{(n+1/2)})))]^2$$

Input interpretation:

$$89 - 5 + 10^3 \times \frac{1}{6} \left(\frac{1}{12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13591409 + 545140134n)}{(n!)^3 (3n)! (640320^3)^{n+1/2}}} \right)^2$$

$n!$ is the factorial function

Result:

4489 028 449 968 712 332 417 573 157 580 308 480 000 000 000 /

$$\left(1651969 144908540723200 \right. \\ \left. {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - 30285563 \right. \\ \left. {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2 + 84 \approx 1728.93$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

1728.93 \approx 1729

Alternate forms:

4489 028 449 968 712 332 417 573 157 580 308 480 000 000 000 /

$$\left(30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) - \right. \\ \left. 1651969 144908540723200 \right. \\ \left. {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) \right)^2 + 84$$

$$\left(4 \left(57309043 170326959 586675 858673568 359 383040000 \right. \right. \\ \left. \left. {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) \right)^2 - \right. \\ \left. 2101294 255711 717051 042644 787200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; \right. \right. \\ \left. \left. -\frac{1}{151931373056000} \right) {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right) + \\ \left. 19261521 850766 349 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2 + \\ \left. 1122257 112492 178083 104393 289395 077 120000000000 \right) / \\ \left(1651969 144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - \right. \\ \left. 30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2$$

$$10^2 * 1/6 * [1 / (((12 * \sum_{n=0}^{\infty} ((-1)^n (6n)! (13591409 + 545140134n))) / (((n!)^3 (3n)! (640320^3)^{(n+1/2}))), n = 0..infinity))]^2 - 29 + 4$$

Input interpretation:

$$10^2 \times \frac{1}{6} \left(\frac{1}{12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13591409 + 545140134n)}{(n!)^3 (3n)! (640320^3)^{n+1/2}}} \right)^2 - 29 + 4$$

Result:

$$\frac{448\,902\,844\,996\,871\,233\,241\,757\,315\,758\,030\,848\,000\,000\,000}{\left(1\,651\,969\,144\,908\,540\,723\,200 \cdot {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) - 30\,285\,563 \cdot {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right)\right)^2 - 25} \approx 139.493$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

139.493

Alternate forms:

$$\frac{448\,902\,844\,996\,871\,233\,241\,757\,315\,758\,030\,848\,000\,000\,000}{\left(30\,285\,563 \cdot {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right) - 1\,651\,969\,144\,908\,540\,723\,200 \cdot {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right)\right)^2 - 25}$$

$$\frac{\left(25 \left(-2\,729\,002\,055\,729\,855\,218\,413\,136\,127\,312\,779\,018\,240\,000 \cdot {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right)^2 + 100\,061\,631\,224\,367\,478\,621\,078\,323\,200 \cdot {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) \cdot {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right) - 917\,215\,326\,226\,969 \cdot {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right)^2 + 17\,956\,113\,799\,874\,849\,329\,670\,292\,630\,321\,233\,920\,000\,000\right)}{\left(1\,651\,969\,144\,908\,540\,723\,200 \cdot {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) - 30\,285\,563 \cdot {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right)\right)^2}$$

$$10^2 * 1/6 * [1 / (((12 * \sum_{n=0}^{\infty} ((-1)^n (6n)! (13591409 + 545140134n)) / ((n!)^3 (3n)! (640320^3)^{n+1/2})))]^2 - 29 - 11 + 1/\text{golden ratio}$$

Input interpretation:

$$10^2 \times \frac{1}{6} \left(\frac{1}{12 \sum_{n=0}^{\infty} \frac{(-1)^n (6n)! (13591409 + 545140134n)}{(n!)^3 (3n)! (640320^3)^{n+1/2}}} \right)^2 - 29 - 11 + \frac{1}{\phi}$$

$n!$ is the factorial function

ϕ is the golden ratio

Result:

$$448902844996871233241757315758030848000000000 / \left(1651969144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - 30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2 + \frac{1}{\phi} - 40 \approx 125.111$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

125.111

Alternate forms:

$$448902844996871233241757315758030848000000000 / \left(1651969144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - 30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2 + \frac{1}{2} (\sqrt{5} - 81)$$

$$448902844996871233241757315758030848000000000 / \left(1651969144908540723200 {}_3F_2 \left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151931373056000} \right) - 30285563 {}_3F_2 \left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151931373056000} \right) \right)^2 - \frac{2(19 + 20\sqrt{5})}{1 + \sqrt{5}}$$

$$\begin{aligned}
& -\left(2 \left(51\,851\,039\,058\,867\,249\,149\,849\,586\,418\,942\,801\,346\,560\,000\right. \right. \\
& \quad \left. \left. {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right)^2 + \right. \right. \\
& \quad 54\,580\,041\,114\,597\,104\,368\,262\,722\,546\,255\,580\,364\,800\,000 \\
& \quad \left. \left. \sqrt{5} {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right)^2 - \right. \right. \\
& \quad 1\,901\,170\,993\,262\,982\,093\,800\,488\,140\,800 \\
& \quad \left. \left. {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) \right. \right. \\
& \quad \left. \left. {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right) - \right. \right. \\
& \quad 2\,001\,232\,624\,487\,349\,572\,421\,566\,464\,000 \sqrt{5} \\
& \quad \left. \left. {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) \right. \right. \\
& \quad \left. \left. {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right) + \right. \right. \\
& \quad 17\,427\,091\,198\,312\,411 {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right)^2 + \\
& \quad 18\,344\,306\,524\,539\,380 \sqrt{5} \\
& \quad \left. \left. {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right)^2 - \right. \right. \\
& \quad 224\,451\,422\,498\,435\,616\,620\,878\,657\,879\,015\,424\,000\,000\,000 - \\
& \quad \left. \left. 224\,451\,422\,498\,435\,616\,620\,878\,657\,879\,015\,424\,000\,000\,000 \sqrt{5}\right)\right) / \\
& \left((1 + \sqrt{5}) \left(1651\,969\,144\,908\,540\,723\,200 \right. \right. \\
& \quad \left. \left. {}_3F_2\left(\frac{1}{6}, \frac{1}{2}, \frac{5}{6}; 1, 1; -\frac{1}{151\,931\,373\,056\,000}\right) - \right. \right. \\
& \quad \left. \left. 30\,285\,563 {}_3F_2\left(\frac{7}{6}, \frac{3}{2}, \frac{11}{6}; 2, 2; -\frac{1}{151\,931\,373\,056\,000}\right) \right)^2 \right)
\end{aligned}$$

From

<https://www.scoop.it/topic/amazing-science/p/4073517830/2017/01/02/ramanujan-and-the-world-of-pi>

we have:

A few formulae (out of the many possible ones)



$$\frac{1}{\pi} = \sum_{n=0}^{\infty} \frac{((2n)!)^3 (42n+5)}{2^{12n+4} (n!)^6} \pi = 4 \left(\sum_{n=0}^{\infty} \frac{(-1)^n (4n)! (1123 + 21460n)}{2^{10n+1} (n!)^4 (441)^{2n+1}} \right)^{-1}$$

$$\pi = \frac{9801}{\sqrt{8}} \left(\sum_{n=0}^{\infty} \frac{(4n)! (1103 + 26390n)}{(n!)^4 (396)^{4n}} \right)^{-1}$$

