

On some Ramanujan equations: mathematical connections between ϕ , $\zeta(2)$, Mock theta functions and various parameters of Particle Physics.

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Abstract

In this paper we have described and analyzed some Ramanujan equations. Furthermore, we have obtained several mathematical connections between ϕ , $\zeta(2)$, Mock theta functions and various parameters of Particle Physics.

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https://link.springer.com/chapter/10.1007/978-81-322-0767-2_12

<https://www.sciencephoto.com/media/228058/view/indian-mathematician-srinivasa-ramanujan>

We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation

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INDUCTIONIS FALLACIS" AND q -TRINOMIAL COEFFICIENTS**
GEORGE E. ANDREWS

We have:

$$\begin{aligned}
E_m(a, b) &= \frac{1}{10} \sum_{\mu=-\infty}^{\infty} \sum_{j=0}^9 \binom{m}{\mu}_2 (\xi^{j(\mu-a)} - \xi^{j(\mu-b)}) \\
&= \frac{1}{10} \sum_{j=0}^9 (1 + \xi^j + \xi^{-j})^m (\xi^{-aj} - \xi^{-bj}) \\
&= \frac{1}{5} \sum_{j=1}^4 (1 + 2 \cos(\pi j/5))^m (\cos(\pi a j/5) - \cos(\pi b j/5)) \\
&\quad + \frac{1}{10} (-1)^m ((-1)^a - (-1)^b) \\
&= \frac{1}{5} \sum_{j=1}^2 \{(1 + 2 \cos(\pi j/5))^m - (1 - 2 \cos(\pi j/5))^m\} \\
&\quad \times (\cos(\pi a j/5) - \cos(\pi b j/5)) + \frac{1}{10} (-1)^m ((-1)^a - (-1)^b) \\
&= \alpha(\phi^2)^m + \beta(\bar{\phi}^2)^m + \gamma\phi^m + \delta\bar{\phi}^m + \varepsilon(-1)^m,
\end{aligned}$$

since $\phi^2 = 1 + 2 \cos(\pi/5)$, $\bar{\phi}^2 = 1 - 2 \cos(2\pi/5)$, $\phi = 1 + 2 \cos(2\pi/5)$, and $\bar{\phi} = 1 - 2 \cos(\pi/5)$.

from

$$\begin{aligned}
&\frac{1}{5} \sum_{j=1}^2 \{(1 + 2 \cos(\pi j/5))^m - (1 - 2 \cos(\pi j/5))^m\} \\
&\quad \times (\cos(\pi a j/5) - \cos(\pi b j/5)) + \frac{1}{10} (-1)^m ((-1)^a - (-1)^b) \\
&= \alpha(\phi^2)^m + \beta(\bar{\phi}^2)^m + \gamma\phi^m + \delta\bar{\phi}^m + \varepsilon(-1)^m
\end{aligned}$$

We obtain, for $m = 1, 2, 3, 4, 5$ and 6 :

$$(1+2\cos(\pi/5))+(1-2\cos(2\pi/5))+(1+2\cos(2\pi/5))+(1-2\cos(\pi/5))+(-1)$$

Input:

$$\left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right) + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right) + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right) + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right) - 1$$

Result:

3

3

$$(1+2\cos(\pi/5))^2+(1-2\cos((2\pi)/5))^2+(1+2\cos((2\pi)/5))^2+(1-2\cos(\pi/5))^2+(-1)^2$$

Input:

$$\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2$$

Result:

11

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Alternative representations:

$$\begin{aligned} &\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2 = \\ &1+\left(1-2\cosh\left(\frac{i\pi}{5}\right)\right)^2+\left(1+2\cosh\left(\frac{i\pi}{5}\right)\right)^2+\left(1-2\cosh\left(\frac{2i\pi}{5}\right)\right)^2+\left(1+2\cosh\left(\frac{2i\pi}{5}\right)\right)^2 \end{aligned}$$

$$\begin{aligned} &\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2 = \\ &1+\left(1-2\cosh\left(-\frac{i\pi}{5}\right)\right)^2+\left(1+2\cosh\left(-\frac{i\pi}{5}\right)\right)^2+ \\ &\left(1-2\cosh\left(-\frac{2i\pi}{5}\right)\right)^2+\left(1+2\cosh\left(-\frac{2i\pi}{5}\right)\right)^2 \end{aligned}$$

$$\begin{aligned} &\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2 = \\ &1+\left(1-\frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^2+\left(1+\frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^2+\left(1-\frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^2+\left(1+\frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^2 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} &\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2 = \\ &5+8T_{\frac{1}{5}}(\cos(\pi))^2+8T_{\frac{2}{5}}(\cos(\pi))^2 \end{aligned}$$

$$\begin{aligned} &\left(1+2\cos\left(\frac{\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1+2\cos\left(\frac{2\pi}{5}\right)\right)^2+\left(1-2\cos\left(\frac{\pi}{5}\right)\right)^2+(-1)^2 = \\ &21-32\cos^2\left(\frac{\pi}{10}\right)+32\cos^4\left(\frac{\pi}{10}\right)-32\cos^2\left(\frac{\pi}{5}\right)+32\cos^4\left(\frac{\pi}{5}\right) \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^2 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^2 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^2 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^2 + (-1)^2 = \\ & 21 - 32 \sin^2\left(\frac{\pi}{10}\right) + 32 \sin^4\left(\frac{\pi}{10}\right) - 32 \sin^2\left(\frac{\pi}{5}\right) + 32 \sin^4\left(\frac{\pi}{5}\right) \end{aligned}$$

$$(1+2\cos(\pi/5))^3+(1-2\cos((2\pi)/5))^3+(1+2\cos((2\pi)/5))^3+(1-2\cos(\pi/5))^3+(-1)^3$$

Input:

$$\left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3$$

Result:

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Alternative representations:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & -1 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^3 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^3 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^3 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^3 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & -1 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^3 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^3 + \\ & \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^3 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^3 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & -1 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^3 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^3 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^3 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^3 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & 3 \left(1 + 8 T_{\frac{1}{5}}(\cos(\pi))^2 + 8 T_{\frac{2}{5}}(\cos(\pi))^2\right) \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & 51 - 96 \cos^2\left(\frac{\pi}{10}\right) + 96 \cos^4\left(\frac{\pi}{10}\right) - 96 \cos^2\left(\frac{\pi}{5}\right) + 96 \cos^4\left(\frac{\pi}{5}\right) \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^3 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^3 + (-1)^3 = \\ & 51 - 96 \sin^2\left(\frac{\pi}{10}\right) + 96 \sin^4\left(\frac{\pi}{10}\right) - 96 \sin^2\left(\frac{\pi}{5}\right) + 96 \sin^4\left(\frac{\pi}{5}\right) \end{aligned}$$

$$(1+2\cos(\pi/5))^4+(1-2\cos((2\pi/5))^4+(1+2\cos((2\pi/5))^4+(1-2\cos(\pi/5))^4+(-1)^4$$

Input:

$$\left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4$$

Result:

55

55

Alternative representations:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & (-1)^4 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^4 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^4 + \\ & \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^4 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^4 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & (-1)^4 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^4 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^4 + \\ & \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^4 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^4 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & (-1)^4 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^4 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^4 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^4 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^4 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & 5 + 48 T_{\frac{1}{5}}(\cos(\pi))^2 + 32 T_{\frac{1}{5}}(\cos(\pi))^4 + 48 T_{\frac{2}{5}}(\cos(\pi))^2 + 32 T_{\frac{2}{5}}(\cos(\pi))^4 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & 1 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^4 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^4 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^4 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^4 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^4 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^4 + (-1)^4 = \\ & 1 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^4 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^4 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^4 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^4 \end{aligned}$$

$$(1+2\cos(\pi/5))^5+(1-2\cos((2\pi/5))^5+(1+2\cos((2\pi/5))^5+(1-2\cos(\pi/5))^5+(-1)^5$$

