The Real Nature of Time

A. Time and Motion

Earliest civilizations date about 10,000 years while the earliest signs of the human species go back as far as 200,000 years ago. We can safely say that humans invented the concept of time way before the first signs of civilization. What we can tell is that they defined time not knowing that they were observing some kind of motion. A day – from sunrise to sunrise – was really the full rotation of the earth on its axis, and a year – the cycle of four seasons – the full rotation of the earth around the sun. Mathematically, what our ancestors did can be represented as a mapping:

Observation of the seasons and the changing daylight → time

Time and motion are then two sides of a coin. It should not be a mystery that the rotation of the needles on a clock mimics the rotation of the earth on its own axis.

Rotations of the needles (clock/time) → rotations of the Earth (motion)

Why we label the first (rotations of the needles) as time is just a convenient, cultural and practical thing.

Now measuring space is easy to deal with – draw two points, use a stick and measure the distance – according to some standard stick kept in a vault somewhere in Paris. But measuring motion requires an entirely different process. What we do, and have done since Galileo, is to compare the motion of a given object with the motion of the internal moving parts (IMPs) of a clock – those moving parts are deliberately set to give regular beats, hence regular motion, that is, the “internal moving parts” (IMPs) travel the same distance after each tick the clock gives out. The need to standardized motion, which was an urgent preoccupation for Galileo, is the result of an observation that there are different kinds of motions – uniform motion versus non-uniform. It is the latter that prompts us to ask: “what causes a non-uniform motion (acceleration)?” To which the notion of force comes into play.

Now we want to take a closer look at what takes place when we decide to actually measure motion. For instance, if the motion of the given object under observation happens to give a constant ratio of distance travelled by the object with respect to the clock’s IMPs, we know automatically that the observed object is in uniform motion, meaning it has a constant speed. This is the only way to measure – that is, quantifying – motion. In fact, here is what we have been doing in calculating this ratio:

(A.1) Ave. speed $= \dfrac{\text{Total distance traveled by the object}}{\text{Total distance traveled by the IMP of the clock}}$

We can symbolize this process of observing the motion of a car, and in trying to figure out its speed (motion) by means of a table (Fig. 1):

1	2	3	4
Clock ticks	Distance of clock's internal moving parts (IMP) in a tick	Distance of moving car in a tick	Ave. speed
t_1	d_{IMP}	d_1	d_1 / d_{IMP}
t_2	d_{IMP}	d_2	$(d_1+d_2)/ 2d_{IMP}$
. . .			

Fig. 1

Note: if the clock gives regular beats, it simply means that its IMPs have travelled the same distance over and over for each beat – Column 2, d_{IMP}. If the ratio is the same throughout – Column 4 – then we know that the moving object is undergoing uniform motion - its velocity is constant, that is, there is no acceleration, and we can deduce from Newton's laws of motion that the net force acting on it is zero.

After every tick of the clock, the IMPs will cover the same distance over and over. This is a standardized distance locked into the device. So every motion that is understudied, its distance is measured against that standardized distance. Just like a stick kept in a vault in Paris is the standardized measure of distance, the second (tick) is then the standardized measure of motion. In other words, a clock is simply a device that provides a standardized motion against which all other motions are measured.

From the data in Fig.1 we can construct the following diagrams for a body in motion:

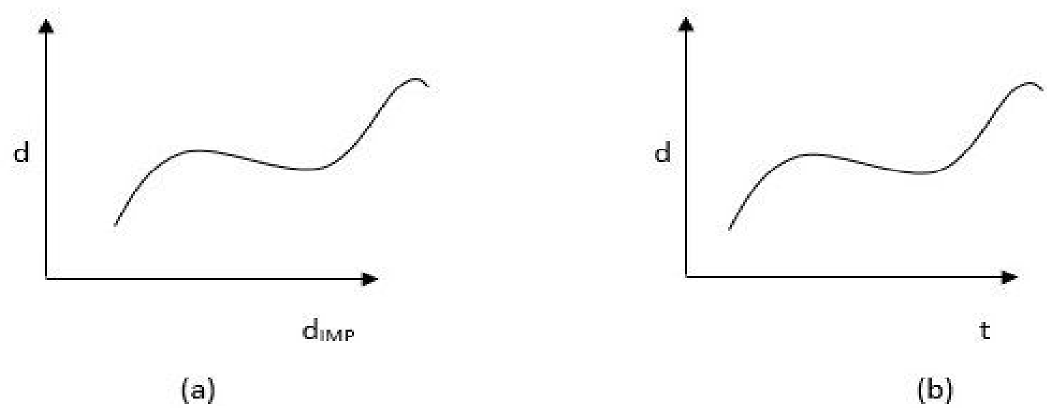

Fig. 2

The diagram in Fig. 2b is the most familiar as this is the one used in textbooks to define the speed of a body in motion. But it is a faithful reproduction of Fig. 2a when you take into consideration the following mapping,

(A.2) Total distance traveled by the IMP of the clock $(d_{IMP}) \rightarrow t$

With this perspective, we get our normal definition of speed ≡ distance/time. We have converted a graph, Distance versus Distance, into a graph, Distance versus Time.

An interesting point to notice is that it turns out our diagram in Fig. 2a is closer to reality – that is, if one should open the clock, one would see the internal moving parts (IMPs) of a clock. On the other hand, time is just a mental construct born out of that particular motion.

Now, Galileo was concerned with precise clocks because his main focus was to identify different types of motion: in particular, uniform motion (constant speed) and uniform acceleration which he studied by rolling balls over an inclined plane, and the acceleration due to gravity. In effect, what he did was a mapping:

Motion (real world) → time (mental construct)

Newton took us on the wrong path by declaring that time was absolute, when in fact, motion is relative, and therefore time can only be relative in kind. But it took nearly 300 years before Einstein corrected that error.

Now in most standard textbook, speed is defined as distance travelled over a time interval. Taking space and time as fundamental, and speed (motion) as a derived notion has been the norm. A contemporary of Einstein, Minkowski, re-interpreted his work in which space-time can play a dynamic role. And that cemented the prevailing thought that space-time was a fundamental concept, maintaining the status of motion as a derived concept.

But what is a clock if nothing but an apparatus made up of a specific type of motion? The fact is when one uses a measure of time (second, day, year, etc.) this is just a translation of a particular motion. “I’ll see you in one hour” means “I’ll see you when the earth will have completed 1/24th of its rotation around its own axis.” Or “I’ll see you in one year” is a translation of “I’ll see you when the earth will have completed one full rotation around the sun”. What about my grey hair at age 70? Isn’t that a testimony of the passage of time? This grey hair is a testimony that I’ve travelled 70 times around the sun. While that travelling took place, stuff happened, one of which is my hair turning grey.

What about the notion that motion is a relative quantity and so, how can it be more fundamental than time?

True that motion is relative with respect to some reference point. An object can be in motion with respect to one point, and at rest with another point. But we must take into consideration the whole picture. For instance you and I can be at rest, but our hearts, lungs and the blood circulating in our body are not. The atoms in our body are in motion, and so are the electrons in those atoms. Now suppose everything that constitute your body and mine – cells, atoms, electrons, etc. – are all at rest. Extend that to every object in the universe. In such a universe where there is no motion, the concept of time is no longer possible as “one moment in time” cannot be distinguished from the next “moment in time”.

The past and future can best be comprehended by examining the trajectory of an object.

Consider a moving object, and since this illustration is static and not a video, pretend that it is right now at position 2, frozen in that position, also labelled “present” in Fig. 3, and positions 1 and 3 are labels for past and future positions respectively.

