

**On some Ramanujan equations: mathematical connections with Prime Number Theorem,  $\phi$ ,  $\zeta(2)$  and various parameters of Particle Physics.**

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**Abstract**

*In this paper we have described and analyzed some Ramanujan equations. We have obtained several mathematical connections between Prime Number Theorem,  $\phi$ ,  $\zeta(2)$  and various parameters of Particle Physics.*

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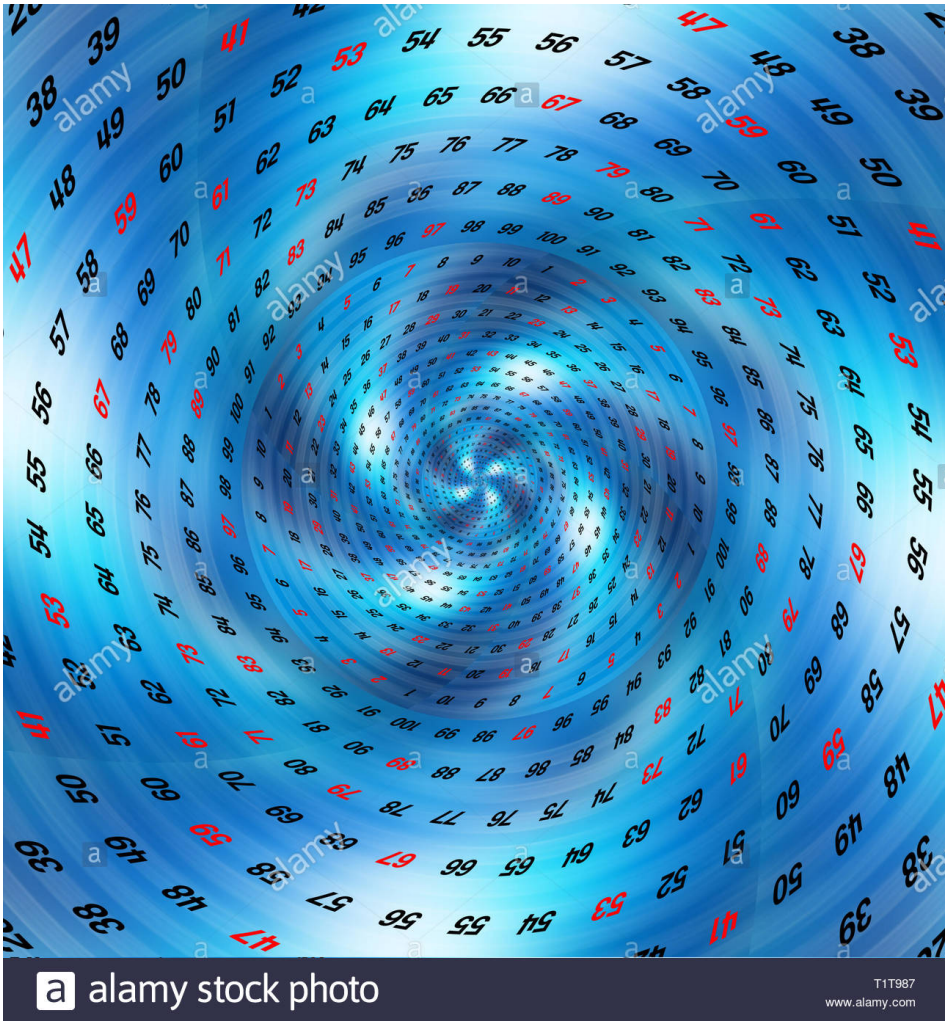
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*An equation means nothing  
to me unless it expresses a  
thought of God.*

*Srinivasa Ramanujan (1887-1920)*

<https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012>



**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

From

**On certain trigonometrical sums and their applications in the theory of numbers – Srinivasa Ramanujan**

Transactions of the Cambridge Philosophical Society, XXII, No.13, 1918, 259 – 276

We have that:

$$s_r(n) = \sum_{\lambda} (-1)^{\frac{1}{2}(\lambda-1)} \sin \frac{2\pi n \lambda}{r},$$

For  $n = 2$ ,  $\lambda = y = 3$  and  $r = 24$ , we have:

Sum  $((-1)^{0.5(y-1)} \sin((2\pi \cdot 2 \cdot y)/24))$ ,  $y = 3..infinity$

**Result:**

$$\sum_{y=3}^{\infty} (-1)^{0.5(y-1)} \sin\left(\frac{2\pi \cdot 2 \cdot y}{24}\right) \text{ (sum does not converge)}$$

**Regularized results:**

**Abel regularization**

$$\lim_{x \rightarrow 1^-} \left( \sum_{y=0}^{\infty} (-1)^{0.5(y+2)} x^y \sin\left(\frac{1}{6} \pi (y + 3)\right) \right) = -0.5 - 0.57735 i$$

**Borel regularization**

$$\lim_{s \rightarrow 1} \int_0^{\infty} e^{-st} \left( \sum_{y=0}^{\infty} \frac{(-1)^{0.5(y+2)} t^y \sin\left(\frac{1}{6} \pi (y + 3)\right)}{y!} \right) dt = -0.5 - 0.57735 i$$

-0.5 - 0.57735 i

**Input:**

-0.5 - 0.57735 i

*i* is the imaginary unit

**Result:**

$$-0.5 - 0.57735 i$$

**Polar coordinates:**

$$r = 0.763762 \text{ (radius), } \theta = -130.893^\circ \text{ (angle)}$$

$$0.763762$$

From which, we obtain:

$$1 + \frac{1}{2} \cdot \frac{1}{(0.763762)}$$

**Input interpretation:**

$$1 + \frac{1}{2} \times \frac{1}{0.763762}$$

**Result:**

$$1.654654198559237039810831122784322864976262238760242064936\dots$$

1.6546541985... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

For  $\lambda = y = 13$ , we have:

$$\text{Sum } (((-1)^{(0.5 \cdot (y-1))} \sin((2\pi \cdot 2 \cdot y)/24))), y = 13..infinity$$

**Result:**

$$\sum_{y=13}^{\infty} (-1)^{0.5(y-1)} \sin\left(\frac{2\pi \cdot 2 \cdot y}{24}\right) \text{ (sum does not converge)}$$

**Regularized results:****Abel regularization**

$$\lim_{x \rightarrow 1^-} \left( \sum_{y=0}^{\infty} (-1)^{0.5(y+12)} x^y \sin\left(\frac{1}{6} \pi (y + 13)\right) \right) = 0.288675 i$$

### Borel regularization

$$\lim_{s \rightarrow 1} \int_0^{\infty} e^{-st} \left( \sum_{y=0}^{\infty} \frac{(-1)^{0.5(y+12)} t^y \sin\left(\frac{1}{6} \pi (y + 13)\right)}{y!} \right) dt = -1.38778 \times 10^{-17} + 0.288675 i$$

We note that:

$$(6 \cdot 0.288675 i)^{12}$$

#### Input interpretation:

$$(6 \times 0.288675 i)^{12}$$

$i$  is the imaginary unit

#### Result:

728.9959212545085857714019266948823195987921623524046096191...

$$728.99592125... \approx 729$$

Thence:

$$10^3 + (6 \cdot 0.288675 i)^{12}$$

#### Input interpretation:

$$10^3 + (6 \times 0.288675 i)^{12}$$

$i$  is the imaginary unit

#### Result:

1728.995921254508585771401926694882319598792162352404609619...

$$1728.99592125... \approx 1729$$

$$(((10^3 + (6 \cdot 0.288675 i)^{12})))^{1/15}$$

#### Input interpretation:

$$\sqrt[15]{10^3 + (6 \times 0.288675 i)^{12}}$$

$i$  is the imaginary unit

#### Result:

1.643814970228915752108776968407202138665028725006260553257...

$$1.6438149702...$$

Now, for  $s = 16$  and  $n = 2$ , from:

if  $s$  is a multiple of 4;

$$(14.7) \quad (1^{-s} + 3^{-s} + 5^{-s} + \dots) \delta'_{2s}(n) = \frac{(\frac{1}{2}\pi)^s}{(s-1)!} (n + \frac{1}{4}s)^{s-1} \left\{ 1^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin(2n + \frac{1}{2}s)\pi} \right) + 3^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{3}(2n + \frac{1}{2}s)\pi} \right) + 5^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{5}(2n + \frac{1}{2}s)\pi} \right) + \dots \right\}$$

From

$$(1^{-s} + 3^{-s} + 5^{-s} + \dots) \delta'_{2s}(n) = \frac{(\frac{1}{2}\pi)^s}{(s-1)!} (n + \frac{1}{4}s)^{s-1} \left\{ 1^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin(2n + \frac{1}{2}s)\pi} \right) + 3^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{3}(2n + \frac{1}{2}s)\pi} \right) + 5^{-s} \left( \frac{\sin(2n + \frac{1}{2}s)\pi}{\sin \frac{1}{5}(2n + \frac{1}{2}s)\pi} \right) + \dots \right\}$$

$$\left( \left( \frac{1}{2}\pi \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) / (16-1)! * \left[ 1^{-16} * \left( \frac{\sin(4+16/2)*\pi}{\sin(4+16/2)*\pi} \right) + 3^{-16} * \left( \frac{\sin(4+16/2)*\pi}{\sin(1/3(4+16/2)\pi)} \right) + 5^{-16} * \left( \frac{\sin(4+16/2)*\pi}{\sin(1/5(4+16/2)*\pi)} \right) \right]$$

**Input:**

$$\frac{\left( \frac{1}{2}\pi \right)^{16} \left( 2 + \frac{16}{4} \right)^{15}}{(16-1)!} \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\sin\left(4 + \frac{16}{2}\right)\pi} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{3^{16}} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{5^{16}} \right)$$

$n!$  is the factorial function

**Exact result:**

$$\frac{19\,683\,\pi^{16} \left( 1 + \frac{7\,632\,981\,758 \sin(12) \csc(1)}{1\,313\,681\,671\,142\,578\,125} \right)}{3\,587\,584\,000}$$

**Decimal approximation:**

493.9547607207878323885481388713700015614845096394662717329...

493.9547607... result practically equal to the rest mass of Kaon meson 493.677

**Alternate forms:**

$$\frac{\pi^{16} (1\ 313\ 681\ 671\ 142\ 578\ 125 + 7\ 632\ 981\ 758 \sin(12) \operatorname{csc}(1))}{239\ 442\ 328\ 125\ 000\ 000\ 000\ 000}$$

$$\pi^{16} \left( \frac{19\ 683}{3\ 587\ 584\ 000} + \frac{3\ 816\ 490\ 879 \sin(12) \operatorname{csc}(1)}{119\ 721\ 164\ 062\ 500\ 000\ 000\ 000} \right)$$

$$\frac{19\ 683 \pi^{16}}{3\ 587\ 584\ 000} + \frac{3\ 816\ 490\ 879 \pi^{16} \sin(12) \operatorname{csc}(1)}{119\ 721\ 164\ 062\ 500\ 000\ 000\ 000}$$

**Alternative representations:**

$$\frac{\left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \left(\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}\right)}{(16-1)!} =$$

$$\frac{\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15} \left( \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} \right)}{(1)_{15}}$$

$$\frac{\left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \left(\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}\right)}{(16-1)!} =$$

$$\frac{\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15} \left( \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} \right)}{14!! \times 15!!}$$

$$\frac{\left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \left(\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}\right)}{(16-1)!} =$$

$$\frac{\left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15} \left( \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12 + \frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1 + \frac{\pi}{2}\right)\right)} \right)}{e^{\log\Gamma(16)}}$$



### Series representations:

$$\frac{\left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}}{(16-1)!} =$$

$$\left( \pi^{16} \left( 1313681671142578125 - \right. \right.$$

$$\left. \left. 15265963516 i \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} 12^{1+2k_2} q^{-1+2k_1}}{(1+2k_2)!} \right) \right) /$$

239442328125000000000000 for  $q = e^i$

$$\frac{\left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}}{(16-1)!} =$$

$$\frac{\pi^{16} \left( 1313681671142578125 + 7632981758 \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 12^{1+2k_2}}{(1+2k_2)! (1-\pi^2 k_1^2)} \right)}{239442328125000000000000}$$

$$\frac{\left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}}{(16-1)!} =$$

$$\left( \pi^{16} \left( 1313681671142578125 - \right. \right.$$

$$\left. \left. 15265963516 i \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} \left(12 - \frac{\pi}{2}\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} \right) \right) /$$

239442328125000000000000 for  $q = e^i$

### Integral representations:

$$\frac{\left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \left(\frac{\pi}{2}\right)^{16} \left(2 + \frac{16}{4}\right)^{15}}{(16-1)!} =$$

$$\frac{\pi^{15} \left( 437893890380859375 \pi + 30531927032 \left( \int_0^{\infty} \frac{\sqrt{t}}{t+t^2} dt \right) \int_0^1 \cos(12t) dt \right)}{79814109375000000000000}$$

$$\frac{\left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16}\left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3}\times 3^{16}\left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5}\times 5^{16}\left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)}\right)\left(\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right)}{(16-1)!} =$$

$$\left(\pi^{29/2}\left(437893890380859375\pi^{3/2}-7632981758i\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-36/s+s}}{s^{3/2}}ds\right)\int_0^\infty\frac{\sqrt{t}}{t+t^2}dt\right)\right)/798141093750000000000000000000 \text{ for } \gamma > 0$$

$$\frac{\left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16}\left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3}\times 3^{16}\left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5}\times 5^{16}\left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)}\right)\left(\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right)}{(16-1)!} =$$

$$\left(\pi^{29/2}\left(1313681671142578125\pi^{3/2}-3816490879i\left(\int_0^\infty\frac{\sqrt{t}}{t+t^2}dt\right)\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{6^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds\right)\right)/239442328125000000000000000000 \text{ for } 0 < \gamma < 1$$

$$\left(\left(\frac{1}{2}\pi\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right) / (16-1)! \left[1^{-16}\left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{\sin\left(4+\frac{16}{2}\right)\pi}\right) + 3^{-16}\left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{\sin\left(\frac{1}{3}\left(4+\frac{16}{2}\right)\pi\right)}\right) + 5^{-16}\left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{\sin\left(\frac{1}{5}\left(4+\frac{16}{2}\right)\pi\right)}\right)\right]\pi + 233 - 55 - \frac{4}{5}$$

**Input:**

$$\frac{\left(\frac{1}{2}\pi\right)^{16}\left(2+\frac{16}{4}\right)^{15}}{(16-1)!} \left(\frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16}} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{3^{16}} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{5^{16}}\right)\pi + 233 - 55 - \frac{4}{5}$$

$n!$  is the factorial function

**Exact result:**

$$\frac{886}{5} + \frac{19683\pi^{17}\left(1 + \frac{7632981758\sin(12)\csc(1)}{1313681671142578125}\right)}{3587584000}$$

$\csc(x)$  is the cosecant function

**Decimal approximation:**

1729.004647486131216583211845677445371379835004793972005662...

