

**On some Ramanujan equations: mathematical connections with  $\phi$ ,  $\zeta(2)$ ,  
Monstrous Moonshine and various parameters of Particle Physics.**

**Michele Nardelli<sup>1</sup>, Antonio Nardelli<sup>2</sup>**

**Abstract**

*In this paper we have described and analyzed some Ramanujan equations. Furthermore, we have obtained several mathematical connections with  $\phi$ ,  $\zeta(2)$ , Monstrous Moonshine and various parameters of Particle Physics.*

---

<sup>1</sup> M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni “R. Caccioppoli” - Università degli Studi di Napoli “Federico II” – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

<sup>2</sup> A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – Sezione Filosofia - scholar of Theoretical Philosophy



[https://link.springer.com/chapter/10.1007/978-81-322-0767-2\\_12](https://link.springer.com/chapter/10.1007/978-81-322-0767-2_12)

<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

From:

**Manuscript Book 3 of Srinivasa Ramanujan**

page 6

$$f(x) = \left( \frac{\log \frac{1 + \sqrt{5}}{2}}{\pi} \right)^2$$

$$x e^{e^x} = \frac{e^x}{1+x} \cdot \frac{e^x}{1+\frac{x}{2}} \cdot \frac{e^x}{1+\frac{x}{3}} \cdot \frac{e^x}{1+\frac{x}{4}} \dots$$

$$(\ln((1+\sqrt{5})/2)/\pi)^2$$

**Input:**

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\log^2\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi^2}$$

**Decimal approximation:**

0.023462421710806909463112025130508650169194928065080959403...

0.02346242171.....

**Alternate forms:**

$$\frac{\operatorname{csch}^{-1}(2)^2}{\pi^2}$$

$$\frac{\log^2\left(\frac{2}{1+\sqrt{5}}\right)}{\pi^2}$$

$$\frac{(\log(1+\sqrt{5})-\log(2))^2}{\pi^2}$$

$\operatorname{csch}^{-1}(x)$  is the inverse hyperbolic cosecant function

**Alternative representations:**

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \left(\frac{\log_e\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2$$

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \left(\frac{\log(a)\log_a\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2$$

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \left(-\frac{\operatorname{Li}_1\left(1+\frac{1}{2}(-1-\sqrt{5})\right)}{\pi}\right)^2$$

**Series representations:**

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \frac{\left(\sum_{k=1}^{\infty} \frac{\left(\frac{1}{2}(1-\sqrt{5})\right)^k}{k}\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \frac{\left(2i\pi\left[\frac{\arg(1+\sqrt{5}-2x)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k (1+\sqrt{5}-2x)^k x^{-k}}{k}\right)^2}{\pi^2}$$

for  $x < 0$

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \frac{\left(2i\pi\left[\frac{\arg\left(\frac{1}{2}(1+\sqrt{5})-x\right)}{2\pi}\right] + \log(x) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{2}\right)^k (1+\sqrt{5}-2x)^k x^{-k}}{k}\right)^2}{\pi^2}$$

for  $x < 0$

### Integral representations:

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = \frac{\left(\int_1^{\frac{1}{2}(1+\sqrt{5})} \frac{1}{t} dt\right)^2}{\pi^2}$$

$$\left(\frac{\log\left(\frac{1}{2}(1+\sqrt{5})\right)}{\pi}\right)^2 = -\frac{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left(-1+\frac{1}{2}(1+\sqrt{5})\right)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds\right)^2}{4\pi^4} \quad \text{for } -1 < \gamma < 0$$

Exp(0.02346242171)/(1+0.02346242171) \*  
 exp(0.02346242171/2)/(1+0.02346242171/2) \*  
 exp(0.02346242171/3)/(1+0.02346242171/3) \*  
 exp(0.02346242171/4)/(1+0.02346242171/4)

### Input interpretation:

$$\frac{\exp(0.02346242171)}{1+0.02346242171} \times \frac{\exp\left(\frac{0.02346242171}{2}\right)}{1+\frac{0.02346242171}{2}} \times \frac{\exp\left(\frac{0.02346242171}{3}\right)}{1+\frac{0.02346242171}{3}} \times \frac{\exp\left(\frac{0.02346242171}{4}\right)}{1+\frac{0.02346242171}{4}}$$

### Result:

1.0003869234...

1.0003869234...

From which:

$$1/(((\exp(0.02346242171)/(1+0.02346242171) * \exp(0.02346242171/2)/(1+0.02346242171/2) * \exp(0.02346242171/3)/(1+0.02346242171/3) * \exp(0.02346242171/4)/(1+0.02346242171/4))))))$$

**Input interpretation:**

$$\frac{1}{\frac{\exp(0.02346242171)}{1+0.02346242171} \times \frac{\exp\left(\frac{0.02346242171}{2}\right)}{1+\frac{0.02346242171}{2}} \times \frac{\exp\left(\frac{0.02346242171}{3}\right)}{1+\frac{0.02346242171}{3}} \times \frac{\exp\left(\frac{0.02346242171}{4}\right)}{1+\frac{0.02346242171}{4}}}$$

**Result:**

0.99961322621...

0.99961322621.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}}}{1 + \sqrt[5]{\sqrt{\varphi^5 \sqrt[4]{5^3}} - 1} - \varphi + 1} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

$$(((1/(((\exp(0.02346242171)/(1+0.02346242171) * \exp(0.02346242171/2)/(1+0.02346242171/2) * \exp(0.02346242171/3)/(1+0.02346242171/3) * \exp(0.02346242171/4)/(1+0.02346242171/4)))))))))^{128}$$

**Input interpretation:**

$$\left( \frac{1}{\frac{\exp(0.02346242171)}{1+0.02346242171} \times \frac{\exp\left(\frac{0.02346242171}{2}\right)}{1+\frac{0.02346242171}{2}} \times \frac{\exp\left(\frac{0.02346242171}{3}\right)}{1+\frac{0.02346242171}{3}} \times \frac{\exp\left(\frac{0.02346242171}{4}\right)}{1+\frac{0.02346242171}{4}}} \right)^{128}$$

**Result:**

0.951689339...

0.951689339... result very near to the spectral index  $n_s$ , to the mesonic Regge slope, to the inflaton value at the end of the inflation 0.9402 and to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{5}}}{\sqrt{(\varphi-1)\sqrt{5}} - \varphi + 1} = 1 - \frac{e^{-\pi}}{1 + \frac{e^{-2\pi}}{1 + \frac{e^{-3\pi}}{1 + \frac{e^{-4\pi}}{1 + \dots}}}}} \approx 0.9568666373$$

1/log base 0.951689339 ((1/(((exp(0.02346242171)/(1+0.02346242171)  
 exp(0.02346242171/2)/(1+0.02346242171/2)  
 exp(0.02346242171/3)/(1+0.02346242171/3)  
 exp(0.02346242171/4)/(1+0.02346242171/4)))))))-e

**Input interpretation:**

$$\log_{0.951689339} \left( \frac{1}{\frac{\exp(0.02346242171)}{1+0.02346242171} \times \frac{\exp\left(\frac{0.02346242171}{2}\right)}{1+\frac{0.02346242171}{2}} \times \frac{\exp\left(\frac{0.02346242171}{3}\right)}{1+\frac{0.02346242171}{3}} \times \frac{\exp\left(\frac{0.02346242171}{4}\right)}{1+\frac{0.02346242171}{4}} \right)$$

$\log_b(x)$  is the base- $b$  logarithm

**Result:**

125.2817...

125.2817...

**Alternative representation:**

$$\begin{aligned}
 & \frac{1}{\log_{0.951689} \left( \frac{1}{\exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \exp\left(\frac{0.0234624}{2}\right)} \right)} - e = \\
 & \left( \begin{aligned}
 & -e + \frac{1}{\log \left( \frac{1}{\exp(0.00782081) \exp(0.0117312) \exp(0.0234624) \exp\left(\frac{0.0234624}{4}\right)} \right)} = \\
 & \frac{1.00782 \times 1.01173 \times 1.02346 \left(1 + \frac{0.0234624}{4}\right)}{\log(0.951689)} \\
 & -e + \frac{\log(0.951689)}{\log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right)}
 \end{aligned} \right)
 \end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & \frac{1}{\log_{0.951689} \left( \frac{1}{\exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \exp\left(\frac{0.0234624}{2}\right)} \right)} - e = \\
 & \frac{\log(0.951689)}{\log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right)} \\
 & - e - \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right)^k}{k}
 \end{aligned}$$

$$\frac{1}{\log_{0.951689} \left( \frac{1}{\left( \frac{\exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} \right)^{-e} =$$

$$-e - 1 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right.$$

$$\left. \left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$\frac{1}{\log_{0.951689} \left( \frac{1}{\left( \frac{\exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} \right)^{-e} =$$

$$-e - 1 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right.$$

$$\left. \left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

27\*1/2\*1/log base 0.951689339 ((1/(((exp(0.0234624217)/(1+0.0234624217) exp(0.0234624217/2)/(1+0.0234624217/2) exp(0.0234624217/3)/(1+0.0234624217/3) exp(0.0234624217/4)/(1+0.0234624217/4)))))))+1

**Input interpretation:**

$$27 \times \frac{1}{2} \times \frac{1}{\log_{0.951689339} \left( \frac{1}{\left( \frac{\exp(0.0234624217)}{1+0.0234624217} \times \frac{\exp\left(\frac{0.0234624217}{2}\right)}{1+\frac{0.0234624217}{2}} \times \frac{\exp\left(\frac{0.0234624217}{3}\right)}{1+\frac{0.0234624217}{3}} \times \frac{\exp\left(\frac{0.0234624217}{4}\right)}{1+\frac{0.0234624217}{4}} \right)} \right)} + 1$$

log<sub>b</sub>(x) is the base- b logarithm

**Result:**

1729.000...  
1729



**Alternative representation:**

$$\begin{aligned}
 & \frac{27}{\log_{0.951689} \left( \frac{1}{\exp\left(\frac{0.0234624}{2}\right) \exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right)} \right)} + 1 = \\
 & \left( 1 + \frac{27}{2 \log \left( \frac{1}{\exp(0.00782081) \exp(0.0117312) \exp(0.0234624) \exp\left(\frac{0.0234624}{4}\right)} \right)} \right) = \\
 & \left( 1 + \frac{27 \log(0.951689)}{2 \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right)} \right)
 \end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & \frac{27}{\log_{0.951689} \left( \frac{1}{\exp\left(\frac{0.0234624}{2}\right) \exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right)} \right)} + 1 = \\
 & 1 - \frac{27 \log(0.951689)}{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right)^k}{k}}
 \end{aligned}$$

$$\frac{27}{+1 = 1 -} \log_{0.951689} \left( \frac{1}{\frac{\exp(\frac{0.0234624}{2}) \exp(0.0234624) \exp(\frac{0.0234624}{3}) \exp(\frac{0.0234624}{4})}{(1 + \frac{0.0234624}{2})(1 + 0.0234624)(1 + \frac{0.0234624}{3})(1 + \frac{0.0234624}{4})}} \right)^2$$

$$13.5 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right)$$

$$\left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$\frac{27}{+1 = 1 -} \log_{0.951689} \left( \frac{1}{\frac{\exp(\frac{0.0234624}{2}) \exp(0.0234624) \exp(\frac{0.0234624}{3}) \exp(\frac{0.0234624}{4})}{(1 + \frac{0.0234624}{2})(1 + 0.0234624)(1 + \frac{0.0234624}{3})(1 + \frac{0.0234624}{4})}} \right)^2$$

$$13.5 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right)$$

$$\left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

1/log base 0.951689339 (((1/(((exp(0.0234624217)/(1+0.0234624217)  
exp(0.0234624217/2)/(1+0.0234624217/2)  
exp(0.0234624217/3)/(1+0.0234624217/3)  
exp(0.0234624217/4)/(1+0.0234624217/4)))))))+11