Input:

$$\left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5$$

Result:

133

133

Alternative representations:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = (-1)^5 + \\ & \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^5 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^5 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^5 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^5 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = \\ & (-1)^5 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^5 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^5 + \\ & \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^5 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^5 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = \\ & (-1)^5 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^5 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^5 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^5 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^5 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = \\ & 3 + 80 T_{\frac{1}{5}}(\cos(\pi))^2 + 160 T_{\frac{1}{5}}(\cos(\pi))^4 + 80 T_{\frac{2}{5}}(\cos(\pi))^2 + 160 T_{\frac{2}{5}}(\cos(\pi))^4 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = \\ & -1 - \left(-3 + 4 \cos^2\left(\frac{\pi}{10}\right)\right)^5 + \left(-1 + 4 \cos^2\left(\frac{\pi}{10}\right)\right)^5 - \left(-3 + 4 \cos^2\left(\frac{\pi}{5}\right)\right)^5 + \left(-1 + 4 \cos^2\left(\frac{\pi}{5}\right)\right)^5 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^5 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^5 + (-1)^5 = \\ & -1 - \left(-3 + 4 \sin^2\left(\frac{\pi}{10}\right)\right)^5 + \left(-1 + 4 \sin^2\left(\frac{\pi}{10}\right)\right)^5 - \left(-3 + 4 \sin^2\left(\frac{\pi}{5}\right)\right)^5 + \left(-1 + 4 \sin^2\left(\frac{\pi}{5}\right)\right)^5 \end{aligned}$$

$$(1+2\cos(\pi/5))^6+(1-2\cos((2\pi)/5))^6+(1+2\cos((2\pi)/5))^6+(1-2\cos(\pi/5))^6+(-1)^6$$

Input:

$$\left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6$$

Result:

341

341

Alternative representations:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = (-1)^6 + \\ & \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = \\ & (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \\ & \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = \\ & (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = \\ & 1 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = \\ & 1 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 \end{aligned}$$

$$\begin{aligned} & \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 = \\ & 1 + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 \end{aligned}$$

We note that, the sum of all results is:

$$341 + 133 + 55 + 21 + 11 + 3 = 564$$

From this value, we obtain:

$$564 - 16 = 548$$

$$\left(\left(\left(3+11+21+55+133+(1+2\cos(\pi/5))^6+(1-2\cos(2\pi/5))^6+(1+2\cos(2\pi/5))^6+(1-2\cos(\pi/5))^6+(-1)^6\right)\right)-16\right)$$

Input:

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \right. \\ & \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right) - 16 \end{aligned}$$

Result:

548

548

Alternative representations:

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 207 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \\ & \quad \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 207 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \\ & \quad \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 207 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)} \right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)} \right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)} \right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)} \right)^6 \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 208 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right) \right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right) \right)^6 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right) \right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right) \right)^6 \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 208 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right) \right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right) \right)^6 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right) \right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right) \right)^6 \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 16 = \\ & 208 + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi)) \right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi)) \right)^6 + \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi)) \right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi)) \right)^6 \end{aligned}$$

$$564 * \pi - 43 = 1728.85825662....$$

$$(((3+11+21+55+133+(1+2\cos(\pi/5))^6+(1-2\cos(2\pi/5))^6+(1+2\cos(2\pi/5))^6+(1-2\cos(\pi/5))^6+(-1)^6))\pi-43)$$

Input:

$$\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right)\pi - 43$$

Exact result:

$$\left(224 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right)\pi - 43$$

Decimal approximation:

1728.858256624643386492930868169639626687203541247559683029...

1728.8582566...

Property:

$$-43 + \left(224 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right)\pi \text{ is a transcendental number}$$

Alternate form:

$$564\pi - 43$$

Alternative representations:

$$\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right)\pi - 43 = -43 + \pi \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6\right)$$

$$\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right)\pi - 43 = -43 + \pi \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6\right)$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) \pi - 43 = \\ & -43 + \pi \left(223 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)} \right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)} \right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)} \right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)} \right)^6 \right) \end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) \pi - 43 = -43 + \\ & \quad \pi \left(224 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right) \right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right) \right)^6 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right) \right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) \pi - 43 = -43 + \\ & \quad \pi \left(224 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right) \right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right) \right)^6 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right) \right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) \pi - 43 = \\ & -43 + \pi \left(224 + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi)) \right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi)) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi)) \right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi)) \right)^6 \right) \end{aligned}$$

$\left(\left((3+11+21+55+133+(1+2\cos(\pi/5))^6+(1-2\cos(2\pi/5))^6+(1+2\cos(2\pi/5))^6+(1-2\cos(\pi/5))^6+(-1)^6) \right) \times \frac{1}{4} - 2 + \frac{1}{\phi} \right)$

Input:

$$\begin{aligned} & \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) \times \frac{1}{4} - 2 + \frac{1}{\phi} \end{aligned}$$

ϕ is the golden ratio

Exact result:

$$\frac{1}{\phi} - 2 + \frac{1}{4} \left(224 + \left(1 + \frac{1}{2} (-1 - \sqrt{5}) \right)^6 + \left(1 + \frac{1}{2} (1 - \sqrt{5}) \right)^6 + \left(1 + \frac{1}{2} (\sqrt{5} - 1) \right)^6 + \left(1 + \frac{1}{2} (1 + \sqrt{5}) \right)^6 \right)$$

Decimal approximation:

139.6180339887498948482045868343656381177203091798057628621...

139.6180339887...

Minimal polynomial:

$$x^2 - 277x + 19181$$

Expanded form:

$$139 + \frac{2}{1 + \sqrt{5}}$$

Alternate forms:

$$\frac{1}{\phi} + 139$$

$$\frac{139\phi + 1}{\phi}$$

$$\frac{1}{2} (277 + \sqrt{5})$$

Alternative representations:

$$\begin{aligned} & \frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = \\ & -2 + \frac{1}{\phi} + \frac{1}{4} \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = \\ & -2 + \frac{1}{\phi} + \frac{1}{4} \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = -2 + \frac{1}{\phi} + \\ \frac{1}{4} \left(223 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 \right)$$

Multiple-argument formulas:

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = \\ \frac{1}{4} \left(216 + \frac{4}{\phi} + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \right. \\ \left. \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 \right)$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = \\ \frac{1}{4} \left(216 + \frac{4}{\phi} + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \right. \\ \left. \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 \right)$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 + \frac{1}{\phi} = \\ \frac{1}{4} \left(216 + \frac{4}{\phi} + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \right. \\ \left. \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 \right)$$

$((3+11+21+55+133+(1+2\cos(\pi/5))^6+(1-2\cos(2\pi/5))^6+(1+2\cos(2\pi/5))^6+(1-2\cos(\pi/5))^6+(-1)^6)) \times 1/4 - 2 \times 8 + 1/\text{golden ratio}$

Input:

$$\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) \times \frac{1}{4} - 2 \times 8 + \frac{1}{\phi}$$

Exact result:

$$\frac{1}{\phi} - 16 + \frac{1}{4} \left(224 + \left(1 + \frac{1}{2} (-1 - \sqrt{5}) \right)^6 + \left(1 + \frac{1}{2} (1 - \sqrt{5}) \right)^6 + \left(1 + \frac{1}{2} (\sqrt{5} - 1) \right)^6 + \left(1 + \frac{1}{2} (1 + \sqrt{5}) \right)^6 \right)$$

Decimal approximation:

125.6180339887498948482045868343656381177203091798057628621...

125.6180339887...

