

**On some Ramanujan integrals concerning Riemann's functions $\xi(s)$ and $\Xi(t)$:
mathematical connections with ϕ , $\zeta(2)$ and various parameters of Particle
Physics. II**

Michele Nardelli¹, Antonio Nardelli²

Abstract

In this paper we have described and analyzed some Ramanujan integrals concerning Riemann's functions $\xi(s)$ and $\Xi(t)$. Furthermore, we have obtained several mathematical connections between ϕ , $\zeta(2)$ and various parameters of Particle Physics.

¹ M.Nardelli studied at Dipartimento di Scienze della Terra Università degli Studi di Napoli Federico II, Largo S. Marcellino, 10 - 80138 Napoli, Dipartimento di Matematica ed Applicazioni "R. Caccioppoli" - Università degli Studi di Napoli "Federico II" – Polo delle Scienze e delle Tecnologie Monte S. Angelo, Via Cintia (Fuorigrotta), 80126 Napoli, Italy

² A. Nardelli studies at the Università degli Studi di Napoli Federico II - Dipartimento di Studi Umanistici – **Sezione Filosofia - scholar of Theoretical Philosophy**



*An equation means nothing
to me unless it expresses a
thought of God.*

Srinivasa Ramanujan (1887-1920)

<https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012>

We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation

From:

New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ – *Srinivasa Ramanujan*
Quarterly Journal of Mathematics, XLVI, 1915, 253 – 260

We have that:

For:

$$\xi(s) = (s - 1)\Gamma\left(1 + \frac{1}{2}s\right)\pi^{-\frac{1}{2}s}\zeta(s).$$

$$\xi\left(\frac{1}{2} + \frac{1}{2}it\right) = \Xi\left(\frac{1}{2}t\right)$$

Thence, for $t = 1$:

$$\left(\frac{1}{2} + \frac{1}{2}i - 1\right) \Gamma\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}i\right)\right) \pi^{-1/2\left(\frac{1}{2} + \frac{1}{2}i\right)} \zeta\left(\frac{1}{2} + \frac{1}{2}i\right)$$

Input:

$$\left(\frac{1}{2} + \frac{1}{2}i - 1\right) \Gamma\left(1 + \frac{1}{2}\left(\frac{1}{2} + \frac{1}{2}i\right)\right) \pi^{-1/2\left(\frac{1}{2} + \frac{1}{2}i\right)} \zeta\left(\frac{1}{2} + \frac{1}{2}i\right)$$

$\Gamma(x)$ is the gamma function

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Exact result:

$$\left(-\frac{1}{2} + \frac{i}{2}\right) \pi^{-1/4-i/4} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) \Gamma\left(\frac{5}{4} + \frac{i}{4}\right)$$

Decimal approximation:

0.494256987910076300380568818360138186867976223134574011846...

(using the principal branch of the logarithm for complex exponentiation)

0.49425698791.....

Alternate forms:

$$-\frac{1}{4} \pi^{-1/4-i/4} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) \Gamma\left(\frac{1}{4} + \frac{i}{4}\right)$$

$$\left(-\frac{4}{13} + \frac{6i}{13}\right) \pi^{-1/4-i/4} \left(\frac{5}{4} + \frac{i}{4}\right)! \zeta\left(\frac{1}{2} + \frac{i}{2}\right)$$

$n!$ is the factorial function

Alternative representations:

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{2}\right) \exp\left(-\log G\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) + \log G\left(2 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right)\right) \pi^{1/2(-1/2-i/2)} \zeta\left(\frac{1}{2} + \frac{i}{2}, 1\right)$$

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{2}\right) (1)_{\frac{1}{2}\left(\frac{1}{2} + \frac{i}{2}\right)} \pi^{1/2(-1/2-i/2)} \zeta\left(\frac{1}{2} + \frac{i}{2}, 1\right)$$

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\frac{\left(-\frac{1}{2} + \frac{i}{2}\right) G\left(2 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(-1/2-i/2)} \zeta\left(\frac{1}{2} + \frac{i}{2}, 1\right)}{G\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right)}$$

Series representations:

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\pi^{-1/4-i/4} \Gamma\left(\frac{5}{4} + \frac{i}{4}\right) \sum_{n=0}^{\infty} \frac{\sum_{k=0}^n (-1)^k (1+k)^{1/2-i/2} \binom{n}{k}}{1+n}$$

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\frac{(1-i) 2^{-1+i/2} \pi^{-1/4-i/4} \Gamma\left(\frac{5}{4} + \frac{i}{4}\right) \sum_{n=0}^{\infty} 2^{-1-n} \sum_{k=0}^n (-1)^k (1+k)^{-1/2-i/2} \binom{n}{k}}{-2^{i/2} + \sqrt{2}}$$

$$\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2(1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right) =$$

$$\left(-\frac{1}{2} + \frac{i}{2}\right) \pi^{-1/4-i/4} \Gamma\left(\frac{5}{4} + \frac{i}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\left(\frac{1}{2} + \frac{i}{2}\right) - s_0\right)^k \zeta^{(k)}(s_0)}{k!} \text{ for } s_0 \neq 1$$

Integral representations:

$$\frac{\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2 (1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right)}{\left(\frac{1}{2} - \frac{i}{2}\right) \pi^{-1/4-i/4} \Gamma\left(\frac{5}{4} + \frac{i}{4}\right)} \int_0^\infty \frac{t^{-1/2+i/2}}{1+e^t} dt$$

$$\frac{\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2 (1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right)}{(1-i) 2^{-1+i/2} \pi^{-1/4-i/4} \left(\int_0^\infty \frac{t^{-1/2+i/2}}{1+e^t} dt\right) \int_0^1 \log^{1/4+i/4}\left(\frac{1}{t}\right) dt}{(-2^{i/2} + \sqrt{2}) \Gamma\left(\frac{1}{2} + \frac{i}{2}\right)}$$

$$\frac{\left(\frac{1}{2} + \frac{i}{2} - 1\right) \Gamma\left(1 + \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \pi^{1/2 (1/2+i/2)(-1)} \zeta\left(\frac{1}{2} + \frac{i}{2}\right)}{(1-i) 2^{-3/2+i} \pi^{-1/4-i/4} \left(\int_0^1 \log^{1/4+i/4}\left(\frac{1}{t}\right) dt\right) \int_0^\infty t^{1/2+i/2} \operatorname{sech}^2(t) dt}{(-2^{i/2} + \sqrt{2}) \Gamma\left(\frac{3}{2} + \frac{i}{2}\right)}$$

From

$$\begin{aligned} & \alpha^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} - 4\alpha \int_0^\infty \left(\frac{3}{3^2+t^2} - \frac{\alpha}{1!} \frac{7x^2}{7^2+t^2} + \frac{\alpha^2}{2!} \frac{11x^4}{11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & + \beta^{-\frac{1}{4}} \left\{ \frac{1}{1+t^2} - 4\beta \int_0^\infty \left(\frac{3}{3^2+t^2} - \frac{\beta}{1!} \frac{7x^2}{7^2+t^2} + \frac{\beta^2}{2!} \frac{11x^4}{11^2+t^2} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & = \frac{1}{4} \pi^{-\frac{3}{4}} \Gamma\left(\frac{-1+it}{4}\right) \Gamma\left(\frac{-1-it}{4}\right) \Xi\left(\frac{1}{2}t\right) \cos\left(\frac{t}{8} \log \frac{\alpha}{\beta}\right). \end{aligned} \tag{9}$$

For $t = 1$, $\alpha = 2$, $\beta = \pi^2 / 2$, and $\Xi(1/2 t) = 0.49425698791$, we obtain:

$$\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{-1+i}{4}\right) \Gamma\left(\frac{-1-i}{4}\right) \times 0.49425698791 \cos\left(\frac{1}{8} \ln(\pi^2)\right)$$

Input interpretation:

$$\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right)$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

0.51792798277...

0.51792798277...

Alternative representations:

$$\begin{aligned} & \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\ & \frac{1}{4} \times 0.494256987910000 \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \\ & \exp\left(-\log G\left(\frac{1}{4}(-1-i)\right) + \log G\left(1 + \frac{1}{4}(-1-i)\right)\right) \\ & \exp\left(-\log G\left(\frac{1}{4}(-1+i)\right) + \log G\left(1 + \frac{1}{4}(-1+i)\right)\right) \pi^{-3/4} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\ & \frac{0.494256987910000 G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\ & \frac{0.494256987910000 G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} \end{aligned}$$

Series representations:

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) = \right. \\ \left. - \left(\left(1.9770279516400 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-3k_1-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{(2k_1)! k_2! k_3!} \right) \right) / \right. \\ \left. \left((-1.000000000000000 + 1.000000000000000 i) \right) \right. \\ \left. \left(1.000000000000000 + 1.000000000000000 i \right) \pi^{3/4} \right)$$

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) = \right. \\ \left. \frac{1}{\pi^{3/4}} 0.123564246977500 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{\left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0)}{(2k_1)! k_2! k_3!} \right. \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \\ \left. \right)$$

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) = \right. \\ \left. - \left(\left(1.9770279516400 \left(1.000000000000000 J_0\left(\frac{\log(\pi^2)}{8}\right) \right) \right. \right. \right. \\ \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} + \right. \\ \left. 2.000000000000000 \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} 4^{-k_2-k_3} \right. \\ \left. \left. \left. (-1-i)^{k_2} (-1+i)^{k_3} J_{2k_1}\left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1) \right) \right) \right) / \right. \\ \left. \left((-1.000000000000000 + 1.000000000000000 i) \right) \right. \\ \left. \left(1.000000000000000 + 1.000000000000000 i \right) \pi^{3/4} \right)$$

$$\begin{aligned}
& \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\
& \frac{1}{\pi^{3/4}} 0.123564246977500 \left(1.000000000000000 J_0\left(\frac{\log(\pi^2)}{8}\right) \right. \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i) - z_0\right)^{k_1} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} + \\
& 2.000000000000000 \\
& \left. \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} J_{2k_1}\left(\frac{\log(\pi^2)}{8}\right) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \right. \\
& \left. \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\
& - \frac{0.494256987910000 \pi^{5/4} \mathcal{A}^2}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\
& - \left(\left(0.0617821234887500 \pi^{5/4} \mathcal{A}^2 \left(-8.000000000000000 + \right. \right. \right. \\
& \left. \left. \left. \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt \right) \right) / \left(\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt \right) \right)
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) = \\
& \frac{0.247128493955000 \sqrt[4]{\pi} \mathcal{A} \sqrt{\pi}}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0
\end{aligned}$$

Multiple-argument formulas:

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) =$$

$$\frac{1}{\pi^{3/4} \sqrt{\pi^2}} 0.0436865584749998$$

$$\left(-0.5000000000000000 + 1.0000000000000000 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) =$$

$$- \frac{1}{\pi^{3/4} \sqrt{\pi^2}} 0.0436865584749998 \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right)$$

$$\Gamma\left(\frac{3+i}{8}\right) \left(-0.5000000000000000 + 1.0000000000000000 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\frac{1}{4} \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \left(0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) =$$

$$\frac{1}{\pi^{3/4} \sqrt{\pi^2}} 0.0873731169499996$$

$$\left(-0.7500000000000000 \cos\left(\frac{\log(\pi^2)}{24}\right) + 1.0000000000000000 \cos^3\left(\frac{\log(\pi^2)}{24}\right) \right)$$

$$\Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

We note that, multiplying by π the above expression and subtracting $(7+2)/10^3$ (where 7 and 2 are primes and Lucas numbers), we obtain:

$$\text{Pi} * (((1/4 * \text{Pi}^{(-3/4)} * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791 * \cos(1/8 * \ln(\text{Pi}^2)))) - (7+2) / 10^3$$

Input interpretation:

$$\pi \left(\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right) \right) - (7+2) \times \frac{1}{10^3}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

1.6181187458...