**Input interpretation:**

$$\frac{1}{+11} \log_{0.951689339} \left( \frac{1}{\frac{\exp(0.0234624217)}{1+0.0234624217} \times \frac{\exp(\frac{0.0234624217}{2})}{1+\frac{0.0234624217}{2}} \times \frac{\exp(\frac{0.0234624217}{3})}{1+\frac{0.0234624217}{3}} \times \frac{\exp(\frac{0.0234624217}{4})}{1+\frac{0.0234624217}{4}}} \right)$$

log<sub>b</sub>(x) is the base- b logarithm

**Result:**

139.0000002874004034825716002379458316885352702508637691818...

[139.0000002874...](#)

**Alternative representation:**

$$\begin{aligned}
 & \frac{1}{\log_{0.951689} \left( \frac{1}{\left( \exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} + 11 = \\
 & \left( \begin{aligned}
 & 11 + \frac{1}{\log \left( \frac{1}{\frac{\exp(0.00782081) \exp(0.0117312) \exp(0.0234624) \exp\left(\frac{0.0234624}{4}\right)}{1.00782 \times 1.01173 \times 1.02346 \left(1 + \frac{0.0234624}{4}\right)}\right)} \right)} = \\
 & 11 + \frac{\log(0.951689)}{\log\left(\frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)}\right)}
 \end{aligned} \right)
 \end{aligned}$$

**Series representations:**

$$\begin{aligned}
 & \frac{1}{\log_{0.951689} \left( \frac{1}{\left( \exp(0.0234624) \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} + 11 = \\
 & 11 - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)}\right)^k}{k}}{\log(0.951689)}
 \end{aligned}$$

$$\frac{1}{\log_{0.951689} \left( \frac{1}{\left( \frac{\exp(0.0234624)}{(1+0.0234624)} \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} + 11 =$$

$$11 - 1 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right.$$

$$\left. \left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } \frac{(-1)^k k}{2(1+k)(2+k)} + G(k) = \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

$$\frac{1}{\log_{0.951689} \left( \frac{1}{\left( \frac{\exp(0.0234624)}{(1+0.0234624)} \exp\left(\frac{0.0234624}{3}\right) \exp\left(\frac{0.0234624}{4}\right) \right) \exp\left(\frac{0.0234624}{2}\right)} \right)} + 11 =$$

$$11 - 1 / \left( \log \left( \frac{1.04969}{\exp(0.00586561) \exp(0.00782081) \exp(0.0117312) \exp(0.0234624)} \right) \right.$$

$$\left. \left( 20.1994 + \sum_{k=0}^{\infty} (-0.0483107)^k G(k) \right) \right)$$

$$\text{for } \left( G(0) = 0 \text{ and } G(k) = \frac{(-1)^{1+k} k}{2(1+k)(2+k)} + \sum_{j=1}^k \frac{(-1)^{1+j} G(-j+k)}{1+j} \right)$$

[27\*1/2\*1/log base 0.951689339 ((1/(((exp(0.023462421)/(1+0.023462421) exp(0.023462421/2)/(1+0.023462421/2) exp(0.023462421/3)/(1+0.023462421/3) exp(0.023462421/4)/(1+0.023462421/4)))))))+1]^1/15

**Input interpretation:**

$$\left( 27 \times \frac{1}{2} \times \frac{1}{\log_{0.951689339} \left( \frac{1}{\left( \frac{\exp(0.023462421)}{1+0.023462421} \times \frac{\exp\left(\frac{0.023462421}{2}\right)}{1+\frac{0.023462421}{2}} \times \frac{\exp\left(\frac{0.023462421}{3}\right)}{1+\frac{0.023462421}{3}} \times \frac{\exp\left(\frac{0.023462421}{4}\right)}{1+\frac{0.023462421}{4}} \right)} \right)} + 1 \right)^{(1/15)}$$

$\log_b(x)$  is the base- $b$  logarithm

## Result:

1.643815235488464938539540963748800525385394061524038520380...

1.643815235...

From:

## Can't you just feel the moonshine?

Ken Ono (Emory University) - <http://people.oregonstate.edu/~petschec/ONTD/Talk2.pdf> - March 30, 2017

### Theorem 5 (Duncan, Mertens, O (2017))

*There is an infinite dimensional graded ON moonshine module. Its MT series are explicit weight  $3/2$  mock modular forms.*

### Remarks (Graded Dimensions)

① If we let  $W := \bigoplus_n W_n$ , then

$\dim W_n =$  “traces of CM disc  $-n$  values of  $J_2$ ”.

② We have

$$\dim W_{163} = \lceil e^{\pi\sqrt{163}} \rceil^2 + \lceil e^{\pi\sqrt{163}} \rceil - 393768,$$

*in terms of Ramanujan's integer*

$$\lceil e^{\pi\sqrt{163}} \rceil = \lceil 262537412640768743.99999999999925\dots \rceil,$$

From

$$\dim W_{163} = \lceil e^{\pi\sqrt{163}} \rceil^2 + \lceil e^{\pi\sqrt{163}} \rceil - 393768$$

we obtain:

$$(e^{(\pi*\sqrt{163})})^2 + e^{(\pi*\sqrt{163})} - 393768$$

**Input:**

$$\left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768$$

**Exact result:**

$$-393768 + e^{\sqrt{163}\pi} + e^{2\sqrt{163}\pi}$$

**Decimal approximation:**

$$6.8925893036109280153623051927318744000000000162988716... \times 10^{34}$$

$$6.8925893... * 10^{34}$$

**Series representations:**

$$\begin{aligned} \left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768 = \\ -393768 + e^{\pi\sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}} + e^{2\pi\sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}} \end{aligned}$$

$$\begin{aligned} \left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768 = \\ -393768 + e^{\pi\sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \exp\left(2\pi\sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right) \end{aligned}$$

$$\begin{aligned} \left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768 = \\ -393768 + \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right) + \\ \exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}}\right) \end{aligned}$$

$$1/\left(\left(\left(\left(\left(\left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768\right)\right)^{1/1024}\right)\right)\right)$$

**Input:**

$$\frac{1}{1024\sqrt{\left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768}}$$

**Exact result:**

$$\frac{1}{1024 \sqrt{-393768 + e^{\sqrt{163} \pi} + e^{2\sqrt{163} \pi}}}$$

**Decimal approximation:**

0.924651635533758880002317102133472072766292304298932508492...

0.924651635.....

We know that  $\alpha'$  is the Regge slope (string tension). With regard the Omega mesons, the values are:

$\omega$	6	$m_{u/d} = 0 - 60$	0.910 - 0.918
$\omega/\omega_3$	5 + 3	$m_{u/d} = 255 - 390$	0.988 - 1.18
$\omega/\omega_3$	5 + 3	$m_{u/d} = 240 - 345$	0.937 - 1.000
$\Psi$	3	$m_c = 1500$	0.979   -0.09

**Series representations:**

$$\frac{1}{1024 \sqrt{(e^{\pi \sqrt{163}})^2 + e^{\pi \sqrt{163}} - 393768}} = \frac{1}{1024 \sqrt{-393768 + e^{\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}} + e^{2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}}}$$

$$\frac{1}{1024 \sqrt{(e^{\pi \sqrt{163}})^2 + e^{\pi \sqrt{163}} - 393768}} = \frac{1}{1024 \sqrt{-393768 + e^{\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/162)^k \binom{-1/2}{k}}{k!}} + \exp\left(2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/162)^k \binom{-1/2}{k}}{k!}\right)}}$$

$$\frac{1}{1024 \sqrt{\left(e^{\pi \sqrt{163}}\right)^2 + e^{\pi \sqrt{163}} - 393768}} =$$

$$1 / \left( \left( -393768 + \exp \left( \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}} \right) \right) + \right.$$

$$\left. \exp \left( \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right) \right)^{(1/1024)}$$

From

**S. Ramanujan to G.H. Hardy** - 12 January 1920  
*University of Madras*

Mock  $\vartheta$ -functions (of 7th order)

$$(i) \quad 1 + \frac{q}{1-q^2} + \frac{q^4}{(1-q^3)(1-q^4)} + \frac{q^9}{(1-q^4)(1-q^5)(1-q^6)} + \dots$$

If  $q = -e^{-t}$ ; for  $t = -0.0594191$ , we obtain:

**Input interpretation:**

$$-e^{-(-0.0594191)}$$

**Result:**

$$-1.06121990481640181110770407907160490464460035719732134267\dots$$

$$-1.0612199\dots = q$$

**Alternative representation:**

$$-e^{-(-1)0.0594191} = -\exp^{-(-1)0.0594191}(z) \text{ for } z = 1$$



**Series representations:**

$$-e^{-(-1)0.0594191} = -\left(\sum_{k=0}^{\infty} \frac{1}{k!}\right)^{0.0594191}$$

$$-e^{-(-1)0.0594191} = -0.95965 \left(\sum_{k=0}^{\infty} \frac{1+k}{k!}\right)^{0.0594191}$$

$$-e^{-(-1)0.0594191} = -\left(\sum_{k=0}^{\infty} \frac{(-1+k)^2}{k!}\right)^{0.0594191}$$

Thence, for  $q = -1.0612199$  we obtain:

$$\left(\left(\left(\left(1+(-1.0612199) / (1-(-1.0612199)^2) + (-1.0612199)^4 / ((1-(-1.0612199)^3)(1-1.0612199)^4)\right)\right)\right) + \left(\left(\left(-1.0612199\right)^9 / ((1-(-1.0612199)^4)(1-(-1.0612199)^5)(1-1.0612199)^6)\right)\right)\right)$$

**Input interpretation:**

$$\left(1 - \frac{1.0612199}{1 - (-1.0612199)^2} + \frac{(-1.0612199)^4}{(1 - (-1.0612199)^3)(1 - (-1.0612199)^4)}\right) + \frac{(-1.0612199)^9}{(1 - (-1.0612199)^4)(1 - (-1.0612199)^5)(1 - (-1.0612199)^6)}$$

**Result:**

0.924652609910869511752313077263733612948126777265979374154...