Minimal polynomial:

$$x^2 - 249x + 15499$$

Expanded form:

$$125 + \frac{2}{1 + \sqrt{5}}$$

Alternate forms:

$$\frac{1}{\phi} + 125$$

$$\frac{125\phi + 1}{\phi}$$

$$\frac{1}{2} (249 + \sqrt{5})$$

Alternative representations:

$$\begin{aligned} & \frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = \\ & -16 + \frac{1}{\phi} + \frac{1}{4} \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right) \right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right) \right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = \\ & -16 + \frac{1}{\phi} + \frac{1}{4} \left(223 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right) \right)^6 + \right. \\ & \quad \left. \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right) \right)^6 \right) \end{aligned}$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = -16 + \frac{1}{\phi} + \\ \frac{1}{4} \left(223 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 \right)$$

Multiple-argument formulas:

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = \\ \frac{1}{4} \left(160 + \frac{4}{\phi} + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \right. \\ \left. \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 \right)$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = \\ \frac{1}{4} \left(160 + \frac{4}{\phi} + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \right. \\ \left. \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 \right)$$

$$\frac{1}{4} \left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \\ \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) - 2 \times 8 + \frac{1}{\phi} = \\ \frac{1}{4} \left(160 + \frac{4}{\phi} + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \right. \\ \left. \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 \right)$$

$\text{sqrt}[\left(\left(\left(3+11+21+55+133+(1+2\cos(\text{Pi}/5))^6+(1-2\cos((2\text{Pi}/5))^6+(1+2\cos((2\text{Pi}/5))^6+(1-2\cos(\text{Pi}/5))^6+(-1)^6)\right)+123-11\right)\right)]$

Input:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \right. \right. \\ \left. \left. \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 \right) + 123 - 11\right)}$$

Result:

26

26 result equal to the dimensions number in Bosonic String Theory

All 2nd roots of $336 + (1 + 1/2 (-1 - \sqrt{5}))^6 + (1 + 1/2 (1 - \sqrt{5}))^6 + (1 + 1/2 (\sqrt{5} - 1))^6 + (1 + 1/2 (1 + \sqrt{5}))^6$:

$$\sqrt{\left(336 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right)} e^0 \approx 26.000 \quad (\text{real, principal root})$$

$$\sqrt{\left(336 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right)} e^{i\pi} \approx -26.000 \quad (\text{real root})$$

Alternative representations:

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right)} = \\ &\sqrt{\left(335 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6\right)} \end{aligned}$$

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right)} = \\ &\sqrt{\left(335 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6\right)} \end{aligned}$$

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right)} = \\ &\sqrt{335 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6} \end{aligned}$$

Series representations:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right) = \sqrt{335 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 \sum_{k=0}^{\infty} \binom{1}{2} \binom{1}{k} \left(335 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6\right)^{-k}}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right) = \sqrt{335 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 \sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(335 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6\right)^{-k} \left(-\frac{1}{2}\right)_k}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right) = \sqrt{\left(336 + \left(1 - 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{4}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 + 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{4}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 - 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 + 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6\right)}$$

Multiple-argument formulas:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 123 - 11\right) = \sqrt{336 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right) + 123 - 11\right) = \sqrt{336 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right) + 123 - 11\right) = \sqrt{\left(336 + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6\right)}$$

$\text{sqrt}[(((3+11+21+55+133+(1+2\cos(\text{Pi}/5))^6+(1-2\cos((2\text{Pi}/5))^6+(1+2\cos((2\text{Pi}/5))^6+(1-2\cos(\text{Pi}/5))^6+(-1)^6)))+12]$

Input:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6\right) + 12\right)}$$

Result:

24

24 value that is linked to the "Ramanujan function" (an elliptic modular function that satisfies the need for "conformal symmetry") that has 24 "modes" corresponding to the physical vibrations of a bosonic string.

All 2nd roots of $236 + (1 + 1/2 (-1 - \text{sqrt}(5)))^6 + (1 + 1/2 (1 - \text{sqrt}(5)))^6 + (1 + 1/2 (\text{sqrt}(5) - 1))^6 + (1 + 1/2 (1 + \text{sqrt}(5)))^6$:

$$\sqrt{\left(236 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right) e^0 \approx 24.000 \text{ (real, principal root)}$$

$$\sqrt{\left(236 + \left(1 + \frac{1}{2}(-1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(1 - \sqrt{5})\right)^6 + \left(1 + \frac{1}{2}(\sqrt{5} - 1)\right)^6 + \left(1 + \frac{1}{2}(1 + \sqrt{5})\right)^6\right)} e^{i\pi} \approx -24.000 \text{ (real root)}$$

Alternative representations:

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right)} = \\ &\sqrt{\left(235 + (-1)^6 + \left(1 - 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(\frac{2i\pi}{5}\right)\right)^6\right)} \end{aligned}$$

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right)} = \\ &\sqrt{\left(235 + (-1)^6 + \left(1 - 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{i\pi}{5}\right)\right)^6 + \left(1 - 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6 + \left(1 + 2 \cosh\left(-\frac{2i\pi}{5}\right)\right)^6\right)} \end{aligned}$$

$$\begin{aligned} &\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right)} = \\ &\sqrt{235 + (-1)^6 + \left(1 - \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{\pi}{5}\right)}\right)^6 + \left(1 - \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6 + \left(1 + \frac{2}{\sec\left(\frac{2\pi}{5}\right)}\right)^6} \end{aligned}$$

Series representations:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) =$$

$$\sqrt{235 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6}$$

$$\sum_{k=0}^{\infty} \binom{\frac{1}{2}}{k} \left(235 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6\right)^{-k}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) =$$

$$\sqrt{235 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6}$$

$$\sum_{k=0}^{\infty} \frac{1}{k!} (-1)^k \left(235 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6\right)^{-k} \left(-\frac{1}{2}\right)_k$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) =$$

$$\sqrt{\left(236 + \left(1 - 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{4}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 + 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{4}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 - 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6 + \left(1 + 2 \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{25}\right)^k \pi^{2k}}{(2k)!}\right)^6\right)}$$

Multiple-argument formulas:

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) =$$

$$\sqrt{236 + \left(1 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \cos^2\left(\frac{\pi}{5}\right)\right)^6}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) = \sqrt{236 + \left(1 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{10}\right)\right)^6 + \left(1 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6 + \left(3 - 4 \sin^2\left(\frac{\pi}{5}\right)\right)^6}$$

$$\sqrt{\left(\left(3 + 11 + 21 + 55 + 133 + \left(1 + 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 + 2 \cos\left(\frac{2\pi}{5}\right)\right)^6 + \left(1 - 2 \cos\left(\frac{\pi}{5}\right)\right)^6 + (-1)^6 + 12\right) = \sqrt{\left(236 + \left(1 - 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{1}{5}}(\cos(\pi))\right)^6 + \left(1 - 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6 + \left(1 + 2 T_{\frac{2}{5}}(\cos(\pi))\right)^6\right)}$$

From:

Theorem 5.1.

$$\prod_{j=1}^m (1 + q^{2j-1}) = \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+2\lambda} (T_0(m, 6\lambda, q) + T_0(m, 6\lambda + 1, q)) - \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+10\lambda+2} (T_0(m, 6\lambda + 2, q) + T_0(m, 6\lambda + 3, q)).$$

If $q = e^{2\pi i \tau}$, for $i\tau = i(1+i)$, we obtain:

$$\exp(2\pi i i(1+i))$$

Input:

$$\exp(2\pi i i(1+i))$$

Exact result:

$$e^{-2\pi}$$

Decimal approximation:

0.001867442731707988814430212934827030393422805002475317199...

0.0018674427...

Product $(1+0.0018674427^{(2j-1)})$, $j=1..6$

Product:

$$\prod_{j=1}^6 (0.00186744^{2^{j-1}} + 1) = 1.00187$$

Product $(1+0.0018674427^{(2j-1)})$, $j=1..7$

Product:

$$\prod_{j=1}^7 (0.00186744^{2^{j-1}} + 1) = 1.00187$$

If $q = e^{2\pi i \tau}$, for $i\tau = 1$, we obtain:

$$e^{(2\pi i)}$$

$$e^{2\pi}$$

535.491655247647365030493295890471814778057976032949155072...