1.6181187458...

Alternative representations:

$$\begin{aligned} & \frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} = \\ & \frac{1}{4} \times 0.494256987910000 \pi \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \\ & \exp\left(-\log\Gamma\left(\frac{1}{4}(-1-i)\right) + \log\Gamma\left(1 + \frac{1}{4}(-1-i)\right)\right) \\ & \exp\left(-\log\Gamma\left(\frac{1}{4}(-1+i)\right) + \log\Gamma\left(1 + \frac{1}{4}(-1+i)\right)\right) \pi^{-3/4} - \frac{9}{10^3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} = \\ & \frac{0.494256987910000 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} - \frac{9}{10^3} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} = \\ & \frac{0.494256987910000 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} - \\ & \frac{9}{10^3} \end{aligned}$$

Series representations:

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-\left(\left(0.009000000000000000 \left(-1.0000000000000000 + 1.0000000000000000 i^2 + \right. \right. \right.$$

$$\left. \left. 219.66977240444 \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} (-1)^{k_1} 4^{-3k_1-k_2-k_3} \right. \right.$$

$$\left. \left. (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1) \right) \right) /$$

$$\left((-1.0000000000000000 + 1.0000000000000000 i) \right)$$

$$\left(1.0000000000000000 + 1.0000000000000000 i \right)$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$0.123564246977500 \left(-0.072836602983052 + 1.0000000000000000 \sqrt[4]{\pi} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} \left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \right.$$

$$\left. \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-\left(\left(0.009000000000000000 \right. \right.$$

$$\left. \left(-1.0000000000000000 + 1.0000000000000000 i^2 + 219.66977240444 \sqrt[4]{\pi} \right. \right.$$

$$\left. \left. J_0\left(\frac{\log(\pi^2)}{8}\right) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} + \right. \right.$$

$$\left. \left. 439.33954480889 \sqrt[4]{\pi} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} (-1)^{k_1} 4^{-k_2-k_3} \right. \right.$$

$$\left. \left. (-1-i)^{k_2} (-1+i)^{k_3} J_2 k_1 \left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1) \right) \right) /$$

$$\left((-1.0000000000000000 + 1.0000000000000000 i) \right)$$

$$\left(1.0000000000000000 + 1.0000000000000000 i \right)$$

Integral representations:

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-\frac{9}{1000} - \frac{0.494256987910000 \pi^{9/4} \mathcal{A}^2}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-\frac{9}{1000} + \frac{\pi^{9/4} \mathcal{A}^2 \left(0.494256987910000 - 0.061782123488750 \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt \right)}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt}$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-\frac{9}{1000} + \frac{0.247128493955000 \pi^{5/4} \mathcal{A} \sqrt{\pi}}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-0.009000000000000000 +$$

$$\frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \left(-0.0218432792374999 + 0.0436865584749998 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$-0.009000000000000000 + \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right)$$

$$\Gamma\left(\frac{3+i}{8}\right) \left(0.0218432792374999 - 0.0436865584749998 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) - \frac{7+2}{10^3} =$$

$$- \frac{9}{1000} + \frac{1}{\sqrt{\pi^2}} 0.0873731169499996 \sqrt[4]{\pi}$$

$$\left(-0.7500000000000000 \cos\left(\frac{\log(\pi^2)}{24}\right) + 1.0000000000000000 \cos^3\left(\frac{\log(\pi^2)}{24}\right) \right)$$

$$\Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

From the same expression, we obtain also:

$$10^3 \left(\pi \left(\left(\left(\frac{1}{4} \pi^{-3/4} \right) * \text{gamma} \left(\frac{-1+i}{4} \right) * \text{gamma} \left(\frac{-1-i}{4} \right) * 0.49425698791 * \cos\left(\frac{1}{8} \ln(\pi^2)\right) \right) \right) + \frac{47-2}{10^3} \right)$$

Input interpretation:

$$10^3 \left(\pi \left(\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right) \right) + \frac{47-2}{10^3} \right)$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

1672.1187458...

1672.1187458... result practically equal to the rest mass of Omega baryon 1672.45

Alternative representations:

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) +$$

$$\frac{47-2}{10^3} = 10^3 \left(\frac{1}{4} \times 0.494256987910000 \pi \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \right)$$

$$\exp\left(-\log G\left(\frac{1}{4}(-1-i)\right) + \log G\left(1 + \frac{1}{4}(-1-i)\right)\right)$$

$$\exp\left(-\log G\left(\frac{1}{4}(-1+i)\right) + \log G\left(1 + \frac{1}{4}(-1+i)\right)\right) \pi^{-3/4} + \frac{45}{10^3}$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 10^3 \left(\frac{0.494256987910000 \pi G\left(1+\frac{1}{4}(-1-i)\right) G\left(1+\frac{1}{4}(-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} + \frac{45}{10^3} \right)$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 10^3 \left(\frac{0.494256987910000 \pi G\left(1+\frac{1}{4}(-1-i)\right) G\left(1+\frac{1}{4}(-1+i)\right) \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} + \frac{45}{10^3} \right)$$

Series representations:

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = \left(45.000000000000000000 \left(-1.000000000000000000 + 1.000000000000000000 i^2 - \frac{43.933954480889 \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-3k_1-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{(2k_1)! k_2! k_3!} \right) \right) / ((-1.000000000000000000 + 1.000000000000000000 i) (1.000000000000000000 + 1.000000000000000000 i))$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} =$$

$$123.564246977500 \left(0.36418301491526 + 1.000000000000000 \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} \left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = \left(45.0000000000000 \right)$$

$$\left(-1.000000000000000 + 1.000000000000000 i^2 - 43.933954480889 \sqrt[4]{\pi} \right)$$

$$J_0\left(\frac{\log(\pi^2)}{8}\right) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} -$$

$$87.867908961778 \sqrt[4]{\pi} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty}$$

$$\frac{(-1)^{k_1} 4^{-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} J_2 k_1 \left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \right)$$

$$/ ((-1.000000000000000 + 1.000000000000000 i)$$

$$(1.000000000000000 + 1.000000000000000 i))$$

Integral representations:

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 45 - \frac{494.256987910000 \pi^{9/4} \mathcal{A}^2}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = \frac{\pi^{9/4} \mathcal{A}^2 \left(494.25698791000 - 61.782123488750 \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt \right)}{45 + \frac{\int_L \phi e^t t^{1/4+i/4} dt \int_L \phi e^t t^{1/4-i/4} dt}{L}}$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 45 + \frac{247.128493955000 \pi^{5/4} \mathcal{A} \sqrt{\pi}}{\int_L \phi e^t t^{1/4+i/4} dt \int_L \phi e^t t^{1/4-i/4} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 45.000000000000000 + \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \left(-21.8432792374999 + 43.6865584749998 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right) \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 45.000000000000000 + \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right) \left(21.8432792374999 - 43.6865584749998 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$10^3 \left(\frac{1}{4} \pi \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \frac{47-2}{10^3} = 45 + \frac{1}{\sqrt{\pi^2}} 87.3731169499996 \sqrt[4]{\pi} \left(-0.750000000000000 \cos\left(\frac{\log(\pi^2)}{24}\right) + 1.000000000000000 \cos^3\left(\frac{\log(\pi^2)}{24}\right) \right) \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8}-\frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

And again, we obtain:

$$76\pi * (((1/4 * \pi^{-3/4}) * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791 * \cos(1/8 * \ln(\pi^2)))) + 2$$

Input interpretation:

$$76 \pi \left(\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right) \right) + 2$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

125.66102468...

[125.66102468...](#)

Alternative representations:

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\ & 2 + \frac{1}{4} \times 37.5635310811600 \pi \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \\ & \quad \exp\left(-\log G\left(\frac{1}{4}(-1-i)\right) + \log G\left(1 + \frac{1}{4}(-1-i)\right)\right) \\ & \quad \exp\left(-\log G\left(\frac{1}{4}(-1+i)\right) + \log G\left(1 + \frac{1}{4}(-1+i)\right)\right) \pi^{-3/4} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\ & 2 + \frac{37.5635310811600 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\ & 2 + \frac{37.5635310811600 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} \end{aligned}$$

Series representations:

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\ & \left(2.000000000000000 \left(-1.000000000000000 + 1.000000000000000 i^2 - \right. \right. \\ & \quad \left. \left. 75.127062162320 \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-3k_1-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{(2k_1)! k_2! k_3!} \right) \right) \\ & \left. \right) / ((-1.000000000000000 + 1.000000000000000 i) \\ & (1.000000000000000 + 1.000000000000000 i)) \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\ & 9.39088277029000 \left(0.212972523342258 + 1.000000000000000 \sqrt[4]{\pi} \right. \\ & \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} \left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \right. \\ & \quad \left. \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\
& \left(\begin{aligned} & 2.000000000000000 \\ & \left(-1.000000000000000 + 1.000000000000000 i^2 - 75.127062162320 \sqrt[4]{\pi} \right. \\ & \quad \left. J_0\left(\frac{\log(\pi^2)}{8}\right) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} - \right. \\ & \quad \left. 150.25412432464 \sqrt[4]{\pi} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} J_2 k_1 \left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \right) \end{aligned} \right) \\
& / ((-1.000000000000000 + 1.000000000000000 i) \\
& (1.000000000000000 + 1.000000000000000 i))
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\
& 2 - \frac{37.5635310811600 \pi^{9/4} \mathcal{A}^2}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\
& 2 + \frac{\pi^{9/4} \mathcal{A}^2 (37.563531081160 - 4.6954413851450 \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt)}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt}
\end{aligned}$$

$$\begin{aligned}
& \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 = \\
& 2 + \frac{18.7817655405800 \pi^{5/4} \mathcal{A} \sqrt{\pi}}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0
\end{aligned}$$

Multiple-argument formulas:

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1 + i)\right) \left(\Gamma\left(\frac{1}{4} (-1 - i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 =$$

$$2.0000000000000000 +$$

$$\frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \left(-1.666008922204999 + 3.32017844409999 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\Gamma\left(\frac{1}{8} (-1 - i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1 + i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1 + i)\right) \left(\Gamma\left(\frac{1}{4} (-1 - i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 =$$

$$2.0000000000000000 + \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \Gamma\left(\frac{1}{8} (-1 - i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1 + i)\right)$$

$$\Gamma\left(\frac{3+i}{8}\right) \left(1.666008922204999 - 3.32017844409999 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1 + i)\right) \left(\Gamma\left(\frac{1}{4} (-1 - i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 2 =$$

$$2 + \frac{1}{\sqrt{\pi^2}} 6.64035688819997 \sqrt[4]{\pi}$$

$$\left(-0.7500000000000000 \cos\left(\frac{\log(\pi^2)}{24}\right) + 1.0000000000000000 \cos^3\left(\frac{\log(\pi^2)}{24}\right) \right)$$

$$\Gamma\left(\frac{1}{8} (-1 - i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1 + i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$76\pi * (((1/4 * \pi^{(-3/4)} * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791 * \cos(1/8 * \ln(\pi^2)))))) + 18 - 2$$

Input interpretation:

$$76 \pi \left(\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4} (-1 + i)\right) \Gamma\left(\frac{1}{4} (-1 - i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right) \right) + 18 - 2$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

Result:

139.66102468...