0.92465260991..... as above

From

$$\left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768$$

we have also:

$$\left(\left(\left(e^{(\pi*\sqrt{163})}\right)^2 + e^{(\pi*\sqrt{163})-393768}\right)\right)^{1/167}$$

**Input:**

$$\sqrt[167]{\left(e^{\pi\sqrt{163}}\right)^2 + e^{\pi\sqrt{163}} - 393768}$$

**Exact result:**

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}}$$

**Decimal approximation:**

1.616639062205656709427665330127047474529504831702121978915...

1.6166390622...

**All 167th roots of  $-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}$ :**

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}} e^{0} \approx 1.6166 \text{ (real, principal root)}$$

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}} e^{(2i\pi)/167} \approx 1.6155 + 0.06081 i$$

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}} e^{(4i\pi)/167} \approx 1.6121 + 0.12153 i$$

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}} e^{(6i\pi)/167} \approx 1.6064 + 0.18209 i$$

$$\sqrt[167]{-393768 + e^{\sqrt{163} \pi} + e^{2 \sqrt{163} \pi}} e^{(8i\pi)/167} \approx 1.5984 + 0.24238 i$$

**Series representations:**

$$\sqrt[167]{\left(e^{\pi \sqrt{163}}\right)^2 + e^{\pi \sqrt{163}} - 393768} =$$

$$\sqrt[167]{-393768 + e^{\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}} + e^{2 \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}}$$

$$\sqrt[167]{\left(e^{\pi \sqrt{163}}\right)^2 + e^{\pi \sqrt{163}} - 393768} =$$

$$\sqrt[167]{-393768 + e^{\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!}} + \exp\left(2 \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}$$

$$167 \sqrt{\left( e^{\pi \sqrt{163}} \right)^2 + e^{\pi \sqrt{163}} - 393768} = \left( -393768 + \exp \left[ \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}} \right] + \exp \left[ \frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{\sqrt{\pi}} \right] \right)^{1/167}$$

### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\operatorname{Re}(a) \text{ and } |\arg(z)| < \pi)$$

We have that:

$$(e^{(\pi \sqrt{163})})$$

### Input:

$$e^{\pi \sqrt{163}}$$

### Decimal approximation:

$$2.625374126407687439999999999999925007259719818568887935... \times 10^{17}$$

$$2.625374126... * 10^{17}$$

### Property:

$e^{\sqrt{163} \pi}$  is a transcendental number

### Constant name:

Ramanujan constant

### Series representations:

$$e^{\pi \sqrt{163}} = \sum_{k=0}^{\infty} \frac{163^{k/2} \pi^k}{k!}$$

$$e^{\pi \sqrt{163}} = \sum_{k=-\infty}^{\infty} I_k(\sqrt{163} \pi)$$

$$e^{\pi \sqrt{163}} = \sum_{k=0}^{\infty} \frac{163^k \pi^{2k} (1 + 2k + \sqrt{163} \pi)}{(1 + 2k)!}$$

### Integral representation:

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \quad \text{for } (0 < \gamma < -\text{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

and:

$$(((e^{(\pi \cdot \sqrt{163})}))^{1/83})$$

### Input:

$$\sqrt[83]{e^{\pi \sqrt{163}}}$$

### Exact result:

$$e^{(\sqrt{163} \pi)/83}$$

### Decimal approximation:

1.621323858200818559036361999666474569023910571353738771381...

1.6213238582...

### Property:

$e^{(\sqrt{163} \pi)/83}$  is a transcendental number

### All 83rd roots of $e^{(\sqrt{163} \pi)}$ :

$$e^{(\sqrt{163} \pi)/83} e^0 \approx 1.6213 \quad (\text{real, principal root})$$

$$e^{(\sqrt{163} \pi)/83} e^{(2i\pi)/83} \approx 1.6167 + 0.12262i$$

$$e^{(\sqrt{163} \pi)/83} e^{(4i\pi)/83} \approx 1.6028 + 0.24454i$$

$$e^{(\sqrt{163} \pi)/83} e^{(6i\pi)/83} \approx 1.5797 + 0.3651i$$

$$e^{(\sqrt{163} \pi)/83} e^{(8i\pi)/83} \approx 1.5476 + 0.4835i$$

**Series representations:**

$$\sqrt[83]{e^{\pi \sqrt{163}}} = \sqrt[83]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}}$$

$$\sqrt[83]{e^{\pi \sqrt{163}}} = \sqrt[83]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/162)^k \binom{-1}{2/k}}{k!}}}$$

$$\sqrt[83]{e^{\pi \sqrt{163}}} = \sqrt[83]{\exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2 \sqrt{\pi}}\right)}$$

**Integral representation:**

$$(1+z)^\alpha = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-\alpha-s)}{z^s} ds}{(2\pi i)\Gamma(-\alpha)} \text{ for } (0 < \gamma < -\operatorname{Re}(\alpha) \text{ and } |\arg(z)| < \pi)$$

(((e^(Pi\*sqrt163))))^(1/(2e))+2Pi+123

**Input:**

$$\sqrt[2e]{e^{\pi \sqrt{163}}} + 2\pi + 123$$

**Exact result:**

$$123 + e^{(\sqrt{163} \pi)/(2e)} + 2\pi$$

**Decimal approximation:**

1729.140172285013148204966205169119698671458437350773465909...

[1729.140172285...](#)

**Series representations:**

$$\sqrt[2e]{e^{\pi \sqrt{163}}} + 2\pi + 123 = 123 + \sqrt[2e]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}} + 2\pi$$

$$\sqrt[2e]{e^{\pi \sqrt{163}}} + 2\pi + 123 = 123 + \sqrt[2e]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/162)^k \binom{-1}{2/k}}{k!}}} + 2\pi$$

$$2e^{\sqrt{e^{\pi\sqrt{163}}}} + 2\pi + 123 = 123 + 2e^{\sqrt{\exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)}} + 2\pi$$

$$\left(\left(\left(\left(e^{\pi\sqrt{163}}\right)\right)\right)\right)^{1/8-11}$$

**Input:**

$$\sqrt[8]{e^{\pi\sqrt{163}}} - 11$$

**Exact result:**

$$e^{(\sqrt{163}\pi)/8} - 11$$

**Decimal approximation:**

139.4523238408903535560817057455564310477133553258664845740...

[139.45232384...](#)

**Property:**

$-11 + e^{(\sqrt{163}\pi)/8}$  is a transcendental number

**Series representations:**

$$\sqrt[8]{e^{\pi\sqrt{163}}} - 11 = -11 + \sqrt[8]{e^{\pi\sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}}$$

$$\sqrt[8]{e^{\pi\sqrt{163}}} - 11 = -11 + \sqrt[8]{e^{\pi\sqrt{162} \sum_{k=0}^{\infty} \frac{(-1/2)^k (-1/2)_k}{k!}}}$$

$$\sqrt[8]{e^{\pi\sqrt{163}}} - 11 = -11 + \sqrt[8]{\exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}{2\sqrt{\pi}}\right)}$$

$$\left(\left(\left(\left(e^{\pi\sqrt{163}}\right)\right)\right)\right)^{1/8-29+4}$$

**Input:**

$$\sqrt[8]{e^{\pi\sqrt{163}}} - 29 + 4$$

**Exact result:**

$$e^{(\sqrt{163} \pi)/8} - 25$$

**Decimal approximation:**

125.4523238408903535560817057455564310477133553258664845740...

125.45232384...

**Property:**

$-25 + e^{(\sqrt{163} \pi)/8}$  is a transcendental number

**Series representations:**

$$\sqrt[8]{e^{\pi \sqrt{163}} - 29 + 4} = -25 + \sqrt[8]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{1/2}{k}}}$$

$$\sqrt[8]{e^{\pi \sqrt{163}} - 29 + 4} = -25 + \sqrt[8]{e^{\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{(-1)^k \binom{-1}{2k}}{k!}}}$$

$$\sqrt[8]{e^{\pi \sqrt{163}} - 29 + 4} = -25 + \sqrt[8]{\exp\left(\frac{\pi \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j} 162^{-s} \Gamma(-\frac{1}{2}-s) \Gamma(s)}{2 \sqrt{\pi}}\right)}$$

Now, from:

**Theorem 2 (Duncan, Griffin, O (2015))**  
*The Monstrous Moonshine module is asymptotically regular. In other words, we have that*

$$\delta(\mathbf{m}_i) = \frac{\dim(\chi_i)}{\sum_{j=1}^{194} \dim(\chi_j)} = \frac{\dim(\chi_i)}{5844076785304502808013602136}$$

We have that:

$$\sum_{j=1}^{194} \dim(\chi_j) = 5844076785304502808013602136$$

From which:

$$(5844076785304502808013602136)^{1/(43+85+5)}$$

where 5, 43 and 85 are Jacobsthal numbers

**Input:**

$$\sqrt[43+85+5]{5844076785304502808013602136}$$

**Result:**

$$2^{3/133} \sqrt[133]{730509598163062851001700267}$$

**Decimal approximation:**

1.617231903238274039725487354922645269827036979564915643308...

[1.6172319032...](#)

$$(5844076785304502808013602136)^{1/8+29-2}$$

**Input:**

$$\sqrt[8]{5844076785304502808013602136} + 29 - 2$$

**Result:**

$$27 + 2^{3/8} \sqrt[8]{730509598163062851001700267}$$

**Decimal approximation:**

2983.919417300206973857944826850744249437083988921034904554...

[2983.9194173...](#) result very near to the rest mass of Charmed eta meson 2980.3

**Minimal polynomial:**

$$x^8 - 216x^7 + 20412x^6 - 1102248x^5 + 37200870x^4 - 803538792x^3 + 10847773692x^2 - 83682825624x - 5844076785304502525584065655$$

$$(5844076785304502808013602136)^{1/9+521-7-\text{golden ratio}}$$

**Input:**

$$\sqrt[9]{5844076785304502808013602136} + 521 - 7 - \phi$$

$\phi$  is the golden ratio



**Result:**

$$-\phi + 514 + \sqrt[3]{2} \sqrt[9]{730509598163062851001700267}$$

**Decimal approximation:**

1729.101994500546054678359462592672937939041758521772193436...

1729.1019945...