$e^{2\pi}$ is a transcendental number

$$q = 535.49165.....$$

thence:

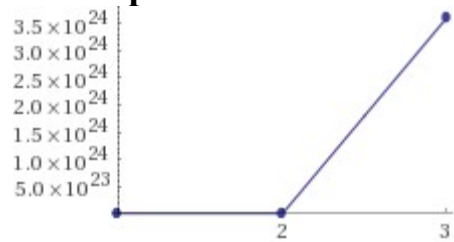
Product $(1+535.14965^{(2j-1)})$, $j=1..3$

Product:

$$\prod_{j=1}^3 (535.15^{2^{j-1}} + 1) = 3.60652 \times 10^{24}$$

$$3.60652 \times 10^{24}$$

Partial products:



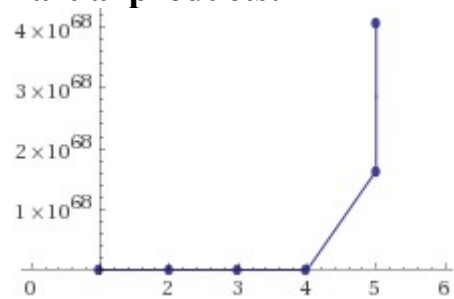
Product (1+535.14965^(2j-1)), j=1..6

Product:

$$\prod_{j=1}^6 (535.15^{2^{j-1}} + 1) = 1.68236 \times 10^{98}$$

$$1.68236e+98$$

Partial products:



((sum ((535.49165^(12x^2+2x)), x=-1..6))) - (((sum (535.49165^(12x^2+10x+2)), x=-1..6)))

Input interpretation:

$$\sum_{x=-1}^6 535.49165^{12x^2+2x} - \sum_{x=-1}^6 535.49165^{12x^2+10x+2}$$

Result:

$$-1.008870404980286 \times 10^{1348}$$

$$-1.008870404980286e+1348$$

Theorem 5.1.

$$\prod_{j=1}^m (1 + q^{2j-1}) = \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+2\lambda} (T_0(m, 6\lambda, q) + T_0(m, 6\lambda + 1, q)) \\ - \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+10\lambda+2} (T_0(m, 6\lambda + 2, q) + T_0(m, 6\lambda + 3, q)).$$

Sum:

$$\sum_{x=-1}^6 535.492^{12x^2+2x} \approx 3.682987307134794285603238931085495395658 \times 10^{1211}$$

Decimal approximation:

$$3.682987307134794285603238931085495395657775330386853... \times 10^{1211}$$

$$3.6829873e+1211$$

Sum:

$$\sum_{x=-1}^6 535.492^{12x^2+10x+2} \approx 1.008870404980244235957178232006758504287 \times 10^{1348}$$

Decimal approximation:

$$1.008870404980244235957178232006758504287081419858406... \times 10^{1348}$$

$$1.00887040498e+1348$$

$$1.68236e+98 = (3.6829873e+1211)x - (1.00887040498e+1348)y$$

Input interpretation:

$$1.68236 \times 10^{98} = (3.6829873 \times 10^{1211})x - (1.00887040498 \times 10^{1348})y$$

Result:

$$1.68236 \times 10^{98} = 3.682987300000000 \times 10^{1211} x - 1.008870404980 \times 10^{1348} y$$

Alternate forms:

$$y = 3.65060495562 \times 10^{-137} x - 1.667567996539012 \times 10^{-1250}$$

$$-3.6829873000000000 \times 10^{1211} x + 1.008870404980 \times 10^{1348} y + 1.68236 \times 10^{98} = 0$$

Real solution:

$$y \approx 3.650604955621641 \times 10^{-137} x - 1.667567996539012 \times 10^{-1250}$$

Solution:

$$y \approx 3.650604955621641 \times 10^{-137} x - 1.667567996539012 \times 10^{-1250}$$

$$(3.6829873e+1211)x - (1.00887040498e+1348)(3.650604955621641 \times 10^{-137} x - 1.667567996539012 \times 10^{-1250})$$

Input interpretation:

$$(3.6829873 \times 10^{1211})x - (1.00887040498 \times 10^{1348}) \left(\frac{3.650604955621641}{10^{137}} x - \frac{1.667567996539012}{10^{1250}} \right)$$

Result:

$$3.6829873000000000 \times 10^{1211} x - 1.008870404980 \times 10^{1348} (3.650604955621641 \times 10^{-137} x - 1.667567996539012 \times 10^{-1250})$$

Alternate forms:

$$3.14498 \times 10^{20} (9.62153750765067 \times 10^{1174} x + 5.34935 \times 10^{77})$$

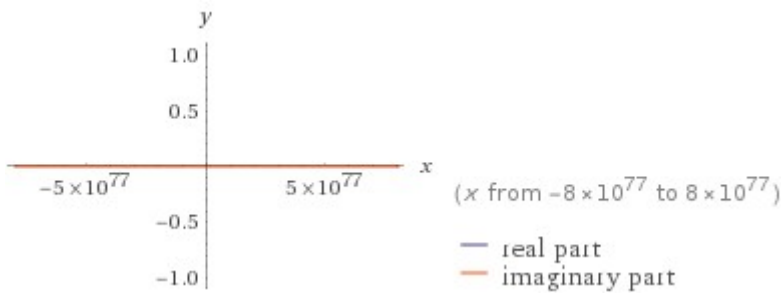
$$3.14498 \times 10^{20} (9.62153750765067 \times 10^{1174} x + 5.34935 \times 10^{77})$$

Input interpretation:

$$3.14498 \times 10^{20} ((9.62153750765067 \times 10^{1174})x + 5.34935 \times 10^{77})$$

Result:

$$3.14498 \times 10^{20} (9.62153750765067 \times 10^{1174} x + 5.34935 \times 10^{77})$$

Plot:**Geometric figure:**

line

Alternate forms:

$$3.025954303081120 \times 10^{1195} x + 1.68236 \times 10^{98}$$

$$3.23442 \times 10^{82} (9.35546482916002 \times 10^{1112} x + 5.20142 \times 10^{15})$$

Root:

$$x \approx -5.559766301120177 \times 10^{-1098}$$

$$-5.559766301120177e-1098$$

$$(3.6829873e+1211)(-5.559766301120177 \times 10^{-1098}) - (1.00887040498e+1348)y$$

Input interpretation:

$$(3.6829873 \times 10^{1211}) \left(-\frac{5.559766301120177}{10^{1098}} \right) - (1.00887040498 \times 10^{1348})y$$

Result:

$$-1.008870404980 \times 10^{1348} y - 2.047654867799359 \times 10^{114}$$

Alternate form:

$$-1.15792 \times 10^{77} (8.712774867653674 \times 10^{1270} y + 1.76839 \times 10^{37})$$

Root:

$$y \approx -2.02965104110 \times 10^{-1234}$$

$$-2.02965104110e-1234 = y$$

$((((3.6829873e+1211)(-5.559766301120177e-1098) - (1.00887040498e+1348)(-2.02965104110e-1234)))) 1/19477.74675331$

Input interpretation:

$$\left((3.6829873 \times 10^{1211}) \left(-\frac{5.559766301120177}{10^{1098}} \right) - (1.00887040498 \times 10^{1348}) \left(-\frac{2.02965104110}{10^{1234}} \right) \right) \times \frac{1}{19477.74675331}$$

Result:

1.6823600000000712813457113081494241919962023289214688... × 10⁹⁸
 1.68236....*10⁹⁸

$8\ln(((((((3.6829873e+1211)(-5.559766301120177e-1098) - (1.00887040498e+1348)(-2.02965104110e-1234)))) 1/19477.74675331)))))-76-4$

