On some Ramanujan formulas: mathematical connections with ϕ, $\zeta(2)$ and several parameters of Quantum Geometry, String Theory and Cosmology. III

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Abstract

In this paper we have described and analyzed some Ramanujan expressions. We have obtained several mathematical connections with ϕ, $\zeta(2)$ and various parameters of Quantum Geometry, String Theory and Cosmology.

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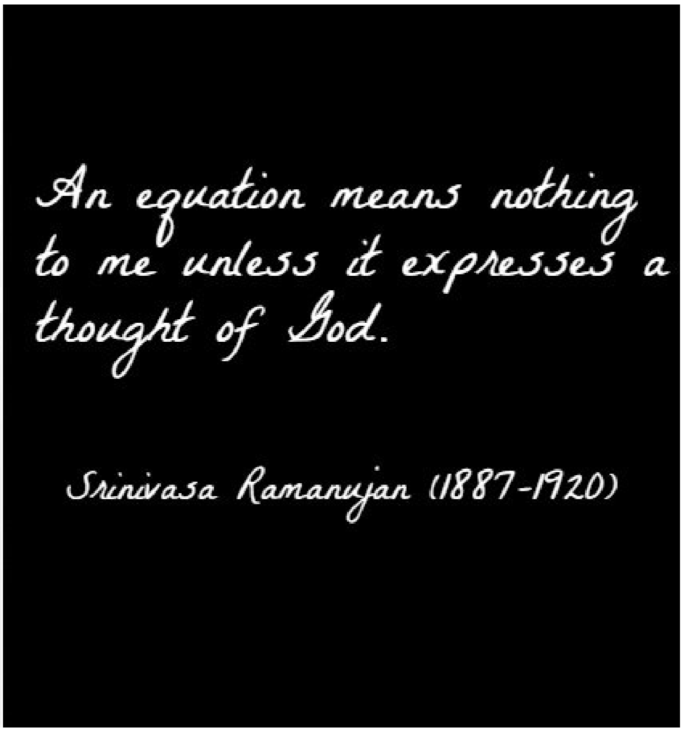


https://mobygeek.com/features/indian-mathematician-srinivasa-ramanujan-quotes-11012

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

From

On Climbing Scalars in String Theory
E. Dudas, N. Kitazawa and A. Sagnotti - arXiv:1009.0874v1 [hep-th] 4 Sep 2010

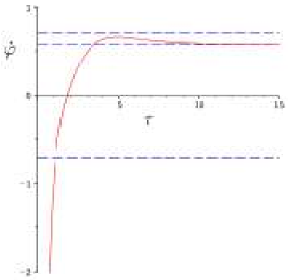


Figure 3: Climbing and inflation for the one–field system with the potential (2.21). Inflation occurs within the strip $|\dot{\varphi}| < 1/\sqrt{2}$, while the lower horizontal line in the upper portion of the plot is the attractor determined by the D3-brane potential, $\dot{\varphi} = 1/\sqrt{3}$.

so that, if the Big Bang occurs at $\tau = 0$, in Region I

$$\dot{\varphi}_{\rm I} = \frac{1}{2\tau} - \frac{1}{2}\,\tau \;, \qquad \varphi_{\rm I} = \varphi^{(0)} + f(\tau) \;. \tag{3.23}$$

In order to enter the well, the scalar field must now reach the top of the barrier while climbing up, and this is possible provided

$$\varphi_1 - \varphi^{(0)} \equiv f(\tau_1) < -\frac{1}{4} \;, \qquad 0 < \tau_1 < 1 \;. \tag{3.24}$$

In a similar fashion, the solution in Region II includes two integration constants, $\tau^{(1)}$ and $\varphi^{(1)}$, and reads

$$\dot{\varphi}_{\rm II} = -\frac{1}{2(\tau - \tau^{(1)})} + \frac{1}{2}\,(\tau - \tau^{(1)}) \;, \qquad \varphi_{\rm II} = \varphi^{(1)} - f(\tau - \tau^{(1)}) \;. \tag{3.25}$$