**Alternate forms:**

$$\frac{1}{2} \left( 1027 - \sqrt{5} + 2 \sqrt[3]{2} \sqrt[9]{730509598163062851001700267} \right)$$

$$\frac{1027}{2} - \frac{\sqrt{5}}{2} + \sqrt[3]{2} \sqrt[9]{730509598163062851001700267}$$

$$514 + \sqrt[3]{2} \sqrt[9]{730509598163062851001700267} + \frac{1}{2} (-1 - \sqrt{5})$$

**Minimal polynomial:**

$$\begin{aligned} &x^{18} - 9243x^{17} + 40343373x^{16} - 110486989236x^{15} + 212756330662110x^{14} - \\ &305900711016200118x^{13} + 340339558201495276914x^{12} - \\ &299595549825403081608384x^{11} + 211532769986864955370893243x^{10} - \\ &132379081424513841031948390617x^9 + \\ &109793974049476057751622225750531x^8 - \\ &131781217163410317031512990916489056x^7 + \\ &139178582043811634726156064318793015602x^6 - \\ &103876547213421484103090651213992024773990x^5 + \\ &52853455896624357913417115643285542337928782x^4 - \\ &18038223428159422232776509921035795483134382964x^3 + \\ &3965213300445543481057774633497325322925214520685x^2 - \\ &508808136269253328732697904627170253286417848146691x + \\ &34182258741917100835753386706007061920978056081927970929 \end{aligned}$$

**Alternative representations:**

$$\begin{aligned} &\sqrt[9]{5844076785304502808013602136} + 521 - 7 - \phi = \\ &514 + \sqrt[9]{5844076785304502808013602136} - 2 \sin(54^\circ) \end{aligned}$$

$$\begin{aligned} &\sqrt[9]{5844076785304502808013602136} + 521 - 7 - \phi = \\ &514 + 2 \cos(216^\circ) + \sqrt[9]{5844076785304502808013602136} \end{aligned}$$

$$\begin{aligned} &\sqrt[9]{5844076785304502808013602136} + 521 - 7 - \phi = \\ &514 + \sqrt[9]{5844076785304502808013602136} + 2 \sin(666^\circ) \end{aligned}$$

From:

**MONSTROUS MOONSHINE** - *J. H. CONWAY AND S. P. NORTON* - [BULL. LONDON MATH. SOC, 11 (1979), 308-339]

In 1973 Bernd Fischer and Bob Griess independently produced evidence for a new simple group  $M$  of order

$$2^{46} \cdot 3^{20} \cdot 5^9 \cdot 7^6 \cdot 11^2 \cdot 13^3 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31 \cdot 41 \cdot 47 \cdot 59 \cdot 71$$

$$= 8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000.$$

We proposed to call this group the MONSTER and conjectured that it had a representation of degree 196883. In a remarkable piece of work, Fischer, Livingstone and Thorne [6] have recently computed the entire character table on this assumption. The MONSTER has not yet been proved to exist, but Thompson [18] has proved its uniqueness on similar assumptions.

(F) McKay noticed that one of the coefficients in the  $q$ -series

$$j = q^{-1} + 744 + 196884q + 21493760q^2 + \dots = \sum a_r q^r, \text{ say,}$$

is  $196883+1$ , and Thompson [17] found that the later  $a_r$  are also simple linear combinations of the character degrees  $f_r$  of  $M$ :—

$$a_{-1} = f_1, a_1 = f_1 + f_2, a_2 = f_1 + f_2 + f_3, a_3 = 2f_1 + 2f_2 + f_3 + f_4.$$

Now, we have that:

$$j = q^{-1} + 744 + 196884q + 21493760q^2 + \dots = \sum a_r q^r.$$

If  $q = e^{2\pi i \tau}$ , for  $i\tau = i(1+i)$ , we obtain:

$$\exp(2\pi i i(1+i))$$

**Input:**

$$\exp(2\pi i i(1+i))$$

**Exact result:**

$$e^{-2\pi}$$

**Decimal approximation:**

$$0.001867442731707988814430212934827030393422805002475317199\dots$$

0.0018674427...

We obtain:

$$1/(0.0018674427)+744+196884*(0.0018674427)+21493760*(0.0018674427^2)$$

**Input interpretation:**

$$\frac{1}{0.0018674427} + 744 + 196\,884 \times 0.0018674427 + 21\,493\,760 \times 0.0018674427^2$$

**Result:**

1722.117350260649287615222360546859081673563531561102249616...

1722.11735026...

If  $q = \exp(-2\pi \times 1.0136)$ , we obtain:

$$\exp(-2\pi \times 1.0136)$$

**Input interpretation:**

$$\exp(-2\pi \times 1.0136)$$

**Result:**

0.00171450...

0.0017145.... = q

and thus:

$$1/(0.0017145)+744+196884*(0.0017145)+21493760*(0.0017145^2)$$

**Input:**

$$\frac{1}{0.0017145} + 744 + 196\,884 \times 0.0017145 + 21\,493\,760 \times 0.0017145^2$$

**Result:**

1727.999171611150819480898221055701370662000583260425780110...

1727.9991716.....  $\approx$  1728

or:

$$(((1/(0.0017145)+744+196884*(0.0017145)+21493760*(0.0017145^2))))+1$$

**Input:**

$$\left(\frac{1}{0.0017145} + 744 + 196884 \times 0.0017145 + 21493760 \times 0.0017145^2\right) + 1$$

**Result:**

1728.999171611150819480898221055701370662000583260425780110...

1728.99917161..... $\approx$  1729

From which:

$$\left[\left(\left(\frac{1}{0.0017145} + 744 + 196884 \times (0.0017145) + 21493760 \times (0.0017145^2)\right)\right) + 1\right]^{1/15}$$

**Input:**

$$\sqrt[15]{\left(\frac{1}{0.0017145} + 744 + 196884 \times 0.0017145 + 21493760 \times 0.0017145^2\right) + 1}$$

**Result:**

1.643815176243676653009593509775264357660701090604715146373...

1.6438151762.....

$$\left[\left(\left(\frac{1}{0.0017145} + 744 + 196884 \times (0.0017145) + 21493760 \times (0.0017145^2)\right)\right) + 1\right]^{1/15} - (21+5) \times \frac{1}{10^3}$$

**Input:**

$$\sqrt[15]{\left(\frac{1}{0.0017145} + 744 + 196884 \times 0.0017145 + 21493760 \times 0.0017145^2\right) + 1} - (21+5) \times \frac{1}{10^3}$$

**Result:**

1.617815176243676653009593509775264357660701090604715146373...

1.6178151762.....

From:

**Modular equations and approximations to  $\pi$**  – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Now, we have:

$$\frac{1 - \frac{3}{\pi\sqrt{n}} - 24 \left( \frac{1}{e^{2\pi\sqrt{n}-1}} + \frac{2}{e^{4\pi\sqrt{n}-1}} + \dots \right)}{1 - 24 \left( \frac{1}{e^{\pi\sqrt{n}+1}} + \frac{3}{e^{3\pi\sqrt{n}+1}} + \dots \right)} = R, \quad (21)$$

For  $n = 163$ , we have:

$$\left[ 1 - \frac{3}{\pi\sqrt{163}} - 24 \left( \frac{1}{\exp(2\pi\sqrt{163})-1} + \frac{2}{\exp(4\pi\sqrt{163})-1} \right) \right] / \left[ 1 - 24 \left( \frac{1}{\exp(\pi\sqrt{163})+1} + \frac{3}{\exp(3\pi\sqrt{163})+1} \right) \right]$$

**Input:**

$$\frac{1 - \frac{3}{\pi\sqrt{163}} - 24 \left( \frac{1}{\exp(2\pi\sqrt{163})-1} + \frac{2}{\exp(4\pi\sqrt{163})-1} \right)}{1 - 24 \left( \frac{1}{\exp(\pi\sqrt{163})+1} + \frac{3}{\exp(3\pi\sqrt{163})+1} \right)}$$

**Exact result:**

$$\frac{1 - 24 \left( \frac{1}{e^{2\sqrt{163}\pi}-1} + \frac{2}{e^{4\sqrt{163}\pi}-1} \right) - \frac{3}{\sqrt{163}\pi}}{1 - 24 \left( \frac{1}{1+e^{\sqrt{163}\pi}} + \frac{3}{1+e^{3\sqrt{163}\pi}} \right)}$$

**Decimal approximation:**

0.925204136593620674554404984124063427002419524808214423829...

0.92520413659...

We know that  $\alpha'$  is the Regge slope (string tension). With regard the Omega mesons, the values are:

$$\omega \quad | \quad 6 \quad | \quad m_{u/d} = 0 - 60 \quad | \quad 0.910 - 0.918$$

$$\omega/\omega_3 \quad | \quad 5 + 3 \quad | \quad m_{u/d} = 255 - 390 \quad | \quad 0.988 - 1.18$$

$$\omega/\omega_3 \mid 5 + 3 \mid m_{u/d} = 240 - 345 \mid 0.937 - 1.000$$

$$\Psi \mid 3 \mid m_c = 1500 \mid 0.979 \mid -0.09$$

**Alternate forms:**

$$\begin{aligned} & -\left( \left( \left( 1 - e^{\sqrt{163} \pi} + e^{2\sqrt{163} \pi} \right) \left( -3\sqrt{163} + 3\sqrt{163} e^{4\sqrt{163} \pi} + \right. \right. \right. \\ & \quad \left. \left. \left. 11899\pi + 3912 e^{2\sqrt{163} \pi} \pi - 163 e^{4\sqrt{163} \pi} \pi \right) \right) \right) / \\ & \quad \left( 163 \left( e^{\sqrt{163} \pi} - 1 \right) \left( 1 + e^{2\sqrt{163} \pi} \right) \left( -95 + 24 e^{\sqrt{163} \pi} - 24 e^{2\sqrt{163} \pi} + e^{3\sqrt{163} \pi} \right) \pi \right) \\ & \frac{24}{47 \left( e^{\sqrt{163} \pi} - 1 \right)} + \frac{24 \left( 24 e^{\sqrt{163} \pi} - 47 \right)}{2785 \left( 1 + e^{2\sqrt{163} \pi} \right)} - \frac{3\sqrt{163} - 163\pi}{163\pi} + \\ & \quad \left( 24 \left( -1570740\sqrt{163} + 392685\sqrt{163} e^{\sqrt{163} \pi} - 392685\sqrt{163} e^{2\sqrt{163} \pi} + \right. \right. \\ & \quad \left. \left. 72019105\pi - 27458002 e^{\sqrt{163} \pi} \pi + 20698066 e^{2\sqrt{163} \pi} \pi \right) \right) / \\ & \quad \left( 21335885 \left( -95 + 24 e^{\sqrt{163} \pi} - 24 e^{2\sqrt{163} \pi} + e^{3\sqrt{163} \pi} \right) \pi \right) \\ & \frac{1}{1 - 24 \left( \frac{1}{1 + e^{\sqrt{163} \pi}} + \frac{3}{1 + e^{3\sqrt{163} \pi}} \right)} - \frac{24}{\left( e^{2\sqrt{163} \pi} - 1 \right) \left( 1 - 24 \left( \frac{1}{1 + e^{\sqrt{163} \pi}} + \frac{3}{1 + e^{3\sqrt{163} \pi}} \right) \right)} - \\ & \quad \frac{\left( e^{4\sqrt{163} \pi} - 1 \right) \left( 1 - 24 \left( \frac{1}{1 + e^{\sqrt{163} \pi}} + \frac{3}{1 + e^{3\sqrt{163} \pi}} \right) \right)}{3} - \\ & \quad \frac{\sqrt{163} \left( 1 - 24 \left( \frac{1}{1 + e^{\sqrt{163} \pi}} + \frac{3}{1 + e^{3\sqrt{163} \pi}} \right) \right) \pi}{\sqrt{163} \left( 1 - 24 \left( \frac{1}{1 + e^{\sqrt{163} \pi}} + \frac{3}{1 + e^{3\sqrt{163} \pi}} \right) \right) \pi} \end{aligned}$$