Input interpretation:

$$8 \log \left(\left((3.6829873 \times 10^{1211}) \left(-\frac{5.559766301120177}{10^{1098}} \right) - (1.00887040498 \times 10^{1348}) \left(-\frac{2.02965104110}{10^{1234}} \right) \right) \times \frac{1}{19477.74675331} \right) - 76 - 4$$

log(x) is the natural logarithm

Result:

1729.24
 1729.24

From which:

$(((((8\ln(((((((3.6829873e+1211)(-5.559766301120177e-1098) - (1.00887040498e+1348)(-2.02965104110e-1234)))) 1/19477.74675331)))))-76-4))))^{1/15}$

Input interpretation:

$$\left(8 \log \left(\left((3.6829873 \times 10^{1211}) \left(-\frac{5.559766301120177}{10^{1098}} \right) - (1.00887040498 \times 10^{1348}) \left(-\frac{2.02965104110}{10^{1234}} \right) \right) \times \frac{1}{19477.74675331} \right) - 76 - 4 \right)^{(1/15)}$$

log(x) is the natural logarithm

Result:

1.64383

1.64383

Corollary 5.2.

$$(5.2) \quad \sum_{j \geq 0} q^{2j^2} \begin{bmatrix} m \\ 2j \end{bmatrix} = \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2-\lambda} \begin{pmatrix} m; 6\lambda; q \\ 6\lambda \end{pmatrix}_2 \\ - \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+7\lambda+1} \begin{pmatrix} m; 6\lambda+2; q \\ 6\lambda+2 \end{pmatrix}_2,$$

$$(5.3) \quad \sum_{j \geq 0} q^{2j^2+2j} \begin{bmatrix} m \\ 2j+1 \end{bmatrix} = \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+5\lambda} \begin{pmatrix} m; 6\lambda+1; q \\ 6\lambda+1 \end{pmatrix}_2 \\ - \sum_{\lambda=-\infty}^{\infty} q^{12\lambda^2+13\lambda+3} \begin{pmatrix} m; 6\lambda+3; q \\ 6\lambda+3 \end{pmatrix}_2.$$

From which:

$$\sum_{j=0}^{\infty} \frac{q^{2j^2}}{(q)_{2j}} = \prod_{n=1}^{\infty} (1+q^{24n-11})(1+q^{24n-13})(1-q^{24n}) \\ - q \sum_{n=1}^{\infty} (1+q^{24n-5})(1+q^{24n-19})(1-q^{24n})$$

$$\sum_{j=0}^{\infty} \frac{q^{2j^2+2j}}{(q)_{2j+1}} = \prod_{n=1}^{\infty} (1+q^{24n-7})(1+q^{24n-17})(1-q^{24n}) \\ - q^2 \prod_{n=1}^{\infty} (1+q^{24n-1})(1+q^{24n-23})(1-q^{24n})$$

If $q = e^{2\pi i \tau}$, we have two results: 535.49165 and 0.0018674427

$$(1+0.0018674427^{37})(1+0.0018674427^{35})(1-0.0018674427^{48})-0.0018674427^2*(1+0.0018674427^{43})(1+0.0018674427^{29})(1-0.0018674427^{48})$$

Input interpretation:

$$(1 + 0.0018674427^{37})(1 + 0.0018674427^{35})(1 - 0.0018674427^{48}) - 0.0018674427^2 (1 + 0.0018674427^{43})(1 + 0.0018674427^{29})(1 - 0.0018674427^{48})$$

Result:

0.9999965126577622167099...

0.9999965126577...

$$(1+535.49165^{37})(1+535.49165^{35})(1-535.49165^{48})-535.49165^2*(1+535.49165^{43})(1+535.49165^{29})(1-535.49165^{48})$$

Input interpretation:

$$(1 + 535.49165^{37})(1 + 535.49165^{35})(1 - 535.49165^{48}) - 535.49165^2 (1 + 535.49165^{43})(1 + 535.49165^{29})(1 - 535.49165^{48})$$

Result:

8.0877603894604129486477657205330064690032985644292203... × 10³³²

8.0877603894... * 10³³²

$$\sum_{j=0}^{\infty} \frac{q^{2j^2+2j}}{(q)_{2j+1}} = \prod_{n=1}^{\infty} (1 + q^{24n-7})(1 + q^{24n-17})(1 - q^{24n}) - q^2 \prod_{n=1}^{\infty} (1 + q^{24n-1})(1 + q^{24n-23})(1 - q^{24n})$$

$$(1+0.0018674427^{41})(1+0.0018674427^{31})(1-0.0018674427^{48})-0.0018674427^2*(1+0.0018674427^{47})(1+0.0018674427^{25})(1-0.0018674427^{48})$$

Input interpretation:

$$(1 + 0.0018674427^{41})(1 + 0.0018674427^{31})(1 - 0.0018674427^{48}) - 0.0018674427^2 (1 + 0.0018674427^{47})(1 + 0.0018674427^{25})(1 - 0.0018674427^{48})$$

Result:

23.2466433673290443458162166457662144499889104945457788879...

23.2466433673...

Alternative representations:

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \cdot 7 = \frac{7}{2} \log_e(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))$$

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \cdot 7 = \frac{7}{2} \log(a) \log_a(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))$$

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \cdot 7 = -\frac{7}{2} \text{Li}_1(1 - \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))$$

Series representations:

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \cdot 7 = \frac{7}{2} \log(-1 + \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) - \frac{7}{2} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (-1 + \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^{-k}$$

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \gamma = 7 i \pi \left[\frac{1}{2\pi} \arg(-x + \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \right] + \frac{7 \log(x)}{2} - \frac{7}{2} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k x^{-k} (-x + \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^k \text{ for } x < 0$$

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \gamma = \frac{7}{2} \left[\frac{1}{2\pi} \arg(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}) - z_0) \right] \log\left(\frac{1}{z_0}\right) + \frac{7 \log(z_0)}{2} + \frac{7}{2} \left[\frac{1}{2\pi} \arg(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}) - z_0) \right] \log(z_0) - \frac{7}{2} \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}) - z_0)^k z_0^{-k}$$

Integral representations:

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \gamma = \frac{7}{2} \int_1^{\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})} \frac{1}{t} dt$$

$$\frac{1}{2} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \gamma = \frac{7}{4 i \pi} \int_{-i \infty + \gamma}^{i \infty + \gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) (-1 + \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^{-s} ds \text{ for } -1 < \gamma < 0$$

$\sqrt{4\pi} \cdot \ln(\ln(8.0877603894604129486477657205330064690032985644292203 \times 10^{332}))$

Input interpretation:

$$\sqrt{4\pi} \log(\log(8.0877603894604129486477657205330064690032985644292203 \times 10^{332}))$$

$\log(x)$ is the natural logarithm

Result:

23.5449157468854497243221574702390591756675863441838597513...

23.5449157468...