Finally, the third region coincides with the second, that the scalar φ retraces after being reflected by the infinite wall, so that φ_{III} takes again the form (3.25), albeit with two different integration constants $\tau^{(2)}$ and $\varphi^{(2)}$:

$$\dot{\varphi}_{\rm III} = -\frac{1}{2(\tau - \tau^{(2)})} + \frac{1}{2}\,(\tau - \tau^{(2)}) \;, \qquad \varphi_{\rm III} = \varphi^{(2)} - f(\tau - \tau^{(2)}) \;. \tag{3.26}$$

From (3.23), we obtain:

$$\varphi_{\rm I} = \varphi^{(0)} + f(\tau)$$

$$\varphi_1 - \varphi^{(0)} \equiv f(\tau_1) < -\frac{1}{4}$$

$\varphi_1 = -1/2$

$$\dot{\varphi}_{\rm I} = \frac{1}{2\tau} - \frac{1}{2}\tau$$

1/(2*5) - 5/2

Input:

$$\frac{1}{2\times 5} - \frac{5}{2}$$

Exact result:

$$-\frac{12}{5}$$

Decimal form:

−2.4

-2.4

$$\dot{\varphi}_{\rm II} = -\frac{1}{2(\tau - \tau^{(1)})} + \frac{1}{2}(\tau - \tau^{(1)}) ,$$

-1/(2(5-3))+1/2(5-3)

Input:

$$-\frac{1}{2(5-3)} + \frac{1}{2}(5-3)$$

Exact result:

$$\frac{3}{4}$$

Decimal form:

0.75

0.75

$$\dot{\varphi}_{\rm III} = -\frac{1}{2(\tau - \tau^{(2)})} + \frac{1}{2}(\tau - \tau^{(2)})$$

-1/(2(5-2))+1/2(5-2)

Input:

$$-\frac{1}{2(5-2)} + \frac{1}{2}(5-2)$$

Exact result:

$$\frac{4}{3}$$

Decimal approximation:

1.3333333333333333333333333333333333333333333333333333333333...

1.3333…

Performing the following calculation, we obtain:

(((1/(2*5) - 5/2))) / (((-1/(2(5-3))+1/2(5-3)))) * (((-1/(2(5-2))+1/2(5-2))))

Input:

$$\frac{\frac{1}{2\times5} - \frac{5}{2}}{-\frac{1}{2(5-3)} + \frac{1}{2}(5-3)}\left(-\frac{1}{2(5-2)} + \frac{1}{2}(5-2)\right)$$

Exact result:

$$-\frac{64}{15}$$

Decimal approximation:

-4.2666666666666666666666666666666666666666666666666666666666...

-4.266666…

From which, multiplying by (-3*5):

(((1/(2*5) - 5/2))) / (((-1/(2(5-3))+1/2(5-3)))) * (((-1/(2(5-2))+1/2(5-2)))) *(-3*5)

Input:

$$\frac{\frac{1}{2\times 5}-\frac{5}{2}}{-\frac{1}{2(5-3)}+\frac{1}{2}(5-3)}\left(-\frac{1}{2(5-2)}+\frac{1}{2}(5-2)\right)(-3\times 5)$$

Exact result:

64

64

From which:

sqrt[(((1/(2*5) - 5/2))) / (((-1/(2(5-3))+1/2(5-3)))) * (((-1/(2(5-2))+1/2(5-2)))) *(-3*5)]

Input:

$$\sqrt{\frac{\frac{1}{2\times 5}-\frac{5}{2}}{-\frac{1}{2(5-3)}+\frac{1}{2}(5-3)}\left(-\frac{1}{2(5-2)}+\frac{1}{2}(5-2)\right)(-3\times 5)}$$

Exact result:

8

8

[(((1/(2*5) - 5/2))) / (((-1/(2(5-3))+1/2(5-3)))) * (((-1/(2(5-2))+1/2(5-2)))) *(-3*5)]^2

Input:

$$\left(\frac{\frac{1}{2\times 5}-\frac{5}{2}}{-\frac{1}{2(5-3)}+\frac{1}{2}(5-3)}\left(-\frac{1}{2(5-2)}+\frac{1}{2}(5-2)\right)(-3\times 5)\right)^2$$

Exact result:

4096

4096

Now, from the Ramanujan equation
(**Modular equations and approximations to π** – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372) :

$$\frac{32}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5} + 29}{64}\left(\frac{1}{2}\right)^3\left(\frac{\sqrt{5} - 1}{2}\right)^8$$

$$+\frac{89\sqrt{5} + 59}{64^2}\left(\frac{1 \cdot 3}{2 \cdot 4}\right)^3\left(\frac{\sqrt{5} - 1}{2}\right)^{16} + \cdots,$$

we obtain:

((5sqrt5)-1)+1/64((47sqrt5)+29) (1/2)^3 ((sqrt5-1)/2)^8 + 1/64^2*(89sqrt5+59) (3/8)^3 ((sqrt5-1)/2)^16

Input:

$$\left(5\sqrt{5} - 1\right) + \left(\frac{1}{64}\left(47\sqrt{5} + 29\right)\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)^8 + \left(\frac{1}{64^2}\left(89\sqrt{5} + 59\right)\right)\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5} - 1\right)\right)^{16}$$

Result:

$$-1 + 5\sqrt{5} + \frac{\left(\sqrt{5} - 1\right)^8\left(29 + 47\sqrt{5}\right)}{131\,072} + \frac{27\left(\sqrt{5} - 1\right)^{16}\left(59 + 89\sqrt{5}\right)}{137\,438\,953\,472}$$

Decimal approximation:

10.18591635745234529933672773439453907823343935074991009694...

10.185916357...

Alternate forms:

$$\frac{15\left(1\,041\,875\sqrt{5} - 905\,609\right)}{2\,097\,152}$$

$$\frac{15\,628\,125\sqrt{5}}{2\,097\,152} - \frac{13\,584\,135}{2\,097\,152}$$

$$\frac{15\,628\,125\sqrt{5} - 13\,584\,135}{2\,097\,152}$$

Minimal polynomial:

$$1\,099\,511\,627\,776\,x^2 + 14\,243\,997\,941\,760\,x - 259\,165\,682\,844\,975$$

From which, we obtain:

32* 1/ (((((5sqrt5)-1)+1/64((47sqrt5)+29) (1/2)^3 ((sqrt5-1)/2)^8 + 1/64^2*(89sqrt5+59) (3/8)^3 ((sqrt5-1)/2)^16)))

Input:

$$32\times 1\Big/\left(\left(5\sqrt{5}-1\right)+\left(\frac{1}{64}\left(47\sqrt{5}+29\right)\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+\left(\frac{1}{64^2}\left(89\sqrt{5}+59\right)\right)\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)$$

Result:

$$\frac{32}{-1+5\sqrt{5}+\frac{\left(\sqrt{5}-1\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(\sqrt{5}-1\right)^{16}\left(59+89\sqrt{5}\right)}{137\,438\,953\,472}}$$

Decimal approximation:

3.141592653722094110325513887398231062068309787569351629942...