### Series representations:

$$\begin{aligned}
& \frac{1 - \frac{3}{\pi \sqrt{163}} - 24 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 24 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} = \\
& \left( \left( 1 + \exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \left( 1 + \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \right. \\
& \quad \left( -3 + 3 \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) + 3 \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) - \\
& \quad 3 \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) + \\
& \quad 73\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} - 49\pi \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \\
& \quad \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} - 25\pi \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \\
& \quad \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} + \pi \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \\
& \quad \left. \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) / \\
& \left( \pi \left( -1 + \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \right. \\
& \quad \left( -95 - 71 \exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) - 23 \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) + \\
& \quad \exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \\
& \quad \left. \left( -1 + \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1 - \frac{3}{\pi \sqrt{163}} - 24 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 24 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} = \\
& \left( \left( 1 + \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \left( 1 + \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \right. \\
& \quad \left( -3 + 3 \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \right. \\
& \quad 3 \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] - 3 \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \\
& \quad \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + 73\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 49\pi \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \\
& \quad 25\pi \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \\
& \quad \pi \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \\
& \quad \left. \left. \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) \right) / \\
& \left( \pi \left( -1 + \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \right) \left( -95 - \right. \\
& \quad 71 \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] - 23 \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \\
& \quad \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \\
& \quad \left( -1 + \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right)
\end{aligned}$$



$$\begin{aligned}
& \frac{1 - \frac{3}{\pi \sqrt{163}} - 24 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 24 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} = \\
& \left( \left( 1 + \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \right. \\
& \quad \left( 1 + \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \\
& \quad \left( -3 + 3 \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) + \right. \\
& \quad 3 \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) - \\
& \quad 3 \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad 73\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} - \\
& \quad 49\pi \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} - \\
& \quad 25\pi \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} + \\
& \quad \pi \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \left. \left. \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( \pi \left( -1 + \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \right. \\
& \quad \left( -95 - 71 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) - \right. \\
& \quad 23 \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \left. \left. \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \right) \\
& \quad \left( -1 + \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \sqrt{z_0} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

Now, if instead of 24, we put  $5280 = 24 * 55 * 4$ , we obtain:

$$\frac{[1 - 3/(\pi \sqrt{163}) - 5280(((1/(\exp(2\pi \sqrt{163}) - 1)) + 2/(\exp(4\pi \sqrt{163}) - 1)))]}{[1 - 5280(((1/(\exp(\pi \sqrt{163}) + 1)) + 3/(\exp(3\pi \sqrt{163}) + 1)))]}$$

**Input:**

$$\frac{1 - \frac{3}{\pi \sqrt{163}} - 5280 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 5280 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)}$$

**Exact result:**

$$\frac{1 - 5280 \left( \frac{1}{e^{2\sqrt{163}\pi} - 1} + \frac{2}{e^{4\sqrt{163}\pi} - 1} \right) - \frac{3}{\sqrt{163}\pi}}{1 - 5280 \left( \frac{1}{1 + e^{\sqrt{163}\pi}} + \frac{3}{1 + e^{3\sqrt{163}\pi}} \right)}$$

**Decimal approximation:**

0.925204136593639197144739909308294525377183635730608795462...

0.92520413659... as above

**Alternate forms:**

$$\frac{-\left( \left( (1 - e^{\sqrt{163}\pi} + e^{2\sqrt{163}\pi}) \left( -3\sqrt{163} + 3\sqrt{163} e^{4\sqrt{163}\pi} + 2582083\pi + 860640 e^{2\sqrt{163}\pi} \pi - 163 e^{4\sqrt{163}\pi} \pi \right) \right) / \left( 163 (e^{\sqrt{163}\pi} - 1) \right) \right) \left( 1 + e^{2\sqrt{163}\pi} \right) \left( -21119 + 5280 e^{\sqrt{163}\pi} - 5280 e^{2\sqrt{163}\pi} + e^{3\sqrt{163}\pi} \right) \pi}{\frac{5280}{10559 (e^{\sqrt{163}\pi} - 1)} + \frac{5280 (5280 e^{\sqrt{163}\pi} - 10559)}{139370881 (1 + e^{2\sqrt{163}\pi})} - \frac{3\sqrt{163} - 163\pi}{163\pi} + \frac{5280 \left( -17659405589748\sqrt{163} + 4414851397437\sqrt{163} e^{\sqrt{163}\pi} - 4414851397437\sqrt{163} e^{2\sqrt{163}\pi} + 815543863399825\pi - 311821586170714 e^{\sqrt{163}\pi} \pi + 239841787642714 e^{2\sqrt{163}\pi} \pi \right)}{\left( 239873592594077 \left( -21119 + 5280 e^{\sqrt{163}\pi} - 5280 e^{2\sqrt{163}\pi} + e^{3\sqrt{163}\pi} \right) \pi \right)}}$$

$$\frac{1}{1 - 5280 \left( \frac{1}{1+e^{\sqrt{163} \pi}} + \frac{3}{1+e^{3\sqrt{163} \pi}} \right)} -$$

$$\frac{\left( e^{2\sqrt{163} \pi} - 1 \right) \left( 1 - 5280 \left( \frac{1}{1+e^{\sqrt{163} \pi}} + \frac{3}{1+e^{3\sqrt{163} \pi}} \right) \right)}{10560} -$$

$$\frac{\left( e^{4\sqrt{163} \pi} - 1 \right) \left( 1 - 5280 \left( \frac{1}{1+e^{\sqrt{163} \pi}} + \frac{3}{1+e^{3\sqrt{163} \pi}} \right) \right)}{3} -$$

$$\sqrt{163} \left( 1 - 5280 \left( \frac{1}{1+e^{\sqrt{163} \pi}} + \frac{3}{1+e^{3\sqrt{163} \pi}} \right) \right) \pi$$

### Series representations:

$$1 - \frac{3}{\pi \sqrt{163}} - 5280 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right) =$$

$$\frac{1 - 5280 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)}{\left( \left( 1 + \exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \left( 1 + \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \right.}$$

$$\left. \left( -3 + 3 \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) + 3 \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) - \right. \right.$$

$$\left. 3 \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) + \right.$$

$$\left. 15841 \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} - 10561 \pi \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right.$$

$$\left. \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} - 5281 \pi \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right.$$

$$\left. \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} + \pi \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right.$$

$$\left. \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) /$$

$$\left( \pi \left( -1 + \exp \left( 2\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \left( -21119 - 15839 \right. \right.$$

$$\left. \exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) - 5279 \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) +$$

$$\exp \left( \pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \exp \left( 3\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right.$$

$$\left. \left( -1 + \exp \left( 4\pi \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right) \right) \sqrt{162} \sum_{k=0}^{\infty} 162^{-k} \binom{\frac{1}{2}}{k} \right)$$

$$\begin{aligned}
& \frac{1 - \frac{3}{\pi \sqrt{163}} - 5280 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 5280 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} = \\
& \left( \left( 1 + \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \left( 1 + \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \right. \\
& \left( -3 + 3 \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \right. \\
& \left. 3 \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] - 3 \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right. \\
& \left. \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + 15841\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \left. 10561\pi \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} - \right. \\
& \left. 5281\pi \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \\
& \left. \pi \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right. \\
& \left. \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right) / \\
& \left( \pi \left( -1 + \exp \left[ 2\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \right. \\
& \left( -21119 - 15839 \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] - \right. \\
& \left. 5279 \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] + \right. \\
& \left. \exp \left[ \pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \exp \left[ 3\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \\
& \left( -1 + \exp \left[ 4\pi \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \right] \right) \sqrt{162} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{162}\right)^k \left(-\frac{1}{2}\right)_k}{k!} \Big)
\end{aligned}$$

$$\begin{aligned}
& \frac{1 - \frac{3}{\pi \sqrt{163}} - 5280 \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right)}{1 - 5280 \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} = \\
& \left( \left( 1 + \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \right. \\
& \quad \left( 1 + \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \\
& \quad \left( -3 + 3 \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) + \\
& \quad 3 \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) - \\
& \quad 3 \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad 15841 \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} - \\
& \quad 10561 \pi \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} - \\
& \quad 5281 \pi \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} + \\
& \quad \pi \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \left. \left. \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) / \\
& \left( \pi \left( -1 + \exp \left( 2\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \right. \\
& \quad \left( -21119 - 15839 \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) - \\
& \quad 5279 \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \quad \exp \left( \pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \\
& \quad \exp \left( 3\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \\
& \quad \left( -1 + \exp \left( 4\pi \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right) \right) \sqrt{z_0} \\
& \quad \left. \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (163 - z_0)^k z_0^{-k}}{k!} \right)
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

From

$$\frac{1 - 5280 \left( \frac{1}{e^{2\sqrt{163}\pi}} + \frac{2}{e^{4\sqrt{163}\pi}} \right) - \frac{3}{\sqrt{163}\pi}}{1 - 5280 \left( \frac{1}{1+e^{\sqrt{163}\pi}} + \frac{3}{1+e^{3\sqrt{163}\pi}} \right)}$$

$$= 0.925204136593639197144739909308294525377183635730608795462\dots$$

we obtain:

$$\left[ 1 - \frac{3}{\pi\sqrt{163}} - \left( \frac{1}{\exp(2\pi\sqrt{163}) - 1} + \frac{2}{\exp(4\pi\sqrt{163}) - 1} \right) \right] \frac{1}{\left[ 1 - \left( \frac{1}{\exp(\pi\sqrt{163}) + 1} + \frac{3}{\exp(3\pi\sqrt{163}) + 1} \right) \right]} x = 5280$$

**Input:**

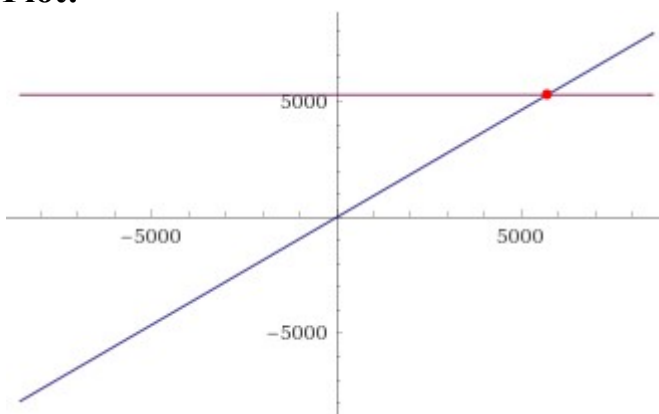
$$\left( 1 - \frac{3}{\pi\sqrt{163}} - \left( \frac{1}{\exp(2\pi\sqrt{163}) - 1} + \frac{2}{\exp(4\pi\sqrt{163}) - 1} \right) \right) x = 5280$$

$$1 - \left( \frac{1}{\exp(\pi\sqrt{163}) + 1} + \frac{3}{\exp(3\pi\sqrt{163}) + 1} \right)$$