By denoting $(x)_n$ the value : $\prod_{i=0}^{n-1} (x+i) = \frac{\Gamma(x+n)}{\Gamma(x)} = x(x+1)\dots(x+n-1)$ (it's Pochhammer's symbol), we get :

$$\pi = 4 \left(\sum_{n=0}^{\infty} \frac{(6n+1) \left(\frac{1}{2}\right)_x^3}{4^n (n!)^3} \right)^{-1} \quad \pi = 32 \left(\sum_{n=0}^{\infty} \frac{(42\sqrt{5}n + 5\sqrt{5} + 30n - 1) \left(\frac{1}{2}\right)_x^3 \left(\frac{\sqrt{5}-1}{2}\right)^{8n}}{64^n (n!)^3} \right)^{-1}$$

$$\pi = \frac{27}{4} \left(\sum_{n=0}^{\infty} \frac{(15n+2) \left(\frac{1}{2}\right)_x \left(\frac{1}{3}\right)_x \left(\frac{2}{3}\right)_x \left(\frac{2}{27}\right)^n}{(n!)^3} \right)^{-1} \quad \pi = \frac{15\sqrt{3}}{2} \left(\sum_{n=0}^{\infty} \frac{(33n+4) \left(\frac{1}{2}\right)_x \left(\frac{1}{3}\right)_x \left(\frac{2}{3}\right)_x \left(\frac{4}{125}\right)^n}{(n!)^3} \right)^{-1}$$

$$\pi = \frac{5\sqrt{5}}{2\sqrt{3}} \left(\sum_{n=0}^{\infty} \frac{(11n+1) \left(\frac{1}{2}\right)_x \left(\frac{1}{6}\right)_x \left(\frac{5}{6}\right)_x \left(\frac{4}{125}\right)^n}{(n!)^3} \right)^{-1} \quad \pi = \frac{85\sqrt{85}}{18\sqrt{3}} \left(\sum_{n=0}^{\infty} \frac{(133n+8) \left(\frac{1}{2}\right)_x \left(\frac{1}{6}\right)_x \left(\frac{5}{6}\right)_x \left(\frac{4}{85}\right)^n}{(n!)^3} \right)^{-1}$$

$$\pi = \frac{4}{\sqrt{3}} \left(\sum_{n=0}^{\infty} \frac{(-1)^n (28n+3) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 3^n 4^{2n+1}} \right)^{-1} \quad \pi = 4 \left(\sum_{n=0}^{\infty} \frac{(-1)^n (20n+3) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 2^{2n+1}} \right)^{-1}$$

$$\pi = \frac{4}{\sqrt{5}} \left(\sum_{n=0}^{\infty} \frac{(-1)^n (644n+41) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 5^n 72^{2n+1}} \right)^{-1} \quad \pi = 4 \left(\sum_{n=0}^{\infty} \frac{(-1)^n (260n+23) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 18^{2n+1}} \right)^{-1}$$

$$\pi = \frac{1}{2\sqrt{2}} \left(\sum_{n=0}^{\infty} \frac{(10n+1) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 9^{2n+1}} \right)^{-1} \quad \pi = 2\sqrt{3} \left(\sum_{n=0}^{\infty} \frac{(8n+1) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 9^n} \right)^{-1}$$

$$\pi = \frac{1}{3\sqrt{3}} \left(\sum_{n=0}^{\infty} \frac{(40n+3) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 49^{2n+1}} \right)^{-1} \quad \pi = \frac{2}{\sqrt{11}} \left(\sum_{n=0}^{\infty} \frac{(280n+19) \left(\frac{1}{2}\right)_x \left(\frac{1}{4}\right)_x \left(\frac{3}{4}\right)_x}{(n!)^3 99^{2n+1}} \right)^{-1}$$

Developing the above formulas, we obtain various expressions.

$$0.0938864 * [2/(\sqrt{11}) 1/((((sum (((280n+19)((1/2)(1/4)(3/4)))))/((n!)^3 99^{(2n+1)})), n = 0..infinity)))]$$

Input interpretation:

$$0.0938864 \left(\frac{2}{\sqrt{11}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(280n+19) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 99^{2n+1}}} \right)$$

n! is the factorial function

Result:

3.14159

3.14159 $\approx \pi$

$$0.0942569 * 1/(3\sqrt{3}) [1/((((sum (((40n+3)((1/2)(1/4)(3/4)))))/((n!)^3 49^{(2n+1)})), n = 0..infinity)))]$$

Input interpretation:

$$0.0942569 \times \frac{1}{3\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(40n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 49^{2n+1}}}$$

n! is the factorial function

Result:

3.14159

3.14159 $\approx \pi$

$$0.105167 * 1/(2\sqrt{2}) [1/((((sum (((10n+1)((1/2)(1/4)(3/4)))))/((n!)^3 9^{(2n+1)})), n = 0..infinity)))]$$

Input interpretation:

$$0.105167 \times \frac{1}{2\sqrt{2}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(10n+1) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 9^{2n+1}}}$$

n! is the factorial function

Result:

3.14159

3.14159 ≈ π

0.516863 (2/(sqrt3)) [1/((((sum (((8n+1) (((1/2)(1/4)(3/4)))))))/(((n!)^3 9^(n))), n = 0..infinity)))]

Input interpretation:

$$0.516863 \times \frac{2}{\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(8n+1) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 9^n}}$$

n! is the factorial function

Result:

3.14159

3.14159 ≈ π

0.0936952*[4/(sqrt5) 1/((((sum (((-1)^n(644n+41) (((1/2)(1/4)(3/4)))))))/(((n!)^3 5^n 72^(2n+1))), n = 0..infinity)))]

Input interpretation:

$$0.0936952 \left(\frac{4}{\sqrt{5}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (644n+41) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 5^n \times 72^{2n+1}}} \right)$$

n! is the factorial function

Result:

3.14159

3.14159 ≈ π

$$0.0905138 * [4 * 1 / (((((\sum_{n=0}^{\infty} ((-1)^n (260n+23) (((1/2)(1/4)(3/4)))))) / (((n!)^3 18^{2n+1}))), n = 0..infinity)))]$$

Input interpretation:

$$0.0905138 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (260n+23) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 18^{2n+1}}}\right)$$

$n!$ is the factorial function

Result:

3.14159

3.14159 $\approx \pi$

$$0.0149102 [4 / (\sqrt{3}) * 1 / (((((\sum_{n=0}^{\infty} ((-1)^n (28n+3) (((1/2)(1/4)(3/4)))))) / (((n!)^3 3^n 4^{n+1}))), n = 0..infinity)))]$$

Input interpretation:

$$0.0149102 \left(\frac{4}{\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (28n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 3^n \times 4^{n+1}}}\right)$$

$n!$ is the factorial function

Result:

3.1416

3.1416 $\approx \pi$

$$-0.0890419 [4 * 1 / (((((\sum_{n=0}^{\infty} ((-1)^n (20n+3) (((1/2)(1/4)(3/4)))))) / (((n!)^3 2^{2n+1}))), n = 0..infinity)))]$$

Input interpretation:

$$-0.0890419 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (20n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 2^{2n+1}}}\right)$$

$n!$ is the factorial function

Result:

3.14159

$3.14159 \approx \pi$

$0.0937526 \left[\frac{5\sqrt{5}}{2\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(11n+1) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{125}\right)^n} \right]$
 $\left(\frac{(((((11n+1)((1/2)(1/6)(5/6))))))}{((n!)^3)} * (4/125)^n \right), n = 0..infinity \right)$

Input interpretation:

$$0.0937526 \left(\frac{5\sqrt{5}}{2\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(11n+1) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{125}\right)^n} \right)$$