[1729.004647486...](#)

**Alternate forms:**

$$\frac{886}{5} + \frac{19\,683\pi^{17}}{3\,587\,584\,000} + \frac{3\,816\,490\,879\pi^{17}\sin(12)\csc(1)}{119\,721\,164\,062\,500\,000\,000\,000}$$

$$(42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125\pi^{17} + 7\,632\,981\,758\pi^{17}\sin(12)\csc(1))/239\,442\,328\,125\,000\,000\,000\,000$$

$$\frac{635\,719\,884\,800 + 19\,683\pi^{17}}{3\,587\,584\,000} + \frac{3\,816\,490\,879\pi^{17}\sin(12)\csc(1)}{119\,721\,164\,062\,500\,000\,000\,000}$$

**Alternative representations:**

$$\frac{\left(\frac{\sin(4+\frac{16}{2})\pi}{1^{16}(\sin(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{3}\times 3^{16}(\sin(1)(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{5}\times 5^{16}(\sin(1)(4+\frac{16}{2})\pi)}\right)\pi\left(\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$178 - \frac{4}{5} + \frac{\pi\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\left(\frac{\pi\cos(-12+\frac{\pi}{2})}{1^{16}(\pi\cos(-12+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{3^{16}(4\pi\cos(-1+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{\frac{1}{5}\times 5^{16}(12\pi\cos(-1+\frac{\pi}{2}))}\right)}{(1)_{15}}$$

$$\frac{\left(\frac{\sin(4+\frac{16}{2})\pi}{1^{16}(\sin(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{3}\times 3^{16}(\sin(1)(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{5}\times 5^{16}(\sin(1)(4+\frac{16}{2})\pi)}\right)\pi\left(\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$178 - \frac{4}{5} + \frac{\pi\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\left(\frac{\pi\cos(-12+\frac{\pi}{2})}{1^{16}(\pi\cos(-12+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{3^{16}(4\pi\cos(-1+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{\frac{1}{5}\times 5^{16}(12\pi\cos(-1+\frac{\pi}{2}))}\right)}{14!!\times 15!!}$$

$$\frac{\left(\frac{\sin(4+\frac{16}{2})\pi}{1^{16}(\sin(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{3}\times 3^{16}(\sin(1)(4+\frac{16}{2})\pi)} + \frac{\sin(4+\frac{16}{2})\pi}{\frac{1}{5}\times 5^{16}(\sin(1)(4+\frac{16}{2})\pi)}\right)\pi\left(\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$178 - \frac{4}{5} + \frac{\pi\left(\frac{\pi}{2}\right)^{16}\left(2+\frac{16}{4}\right)^{15}\left(\frac{\pi\cos(-12+\frac{\pi}{2})}{1^{16}(\pi\cos(-12+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{3^{16}(4\pi\cos(-1+\frac{\pi}{2}))} + \frac{\pi\cos(-12+\frac{\pi}{2})}{\frac{1}{5}\times 5^{16}(12\pi\cos(-1+\frac{\pi}{2}))}\right)}{e^{\log\Gamma(16)}}$$

### Series representations:

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125\,\pi^{17} -$$

$$15\,265\,963\,516\,i\pi^{17} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} 12^{1+2k_2} q^{-1+2k_1}}{(1+2k_2)!} \right) /$$

$$239\,442\,328\,125\,000\,000\,000\,000 \text{ for } q = e^i$$

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125\,\pi^{17} +$$

$$7\,632\,981\,758\,\pi^{17} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 12^{1+2k_2}}{(1+2k_2)! (1-\pi^2 k_1^2)} \right) /$$

$$239\,442\,328\,125\,000\,000\,000\,000$$

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125\,\pi^{17} -$$

$$15\,265\,963\,516\,i\pi^{17} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} \left(12 - \frac{\pi}{2}\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} \right) /$$

$$239\,442\,328\,125\,000\,000\,000\,000 \text{ for } q = e^i$$

## Integral representations:

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 14\,143\,060\,181\,250\,000\,000\,000\,000 + 437\,893\,890\,380\,859\,375 \pi^{17} + \right.$$

$$\left. 30\,531\,927\,032 \pi^{16} \left( \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) \int_0^1 \cos(12t) dt \right) /$$

$$79\,814\,109\,375\,000\,000\,000\,000$$

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 14\,143\,060\,181\,250\,000\,000\,000\,000 + 437\,893\,890\,380\,859\,375 \pi^{17} - \right.$$

$$\left. 7\,632\,981\,758 i \pi^{31/2} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-36/s+s}}{s^{3/2}} ds \right) \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) /$$

$$79\,814\,109\,375\,000\,000\,000\,000 \text{ for } \gamma > 0$$

$$\frac{\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} +$$

$$233 - 55 - \frac{4}{5} =$$

$$\left( 42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125 \pi^{17} - \right.$$

$$\left. 3\,816\,490\,879 i \pi^{31/2} \left( \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds \right) /$$

$$239\,442\,328\,125\,000\,000\,000\,000 \text{ for } 0 < \gamma < 1$$

$$\begin{aligned} & (((((((((1/2*\pi)^{16} (2+16/4)^{15})) / (16-1)! [1^{-16} \\ & (((\sin(4+16/2)*\pi)/(\sin(4+16/2)*\pi))) + 3^{-16} \\ & (\sin(4+16/2)*\pi)/(\sin 1/3(4+16/2)\pi)+5^{-16} \\ & 16(\sin(4+16/2)*\pi)/(\sin 1/5(4+16/2)*\pi)]*\pi+233-55-4/5))))^{1/15} \end{aligned}$$

**Input:**

$$\sqrt[15]{\frac{\left(\frac{1}{2}\pi\right)^{16} \left(2 + \frac{16}{4}\right)^{15}}{(16-1)!} \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16}} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{3^{16}} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{5^{16}} \right) \pi + 233 - 55 - \frac{4}{5}}$$

$n!$  is the factorial function

**Exact result:**

$$\sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \operatorname{csc}(1)}{131368167142578125}\right)}{3587584000}}$$

$\operatorname{csc}(x)$  is the cosecant function

**Decimal approximation:**

1.643815523315872719740459558951017206182900655629199501415...

1.6438155233...

**Alternate forms:**

$$\sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 - \frac{15265963516 \sin(1) \sin(12)}{131368167142578125 (\cos(2)-1)}\right)}{3587584000}}$$

$$\sqrt[15]{\frac{886}{5} + \frac{19683 \left(1 + \frac{7632981758 (e^{-12i} - e^{12i})}{131368167142578125 (e^{-i} - e^i)}\right) \pi^{17}}{3587584000}}$$

$$\left( \left( \frac{1}{143} (424291805437500000000000 + 131368167142578125 \pi^{17} + 7632981758 \pi^{17} \sin(12) \operatorname{csc}(1)) \right)^{(1/15)} \right) / \left( 5 \times 2^{4/5} \times 3^{7/15} \sqrt[5]{5} 7^{2/15} \right)$$

**All 15th roots of  $886/5 + (19683 \pi^{17} (1 + (7632981758 \sin(12) \csc(1))/1313681671142578125))/3587584000$ :**

$$e^{0/15} \sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \csc(1)}{1313681671142578125}\right)}{3587584000}} \approx 1.6438 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15} \sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \csc(1)}{1313681671142578125}\right)}{3587584000}} \approx 1.5017 + 0.6686 i$$

$$e^{(4i\pi)/15} \sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \csc(1)}{1313681671142578125}\right)}{3587584000}} \approx 1.0999 + 1.2216 i$$

$$e^{(6i\pi)/15} \sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \csc(1)}{1313681671142578125}\right)}{3587584000}} \approx 0.5080 + 1.5634 i$$

$$e^{(8i\pi)/15} \sqrt[15]{\frac{886}{5} + \frac{19683 \pi^{17} \left(1 + \frac{7632981758 \sin(12) \csc(1)}{1313681671142578125}\right)}{3587584000}} \approx -0.17183 + 1.6348 i$$

**Addition formulas:**

$$z^{a_1+a_2} = z^{a_1} z^{a_2}$$

$$z^{a_1+a_2+\dots+a_m} = \prod_{k=1}^m z^{a_k}$$

$$(z_1 + z_2)^n = \sum_{k=0}^n \binom{n}{k} z_1^k z_2^{n-k} \text{ for } (n \in \mathbb{Z} \text{ and } n > 0)$$

## Alternative representations:

$$\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) \right. \\ \left. \frac{(16-1)!}{233 - 55 - \frac{4}{5}} \right)^{\wedge} (1/15) = \left( 178 - \frac{4}{5} + \right. \\ \left. \frac{\pi \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \left( \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} \right)}{(1)_{15}} \right)^{\wedge} \\ (1/15)$$

$$\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) \right. \\ \left. \frac{(16-1)!}{233 - 55 - \frac{4}{5}} \right)^{\wedge} (1/15) = \left( 178 - \frac{4}{5} + \right. \\ \left. \frac{\pi \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \left( \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} \right)}{14!! \times 15!!} \right)^{\wedge} \\ (1/15)$$

$$\left( \left( \frac{\sin\left(4+\frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4+\frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4+\frac{16}{2}\right)\pi\right)} \right) \pi \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) \right. \\ \left. \frac{(16-1)!}{233 - 55 - \frac{4}{5}} \right)^{\wedge} (1/15) = \left( 178 - \frac{4}{5} + \right. \\ \left. \frac{\pi \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \left( \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{1^{16} \left(\pi \cos\left(-12+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{3^{16} \left(4 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} + \frac{\pi \cos\left(-12+\frac{\pi}{2}\right)}{\frac{1}{5} \times 5^{16} \left(12 \pi \cos\left(-1+\frac{\pi}{2}\right)\right)} \right)}{e^{\log\Gamma(16)}} \right)^{\wedge} \\ (1/15)$$



## Series representations:

$$\left( \frac{1}{(16-1)!} \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) + 233 - 55 - \frac{4}{5} \Big)^{\wedge (1/15)} =$$

$$\left( \left( 424291805437500000000000000 + 1313681671142578125\pi^{17} - 15265963516i\pi^{17} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} 12^{1+2k_2} q^{-1+2k_1}}{(1+2k_2)!} \right)^{\wedge (1/15)} \right) / \left( 5 \times 2^{4/5} \times 3^{7/15} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right) \text{ for } q = e^i$$

$$\left( \frac{\left( \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} + 233 - 55 - \frac{4}{5} \right)^{\wedge (1/15)} =$$

$$\left( \left( 424291805437500000000000000 + 1313681671142578125\pi^{17} + 7632981758\pi^{17} \sum_{k_1=-\infty}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} 12^{1+2k_2}}{(1+2k_2)!(1-\pi^2 k_1^2)} \right)^{\wedge (1/15)} \right) / \left( 5 \times 2^{4/5} \times 3^{7/15} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right)$$

$$\left( \frac{1}{(16-1)!} \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) + 233 - 55 - \frac{4}{5} \Big)^{\wedge (1/15)} =$$

$$\left( \left( 424291805437500000000000000 + 1313681671142578125\pi^{17} - 15265963516i\pi^{17} \sum_{k_1=1}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_2} \left(12 - \frac{\pi}{2}\right)^{2k_2} q^{-1+2k_1}}{(2k_2)!} \right)^{\wedge (1/15)} \right) / \left( 5 \times 2^{4/5} \times 3^{7/15} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right) \text{ for } q = e^i$$

## Integral representations:

$$\left( \frac{\left( \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right)}{(16-1)!} + \right. \\ \left. 233 - 55 - \frac{4}{5} \right)^{\wedge (1/15)} =$$

$$\left( \left( 14\,143\,060\,181\,250\,000\,000\,000\,000 + 437\,893\,890\,380\,859\,375 \pi^{17} + \right. \right. \\ \left. \left. 30531927032 \pi^{16} \left( \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) \int_0^1 \cos(12t) dt \right) \right)^{\wedge (1/15)} / \left( 5 \times 2^{4/5} \times 3^{2/5} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right)$$

$$\left( \frac{1}{(16-1)!} \left( \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \right. \\ \left. \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) + 233 - 55 - \frac{4}{5} \right)^{\wedge (1/15)} =$$

$$\left( \left( 14\,143\,060\,181\,250\,000\,000\,000\,000 + 437\,893\,890\,380\,859\,375 \pi^{17} - \right. \right. \\ \left. \left. 7632981758 i \pi^{31/2} \left( \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-36/s+s}}{s^{3/2}} ds \right) \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) \right)^{\wedge (1/15)} / \\ \left( 5 \times 2^{4/5} \times 3^{2/5} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right) \text{ for } \gamma > 0$$

$$\left( \frac{1}{(16-1)!} \left( \left( \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{1^{16} \left(\sin\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{3} \times 3^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} + \frac{\sin\left(4 + \frac{16}{2}\right)\pi}{\frac{1}{5} \times 5^{16} \left(\sin(1)\left(4 + \frac{16}{2}\right)\pi\right)} \right) \pi \right) \right. \\ \left. \left( \left( \frac{\pi}{2} \right)^{16} \left( 2 + \frac{16}{4} \right)^{15} \right) + 233 - 55 - \frac{4}{5} \right)^{\wedge (1/15)} =$$

$$\left( \left( 42\,429\,180\,543\,750\,000\,000\,000\,000 + 1\,313\,681\,671\,142\,578\,125 \pi^{17} - \right. \right. \\ \left. \left. 3816490879 i \pi^{31/2} \left( \int_0^\infty \frac{\sqrt[3]{t}}{t+t^2} dt \right) \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{6^{1-2s} \Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)} ds \right) \right)^{\wedge (1/15)} / \\ \left( 5 \times 2^{4/5} \times 3^{7/15} \sqrt[5]{5} 7^{2/15} \sqrt[15]{143} \right) \text{ for } 0 < \gamma < 1$$

We have that:

It follows that

$$(17.5) \quad \begin{aligned} \sigma_s(1) + \sigma_s(2) + \cdots + \sigma_s(n) &= n^s \{ \sigma_{-s}(1) + \sigma_{-s}(2) + \cdots + \sigma_{-s}(n) \} \\ &= \frac{sn^{1+s}}{1+s} \zeta(1+s) + \frac{1}{2} n^s \zeta(s) - \frac{sn}{1-s} \zeta(1-s) + O(m) \end{aligned}$$

if  $s > 0$ ,  $m$  being the same as in (17.4). If  $s = 1$ , (17.5) reduces to

$$(17.6) \quad \begin{aligned} (n-1)\sigma_{-1}(1) + (n-2)\sigma_{-1}(2) + \cdots + (n-n)\sigma_{-1}(n) \\ = \frac{\pi^2}{12} n^2 - \frac{1}{2} n (\gamma - 1 + \log 2n\pi) + O(\sqrt{n})^* \end{aligned}$$

From

$$\begin{aligned} (n-1)\sigma_{-1}(1) + (n-2)\sigma_{-1}(2) + \cdots + (n-n)\sigma_{-1}(n) \\ = \frac{\pi^2}{12} n^2 - \frac{1}{2} n (\gamma - 1 + \log 2n\pi) + O(\sqrt{n})^* \end{aligned}$$

We obtain, for  $n = 2$ :

$$(4\pi^2)/12 - 1/2 * 2(\text{euler constant} - 1 + \ln(4\pi)) + (\text{sqrt}(2))$$

**Input:**

$$\frac{1}{12} (4\pi^2) - \frac{1}{2} \times 2 (\gamma - 1 + \log(4\pi)) + \sqrt{2}$$

$\log(x)$  is the natural logarithm

$\gamma$  is the Euler-Mascheroni constant

**Exact result:**

$$1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)$$

**Decimal approximation:**

2.595841784198724268162115373149934178167117260214799270087...