**Exact result:**

$$\frac{\left( 1 - \frac{1}{e^{2\sqrt{163}\pi}} - \frac{2}{e^{4\sqrt{163}\pi}} - \frac{3}{\sqrt{163}\pi} \right) x}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}} = 5280$$

**Plot:**



$$\frac{\left( 1 + \frac{1}{1 - e^{2\sqrt{163}\pi}} - \frac{2}{e^{4\sqrt{163}\pi}} - \frac{3}{\sqrt{163}\pi} \right) x}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}} = 5280$$

**Alternate forms:**

$$\begin{aligned}
 & -\left( \left( (1 - e^{\sqrt{163} \pi} + e^{2\sqrt{163} \pi}) \left( -3\sqrt{163} + 3\sqrt{163} e^{4\sqrt{163} \pi} + 652\pi + \right. \right. \right. \\
 & \quad \left. \left. \left. 163 e^{2\sqrt{163} \pi} \pi - 163 e^{4\sqrt{163} \pi} \pi \right) x \right) / \left( 163 (e^{\sqrt{163} \pi} - 1) \right) \right. \\
 & \quad \left. \left( 1 + e^{2\sqrt{163} \pi} \right) \left( -3 + e^{\sqrt{163} \pi} - e^{2\sqrt{163} \pi} + e^{3\sqrt{163} \pi} \right) \pi \right) = 5280 \\
 & -\left( \left( \left( 24\sqrt{163} - 6\sqrt{163} e^{\sqrt{163} \pi} + 6\sqrt{163} e^{2\sqrt{163} \pi} - 815\pi + 326 e^{\sqrt{163} \pi} \pi + \right. \right. \right. \\
 & \quad \left. \left. \left. 163 e^{2\sqrt{163} \pi} \pi \right) x \right) / \left( 326 \left( -3 + e^{\sqrt{163} \pi} - e^{2\sqrt{163} \pi} + e^{3\sqrt{163} \pi} \right) \pi \right) \right) - \\
 & \quad \frac{(3\sqrt{163} - 163\pi)x}{163\pi} + \frac{(e^{\sqrt{163} \pi} - 1)x}{2(1 + e^{2\sqrt{163} \pi})} + \frac{x}{e^{\sqrt{163} \pi} - 1} = 5280 \\
 & -\frac{3x}{\sqrt{163} \left( 1 - \frac{1}{1+e^{\sqrt{163} \pi}} - \frac{3}{1+e^{3\sqrt{163} \pi}} \right) \pi} - \frac{2x}{(e^{4\sqrt{163} \pi} - 1) \left( 1 - \frac{1}{1+e^{\sqrt{163} \pi}} - \frac{3}{1+e^{3\sqrt{163} \pi}} \right)} \\
 & \quad + \frac{(e^{2\sqrt{163} \pi} - 1) \left( 1 - \frac{1}{1+e^{\sqrt{163} \pi}} - \frac{3}{1+e^{3\sqrt{163} \pi}} \right)}{x} - 5280 = 0 \\
 & \quad 1 - \frac{1}{1+e^{\sqrt{163} \pi}} - \frac{3}{1+e^{3\sqrt{163} \pi}}
 \end{aligned}$$

**Solution:**

$$x = 860640$$

$$\begin{aligned}
 & \left( 3 - 4 e^{\sqrt{163} \pi} + 5 e^{2\sqrt{163} \pi} - 6 e^{3\sqrt{163} \pi} + 3 e^{4\sqrt{163} \pi} - 2 e^{5\sqrt{163} \pi} + e^{6\sqrt{163} \pi} \right) \\
 & \pi / \left( (1 - e^{\sqrt{163} \pi} + e^{2\sqrt{163} \pi}) \right) \\
 & \left( 3\sqrt{163} - 652\pi - 163 e^{2\sqrt{163} \pi} \pi + e^{4\sqrt{163} \pi} (163\pi - 3\sqrt{163}) \right)
 \end{aligned}$$

thence:

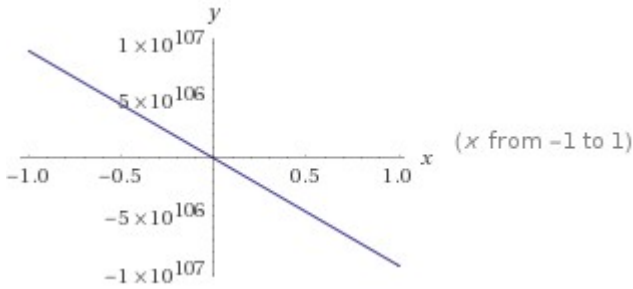
**Input:**

$$\begin{aligned}
 & \left( 1 - \frac{3}{\pi \sqrt{163}} - \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right) \right) \times \\
 & \quad \frac{1}{1 - \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)} x
 \end{aligned}$$

**Exact result:**

$$\frac{\left(1 - \frac{1}{e^{2\sqrt{163}\pi}} - \frac{2}{e^{4\sqrt{163}\pi}} - \frac{3}{\sqrt{163}\pi}\right)x}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}}$$

**Plot:**



**Geometric figure:**

line

**Alternate forms:**

$$\text{Factor} \left[ \frac{\left(1 - \frac{1}{e^{2\sqrt{163}\pi}} - \frac{2}{e^{4\sqrt{163}\pi}} - \frac{3}{\sqrt{163}\pi}\right)x}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}}, \text{Extension} \rightarrow e^{\sqrt{163}\pi} \right]$$

$$- \left( \left( 24\sqrt{163} - 6\sqrt{163}e^{\sqrt{163}\pi} + 6\sqrt{163}e^{2\sqrt{163}\pi} - 815\pi + 326e^{\sqrt{163}\pi}\pi + 163e^{2\sqrt{163}\pi}\pi \right)x \right) / \left( 326(-3 + e^{\sqrt{163}\pi} - e^{2\sqrt{163}\pi} + e^{3\sqrt{163}\pi})\pi \right) -$$

$$\frac{(3\sqrt{163} - 163\pi)x}{163\pi} + \frac{(e^{\sqrt{163}\pi} - 1)x}{2(1 + e^{2\sqrt{163}\pi})} + \frac{x}{e^{\sqrt{163}\pi} - 1}$$

$$- \left( \left( (1 - e^{\sqrt{163}\pi} + e^{2\sqrt{163}\pi})(-3\sqrt{163} + 3\sqrt{163}e^{4\sqrt{163}\pi} + 652\pi + 163e^{2\sqrt{163}\pi}\pi - 163e^{4\sqrt{163}\pi}\pi)x \right) / \right.$$

$$\left. (163(e^{\sqrt{163}\pi} - 1)(1 + e^{2\sqrt{163}\pi})(-3 + e^{\sqrt{163}\pi} - e^{2\sqrt{163}\pi} + e^{3\sqrt{163}\pi})\pi) \right)$$

**Expanded form:**

$$-\frac{3x}{\sqrt{163} \left(1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}\right)\pi} - \frac{2x}{\left(e^{4\sqrt{163}\pi} - 1\right)\left(1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}\right)\pi}$$

$$+ \frac{\left(e^{2\sqrt{163}\pi} - 1\right)\left(1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}\right)}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}}$$



**Properties as a real function:**

**Domain**

$\mathbb{R}$  (all real numbers)

**Range**

$\mathbb{R}$  (all real numbers)

**Bijectivity**

bijjective from its domain to  $\mathbb{R}$

**Parity**

odd

$\mathbb{R}$  is the set of real numbers

**Derivative:**

$$\frac{d}{dx} \left( \frac{\left( 1 - \frac{3}{\pi \sqrt{163}} - \left( \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right) \right) X}{1 - \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1}} \right) =$$

$$\frac{1 - \frac{1}{e^{2\sqrt{163}\pi} - 1} - \frac{2}{e^{4\sqrt{163}\pi} - 1} - \frac{3}{\sqrt{163}\pi}}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}}$$

**Indefinite integral:**

$$\int \frac{\left( 1 - \frac{1}{-1+e^{2\sqrt{163}\pi}} - \frac{2}{-1+e^{4\sqrt{163}\pi}} - \frac{3}{\sqrt{163}\pi} \right) X}{1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}}} dx =$$

$$\frac{-\frac{3X^2}{2\sqrt{163} \left( 1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}} \right) \pi} - \frac{\left( e^{4\sqrt{163}\pi} - 1 \right) \left( 1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}} \right)}{X^2} - \frac{2 \left( e^{2\sqrt{163}\pi} - 1 \right) \left( 1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}} \right)}{X^2} + \frac{2 \left( 1 - \frac{1}{1+e^{\sqrt{163}\pi}} - \frac{3}{1+e^{3\sqrt{163}\pi}} \right)}{X^2}}{+ \text{constant}}$$

Thence:

**Input:**

$$\left( 1 - \frac{3}{\pi \sqrt{163}} - \frac{1}{\exp(2\pi \sqrt{163}) - 1} + \frac{2}{\exp(4\pi \sqrt{163}) - 1} \right) \times \frac{1}{1 - \left( \frac{1}{\exp(\pi \sqrt{163}) + 1} + \frac{3}{\exp(3\pi \sqrt{163}) + 1} \right)}$$

**Exact result:**

$$\frac{1 - \frac{1}{e^{2\sqrt{163}\pi} - 1} - \frac{2}{e^{4\sqrt{163}\pi} - 1} - \frac{3}{\sqrt{163}\pi}}{1 - \frac{1}{1 + e^{\sqrt{163}\pi}} - \frac{3}{1 + e^{3\sqrt{163}\pi}}}$$

**Decimal approximation:**

0.925204136593620593500451844232383236822496245047348795521...