[139.66102468...](#)

Alternative representations:

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 = \\ & 16 + \frac{1}{4} \times 37.5635310811600 \pi \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \\ & \exp\left(-\log G\left(\frac{1}{4} (-1-i)\right) + \log G\left(1 + \frac{1}{4} (-1-i)\right)\right) \\ & \exp\left(-\log G\left(\frac{1}{4} (-1+i)\right) + \log G\left(1 + \frac{1}{4} (-1+i)\right)\right) \pi^{-3/4} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 = \\ & 16 + \frac{37.5635310811600 \pi G\left(1 + \frac{1}{4} (-1-i)\right) G\left(1 + \frac{1}{4} (-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4} (-1-i)\right) G\left(\frac{1}{4} (-1+i)\right)} \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 = \\ & 16 + \frac{37.5635310811600 \pi G\left(1 + \frac{1}{4} (-1-i)\right) G\left(1 + \frac{1}{4} (-1+i)\right) \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4} (-1-i)\right) G\left(\frac{1}{4} (-1+i)\right)} \end{aligned}$$

Series representations:

$$\begin{aligned} & \frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 = \\ & \left(16.00000000000000 \left(-1.000000000000000 + 1.000000000000000 i^2 - \right. \right. \\ & \quad \left. \left. 9.3908827702900 \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-3k_1-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{(2k_1)! k_2! k_3!} \right) \right) \\ & \left. \right) / ((-1.000000000000000 + 1.000000000000000 i) \\ & (1.000000000000000 + 1.000000000000000 i)) \end{aligned}$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$9.39088277029000 \left(1.70378018673807 + 1.000000000000000 \sqrt[4]{\pi} \right.$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} \left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2}$$

$$\left. \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$\left(16.00000000000000 \right.$$

$$\left. \left(-1.000000000000000 + 1.000000000000000 i^2 - 9.3908827702900 \sqrt[4]{\pi} \right. \right.$$

$$J_0\left(\frac{\log(\pi^2)}{8}\right) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} -$$

$$18.781765540580 \sqrt[4]{\pi} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} J_{2k_1}\left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \left. \right) \right)$$

$$/ ((-1.000000000000000 + 1.000000000000000 i)$$

$$(1.000000000000000 + 1.000000000000000 i))$$

Integral representations:

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16 - \frac{37.5635310811600 \pi^{9/4} \mathcal{A}^2}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16 + \frac{\pi^{9/4} \mathcal{A}^2 \left(37.563531081160 - 4.6954413851450 \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt \right)}{\oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt}$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16 + \frac{18.7817655405800 \pi^{5/4} \mathcal{A} \sqrt{\pi}}{\int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0} \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma} \frac{e^{s-\log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

Multiple-argument formulas:

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16.000000000000000 +$$

$$\frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \left(-1.66608922204999 + 3.32017844409999 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\Gamma\left(\frac{1}{8} (-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16.000000000000000 + \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \Gamma\left(\frac{1}{8} (-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1+i)\right)$$

$$\Gamma\left(\frac{3+i}{8}\right) \left(1.66608922204999 - 3.32017844409999 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right)$$

$$\frac{1}{4} (76 \pi) \pi^{-3/4} \left(\Gamma\left(\frac{1}{4} (-1+i)\right) \left(\Gamma\left(\frac{1}{4} (-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + 18 - 2 =$$

$$16 + \frac{1}{\sqrt{\pi^2}} 6.64035688819997 \sqrt[4]{\pi}$$

$$\left(-0.750000000000000 \cos\left(\frac{\log(\pi^2)}{24}\right) + 1.000000000000000 \cos^3\left(\frac{\log(\pi^2)}{24}\right) \right)$$

$$\Gamma\left(\frac{1}{8} (-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8} (-1+i)\right) \Gamma\left(\frac{3+i}{8}\right)$$

$$27*1/2(((76Pi*(((1/4*Pi^(-3/4) * gamma((-1+i)/4) * gamma((-1-i)/4) * 0.49425698791 * cos(1/8*ln(Pi^2)))))+5-1/golden ratio)))+1/2$$

Input interpretation:

$$27 \times \frac{1}{2}$$

$$\left(76 \pi \left(\frac{1}{4} \pi^{-3/4} \Gamma\left(\frac{1}{4} (-1+i)\right) \Gamma\left(\frac{1}{4} (-1-i)\right) \times 0.49425698791 \cos\left(\frac{1}{8} \log(\pi^2)\right) \right) + 5 - \frac{1}{\phi} \right) +$$

$$\frac{1}{2}$$

$\Gamma(x)$ is the gamma function

$\log(x)$ is the natural logarithm

i is the imaginary unit

ϕ is the golden ratio

Result:

1729.0803743...

1729.0803743...

Alternative representations:

$$\frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \right. \\ \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = \frac{1}{2} + \frac{27}{2} \left(5 - \frac{1}{\phi} + \frac{1}{4} \times 37.5635310811600 \pi \right. \\ \left. \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \exp\left(-\log G\left(\frac{1}{4}(-1-i)\right) + \log G\left(1 + \frac{1}{4}(-1-i)\right)\right) \right. \\ \left. \exp\left(-\log G\left(\frac{1}{4}(-1+i)\right) + \log G\left(1 + \frac{1}{4}(-1+i)\right)\right) \pi^{-3/4} \right)$$

$$\frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \right. \\ \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = \frac{1}{2} + \frac{27}{2} \left(5 - \frac{1}{\phi} + \frac{37.5635310811600 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \cosh\left(\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4}}{4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right)} \right)$$

$$\frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \right. \\ \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = \\ \frac{1}{2} + \frac{27}{2} \left(5 - \frac{1}{\phi} + \left(37.5635310811600 \pi G\left(1 + \frac{1}{4}(-1-i)\right) G\left(1 + \frac{1}{4}(-1+i)\right) \right. \right. \\ \left. \left. \cosh\left(-\frac{1}{8} i \log(\pi^2)\right) \pi^{-3/4} \right) / \left(4 G\left(\frac{1}{4}(-1-i)\right) G\left(\frac{1}{4}(-1+i)\right) \right) \right)$$

Series representations:

$$\begin{aligned} & \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \\ & \quad \left(5 - \frac{1}{\phi} \right) + \frac{1}{2} = \\ & \left(68.00000000000000 \left(0.198529411764706 - 1.000000000000000 \phi - \right. \right. \\ & \quad 0.198529411764706 i^2 + 1.000000000000000 \phi i^2 - \\ & \quad 29.829862917392 \phi \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \\ & \quad \left. \frac{(-1)^{k_1} 4^{-3k_1-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} \log^{2k_1}(\pi^2) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{(2k_1)! k_2! k_3!} \right) \\ & \left. \right) / (\phi(-1.000000000000000 + 1.000000000000000 i) \\ & (1.000000000000000 + 1.000000000000000 i)) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \\ & \quad \left(5 - \frac{1}{\phi} \right) + \frac{1}{2} = \frac{1}{\phi} 126.77691739892 \\ & \left(-0.106486261671129 + 0.53637524397310 \phi + 1.000000000000000 \phi \right. \\ & \quad \sqrt[4]{\pi} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{(2k_1)! k_2! k_3!} \left(-\frac{1}{64}\right)^{k_1} \log^{2k_1}(\pi^2) \left(\frac{1}{4}(-1+i) - z_0\right)^{k_2} \\ & \quad \left. \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0) \end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) \right) + \\
& \quad \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = \\
& \left(68.0000000000000 \left[0.19852941176471 - 1.000000000000000 \phi - \right. \right. \\
& \quad 0.19852941176471 i^2 + 1.000000000000000 \phi i^2 - 29.829862917392 \phi \\
& \quad \left. \sqrt[4]{\pi} J_0\left(\frac{\log(\pi^2)}{8}\right) \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} - \right. \\
& \quad \left. 59.659725834784 \phi \sqrt[4]{\pi} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(-1)^{k_1} 4^{-k_2-k_3} (-1-i)^{k_2} (-1+i)^{k_3} J_2 k_1 \left(\frac{\log(\pi^2)}{8}\right) \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \right) \\
& \quad / (\phi(-1.000000000000000 + 1.000000000000000 i) \\
& \quad (1.000000000000000 + 1.000000000000000 i))
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) \right) + 5 - \\
& \quad \left. \frac{1}{\phi} \right) + \frac{1}{2} = 68 - \frac{27}{2\phi} - \frac{507.107669595660 \pi^{9/4} \mathcal{A}^2}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt} \int_{\frac{\pi}{2}}^{\frac{\log(\pi^2)}{8}} \sin(t) dt
\end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) \right) + \\
& \quad \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = 68 - \frac{27}{2\phi} + \\
& \quad \frac{\pi^{9/4} \mathcal{A}^2 (507.10766959566 - 63.388458699458 \log(\pi^2) \int_0^1 \sin\left(\frac{1}{8} t \log(\pi^2)\right) dt)}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt}
\end{aligned}$$

$$\begin{aligned}
& \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \left(\Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) \right) + \\
& \quad \left. 5 - \frac{1}{\phi} \right) + \frac{1}{2} = \\
& \quad 68 - \frac{27}{2\phi} + \frac{253.553834797830 \pi^{5/4} \mathcal{A} \sqrt{\pi}}{\int_L \oint e^t t^{1/4+i/4} dt \int_L \oint e^t t^{1/4-i/4} dt} \int_{-\mathcal{A} \infty + \gamma}^{\mathcal{A} \infty + \gamma} \frac{e^{s - \log^2(\pi^2)/(256s)}}{\sqrt{s}} ds \text{ for } \gamma > 0
\end{aligned}$$

Multiple-argument formulas:

$$\begin{aligned} & \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \\ & \quad 5 - \frac{1}{\phi} + \frac{1}{2} = 68.00000000000000 - \frac{13.50000000000000}{\phi} + \\ & \quad \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \left(-22.4112044976749 + 44.8224089953498 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right) \\ & \quad \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \\ & \quad 5 - \frac{1}{\phi} + \frac{1}{2} = 68.00000000000000 - \frac{13.50000000000000}{\phi} + \\ & \quad \frac{1}{\sqrt{\pi^2}} \sqrt[4]{\pi} \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{3}{8} - \frac{i}{8}\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \Gamma\left(\frac{3+i}{8}\right) \\ & \quad \left(22.4112044976749 - 44.8224089953498 \sin^2\left(\frac{\log(\pi^2)}{16}\right) \right) \end{aligned}$$

$$\begin{aligned} & \frac{27}{2} \left(\frac{76}{4} \pi \left(\pi^{-3/4} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \cos\left(\frac{\log(\pi^2)}{8}\right) \right) \right) + \\ & \quad 5 - \frac{1}{\phi} + \frac{1}{2} = \\ & \quad \frac{1}{2} + \frac{27}{2} \left(5 - \frac{1}{\phi} + \frac{1}{\sqrt{\pi^2}} 9.39088277029000 \times 2^{-2+1/4(-1-i)+1/4(-1+i)} \right. \\ & \quad \left. \sqrt[4]{\pi} \left(-1 + 2 \cos^2\left(\frac{\log(\pi^2)}{16}\right) \right) \Gamma\left(\frac{1}{2} + \frac{1}{8}(-1-i)\right) \right. \\ & \quad \left. \Gamma\left(\frac{1}{2} + \frac{1}{8}(-1+i)\right) \Gamma\left(\frac{1}{8}(-1-i)\right) \Gamma\left(\frac{1}{8}(-1+i)\right) \right) \end{aligned}$$