3.141592653722... ≈ π

Alternate forms:

$$\frac{16\,777\,216\left(905\,609+1\,041\,875\sqrt{5}\right)}{17\,277\,712\,189\,665}$$

$$\frac{15\,193\,597\,804\,544+17\,479\,761\,920\,000\sqrt{5}}{17\,277\,712\,189\,665}$$

$$\frac{3\,495\,952\,384\,000\sqrt{5}}{3\,455\,542\,437\,933}+\frac{15\,193\,597\,804\,544}{17\,277\,712\,189\,665}$$

Minimal polynomial:

$$259\,165\,682\,844\,975\,x^2 - 455\,807\,934\,136\,320\,x - 1\,125\,899\,906\,842\,624$$

and:

2Pi * (((((5sqrt5)-1)+1/64((47sqrt5)+29) (1/2)^3 ((sqrt5-1)/2)^8 + 1/64^2*(89sqrt5+59) (3/8)^3 ((sqrt5-1)/2)^16)))

Input:

$$2\pi\left(\left(5\sqrt{5}-1\right)+\left(\frac{1}{64}\left(47\sqrt{5}+29\right)\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+\left(\frac{1}{64^2}\left(89\sqrt{5}+59\right)\right)\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)$$

Result:

$$2\left(-1+5\sqrt{5}+\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072}+\frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472}\right)\pi$$

Decimal approximation:

63.99999999730478877037356118727778112992174448981692286347...

63.9999... ≈ 64

Property:

$$2\left(-1+5\sqrt{5}+\frac{(-1+\sqrt{5})^8(29+47\sqrt{5})}{131072}+\frac{27(-1+\sqrt{5})^{16}(59+89\sqrt{5})}{137438953472}\right)\pi$$

is a transcendental number

Alternate forms:

$$\frac{\pi\,15\,(1041875\sqrt{5}-905609)}{1048576}$$

$$\frac{(15628125\sqrt{5}-13584135)\pi}{1048576}$$

$$\frac{15628125\sqrt{5}\,\pi}{1048576}-\frac{13584135\,\pi}{1048576}$$

Series representations:

$$2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)=$$

$$2\pi\left(-1+5\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\frac{\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^8\left(29+47\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{131072}+\frac{27\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{16}\left(59+89\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{137438953472}\right)$$

$$2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)=2\pi\left(-1+5\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\frac{\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8\left(29+47\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{131072}+\frac{27\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}\left(59+89\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{137438953472}\right)$$

$$2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)=$$

$$2\pi\left(-1+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\frac{\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^8\left(29+47\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)}{131\,072}+\frac{1}{137\,438\,953\,472}\,27\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{16}\left(59+89\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

and again:

sqrt[2Pi * (((((5sqrt5)-1)+1/64((47sqrt5)+29) (1/2)^3 ((sqrt5-1)/2)^8 + 1/64^2*(89sqrt5+59) (3/8)^3 ((sqrt5-1)/2)^16)))]

Input:

$$\sqrt{2\pi\left(\left(5\sqrt{5}-1\right)+\left(\frac{1}{64}\left(47\sqrt{5}+29\right)\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+\left(\frac{1}{64^2}\left(89\sqrt{5}+59\right)\right)\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)}$$

Exact result:

$$\sqrt{2\left(-1+5\sqrt{5}+\frac{\left(\sqrt{5}-1\right)^8\left(29+47\sqrt{5}\right)}{131\,072}+\frac{27\left(\sqrt{5}-1\right)^{16}\left(59+89\sqrt{5}\right)}{137\,438\,953\,472}\right)\pi}$$

Decimal approximation:

7.999999999831549298146574096770179334055760362934256228009...

7.9999…≈ 8

Property:

$$\sqrt{2\left(-1+5\sqrt{5}+\frac{(-1+\sqrt{5})^8(29+47\sqrt{5})}{131\,072}+\frac{27(-1+\sqrt{5})^{16}(59+89\sqrt{5})}{137\,438\,953\,472}\right)\pi}$$

is a transcendental number

Alternate forms:

$$\sqrt{2\left(\frac{15\,628\,125\sqrt{5}}{2\,097\,152}-\frac{13\,584\,135}{2\,097\,152}\right)\pi}$$

$$\frac{\sqrt{(15\,628\,125\sqrt{5}-13\,584\,135)\pi}}{1024}$$

$$\frac{\sqrt{15(1\,041\,875\sqrt{5}-905\,609)\pi}}{1024}$$

All 2nd roots of 2 (-1 + 5 sqrt(5) + ((sqrt(5) - 1)^8 (29 + 47 sqrt(5)))/131072 + (27 (sqrt(5) - 1)^16 (59 + 89 sqrt(5)))/137438953472) π:

$$\sqrt{2\left(-1+5\sqrt{5}+\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131\,072}+\frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137\,438\,953\,472}\right)\pi}\;e^0\approx 8.000$$