0.92520413659..... as above

For

$$x = \left( 860640 \frac{\left( 3 - 4e^{\sqrt{163}\pi} + 5e^{2\sqrt{163}\pi} - 6e^{3\sqrt{163}\pi} + 3e^{4\sqrt{163}\pi} - 2e^{5\sqrt{163}\pi} + e^{6\sqrt{163}\pi} \right) \pi}{\left( \left( 1 - e^{\sqrt{163}\pi} + e^{2\sqrt{163}\pi} \right) \left( 3\sqrt{163} - 652\pi - 163e^{2\sqrt{163}\pi} + e^{4\sqrt{163}\pi} \left( 163\pi - 3\sqrt{163} \right) \right) \right)} \right)$$

$x \approx 5706.8$

5706.8486738934

We obtain:

(0.9252041365936205935 \* 5706.8486738934)

**Input interpretation:**

0.9252041365936205935 × 5706.8486738934

**Result:**

5279.999999999917999136880637329

5279.9999999..... = 5280

From:

The International Journal Of Engineering And Science (IJES) || Volume || 3 || Issue || 5 || Pages || 25-36 || 2014 || ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805  
www.theijes.com The IJES Page 25

**Some Moonshine connections between Fischer-Griess Monster group (M) and Number theory - Amina Muhammad Lawan**

We have that:

$$A = e^{\pi\sqrt{67}} \approx A_e = 147197952744 = 5280^3 + 744$$

$$A_{e_1} = 5280^3 + 744 - \frac{196884}{5280^3}$$

$$A_{e_2} - A = -7.715679939894815927873779... \times 10^{-15}$$

(((5280^3+744-196884/5280^3)))

**Input:**

$$5280^3 + 744 - \frac{196884}{5280^3}$$



$$5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}$$

**Input:**

$$5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}$$

**Exact result:**

$$\frac{6589613509842273573950927194751}{44767018745856000000}$$

**Decimal approximation:**

$$1.4719795274399999866245422450000002080388701528647823... \times 10^{11}$$

$$1.47197952743... \cdot 10^{11}$$

$$\ln(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}) / \sqrt{67}$$

**Input:**

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}}$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{\log\left(\frac{6589613509842273573950927194751}{44767018745856000000}\right)}{\sqrt{67}}$$

**Decimal approximation:**

$$3.141592653589793238462643383273834839888336891556723534192...$$

$$3.141592653... \approx \pi$$

**Property:**

$$\frac{\log\left(\frac{6589613509842273573950927194751}{44767018745856000000}\right)}{\sqrt{67}} \text{ is a transcendental number}$$

**Alternate forms:**

$$-\frac{1}{\sqrt{67}} (28 \log(2) + 6 \log(3) + 6 \log(5) - \log(7) + 4 \log(11) - \log(157) - \log(5996008653177682960828869149))$$

$$\frac{-\frac{28 \log(2)}{\sqrt{67}} - \frac{6 \log(3)}{\sqrt{67}} - \frac{6 \log(5)}{\sqrt{67}} + \frac{\log(7)}{\sqrt{67}} - \frac{4 \log(11)}{\sqrt{67}} + \frac{\log(157)}{\sqrt{67}} + \frac{\log(5\,996\,008\,653\,177\,682\,960\,828\,869\,149)}{\sqrt{67}}}{\sqrt{67}}$$

### Alternative representations:

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \frac{\log_e\left(744 + 5280^3 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}}$$

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \frac{\log(a) \log_a\left(744 + 5280^3 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}}$$

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = -\frac{\text{Li}_1\left(-743 - 5280^3 + \frac{196884}{5280^3} - \frac{167827484}{5280^6}\right)}{\sqrt{67}}$$

### Series representations:

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \frac{\log\left(\frac{6589613509797506555205071194751}{44767018745856000000}\right)}{\sqrt{67}} - \frac{\sum_{k=1}^{\infty} \left(\frac{44767018745856000000}{6589613509797506555205071194751}\right)^k}{k \sqrt{67}}$$

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \frac{2i\pi \left[ \frac{\arg\left(\frac{6589613509842273573950927194751}{44767018745856000000} - x\right)}{2\pi} \right]}{\sqrt{67}} + \frac{\log(x)}{\sqrt{67}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{6589613509842273573950927194751}{44767018745856000000} - x\right)^k x^{-k}}{k}}{\sqrt{67}} \quad \text{for } x < 0$$

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \frac{2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right]}{\sqrt{67}} +$$

$$\frac{\log(z_0)}{\sqrt{67}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k \left( \frac{6589613509842273573950927194751}{44767018745856000000} \right)^k z_0^{-k}}{k}}{\sqrt{67}}$$

**Integral representations:**

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} =$$

$$\frac{1}{\sqrt{67}} \int_1^{\frac{6589613509842273573950927194751}{44767018745856000000}} \frac{1}{t} dt$$

$$\frac{\log\left(5280^3 + 744 - \frac{196884}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = -\frac{i}{2\sqrt{67}\pi}$$

$$\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\left( \frac{6589613509842273573950927194751}{44767018745856000000} \right)^s \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$(((\ln(5280^3+744-((x*108)+96*108-1*108)/5280^3+167827484/5280^6)/\text{sqrt}(67)))) = \text{Pi}$

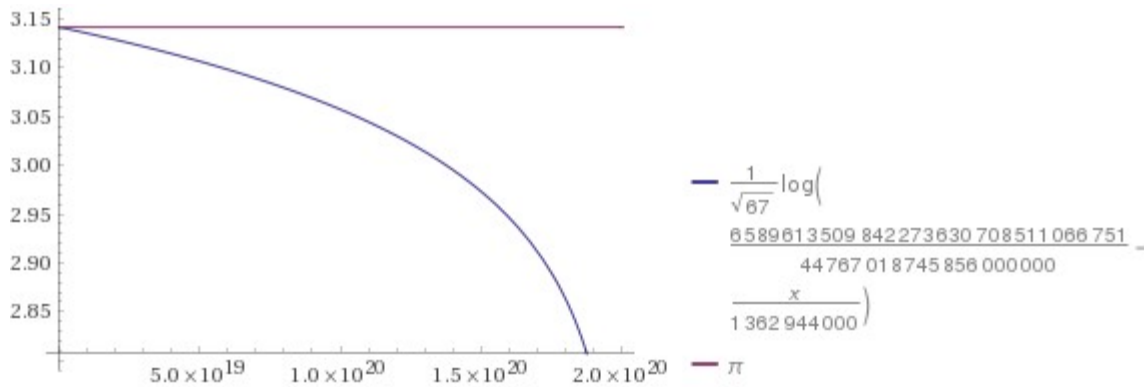
**Input:**

$$\frac{\log\left(5280^3 + 744 - \frac{x \times 108 + 96 \times 108 - 1 \times 108}{5280^3} + \frac{167827484}{5280^6}\right)}{\sqrt{67}} = \pi$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\log\left(\frac{-108x - 10260}{147197952000} + \frac{6589613509842273633828864346751}{44767018745856000000}\right)}{\sqrt{67}} = \pi$$

**Plot:****Alternate form:**

$$\frac{\log\left(\frac{6589613509842273630708511066751}{44767018745856000000} - \frac{x}{1362944000}\right)}{\sqrt{67}} = \pi$$

**Solution:**

$$x = \frac{6589613509842273630708511066751}{32845824000} - 1362944000 e^{\sqrt{67} \pi}$$

**Solution:**

$$x \approx 1728.0$$

1728

We note that:

$$e^{(\pi \cdot \sqrt{67})}$$

**Input:**

$$e^{\pi \sqrt{67}}$$

**Decimal approximation:**

$$1.4719795274399999866245422450682926131257862850818331... \times 10^{11}$$

$$1.471979527439... \cdot 10^{11}$$

**Property:**

$e^{\sqrt{67} \pi}$  is a transcendental number



### Series representations:

$$e^{\pi\sqrt{67}} = \sum_{k=0}^{\infty} \frac{67^{k/2} \pi^k}{k!}$$

$$e^{\pi\sqrt{67}} = \sum_{k=-\infty}^{\infty} I_k(\sqrt{67} \pi)$$

$$e^{\pi\sqrt{67}} = \sum_{k=0}^{\infty} \frac{67^k \pi^{2k} (1 + 2k + \sqrt{67} \pi)}{(1 + 2k)!}$$

### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

and that

$$5280^3$$

**Input:**

$$5280^3$$

**Result:**

$$147197952000$$

**Scientific notation:**

$$1.47197952 \times 10^{11}$$

1.47197952 \* 10<sup>11</sup> result that is very near to 1.471979527439....\*10<sup>11</sup>

We have also:

$$e^{(\pi*\text{sqrt}163)}$$

**Input:**

$$e^{\pi \sqrt{163}}$$

**Decimal approximation:**

$$2.6253741264076874399999999999925007259719818568887935... \times 10^{17}$$

$$2.625374126407687439... * 10^{17}$$

$$\approx 262537412640768743$$

**Property:**

$e^{\sqrt{163} \pi}$  is a transcendental number

**Constant name:**

Ramanujan constant

We know that (from <https://mathworld.wolfram.com/RamanujanConstant.html>):

The irrational constant

$$R = e^{\pi \sqrt{163}} \tag{1}$$

$$= 262537412640768743.999999999992500 \dots \tag{2}$$

(OEIS A060295), which is very close to an integer. Numbers such as the Ramanujan constant can be found using the theory of modular functions. In fact, the nine Heegner numbers (which include 163) share a deep number theoretic property related to some amazing properties of the  $j$ -function that leads to this sort of near-identity.

and:

$$(640320)^3$$

**Input:**

$$640320^3$$

**Result:**

262537412640768000

**Scientific notation:** $2.62537412640768 \times 10^{17}$ 2.62537412640768 \* 10<sup>17</sup> a result very near to 2.625374126407687439... \* 10<sup>17</sup>

Now, we have that:

$$\pi \approx \frac{\ln(640320^3 + 744)^2 - 2 \cdot 196884}{2 \cdot \sqrt{163}}$$

Thence, we have:

$$\ln((640320^3 + 744)^2 - 2 \cdot 196884) / (2 \cdot \sqrt{163})$$

**Input:**

$$\frac{\log((640320^3 + 744)^2 - 2 \times 196884)}{2 \sqrt{163}}$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\log(68925893036109279891085639286943768)}{2 \sqrt{163}}$$

**Decimal approximation:**

3.141592653589793238462643383279502884197169399282071114789...