Alternative representations:

$$\begin{aligned} &\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \\ &\log_e(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \sqrt{4\pi} \end{aligned}$$

$$\begin{aligned} &\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \log(a) \log_a(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \sqrt{4\pi} \end{aligned}$$

$$\begin{aligned} &\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = -\text{Li}_1(1 - \log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \sqrt{4\pi} \end{aligned}$$

Series representations:

$$\begin{aligned} &\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \\ &\log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) \sqrt{-1+4\pi} \sum_{k=0}^{\infty} (-1+4\pi)^{-k} \binom{\frac{1}{2}}{k} \end{aligned}$$

$$\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \sqrt{-1+4\pi} \left(\sum_{k=0}^{\infty} (-1+4\pi)^{-k} \binom{\frac{1}{2}}{k} \right) \left(\log(-1+\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (-1+\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^{-k} \right)$$

$$\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \sqrt{-1+4\pi} \left(\log(-1+\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) - \sum_{k=1}^{\infty} \frac{1}{k} (-1)^k (-1+\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^{-k} \right)$$

$$\sum_{k=0}^{\infty} \frac{(-1)^k (-1+4\pi)^{-k} \binom{-\frac{1}{2}}{k}}{k!}$$

Integral representations:

$$\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \sqrt{4\pi} \int_1^{\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})} \frac{1}{t} dt$$

$$\sqrt{4\pi} \log(\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332})) = \frac{\sqrt{4\pi}}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{1}{\Gamma(1-s)} \Gamma(-s)^2 \Gamma(1+s) (-1+\log(8.08776038946041294864776572053300646900329856442922030000 \times 10^{332}))^{-s} ds \text{ for } -1 < \gamma < 0$$

$$(8.0877603894604129486 \times 10^{332})^{1/(24 \cdot 64)}$$

Input interpretation:

$$\sqrt[24 \cdot 64]{8.0877603894604129486 \times 10^{332}}$$

Result:

1.6471640966542543534980...

1.647164096654...

$$\text{sqrt}(\text{6}(\text{8.0877603894604129486} \times 10^{332})^{1/(24 \cdot 64)}))$$

Input interpretation:

$$\sqrt{6 \sqrt[24 \cdot 64]{8.0877603894604129486 \times 10^{332}}}$$

Result:

3.1437214539340991240670...

3.143721453...

Furthermore, we obtain also

$$(8.0877603894604129486 \times 10^{332})^{1/(24 \cdot 64)} - \frac{29}{10^3}$$

Input interpretation:

$$\sqrt[24 \cdot 64]{8.0877603894604129486 \times 10^{332}} - \frac{29}{10^3}$$

Result:

1.6181640966542543534980...

1.618164096...

$$(8.0877603894604129486 \times 10^{332})^{1/103 + (29-7) + 1/2}$$

where 103 is a Prime number

Input interpretation:

$$\sqrt[103]{8.0877603894604129486 \times 10^{332}} + (29 - 7) + \frac{1}{2}$$

Result:

1729.033576797197483393...

1729.0335767...

$$(8.0877603894604129486 \times 10^{332})^{1/156} + \pi$$

where $156 = 2^2 * 3 * 13$ **Input interpretation:**

$$\sqrt[156]{8.0877603894604129486 \times 10^{332}} + \pi$$

Result:

139.2937540931739855749...

139.293754...

Alternative representations:

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 180^\circ + \sqrt[156]{8.08776038946041294860000 \times 10^{332}}$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = -i \log(-1) + \sqrt[156]{8.08776038946041294860000 \times 10^{332}}$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = \cos^{-1}(-1) + \sqrt[156]{8.08776038946041294860000 \times 10^{332}}$$

Series representations:

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 136.152161439584192336437570 + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1 + 2k}$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 134.152161439584192336437570 + 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 136.152161439584192336437570 + \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

Integral representations:

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 136.152161439584192336437570 + 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 136.152161439584192336437570 + 4 \int_0^1 \sqrt{1-t^2} dt$$

$$\sqrt[156]{8.08776038946041294860000 \times 10^{332}} + \pi = 136.152161439584192336437570 + 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$(8.0877603894604129486 \times 10^{332})^{1/156} - 11$$

Input interpretation:

$$\sqrt[156]{8.0877603894604129486 \times 10^{332}} - 11$$

Result:

125.1521614395841923364...

125.152161...

$$(8.0877603894604129486 \times 10^{332})^{1/248} + 2$$

Input interpretation:

$$\sqrt[248]{8.0877603894604129486 \times 10^{332}} + 2$$

Result:

23.99734546070295063813...

23.997345...

or:

$$(8.0877603894604129486 \times 10^{332})^{1/241}$$

where 241 is a Prime number

Input interpretation:

$$\sqrt[241]{8.0877603894604129486 \times 10^{332}}$$

Result:

24.06358246006350275094...

24.06358246...

Now, we have that:

Theorem 6.1.

$$(6.10) \quad R_m(q) = \sum_{\lambda=-\infty}^{\infty} q^{40\lambda^2+4\lambda} U(m, 10\lambda, q) - \sum_{\lambda=-\infty}^{\infty} q^{40\lambda^2+36\lambda+8} U(m, 10\lambda + 4, q),$$

$$(6.11) \quad S_m(q) = \sum_{\lambda=-\infty}^{\infty} q^{40\lambda^2+12\lambda} U(m, 10\lambda + 1, q) - \sum_{\lambda=-\infty}^{\infty} q^{40\lambda^2+28\lambda+4} U(m, 10\lambda + 3, q).$$

If $q = e^{2\pi i \tau}$, we have two results: 535.49165 and 0.0018674427. For $\lambda = 2$, we obtain:

$$0.0018674427^{(40 \times 4 + 8)}(1, 20, 0.0018674427) - (0.0018674427^{(40 \times 4 + 36 \times 2 + 8)}(1, 24, 0.0018674427))$$

Input interpretation:

$$0.0018674427^{40 \times 4 + 8} (1, 20, 0.0018674427) - 0.0018674427^{40 \times 4 + 36 \times 2 + 8} (1, 24, 0.0018674427)$$

Result:

$(3.711452188720053 \times 10^{-459}, 7.42290437744011 \times 10^{-458}, 6.930924296224285 \times 10^{-462})$

Vector length:

$7.43217724832287 \times 10^{-458}$

$7.43217724832287 * 10^{-458}$

Normalized vector:

$(0.0499376167267481, 0.998752334534963, 0.0000932556378117637)$

Spherical coordinates (radial, polar, azimuthal):

$r \approx 7.43217724832287 \times 10^{-458}, \theta \approx 89.994656846^\circ, \phi \approx 87.13759477388825^\circ$

$0.0018674427^{(40*4+12*2)}(1, 21, 0.0018674427) -$
 $(0.0018674427^{(40*4+28*2+4)}(1, 24, 0.0018674427))$

Input interpretation:

$0.0018674427^{40 \times 4 + 12 \times 2} (1, 21, 0.0018674427) -$
 $0.0018674427^{40 \times 4 + 28 \times 2 + 4} (1, 24, 0.0018674427)$

Result:

$(8.11896051640741 \times 10^{-503}, 1.704981708445556 \times 10^{-501}, 1.516169354795325 \times 10^{-505})$

Vector length:

$1.706913706403555 \times 10^{-501}$

$1.706913706403555 * 10^{-501}$

Normalized vector:

$(0.0475651492278069, 0.998868133783945, 0.0000888251906998787)$

Spherical coordinates (radial, polar, azimuthal):

$$r \approx 1.706913706403555 \times 10^{-501}, \quad \theta \approx 89.994910691^\circ, \quad \phi \approx 87.27368900609373^\circ$$

$$(7.43217724832287 \times 10^{-458} * 1 / 1.706913706403555 \times 10^{-501})$$

Input interpretation:

$$\frac{7.43217724832287}{10^{458}} \times \frac{1}{\frac{1.706913706403555}{10^{501}}}$$

Result:

$$4.3541610922923402434733069236044494123036858533595875... \times 10^{43}$$

$$4.35416109229.... * 10^{43}$$

We note that:

$$(7.43217724832287 \times 10^{-458} * 1 / 1.706913706403555 \times 10^{-501})^{1/209}$$

where 209 is an Ulam number

[A002858](#) **Ulam numbers:** a(1) = 1; a(2) = 2; for n>2, a(n) = least number > a(n-1) which is a unique sum of two distinct earlier terms. (Formerly M0557 N0201)

1, 2, 3, 4, 6, 8, 11, 13, 16, 18, 26, 28, 36, 38, 47, 48, 53, 57, 62, 69, 72, 77, 82, 87, 97, 99, 102, 106, 114, 126, 131, 138, 145, 148, 155, 175, 177, 180, 182, 189, 197, 206, 209, 219, 221, 236, 238, 241, 243, 253, 258, 260, 273, 282, 309, 316, 319, 324, 339 ([list](#); [graph](#); [refs](#); [listen](#); [history](#); [text](#); [internal format](#))

Input interpretation:

$$\sqrt[209]{\frac{7.43217724832287}{10^{458}} \times \frac{1}{\frac{1.706913706403555}{10^{501}}}}$$

Result:

$$1.6173297903332907...$$

$$1.6173297903.....$$

We have also:

$$(7.43217724832287 \times 10^{-458} * 1 / 1.706913706403555 \times 10^{-501})^{1/20-11\text{-golden ratio}}$$

Input interpretation:

$$\sqrt[20]{\frac{7.43217724832287}{10^{458}} \times \frac{1}{\frac{1.706913706403555}{10^{501}}}} - 11 - \phi$$

ϕ is the golden ratio

Result:

139.4175422980254...