**2.595841784...**

**Alternate forms:**

$$\frac{1}{3} \left( 3 + 3\sqrt{2} - 3\gamma + \pi^2 - 3 \log(4\pi) \right)$$

$$\frac{1}{3} \left( -3\gamma + \pi^2 - 3 \left( -1 - \sqrt{2} + 2 \log(2) \right) - 3 \log(\pi) \right)$$

$$1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \log\left(\frac{1}{4\pi}\right)$$

**Alternative representations:**

$$\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}$$

$$\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}$$

$$\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 - \gamma + \text{Li}_1(1 - 4\pi) + \frac{4\pi^2}{12} + \sqrt{2}$$

**Series representations:**

$$\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-4\pi}\right)^k}{k}$$

$$\begin{aligned} \frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = \\ 1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - 2i\pi \left[ \frac{\arg(4\pi - x)}{2\pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi - x)^k x^{-k}}{k} \quad \text{for } x < 0 \end{aligned}$$

$$\begin{aligned} \frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = \\ 1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - 2i\pi \left[ \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right] - \log(z_0) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi - z_0)^k z_0^{-k}}{k} \end{aligned}$$

**Integral representations:**

$$\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \int_1^{4\pi} \frac{1}{t} dt$$

$$\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2} = 1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \quad \text{for } -1 < \gamma < 0$$

$$\left(\left(\left(\left(4\pi^2\right)/12 - 1/2 * 2(\text{euler constant} - 1 + \ln(4\pi)) + (\text{sqrt}(2))\right)\right)\right)^{1/2}$$

**Input:**

$$\sqrt{\frac{1}{12}(4\pi^2) - \frac{1}{2} \times 2(\gamma - 1 + \log(4\pi)) + \sqrt{2}}$$

$\log(x)$  is the natural logarithm

$\gamma$  is the Euler-Mascheroni constant

**Exact result:**

$$\sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)}$$

**Decimal approximation:**

1.611161625721865394057230517147071909233641376206228980311...

1.61116162572...

**Alternate forms:**

$$\sqrt{\frac{1}{3} \left( 3 + 3\sqrt{2} - 3\gamma + \pi^2 - 3\log(4\pi) \right)}$$

$$\sqrt{\frac{1}{-3\gamma + \pi^2 - 3(-1 - \sqrt{2} + 2\log(2)) - 3\log(\pi)}}$$

$$\sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \log\left(\frac{1}{4\pi}\right)}$$

**All 2nd roots of  $1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)$ :**

$$e^0 \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)} \approx 1.6112 \quad (\text{real, principal root})$$

$$e^{i\pi} \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)} \approx -1.6112 \quad (\text{real root})$$

**Alternative representations:**

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}$$

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}$$

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 - \gamma + \text{Li}_1(1 - 4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}$$

**Series representations:**

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-4\pi}\right)^k}{k}}$$

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - 2i\pi \left[ \frac{\arg(4\pi - x)}{2\pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi - x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{\left(1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(z_0) - \left[ \frac{\arg(4\pi - z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi - z_0)^k z_0^{-k}}{k} \right)}$$

**Integral representations:**

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \int_1^{4\pi} \frac{1}{t} dt}$$

$$\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}} = \sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \text{ for } -1 < \gamma < 0$$

$$1/((((4\pi^2)/12-1/2*2(\text{euler constant}- 1 +\ln (4\pi))+(\text{sqrt}(2))))^1/64$$

**Input:**

$$\frac{1}{\sqrt[64]{\frac{1}{12}(4\pi^2) - \frac{1}{2} \times 2(\gamma - 1 + \log(4\pi)) + \sqrt{2}}}$$

log(x) is the natural logarithm

γ is the Euler-Mascheroni constant

**Exact result:**

$$\frac{1}{\sqrt[64]{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(4\pi)}}$$

**Decimal approximation:**

0.985205670521153006968489609998707694558697555325715632043...

0.9852056705.... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} - \varphi + 1 \approx 0.9991104684$$

and to the Omega mesons ( $\omega/\omega_3 \mid 5 + 3 \mid m_{u/d} = 255 - 390 \mid 0.988 - 1.18$  ) Regge slope value (0.988) connected to the dilaton scalar field  $0.989117352243 = \phi$

### Alternate forms:

$${}^{64}\sqrt{\frac{3}{3 + 3\sqrt{2} - 3\gamma + \pi^2 - 3\log(4\pi)}}$$

$${}^{64}\sqrt{\frac{3}{-3\gamma + \pi^2 - 3(-1 - \sqrt{2} + 2\log(2)) - 3\log(\pi)}}$$

$$\frac{1}{{}^{64}\sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \log\left(\frac{1}{4\pi}\right)}}$$

### Alternative representations:

$$\frac{1}{{}^{64}\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{{}^{64}\sqrt{1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}}$$

$$\frac{1}{{}^{64}\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{{}^{64}\sqrt{1 - \gamma + \text{Li}_1(1 - 4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}}$$

$$\frac{1}{{}^{64}\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{{}^{64}\sqrt{1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}}}$$

### Series representations:

$$\frac{1}{{}^{64}\sqrt{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{{}^{64}\sqrt{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{\left(\frac{1}{1-4\pi}\right)^k}{k}}}$$



$$\frac{1}{\sqrt[64]{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{\sqrt[64]{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - 2i\pi \left[ \frac{\arg(4\pi-x)}{2\pi} \right] - \log(x) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi-x)^k x^{-k}}{k}}} \quad \text{for } x < 0$$

$$\frac{1}{\sqrt[64]{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{\sqrt[64]{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \log(z_0) - \left[ \frac{\arg(4\pi-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) + \sum_{k=1}^{\infty} \frac{(-1)^k (4\pi-z_0)^k z_0^{-k}}{k}}}$$

**Integral representations:**

$$\frac{1}{\sqrt[64]{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{\sqrt[64]{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} - \int_1^{4\pi} \frac{1}{t} dt}}$$

$$\frac{1}{\sqrt[64]{\frac{4\pi^2}{12} - \frac{2}{2}(\gamma - 1 + \log(4\pi)) + \sqrt{2}}} = \frac{1}{\sqrt[64]{1 + \sqrt{2} - \gamma + \frac{\pi^2}{3} + \frac{i}{2\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}} \quad \text{for } -1 < \gamma < 0$$

27\*1/2\*2 log base 0.985205670521 (((1/(((4Pi^2)/12-1/2\*2(euler constant- 1 +ln (4Pi)))+(sqrt(2))))))))+1

**Input interpretation:**

$$27 \times \frac{1}{2} \times 2 \log_{0.985205670521} \left( \frac{1}{\frac{1}{12} (4\pi^2) - \frac{1}{2} \times 2 (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1$$

log(x) is the natural logarithm

log<sub>b</sub>(x) is the base- b logarithm

γ is the Euler-Mascheroni constant

**Result:**

1729.000000...

1729

**Alternative representations:**

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + \frac{27 \log \left( \frac{1}{1 - \gamma - \log(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right)}{\log(0.9852056705210000)}$$

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right)$$

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right)$$

**Series representations:**

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 - \frac{27 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3}{3 - 3\gamma + \pi^2 - 3 \log(4\pi) + 3\sqrt{2}} \right)^k}{k}}{\log(0.9852056705210000)}$$

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \exp\left(i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \right. \right.$$

$$\left. \left. \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \right. \right.$$

$$\left. \left. \left( \frac{1}{z_0} \right)^{1/2 [\arg(2-z_0)/(2\pi)]} z_0^{1/2 (1+[\arg(2-z_0)/(2\pi)])} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right) \right)$$

### Integral representations:

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( \frac{3}{3 - 3\gamma + \pi^2 - 3 \int_1^{4\pi} \frac{1}{t} dt + 3\sqrt{2}} \right)$$

$$\frac{27}{2} \times 2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1 =$$

$$1 + 27 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma + \frac{\pi^2}{3} - \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \sqrt{2}} \right)$$

for  $-1 < \gamma < 0$

$(((((27 \times \frac{1}{2} \times 2 \log_{\text{base } 0.985205670521} (((1/(((4\pi^2)/12 - 1/2 \times 2(\text{euler constant} - 1 + \ln(4\pi)) + (\text{sqrt}(2)))))))))) + 1))))^{1/15}$

**Input interpretation:**

$$\sqrt[15]{27 \times \frac{1}{2} \times 2 \log_{0.985205670521} \left( \frac{1}{\frac{1}{12} (4\pi^2) - \frac{1}{2} \times 2 (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 1}$$

$\log(x)$  is the natural logarithm  
 $\log_b(x)$  is the base- $b$  logarithm  
 $\gamma$  is the Euler-Mascheroni constant

**Result:**

1.643815228747586916876220187768741933757775158298989979528...

[1.6438152287...](#)

$2 \log_{\text{base } 0.985205670521} (((1/(((4\pi^2)/12 - 1/2 \times 2(\text{euler constant} - 1 + \ln(4\pi)) + (\text{sqrt}(2)))))))) - \pi + 1/\text{golden ratio}$

**Input interpretation:**

$$2 \log_{0.985205670521} \left( \frac{1}{\frac{1}{12} (4\pi^2) - \frac{1}{2} \times 2 (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm  
 $\log_b(x)$  is the base- $b$  logarithm  
 $\gamma$  is the Euler-Mascheroni constant  
 $\phi$  is the golden ratio

**Result:**

125.4764413...

[125.4764413...](#)

### Alternative representations:

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + \frac{1}{\phi} + \frac{2 \log \left( \frac{1}{1 - \gamma - \log(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right)}{\log(0.9852056705210000)}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right) + \frac{1}{\phi}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$-\pi + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right) + \frac{1}{\phi}$$

### Series representations:

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3}{3 - 3\gamma + \pi^2 - 3 \log(4\pi) + 3\sqrt{2}} \right)^k}{k}}{\log(0.9852056705210000)}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$-\frac{1}{\phi} \left( -1 + \phi \pi - 2 \phi \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \right. \right. \right.$$

$$\left. \left. \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \exp \left( i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] \right) \sqrt{x} \right. \right.$$

$$\left. \left. \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left( -\frac{1}{2} \right)_k}{k!} \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$-\frac{1}{\phi} \left( -1 + \phi \pi - 2 \phi \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \left( \frac{1}{z_0} \right)^{1/2 \lfloor \arg(2-z_0)/(2\pi) \rfloor} \right. \right. \right.$$

$$\left. \left. \left. z_0^{1/2 (1 + \lfloor \arg(2-z_0)/(2\pi) \rfloor)} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (2 - z_0)^k z_0^{-k}}{k!} \right) \right) \right)$$

**Integral representations:**

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \log_{0.9852056705210000} \left( \frac{3}{3 - 3\gamma + \pi^2 - 3 \int_1^{4\pi} \frac{1}{t} dt + 3\sqrt{2}} \right)$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) - \pi + \frac{1}{\phi} =$$

$$\frac{1}{\phi} - \pi + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma + \frac{\pi^2}{3} - \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \sqrt{2}} \right)$$

for  $-1 < \gamma < 0$

2 log base 0.985205670521 (((1/(((4Pi^2)/12-1/2\*2(euler constant- 1 +ln (4Pi)))+(sqrt(2))))))))+11+1/golden ratio

**Input interpretation:**

$$2 \log_{0.985205670521} \left( \frac{1}{\frac{1}{12} (4\pi^2) - \frac{1}{2} \times 2 (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi}$$

$\log(x)$  is the natural logarithm

$\log_b(x)$  is the base- $b$  logarithm

$\gamma$  is the Euler-Mascheroni constant

$\phi$  is the golden ratio

**Result:**

139.6180340...

139.618034...

**Alternative representations:**

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + \frac{2 \log \left( \frac{1}{1 - \gamma - \log(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right)}{\log(0.9852056705210000)}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log_e(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right) + \frac{1}{\phi}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma - \log(a) \log_a(4\pi) + \frac{4\pi^2}{12} + \sqrt{2}} \right) + \frac{1}{\phi}$$

**Series representations:**

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} - \frac{2 \sum_{k=1}^{\infty} \frac{(-1)^k \left( -1 + \frac{3}{3 - 3\gamma + \pi^2 - 3 \log(4\pi) + 3\sqrt{2}} \right)^k}{k}}{\log(0.9852056705210000)}$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} \left( 1 + 11\phi + 2\phi \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \exp\left(i\pi \left[ \frac{\arg(2-x)}{2\pi} \right]\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (2-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!} \right) \right) \right) \text{ for } (x \in \mathbb{R} \text{ and } x < 0)$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$\frac{1}{\phi} \left( 1 + 11\phi + 2\phi \log_{0.9852056705210000} \left( 1 / \left( 1 - \gamma + \frac{\pi^2}{3} - \log(-1 + 4\pi) + \sum_{k=1}^{\infty} \frac{(-1)^k (-1 + 4\pi)^{-k}}{k} + \left(\frac{1}{z_0}\right)^{1/2 [\arg(2-z_0)/(2\pi)]} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (2-z_0)^k z_0^{-k}}{k!} \right) \right) \right)$$

### Integral representations:

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \log_{0.9852056705210000} \left( \frac{3}{3 - 3\gamma + \pi^2 - 3 \int_1^{4\pi} \frac{1}{t} dt + 3\sqrt{2}} \right)$$

$$2 \log_{0.9852056705210000} \left( \frac{1}{\frac{4\pi^2}{12} - \frac{2}{2} (\gamma - 1 + \log(4\pi)) + \sqrt{2}} \right) + 11 + \frac{1}{\phi} =$$

$$11 + \frac{1}{\phi} + 2 \log_{0.9852056705210000} \left( \frac{1}{1 - \gamma + \frac{\pi^2}{3} - \frac{1}{2i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{(-1+4\pi)^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds + \sqrt{2}} \right) \text{ for } -1 < \gamma < 0$$



From

**The normal number of prime factors of a number n – Srinivasa Ramanujan - Quarterly Journal of Mathematics, XLVIII, 1917, 76 – 92**

Now, we have that:

But

$$(2.25) \quad \frac{1}{\log x - \log p} = \frac{1}{\log x} + \frac{\log p}{(\log x)^2} \left\{ 1 + \frac{\log p}{\log x} + \dots \right\} \leq \frac{1}{\log x} + \frac{2 \log p}{(\log x)^2},$$

since  $\log p \leq \frac{1}{2} \log x$ ; and so

$$(2.26) \quad \sum_{p^2 \leq x} \frac{1}{p \log(x/p)} < \frac{1}{\log x} \sum_{p^2 \leq x} \frac{1}{p} + \frac{2}{(\log x)^2} \sum_{p^2 \leq x} \frac{\log p}{p} \\ < \frac{\log \log x + B}{\log x} + \frac{H}{\log x} < \frac{\log \log x + C}{\log x}.$$

For  $x = 3$  and  $p = \sqrt{3}$

$$1/(\ln(3)) + (2 \ln(\sqrt{3})) / ((\ln 3)^2)$$

**Input:**

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)}$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$\frac{2}{\log(3)}$$

**Decimal approximation:**

1.820478453253674787228480331472214001225272114510423489452...