Now, we have that:

$$\begin{aligned} & \alpha^{-\frac{1}{4}} \left\{ \frac{1}{1-s} - 4\alpha \int_0^{\infty} \left(\frac{1}{1+s} - \frac{\alpha x^2}{1! 3+s} + \frac{\alpha^2 x^4}{2! 5+s} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & + \beta^{-\frac{1}{4}} \left\{ \frac{1}{s} - 4\beta \int_0^{\infty} \left(\frac{1}{2-s} - \frac{\beta x^2}{1! 4-s} + \frac{\beta^2 x^4}{2! 6-s} - \dots \right) \frac{x dx}{e^{2\pi x} - 1} \right\} \\ & = \frac{1}{2} \pi^{-\frac{3}{4}} \left(\frac{\alpha}{\beta} \right)^{\frac{1}{8} - \frac{1}{4}s} \Gamma\left(-\frac{s}{2}\right) \Gamma\left(\frac{s-1}{2}\right) \xi(s). \end{aligned} \quad (8)$$

For $t = 1$, $\alpha = 2$, $\beta = \pi^2 / 2$, and $\Xi(1/2 t) = \xi(s) = 0.49425698791$, $s = (1/2 + 1/2i)$, we obtain:

$$\frac{1}{2} \pi^{-3/4} * ((2/(\pi^2)/2))^{(1/8 - 1/4 * (1/2 + 1/2i))} * ((\Gamma(-1/2 + 1/2i) * 1/2)) * ((\Gamma(((1/2 + 1/2i) - 1) * 1/2))) * ((0.49425698791))$$

Input interpretation:

$$\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8 - 1/4 * (1/2 + 1/2i)} \Gamma\left(-\left(\frac{1}{2} + \frac{1}{2}i\right) \times \frac{1}{2}\right) \Gamma\left(\left(\left(\frac{1}{2} + \frac{1}{2}i\right) - 1\right) \times \frac{1}{2}\right) \times 0.49425698791$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$1.0358559655... + 0.30481099408... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 1.0797718849 \text{ (radius), } \theta = 16.397047785^\circ \text{ (angle)}$$

1.0797718849

Alternative representations:

$$\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right)\right) 0.494256987910000 \right) =$$

$$0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8}$$

$$\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right)\right) 0.494256987910000 \right) =$$

$$0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8}$$

$$\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right)\right) 0.494256987910000 \right) =$$

$$0.247128493955000 e^{\log \Gamma(1/2(-1/2-i/2))} e^{\log \Gamma(1/2(-1/2+i/2))} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8}$$

From which, we obtain:

$$\left(\left(\left(\left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \right) \right) \right) \Gamma\left(-\left(\frac{1}{2} + \frac{i}{2}\right)\right) \right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right)\right) 0.49425698791 \right) \right)^{2\pi}$$

Input interpretation:

$$\left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \right) \Gamma\left(-\left(\frac{1}{2} + \frac{i}{2}\right)\right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right)\right) 0.49425698791 \right)^{2\pi}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-0.3650579786... + 1.578011482... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 1.619687490 \text{ (radius), } \theta = 103.0256897^\circ \text{ (angle)}$$

1.619687490

Alternative representations:

$$\left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} = \\ \left(0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \right. \\ \left. \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

$$\left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} = \\ \left(0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

$$\left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} = \\ \left(0.247128493955000 e^{\log \Gamma(1/2(-1/2-i/2))} e^{\log \Gamma(1/2(-1/2+i/2))} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

And again:

$$10^3 * (((((1/2 * \pi^{-3/4} * ((2/(\pi^2)/2))^{1/8-1/4*(1/2+1/2i)} * ((\text{gamma} (- \\ (1/2+1/2i)*1/2))) * ((\text{gamma} (((1/2+1/2i)-1)*1/2))) * (((0.49425698791)))))))))^{2\pi} \\ +(123-11)i$$

Input interpretation:

$$10^3 \left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \Gamma\left(-\left(\frac{1}{2} + \frac{1}{2}i\right) \times \frac{1}{2}\right) \right. \\ \left. \Gamma\left(\left(\left(\frac{1}{2} + \frac{1}{2}i\right) - 1\right) \times \frac{1}{2}\right) \times 0.49425698791 \right)^{2\pi} + (123 - 11)i$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-365.0579786... + 1690.011482... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 1728.989918 \text{ (radius), } \theta = 102.1891352^\circ \text{ (angle)}$$

$$1728.989918 \approx 1729$$

Alternative representations:

$$10^3 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} + (123 - 11)i = \\ 112i + 10^3 \left(0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \right) \\ \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

$$10^3 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} + (123 - 11)i = 112i + \\ 10^3 \left(0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

$$10^3 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{2\pi} + (123 - 11)i = \\ 112i + 10^3 \left(0.247128493955000 e^{\log\Gamma(1/2(-1/2-i/2))} e^{\log\Gamma(1/2(-1/2+i/2))} \right) \\ \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{2\pi}$$

$$21 \times 2 \left(\left(\left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \right) \Gamma \left(-\left(\frac{1}{2} + \frac{1}{2} i \right) \times \frac{1}{2} \right) \right) \right. \\ \left. \left(\frac{1}{2} + \frac{1}{2} i \right) \right) \times \left(\Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2} i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right) \right)^{16+4i}$$

Input interpretation:

$$21 \times 2 \left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \Gamma \left(-\left(\frac{1}{2} + \frac{1}{2} i \right) \times \frac{1}{2} \right) \right. \\ \left. \Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2} i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right)^{16+4i}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-19.0832864... - 138.128677... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 139.440679 \text{ (radius), } \theta = -97.8659541^\circ \text{ (angle)}$$

139.440679

Alternative representations:

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16+4i} = \\ 4i + 42 \left(0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \right. \\ \left. \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}$$

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16+4i} = \\ 4i + 42 \left(0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}$$

$$\begin{aligned}
& 21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\
& \quad \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 4i = \\
& 4i + 42 \left(0.247128493955000 e^{\log \Gamma(1/2(-1/2-i/2))} e^{\log \Gamma(1/2(-1/2+i/2))} \right. \\
& \quad \left. \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}
\end{aligned}$$

Series representations:

$$\begin{aligned}
& 21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\
& \quad \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 4i = \\
& 4i + \left(34.911966473250 \left(\frac{1}{\pi^2} \right)^{2+4(-1/2-i/2)} \left(\sum_{k=0}^{\infty} \frac{2^{-k} \left(-\frac{1}{2} - \frac{i}{2} \right)^k \Gamma^{(k)}(1)}{k!} \right)^{16} \right. \\
& \quad \left. \left(\sum_{k=0}^{\infty} \frac{2^{-k} \left(-\frac{1}{2} + \frac{i}{2} \right)^k \Gamma^{(k)}(1)}{k!} \right)^{16} \right) / \left(\left(-\frac{1}{2} - \frac{i}{2} \right)^{16} \left(-\frac{1}{2} + \frac{i}{2} \right)^{16} \pi^{12} \right)
\end{aligned}$$

$$\begin{aligned}
& 21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\
& \quad \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 4i = \\
& \frac{1}{\pi^{12}} 4.000000000000 \left(\frac{1}{\pi^2} \right)^{-2i} \left(1.000000000000 i \left(\frac{1}{\pi^2} \right)^{2i} \pi^{12} + \right. \\
& \quad \left. 2.032143906297 \times 10^{-9} \left(\sum_{k=0}^{\infty} \frac{4^{-k} (-1+i-4z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)^{16} \right. \\
& \quad \left. \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} - \frac{i}{4} - z_0 \right)^k \Gamma^{(k)}(z_0)}{k!} \right)^{16} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 4i \propto \\ 4i + \left(8.128575625189 \times 10^{-9} \times 2^{16+8(1/2-i/2)+8(1/2+i/2)} e^{8(1/2-i/2)+8(1/2+i/2)} \right. \\ \left. \left(-\frac{1}{2} - \frac{i}{2} \right)^{-8+8(-1/2-i/2)} \left(-\frac{1}{2} + \frac{i}{2} \right)^{-8+8(-1/2+i/2)} \left(\frac{1}{\pi^2} \right)^{2+4(-1/2-i/2)} \sqrt{2\pi}^{32} \right) / \\ \left(\pi^{12} \exp^{16} \left[-\sum_{k=0}^{\infty} \frac{2^{2k} \left(-\frac{1}{2} - \frac{i}{2} \right)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)} \right] \right) \\ \exp^{16} \left[-\sum_{k=0}^{\infty} \frac{2^{2k} \left(-\frac{1}{2} + \frac{i}{2} \right)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)} \right] \Bigg) \text{ for } \infty \rightarrow \frac{1}{2\sqrt{2}}$$

\mathbb{Z} is the set of integers

B_n is the n^{th} Bernoulli number

$$21*2((((1/2*Pi^(-3/4)) * ((2/(Pi^2)/2))^(1/8-1/4*(1/2+1/2i)) * ((gamma (-1/2+1/2i)*1/2))) * ((gamma (((1/2+1/2i)-1)*1/2))) * (((0.49425698791))))))^(16+18i)$$

Input interpretation:

$$21 \times 2 \left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \Gamma \left(-\left(\frac{1}{2} + \frac{1}{2}i \right) \times \frac{1}{2} \right) \right. \\ \left. \Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2}i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right)^{16} + 18i$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$-19.0832864... - 124.128677... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 125.587022 \text{ (radius), } \theta = -98.7401050^\circ \text{ (angle)}$$

$$125.587022i$$

Alternative representations:

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i = \\ 18 i + 42 \left(0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \right. \\ \left. \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}$$

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i = 18 i + \\ 42 \left(0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}$$

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i = \\ 18 i + 42 \left(0.247128493955000 e^{\log \Gamma(1/2(-1/2-i/2))} e^{\log \Gamma(1/2(-1/2+i/2))} \right. \\ \left. \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)^{16}$$