$n!$ is the factorial function

Result:

3.14159

$3.14159 \approx \pi$

$0.127686 \left[\frac{85\sqrt{85}}{18\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(133n+8) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{85}\right)^n} \right]$
 $\left(\frac{(((((133n+8)((1/2)(1/6)(5/6))))))}{((n!)^3)} * (4/85)^n \right), n = 0..infinity \right)$

Input interpretation:

$$0.127686 \left(\frac{85\sqrt{85}}{18\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(133n+8) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{85}\right)^n} \right)$$

$n!$ is the factorial function

Result:

3.14159

$3.14159 \approx \pi$

$$0.169687 * [(27/4)^* 1/((((sum$$

$$((((15n+2)((1/2)(1/3)(2/3)))))/(((n!)^3))*(2/27)^n), n = 0..infinity)))]$$

Input interpretation:

$$0.169687 \left(\frac{27}{4} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(15n+2) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}\right)}{(n!)^3} \left(\frac{2}{27}\right)^n} \right)$$

$n!$ is the factorial function

Result:

3.14159

3.14159 $\approx \pi$

$$0.139541 * [((15\sqrt{3})/2)^* 1/((((sum$$

$$((((33n+4)((1/2)(1/3)(2/3)))))/(((n!)^3))*(4/125)^n), n = 0..infinity)))]$$

Input interpretation:

$$0.139541 \left(\frac{1}{2} (15\sqrt{3}) \times \frac{1}{\sum_{n=0}^{\infty} \frac{(33n+4) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}\right)}{(n!)^3} \left(\frac{4}{125}\right)^n} \right)$$

$n!$ is the factorial function

Result:

3.14159

3.14159 $\approx \pi$

From this expression, we obtain:

$$x/10^3 [((15\sqrt{3})/2)^* 1/((((sum$$

$$((((33n+4)((1/2)(1/3)(2/3)))))/(((n!)^3))*(4/125)^n), n = 0..infinity)))] = \pi$$

Input interpretation:

$$\frac{x}{10^3} \left(\frac{1}{2} (15 \sqrt{3}) \right) \times \frac{1}{\sum_{n=0}^{\infty} \frac{(33n+4) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}\right) \left(\frac{4}{125}\right)^n}{(n!)^3}} = \pi$$

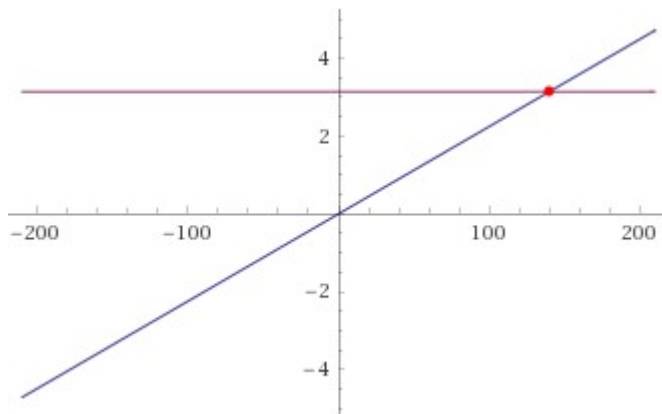
$n!$ is the factorial function

Result:

$$\frac{3 \sqrt{3} x}{400 \left(\frac{{}_4F_2\left(; 1, 1; \frac{4}{125}\right)}{9} + \frac{{}_4F_2\left(; 2, 2; \frac{4}{125}\right)}{375} \right)} = \pi$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

Plot:



— $\frac{135 \sqrt{3} x}{8000 {}_0F_2\left(; 1, 1; \frac{4}{125}\right) + 2112 {}_0F_2\left(; 2, 2; \frac{4}{125}\right)}$
 — π

Alternate forms:

$$\frac{135 \sqrt{3} x}{8000 {}_0F_2\left(; 1, 1; \frac{4}{125}\right) + 2112 {}_0F_2\left(; 2, 2; \frac{4}{125}\right)} = \pi$$

$$\frac{135 \sqrt{3} x}{64 \left(125 {}_0F_2\left(; 1, 1; \frac{4}{125}\right) + 33 {}_0F_2\left(; 2, 2; \frac{4}{125}\right) \right)} = \pi$$

Solution:

$x \approx 139.541$

139.541

$$0.280087 [4 * 1 / \sum ((6n+1)(1/2)^3) / ((4^n (n!)^3)), n = 0..infinity]$$

Input interpretation:

$$0.280087 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(6n+1) \left(\frac{1}{2}\right)^3}{4^n (n!)^3}} \right)$$

$n!$ is the factorial function

Result:

3.14159

$$3.14159 \approx \pi$$

In conclusion, we have:

$$[32 * 1 / \sum (((42\sqrt{5})n + 5\sqrt{5} + 30n - 1)(1/2)^3) / ((64^n (n!)^3)) ((\sqrt{5}-1)/2)^{(8n)}, n = 0..infinity] x = \pi$$

Input interpretation:

$$\left(32 \times \frac{1}{\sum_{n=0}^{\infty} \frac{((42\sqrt{5})n + 5\sqrt{5} + 30n - 1) \left(\frac{1}{2}\right)^3}{64^n (n!)^3} \left(\frac{1}{2}(\sqrt{5}-1)\right)^{8n}} \right) x = \pi$$

$n!$ is the factorial function

Result:

$$(4096 x) / \left(-16 {}_0F_2 \left(; 1, 1; \frac{(-1+\sqrt{5})^8}{16384} \right) + 80 \sqrt{5} {}_0F_2 \left(; 1, 1; \frac{(-1+\sqrt{5})^8}{16384} \right) - 375 {}_0F_2 \left(; 2, 2; \frac{(-1+\sqrt{5})^8}{16384} \right) + 168 \sqrt{5} {}_0F_2 \left(; 2, 2; \frac{(-1+\sqrt{5})^8}{16384} \right) \right) = \pi$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

Solution:

$$x = \frac{\pi \left(16(5\sqrt{5}-1) {}_0F_2 \left(; 1, 1; \frac{(-1+\sqrt{5})^8}{16384} \right) + 3(56\sqrt{5}-125) {}_0F_2 \left(; 2, 2; \frac{(-1+\sqrt{5})^8}{16384} \right) \right)}{4096}$$

Solution:

$x \approx 0.125479$

$0.125479 [32 * 1 / \sum_{n=0}^{\infty} (((42\sqrt{5})n + 5\sqrt{5} + 30n - 1)(1/2)^3) / ((64^n (n!)^3)) ((\sqrt{5} - 1)/2)^{(8n)}, n = 0..infinity]$

Input interpretation:

$$0.125479 \left(32 \times \frac{1}{\sum_{n=0}^{\infty} \frac{((42\sqrt{5})n + 5\sqrt{5} + 30n - 1)\left(\frac{1}{2}\right)^3}{64^n (n!)^3} \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8n}} \right)$$

n! is the factorial function

Result:

3.14159

$3.14159 \approx \pi$

From this expression, we obtain:

$x/10^3 [32 * 1 / \sum_{n=0}^{\infty} (((42\sqrt{5})n + 5\sqrt{5} + 30n - 1)(1/2)^3) / ((64^n (n!)^3)) ((\sqrt{5} - 1)/2)^{(8n)}, n = 0..infinity] = \pi$