1.82047845325.....

**Property:**

$\frac{2}{\log(3)}$  is a transcendental number

### Alternative representations:

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{1}{\log_e(3)} + \frac{2 \log_e(\sqrt{3})}{\log_e^2(3)}$$

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{1}{\log(a) \log_a(3)} + \frac{2 \log(a) \log_a(\sqrt{3})}{(\log(a) \log_a(3))^2}$$

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = -\frac{1}{\text{Li}_1(-2)} - \frac{2 \text{Li}_1(1 - \sqrt{3})}{(-\text{Li}_1(-2))^2}$$

### Series representations:

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{2}{\log(2) - \sum_{k=1}^{\infty} \frac{(-\frac{1}{2})^k}{k}}$$

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{2}{2 i \pi \left[ \frac{\text{arg}(3-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{2}{\log(z_0) + \left[ \frac{\text{arg}(3-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (3-z_0)^k z_0^{-k}}{k}}$$

### Integral representations:

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \int_1^3 \frac{1}{t} dt$$

$$\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} = \frac{4 i \pi}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$(\ln \ln(3) + 1) / \ln(3)$$

**Input:**

$$\frac{\log(\log(3)) + 1}{\log(3)}$$

$\log(x)$  is the natural logarithm

**Decimal approximation:**

0.995845248502595625910060697024710370192775541553465572766...

0.9958452485025..... result very near to the value of the following Rogers-Ramanujan continued fraction:

$$\frac{e^{-\frac{\pi}{\sqrt{5}}}}{\sqrt{5}} = 1 - \frac{e^{-\pi\sqrt{5}}}{1 + \frac{e^{-2\pi\sqrt{5}}}{1 + \frac{e^{-3\pi\sqrt{5}}}{1 + \frac{e^{-4\pi\sqrt{5}}}{1 + \dots}}}} \approx 0.9991104684$$

and to the Omega mesons (  $\omega/\omega_3 \mid 5 + 3 \mid m_{u/d} = 255 - 390 \mid 0.988 - 1.18$  ) Regge slope value (0.988) connected to the dilaton scalar field **0.989117352243 =  $\phi$**

**Alternate form:**

$$\frac{1}{\log(3)} + \frac{\log(\log(3))}{\log(3)}$$

**Alternative representations:**

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{1 + \log_e(\log(3))}{\log_e(3)}$$

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{1 + \log(a) \log_a(\log(3))}{\log(a) \log_a(3)}$$

$$\frac{\log(\log(3)) + 1}{\log(3)} = -\frac{1 - \text{Li}_1(1 - \log(3))}{\text{Li}_1(-2)}$$

### Series representations:

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{-i + 2\pi \left[ \frac{\text{arg}(-x + \log(3))}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(3))^k}{k}}{2\pi \left[ \frac{\text{arg}(3-x)}{2\pi} \right] - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{-i + 2\pi \left[ \frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(3) - z_0)^k z_0^{-k}}{k}}{2\pi \left[ \frac{\pi - \text{arg}\left(\frac{1}{z_0}\right) - \text{arg}(z_0)}{2\pi} \right] - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}}$$

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{1 + \left[ \frac{\text{arg}(\log(3) - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\text{arg}(\log(3) - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(3) - z_0)^k z_0^{-k}}{k}}{\left[ \frac{\text{arg}(3 - z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\text{arg}(3 - z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}}$$

### Integral representations:

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{1 + \int_1^{\log(3)} \frac{1}{t} dt}{\int_1^3 \frac{1}{t} dt}$$

$$\frac{\log(\log(3)) + 1}{\log(3)} = \frac{2i\pi + \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1 + \log(3))^{-s}}{\Gamma(1-s)} ds}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds} \quad \text{for } -1 < \gamma < 0$$

$$2\left(\frac{1}{\ln(3)} + \frac{2 \ln(\sqrt{3})}{\ln^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)}\right)$$

### Input:

$$2\left(\frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)}\right)$$

$\log(x)$  is the natural logarithm

**Exact result:**

$$2 \left( \frac{2}{\log(3)} - \frac{1 + \log(\log(3))}{\log(3)} \right)$$

**Decimal approximation:**

1.649266409502158322636839268895007262064993145913915833372...

1.6492664095021583.....

**Alternate forms:**

$$\frac{2 (\log(\log(3)) - 1)}{\log(3)}$$

$$\frac{2 - 2 \log(\log(3))}{\log(3)}$$

$$\frac{2}{\log(3)} - \frac{2 \log(\log(3))}{\log(3)}$$

**Alternative representations:**

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) =$$

$$2 \left( \frac{1}{\log(a) \log_a(3)} - \frac{1 + \log(a) \log_a(\log(3))}{\log(a) \log_a(3)} + \frac{2 \log(a) \log_a(\sqrt{3})}{(\log(a) \log_a(3))^2} \right)$$

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = 2 \left( \frac{1}{\log_e(3)} - \frac{1 + \log_e(\log(3))}{\log_e(3)} + \frac{2 \log_e(\sqrt{3})}{\log_e^2(3)} \right)$$

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) =$$

$$2 \left( -\frac{1}{\text{Li}_1(-2)} - \frac{1 - \text{Li}_1(1 - \log(3))}{\text{Li}_1(-2)} - \frac{2 \text{Li}_1(1 - \sqrt{3})}{(-\text{Li}_1(-2))^2} \right)$$

### Series representations:

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = \frac{2 \left( i + 2\pi \left\lfloor \frac{\arg(-x + \log(3))}{2\pi} \right\rfloor - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k} (-x + \log(3))^k}{k} \right)}{2\pi \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor - i \log(x) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}}{k}} \quad \text{for } x < 0$$

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = \frac{2 \left( i + 2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (\log(3) - z_0)^k z_0^{-k}}{k} \right)}{2\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor - i \log(z_0) + i \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k}}$$

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = - \left( \left( 2 \left( -1 + \left\lfloor \frac{\arg(\log(3) - z_0)}{2\pi} \right\rfloor \right) \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(\log(3) - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (\log(3) - z_0)^k z_0^{-k}}{k} \right) \right) / \left( \left\lfloor \frac{\arg(3 - z_0)}{2\pi} \right\rfloor \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left\lfloor \frac{\arg(3 - z_0)}{2\pi} \right\rfloor \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (3 - z_0)^k z_0^{-k}}{k} \right)$$

### Integral representations:

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = - \frac{2 \left( -1 + \int_1^{\log(3)} \frac{1}{t} dt \right)}{\int_1^3 \frac{1}{t} dt}$$

$$2 \left( \frac{1}{\log(3)} + \frac{2 \log(\sqrt{3})}{\log^2(3)} - \frac{\log(\log(3)) + 1}{\log(3)} \right) = \frac{2i \left( 2\pi + i \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s) (-1+\log(3))^{-s}}{\Gamma(1-s)} ds \right)}{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{2^{-s} \Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds}$$

for  $-1 < \gamma < 0$

$\Gamma(x)$  is the gamma function

# Ulam Spiral

From:

[http://primorial-sieve.com/3\\_Prime%20number%20pattern.php](http://primorial-sieve.com/3_Prime%20number%20pattern.php)

100	99	98	97	96	95	94	93	92	91
65	64	63	62	61	60	59	58	57	90
66	37	36	35	34	33	32	31	56	89
67	38	17	16	15	14	13	30	55	88
68	39	18	5	4	3	12	29	54	87
69	40	19	6	1	2	11	28	53	86
70	41	20	7	8	9	10	27	52	85
71	42	21	22	23	24	25	26	51	84
72	43	44	45	46	47	48	49	50	83
73	74	75	76	77	78	79	80	81	82

$$[(97/89 + 89/83 + 83/79 + 79/73 + 73/71 + 71/67 + 67/61 + 61/59 + 59/53 + 53/47 + 47/43 + 43/41 + 41/37 + 37/31 + 31/29 + 29/23 + 23/19 + 19/17 + 17/13 + 13/11 + 11/7 + 7/5 + 5/3 + 3/2) \times 1/24]$$

**Input:**

$$\left( \frac{97}{89} + \frac{89}{83} + \frac{83}{79} + \frac{79}{73} + \frac{73}{71} + \frac{71}{67} + \frac{67}{61} + \frac{61}{59} + \frac{59}{53} + \frac{53}{47} + \frac{47}{43} + \frac{43}{41} + \frac{41}{37} + \frac{37}{31} + \frac{31}{29} + \frac{29}{23} + \frac{23}{19} + \frac{19}{17} + \frac{17}{13} + \frac{13}{11} + \frac{11}{7} + \frac{7}{5} + \frac{5}{3} + \frac{3}{2} \right) \times \frac{1}{24}$$

**Exact result:**

677 054 336 195 724 798 111 142 246 030 764 679  
 570 449 805 512 293 218 495 612 902 432 599 440

**Decimal approximation:**

1.186878020911402076501256271953137905236464024789974791971...

1.186878020911...

$(1.186878020911402076501256271953137905236464024789974791971)^{(1.5236+1.2683)}$

**Input interpretation:**

1.186878020911402076501256271953137905236464024789974791971<sup>1.5236+1.2683</sup>

**Result:**

1.613371406015215413838501589685286116664864943342145497410...

1.613371406...

$[(97/89 + 89/83 + 83/79 + 79/73 + 73/71 + 71/67 + 67/61 + 61/59 + 59/53 + 53/47 + 47/43 + 43/41 + 41/37 + 37/31 + 31/29 + 29/23 + 23/19 + 19/17 + 17/13 + 13/11 + 11/7 + 7/5 + 5/3 + 3/2) \times 1/24]^6$

**Input:**

$$\left( \left( \frac{97}{89} + \frac{89}{83} + \frac{83}{79} + \frac{79}{73} + \frac{73}{71} + \frac{71}{67} + \frac{67}{61} + \frac{61}{59} + \frac{59}{53} + \frac{53}{47} + \frac{47}{43} + \frac{43}{41} + \frac{41}{37} + \frac{37}{31} + \frac{31}{29} + \frac{29}{23} + \frac{23}{19} + \frac{19}{17} + \frac{17}{13} + \frac{13}{11} + \frac{11}{7} + \frac{7}{5} + \frac{5}{3} + \frac{3}{2} \right) \times \frac{1}{24} \right)^6$$

**Exact result:**

96 325 471 443 063 196 617 901 077 941 726 536 246 063 882 958 624 743 397 889 %  
631 504 474 952 879 671 301 182 457 469 946 513 948 491 944 813 313 111 669 911 %  
874 191 586 578 516 625 124 201 111 049 898 143 476 838 664 995 154 192 486 608 %  
233 567 091 703 128 193 322 294 407 912 907 921 /  
34 459 154 590 074 954 903 194 170 060 841 016 777 864 373 477 487 011 884 917 %  
413 317 144 424 040 513 526 493 159 794 469 830 380 742 770 355 432 004 605 %  
726 509 916 586 664 358 111 726 303 906 102 790 874 799 443 651 283 642 980 %  
411 569 997 399 792 275 699 005 063 862 419 456 000 000

**Decimal approximation:**

2.795352137594437463215607882784766428695746202551642565502...

2.7953521375...



sqrt(2.795352137594437463215607882784766428695746202551642565502)

**Input interpretation:**

$$\sqrt{2.795352137594437463215607882784766428695746202551642565502}$$

**Result:**

1.671930661718492877049814408665075227093289566919910534188...

1.6719306617...

We have also:

$$[(97/89+89/83+83/79+79/73+73/71+71/67+67/61+61/59+59/53+53/47+47/43+43/41+41/37+37/31+31/29+29/23+23/19+19/17+17/13+13/11+11/7+7/5+5/3+3/2)(11.8458)^{1/8} * 1/(24)]$$

Where 11.8458 is an entropy deriving from ln(139503)

**Input interpretation:**

$$\left( \frac{97}{89} + \frac{89}{83} + \frac{83}{79} + \frac{79}{73} + \frac{73}{71} + \frac{71}{67} + \frac{67}{61} + \frac{61}{59} + \frac{59}{53} + \frac{53}{47} + \frac{47}{43} + \frac{43}{41} + \frac{41}{37} + \frac{37}{31} + \frac{31}{29} + \frac{29}{23} + \frac{23}{19} + \frac{19}{17} + \frac{17}{13} + \frac{13}{11} + \frac{11}{7} + \frac{7}{5} + \frac{5}{3} + \frac{3}{2} \right)^{\frac{1}{8} \sqrt{11.8458}} \times \frac{1}{24}$$

**Result:**

1.616596510290705942928488832563501790747715820085286562000...

1.61659651029...

We have that:

A fractal is a mathematically defined, self similar object which has similarity and symmetry on a variety of scales. The Julia Set Fractal is a type of fractal defined by the behavior of a function that operates on input complex numbers. More explicitly, upon iterative updating of input complex number, the Julia Set Fractal represents the set of inputs whose resulting outputs either tend towards infinity or remain bounded.

**Mathematics of the Julia Set Fractal**

The Julia Set Fractal is dependent upon complex numbers - numbers which have both a real and 'imaginary' component *i*, *i* being defined as the square root of -1. A complex number can formally be expressed as:

$$c = r + b * i$$

Where  $c$  is the complex number,  $r$  is the real component and  $b$  the imaginary component. To create the bounded set, we first create a mathematical function  $f(z)$  which accepts a complex number, a simple example is the following equation...

$$z = z^2 + c$$

...where  $c$  is a constant complex number. The complex number  $z$  can be updated iteratively (here defined as  $F(z)$ ):

- Initialization of the complex number variable  $z$ .
- Iteratively updating the value of  $z$  based upon the function  $f(z)$ .

Often, we set a threshold to prevent infinite iteration, which can be one or both of a) we surpass a value of  $z$  (in the examples below, iteration stops when absolute value of  $z$  exceeds 2) b) and/or b) we surpass a predefined number of iterations. Based upon either method,  $z$  can be defined as bounded or unbounded (iteration trends towards infinity).

The **Douady rabbit** is any of various particular filled Julia sets associated with the parameter near the center period 3 buds of Mandelbrot set for complex quadratic map.