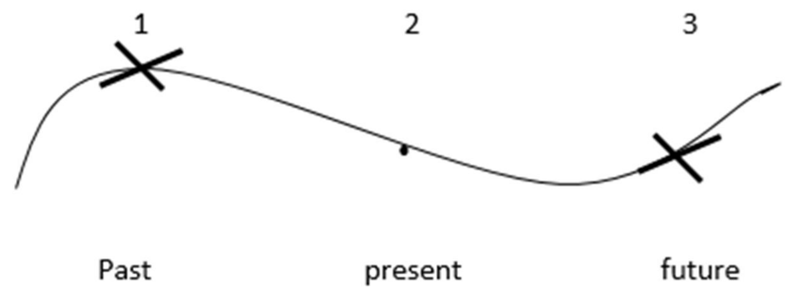

Fig. 3

Sometimes this is mistaken as the “flow” of time. That’s because when you are thinking of time, you are really thinking of motion. If you’re asking, “Is time a one-way street?” you are asking the wrong question. Grabbing the object presently at position 2 and returning it to position 1 would not be a case for traveling back into the past – we’re just altering the motion of that object to a different trajectory. The past doesn’t exist, except as images of that motion either in our memory or on some form of media. What existed at position 1 was a motion that has continued into the present (position 2). The future (position 3) represents a motion of the object along its trajectory. It hasn’t happened yet and therefore like the past, the future does not exist in the present in the sense that its content hasn’t come about. In Fig. 3, position 3 represents a possible outcome. That such points as 1 and 3 are designated as past and future respectively does not endow these positions with existence. They are just a convenient way to talk about motion. Time is just a label for arbitrary points on a trajectory. Note that this is again what we fundamentally observe in the world: “matter moving through space”. Time is a concept that helps us to visualize that fundamental observation.

Conclusion

This new proposal that time is a derived concept demands a rethinking of an idea that has stretched back to the Ancients. And so this is difficult to absorb as we were trained to think of time differently. Many live by the calendar, which is just a gadget based on the motion of our planet around its sun. More importantly we must keep in mind that a universe without motion is a universe without time. Motion is the fundamental thing that we observe. Without motion, the idea of time would be inconceivable. And so it must be acknowledged that motion is a fundamental concept. And time is a derived concept, which is nevertheless indispensable in making the measurement of motion possible. With its IMPs, a clock and its regulated time measured in seconds is basically standardized motion against which all other motions are measured. The fact is that in just about every situation the word "time" can be replaced by an object undergoing a specific motion, and there would be no loss in meaning. The bottom line is that we measure the motion of a body with a standardized motion, which is what a clock offers.

The next occasion you get to witness a sunset, instead of thinking that time is flying fast and another day is gone, think of yourself standing on the surface of a spinning ball moving away from the sun. Motion is about the real world, time is what is in our head.

B. Time and Special Relativity

No one can doubt that time plays an important role in Special Relativity. So what more can be said in terms of what we are proposing? We will now review the basic features of the theory, and hopefully bare to light with some new insights.

First, Special Relativity is based on two basic postulates:

Postulate 1 – The laws of physics are the same in all inertial frames.

Postulate 2 – The speed of light is invariant in every inertial frame.

Note 1: The main point for postulate 1 is about observers in different frames. Two observers can choose any coordinate system – both chooses the same coordinate system (Cartesian) or different (one, Cartesian; the other, polar) – but what we must keep in mind is that we are dealing with different frames, regardless of the choice of a coordinate system.

Note 2: "Invariant" means that a measurement in one inertial frame will be the same in a different inertial frame, while "constant" means that it doesn't change with respect to time in any frame.

To see this more clearly, consider the 3-D momentum, p_i with i = 1,2,3. In an elastic collision, the total momentum before collision is equal to the total momentum after collision. The total momentum is said to be a constant – that is, time independent. In a different frame, a second observer will also observe that the total momentum before collision is equal to the total momentum after collision. But this constant will be different than the constant observed by the first observer. The 3-D momentum is not a Lorentz invariant. However, the space-time interval, which we will define more precisely, is a Lorentz invariant but it is not a constant quantity. In the case of the speed of light, it is both a constant (time-independent) and a Lorentz invariant (the same in every inertial frame).

At the heart of these two postulates are the Lorentz transformation laws to which we now turn our attention. Since light moves in a spherical wave front, it can be described by observer O as a sphere in that inertial frame,

(B.1) $c^2t^2 = x^2 + y^2 + z^2$

We can rewrite this as,

(B.2) $c^2t^2 - x^2 - y^2 - z^2 = 0$

A second observer O' in a different inertial frame will also write,

(B.3) $c^2t'^2 - x'^2 - y'^2 - z'^2 = 0$

Since these two equation are equal to zero, they are equal to each other. We have,