Input interpretation:

$$\frac{x}{10^3} \left(32 \times \frac{1}{\sum_{n=0}^{\infty} \frac{((42\sqrt{5})n + 5\sqrt{5} + 30n - 1)\left(\frac{1}{2}\right)^3}{64^n (n!)^3} \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8n}} \right) = \pi$$

n! is the factorial function

Result:

$$(512x) / \left(125 \left(-16 {}_0F_2 \left(; 1, 1; \frac{(-1+\sqrt{5})^8}{16384} \right) + 80\sqrt{5} {}_0F_2 \left(; 1, 1; \frac{(-1+\sqrt{5})^8}{16384} \right) - 375 {}_0F_2 \left(; 2, 2; \frac{(-1+\sqrt{5})^8}{16384} \right) + 168\sqrt{5} {}_0F_2 \left(; 2, 2; \frac{(-1+\sqrt{5})^8}{16384} \right) \right) \right) = \pi$$

${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function

Solution:

$$x \approx 125.479$$

125.479

$$(9801/\sqrt{8}) \frac{1}{\sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{(26390n+1103)}{396^{4n}}} = \pi$$

Input interpretation:

$$\frac{9801}{\sqrt{8}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n+1103}{396^{4n}}}$$

$n!$ is the factorial function

Result:

$$\pi \approx 3.14159$$

3.14159 $\approx \pi$

From the sum of all 15 results, we obtain:

$$\left[0.0938864 \left(\frac{2}{\sqrt{11}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(280n+19) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 99^{2n+1}} \right) \right] + \left[0.105167 \times \frac{1}{2\sqrt{2}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(10n+1) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 9^{2n+1}} \right] +$$

$$\left[0.0942569 \times \frac{1}{3\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(40n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 49^{2n+1}} \right] + \left[0.516863 \times \frac{2}{\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(8n+1) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 9^n} \right] +$$

$$\left[0.0936952 \left(\frac{4}{\sqrt{5}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (644n+41) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 5^n \times 72^{2n+1}} \right) \right] + \left[0.0905138 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (260n+23) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 18^{2n+1}} \right) \right] +$$

$$\left[0.0149102 \left(\frac{4}{\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (28n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 3^n \times 4^{n+1}} \right) \right] + \left[-0.0890419 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(-1)^n (20n+3) \left(\frac{1}{2} \times \frac{1}{4} \times \frac{3}{4}\right)}{(n!)^3 \times 2^{2n+1}} \right) \right] +$$

$$\left[0.0937526 \left(\frac{5\sqrt{5}}{2\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(11n+1) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{125}\right)^n} \right) \right] + \left[0.127686 \left(\frac{85\sqrt{85}}{18\sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(133n+8) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{85}\right)^n} \right) \right] +$$

$$\left[0.169687 \left(\frac{27}{4} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(15n+2) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}\right)}{(n!)^3} \left(\frac{2}{27}\right)^n} \right) \right] + \left[0.139541 \left(\frac{1}{2} (15\sqrt{3}) \right) \times \frac{1}{\sum_{n=0}^{\infty} \frac{(33n+4) \left(\frac{1}{2} \times \frac{1}{3} \times \frac{2}{3}\right)}{(n!)^3} \left(\frac{4}{125}\right)^n} \right) \right] +$$

$$\left[0.280087 \left(4 \times \frac{1}{\sum_{n=0}^{\infty} \frac{(6n+1) \left(\frac{1}{2}\right)^3}{4^n (n!)^3} \right) \right] + \left[0.125479 \left(32 \times \frac{1}{\sum_{n=0}^{\infty} \frac{((42\sqrt{5})n + 5\sqrt{5} + 30n - 1) \left(\frac{1}{2}\right)^3}{64^n (n!)^3} \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{8n} \right) \right] +$$

$$\left[\frac{9801}{\sqrt{8}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(4n)!}{(n!)^4} \times \frac{26390n + 1103}{396^{4n}}} \right] = 14 * 3.14159 + 3.1416 = 47.12386$$

(note that $47.12386 \approx 47 = \text{Lucas number}$)

and:

$$(14 \cdot 3.14159 + 3.1416)^{1/8}$$

Input interpretation:

$$\sqrt[8]{14 \times 3.14159 + 3.1416}$$

Result:

1.618658033891464475132482633593636548278222727572148834138...

1.6186580338...

If we take the complete result, i.e. π , we obtain:

$$(15 \cdot \pi)^{1/8}$$

Input:

$$\sqrt[8]{15 \pi}$$

Decimal approximation:

1.618658161858016012453671344191568700313194369911074132746...

1.61865816...

Property:

$\sqrt[8]{15 \pi}$ is a transcendental number

All 8th roots of 15π :

$$\sqrt[8]{15 \pi} e^{i0} \approx 1.61866 \quad (\text{real, principal root})$$

$$\sqrt[8]{15 \pi} e^{(i\pi)/4} \approx 1.14456 + 1.14456 i$$

$$\sqrt[8]{15 \pi} e^{(i\pi)/2} \approx 1.61866 i$$

$$\sqrt[8]{15 \pi} e^{(3i\pi)/4} \approx -1.1446 + 1.14456 i$$

$$\sqrt[8]{15 \pi} e^{i\pi} \approx -1.6187 \quad (\text{real root})$$

Alternative representations:

$$\sqrt[8]{15 \pi} = \sqrt[8]{2700^\circ}$$

$$\sqrt[8]{15\pi} = \sqrt[8]{-15i \log(-1)}$$

$$\sqrt[8]{15\pi} = \sqrt[8]{15 \cos^{-1}(-1)}$$

Series representations:

$$\sqrt[8]{15\pi} = \sqrt[4]{2} \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\sqrt[8]{15\pi} = \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$\sqrt[8]{15\pi} = \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)}$$

Integral representations:

$$\sqrt[8]{15\pi} = \sqrt[8]{30} \sqrt[8]{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\sqrt[8]{15\pi} = \sqrt[8]{30} \sqrt[8]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

$$\sqrt[8]{15\pi} = \sqrt[4]{2} \sqrt[8]{15} \sqrt[8]{\int_0^1 \sqrt{1-t^2} dt}$$

Furthermore, we obtain:

$$\left(\left(\left(15 \cdot 0.127686 \left[\frac{(85\sqrt{85})}{(18\sqrt{3})} \cdot \frac{1}{\left(\left(\left(\sum_{n=0}^{\infty} \frac{((133n+8)((1/2)(1/6)(5/6)))}{((n!)^3)}\right) \cdot (4/85)^n\right)}\right)\right]\right)\right)^{1/8} + \frac{\pi}{85}\right)$$

Input interpretation:

$$\sqrt[8]{15 \times 0.127686 \left(\frac{85 \sqrt{85}}{18 \sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(133n+8) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right) \left(\frac{4}{85}\right)^n}{(n!)^3}} \right) + \frac{\pi}{85}}$$

$n!$ is the factorial function

Result:

1.65562

1.65562 result practically equal to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

That is:

$$(15 \cdot \pi)^{1/8} + \pi/85$$

Input:

$$\sqrt[8]{15 \pi} + \frac{\pi}{85}$$

Decimal approximation:

1.655618075429660638788525972230151087186102245197840083581...