We have:

$$\left[ \left( \frac{97}{89} + \frac{89}{83} + \frac{83}{79} + \frac{79}{73} + \frac{73}{71} + \frac{71}{67} + \frac{67}{61} + \frac{61}{59} + \frac{59}{53} + \frac{53}{47} + \frac{47}{43} + \frac{43}{41} + \frac{41}{37} + \frac{37}{31} + \frac{31}{29} + \frac{29}{23} + \frac{23}{19} + \frac{19}{17} + \frac{17}{13} + \frac{13}{11} + \frac{11}{7} + \frac{7}{5} + \frac{5}{3} + \frac{3}{2} \right) \left( 2.06 - \frac{1.3934}{2} \right) \times \frac{1}{24} \right]$$

Where 2.06 and 1.3934 are two Hausdorff dimension, i.e. 1.3934 for the Julia set for  $c = -0,123 + 0.745i$ , while 2.06 for the Lorenz attractor

**Input interpretation:**

$$\left( \frac{97}{89} + \frac{89}{83} + \frac{83}{79} + \frac{79}{73} + \frac{73}{71} + \frac{71}{67} + \frac{67}{61} + \frac{61}{59} + \frac{59}{53} + \frac{53}{47} + \frac{47}{43} + \frac{43}{41} + \frac{41}{37} + \frac{37}{31} + \frac{31}{29} + \frac{29}{23} + \frac{23}{19} + \frac{19}{17} + \frac{17}{13} + \frac{13}{11} + \frac{11}{7} + \frac{7}{5} + \frac{5}{3} + \frac{3}{2} \right) \left( 2.06 - \frac{1.3934}{2} \right) \times \frac{1}{24}$$

**Result:**

1.618070805908514450894162675553712906208871404996172633895...

1.61807080590851445.....

Now, from 101 to 200, we have the following prime numbers:

101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

Thence,

$(\frac{199}{197} + \frac{197}{193} + \frac{193}{191} + \frac{191}{181} + \frac{181}{179} + \frac{179}{173} + \frac{173}{167} + \frac{167}{163} + \frac{163}{157} + \frac{157}{151} + \frac{151}{149} + \frac{149}{139} + \frac{139}{137} + \frac{137}{131} + \frac{131}{127} + \frac{127}{113} + \frac{113}{109} + \frac{109}{107} + \frac{107}{103} + \frac{103}{101}) \times \frac{1}{20}$

**Input:**

$$\left( \frac{199}{197} + \frac{197}{193} + \frac{193}{191} + \frac{191}{181} + \frac{181}{179} + \frac{179}{173} + \frac{173}{167} + \frac{167}{163} + \frac{163}{157} + \frac{157}{151} + \frac{151}{149} + \frac{149}{139} + \frac{139}{137} + \frac{137}{131} + \frac{131}{127} + \frac{127}{113} + \frac{113}{109} + \frac{109}{107} + \frac{107}{103} + \frac{103}{101} \right) \times \frac{1}{20}$$

**Exact result:**

175 920 640 480 325 044 604 603 282 299 425 538 139 497 733  
170 004 045 693 312 436 240 693 405 065 149 782 450 812 170

**Decimal approximation:**

1.034802670506360505054235319585467414133742213225363599994...

1.03480267050636....

Performing the mean with the previous result for the primes between 1 to 10, we obtain:

$$(1.03480267050636+1.186878020911)/2$$

**Input interpretation:**

$$\frac{1.03480267050636 + 1.186878020911}{2}$$

**Result:**

1.11084034570868

**1.11084034570868**

From which, we obtain the following result:

$$1+1/((((1.03480267050636+1.186878020911)/2)))^4$$

**Input interpretation:**

$$1 + \frac{1}{\left(\frac{1.03480267050636+1.186878020911}{2}\right)^4}$$

**Result:**

1.656739926860039435540872040193724995228204057232412381480...

1.65673992686.... result very near to the 14th root of the following Ramanujan's class invariant  $Q = (G_{505}/G_{101/5})^3 = 1164.2696$  i.e. 1.65578...

From

## II

### RAMANUJAN AND THE THEORY OF PRIME NUMBERS

16 Jan. 1913

$$\phi_2(y) + \log 2 \left( 1 - \frac{y}{3 \cdot 1!} + \frac{y^2}{7 \cdot 2!} - \frac{y^3}{15 \cdot 3!} + \dots \right) = \frac{1}{y} + F(y),$$

where  $y F(y) = 0.0000098844 \cos \left( \frac{2\pi \log y}{\log 2} + 0.872811 \right)$

$$0.0000098844 \cos \left( \frac{2\pi \ln(x)}{\ln 2} + 0.872811 \right) = - \left( \frac{\ln 2 \left( \left( 1 - \frac{x}{(3 \cdot 1)!} + \frac{x^2}{(7 \cdot 2)!} - \frac{x^3}{(15 \cdot 3)!} \right) \right)}{x^3} \right)$$

**Input interpretation:**

$$9.8844 \times 10^{-6} \cos \left( \frac{2\pi \log(x)}{\log(2)} + 0.872811 \right) = - \left( \log(2) \left( 1 - \frac{x}{(3 \times 1)!} + \frac{x^2}{(7 \times 2)!} - \frac{x^3}{(15 \times 3)!} \right) \right)$$

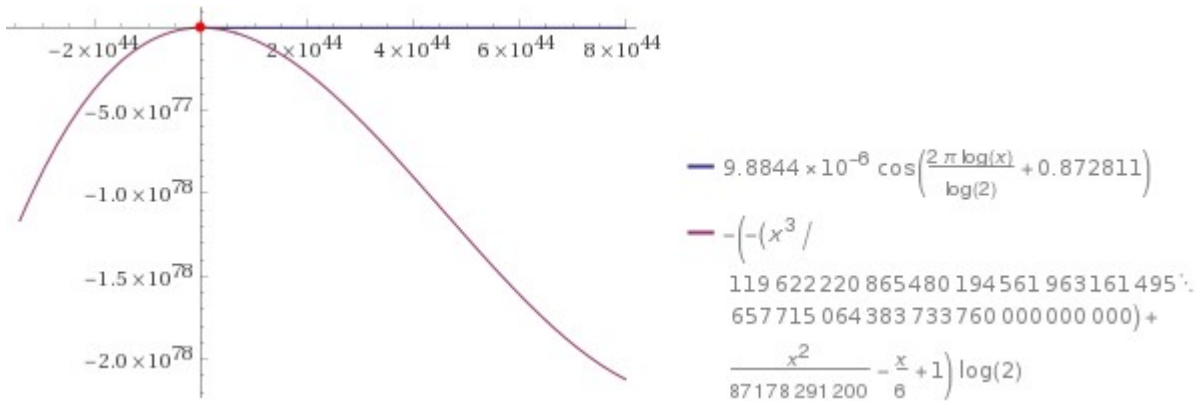
log(x) is the natural logarithm

n! is the factorial function

**Result:**

$$9.8844 \times 10^{-6} \cos \left( \frac{2\pi \log(x)}{\log(2)} + 0.872811 \right) = - \left( \frac{x^3}{119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000} + \frac{x^2}{87\,178\,291\,200} - \frac{x}{6} + 1 \right) \log(2)$$

**Plot:**



**Alternate forms:**

$$-5.86224 \times 10^{-52} x^3 + 8.0439 \times 10^{-7} x^2 - 11\,687.6 x + \cos \left( \frac{2\pi \log(x)}{\log(2)} + 0.872811 \right) = -70\,125.4$$

$$\begin{aligned}
& 9.8844 \times 10^{-6} \cos\left(\frac{2\pi \log(x)}{\log(2)} + 0.872811\right) = \\
& \left( (x^3 - 13721560633833597619537094287030222848000000000x^2 + \right. \\
& \quad 1993703681091336576032719358260961917739728896000000000x - \\
& \quad \left. 11962222086548019456196316149565771506438373376000000000000) \log(2) \right) / \\
& \quad 11962222086548019456196316149565771506438373376000000000000 \\
& \left( (3.17623 \times 10^{-6} + 3.78641 \times 10^{-6}i) x^{(2i\pi)/\log(2)} + \right. \\
& \quad \left. (3.17623 \times 10^{-6} - 3.78641 \times 10^{-6}i) x^{-(2i\pi)/\log(2)} \right) = \\
& \quad \frac{x^3 \log(2)}{11962222086548019456196316149565771506438373376000000000000} - \\
& \quad \frac{x^2 \log(2)}{87178291200} + \frac{1}{6} x \log(2) - \log(2)
\end{aligned}$$

### Alternate form assuming x is positive:

$$\begin{aligned}
& x((8.0439 \times 10^{-7} - 5.86224 \times 10^{-52}x)x - 11687.6) + \\
& \cos(9.06472 \log(x) + 0.872811) + 70125.4 = 0
\end{aligned}$$

### Expanded form:

$$\begin{aligned}
& 9.8844 \times 10^{-6} \cos\left(\frac{2\pi \log(x)}{\log(2)} + 0.872811\right) = \\
& \quad \frac{x^3 \log(2)}{11962222086548019456196316149565771506438373376000000000000} - \\
& \quad \frac{x^2 \log(2)}{87178291200} + \frac{1}{6} x \log(2) - \log(2)
\end{aligned}$$

### Alternate form assuming x>0:

$$\begin{aligned}
& 9.8844 \times 10^{-6} \cos(9.06472 \log(x) + 0.872811) = \\
& \quad \frac{x^3 \log(2)}{11962222086548019456196316149565771506438373376000000000000} - \\
& \quad \frac{x^2 \log(2)}{87178291200} + \frac{1}{6} x \log(2) - \log(2)
\end{aligned}$$

### Numerical solution:

$$x \approx 5.99998601882105...$$

5.999986...

$$0.0000098844 \cos((2\pi \cdot \ln(5.99998601882105))/\ln 2 + 0.872811)$$

**Input interpretation:**

$$9.8844 \times 10^{-6} \cos\left(\frac{2 \pi \log(5.99998601882105)}{\log(2)} + 0.872811\right)$$

$\log(x)$  is the natural logarithm

**Result:**

$$-1.61546... \times 10^{-6}$$

$$-1.61546... * 10^{-6}$$

**Addition formulas:**

$$\begin{aligned} 9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) = \\ 9.8844 \times 10^{-6} \left( \cos(0.872811) \cos\left(-\frac{2 \pi \log(5.999986018821050000)}{\log(2)}\right) + \right. \\ \left. \sin(0.872811) \sin\left(-\frac{2 \pi \log(5.999986018821050000)}{\log(2)}\right) \right) \end{aligned}$$

$$\begin{aligned} 9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) = \\ 9.8844 \times 10^{-6} \cos(0.872811) \cos\left(\frac{2 \pi \log(5.999986018821050000)}{\log(2)}\right) - \\ 9.8844 \times 10^{-6} \sin(0.872811) \sin\left(\frac{2 \pi \log(5.999986018821050000)}{\log(2)}\right) \end{aligned}$$

$$\begin{aligned} 9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) = \\ 9.8844 \times 10^{-6} \left( \cosh\left(\frac{2 i \pi \log(5.999986018821050000)}{\log(2)}\right) \cos(0.872811) + \right. \\ \left. i \sinh\left(\frac{2 i \pi \log(5.999986018821050000)}{\log(2)}\right) \sin(0.872811) \right) \end{aligned}$$

$$\begin{aligned} 9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) = \\ 9.8844 \times 10^{-6} \cosh\left(-\frac{2 i \pi \log(5.999986018821050000)}{\log(2)}\right) \cos(0.872811) - \\ 9.8844 \times 10^{-6} i \sinh\left(-\frac{2 i \pi \log(5.999986018821050000)}{\log(2)}\right) \sin(0.872811) \end{aligned}$$

### Alternative representations:

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$9.8844 \times 10^{-6} \cosh\left(i \left(0.872811 + \frac{2 \pi \log(5.999986018821050000))}{\log(2)}\right)\right)$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$9.8844 \times 10^{-6} \cosh\left(-i \left(0.872811 + \frac{2 \pi \log(5.999986018821050000))}{\log(2)}\right)\right)$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$4.9422 \times 10^{-6} \left( e^{-i (0.872811 + (2 \pi \log(5.999986018821050000)) / \log(2))} + e^{i (0.872811 + (2 \pi \log(5.999986018821050000)) / \log(2))} \right)$$

### Series representations:

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$9.8844 \times 10^{-6} \sum_{k=0}^{\infty} \frac{(-1)^k \left(0.872811 + \frac{2 \pi \log(5.999986018821050000))}{\log(2)}\right)^{2k}}{(2k)!}$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$-9.8844 \times 10^{-6} \sum_{k=0}^{\infty} \frac{(-1)^k \left(0.872811 + \pi \left(-\frac{1}{2} + \frac{2 \log(5.999986018821050000))}{\log(2)}\right)\right)^{1+2k}}{(1+2k)!}$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2 (\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$9.8844 \times 10^{-6} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{k\pi}{2} + z_0\right) \left(0.872811 + \frac{2 \pi \log(5.999986018821050000))}{\log(2)} - z_0\right)^k}{k!}$$



**Integral representations:**

$$9.8844 \times 10^{-6} \cos\left(\frac{2(\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$-9.8844 \times 10^{-6} \int_{\frac{\pi}{2}}^{0.872811 + \frac{2\pi \log(5.999986018821050000)}{\log(2)}} \sin(t) dt$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2(\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$9.8844 \times 10^{-6} + \int_0^1 \frac{1}{\log(2)}$$

$$\left(-8.62721 \times 10^{-6} \log(2) - 0.0000197688 \pi \log(5.999986018821050000)\right)$$

$$\sin\left(t \left(0.872811 + \frac{2\pi \log(5.999986018821050000)}{\log(2)}\right)\right) dt$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2(\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$\frac{4.9422 \times 10^{-6} \sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{e^{-\frac{(0.436406 \log(2) + \pi \log(5.999986018821050000))^2}{s \log^2(2)}}}{\sqrt{s}} ds \text{ for } \gamma > 0$$

$$9.8844 \times 10^{-6} \cos\left(\frac{2(\pi \log(5.999986018821050000))}{\log(2)} + 0.872811\right) =$$

$$\frac{4.9422 \times 10^{-6} \sqrt{\pi}}{i\pi} \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{4^s \Gamma(s) \left(0.872811 + \frac{2\pi \log(5.999986018821050000)}{\log(2)}\right)^{-2s}}{\Gamma\left(\frac{1}{2} - s\right)} ds$$

for  $0 < \gamma < \frac{1}{2}$

$$-(((\ln 2(((1-(5.99998601882105)/(3*1)!)+(5.99998601882105)^2/(7*2)!-(5.99998601882105)^3/(15*3)!))))))$$

**Input interpretation:**

$$-\left(\log(2) \left(1 - \frac{5.99998601882105}{(3 \times 1)!} + \frac{5.99998601882105^2}{(7 \times 2)!} - \frac{5.99998601882105^3}{(15 \times 3)!}\right)\right)$$

$\log(x)$  is the natural logarithm

$n!$  is the factorial function

**Result:**

$$-1.61545536... \times 10^{-6}$$

$$-1.61545536... * 10^{-6}$$

**Alternative representations:**

$$-\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) =$$

$$-\log(a) \log_a(2) \left( 1 - \frac{5.999986018821050000}{\Gamma(4)} + \frac{5.999986018821050000^2}{\Gamma(15)} - \frac{5.999986018821050000^3}{\Gamma(46)} \right)$$

$$-\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) =$$

$$-\log_e(2) \left( 1 - \frac{5.999986018821050000}{(1)_3} + \frac{5.999986018821050000^2}{(1)_{14}} - \frac{5.999986018821050000^3}{(1)_{45}} \right)$$

$$-\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) =$$

$$-\log(a) \log_a(2) \left( 1 - \frac{5.999986018821050000}{(1)_3} + \frac{5.999986018821050000^2}{(1)_{14}} - \frac{5.999986018821050000^3}{(1)_{45}} \right)$$