(B.4) $c^2t^2 - x^2 - y^2 - z^2 = c^2t'^2 - x'^2 - y'^2 - z'^2$

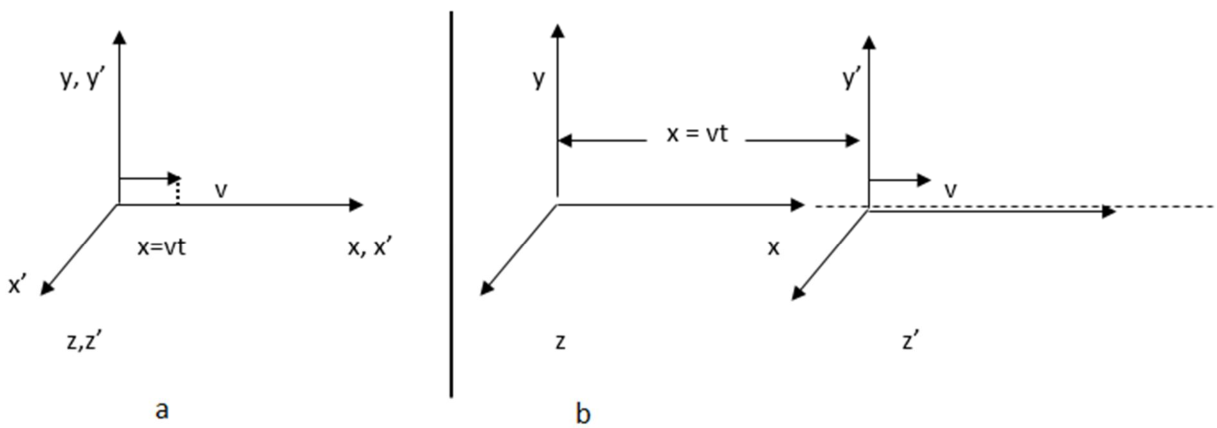

Fig. 4

In Fig. 4b, the two frames are at a certain distance away from each other as the primed frame is moving at a speed v going towards the right with respect to the unprimed frame. We can always arrange our two frames such that initially we have y' = y and z' = z, (Fig 4a).

Equation B.4 then becomes,

(B.5) $c^2t'^2 - x'^2 = c^2t^2 - x^2$

Another assumption is that the transformation laws that relate the two frames O and O' are linear. Therefore, we can state that,

(B.6a) $x' = Ax + Bct$

(B.6b) $ct' = Cx + Dct$

Where A, B, C, and D are to be determined. Those two equations can be put as a single matrix equation such as,

(B.7) $\begin{bmatrix} x' \\ ct' \end{bmatrix} = \Lambda \begin{bmatrix} x \\ ct \end{bmatrix}$

Where

(B.8) $\Lambda = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$

Substituting equations B.6 into the LHS of equation B.5, we get

$\rightarrow c^2 t'^2 - x'^2 = (Cx + Dct)^2 - (Ax + Bct)^2$

Expanding and collecting similar terms we have,

$$\rightarrow c^2 t'^2 - x'^2 = (D^2 - B^2)c^2 t^2 - (A^2 - C^2)x^2$$
$$+2(DC - AB)xct$$

In order to satisfy the RHS of equation B.5, we need

$$D^2 - B^2 = 1$$

$$A^2 - C^2 = 1$$

$$DC = AB$$

This can be satisfied by using the following identity,

(B.9) $cosh^2\theta - sinh^2\theta = 1$

And then set,

(B.10) $A = D = cosh\theta, \; B = C = -sinh\theta$

Other settings would depict motions different than the one in Fig. 4.

We can now solve for the angle θ. We note that when the two origins are coincident (Fig. 4a), we get the followings: x' = 0 and x = vt. Substitute that with equations B.10 into B.6a, we have,

$\rightarrow x' = 0 = cosh\theta x - sinh\theta \, ct$

$= cosh\theta \, vt - sinh\theta \, ct$

Or

(B.11) $\tanh\theta = \frac{v}{c}$

Using the identity,

(B.12) $\cosh\theta = \frac{1}{\sqrt{1-tanh^2\theta}}$

We get

(B.13) $\cosh\theta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \equiv \gamma$;

$\sinh\theta = \gamma(\frac{v}{c})$

Note that for massless particle, *v = c* and *γ = 1*; for massive particle, v < c and γ > 1.