1.655618075429... as above

Alternate forms:

$$\frac{1}{85} \left(\pi + 85 \sqrt[8]{15 \pi} \right)$$

$$\frac{1}{85} \left(85 \sqrt[8]{15} + \pi^{7/8} \right) \sqrt[8]{\pi}$$

Alternative representations:

$$\sqrt[8]{15 \pi} + \frac{\pi}{85} = \frac{180^\circ}{85} + \sqrt[8]{2700^\circ}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = -\frac{1}{85} i \log(-1) + \sqrt[8]{-15 i \log(-1)}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \cos^{-1}(-1) + \sqrt[8]{15 \cos^{-1}(-1)}$$

Series representations:

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \left(85 \sqrt[4]{2} \sqrt[8]{15} + 4 \left(\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)^{7/8} \right) \sqrt[8]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \sum_{k=0}^{\infty} \frac{4(-1)^k (956 \times 5^{-2k} - 5 \times 239^{-2k})}{101575 (1+2k)} + \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \left(85 \sqrt[8]{15} + \left(\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right) \right)^{7/8} \right) \sqrt[8]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k} \right)}$$

Integral representations:

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \left(85 \sqrt[8]{30} + 2 \left(\int_0^{\infty} \frac{1}{1+t^2} dt \right)^{7/8} \right) \sqrt[8]{\int_0^{\infty} \frac{1}{1+t^2} dt}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \left(85 \sqrt[8]{30} + 2 \left(\int_0^{\infty} \frac{\sin(t)}{t} dt \right)^{7/8} \right) \sqrt[8]{\int_0^{\infty} \frac{\sin(t)}{t} dt}$$

$$\sqrt[8]{15\pi} + \frac{\pi}{85} = \frac{1}{85} \left(85 \sqrt[8]{30} + 2 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^{7/8} \right) \sqrt[8]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt}$$

From which:

$$10^3 * (((15 * \pi)^{1/8} + \pi/85)) + 76 - 2$$

Input:

$$10^3 \left(\sqrt[8]{15 \pi} + \frac{\pi}{85} \right) + 76 - 2$$

Exact result:

$$74 + 1000 \left(\frac{\pi}{85} + \sqrt[8]{15 \pi} \right)$$

Decimal approximation:

1729.618075429660638788525972230151087186102245197840083581...

[1729.618075429...](#)

Alternate forms:

$$74 + \frac{200 \pi}{17} + 1000 \sqrt[8]{15 \pi}$$

$$74 + \frac{200}{17} \left(\pi + 85 \sqrt[8]{15 \pi} \right)$$

$$\frac{2}{17} \left(629 + 100 \pi + 8500 \sqrt[8]{15 \pi} \right)$$

Alternative representations:

$$10^3 \left(\sqrt[8]{15 \pi} + \frac{\pi}{85} \right) + 76 - 2 = 74 + 10^3 \left(\frac{180^\circ}{85} + \sqrt[8]{2700^\circ} \right)$$

$$10^3 \left(\sqrt[8]{15 \pi} + \frac{\pi}{85} \right) + 76 - 2 = 74 + 10^3 \left(-\frac{1}{85} i \log(-1) + \sqrt[8]{-15 i \log(-1)} \right)$$

$$10^3 \left(\sqrt[8]{15 \pi} + \frac{\pi}{85} \right) + 76 - 2 = 74 + 10^3 \left(\frac{1}{85} \cos^{-1}(-1) + \sqrt[8]{15 \cos^{-1}(-1)} \right)$$

Series representations:

$$10^3 \left(\sqrt[8]{15 \pi} + \frac{\pi}{85} \right) + 76 - 2 = \frac{2}{17} \left(629 + 8500 \sqrt[4]{2} \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}} + 400 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 =$$

$$\frac{2}{17} \left(629 + 8500 \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)} \right) +$$

$$100 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 =$$

$$74 + 1000 \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}} +$$

$$\sum_{k=0}^{\infty} \frac{160(-1)^k 239^{-1-2k} (-5 + 4 \times 5^{-2k} \times 239^{1+2k})}{17(1+2k)}$$

Integral representations:

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 =$$

$$\frac{2}{17} \left(629 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^{\infty} \frac{1}{1+t^2} dt + 200 \int_0^{\infty} \frac{1}{1+t^2} dt} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 =$$

$$\frac{2}{17} \left(629 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt + 200 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 =$$

$$\frac{2}{17} \left(629 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^{\infty} \frac{\sin(t)}{t} dt + 200 \int_0^{\infty} \frac{\sin(t)}{t} dt} \right)$$

and again:

$$10^3 * (((15 * \text{Pi})^{1/8} + \text{Pi}/85)) + 76 - 2 - (10^3 + 9^3 - 1)$$

Input:

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1)$$

Exact result:

$$1000 \left(\frac{\pi}{85} + \sqrt[8]{15\pi} \right) - 1654$$

Decimal approximation:

1.618075429660638788525972230151087186102245197840083581498...

1.618075429...

Alternate forms:

$$-1654 + \frac{200\pi}{17} + 1000 \sqrt[8]{15\pi}$$

$$\frac{200}{17} \left(\pi + 85 \sqrt[8]{15\pi} \right) - 1654$$

$$\frac{2}{17} \left(-14059 + 100\pi + 8500 \sqrt[8]{15\pi} \right)$$

Alternative representations:

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) = 75 - 9^3 - 10^3 + 10^3 \left(\frac{180^\circ}{85} + \sqrt[8]{2700^\circ} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) = 75 - 9^3 - 10^3 + 10^3 \left(-\frac{1}{85} i \log(-1) + \sqrt[8]{-15 i \log(-1)} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) = 75 - 9^3 - 10^3 + 10^3 \left(\frac{1}{85} \cos^{-1}(-1) + \sqrt[8]{15 \cos^{-1}(-1)} \right)$$

Series representations:

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) = \frac{2}{17} \left(-14059 + 8500 \sqrt[4]{2} \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}} + 400 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) =$$

$$\frac{2}{17} \left(-14\,059 + 8500 \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)} + \right.$$

$$\left. 100 \sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right) \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) =$$

$$-1654 + 1000 \sqrt[8]{15} \sqrt[8]{\sum_{k=0}^{\infty} -\frac{4(-1)^k 1195^{-1-2k} (5^{1+2k} - 4 \times 239^{1+2k})}{1+2k}} +$$

$$\sum_{k=0}^{\infty} \frac{160(-1)^k 239^{-1-2k} (-5 + 4 \times 5^{-2k} \times 239^{1+2k})}{17(1+2k)}$$

Integral representations:

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) =$$

$$\frac{2}{17} \left(-14\,059 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^{\infty} \frac{1}{1+t^2} dt} + 200 \int_0^{\infty} \frac{1}{1+t^2} dt \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) =$$

$$\frac{2}{17} \left(-14\,059 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^1 \frac{1}{\sqrt{1-t^2}} dt} + 200 \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)$$

$$10^3 \left(\sqrt[8]{15\pi} + \frac{\pi}{85} \right) + 76 - 2 - (10^3 + 9^3 - 1) =$$

$$\frac{2}{17} \left(-14\,059 + 8500 \sqrt[8]{30} \sqrt[8]{\int_0^{\infty} \frac{\sin(t)}{t} dt} + 200 \int_0^{\infty} \frac{\sin(t)}{t} dt \right)$$

From:

Primordial Black Holes - Perspectives in Gravitational Wave Astronomy -
Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama -
 arXiv:1801.05235v1 [astro-ph.CO] 16 Jan 2018

4.4.3 Mass distribution

Masses of the individual BHs before the merger are $(m_1, m_2) = (35, 30)$ for GW150914, $(m_1, m_2) = (14, 8)$ for GW151226, $(m_1, m_2) = (31, 19)$ for GW170104, $(m_1, m_2) = (12, 7)$ for GW170608, and $(m_1, m_2) = (30, 25)$ for GW170814 in units of solar mass. Obviously, there is some spread in the mass distribution. It is natural to think that the event rate distribution in the 2-dimensional mass plane should reflect to a certain degree the formation mechanism of the BH binaries and its statistical nature can be used to discriminate different formation scenarios. Although merger

and, we have that:

Thus, the energy loss during one orbital period T becomes

$$\Delta E = -T \left\langle \frac{dE}{dt} \right\rangle = \frac{64\pi \sqrt{G(m_1 + m_2)} G^3 m_1^2 m_2^2}{5r_p^{7/2} (1+e)^{7/2}} \left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4 \right). \quad (142)$$

where we have used the Kepler's third law and $r_p = a(1-e)$. We can approximate the trajectory of the close encounter by the ellipse with $e = 1$ since the true trajectory is physically indistinguishable from the ellipse near the periastron where the dominant emission of GWs occurs. Then, the energy loss by the close-encounter is obtained by plugging $e = 1$ into the above equation,

$$\Delta E = \frac{85\pi \sqrt{G(m_1 + m_2)} G^3 m_1^2 m_2^2}{12\sqrt{2}r_p^{7/2}}. \quad (143)$$

If this energy is greater than the kinetic energy $\mu v^2/2$, where μ is the reduced mass and v is the relative velocity at large separation, then the PBHs form a binary. This imposes a condition on r_p as

$$r_p < r_{p,\max} = \left[\frac{85\pi G^{7/2} (m_1 + m_2)^{3/2} m_1 m_2}{6\sqrt{2} v^2} \right] \quad (144)$$

In the Newtonian approximation, relation between b and r_p is given by

$$b^2(r_p) = r_p^2 + \frac{2GM r_p}{v^2}. \quad (145)$$

The encounter with the impact parameter less than $b(r_{p,\max})$ yields a binary. In the limit of the strong gravitational focusing ($r_p \ll b$), which we are interested in, the cross section for forming a binary becomes

$$\sigma = \pi b^2(r_{p,\max}) \simeq \left(\frac{85\pi}{3} \right)^{2/7} \frac{\pi (2GM_{\text{PBH}})^2}{v^{18/7}}. \quad (146)$$

Contrary to the PBH binaries that are formed in the radiation dominated epoch, the PBH binaries produced by the present mechanism merge in less than the age of the Universe [247].

For $m_1 = 30$ and $m_2 = 25$, from the above formulas, i.e.

$$\Delta E = \frac{85\pi\sqrt{G(m_1 + m_2)}G^3m_1^2m_2^2}{12\sqrt{2}r_p^{7/2}}. \quad (143)$$

and

$$r_p < r_{p,\max} = \left[\frac{85\pi G^{7/2}(m_1 + m_2)^{3/2}m_1m_2}{6\sqrt{2}v^2} \right] \quad (144)$$

we obtain:

$$\frac{(((85\pi * ((\sqrt{6.67408e-11(30+25)})) * (6.67408e-11)^3 * (30^2 * 25^2))))}{(((12\sqrt{2} * [(85 * \pi) / (6\sqrt{2})] * (6.67408e-11)^{3.5} (30+25)^{1.5} (30 * 25) / (v^2)]^{3.5}))}}$$

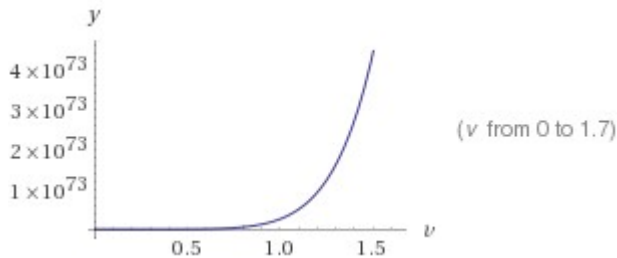
Input interpretation:

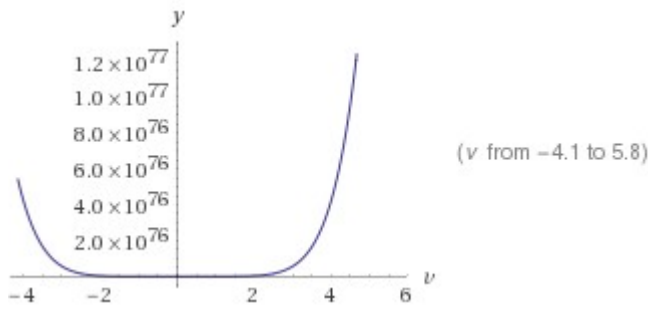
$$\frac{85\pi\sqrt{6.67408 \times 10^{-11}(30+25)}(6.67408 \times 10^{-11})^3(30^2 \times 25^2)}{12\sqrt{2}\left(\frac{85\pi}{6\sqrt{2}}(6.67408 \times 10^{-11})^{3.5}(30+25)^{1.5} \times \frac{30 \times 25}{v^2}\right)^{3.5}}$$

Result:

$$\frac{2.57914 \times 10^{72}}{\left(\frac{1}{v^2}\right)^{3.5}}$$

Plots:





Alternate form:

$$\frac{2.57914 \times 10^{72}}{\left(\frac{1}{v^2}\right)^{7/2}}$$

Alternate form assuming v>0:

$$2.57914 \times 10^{72} v^7$$

Alternate forms assuming v is real:

$$\frac{2.57914 \times 10^{72}}{\left(\frac{1}{v^4}\right)^{1.75}}$$

$$2.57914 \times 10^{72} |v|^7$$

|z| is the absolute value of z

Roots:

(no roots exist)

Property as a function:

Parity

even

Derivative:

$$\frac{d}{dv} \left(2579135372783871835320018702343985859009863671735176829617 \cdot \frac{662840855855104}{\left(\frac{1}{v^2}\right)^{3.5}} \right) = 18053947609487101670283555531405257793858528850708784788131 \cdot \frac{994048984514560}{\left(\left(\frac{1}{v^2}\right)^{4.5} v^3\right)}$$

Indefinite integral:

$$\int \frac{85 \pi \sqrt{6.67408 \times 10^{-11} (30 + 25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{(85 \pi) (6.67408 \times 10^{-11})^{3.5} (30+25)^{1.5} (30 \times 25)}{(6 \sqrt{2}) v^2} \right)^{3.5}} dv =$$

$$\frac{3.22392 \times 10^{71} v}{\left(\frac{1}{v^2}\right)^{7/2}} + \text{constant}$$

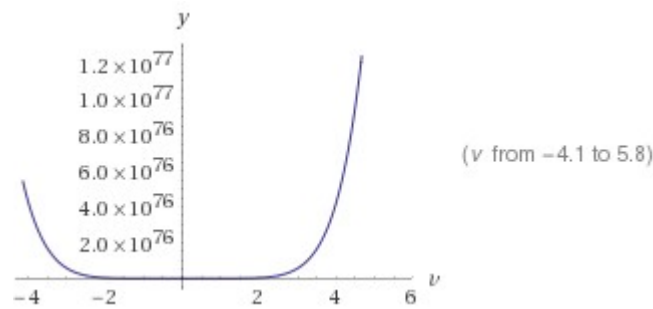
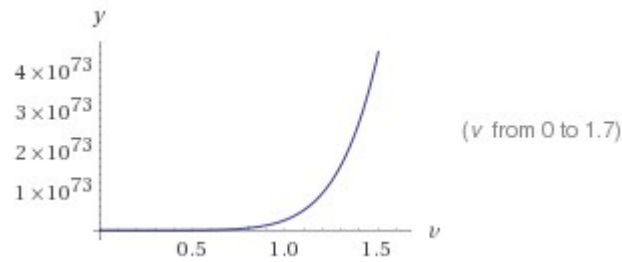
Local minimum:

$$\min\left\{2579\ 135\ 372\ 783\ 871\ 835\ 320\ 018\ 702\ 343\ 985\ 859\ 009\ 863\ 671\ 735\ 176\ 829\ ;\right.$$

$$\left.617\ 662\ 840\ 855\ 855\ 104 / \left(\frac{1}{v^2}\right)^{3.5}\right\} = 0 \text{ at } v = 0$$

