Velocity Transformation of Standing Wave

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A standing wave can be formed in a microwave resonator if the length of the resonator cavity is equal to multiple half wavelengths. The stationary standing wave becomes a moving standing wave in another inertial reference frame. The covariance property of the moving standing wave verifies that the frequencies of two microwaves forming the standing wave become different in the new reference frame while the wavelengths remain identical. Hence, the apparent speed of the microwave appears to be different in a different inertial reference frame.

I. INTRODUCTION

A standing wave can be formed between the wave transmitter and the reflector if the distance between the transmitter and the reflector is equal to multiple half wavelengths. The standing wave exhibits the first harmonic if the distance is equal to one half of the wavelength. The standing wave exhibits the second harmonic if the distance is equal to the wavelength.

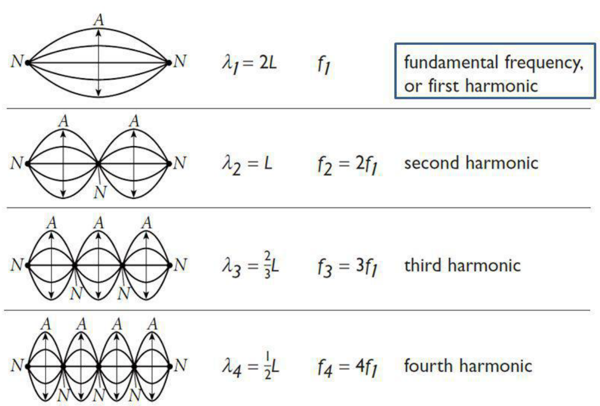

FIG. 1. Standing Wave harmonic

The stationary standing wave becomes a moving standing wave in another inertial reference frame. The frequencies and the wavelengths of both waves that form the standing wave may vary. However, the standing wave formation is conserved in inertial reference frames. The physics of the standing wave can not be altered by the choice of inertial reference frame. The new frequency and wavelength in a new inertial reference frame can be derived from the same physics and wave formation precisely.

II. PROOF

Consider one dimensional motion.

A. Standing Wave

A standing wave formed by two identical waves travelling in the opposite direction can be represented as

$$sin(k'x' - w't') + sin(-k'x' - w't') \tag{1}$$

Let F_2 be another inertial reference frame moving at a velocity of -V relative to F_1, the rest frame for all nodes of the standing wave. In F_2, the standing wave can be represented as

$$sin(k_1x - W_1t) + sin(-k_2x - W_2t) \tag{2}$$

Define W_+ and W_- as

$$W_+ = \frac{W_1 + W_2}{2} \tag{3}$$

$$W_- = \frac{W_1 - W_2}{2} \tag{4}$$

From equations (2,3,4), the standing wave is

$$sin(k_1x - W_+t - W_-t) \tag{5}$$

$$+sin(-k_2x - W_+t + W_-t) \tag{6}$$

Define k_+ and k_- as

$$k_+ = \frac{k_1 + k_2}{2} \tag{7}$$

$$k_- = \frac{k_1 - k_2}{2} \tag{8}$$

From equations (5,6,7,8), the standing wave is

$$sin(k_+x + k_-x - W_+t - W_-t) \tag{9}$$

$$+sin(-k_+x + k_-x - W_+t + W_-t) \tag{10}$$

Apply trigonometry identity.

$$sin(A + B) = sin(A)cos(B) + sin(B)cos(A) \tag{11}$$

From equations (9,10,11), the standing wave in F_2 is

$$2sin(k_- x - W_+ t)cos(k_+ x - W_- t) \tag{12}$$

From equations (1,11), the standing wave in F_1 is

$$2sin(-w't')cos(k'x') \tag{13}$$

$2sin(-w't')$ is the amplitude of the static wave $cos(k'x')$.

In F_2, all nodes travel at the velocity of V. The static wave of equation (13) can be represented in F_2 as

$$2sin(-Wt)cos(kx - wt) \tag{14}$$

$$\frac{dx}{dt} = \frac{w}{k} = V \tag{15}$$

From equations (12,14) under the initial condition that $0 = x = x' = t = t'$,

$$k_- = 0 \tag{16}$$

$$W = W_+ \tag{17}$$

$$k = k_+ \tag{18}$$

$$w = W_- \tag{19}$$

B. Microwave Resonance

Let a microwave transmitter and a reflector plate be stationary relative to F_1. The microwave is emitted in the positive x direction toward the reflector which is in the y-z plane. A standing wave can be formed by adjusting the distance between the transmitter and the reflector until the distance is equal to multiple half wavelengths[1].

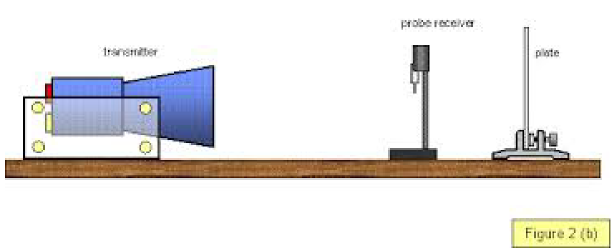

FIG. 2. Microwave Transmitter and Reflector Plate

In F_2, the standing wave and the reflector are moving at the velocity of V. One microwave is represented by k_1 and W_1. The other microwave is represented by k_2 and W_2.

From equations (7,8,16,18),

$$k_1 = k_2 = k \tag{20}$$

The wavelengths of both microwaves are identical in F_2.

From equations (3,17),

$$W_1 + W_2 = 2W \tag{21}$$

From equations (4,19),

$$W_1 - W_2 = 2w \tag{22}$$

From equations (21,22),

$$W_1 = W + w \tag{23}$$

$$W_2 = W - w \tag{24}$$

One microwave travels at the velocity of v_1.

$$v_1 = \frac{W_1}{k} \tag{25}$$

The other microwave travels at the velocity of v_2.

$$v_2 = \frac{W_2}{k} \tag{26}$$

From equations (15,20,22,25,26),

$$v_1 - v_2 = 2V \tag{27}$$

Upon reflection, the speed of microwave changes by twice the speed of the reflector.

C. Velocity Transformation

From equation (27),

$$v_1 - V = v_2 + V \tag{28}$$

Define C as

$$C = v_1 - V \tag{29}$$

From equations (28,29),

$$v_1 = C + V \tag{30}$$

$$v_2 = C - V \tag{31}$$

If V is reduced to zero, F_2 becomes F_1. Both microwaves travel at the same speed in F_1.

$$v_1 = C = v_2 \tag{32}$$

C is the speed of microwave in F_1. From equations (1),

$$C = \frac{w'}{k'} \tag{33}$$

The speed of microwave in F_2 depends on the the speed of microwave in F_1 as well as the relative speed between F_1 and F_2.

III. CONCLUSION

The apparent speed of microwave is different in a different reference frame. The speed of microwave in the rest frame of the observer depends on the relative motion between the observer and the transmitter.

The covariance property of the standing wave verifies that the wavelengths of both microwaves forming the standing wave are equal to each other in any inertial reference frame. The frequency of each microwave depends on the choice of reference frame. Hence, these two microwaves travel at different speeds in another reference frame.

Consequently, the speed of the microwave is different in a different inertial reference frame.

[1] R. E. Collin, Foundations for Microwave Engineering, 2nd edition, IEEE Press, New York, NY, 2001.

[2] Eric Su: List of Publications, http://vixra.org/author/eric_su