We can now express the Lorentz transformation equations (B.6a-b) in the familiar form:

(B.14) $x' = \gamma(x - vt)$;

$ct' = \gamma\left(ct - \left(\frac{v}{c}\right)x\right)$;

$y' = y$;

$z' = z$

For instance: $x' = Ax + Bct = cosh\theta x - sinh\theta ct$

$= cosh\theta\left(x - \frac{sinh\theta}{cosh\theta}ct\right) = \gamma(x - \tanh\theta\, ct)$

$= \gamma\left(x - (\frac{v}{c})ct\right) = \gamma(x - vt)$

Similarly for ct'.

Before we proceed, we must carefully examine equation B.5, reproduced below as,

(B.15) $(c^2t^2)_{light} = (x^2 + y^2 + z^2)_{light}$

We must reiterate that both sides of the equation referred to the distance squared traveled by light in the unprimed frame. This was done in a 3-D Cartesian coordinate system (also being a flat space) with time as a parameter.

Now an important concept is the space-time interval based on equation B.1, which is defined as,

(B.16) $ds^2 \equiv -c^2dt^2 + dx_i\, dx^i$

We used differentials since we are going to be dealing with distances, rather than positions. The Einstein summation is applied, and we set $x = x^1, y = x^2, z = x^3$ $ct = x^0$.

The metric tensor is a 4×4 matrix defined as, (B.17) $\eta_{\mu\nu} \equiv \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

With these tools in hand we can rewrite equation B.16 as,

(B.18) $ds^2 = \eta_{\mu\nu}\, dx^\mu\, dx^\nu$ with μ,ν = 0,1,2,3

C. The Invariance of the Space-Time Interval

The invariance of the space-time interval is so important that it crystalizes everything about SR. No physical theory of the real world can survive without it.

We rewrite equation B.7 for differentials with the indices in the right places.

(C.1) $dx'^\mu = \Lambda^\mu_\nu\, dx^\nu$

Our assumption that the transformation laws relating the two frames O and O' to be linear demands that the Λ's have an inverse expressed by this restriction:

(C.2) $\Lambda^\mu_\gamma \Lambda^\nu_\delta \eta_{\mu\nu} = \eta_{\gamma\delta}$

In the primed frame, observer O' writes down her own equation for the space-time interval as,

(C.3) $ds'^2 = \eta_{\mu\nu}\, dx'^\mu dx'^\nu$

Note that the metric tensor is composed of 0's and 1's, and so is invariant. This will no longer be the case in GR as the metric tensor will be replaced by $\eta_{\mu\nu} \to g_{\mu\nu}$, a quantity to be determined on a case by case.

Substituting equation C.1 into C.3, we get

(C.4) $ds'^2 = \eta_{\mu\nu}\, (\Lambda^\mu_\gamma\, dx^\gamma)(\Lambda^\nu_\delta\, dx^\delta)$

Using the above restriction (equation C.2), we obtain

(C.5) $ds'^2 = \eta_{\gamma\delta}\, dx^\gamma dx^\delta$

Now the indices in the Einstein summation are dummy indices, the RHS is just equation B.18. Therefore,

(C.6) $ds'^2 = ds^2$

We now see the power of the 4-vector formalism. In that mathematical framework, the interval ds is a manifestly invariant quantity: any observer in any initial frame will measure the same space-time interval. The above considerations was for light, which is a massless particle. For a massive particle, moving along a straight line in the x-direction, we have,

(C.7) $ds^2 = -c^2dt^2 + dx^2 < 0$

That's true because no massive particle can travel faster than light, that is, dx < cdt.

D. Time Dilation, Twin Paradox, Length Contraction

To illustrate the time dilation phenomenon, consider again two observers O and O', Fig. 4. In the primed frame, the clock is at rest with respect to O', and necessarily moving with respect to O.