We have that:

Plots:



For $v = 1$; we obtain: (a)

Input interpretation:

$$\frac{85 \pi \sqrt{6.67408 \times 10^{-11} (30 + 25) (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} (30 \times 25) \right)^{3.5}}$$

Result:

$$2.57914... \times 10^{72}$$

$$2.57914 * 10^{72}$$

For $v = 1.7$, we obtain:

Input interpretation:

$$\frac{85 \pi \sqrt{6.67408 \times 10^{-11} (30 + 25) (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \times \frac{30 \times 25}{1.7^2} \right)^{3.5}}$$

Result:

$$1.0583189863554945721218470814322020536231213103000582... \times 10^{74}$$

$$1.0583189... * 10^{74}$$

For $v = -4.1$, we obtain:

Input interpretation:

$$\frac{85 \pi \sqrt{6.67408 \times 10^{-11} (30 + 25) (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \left(-\frac{30 \times 25}{4.1^2} \right) \right)^{3.5}}$$

Result:

$$5.02298... \times 10^{76} i$$

Polar coordinates:

$$r = 5.02298 \times 10^{76} \text{ (radius), } \theta = 90.^\circ \text{ (angle)}$$

$$5.02298 * 10^{76}$$

For $v = 5.8$, we obtain: (b)

Input interpretation:

$$\frac{85 \pi \sqrt{6.67408 \times 10^{-11} (30 + 25) (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \times \frac{30 \times 25}{5.8^2} \right)^{3.5}}$$

Result:

$$5.69469... \times 10^{77}$$

$$5.69469 * 10^{77}$$

From (a), we obtain:

$$\frac{(((85x * ((\sqrt{6.67408e-11(30+25)})) * (6.67408e-11)^3 * (30^2 * 25^2))))}{(((12\sqrt{2} * [(85 * \pi) / (6\sqrt{2}) * (6.67408e-11)^{3.5} (30+25)^{1.5} (30 * 25)^{3.5}])))} = 2.57913537e+72$$

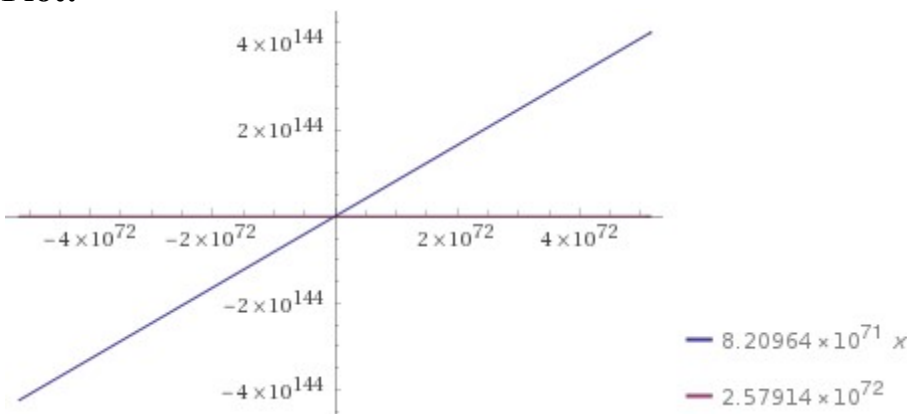
Input interpretation:

$$\frac{85 x \sqrt{6.67408 \times 10^{-11} (30 + 25) (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} (30 \times 25) \right)^{3.5}} = 2.57913537 \times 10^{72}$$

Result:

$$8.20964 \times 10^{71} x = 2.57914 \times 10^{72}$$

Plot:



Alternate form:

$$8.20964 \times 10^{71} x - 2.57914 \times 10^{72} = 0$$

Alternate form assuming x is real:

$$8.20964 \times 10^{71} x + 0 = 2.57914 \times 10^{72}$$

Solution:

$$x \approx 3.14159$$

$$3.14159 \approx \pi$$

$$\left(\frac{\frac{x-29}{20} \pi \sqrt{6.67408 \times 10^{-11} (30+25)}}{(6.67408 \times 10^{-11})^3 (30^2 \times 25^2)} \right) / \left(\frac{12 \sqrt{2} \left[\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30+25)^{1.5} (30 \times 25) \right]^{3.5}}{(6.67408 \times 10^{-11})^3 (30^2 \times 25^2)} \right) = 2.57913537 \times 10^{72}$$

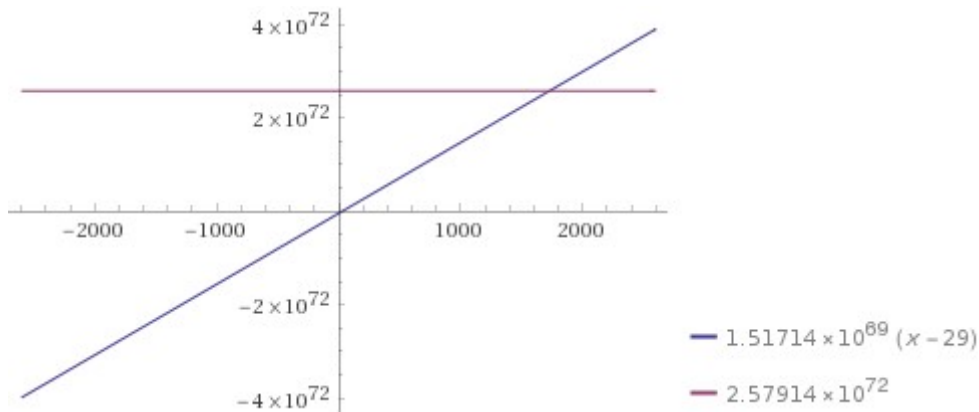
Input interpretation:

$$\frac{\frac{x-29}{20} \pi \sqrt{6.67408 \times 10^{-11} (30+25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30+25)^{1.5} (30 \times 25) \right)^{3.5}} = 2.57913537 \times 10^{72}$$

Result:

$$1.51714 \times 10^{69} (x - 29) = 2.57914 \times 10^{72}$$

Plot:



Alternate forms:

$$1.51714 \times 10^{69} x - 2.62313 \times 10^{72} = 0$$

$$1.51714 \times 10^{69} x - 4.3997 \times 10^{70} = 2.57914 \times 10^{72}$$

From (b), we obtain:

$$\begin{aligned} & \left(\left(\left(85x \cdot \left(\sqrt{6.67408e-11(30+25)} \right) \right) \cdot \left(6.67408e-11 \right)^3 \cdot \left(30^2 \cdot 25^2 \right) \right) \right) / \\ & \left(\left(\left(12\sqrt{2} \cdot \left[\frac{85 \cdot \pi}{6\sqrt{2}} \right] \cdot \left(6.67408e-11 \right)^{3.5} \cdot \left(30+25 \right)^{1.5} \cdot \left(30 \cdot 25 \right) / \left(5.8^2 \right) \right]^{3.5} \right) \right) \right) \\ & = 5.69469e+77 \end{aligned}$$