Series representations:

$$21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\ \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i = \\ 18 i + \left(34.911966473250 \left(\frac{1}{\pi^2} \right)^{2+4(-1/2-i/2)} \left(\sum_{k=0}^{\infty} \frac{2^{-k} \left(-\frac{1}{2} - \frac{i}{2} \right)^k \Gamma^{(k)}(1)}{k!} \right) \right)^{16} \\ \left(\sum_{k=0}^{\infty} \frac{2^{-k} \left(-\frac{1}{2} + \frac{i}{2} \right)^k \Gamma^{(k)}(1)}{k!} \right)^{16} \Big/ \left(\left(-\frac{1}{2} - \frac{i}{2} \right)^{16} \left(-\frac{1}{2} + \frac{i}{2} \right)^{16} \pi^{12} \right)$$

$$\begin{aligned}
& 21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\
& \quad \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i = \\
& \frac{1}{\pi^{12}} 18.0000000000 \left(\frac{1}{\pi^2} \right)^{-2i} \left(1.000000000000 i \left(\frac{1}{\pi^2} \right)^{2i} \pi^{12} + \right. \\
& \quad 4.515875347327 \times 10^{-10} \left(\sum_{k=0}^{\infty} \frac{4^{-k} (-1+i-4z_0)^k \Gamma^{(k)}(z_0)}{k!} \right)^{16} \\
& \quad \left. \left(\sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4} - \frac{i}{4} - z_0 \right)^k \Gamma^{(k)}(z_0)}{k!} \right)^{16} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

$$\begin{aligned}
& 21 \times 2 \left(\frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \\
& \quad \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right)^{16} + 18 i \propto \\
& 18 i + \left(8.128575625189 \times 10^{-9} \times 2^{16+8(1/2-i/2)+8(1/2+i/2)} e^{8(1/2-i/2)+8(1/2+i/2)} \right. \\
& \quad \left. \left(-\frac{1}{2} - \frac{i}{2} \right)^{-8+8(-1/2-i/2)} \left(-\frac{1}{2} + \frac{i}{2} \right)^{-8+8(-1/2+i/2)} \left(\frac{1}{\pi^2} \right)^{2+4(-1/2-i/2)} \sqrt{2\pi}^{32} \right) / \\
& \left(\pi^{12} \exp^{16} \left[-\sum_{k=0}^{\infty} \frac{2^{2k} \left(-\frac{1}{2} - \frac{i}{2} \right)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)} \right] \right. \\
& \quad \left. \exp^{16} \left[-\sum_{k=0}^{\infty} \frac{2^{2k} \left(-\frac{1}{2} + \frac{i}{2} \right)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)} \right] \right) \text{ for } \infty \rightarrow \frac{1}{2\sqrt{2}}
\end{aligned}$$

\mathbb{Z} is the set of integers

B_n is the n^{th} Bernoulli number

From the sum of the two results, we have:

$$1 + 1 / \left(\left(\left(\left(\left(0.51792798277 + \left(\left(\left(\frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8 - 1/4 (1/2 + 1/2 i)} \right) \right) \right) \right) \right) \right) \right) \Gamma \left(- \left(\frac{1}{2} + \frac{1}{2} i \right) \times \frac{1}{2} \right) \Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2} i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right) \right)$$

Input interpretation:

$$1 + 1 / \left(\left(\left(\left(\left(0.51792798277 + \frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8 - 1/4 (1/2 + 1/2 i)} \right) \right) \right) \right) \right) \Gamma \left(- \left(\frac{1}{2} + \frac{1}{2} i \right) \times \frac{1}{2} \right) \Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2} i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$1.6197400568... - 0.12157647978... i$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$$r = 1.6242963683 \text{ (radius)}, \quad \theta = -4.292529308^\circ \text{ (angle)}$$

[1.6242963683](#)

Alternative representations:

$$1 + 1 / \left(\left(\left(\left(\left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8 - 1/4 (1/2 + i/2)} \right) \right) \right) \right) \right) \right) \Gamma \left(- \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) =$$

$$1 + 1 / \left(\left(\left(\left(\left(0.517927982770000 + 0.247128493955000 \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) \right) \right) \right) \right) \right) \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) \right)! \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4 (1/2 + i/2) + 1/8}$$

$$1 + 1 / \left(\left(\left(\left(\left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8 - 1/4 (1/2 + i/2)} \right) \right) \right) \right) \right) \right) \Gamma \left(- \frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) =$$

$$1 + 1 / \left(\left(\left(\left(\left(0.517927982770000 + 0.247128493955000 (1)_{-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right)} \right) \right) \right) \right) \right) (1)_{-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right)} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4 (1/2 + i/2) + 1/8}$$

$$1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\ \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) = \\ 1 + 1 / \left(0.517927982770000 + 0.247128493955000 e^{\log \Gamma(1/2(-1/2-i/2))} \right. \\ \left. e^{\log \Gamma(1/2(-1/2+i/2))} \pi^{-3/4} \left(\frac{1}{\pi^2} \right)^{-1/4(1/2+i/2)+1/8} \right)$$

Series representations:

$$1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\ \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) \propto \\ 1 + 1 / \left(0.517927982770000 + \frac{1}{\pi^{3/4}} 0.698984935599997 \sqrt{e} \left(-\frac{1}{2} - \frac{i}{2}\right)^{-3/4-i/4} \right. \\ \left. \left(-\frac{1}{2} + \frac{i}{2}\right)^{-3/4+i/4} \left(\frac{1}{\pi^2}\right)^{-i/8} \sqrt{2\pi^2} \left(1 + \sum_{k=1}^{\infty} \sum_{j=1}^{2k} \frac{\left(-\frac{1}{2}\right)^j \left(-\frac{1}{2} - \frac{i}{2}\right)^{-k} \mathcal{D}_2(j+k,j)}{(j+k)!} \right) \right. \\ \left. \left(1 + \sum_{k=1}^{\infty} \sum_{j=1}^{2k} \frac{\left(-\frac{1}{2}\right)^j \left(-\frac{1}{2} + \frac{i}{2}\right)^{-k} \mathcal{D}_2(j+k,j)}{(j+k)!} \right) \right)$$

for $\left(\left(\infty \rightarrow \frac{1}{2\sqrt{2}} \right. \right. \text{ and } \mathcal{D}_{n,j} = (-1+n)((-2+n)\mathcal{D}_{-3+n,-1+j} + \mathcal{D}_{-1+n,j}) \text{ and}$
 $\left. \left. \mathcal{D}_{0,0} = 1 \text{ and } \mathcal{D}_{n,1} = (-1+n)! \text{ and } \mathcal{D}_{n,j} = 0 \right) \text{ for } n \leq -1+3j \right)$

$$1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\ \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) = \\ \left(2.930770364350 \left[-1.000000000000000 \left(\frac{1}{\pi^2}\right)^{i/8} \pi^{3/4} + \right. \right. \\ \left. \left. 1.000000000000000 i^2 \left(\frac{1}{\pi^2}\right)^{i/8} \pi^{3/4} - 2.6049034922358 \right. \right. \\ \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{2^{-k_1-k_2} \left(-\frac{1}{2} - \frac{i}{2}\right)^{k_1} \left(-\frac{1}{2} + \frac{i}{2}\right)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} \right] \right) / \\ \left(-1.000000000000000 \left(\frac{1}{\pi^2}\right)^{i/8} \pi^{3/4} + 1.000000000000000 i^2 \left(\frac{1}{\pi^2}\right)^{i/8} \pi^{3/4} - \right. \\ \left. 7.634373957037 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{2^{-k_1-k_2} \left(-\frac{1}{2} - \frac{i}{2}\right)^{k_1} \left(-\frac{1}{2} + \frac{i}{2}\right)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} \right)$$

$$\begin{aligned}
& 1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\
& \quad \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) = \\
& \left(2.930770364350 \left[1.000000000000000 \left(\frac{1}{\pi^2} \right)^{i/8} \pi^{3/4} + 0.16280646826474 \right. \right. \\
& \quad \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) - z_0 \right)^{k_1} \left(\frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) - z_0 \right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right] \right) / \\
& \left(1.000000000000000 \left(\frac{1}{\pi^2} \right)^{i/8} \pi^{3/4} + 0.47714837231481 \right. \\
& \quad \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2} \right) - z_0 \right)^{k_1} \left(\frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2} \right) - z_0 \right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right) \\
& \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)
\end{aligned}$$

Integral representations:

$$\begin{aligned}
& 1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\
& \quad \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) = \\
& 1 + 1 / \left(0.517927982770000 + \frac{1}{\pi^{3/4}} 0.247128493955000 \left(\frac{1}{\pi^2} \right)^{-i/8} \right. \\
& \quad \left(\int_1^{\infty} e^{-t} t^{-5/4-i/4} dt + \sum_{k=0}^{\infty} -\frac{4(-1)^k}{(1+i-4k)k!} \right) \\
& \quad \left. \left(\int_1^{\infty} e^{-t} t^{1/4(-5+i)} dt + \sum_{k=0}^{\infty} \frac{4(-1)^k}{(-1+i+4k)k!} \right) \right)
\end{aligned}$$

$$\begin{aligned}
& 1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\
& \quad \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right)\right) \Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1\right)\right) 0.494256987910000 \right) \right) = \\
& 1 + \frac{1}{0.517927982770000 + \frac{0.988513975820000 \left(\frac{1}{\pi^2} \right)^{1/8+1/4(-1/2-i/2)} \pi^{5/4} \pi^2}{\int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/2(1/2-i/2)} dt}}
\end{aligned}$$

$$\begin{aligned}
& 1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \\
& \quad \left. \left. \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right) = \\
& \left(2.93077036435 \left(1.00000000000000 \left(\frac{1}{\pi^2} \right)^{i/8} \pi^{3/4} + \right. \right. \\
& \quad 0.1628064682647 \left(\int_0^\infty e^{-t} t^{-5/4-i/4} \left(1 - e^{-t} \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \\
& \quad \left. \left. \int_0^\infty e^{-t} t^{1/4(-5+i)} \left(1 - e^{-t} \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \right) / \\
& \left(1.00000000000000 \left(\frac{1}{\pi^2} \right)^{i/8} \pi^{3/4} + 0.477148372315 \right. \\
& \quad \left(\int_0^\infty e^{-t} t^{-5/4-i/4} \left(1 - e^{-t} \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \\
& \quad \left. \int_0^\infty e^{-t} t^{1/4(-5+i)} \left(1 - e^{-t} \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4} \right)
\end{aligned}$$

or:

$$\begin{aligned}
& (((1+1/((((0.51792798277+(((1/2*\text{Pi}^{-3/4})*((2/(\text{Pi}^2)/2))^{1/8- \\
& 1/4*(1/2+1/2i))*((\text{gamma}(-(1/2+1/2i)*1/2)))*((\text{gamma}(((1/2+1/2i)- \\
& 1)*1/2)))*((0.49425698791)))))))))))-6/10^3)+(1/10^3)i
\end{aligned}$$

Input interpretation:

$$\begin{aligned}
& \left(1 + 1 / \left(0.51792798277 + \frac{1}{2} \pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+1/2i)} \right. \right. \\
& \quad \left. \left. \Gamma \left(-\left(\frac{1}{2} + \frac{1}{2} i \right) \times \frac{1}{2} \right) \right. \right. \\
& \quad \left. \left. \Gamma \left(\left(\left(\frac{1}{2} + \frac{1}{2} i \right) - 1 \right) \times \frac{1}{2} \right) \times 0.49425698791 \right) \right) - \frac{6}{10^3} + \frac{1}{10^3} i
\end{aligned}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$$\begin{aligned}
& 1.6137400568... - \\
& 0.12057647978... i
\end{aligned}$$

(using the principal branch of the logarithm for complex exponentiation)

Polar coordinates:

$r = 1.6182384430$ (radius), $\theta = -4.273123039^\circ$ (angle)

1.6182384430

Alternative representations:

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2}\right)^{1/8-1/4(1/2+i/2)}\right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \right. \right. \\ \left. \left. \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right) 0.494256987910000\right)\right)\right) - \frac{6}{10^3} + \frac{i}{10^3} = \\ 1 - \frac{6}{10^3} + \frac{i}{10^3} + 1 / \left(0.517927982770000 + 0.247128493955000 \right. \\ \left. \left(-1 + \frac{1}{2} \left(-\frac{1}{2} - \frac{i}{2}\right)\right)! \left(-1 + \frac{1}{2} \left(-\frac{1}{2} + \frac{i}{2}\right)\right)! \pi^{-3/4} \left(\frac{1}{\pi^2}\right)^{-1/4(1/2+i/2)+1/8}\right)$$