Recall equation B.5 reproduced below with some modifications:

(D.1) $c^2\Delta\tau'^2 - \Delta x'^2 = c^2\Delta t^2 - \Delta x^2$

We are using the Δ symbol to indicate that we are looking at intervals of time and position; and the symbol τ for the proper time, which is defined for a clock at rest. From the point of view of O', the clock is at rest, and so we have $\Delta x'^2 = 0$. Therefore,

(D.2) $c^2\Delta\tau'^2 = c^2\Delta t^2 - \Delta x^2$

Divide both sides by c^2, and factor out Δt^2. We get,

(D.3) $\Delta\tau'^2 = \Delta t^2(1 - \frac{\Delta x^2}{c^2\Delta t^2})$

Using $v = \frac{\Delta x}{\Delta t}$ and taking the square-root on both sides, the result is

(D.4) $\Delta\tau' = \Delta t \left(1 - \frac{v^2}{c^2}\right)^{½}$

From equation B.13, we have

(D.5) $\Delta\tau' = \Delta t/\gamma$

With γ > 1, the proper time $\Delta\tau' < \Delta t$, we say that moving clocks slow down.

In the twin paradox, the frequent question asked is: how do we determine which clock registers the proper time, since a clock can be at rest with respect to one observer but is in motion with respect to another observer.

Consider Alice is in a rocket ship about to undergo a trip to the planet in the nearest star to our sun, say planet Proxima Centauri in the Alpha Centauri star system – about 4.22 light-years. Her twin brother Bob stays back home on planet Earth. In this case, the clock in Alice's ship will register the proper time. How do we come to that conclusion? Alice will register two events with one clock: her departure from earth and her arrival on Proxima Centauri. Her clock is at rest with respect to Alice but not with respect to the two events – it has to move from event one (departure) to event two (arrival). On the other hand, Bob will need two clocks to measure the time taken by his sister of these two events: one clock on planet earth for her departue, and another one on Proxima Centauri – he needs his good friend Cortney already on Proxima Centauri to record Alice's arrival. So we can see that Bob's clock is NOT the moving clock as it stays back home, registering just one of those two events. Therefore it is Alice who will look younger in age than her twin brother Bob as her clock and everything in her rocket ship will experience this time dilation. Note that the twin paradox is not about traveling to the future but it's all about aging.

This phenomenon has been confirmed in the decay of muons. Comparing the half-life decay between muons producing in the lab with those arriving at very high speed from outer space was a triumph of SR.

In the case of length contraction, we use a car moving at a speed v. Observer O will measure the length L of that car with one clock by registering the time as both front and end of that car will pass in front of him.

(D.6) $L = v\,(t_2 - t_1) = v\,\Delta t$

Observer O' is sitting in the car, but in this case, he needs two clocks, one in the front and another in the back of the car, as his car passes through right in front of observer O. He will also measure,

(D.7) $L' = v\,({t'}_2 - {t'}_1) = v\,\Delta t'$

We can see that it is observer O who registers the proper time (he uses one clock for both events),

Hence equation D.6 becomes,

(D.8) $L = v\,(t_2 - t_1) = v\,\Delta t'/\gamma$

Divide equation D.8 by D.7 yields the result for length contraction,

(D.9) $L = L'/\gamma$

With $\gamma > 1$, $L < L'$. The moving car (the ruler) is contracted, hence moving rulers contract.

According to Bob, Alice will have a slower clock but will also measure her distance from planet Earth to planet Proxima Centauri with a shrinking ruler. Therefore with this dual effect taking place, both Alice and Bob will agree that the rocket ship (the frame) in which Alice is traveling moves at the speed v. This is an important result. We have established that the space-time interval is Lorentz invariant but speed in general is not. However in the case of the speed of the frame, because of this dual effect, it is invariant. Considering that we chose the observer O at rest and observer O' moving to the right (Fig. 4) that choice was arbitrary as we could have easily chosen O' at rest and O moving to the left. Had the case be that the speed of the frame was not invariant – that is, O and O' would obtain different results for their relative speed – SR as a physical theory would collapse.

As we have argued in section A, the speed of an object is defined as, (equations A.1 and A.2 reproduced below):

$$\text{Ave. speed} \ = \frac{\text{Total distance traveled by the object}}{\text{Total distance traveled by the IMP of the clock}}$$

Where total distance traveled by the IMP of the clock $(d_{IMP}) \ \rightarrow \ t$

From this we can say that time dilation is length contraction in disguise – they are the two sides of the same coin. Hence the γ factor, top and bottom of the ratio, cancels out. One of the great result is that velocity (motion) is unitless – in SR, the speed of light is taken to be 1, a fitting convention to the real nature of motion as a fundamental concept. What about time? It is still, without any doubt, one of the greatest inventions of the human mind.