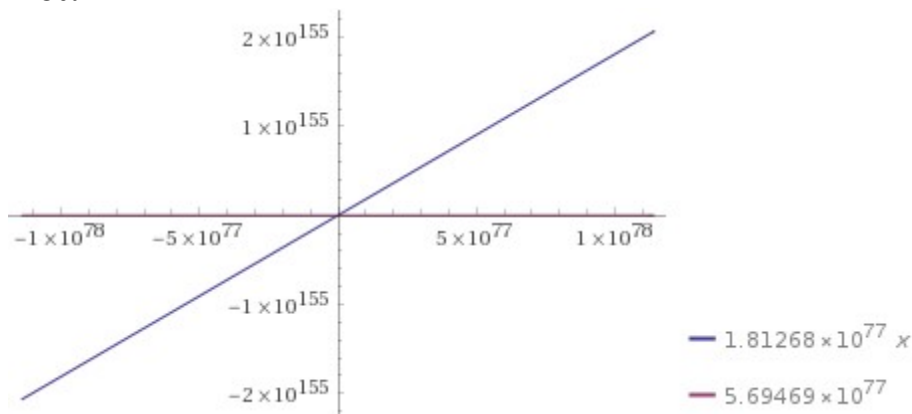
Input interpretation:

$$\frac{85x \sqrt{6.67408 \times 10^{-11} (30 + 25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{85 \pi}{6\sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \times \frac{30 \cdot 25}{5.8^2} \right)^{3.5}} = 5.69469 \times 10^{77}$$

Result:

$$1.81268 \times 10^{77} x = 5.69469 \times 10^{77}$$

Plot:



Alternate form:

$$1.81268 \times 10^{77} x - 5.69469 \times 10^{77} = 0$$

Alternate form assuming x is real:

$$1.81268 \times 10^{77} x + 0 = 5.69469 \times 10^{77}$$

Solution:

$$x \approx 3.14159$$

$$3.14159 \approx \pi$$

$$\frac{\left(\frac{(x-29)}{20}\pi\sqrt{6.67408e-11(30+25)}\right)^3(6.67408e-11)^3(30^2 \times 25^2)}{\left(\frac{12\sqrt{2}\left(\frac{85\pi}{6\sqrt{2}}\right)(6.67408e-11)^{3.5}(30+25)^{1.5}\left(\frac{30 \times 25}{5.8^2}\right)^{3.5}}\right)} = 5.69469e+77$$

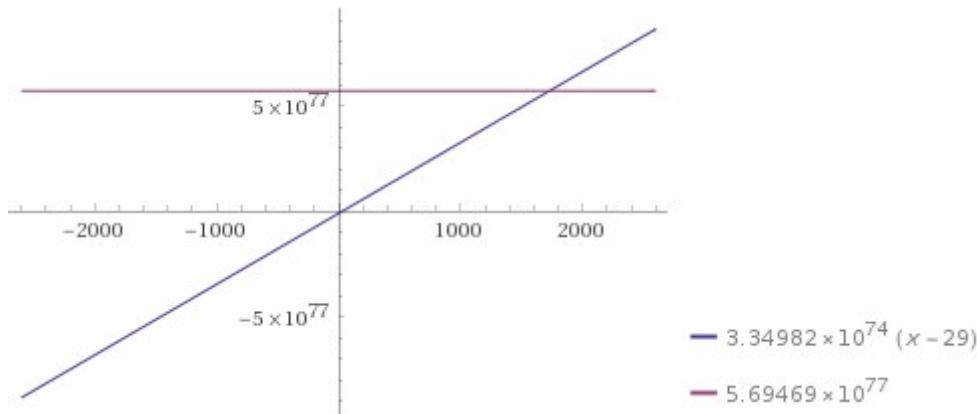
Input interpretation:

$$\frac{\frac{x-29}{20} \pi \sqrt{6.67408 \times 10^{-11} (30 + 25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \times \frac{30 \times 25}{5.8^2} \right)^{3.5}} = 5.69469 \times 10^{77}$$

Result:

$$3.34982 \times 10^{74} (x - 29) = 5.69469 \times 10^{77}$$

Plot:



Solution:

$$x \approx 1729.$$

1729

$$\frac{\left(\frac{(x^{15}-29)}{20}\pi\sqrt{6.67408e-11(30+25)}\right)^3(6.67408e-11)^3(30^2 \times 25^2)}{\left(\frac{12\sqrt{2}\left(\frac{85\pi}{6\sqrt{2}}\right)(6.67408e-11)^{3.5}(30+25)^{1.5}\left(\frac{30 \times 25}{5.8^2}\right)^{3.5}}\right)} = 5.69469e+77$$

Input interpretation:

$$\frac{\left(\frac{1}{20} (x^{15} - 29)\right) \pi \sqrt{6.67408 \times 10^{-11} (30 + 25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{85 \pi}{6 \sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} \times \frac{30 \times 25}{5.8^2} \right)^{3.5}} = 5.69469 \times 10^{77}$$

Result:

$$3.34982 \times 10^{74} (x^{15} - 29) = 5.69469 \times 10^{77}$$

Alternate forms:

$$3.34982 \times 10^{74} x^{15} - 5.79183 \times 10^{77} = 0$$

$$3.34982 \times 10^{74} x^{15} - 9.71447 \times 10^{75} = 5.69469 \times 10^{77}$$

$$5.02168 \times 10^{58} (6.67071 \times 10^{15} x^{15} - 1.93451 \times 10^{17}) = 5.69469 \times 10^{77}$$

Real solution:

$$x \approx 1.64382$$

$$1.64382$$

Complex solutions:

$$x = -1.60789 - 0.341768 i$$

$$x = -1.60789 + 0.341768 i$$

$$x = -1.32987 - 0.96621 i$$

$$x = -1.32987 + 0.96621 i$$

$$x = -0.821908 - 1.42359 i$$

Thence, we obtain the following possible and interesting mathematical connection:

$$\left(\frac{85 x \sqrt{6.67408 \times 10^{-11} (30 + 25)} (6.67408 \times 10^{-11})^3 (30^2 \times 25^2)}{12 \sqrt{2} \left(\frac{85 \pi}{6\sqrt{2}} (6.67408 \times 10^{-11})^{3.5} (30 + 25)^{1.5} (30 \times 25) \right)^{3.5}} = 2.57913537 \times 10^{72} \right) \Rightarrow$$

$$\Rightarrow x \approx 3.14159$$

$$3.14159$$

$$\Rightarrow \left(0.127686 \left(\frac{85 \sqrt{85}}{18 \sqrt{3}} \times \frac{1}{\sum_{n=0}^{\infty} \frac{(133n + 8) \left(\frac{1}{2} \times \frac{1}{6} \times \frac{5}{6}\right)}{(n!)^3} \left(\frac{4}{85}\right)^n} \right) \right) =$$

$$= 3.14159$$

$$3.14159$$

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the

golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the

second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

https://twitter.com/winkjs_org/status/973840788902866944

<https://www.scoop.it/topic/amazing-science/p/4073517830/2017/01/02/ramanujan-and-the-world-of-pi>

Primordial Black Holes - Perspectives in Gravitational Wave Astronomy -
Misao Sasaki, Teruaki Suyama, Takahiro Tanaka, and Shuichiro Yokoyama -
arXiv:1801.05235v1 [astro-ph.CO] 16 Jan 2018