3.1415926535...  $\approx \pi$ **Property:**

$$\frac{\log(68925893036109279891085639286943768)}{2 \sqrt{163}} \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{3 \log(2) + 2 \log(3) + \log(3374739421) + \log(283667551926807631500839)}{2 \sqrt{163}}$$

$$\frac{3 \log(2)}{2\sqrt{163}} + \frac{\log(3)}{\sqrt{163}} + \frac{\log(3\,374\,739\,421)}{2\sqrt{163}} + \frac{\log(283\,667\,551\,926\,807\,631\,500\,839)}{2\sqrt{163}}$$

### Alternative representations:

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = \frac{\log_e(-393\,768 + (744 + 640\,320^3)^2)}{2\sqrt{163}}$$

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = \frac{\log(a) \log_a(-393\,768 + (744 + 640\,320^3)^2)}{2\sqrt{163}}$$

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = -\frac{\text{Li}_1(393\,769 - (744 + 640\,320^3)^2)}{2\sqrt{163}}$$

### Series representations:

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = \frac{\log(68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,767)}{2\sqrt{163}} - \frac{\sum_{k=1}^{\infty} \left( \frac{-\frac{1}{68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,767}}{k} \right)^k}{2\sqrt{163}}$$

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = \frac{i\pi \left[ \frac{\arg(68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,768 - x)}{2\pi} \right]}{\sqrt{163}} + \frac{\log(x)}{2\sqrt{163}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,768 - x)^k x^{-k}}{k}}{2\sqrt{163}} \quad \text{for } x < 0$$

$$\frac{\log((640\,320^3 + 744)^2 - 2 \times 196\,884)}{2\sqrt{163}} = \frac{1}{2\sqrt{163}} \left( \log(z_0) + \left[ \frac{\arg(68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,768 - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (68\,925\,893\,036\,109\,279\,891\,085\,639\,286\,943\,768 - z_0)^k z_0^{-k}}{k} \right)$$

**Integral representations:**

$$\frac{\log((640320^3 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = \frac{1}{2\sqrt{163}} \int_1^{68925893036109279891085639286943768} \frac{1}{t} dt$$

$$\frac{\log((640320^3 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = -\frac{i}{4\sqrt{163}\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{68925893036109279891085639286943767^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

or:

$$\ln((262537412640768744+744)^2-2*196884) / (2*\sqrt{163})$$

**Input:**

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}}$$

log(x) is the natural logarithm

**Exact result:**

$$\frac{\log(68925893036109670546755648751388376)}{2\sqrt{163}}$$

**Decimal approximation:**

3.141592653589793460429398113361876729587656644445051041838...

$$3.141592653... \approx \pi$$

**Property:**

$\frac{\log(68925893036109670546755648751388376)}{2\sqrt{163}}$  is a transcendental number

**Alternate forms:**

$$\frac{1}{2\sqrt{163}} (3 \log(2) + 2 \log(3) + \log(7) + \log(41) + \log(14851) + \log(54605790089) + \log(4113141167321831))$$

$$\frac{3 \log(2)}{2 \sqrt{163}} + \frac{\log(3)}{\sqrt{163}} + \frac{\log(957\,304\,069\,945\,967\,646\,482\,717\,343\,769\,283)}{2 \sqrt{163}}$$

$$\frac{3 \log(2)}{2 \sqrt{163}} + \frac{\log(3)}{\sqrt{163}} + \frac{\log(7)}{2 \sqrt{163}} + \frac{\log(41)}{2 \sqrt{163}} + \frac{\log(14\,851)}{2 \sqrt{163}} + \frac{\log(54\,605\,790\,089)}{2 \sqrt{163}} + \frac{\log(4\,113\,141\,167\,321\,831)}{2 \sqrt{163}}$$

**Alternative representations:**

$$\frac{\log((262\,537\,412\,640\,768\,744 + 744)^2 - 2 \times 196\,884)}{2 \sqrt{163}} = \frac{\log_e(-393\,768 + 262\,537\,412\,640\,769\,488^2)}{2 \sqrt{163}}$$

$$\frac{\log((262\,537\,412\,640\,768\,744 + 744)^2 - 2 \times 196\,884)}{2 \sqrt{163}} = \frac{\log(a) \log_a(-393\,768 + 262\,537\,412\,640\,769\,488^2)}{2 \sqrt{163}}$$

$$\frac{\log((262\,537\,412\,640\,768\,744 + 744)^2 - 2 \times 196\,884)}{2 \sqrt{163}} = \frac{\text{Li}_1(393\,769 - 262\,537\,412\,640\,769\,488^2)}{2 \sqrt{163}}$$

**Series representations:**

$$\frac{\log((262\,537\,412\,640\,768\,744 + 744)^2 - 2 \times 196\,884)}{2 \sqrt{163}} = \frac{\log(68\,925\,893\,036\,109\,670\,546\,755\,648\,751\,388\,375)}{2 \sqrt{163}} - \frac{\sum_{k=1}^{\infty} \left( \frac{-\frac{1}{68\,925\,893\,036\,109\,670\,546\,755\,648\,751\,388\,375}}{k} \right)^k}{2 \sqrt{163}}$$

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = \frac{i\pi \left[ \frac{\arg(68925893036109670546755648751388376-x)}{2\pi} \right]}{\sqrt{163}} + \frac{\log(x)}{2\sqrt{163}} - \frac{\sum_{k=1}^{\infty} \frac{(-1)^k (68925893036109670546755648751388376-x)^k x^{-k}}{k}}{2\sqrt{163}} \quad \text{for } x < 0$$

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = \frac{1}{2\sqrt{163}} \left( \log(z_0) + \left[ \frac{\arg(68925893036109670546755648751388376 - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (68925893036109670546755648751388376 - z_0)^k z_0^{-k}}{k} \right)$$

### Integral representations:

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = \frac{1}{2\sqrt{163}} \int_1^{68925893036109670546755648751388376} \frac{1}{t} dt$$

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} = -\frac{i}{4\sqrt{163}\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{68925893036109670546755648751388375^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds$$

for  $-1 < \gamma < 0$

Now, we obtain:

$$\left( \frac{\ln((262537412640768744+744)^2-2*196884)}{2*\sqrt{163}} \right) * \left( \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376)} \right)$$

### Input:

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2\sqrt{163}} \times \frac{3456\sqrt{163}}{\log(68925893036109670546755648751388376)}$$

**Result:**

1728

1728

**Alternative representations:**

$$\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376) (2 \sqrt{163})} = \frac{3456 \log_e(-393768 + 262537412640769488^2) \sqrt{163}}{\log_e(68925893036109670546755648751388376) (2 \sqrt{163})}$$

$$\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376) (2 \sqrt{163})} = \frac{3456 \log(\alpha) \log_\alpha(-393768 + 262537412640769488^2) \sqrt{163}}{(\log(\alpha) \log_\alpha(68925893036109670546755648751388376)) (2 \sqrt{163})}$$

$$\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376) (2 \sqrt{163})} = \frac{-3456 \operatorname{Li}_1(393769 - 262537412640769488^2) \sqrt{163}}{-\operatorname{Li}_1(-68925893036109670546755648751388375) (2 \sqrt{163})}$$

and:

$$\left( \frac{\ln((262537412640768744+744)^2-2*196884)}{(2*\sqrt{163})} \right) * (3456 \sqrt{163}) / \log(68925893036109670546755648751388376) + 1$$

**Input:**

$$\frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2 \sqrt{163}} \times \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376)} + 1$$



log(x) is the natural logarithm

**Result:**

1729

1729

From which:

$$\left( \frac{\ln((262537412640768744+744)^2 - 2 \times 196884)}{2 \sqrt{163}} \right) \times \left( \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376) + 1} \right) \times (3456 \sqrt{163}) \times \log(68925893036109670546755648751388376) + 1 \right)^{1/15}$$

**Input:**

$$\left( \frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2 \sqrt{163}} \times \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376) + 1} + 1 \right)^{(1/15)}$$

log(x) is the natural logarithm

**Exact result:**

$$\sqrt[15]{1729}$$

**Decimal approximation:**

1.643815228748728130580088031324769514329283143699940172645...

1.6438152287...

and:

$$\left( \frac{\ln((262537412640768744+744)^2 - 2 \times 196884)}{2 \sqrt{163}} \right) \times \left( \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376) + 1} \right) \times (3456 \sqrt{163}) \times \log(68925893036109670546755648751388376) + 1 \right)^{1/15} - (21+5)1/10^3$$

**Input:**

$$\left( \frac{\log((262537412640768744 + 744)^2 - 2 \times 196884)}{2 \sqrt{163}} \times \frac{3456 \sqrt{163}}{\log(68925893036109670546755648751388376) + 1} + 1 \right)^{(1/15) - (21+5) \times \frac{1}{10^3}}$$

**Exact result:**

$$\sqrt[15]{1729} - \frac{13}{500}$$

**Decimal approximation:**

1.617815228748728130580088031324769514329283143699940172645...

1.6178152287...

**Alternate forms:**

$$\frac{1}{500} \left( 500 \sqrt[15]{1729} - 13 \right)$$

$$\frac{1}{500} \left( 500 \left( \text{root of } \begin{array}{l} 31\,250\,000\,000\,000\,x^5 + 686\,562\,500\,000\,000\,x^4 + 6\,033\,511\,250\,000\,000\,x^3 + \\ 26\,511\,248\,432\,500\,000\,x^2 + 58\,245\,212\,806\,202\,500\,x - \\ 52\,764\,892\,578\,124\,999\,999\,999\,999\,948\,814\,106\,985\,909\,243 \\ \text{near } x = 1.11045 \times 10^6 \end{array} + 2197 \right)^{(1/3)} - 13 \right)$$

**Alternative representations:**

$$\sqrt[15]{\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376)(2 \sqrt{163})} + 1} - \frac{21+5}{10^3} =$$

$$-\frac{26}{10^3} + \sqrt[15]{1 + \frac{3456 \log_e(-393768 + 262537412640769488^2) \sqrt{163}}{\log_e(68925893036109670546755648751388376)(2 \sqrt{163})}}$$

$$\sqrt[15]{\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376)(2 \sqrt{163})} + 1} - \frac{21+5}{10^3} =$$

$$-\frac{26}{10^3} + \sqrt[15]{1 + \frac{3456 \log(a) \log_a(-393768 + 262537412640769488^2) \sqrt{163}}{(\log(a) \log_a(68925893036109670546755648751388376))(2 \sqrt{163})}}$$

$$\sqrt[15]{\frac{(3456 \sqrt{163}) \log((262537412640768744 + 744)^2 - 2 \times 196884)}{\log(68925893036109670546755648751388376)(2 \sqrt{163})} + 1 - \frac{21+5}{10^3}} =$$

$$-\frac{26}{10^3} + \sqrt[15]{1 - \frac{3456 \operatorname{Li}_1(393769 - 262537412640769488^2) \sqrt{163}}{\operatorname{Li}_1(-68925893036109670546755648751388375)(2 \sqrt{163})}}$$

## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJlQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:*

125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

### Manuscript Book 3 of Srinivasa Ramanujan

#### Can't you just feel the moonshine?

Ken Ono (Emory University) - <http://people.oregonstate.edu/~petschec/ONTD/Talk2.pdf> - March 30, 2017

#### S. Ramanujan to G.H. Hardy - 12 January 1920

*University of Madras*

**MONSTROUS MOONSHINE** - J. H. CONWAY AND S. P. NORTON - [BULL. LONDON MATH. SOC, 11 (1979), 308-339]

**Modular equations and approximations to  $\pi$**  – Srinivasa Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

The International Journal Of Engineering And Science (IJES) || Volume || 3 || Issue || 5 || Pages || 25-36 || 2014 || ISSN (e): 2319 – 1813 ISSN (p): 2319 – 1805  
www.theijes.com The IJES Page 25

**Some Moonshine connections between Fischer-Griess Monster group (M) and Number theory** - Amina Muhammad Lawan