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2}\right)^{1/8-1/4(1/2+i/2)}\right) \right. \right. \\ \left. \left. \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right) 0.494256987910000\right)\right)\right) - \\ \frac{6}{10^3} + \frac{i}{10^3} = 1 - \frac{6}{10^3} + \frac{i}{10^3} + 1 / \left(0.517927982770000 + \right. \\ \left. 0.247128493955000 (1)_{-1+\frac{1}{2}(-\frac{1}{2}-\frac{i}{2})} (1)_{-1+\frac{1}{2}(-\frac{1}{2}+\frac{i}{2})} \pi^{-3/4} \left(\frac{1}{\pi^2}\right)^{-1/4(1/2+i/2)+1/8}\right)$$

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2}\right)^{1/8-1/4(1/2+i/2)}\right) \Gamma\left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2}\right)\right) \right. \right. \\ \left. \left. \left(\Gamma\left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right) 0.494256987910000\right)\right)\right) - \frac{6}{10^3} + \frac{i}{10^3} = \\ 1 - \frac{6}{10^3} + \frac{i}{10^3} + 1 / \left(0.517927982770000 + 0.247128493955000 \right. \\ \left. e^{\log\Gamma(1/2(-1/2-i/2))} e^{\log\Gamma(1/2(-1/2+i/2))} \pi^{-3/4} \left(\frac{1}{\pi^2}\right)^{-1/4(1/2+i/2)+1/8}\right)$$

Series representations:

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} 2\right)^{1/8-1/4(1/2+i/2)}\right)\right.\right. \\ \left.\left.\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{2}\right)\right)\left(\Gamma\left(\frac{1}{2}\left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right)0.494256987910000\right)\right)\right) - \\ \frac{6}{10^3} + \frac{i}{10^3} \propto \frac{497}{500} + \frac{i}{1000} + 1 / \left(0.517927982770000 + \right. \\ \left. \frac{1}{\pi^{3/4}} 0.698984935599997 \sqrt{e} \left(-\frac{1}{2} - \frac{i}{2}\right)^{-3/4-i/4} \right. \\ \left. \left(-\frac{1}{2} + \frac{i}{2}\right)^{-3/4+i/4} \left(\frac{1}{\pi^2}\right)^{-i/8} \exp\left(\sum_{k=0}^{\infty} \frac{2^{1+4k} (-1-i)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)}\right)\right) \\ \left.\exp\left(\sum_{k=0}^{\infty} \frac{2^{1+4k} (-1+i)^{-1-2k} B_{2+2k}}{1+3k+2k^2}\right) \sqrt{2\pi^2}\right) \text{ for } \infty \rightarrow \frac{1}{2\sqrt{2}}$$

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} 2\right)^{1/8-1/4(1/2+i/2)}\right)\right.\right. \\ \left.\left.\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{2}\right)\right)\left(\Gamma\left(\frac{1}{2}\left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right)0.494256987910000\right)\right)\right) - \\ \frac{6}{10^3} + \frac{i}{10^3} \propto \frac{497}{500} + \frac{i}{1000} + 1 / \left(0.517927982770000 + \frac{1}{\pi^{3/4}} \right. \\ \left. 0.698984935599997 \sqrt{e} \left(-\frac{1}{2} - \frac{i}{2}\right)^{-3/4-i/4} \left(-\frac{1}{2} + \frac{i}{2}\right)^{-3/4+i/4} \right. \\ \left. \left(\frac{1}{\pi^2}\right)^{-i/8} \sqrt{2\pi^2} \left(1 + \sum_{k=1}^{\infty} \sum_{j=1}^{2k} \frac{\left(-\frac{1}{2}\right)^j \left(-\frac{1}{2} - \frac{i}{2}\right)^{-k} \mathcal{D}_2(j+k,j)}{(j+k)!}\right)\right) \\ \left.\left(1 + \sum_{k=1}^{\infty} \sum_{j=1}^{2k} \frac{\left(-\frac{1}{2}\right)^j \left(-\frac{1}{2} + \frac{i}{2}\right)^{-k} \mathcal{D}_2(j+k,j)}{(j+k)!}\right)\right) \\ \text{for } \left(\left(\infty \rightarrow \frac{1}{2\sqrt{2}} \text{ and } \mathcal{D}_{n,j} = (-1+n)(-2+n)\mathcal{D}_{-3+n,-1+j} + \mathcal{D}_{-1+n,j}\right) \text{ and } \right. \\ \left. \mathcal{D}_{0,0} = 1 \text{ and } \mathcal{D}_{n,1} = (-1+n)! \text{ and } \mathcal{D}_{n,j} = 0 \text{ for } n \leq -1+3j\right)$$

$$\left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} 2\right)^{1/8-1/4(1/2+i/2)}\right)\right.\right. \\ \left.\left.\Gamma\left(-\frac{1}{2}\left(\frac{1}{2} + \frac{i}{2}\right)\right)\left(\Gamma\left(\frac{1}{2}\left(\left(\frac{1}{2} + \frac{i}{2}\right) - 1\right)\right)0.494256987910000\right)\right)\right) - \frac{6}{10^3} + \frac{i}{10^3} \propto \\ \frac{497}{500} + \frac{i}{1000} + 1 / \left(0.517927982770000 + \frac{0.698984935599997 \sqrt{e} \left(-\frac{1}{2} - \frac{i}{2}\right)^{-3/4-i/4} \left(-\frac{1}{2} + \frac{i}{2}\right)^{-3/4+i/4} \left(\frac{1}{\pi^2}\right)^{-i/8} \sqrt{2\pi^2}}{\pi^{3/4} \exp\left(-\sum_{k=0}^{\infty} \frac{2^{1+4k} (-1-i)^{-1-2k} B_{2+2k}}{(1+k)(1+2k)}\right) \exp\left(-\sum_{k=0}^{\infty} \frac{2^{1+4k} (-1+i)^{-1-2k} B_{2+2k}}{1+3k+2k^2}\right)}\right) \text{ for } \\ \infty \rightarrow \frac{1}{2\sqrt{2}}$$

B_n is the n^{th} Bernoulli number

Integral representations:

$$\begin{aligned} & \left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right. \right. \right. \\ & \quad \left. \left. \left. \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right) \right) \right) - \\ & \frac{6}{10^3} + \frac{i}{10^3} = \frac{497}{500} + \frac{i}{1000} + 1 / \left(0.517927982770000 + \frac{1}{\pi^{3/4}} \right. \\ & \quad \left. 0.247128493955000 \left(\frac{1}{\pi^2} \right)^{-i/8} \left(\int_1^\infty e^{-t} t^{-5/4-i/4} dt + \sum_{k=0}^\infty -\frac{4(-1)^k}{(1+i-4k)k!} \right) \right. \\ & \quad \left. \left(\int_1^\infty e^{-t} t^{1/4(-5+i)} dt + \sum_{k=0}^\infty \frac{4(-1)^k}{(-1+i+4k)k!} \right) \right) \end{aligned}$$

$$\begin{aligned} & \left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \right. \\ & \quad \left. \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right) \right) - \frac{6}{10^3} + \frac{i}{10^3} = \\ & \frac{497}{500} + \frac{i}{1000} + \frac{1}{0.517927982770000 + \frac{0.988513975820000 \left(\frac{1}{\pi^2} \right)^{1/8+1/4(-1/2-i/2)} \pi^{5/4} \mathcal{A}^2}{\int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/2(1/2-i/2)} dt}} \end{aligned}$$

$$\begin{aligned} & \left(1 + 1 / \left(0.517927982770000 + \frac{1}{2} \left(\pi^{-3/4} \left(\frac{2}{\pi^2} \right)^{1/8-1/4(1/2+i/2)} \right) \Gamma \left(-\frac{1}{2} \left(\frac{1}{2} + \frac{i}{2} \right) \right) \right. \right. \\ & \quad \left. \left. \left(\Gamma \left(\frac{1}{2} \left(\left(\frac{1}{2} + \frac{i}{2} \right) - 1 \right) \right) 0.494256987910000 \right) \right) \right) - \frac{6}{10^3} + \frac{i}{10^3} = \\ & \frac{497}{500} + \frac{i}{1000} + \frac{1}{0.517927982770000 + \frac{0.988513975820000 \left(\frac{1}{\pi^2} \right)^{1/8+1/4(-1/2-i/2)} \pi^{5/4}}{\mathcal{A}^2 \int_L e^{-t} (-t)^{1/4+i/4} dt \int_L e^{-t} (-t)^{1/2(1/2-i/2)} dt}} \end{aligned}$$

Now, we have that:

$$\begin{aligned} & \int_0^\infty \left\{ e^{-z} - 4\pi \int_0^\infty \frac{x e^{-3z-\pi x^2 e^{-4z}}}{e^{2\pi x} - 1} dx \right\} \cos tz dz \\ & = \frac{1}{8\sqrt{\pi}} \Gamma \left(\frac{-1+it}{4} \right) \Gamma \left(\frac{-1-it}{4} \right) \Xi \left(\frac{1}{2} t \right). \end{aligned} \tag{12}$$

For $t = 1$ and $\Xi(1/2 t) = 0.49425698791$, we obtain:

$$1/(8\sqrt{\pi}) * \Gamma((-1+i)/4) * \Gamma((-1-i)/4) * 0.49425698791$$

Input interpretation:

$$\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

0.35938462381...

0.35938462381...

Alternative representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8\sqrt{\pi}} = \frac{0.494256987910000 \left(-1 + \frac{1}{4}(-1-i)\right)! \left(-1 + \frac{1}{4}(-1+i)\right)!}{8\sqrt{\pi}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8\sqrt{\pi}} = \frac{0.494256987910000 (1)_{-1+\frac{1}{4}(-1-i)} (1)_{-1+\frac{1}{4}(-1+i)}}{8\sqrt{\pi}}$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8\sqrt{\pi}} = \frac{0.494256987910000 e^{\log\Gamma(1/4(-1-i))} e^{\log\Gamma(1/4(-1+i))}}{8\sqrt{\pi}}$$

Series representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = \frac{0.0617821234887500 \Gamma\left(\frac{1}{4}(-1-i)\right)\Gamma\left(\frac{1}{4}(-1+i)\right)}{\exp\left(\pi \mathcal{A}\left[\frac{\operatorname{arctg}(\pi-x)}{2\pi}\right]\right)\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R} \text{ and } x < 0)$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = -\left(\left(0.9885139758200 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1)\Gamma^{(k_2)}(1)}{k_1!k_2!}\right) / \left((-1.0000000000000000 + 1.0000000000000000 i) (1.0000000000000000 + 1.0000000000000000 i) \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}\right)\right)$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = \frac{0.0617821234887500 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i)-z_0\right)^{k_1} \left(-\frac{1}{4}-\frac{i}{4}-z_0\right)^{k_2} \Gamma^{(k_1)}(z_0)\Gamma^{(k_2)}(z_0)}{k_1!k_2!}}{\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}}$$

for $(z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$

Integral representations:

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = \frac{1}{\sqrt{\pi}} 0.0617821234887500 \csc\left(\frac{1}{8}(-1+i)\pi\right) \csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt\right) \int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = \frac{1}{\sqrt{\pi}} 0.0617821234887500 \left(\int_0^{\infty} t^{-5/4-i/4} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!}\right) dt\right) \int_0^{\infty} t^{1/4(-5+i)} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!}\right) dt \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4}\right)$$

$$\frac{\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000}{8\sqrt{\pi}} = \frac{0.247128493955000\pi^2\mathcal{A}^2}{\sqrt{\pi}\int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/4-i/4} dt}$$

$\operatorname{csc}(x)$ is the cosecant function

From which, we obtain:

$$(2+\sqrt{7}) * (((1/(8\sqrt{\pi}) * \Gamma((-1+i)/4) * \Gamma((-1-i)/4) * 0.49425698791)) - (29-4)/10^3)$$

Input interpretation:

$$(2+\sqrt{7})\left(\frac{1}{8\sqrt{\pi}}\Gamma\left(\frac{1}{4}(-1+i)\right)\Gamma\left(\frac{1}{4}(-1-i)\right)\times 0.49425698791\right) - (29-4)\times\frac{1}{10^3}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.6446115872...