139.41754229.....

$(7.43217724832287 \times 10^{-458} * 1 / 1.706913706403555 \times 10^{-501})^{1/20-29+4}$ -golden ratio

Input interpretation:

$$\sqrt[20]{\frac{7.43217724832287}{10^{458}} \times \frac{1}{\frac{1.706913706403555}{10^{501}}}} - 29 + 4 - \phi$$

ϕ is the golden ratio

Result:

125.4175422980254...

125.41754229....

$(7.43217724832287 \times 10^{-458} * 1 / 1.706913706403555 \times 10^{-501})^{1/13-521-29+3}$ +golden ratio

Input interpretation:

$$\sqrt[13]{\frac{7.43217724832287}{10^{458}} \times \frac{1}{\frac{1.706913706403555}{10^{501}}}} - 521 - 29 + 3 + \phi$$

ϕ is the golden ratio

Result:

1728.871195624040...

1728.871195624....

Now, we have that:

Finally we note that if $m \rightarrow \infty$ in (6.10), then we obtain (using (4.16) and Jacobi's triple product [2, p. 21])

$$(6.20) \quad \sum_{j=0}^{\infty} \frac{q^{j^2}}{(q)_{2j}} = \prod_{n=1}^{\infty} \frac{(1+q^{2n-1})(1-q^{20n-12})(1-q^{20n-8})(1-q^{20n})}{(1-q^{2n})},$$

which is Slater's equation (79) [14, p. 160]. If $m \rightarrow \infty$ in (6.11) we obtain (using (4.16) and Jacobi's triple product [2, p. 21])

$$(6.21) \quad \sum_{j=0}^{\infty} \frac{q^{j^2+2j}}{(q)_{2j+1}} = \prod_{n=1}^{\infty} \frac{(1+q^{2n-1})(1-q^{20n-4})(1-q^{20n-16})(1-q^{20n})}{(1-q^{2n})},$$

From (6.20)

$$\sum_{j=0}^{\infty} \frac{q^{j^2}}{(q)_{2j}} = \prod_{n=1}^{\infty} \frac{(1+q^{2n-1})(1-q^{20n-12})(1-q^{20n-8})(1-q^{20n})}{(1-q^{2n})}$$

For $q = 0.0018674427$, we obtain:

Product $((1+0.0018674427^{(2n-1)})(1-0.0018674427^{(20n-12)})(1-0.0018674427^{(20n-8)})(1-0.0018674427^{(20n)}) / ((1-0.0018674427^{(2n)}))$, $n = 1..infinity$

Input interpretation:

$$\prod_{n=1}^{\infty} \frac{1}{1 - 0.0018674427^{2n}} (1 + 0.0018674427^{2n-1}) (1 - 0.0018674427^{20n-12}) (1 - 0.0018674427^{20n-8}) (1 - 0.0018674427^{20n})$$

Infinite product:

$$\prod_{n=1}^{\infty} \frac{1}{1 - 0.00186744^{2n}} (1 - 0.00186744^{20n}) (0.00186744^{2n-1} + 1) (1 - 0.00186744^{20n-12}) (1 - 0.00186744^{20n-8}) = 1.00187$$

1.00187

and:

$((\text{Product } (((1+0.0018674427^{(2n-1)})(1-0.0018674427^{(20n-12)})(1-0.0018674427^{(20n-8)})(1-0.0018674427^{(20n)})) / (((1-0.0018674427^{(2n)}))))), n = 1..infinity))^{257}$

with 257 that is a prime number

Input interpretation:

$$\left(\prod_{n=1}^{\infty} \frac{((1 + 0.0018674427^{2n-1})(1 - 0.0018674427^{20n-12})(1 - 0.0018674427^{20n-8})(1 - 0.0018674427^{20n}))}{(1 - 0.0018674427^{2n})} \right)^{257}$$

Result:

1.61669

1.61669

From (6.21)

$$\sum_{j=0}^{\infty} \frac{q^{j^2+2j}}{(q)_{2j+1}} = \prod_{n=1}^{\infty} \frac{(1 + q^{2n-1})(1 - q^{20n-4})(1 - q^{20n-16})(1 - q^{20n})}{(1 - q^{2n})}$$

We obtain:

For $q = 0.0018674427$, we obtain:

Product ((((((1+0.0018674427⁽²ⁿ⁻¹⁾)(1-0.0018674427⁽²⁰ⁿ⁻⁴⁾)(1-0.0018674427⁽²⁰ⁿ⁻¹⁶⁾)(1-0.0018674427⁽²⁰ⁿ⁾)))))) / ((((((1-0.0018674427⁽²ⁿ⁾))))))), $n = 1..1000$

Product:

$$\prod_{n=1}^{1000} \frac{1}{1 - 0.00186744^{2n}} (1 - 0.00186744^{20n})(0.00186744^{2n-1} + 1) (1 - 0.00186744^{20n-16})(1 - 0.00186744^{20n-4}) = 1.00187$$

1.00187 as above.

Now, we have that:

$$(6.22) \quad \sum_{j=0}^{\infty} \frac{q^{j^2}}{(q)_j} = \frac{1}{(q^2; q^2)_{\infty}} \left\{ \sum_{\lambda=-\infty}^{\infty} (q^{60\lambda^2-4\lambda} - q^{60\lambda^2+44\lambda+8}) + q \sum_{\lambda=-\infty}^{\infty} (q^{60\lambda^2+16\lambda} - q^{60\lambda^2+64\lambda+16}) \right\}.$$

$$(6.23) \quad \sum_{j=0}^{\infty} \frac{q^{j^2+j}}{(q)_j} = \frac{1}{(q^2; q^2)_{\infty}} \left\{ \sum_{\lambda=-\infty}^{\infty} (q^{60\lambda^2+8\lambda} - q^{60\lambda^2+32\lambda+4}) + q \sum_{\lambda=-\infty}^{\infty} (q^{60\lambda^2+28\lambda+2} - q^{60\lambda^2+52\lambda+10}) \right\}.$$

From (6.22), for $q = 0.0018674427$, and $\lambda = 2$, we obtain:

$$(0.0018674427^{(60*4-8)} - 0.0018674427^{(60*4+44*2+8)}) + 0.0018674427(((0.0018674427^{(60*4+16*2)} - 0.0018674427^{(60*4+64*2+16)}))$$

Input interpretation:

$$0.0018674427^{60 \times 4 - 8} - 0.0018674427^{60 \times 4 + 44 \times 2 + 8} + 0.0018674427(0.0018674427^{60 \times 4 + 16 \times 2} - 0.0018674427^{60 \times 4 + 64 \times 2 + 16})$$

Result:

$$8.499044567445564885331289280121443414585753891627879... \times 10^{-634}$$

$$8.499044567445... * 10^{-634}$$

From (6.23), we obtain:

$$(0.0018674427^{(60*4+16)} - 0.0018674427^{(60*4+32*2+4)}) + 0.0018674427(((0.0018674427^{(60*4+28*2+2)} - 0.0018674427^{(60*4+52*2+10)}))$$