## Series representations:

$$\begin{aligned}
 & -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
 & \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
 & -\log(2) + \frac{5.999986018821050000 \log(2)}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \frac{35.99983222604807336 \log(2)}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \\
 & \frac{215.9984900361919178 \log(2)}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}}
 \end{aligned}$$

for  $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \rightarrow 3 \text{ and } n_0 \rightarrow 14 \text{ and } n_0 \rightarrow 45)$

$$\begin{aligned}
 & -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
 & \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
 & - \left( 2 i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \left( 1 - \frac{5.999986018821050000}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \right. \\
 & \quad \left. \frac{35.99983222604807336}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \frac{215.9984900361919178}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right)
 \end{aligned}$$

for  $(x < 0 \text{ and } (n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \rightarrow 3 \text{ and } n_0 \rightarrow 14 \text{ and } n_0 \rightarrow 45)$

$$\begin{aligned}
 & -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
 & \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
 & - \left( \log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \\
 & \left( 1 - \frac{5.999986018821050000}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \right. \\
 & \quad \left. \frac{35.99983222604807336}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \frac{215.9984900361919178}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right)
 \end{aligned}$$

for  $((n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z}) \text{ and } n_0 \rightarrow 3 \text{ and } n_0 \rightarrow 14 \text{ and } n_0 \rightarrow 45)$

$$\begin{aligned}
& -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
& \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
& - \left( 2 i \pi \left[ -\frac{-\pi + \arg\left(\frac{2}{z_0}\right) + \arg(z_0)}{2 \pi} \right] + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \\
& \left( 1 - \frac{5.999986018821050000}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \right. \\
& \quad \left. \frac{35.99983222604807336}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \frac{215.9984900361919178}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

### Integral representations:

$$\begin{aligned}
& -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
& \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
& -\log(2) + \frac{5.999986018821050000 \log(2)}{\int_1^{\infty} e^{-t} t^3 dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(4+k)k!}} - \frac{35.99983222604807336 \log(2)}{\int_1^{\infty} e^{-t} t^{14} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(15+k)k!}} + \\
& \frac{215.9984900361919178 \log(2)}{\int_1^{\infty} e^{-t} t^{45} dt + \sum_{k=0}^{\infty} \frac{(-1)^k}{(46+k)k!}}
\end{aligned}$$

$$\begin{aligned}
& -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \right. \\
& \quad \left. \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
& - \left( \left( 1.000000000000000000 \left( \int_1^2 \frac{1}{t} dt \right) \left( \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) dt_2 dt_1 + \right. \right. \right. \\
& \quad \left. \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{45}\left(\frac{1}{t_2}\right) dt_2 dt_1 + \int_0^1 \int_0^1 \log^{14}\left(\frac{1}{t_1}\right) \log^{45}\left(\frac{1}{t_2}\right) dt_2 \right. \\
& \quad \left. \left. dt_1 + \int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \right) \right) / \\
& \left( \left( \int_0^1 \log^3\left(\frac{1}{t}\right) dt \right) \left( \int_0^1 \log^{14}\left(\frac{1}{t}\right) dt \right) \int_0^1 \log^{45}\left(\frac{1}{t}\right) dt \right)
\end{aligned}$$

$$\begin{aligned}
& -\log(2) \left( 1 - \frac{5.999986018821050000}{(3 \times 1)!} + \frac{5.999986018821050000^2}{(7 \times 2)!} - \frac{5.999986018821050000^3}{(15 \times 3)!} \right) = \\
& -\int_1^2 \frac{1}{t} dt + \frac{5.999986018821050000 \int_1^2 \frac{1}{t} dt}{\int_1^\infty e^{-t} t^3 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(4+k)k!}} - \frac{35.99983222604807336 \int_1^2 \frac{1}{t} dt}{\int_1^\infty e^{-t} t^{14} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(15+k)k!}} + \\
& \frac{215.9984900361919178 \int_1^2 \frac{1}{t} dt}{\int_1^\infty e^{-t} t^{45} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(46+k)k!}}
\end{aligned}$$

$$1/(((((((\ln 2(((1-(5.99998601882105)/(3*1)!+(5.99998601882105)^2/(7*2)!-(5.99998601882105)^3/(15*3)!)))))))))))))$$

Approximating 5.99998601882105 to 5.99998604273, we obtain:

$$1/(((((((\ln 2(((1-(5.99998604273)/(3*1)!+(5.99998604273)^2/(7*2)!-(5.99998604273)^3/(15*3)!)))))))))))))$$

**Input interpretation:**

$$\frac{1}{\log(2) \left( 1 - \frac{5.99998604273}{(3 \times 1)!} + \frac{5.99998604273^2}{(7 \times 2)!} - \frac{5.99998604273^3}{(15 \times 3)!} \right)}$$

$\log(x)$  is the natural logarithm

$n!$  is the factorial function

**Result:**

620080.7099280641834039998686573842181852974296511048637519...

620080.709928..... result that is practically equal to the following algebraic sum concerning the Prime Number Theorem:

x	$\pi(x)$	$\pi(x) - x / \ln x$	$\pi(x) / (x / \ln x)$	$\text{Li}(x) - \pi(x)$	$\pi(x) / \text{Li}(x)$	$x / \pi(x)$
$10^7$	664 579	44 158	1,071	339	0,999490163	15,047

$$664579 - 44158 - 1.071 - 339 = 620080.929$$

### Alternative representations:

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)}$$

$$\frac{1}{\log(a) \log_a(2) \left( 1 - \frac{5.999986042730000}{\Gamma(4)} + \frac{5.999986042730000^2}{\Gamma(15)} - \frac{5.999986042730000^3}{\Gamma(46)} \right)}$$

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)}$$

$$\frac{1}{\log_e(2) \left( 1 - \frac{5.999986042730000}{(1)_3} + \frac{5.999986042730000^2}{(1)_{14}} - \frac{5.999986042730000^3}{(1)_{45}} \right)}$$

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)}$$

$$\frac{1}{\log(a) \log_a(2) \left( 1 - \frac{5.999986042730000}{(1)_3} + \frac{5.999986042730000^2}{(1)_{14}} - \frac{5.999986042730000^3}{(1)_{45}} \right)}$$

### Series representations:

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)}$$

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \frac{35.99983251295481}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \frac{215.9984926183465}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right)}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

$$\begin{aligned}
& \frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)} = \\
& \left( \frac{0.5000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right) \\
& / \\
& \left( \left( 1.0000000000000000 i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] + 0.5000000000000000 \log(x) - \right. \right. \\
& \left. \left. 0.5000000000000000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \left( -215.998492618346 \right. \right. \\
& \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. 35.9998325129548 \right. \right. \\
& \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \right. \right. \\
& \left. \left. 5.99998604273000 \right. \right. \\
& \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. 1.0000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} \right. \right. \\
& \left. \left. (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right)
\end{aligned}$$

for  $(x < 0$  and  $(n_0 \geq 0$  or  $n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$   
and  
 $n_0 \rightarrow 45)$

$$\begin{aligned}
& \log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right) = \\
& \left( 0.5000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right) \\
& / \left( \left( 1.0000000000000000 i \pi \left[ -\frac{-\pi + \arg\left(\frac{2}{z_0}\right) + \arg(z_0)}{2\pi} \right] + \right. \right. \\
& 0.5000000000000000 \log(z_0) - \\
& 0.5000000000000000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \left. \left( -215.998492618346 \right. \right. \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \\
& 35.9998325129548 \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \\
& 5.99998604273000 \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \\
& 1.0000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} \\
& \left. \left. \left. (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right) \right)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$   
and  
 $n_0 \rightarrow 45$ )



$$\begin{aligned}
& \frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)} = \\
& \left( \frac{1.0000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!}}{\right. \\
& \left. \left( \left( 1.0000000000000000 \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log\left(\frac{1}{z_0}\right) + 1.0000000000000000 \log(z_0) + \right. \right. \right. \\
& \quad 1.0000000000000000 \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \\
& \quad \left. \left. 1.0000000000000000 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \left( -215.998492618346 \right. \right. \\
& \quad \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \quad 35.9998325129548 \\
& \quad \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \right. \right. \\
& \quad 5.99998604273000 \\
& \quad \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \quad \left. \left. 1.0000000000000000 \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} \right. \right. \\
& \quad \left. \left. (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right) \Bigg)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

### Integral representations:

$$\begin{aligned}
& \frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)} = \\
& \int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \\
& \frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)} = \\
& \int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \text{ for } -1 < \gamma < 0
\end{aligned}$$

$$\frac{1}{\log(2) \left( 1 - \frac{5.999986042730000}{(3 \times 1)!} + \frac{5.999986042730000^2}{(7 \times 2)!} - \frac{5.999986042730000^3}{(15 \times 3)!} \right)} =$$

$$1 / \left( \log(2) \left( 1 - \frac{5.999986042730000}{\int_1^\infty e^{-t} t^3 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(4+k)k!}} + \right. \right.$$

$$\left. \left. \frac{35.99983251295481}{\int_1^\infty e^{-t} t^{14} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(15+k)k!}} - \frac{215.9984926183465}{\int_1^\infty e^{-t} t^{45} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(46+k)k!}} \right) \right)$$

$$-(((\ln 2(((1-x/(3*1)!)+x^2/(7*2)!-x^3/(15*3)!))))))$$

**Input:**

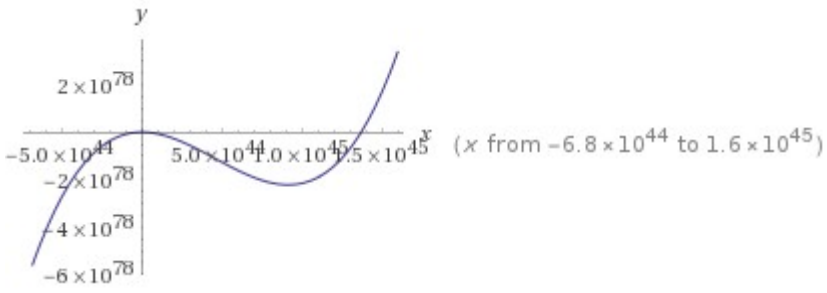
$$-\left( \log(2) \left( 1 - \frac{x}{(3 \times 1)!} + \frac{x^2}{(7 \times 2)!} - \frac{x^3}{(15 \times 3)!} \right) \right)$$

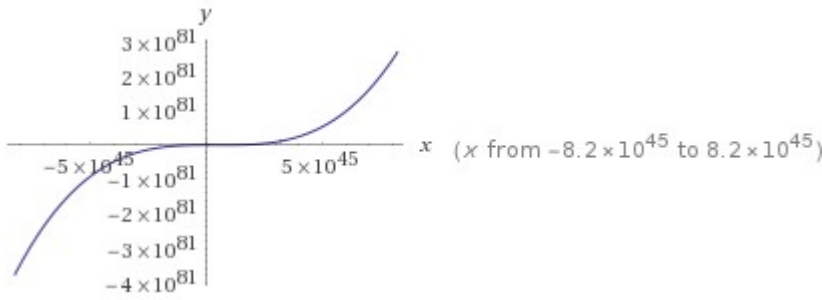
log(x) is the natural logarithm  
n! is the factorial function

**Exact result:**

$$-\left( -\frac{x^3}{119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000} + \frac{x^2}{87\,178\,291\,200} - \frac{x}{6} + 1 \right) \log(2)$$

**Plots:**





**Alternate forms:**

$$x \left( x \left( x \log(2) \right) / \right. \\ \left. 119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760 \cdot \right. \\ \left. 000\,000\,000 - \frac{\log(2)}{87\,178\,291\,200} \right) + \frac{\log(2)}{6} \Big) - \log(2) \\ ((x^3 - 1\,372\,156\,063\,383\,359\,761\,953\,709\,428\,703\,022\,284\,800\,000\,000\,000\,000\,000\,000\,000\,000 \cdot \\ 19\,937\,036\,810\,913\,365\,760\,327\,193\,582\,609\,619\,177\,397\,288\,960\,000\,000\,000\,000\,000 \cdot \\ 000\,000 \cdot \\ 119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000 \cdot \\ 000) \log(2) / \\ 119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000\,000 \\ - \left( - \left( (x - 457\,385\,354\,461\,119\,920\,651\,236\,476\,234\,340\,761\,600\,000\,000) \right)^3 / \right. \\ \left. 119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760 \cdot \right. \\ \left. 000\,000\,000) + \frac{1}{6} \\ 31\,479\,306\,246\,905\,646\,206\,419\,550\,207\,999\,999 \\ (x - 457\,385\,354\,461\,119\,920\,651\,236\,476\,234\,340\,761\,600\,000\,000) + \\ 1599\,797\,071\,770\,120\,621\,494\,563\,038\,355\,093\,662\,753\,539\,433\,181\,734\,915 \cdot \\ \left. 113\,049\,340\,143\,206\,400\,000\,001 \right) \log(2)$$

**Expanded form:**

$$\frac{x^3 \log(2)}{119\,622\,220\,865\,480\,194\,561\,963\,161\,495\,657\,715\,064\,383\,733\,760\,000\,000\,000\,000} - \\ \frac{x^2 \log(2)}{87\,178\,291\,200} + \frac{1}{6} x \log(2) - \log(2)$$

**Roots:**

$x \approx 6$

$x \approx 14529715194$

[14529715194](#)

$$x \approx 1\,372\,156\,063\,383\,359\,761\,953\,709\,430\,358\,354\,116\,132\,995\,072$$

**Polynomial discriminant:**

$$\Delta_x = \frac{(1\,937\,049\,649\,351\,556\,898\,123\,342\,031\,656\,389\,549\,306\,638\,026\,036\,939\,622\,320 \cdot 880\,160\,809\,843\,296\,514\,867\,199\,999\,999 \log^4(2)) / (529\,980\,582\,399\,619\,452\,893\,477\,372\,284\,140\,772\,307\,073\,481\,102\,003\,002\,926 \cdot 358\,239\,420\,364\,608\,942\,414\,147\,143\,650\,508\,800\,000\,000\,000\,000\,000\,000)}{x^2} + \left( \frac{x}{43\,589\,145\,600} - \frac{1}{6} \right) \log(2)$$

**Derivative:**

$$\frac{d}{dx} \left( - \left( \log(2) \left( 1 - \frac{x}{(3 \times 1)!} + \frac{x^2}{(7 \times 2)!} - \frac{x^3}{(15 \times 3)!} \right) \right) \right) = - \left( \frac{39\,874\,073\,621\,826\,731\,520\,654\,387\,165\,219\,238\,354\,794\,577\,920\,000\,000\,000}{x^2} + \frac{x}{43\,589\,145\,600} - \frac{1}{6} \right) \log(2)$$