1.6446115872...

Alternative representations:

$$\frac{(2+\sqrt{7})\Gamma\left(\frac{1}{4}(-1+i)\right)\left(\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000\right)}{8\sqrt{\pi}} - \frac{29-4}{10^3} = -\frac{25}{10^3} + \frac{0.494256987910000\left(-1+\frac{1}{4}(-1-i)\right)!\left(-1+\frac{1}{4}(-1+i)\right)!(2+\sqrt{7})}{8\sqrt{\pi}}$$

$$\frac{(2+\sqrt{7})\Gamma\left(\frac{1}{4}(-1+i)\right)\left(\Gamma\left(\frac{1}{4}(-1-i)\right)0.494256987910000\right)}{8\sqrt{\pi}} - \frac{29-4}{10^3} = -\frac{25}{10^3} + \frac{0.494256987910000(1)_{-1+\frac{1}{4}(-1-i)}(1)_{-1+\frac{1}{4}(-1+i)}(2+\sqrt{7})}{8\sqrt{\pi}}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29 - 4}{10^3} =$$

$$-\frac{25}{10^3} + \frac{0.494256987910000 e^{\log \Gamma(1/4(-1-i))} e^{\log \Gamma(1/4(-1+i))} (2 + \sqrt{7})}{8 \sqrt{\pi}}$$

Series representations:

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29 - 4}{10^3} =$$

$$-\left(0.0250000000000000 \left(-4.9425698791000 \Gamma\left(\frac{1}{4}(-1 - i)\right) \Gamma\left(\frac{1}{4}(-1 + i)\right) -\right.\right.$$

$$2.47128493955000 \exp\left(\pi \mathcal{A}\left[\frac{\arg(7 - x)}{2\pi}\right]\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) \Gamma\left(\frac{1}{4}(-1 + i)\right)$$

$$\left.\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (7 - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + 1.0000000000000000\right.$$

$$\left.\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi - x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) /$$

$$\left(\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi - x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi - x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}\right) \text{ for } (x \in$$

\mathbb{R} and $x < 0$)

$$\begin{aligned}
& \frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29 - 4}{10^3} = \\
& - \left(\left(0.0250000000000000 \left(-1.000000000000000 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} \right) + \right. \right. \\
& \quad 1.000000000000000 i^2 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} + 79.08111806560 \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1 - k_2} (-1 - i)^{k_1} (-1 + i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} + \\
& \quad 39.540559032800 \sqrt{6} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \\
& \quad \left. \frac{2^{-k_1 - 2k_2 - 2k_3} \times 3^{-k_1} (-1 - i)^{k_2} (-1 + i)^{k_3} \binom{\frac{1}{2}}{k_1} \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \right) \\
& \left. \right) / \left((-1.000000000000000 + 1.000000000000000 i) \right. \\
& \left. \left. (1.000000000000000 + 1.000000000000000 i) \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} \right) \right)
\end{aligned}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29-4}{10^3} =$$

$$-\left(0.02500000000000000000\right)$$

$$\left(1.000000000000000000 \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} - 4.9425698791000\right)$$

$$\sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i)-z_0\right)^{k_1} \left(-\frac{1}{4}-\frac{i}{4}-z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} -$$

$$2.47128493955000 \sqrt{6} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} 6^{-k_1} \binom{\frac{1}{2}}{k_1}$$

$$\left(\frac{1}{4}(-1+i)-z_0\right)^{k_2} \left(-\frac{1}{4}-\frac{i}{4}-z_0\right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \Big/$$

$$\left(\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k}\right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29-4}{10^3} =$$

$$-\frac{1}{40} + \frac{1}{\sqrt{\pi}} 0.0617821234887500 \left(2.0000000000000000 + \sqrt{7}\right)$$

$$\left(\int_1^{\infty} e^{-t} t^{-5/4-i/4} dt - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+i-4k)k!}\right)$$

$$\left(\int_1^{\infty} e^{-t} t^{1/4(-5+i)} dt + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{(-1+i+4k)k!}\right)$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29-4}{10^3} =$$

$$\frac{1}{\sqrt{\pi}} 0.06178212348875 \left(2.0000000000000000 \csc\left(\frac{1}{8}(-1+i)\pi\right)\right)$$

$$\csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt\right) \int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt +$$

$$1.0000000000000000 \csc\left(\frac{1}{8}(-1+i)\pi\right) \csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt\right)$$

$$\left(\int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt\right) \sqrt{7} - 0.40464779435029 \sqrt{\pi}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{29 - 4}{10^3} =$$

$$-\frac{1}{40} + \frac{0.247128493955000 \pi^2 \mathcal{A}^2(2 + \sqrt{7})}{\sqrt{\pi} \int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/4-i/4} dt}$$

$$(2+\text{sqrt}7) * (((1/(8\text{sqrt}Pi) * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791)))-$$

$$(55-3)1/10^3$$

Input interpretation:

$$(2 + \sqrt{7}) \left(\frac{1}{8 \sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) \times 0.49425698791 \right) - (55 - 3) \times \frac{1}{10^3}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.6176115872...

[1.6176115872...](#)

Alternative representations:

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55 - 3}{10^3} =$$

$$-\frac{52}{10^3} + \frac{0.494256987910000 \left(-1 + \frac{1}{4}(-1 - i)\right)! \left(-1 + \frac{1}{4}(-1 + i)\right)! (2 + \sqrt{7})}{8 \sqrt{\pi}}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55 - 3}{10^3} =$$

$$-\frac{52}{10^3} + \frac{0.494256987910000 (1)_{-1+\frac{1}{4}(-1-i)} (1)_{-1+\frac{1}{4}(-1+i)} (2 + \sqrt{7})}{8 \sqrt{\pi}}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55 - 3}{10^3} =$$

$$-\frac{52}{10^3} + \frac{0.494256987910000 e^{\log \Gamma(1/4(-1-i))} e^{\log \Gamma(1/4(-1+i))} (2 + \sqrt{7})}{8 \sqrt{\pi}}$$

Series representations:

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55 - 3}{10^3} =$$

$$-\left(\left(0.0520000000000000 \left(-2.37623551879808 \Gamma\left(\frac{1}{4}(-1 - i)\right) \Gamma\left(\frac{1}{4}(-1 + i)\right) - \right. \right. \right.$$

$$1.18811775939904 \exp\left(\pi \mathcal{A}\left[\frac{\arg(7-x)}{2\pi}\right]\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) \Gamma\left(\frac{1}{4}(-1 + i)\right)$$

$$\sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (7-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} + 1.0000000000000000$$

$$\left. \left. \left. \exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right) /$$

$$\left(\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \text{ for } (x \in$$

\mathbb{R} and $x < 0$)

$$\begin{aligned}
& \frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55 - 3}{10^3} = \\
& - \left(\left(0.0520000000000000 \left[-1.0000000000000000 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} + \right. \right. \right. \\
& \quad 1.0000000000000000 i^2 \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} + 38.019768300769 \\
& \quad \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} + \\
& \quad 19.009884150385 \sqrt{6} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \\
& \quad \left. \left. \frac{2^{-k_1-2k_2-2k_3} \times 3^{-k_1} (-1-i)^{k_2} (-1+i)^{k_3} \binom{\frac{1}{2}}{k_1} \Gamma^{(k_2)}(1) \Gamma^{(k_3)}(1)}{k_2! k_3!} \right) \right) \\
& \left. \left. \left. \right) / \left(-1.0000000000000000 + 1.0000000000000000 i \right) \right) \right) \\
& \left(1.0000000000000000 + 1.0000000000000000 i \right) \sqrt{-1 + \pi} \sum_{k=0}^{\infty} (-1 + \pi)^{-k} \binom{\frac{1}{2}}{k} \left. \right) \left. \right)
\end{aligned}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55-3}{10^3} =$$

$$-\left(\left(0.0520000000000000 \right. \right.$$

$$\left. \left(1.0000000000000000 \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} - 2.37623551879808 \right. \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i)-z_0\right)^{k_1} \left(-\frac{1}{4}-\frac{i}{4}-z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} - \right.$$

$$1.18811775939904 \sqrt{6} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_2! k_3!} 6^{-k_1} \binom{\frac{1}{2}}{k_1}$$

$$\left. \left. \left. \left(\frac{1}{4}(-1+i)-z_0 \right)^{k_2} \left(-\frac{1}{4}-\frac{i}{4}-z_0 \right)^{k_3} \Gamma^{(k_2)}(z_0) \Gamma^{(k_3)}(z_0) \right) \right) \right)$$

$$\left(\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55-3}{10^3} =$$

$$-\frac{13}{250} + \frac{1}{\sqrt{\pi}} 0.0617821234887500 \left(2.0000000000000000 + \sqrt{7} \right)$$

$$\left(\int_1^{\infty} e^{-t} t^{-5/4-i/4} dt - 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{(1+i-4k)k!} \right)$$

$$\left(\int_1^{\infty} e^{-t} t^{1/4(-5+i)} dt + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{(-1+i+4k)k!} \right)$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - \frac{55-3}{10^3} =$$

$$\frac{1}{\sqrt{\pi}} 0.06178212348875 \left(2.0000000000000000 \csc\left(\frac{1}{8}(-1+i)\pi\right) \right)$$

$$\csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt \right) \int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt +$$

$$1.0000000000000000 \csc\left(\frac{1}{8}(-1+i)\pi\right) \csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt \right)$$

$$\left(\int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt \right) \sqrt{7} - 0.84166741224861 \sqrt{\pi}$$

$$\frac{(2 + \sqrt{7}) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8\sqrt{\pi}} - \frac{55 - 3}{10^3} =$$

$$-\frac{13}{250} + \frac{0.247128493955000 \pi^2 \mathcal{A}^2 (2 + \sqrt{7})}{\sqrt{\pi} \int_L \phi e^t t^{1/4+i/4} dt \int_L \phi e^t t^{1/4-i/4} dt}$$

$5 * 10^3(((1/(8\sqrt{\pi}) * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791)))-$
 $76+7+1/\text{golden ratio}+1/2$

Input interpretation:

$$5 \times 10^3 \left(\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) \times 0.49425698791 \right) - 76 + 7 + \frac{1}{\phi} + \frac{1}{2}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

ϕ is the golden ratio

Result:

1729.0411530...