Input interpretation:

$$0.0018674427^{60 \times 4 + 16} - 0.0018674427^{60 \times 4 + 32 \times 2 + 4} + 0.0018674427 \left(0.0018674427^{60 \times 4 + 28 \times 2 + 2} - 0.0018674427^{60 \times 4 + 52 \times 2 + 10} \right)$$

Result:

$$2.749824298400822485215839373214400835456168068419920... \times 10^{-699}$$

$$2.749824298400822..... * 10^{-699}$$

Thence, we have:

$$(8.49904456744556488533 \times 10^{-634} * 1 / 2.74982429840082248521 \times 10^{-699})$$

Input interpretation:

$$\frac{8.49904456744556488533}{10^{634}} \times \frac{1}{\frac{2.74982429840082248521}{10^{699}}}$$

Result:

$$3.0907591340974903000931793670159733062124403693944336... \times 10^{65}$$

$$3.090759134097.... * 10^{65}$$

and:

$$(8.49904456744556488533 \times 10^{-634} * 1 / 2.74982429840082248521 \times 10^{-699})^{1/314}$$

Input interpretation:

$$\sqrt[314]{\frac{8.49904456744556488533}{10^{634}} \times \frac{1}{\frac{2.74982429840082248521}{10^{699}}}}$$

Result:

$$1.6164679064626918866244...$$

$$1.6164679064.....$$

We have also:

$$(8.49904456744556488533 \times 10^{-634} * 1 / 2.74982429840082248521 \times 10^{-699})^{1/20-144-8-1/2}$$

Input interpretation:

$$\sqrt[20]{\frac{8.49904456744556488533}{10^{634}} \times \frac{1}{\frac{2.74982429840082248521}{10^{699}}}} - 144 - 8 - \frac{1}{2}$$

Result:

1728.995821112703059750...

1728.995821....

$$(8.49904456744556488533 \times 10^{-634} * 1 / 2.74982429840082248521 \times 10^{-699})^{1/30-13}$$

Input interpretation:

$$\sqrt[30]{\frac{8.49904456744556488533}{10^{634}} \times \frac{1}{\frac{2.74982429840082248521}{10^{699}}}} - 13$$

Result:

139.4060376093635962838...

139.406037.....

$$(8.49904456744556488533 \times 10^{-634} * 1 / 2.74982429840082248521 \times 10^{-699})^{1/31-4}$$

Input interpretation:

$$\sqrt[31]{\frac{8.49904456744556488533}{10^{634}} \times \frac{1}{\frac{2.74982429840082248521}{10^{699}}}} - 4$$

Result:

125.5933591248055847515...

125.5933591....

For $q = 535.49165$, we obtain:

$$(535.49165^{(60 \cdot 4 - 8)} - 535.49165^{(60 \cdot 4 + 44 \cdot 2 + 8)} + 535.49165(((535.49165^{(60 \cdot 4 + 16 \cdot 2)} - 535.49165^{(60 \cdot 4 + 64 \cdot 2 + 16)}))))$$

Input interpretation:

$$535.49165^{60 \cdot 4 - 8} - 535.49165^{60 \cdot 4 + 44 \cdot 2 + 8} + 535.49165(535.49165^{60 \cdot 4 + 16 \cdot 2} - 535.49165^{60 \cdot 4 + 64 \cdot 2 + 16})$$

Result:

$$-3.71356246215462421350282267407390066977550351977676... \times 10^{1050}$$

$$-3.713562462154... \cdot 10^{1050}$$

$$(535.49165^{(60 \cdot 4 + 16)} - 535.49165^{(60 \cdot 4 + 32 \cdot 2 + 4)} + 535.49165(((535.49165^{(60 \cdot 4 + 28 \cdot 2 + 2)} - 535.49165^{(60 \cdot 4 + 52 \cdot 2 + 10)}))))$$

Input interpretation:

$$535.49165^{60 \cdot 4 + 16} - 535.49165^{60 \cdot 4 + 32 \cdot 2 + 4} + 535.49165(535.49165^{60 \cdot 4 + 28 \cdot 2 + 2} - 535.49165^{60 \cdot 4 + 52 \cdot 2 + 10})$$

Result:

$$-5.095767880690636696076300664565341753522391138738053... \times 10^{968}$$

$$-5.0957678806... \cdot 10^{968}$$

$$(-3.713562462154624 \times 10^{1050} / -5.095767880690636 \times 10^{968})$$

Input interpretation:

$$\frac{-(3.713562462154624 \times 10^{1050})}{-(5.095767880690636 \times 10^{968})}$$

Result:

$$7.2875424256006733276384742579155851762247482740977685... \times 10^{81}$$

$$7.2875424256... \cdot 10^{81}$$

$$(-3.713562462154624 \times 10^{1050} / -5.095767880690636 \times 10^{968})^{1/392}$$

Input interpretation:

$$\sqrt[392]{\frac{-(3.713562462154624 \times 10^{1050})}{-(5.095767880690636 \times 10^{968})}}$$

Result:

1.61745838262289048...

1.6174583826...

$$(-3.713562462154624 \times 10^{1050} / -5.095767880690636 \times 10^{968})^{1/39}$$

Input interpretation:

$$\sqrt[39]{\frac{-(3.713562462154624 \times 10^{1050})}{-(5.095767880690636 \times 10^{968})}}$$

Result:

125.61472182212744...

125.614721...

$$(-3.713562462154624 \times 10^{1050} / -5.095767880690636 \times 10^{968})^{1/39+18-4}$$

Input interpretation:

$$\sqrt[39]{\frac{-(3.713562462154624 \times 10^{1050})}{-(5.095767880690636 \times 10^{968})}} + 18 - 4$$

Result:

139.61472182212744...

139.614721...

$$27 \times \frac{1}{2} \left(\left(\frac{-3.713562462154624 \times 10^{1050}}{-5.095767880690636 \times 10^{968}} \right)^{1/39} + \text{golden ratio}^2 \right) - 2$$

Input interpretation:

$$27 \times \frac{1}{2} \left(\sqrt[39]{\frac{-(3.713562462154624 \times 10^{1050})}{-(5.095767880690636 \times 10^{968})}} + \phi^2 \right) - 2$$

ϕ is the golden ratio

Result:

1729.1422034468440...

1729.1422...

Alternative representations:

$$\begin{aligned} & \frac{27}{2} \left(\sqrt[39]{\frac{-(3.7135624621546240000 \times 10^{1050})}{-(5.0957678806906360000 \times 10^{968})}} + \phi^2 \right) - 2 = \\ & -2 + \frac{27}{2} \left(\sqrt[39]{\frac{-3.7135624621546240000 \times 10^{1050}}{-5.0957678806906360000 \times 10^{968}}} + (2 \sin(54^\circ))^2 \right) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\sqrt[39]{\frac{-(3.7135624621546240000 \times 10^{1050})}{-(5.0957678806906360000 \times 10^{968})}} + \phi^2 \right) - 2 = \\ & -2 + \frac{27}{2} \left((-2 \cos(216^\circ))^2 + \sqrt[39]{\frac{-3.7135624621546240000 \times 10^{1050}}{-5.0957678806906360000 \times 10^{968}}} \right) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\sqrt[39]{\frac{-(3.7135624621546240000 \times 10^{1050})}{-(5.0957678806906360000 \times 10^{968})}} + \phi^2 \right) - 2 = \\ & -2 + \frac{27}{2} \left(\sqrt[39]{\frac{-3.7135624621546240000 \times 10^{1050}}{-5.0957678806906360000 \times 10^{968}}} + (-2 \sin(666^\circ))^2 \right) \end{aligned}$$

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:

125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

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