**Indefinite integral:**

$$\int - \left( \log(2) \left( 1 - \frac{x}{(3 \times 1)!} + \frac{x^2}{(7 \times 2)!} - \frac{x^3}{(15 \times 3)!} \right) \right) dx = - \left( \frac{x^4}{478\,488\,883\,461\,920\,778\,247\,852\,645\,982\,630\,860\,257\,534\,935 \cdot 040\,000\,000\,000} + \frac{x^3}{261\,534\,873\,600} - \frac{x^2}{12} + x \right) \log(2) + \text{constant}$$

$$1 / \left( \left( \ln 2 \left( \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right) \right) \right) \right)$$

**Input:**

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)}$$

log(x) is the natural logarithm

n! is the factorial function

**Exact result:**

$$\frac{553\,806\,578\,080\,926\,826\,675\,755\,377\,294\,711\,643\,816\,591\,360\,000\,000\,000}{228\,692\,677\,230\,559\,946\,124\,663\,168\,493\,930\,073\,073\,062\,401 \log(2)}$$

**Decimal approximation:**

$$3.49365801076149907230323661978603930408690860629724192... \times 10^9$$

**Input interpretation:**

$$3.49365801076149907230323661978603930408690860629724192 \times 10^9$$

**Decimal form:**

3493658010.76149907230323661978603930408690860629724192

3493658010.76149..... result very near to the following algebraic sum concerning the Prime Number Theorem:

$10^{11}$	4 118 054 813	169 923 159	1,043	11 588
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$10^{10}$	455 052 511	20 758 029	1,048	3 104
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$10^9$	50 847 534	2 592 592	1,054	1.701
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$10^7$	664 579	44 158	1,071
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$$(4118054813 - 169923159 - 1.043 - 11588 - 455052511 - 1.048 - 3104 - 1.054 - 1701 + 664579 - 44158)$$

**Input:**

$$4118054813 - 169923159 - 1.043 - 11588 - 455052511 - 1.048 - 3104 - 1.054 - 1701 + 664579 - 44158$$

**Result:**

$3.493683167855 \times 10^9$

3493683167.855

**Property:**

553 806 578 080 926 826 675 755 377 294 711 643 816 591 360 000 000 000

228 692 677 230 559 946 124 663 168 493 930 073 073 062 401  $\log(2)$

is a transcendental number

**Alternative representations:**

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{1}{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{\Gamma(4)} + \frac{14529715194^2}{\Gamma(15)} - \frac{14529715194^3}{\Gamma(46)} \right)}$$

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{1}{\log_e(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}$$

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{1}{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}$$

**Series representations:**

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

553 806 578 080 926 826 675 755 377 294 711 643 816 591 360 000 000 000 /

( 228 692 677 230 559 946 124 663 168 493 930 073 073 062 401

$$\left( 2i\pi \left[ \frac{\arg(2-x)}{2\pi} \right] + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \text{ for } x < 0$$

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{55380657808092682667575537729471164381659136000000000}{\left( 228692677230559946124663168493930073073073062401 \right.}$$

$$\left. \left( \log(z_0) + \left\lfloor \frac{\arg(2 - z_0)}{2\pi} \right\rfloor \left( \log\left(\frac{1}{z_0}\right) + \log(z_0) \right) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right)$$

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{55380657808092682667575537729471164381659136000000000}{\left( 228692677230559946124663168493930073073073062401 \right.}$$

$$\left. \left( 2i\pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2 - z_0)^k z_0^{-k}}{k} \right) \right)$$

### Integral representations:

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{55380657808092682667575537729471164381659136000000000}{228692677230559946124663168493930073073073062401 \int_1^2 \frac{1}{t} dt}$$

$$\frac{1}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$(1107613156161853653351510754589423287633182720000000000i\pi) /$$

$$\left( 228692677230559946124663168493930073073073062401 \right.$$

$$\left. \int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(-s)^2 \Gamma(1+s)}{\Gamma(1-s)} ds \right) \text{ for } -1 < \gamma < 0$$

We note that, we obtain also:

$$(3.426486 * \pi) / (((\ln 2(((1 - (14529715194) / (3 * 1)! + (14529715194)^2 / (7 * 2)! - (14529715194)^3 / (15 * 3)!))))))$$

**Input interpretation:**

$$\frac{3.426486 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)}$$

log(x) is the natural logarithm

n! is the factorial function

**Result:**

$$3.7607912233521212299789337310144491506028977634701717... \times 10^{10}$$

$$3.76079122... * 10^{10} \text{ value very near to } 3.7607912018 * 10^{10}$$

**Alternative representations:**

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \frac{3.42649 \pi}{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{\Gamma(4)} + \frac{14529715194^2}{\Gamma(15)} - \frac{14529715194^3}{\Gamma(46)} \right)}$$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \frac{3.42649 \pi}{\log_e(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}$$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \frac{3.42649 \pi}{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}$$



### Series representations:

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$(3.42649 \pi) / \left( \log(2) \left( 1 - \frac{14529715194}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} + \frac{211112623618754457636}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!} - \frac{3067406295038619906469018521384}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right) \right)$$

for  $(n_0 \geq 0$  or  $n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45)$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\left( 1.71324 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right)$$

$$/ \left( \left( i \pi \left[ \frac{\arg(2-x)}{2\pi} \right] + 0.5 \log(x) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right.$$

$$\left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right.$$

$$2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} -$$

$$1.45297 \times 10^{10} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} +$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right)$$

for  $(x < 0$  and  $(n_0 \geq 0$  or  $n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45)$

$$\begin{aligned}
& \frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \\
& \left( \frac{1.71324 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!}}{\left( \left( i \pi \left[ -\frac{-\pi + \arg\left(\frac{2}{z_0}\right) + \arg(z_0)}{2\pi} \right] + 0.5 \log(z_0) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right.} \right. \\
& \left. \left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. 2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \right. \right. \\
& \left. \left. 1.45297 \times 10^{10} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \right. \right. \\
& \left. \left. \left. \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right) \right)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\left( \frac{3.42649 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!}}{\left( \left[ \left| \frac{\arg(2-z_0)}{2\pi} \right| \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right.}$$

$$\left. \left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right.$$

$$\left. 2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \right.$$

$$\left. 1.45297 \times 10^{10} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right.$$

$$\left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right)$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

### Integral representations:

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1$$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \text{ for } -1 < \gamma < 0$$

$$\frac{3.42649 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = (3.42649 \pi) /$$

$$\left( \log(2) \left( 1 - \frac{14529715194}{\int_1^\infty e^{-t} t^3 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(4+k)k!}} + \frac{211112623618754457636}{\int_1^\infty e^{-t} t^{14} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(15+k)k!}} - \frac{3067406295038619906469018521384}{\int_1^\infty e^{-t} t^{45} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(46+k)k!}} \right) \right)$$

and:

$$(2719.2 * \text{Pi}) / (((\ln 2(((1 - (14529715194)/(3*1)! + (14529715194)^2/(7*2)! - (14529715194)^3/(15*3)!))))))$$

### Input interpretation:

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)}$$

$\log(x)$  is the natural logarithm

$n!$  is the factorial function

### Result:

$$2.9844988406603990352094584951972633567799196023062959... \times 10^{13}$$

$2.984498840... * 10^{13}$  result very near to the value  $2.9844570422669 * 10^{13}$

### Alternative representations:

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{2719.2 \pi}{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{\Gamma(4)} + \frac{14529715194^2}{\Gamma(15)} - \frac{14529715194^3}{\Gamma(46)} \right)}$$

$$\frac{2719.2 \pi}{\log_e(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}$$

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\frac{\log(a) \log_a(2) \left( 1 - \frac{14529715194}{(1)_3} + \frac{14529715194^2}{(1)_{14}} - \frac{14529715194^3}{(1)_{45}} \right)}{2719.2 \pi}$$

### Series representations:

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$(2719.2 \pi) / \left( \log(2) \left( 1 - \frac{14529715194}{\sum_{k=0}^{\infty} \frac{(3-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} + \frac{211112623618754457636}{\sum_{k=0}^{\infty} \frac{(14-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} - \right. \right.$$

$$\left. \left. \frac{3067406295038619906469018521384}{\sum_{k=0}^{\infty} \frac{(45-n_0)^k \Gamma^{(k)}(1+n_0)}{k!}} \right) \right)$$

for  $(n_0 \geq 0$  or  $n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$ )

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$\left( 1359.6 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right)$$

$$/ \left( \left( i \pi \left\lfloor \frac{\arg(2-x)}{2\pi} \right\rfloor + 0.5 \log(x) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-x)^k x^{-k}}{k} \right) \right.$$

$$\left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right.$$

$$2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} -$$

$$1.45297 \times 10^{10} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} +$$

$$\left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right)$$

for  $(x < 0$  and  $(n_0 \geq 0$  or  $n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$ )

$$\begin{aligned}
& \frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \\
& \left( 1359.6 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!} \right) \\
& / \\
& \left( \left( i \pi \left[ -\frac{-\pi + \arg\left(\frac{2}{z_0}\right) + \arg(z_0)}{2\pi} \right] + 0.5 \log(z_0) - 0.5 \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right. \\
& \left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \\
& 2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \\
& 1.45297 \times 10^{10} \\
& \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \\
& \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \right. \\
& \left. \left. \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$

$$\begin{aligned}
& \frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \\
& \left( \frac{2719.2 \pi \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0)}{k_1! k_2! k_3!}}{\left( \left[ \left| \frac{\arg(2-z_0)}{2\pi} \right| \log\left(\frac{1}{z_0}\right) + \log(z_0) + \left[ \frac{\arg(2-z_0)}{2\pi} \right] \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k (2-z_0)^k z_0^{-k}}{k} \right) \right.} \\
& \left. \left( -3.06741 \times 10^{30} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (14-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. 2.11113 \times 10^{20} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(3-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} - \right. \right. \\
& \left. \left. 1.45297 \times 10^{10} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(14-n_0)^{k_1} (45-n_0)^{k_2} \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0)}{k_1! k_2!} + \right. \right. \\
& \left. \left. \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{k_3=0}^{\infty} \frac{1}{k_1! k_2! k_3!} (3-n_0)^{k_1} (14-n_0)^{k_2} (45-n_0)^{k_3} \right. \right. \\
& \left. \left. \left. \Gamma^{(k_1)}(1+n_0) \Gamma^{(k_2)}(1+n_0) \Gamma^{(k_3)}(1+n_0) \right) \right) \right)
\end{aligned}$$

for  $(n_0 \geq 0 \text{ or } n_0 \notin \mathbb{Z})$  and  $n_0 \rightarrow 3$  and  $n_0 \rightarrow 14$  and  $n_0 \rightarrow 45$ )

## Integral representations:

$$\begin{aligned}
& \frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \\
& \int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \\
& \frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} = \\
& \int_0^1 \int_0^1 \int_0^1 \log^3\left(\frac{1}{t_1}\right) \log^{14}\left(\frac{1}{t_2}\right) \log^{45}\left(\frac{1}{t_3}\right) dt_3 dt_2 dt_1 \text{ for } -1 < \gamma < 0
\end{aligned}$$

$$\frac{2719.2 \pi}{\log(2) \left( 1 - \frac{14529715194}{(3 \times 1)!} + \frac{14529715194^2}{(7 \times 2)!} - \frac{14529715194^3}{(15 \times 3)!} \right)} =$$

$$(2719.2 \pi) / \left( \log(2) \left( 1 - \frac{14529715194}{\int_1^\infty e^{-t} t^3 dt + \sum_{k=0}^\infty \frac{(-1)^k}{(4+k)k!}} + \frac{211112623618754457636}{\int_1^\infty e^{-t} t^{14} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(15+k)k!}} - \frac{3067406295038619906469018521384}{\int_1^\infty e^{-t} t^{45} dt + \sum_{k=0}^\infty \frac{(-1)^k}{(46+k)k!}} \right) \right)$$

From the ratio of the two results, performing the 18<sup>th</sup> root, we obtain:

$$(3493683167.855 / 620080.709928)^{1/18}$$

**Input interpretation:**

$$\sqrt[18]{\frac{3.493683167855 \times 10^9}{620080.709928}}$$

**Result:**

1.615770738004...

[1.615770738004...](#)

Or, from the following values concerning Prime Number Theorem

x	$\pi(x)$	$\pi(x) - x / \ln x$
$10^{11}$	4 118 054 813	169 923 159
$10^7$	664 579	44 158

we obtain:



$$(4118054813 / 664579)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{4\,118\,054\,813}{664\,579}}$$

**Decimal approximation:**

1.624331814232483790805056207712866227532386597551114202241...

1.62433181423...

**Alternate form:**

$$\frac{\sqrt[18]{4\,118\,054\,813} \cdot 664\,579^{17/18}}{664\,579}$$

From the mean of two expressions, we obtain:

$$((3493683167.855 / 620080.709928)^{1/18} + (4118054813 / 664579)^{1/18})/2$$

**Input interpretation:**

$$\frac{1}{2} \left( \sqrt[18]{\frac{3.493683167855 \times 10^9}{620\,080.709928}} + \sqrt[18]{\frac{4\,118\,054\,813}{664\,579}} \right)$$

**Result:**

1.6200512761183...

1.6200512761183...

We have also:

$$(664579/168)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{664\,579}{168}}$$

**Result:**

$$\frac{\sqrt[18]{\frac{664579}{21}}}{\sqrt[6]{2}}$$

**Decimal approximation:**

1.584333183323270308087964453492573173106589909653941136428...

1.58433318332...