[1729.0411530...](#)

Alternative representations:

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8\sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \frac{2.47128493955000 \left(-1 + \frac{1}{4}(-1 - i)\right)! \left(-1 + \frac{1}{4}(-1 + i)\right)! 10^3}{8\sqrt{\pi}}$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1 + i)\right) \Gamma\left(\frac{1}{4}(-1 - i)\right) 0.494256987910000}{8\sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \frac{2.47128493955000 (1)_{-1+\frac{1}{4}(-1-i)} (1)_{-1+\frac{1}{4}(-1+i)} 10^3}{8\sqrt{\pi}}$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \frac{2.47128493955000 \times 10^3 e^{\log \Gamma(1/4(-1-i))} e^{\log \Gamma(1/4(-1+i))}}{8 \sqrt{\pi}}$$

Series representations:

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \frac{308.910617443750 \Gamma\left(\frac{1}{4}(-1-i)\right) \Gamma\left(\frac{1}{4}(-1+i)\right)}{\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\left(\left(68.500000000000 \left(-0.0145985401459854 \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) + \right. \right.$$

$$1.00000000000000 \phi \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} -$$

$$4.5096440502737 \phi$$

$$\left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i) - z_0\right)^{k_1} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right) \right) /$$

$$\left(\phi \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \left(4942.56987910000 \left(\frac{1}{z_0}\right)^{-1/2 [\arg(\pi-z_0)/(2\pi)]} z_0^{1/2 (-1 - [\arg(\pi-z_0)/(2\pi)])} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} \right) /$$

$$\left((-1-i)(-1+i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi-z_0)^k z_0^{-k}}{k!} \right)$$

Integral representations:

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$\frac{1}{\phi \sqrt{\pi}} 308.91061744375 \left(1.000000000000000 \phi \csc\left(\frac{1}{8}(-1+i)\pi\right) \right.$$

$$\left. \csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^\infty t^{-5/4-i/4} \sin(t) dt \right) \int_0^\infty t^{1/4(-5+i)} \sin(t) dt + \right.$$

$$\left. 0.0032371823548023 \sqrt{\pi} - 0.22174699130396 \phi \sqrt{\pi} \right)$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{1}{\phi \sqrt{\pi}} 68.50000000000 \left(-4.509644050274 \phi \left(\int_0^\infty t^{-5/4-i/4} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \right.$$

$$\left. \int_0^\infty t^{1/4(-5+i)} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt - 0.014598540145985 \sqrt{\pi} + \right.$$

$$\left. 1.000000000000000 \phi \sqrt{\pi} \right) \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4} \right)$$

$$\frac{(5 \times 10^3) \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000}{8 \sqrt{\pi}} - 76 + 7 + \frac{1}{\phi} + \frac{1}{2} =$$

$$-\frac{137}{2} + \frac{1}{\phi} + \frac{1235.64246977500 \pi^2 \mathcal{A}^2}{\sqrt{\pi} \int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/4-i/4} dt}$$

$$1/3 * 10^3(((1/(8\sqrt{\pi}) * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791))) + 21 + 2 - \pi$$

Input interpretation:

$$\frac{1}{3} \times 10^3 \left(\frac{1}{8 \sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \right) + 21 + 2 - \pi$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

139.65328195...

139.65328195...

Alternative representations:

$$\begin{aligned}
 & \frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi = \\
 & 23 - \pi + \frac{0.164752329303333 \left(-1 + \frac{1}{4}(-1-i)\right)! \left(-1 + \frac{1}{4}(-1+i)\right)! 10^3}{8\sqrt{\pi}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi = \\
 & 23 - \pi + \frac{0.164752329303333 (1)_{-1+\frac{1}{4}(-1-i)} (1)_{-1+\frac{1}{4}(-1+i)} 10^3}{8\sqrt{\pi}}
 \end{aligned}$$

$$\begin{aligned}
 & \frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi = \\
 & 23 - \pi + \frac{0.164752329303333 \times 10^3 e^{\log\Gamma(1/4(-1-i))} e^{\log\Gamma(1/4(-1+i))}}{8\sqrt{\pi}}
 \end{aligned}$$

Series representations:

$$\begin{aligned}
 & \frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi = \\
 & 23 - \pi + \frac{20.5940411629167 \Gamma\left(\frac{1}{4}(-1-i)\right) \Gamma\left(\frac{1}{4}(-1+i)\right)}{\exp\left(\pi \mathcal{A}\left[\frac{\text{arg}(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)
 \end{aligned}$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi =$$

$$-\left(1.000000000000000 \right.$$

$$\left. \left(-23.000000000000000 \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} + 1.000000000000000 \right. \right.$$

$$\left. \left. \pi \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} - 20.5940411629167 \right. \right.$$

$$\left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i) - z_0\right)^{k_1} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right) \right)$$

$$\left(\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi =$$

$$23 - \pi + \left(329.504658606667 \left(\frac{1}{z_0}\right)^{-1/2 \lfloor \arg(\pi - z_0)/(2\pi) \rfloor} z_0^{1/2(-1 - \lfloor \arg(\pi - z_0)/(2\pi) \rfloor)} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1 - k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} \right)$$

$$\left((-1-i)(-1+i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi - z_0)^k z_0^{-k}}{k!} \right)$$

Integral representations:

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi =$$

$$-\frac{1}{\sqrt{\pi}} 1.000000000000000 \left(-20.5940411629167 \csc\left(\frac{1}{8}(-1+i)\pi\right) \right.$$

$$\left. \csc\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt \right) \int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt - \right.$$

$$\left. 23.000000000000000 \sqrt{\pi} + 1.000000000000000 \pi \sqrt{\pi} \right)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi =$$

$$-\frac{1}{\sqrt{\pi}} 1.000000000000000 \left(-20.594041162917 \left(\int_0^\infty t^{-5/4-i/4} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \right)$$

$$\int_0^\infty t^{1/4(-5+i)} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt - 23.0000000000000 \sqrt{\pi} +$$

$$1.000000000000000 \pi \sqrt{\pi} \left) \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4} \right)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 21 + 2 - \pi =$$

$$23 - \pi + \frac{82.3761646516667 \pi^2 \mathcal{A}^2}{\sqrt{\pi} \int_L e^t t^{1/4+i/4} dt \int_L e^t t^{1/4-i/4} dt}$$

$1/3 * 10^3(((1/(8\sqrt{\pi}) * \text{gamma}((-1+i)/4) * \text{gamma}((-1-i)/4) * 0.49425698791)))) + 7 - \text{golden ratio}$

Input interpretation:

$$\frac{1}{3} \times 10^3 \left(\frac{1}{8\sqrt{\pi}} \Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) \times 0.49425698791 \right) + 7 - \phi$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

ϕ is the golden ratio

Result:

125.17684061...

[125.17684061...](#)

Alternative representations:

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \frac{0.164752329303333 \left(-1 + \frac{1}{4}(-1-i)\right)! \left(-1 + \frac{1}{4}(-1+i)\right)! 10^3}{8\sqrt{\pi}}$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \frac{0.164752329303333 (1)_{-1+\frac{1}{4}(-1-i)} (1)_{-1+\frac{1}{4}(-1+i)} 10^3}{8\sqrt{\pi}}$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \frac{0.164752329303333 \times 10^3 e^{\log\Gamma(1/4(-1-i))} e^{\log\Gamma(1/4(-1+i))}}{8\sqrt{\pi}}$$

Series representations:

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \frac{20.5940411629167 \Gamma\left(\frac{1}{4}(-1-i)\right) \Gamma\left(\frac{1}{4}(-1+i)\right)}{\exp\left(\pi \mathcal{A}\left[\frac{\arg(\pi-x)}{2\pi}\right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (\pi-x)^k x^{-k} \binom{-\frac{1}{2}}{k}}{k!}} \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$-\left(\left(1.000000000000000 \left(-7.000000000000000 \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) + \right. \right.$$

$$1.000000000000000 \phi \sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} -$$

$$20.5940411629167$$

$$\left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{\left(\frac{1}{4}(-1+i) - z_0\right)^{k_1} \left(-\frac{1}{4} - \frac{i}{4} - z_0\right)^{k_2} \Gamma^{(k_1)}(z_0) \Gamma^{(k_2)}(z_0)}{k_1! k_2!} \right) \right) /$$

$$\left(\sqrt{-1+\pi} \sum_{k=0}^{\infty} (-1+\pi)^{-k} \binom{\frac{1}{2}}{k} \right) \text{ for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \left(329.504658606667 \left(\frac{1}{z_0} \right)^{-1/2 \lfloor \arg(\pi - z_0) / (2\pi) \rfloor} z_0^{1/2(-1 - \lfloor \arg(\pi - z_0) / (2\pi) \rfloor)} \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{4^{-k_1-k_2} (-1-i)^{k_1} (-1+i)^{k_2} \Gamma^{(k_1)}(1) \Gamma^{(k_2)}(1)}{k_1! k_2!} \right) /$$

$$\left((-1-i)(-1+i) \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (\pi - z_0)^k z_0^{-k}}{k!} \right)$$

Integral representations:

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$-\frac{1}{\sqrt{\pi}} 1.0000000000000000 \left(-20.5940411629167 \operatorname{csc}\left(\frac{1}{8}(-1+i)\pi\right) \right.$$

$$\left. \operatorname{csc}\left(-\frac{1}{8}(1+i)\pi\right) \left(\int_0^{\infty} t^{-5/4-i/4} \sin(t) dt \right) \int_0^{\infty} t^{1/4(-5+i)} \sin(t) dt - \right.$$

$$\left. 7.0000000000000000 \sqrt{\pi} + 1.0000000000000000 \phi \sqrt{\pi} \right)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$-\frac{1}{\sqrt{\pi}} 1.0000000000000000 \left(-20.594041162917 \left(\int_0^{\infty} t^{-5/4-i/4} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt \right) \right.$$

$$\left. \int_0^{\infty} t^{1/4(-5+i)} \left(e^{-t} - \sum_{k=0}^n \frac{(-t)^k}{k!} \right) dt - 7.0000000000000000 \sqrt{\pi} + \right.$$

$$\left. 1.0000000000000000 \phi \sqrt{\pi} \right) \text{ for } \left(n \in \mathbb{Z} \text{ and } 0 \leq n < \frac{1}{4} \right)$$

$$\frac{10^3 \left(\Gamma\left(\frac{1}{4}(-1+i)\right) \Gamma\left(\frac{1}{4}(-1-i)\right) 0.494256987910000 \right)}{(8\sqrt{\pi})^3} + 7 - \phi =$$

$$7 - \phi + \frac{82.3761646516667 \pi^2 \mathcal{A}^2}{\sqrt{\pi} \oint_L e^t t^{1/4+i/4} dt \oint_L e^t t^{1/4-i/4} dt}$$

Observations

From:

https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJIQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that $p(9) = 30$, $p(9 + 5) = 135$, $p(9 + 10) = 490$, $p(9 + 15) = 1,575$ and so on are all divisible by 5. Note that here the n 's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of $p(n)$ that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n 's separated by $5^3 = 125$ units, saying that the corresponding $p(n)$'s should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for $T = 0$ and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and $4096 = 64^2$

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π , ϕ , $1/\phi$, the Fibonacci and Lucas numbers, linked to the

golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.

In mathematics, the Fibonacci numbers, commonly denoted F_n , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the n th Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.

Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.^[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the

second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ , the golden ratio.^[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies^[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **$f_0(1710)$ scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson π) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

References

New expressions for Riemann's functions $\xi(s)$ and $\Xi(t)$ – Srinivasa Ramanujan
Quarterly Journal of Mathematics, XLVI, 1915, 253 – 260