**Alternate form:**

root of  $168x^{18} - 664579$  near  $x = 1.58433$

**All 18th roots of 664579/168:**

$$\frac{\sqrt[18]{\frac{664579}{21}} e^{i0}}{\sqrt[6]{2}} \approx 1.58433 \quad (\text{real, principal root})$$

$$\frac{\sqrt[18]{\frac{664579}{21}} e^{i\pi/9}}{\sqrt[6]{2}} \approx 1.48879 + 0.5419 i$$

$$\frac{\sqrt[18]{\frac{664579}{21}} e^{2i\pi/9}}{\sqrt[6]{2}} \approx 1.2137 + 1.0184 i$$

$$\frac{\sqrt[18]{\frac{664579}{21}} e^{i\pi/3}}{\sqrt[6]{2}} \approx 0.79217 + 1.3721 i$$

$$\frac{\sqrt[18]{\frac{664579}{21}} e^{4i\pi/9}}{\sqrt[6]{2}} \approx 0.27512 + 1.56026 i$$

We note that, from the following Table (from Wikipedia):

$x$	$\pi(x)$	$\pi(x) - x / \ln x$	$\frac{\pi(x)}{x / \ln x}$	$\text{Li}(x) - \pi(x)$	$\pi(x) / \text{Li}(x)$	$\frac{x}{\pi(x)}$
10	4	-0,3	0,921	2,2	0,64516129	2,500
$10^2$	25	3,3	1,151	5,1	0,830564784	4,000
$10^3$	168	23	1,161	10	0,943820225	5,952
$10^4$	1 229	143	1,132	17	0,98635634	8,137
$10^5$	9 592	906	1,104	38	0,996053998	10,425
$10^6$	78 498	6 116	1,084	130	0,998346645	12,740
$10^7$	664 579	44 158	1,071	339	0,999490163	15,047
$10^8$	5 761 455	332 774	1,061	754	0,999869147	17,357
$10^9$	50 847 534	2 592 592	1,054	1.701	0,999966548	19,667
$10^{10}$	455 052 511	20 758 029	1,048	3 104	0,999993179	21,975
$10^{11}$	4 118 054 813	169 923 159	1,043	11 588	0,999993179	24,283
$10^{12}$	37 607 912 018	1 416 705 193	1,039	38 263	0,999997186	26,590
$10^{13}$	346 065 536 839	11 992 858 452	1,034	108 971	0,999998983	28,896
$10^{14}$	3 204 941 750 802	102 838 308 636	1,033	314 890	0,999999685	31,202
$10^{15}$	29 844 570 422 669	891 604 962 452	1,031	1 052 619	0,999999902	33,507
$10^{16}$	279 238 341 033 925	7 804 289 844 393	1,029	3 214 632	0,999999965	35,812
$10^{17}$	2 623 557 157 654 233	68 883 734 693 281	1,027	7 956 589	0,999999988	38,116
$10^{18}$	24 739 954 287 740 860	612 483 070 893 536	1,025	21 949 555	0,999999997	40,420
$10^{19}$	234 057 667 276 344 607	5 481 624 169 369 960	1,024	99 877 775	0,999999999	42,725
$10^{20}$	2 220 819 602 560 918 840	49 347 193 044 659 701	1,023	222 744 644	1,000000000	45,028
$10^{21}$	21 127 269 486 018 731 928	446 579 871 578 168 707	1,022	597 394 254	1,000000000	47,332
$10^{22}$	201 467 286 689 315 906 290	4 060 704 006 019 620 994	1,021	1 932 355 208	1,000000000	49,636
$10^{23}$	1 925 320 391 606 818 006 727	37 083 513 766 592 669 113	1,020	7 236 148 412	1,000000000	51,939

We obtain the following interesting mathematical connections:

$$(9592/4)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{9592}{4}}$$

**Result:**

$$\sqrt[18]{2398}$$

**Decimal approximation:**

1.540882059926678322421659818907171314837670343186868694886...

1.5408820599266783....

**All 18th roots of 2398:**

$$\sqrt[18]{2398} e^0 \approx 1.540882 \text{ (real, principal root)}$$

$$\sqrt[18]{2398} e^{(i\pi)/9} \approx 1.44796 + 0.5270 i$$

$$\sqrt[18]{2398} e^{(2i\pi)/9} \approx 1.1804 + 0.9905 i$$

$$\sqrt[18]{2398} e^{(i\pi)/3} \approx 0.77044 + 1.3344 i$$

$$\sqrt[18]{2398} e^{(4i\pi)/9} \approx 0.26757 + 1.51747 i$$

$$(78498/25)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{78\,498}{25}}$$

**Exact result:**

$$\sqrt[9]{\frac{21}{5}} \sqrt[18]{178}$$

**Decimal approximation:**

1.564131472120657585251775339556746589935426988024405402709...

1.56413147212065....

$$(50847534 / 9592)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{50\,847\,534}{9592}}$$

**Result:**

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941\,621}{1199}}}{\sqrt[9]{2}}$$

**Decimal approximation:**

1.610307920928417017761430919405480608489523266418111241803...

1.610307920928417....

**Alternate form:**root of  $4796 x^{18} - 25423767$  near  $x = 1.61031$ **All 18th roots of 25423767/4796:**

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941621}{1199}} e^0}{\sqrt[9]{2}} \approx 1.61031 \text{ (real, principal root)}$$

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941621}{1199}} e^{(i\pi)/9}}{\sqrt[9]{2}} \approx 1.51319 + 0.5508 i$$

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941621}{1199}} e^{(2i\pi)/9}}{\sqrt[9]{2}} \approx 1.2336 + 1.0351 i$$

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941621}{1199}} e^{(i\pi)/3}}{\sqrt[9]{2}} \approx 0.8052 + 1.3946 i$$

$$\frac{\sqrt[6]{3} \sqrt[18]{\frac{941621}{1199}} e^{(4i\pi)/9}}{\sqrt[9]{2}} \approx 0.27963 + 1.58584 i$$

$(3204941750802 / 455052511)^{1/18}$

**Input:**

$$\sqrt[18]{\frac{3\,204\,941\,750\,802}{455\,052\,511}}$$

**Result:**

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}}$$

**Decimal approximation:**

1.635928686983868827528162061384938650768886510848504451936...

1.6359286869838.....

**All 18th roots of 3204941750802/455052511**

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}} e^0 \approx 1.63593 \text{ (real, principal root)}$$

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}} e^{(i\pi)/9} \approx 1.53727 + 0.5595 i$$

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}} e^{(2i\pi)/9} \approx 1.2532 + 1.0516 i$$

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}} e^{(i\pi)/3} \approx 0.81796 + 1.4168 i$$

$$\sqrt[6]{3} \sqrt[18]{\frac{118\,701\,546\,326}{455\,052\,511}} e^{(4i\pi)/9} \approx 0.28408 + 1.61108 i$$

$(21127269486018731928 / 2623557157654233)^{1/18}$

**Input:**

$$\sqrt[18]{\frac{21\,127\,269\,486\,018\,731\,928}{2\,623\,557\,157\,654\,233}}$$

**Result:**

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}}$$

**Decimal approximation:**

1.648152448906887728782722352713932534909452975975684199033...

1.6481524489068877....

**All 18th roots of 7042423162006243976/874519052551411:**

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}} e^0 \approx 1.64815 \text{ (real, principal root)}$$

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}} e^{(i\pi)/9} \approx 1.54876 + 0.5637 i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}} e^{(2i\pi)/9} \approx 1.2626 + 1.0594i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}} e^{(i\pi)/3} \approx 0.82408 + 1.4273i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{880\,302\,895\,250\,780\,497}{874\,519\,052\,551\,411}} e^{(4i\pi)/9} \approx 0.28620 + 1.62311i$$

$(1925320391606818006727 / 234057667276344607)^{1/18}$

**Input:**

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}}$$

**Decimal approximation:**

1.650099024997094439272719440817175234129925511065623775089...

1.650099024997094439.....

**Alternate form:**

$$\frac{\sqrt[18]{1\,925\,320\,391\,606\,818\,006\,727} \sqrt[17]{234\,057\,667\,276\,344\,607}}{234\,057\,667\,276\,344\,607}$$

**All 18th roots of 1925320391606818006727/234057667276344607:**

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}} e^0 \approx 1.650099 \text{ (real, principal root)}$$

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}} e^{(i\pi)/9} \approx 1.55059 + 0.5644i$$

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}} e^{(2i\pi)/9} \approx 1.2640 + 1.0607i$$

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}} e^{(i\pi)/3} \approx 0.82505 + 1.4290 i$$

$$\sqrt[18]{\frac{1\,925\,320\,391\,606\,818\,006\,727}{234\,057\,667\,276\,344\,607}} e^{(4i\pi)/9} \approx 0.28654 + 1.62503 i$$

$$(234057667276344607 / 29844570422669)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{234\,057\,667\,276\,344\,607}{29\,844\,570\,422\,669}}$$

**Result:**

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}}}{\sqrt[9]{19}}$$

**Decimal approximation:**

1.645730630078931997881403228219966724829355307607447620548...

1.64573063007893199....

**All 18th roots of 234057667276344607/29844570422669:**

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}} e^0}{\sqrt[9]{19}} \approx 1.64573 \text{ (real, principal root)}$$

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}} e^{(i\pi)/9}}{\sqrt[9]{19}} \approx 1.54648 + 0.5629 i$$

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}} e^{(2i\pi)/9}}{\sqrt[9]{19}} \approx 1.2607 + 1.0579 i$$

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}} e^{(i\pi)/3}}{\sqrt[9]{19}} \approx 0.82287 + 1.4252 i$$

$$\frac{\sqrt[18]{\frac{234057\,667276\,344607}{82\,671940\,229}} e^{(4i\pi)/9}}{\sqrt[9]{19}} \approx 0.28578 + 1.62073 i$$



$$(29844570422669 / 4118054813)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{29\,844\,570\,422\,669}{4\,118\,054\,813}}$$

**Result:**

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}}$$

**Decimal approximation:**

1.638528754376369092359768164613456552143708993760130791940...

1.638528754376369.....

**All 18th roots of 1570766864351/216739727:**

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}} e^{0} \approx 1.638529 \quad (\text{real, principal root})$$

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}} e^{(i\pi)/9} \approx 1.53971 + 0.5604 i$$

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}} e^{(2i\pi)/9} \approx 1.2552 + 1.0532 i$$

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}} e^{(i\pi)/3} \approx 0.81926 + 1.4190 i$$

$$\sqrt[18]{\frac{1\,570\,766\,864\,351}{216\,739\,727}} e^{(4i\pi)/9} \approx 0.28453 + 1.61364 i$$

The mean of all results is:

$$(1.62433181423 + 1.5843331833232703080 + 1.650099024997094439 + 1.64573063007893199 + 1.638528754376369)/5$$

**Input interpretation:**

$$\frac{1}{5} (1.62433181423 + 1.5843331833232703080 + 1.650099024997094439 + 1.64573063007893199 + 1.638528754376369)$$

**Result:**

1.6286046814011331474

1.6286046814011331474

(1.62433181423+ 1.5843331833232703080 +  
 1.650099024997094439+1.64573063007893199+1.638528754376369)/5 -  
 11/10<sup>3</sup>+ 5/10<sup>4</sup>

**Input interpretation:**

$$\frac{1}{5} (1.62433181423 + 1.5843331833232703080 + 1.650099024997094439 +$$

$$1.64573063007893199 + 1.638528754376369) - \frac{11}{10^3} + \frac{5}{10^4}$$

**Result:**

1.6181046814011331474

1.6181046814011331474

And also:

(201467286689315906290 / 24739954287740860)<sup>1/18</sup>**Input:**

$$\sqrt[18]{\frac{201\ 467\ 286\ 689\ 315\ 906\ 290}{24\ 739\ 954\ 287\ 740\ 860}}$$

**Result:**

$$\sqrt[18]{\frac{20\ 146\ 728\ 668\ 931\ 590\ 629}{2\ 473\ 995\ 428\ 774\ 086}}$$

**Decimal approximation:**

1.649175902220077641794167141875437150461886006342390439783...

1.64917590222007764.....

**Alternate form:**

$$\frac{\sqrt[18]{20\ 146\ 728\ 668\ 931\ 590\ 629\ 2\ 473\ 995\ 428\ 774\ 086^{17/18}}}{2473\ 995\ 428\ 774\ 086}$$

**All 18th roots of 20146728668931590629/2473995428774086:**

$$\sqrt[18]{\frac{20\,146\,728\,668\,931\,590\,629}{2\,473\,995\,428\,774\,086}} e^0 \approx 1.649176 \text{ (real, principal root)}$$

$$\sqrt[18]{\frac{20\,146\,728\,668\,931\,590\,629}{2\,473\,995\,428\,774\,086}} e^{(i\pi)/9} \approx 1.54972 + 0.5641 i$$

$$\sqrt[18]{\frac{20\,146\,728\,668\,931\,590\,629}{2\,473\,995\,428\,774\,086}} e^{(2i\pi)/9} \approx 1.2633 + 1.0601 i$$

$$\sqrt[18]{\frac{20\,146\,728\,668\,931\,590\,629}{2\,473\,995\,428\,774\,086}} e^{(i\pi)/3} \approx 0.82459 + 1.4282 i$$

$$\sqrt[18]{\frac{20\,146\,728\,668\,931\,590\,629}{2\,473\,995\,428\,774\,086}} e^{(4i\pi)/9} \approx 0.28638 + 1.62412 i$$

$$(2623557157654233 / 346065536839)^{1/18}$$

**Input:**

$$\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{346\,065\,536\,839}}$$

**Result:**

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}}}{\sqrt[9]{11}}$$

**Decimal approximation:**

1.642633503621954394450749140924515710577984938855983356254...

1.64263350362195.....

**Alternate form:**

root of $346\,065\,536\,839 x^{18} - 2\,623\,557\,157\,654\,233$ near $x = 1.64263$
---

**All 18th roots of 2623557157654233/346065536839:**

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}} e^0}{\sqrt[9]{11}} \approx 1.64263 \text{ (real, principal root)}$$

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}} e^{(i\pi)/9}}{\sqrt[9]{11}} \approx 1.54357 + 0.5618 i$$

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}} e^{(2i\pi)/9}}{\sqrt[9]{11}} \approx 1.2583 + 1.0559 i$$

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}} e^{(i\pi)/3}}{\sqrt[9]{11}} \approx 0.82132 + 1.4226 i$$

$$\frac{\sqrt[18]{\frac{2\,623\,557\,157\,654\,233}{2\,860\,045\,759}} e^{(4i\pi)/9}}{\sqrt[9]{11}} \approx 0.28524 + 1.61768 i$$

$(2220819602560918840 / 279238341033925)^{1/18}$

**Input:**

$$\sqrt[18]{\frac{2\,220\,819\,602\,560\,918\,840}{279\,238\,341\,033\,925}}$$

**Result:**

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}}$$

**Decimal approximation:**

1.647011260802406796615541167014174488180388373572217984617...

1.6470112608024....

**All 18th roots of 34166455424014136/4295974477445:**

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}} e^0 \approx 1.64701 \text{ (real, principal root)}$$

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}} e^{(i\pi)/9} \approx 1.54768 + 0.5633 i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}} e^{(2i\pi)/9} \approx 1.2617 + 1.0587 i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}} e^{(i\pi)/3} \approx 0.82351 + 1.4264 i$$

$$\sqrt[6]{2} \sqrt[18]{\frac{4\,270\,806\,928\,001\,767}{4\,295\,974\,477\,445}} e^{(4i\pi)/9} \approx 0.28600 + 1.62199 i$$

The final mean is:

$$(1.5408820599266783+1.56413147212065+1.610307920928417+1.6359286869838+1.6481524489068877+1.650099024997094439+1.64573063007893199+1.638528754376369+1.64917590222007764+1.64263350362195+1.6470112608024)/11$$

1.62478015136029600627 (period 2)

1.62478015136029600627...

From the previous Table that compares the three functions  $\pi(x)$ ,  $x/\ln(x)$  and  $\text{Li}(x)$ , we have obtained, performing the above ratio, a value that approximates the golden ratio. It is practically an average between  $\phi$  and  $\zeta(2)$ . What has been obtained could indicate a connection between the Prime Number Theorem,  $\zeta(2)$  and  $\phi$ .

## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJIQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJIQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the

*golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the*



*second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

*2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).*

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson  $\pi$ ) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

## References

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## II

### RAMANUJAN AND THE THEORY OF PRIME NUMBERS

16 Jan. 1913