(real, principal root)

$$\sqrt{2\left(-1+5\sqrt{5}+\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131\,072}+\frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137\,438\,953\,472}\right)\pi}\;e^{i\pi}$$

≈ -8.000 (real root)

Series representations:

$$\sqrt{\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)} = \sqrt{-1+2\pi\left(-1+5\sqrt{5}+\frac{\left(-1+\sqrt{5}\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(-1+\sqrt{5}\right)^{16}\left(59+89\sqrt{5}\right)}{137438953472}\right)} \sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+2\pi\left(-1+5\sqrt{5}+\frac{\left(-1+\sqrt{5}\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(-1+\sqrt{5}\right)^{16}\left(59+89\sqrt{5}\right)}{137438953472}\right)\right)^{-k}$$

$$\sqrt{\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)} = \sqrt{-1+2\pi\left(-1+5\sqrt{5}+\frac{\left(-1+\sqrt{5}\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(-1+\sqrt{5}\right)^{16}\left(59+89\sqrt{5}\right)}{137438953472}\right)} \sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+2\pi\left(-1+5\sqrt{5}+\frac{\left(-1+\sqrt{5}\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(-1+\sqrt{5}\right)^{16}\left(59+89\sqrt{5}\right)}{137438953472}\right)\right)^{-k}$$

$$\sqrt{\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)} = \sqrt{z_0}\sum_{k=0}^{\infty}\frac{1}{k!}(-1)^k\left(-\frac{1}{2}\right)_k\left(2\pi\left(-1+5\sqrt{5}+\frac{\left(-1+\sqrt{5}\right)^8\left(29+47\sqrt{5}\right)}{131072}+\frac{27\left(-1+\sqrt{5}\right)^{16}\left(59+89\sqrt{5}\right)}{137438953472}\right)-z_0\right)^k z_0^{-k} \quad \text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty < z_0 \le 0))$$

and:

[2Pi * (((((5sqrt5)-1)+1/64((47sqrt5)+29) (1/2)^3 ((sqrt5-1)/2)^8 + 1/64^2*(89sqrt5+59) (3/8)^3 ((sqrt5-1)/2)^16)))]^2

Input:

$$\left(2\pi\left(\left(5\sqrt{5}-1\right)+\left(\frac{1}{64}\left(47\sqrt{5}+29\right)\right)\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8+\left(\frac{1}{64^2}\left(89\sqrt{5}+59\right)\right)\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\right)^2$$

Result:

$$4\left(-1+5\sqrt{5}+\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072}+\frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472}\right)^2\pi^2$$

Decimal approximation:

4095.99999655012962615079995543860444916036424995512162450...

4095.9999…≈ 4096

Property:

$$4\left(-1+5\sqrt{5}+\frac{(-1+\sqrt{5})^8(29+47\sqrt{5})}{131072}+\frac{27(-1+\sqrt{5})^{16}(59+89\sqrt{5})}{137438953472}\right)^2\pi^2$$

is a transcendental number

Alternate forms:

$$\frac{225\pi^2(3123822619503-943531376875\sqrt{5})}{549755813888}$$

$$\frac{(702860089388175-212294559796875\sqrt{5})\pi^2}{549755813888}$$

$$\frac{225(905609-1041875\sqrt{5})^2\pi^2}{1099511627776}$$

Series representations:

$$\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)^2 =$$

$$4\pi^2\left(-1+5\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}+\frac{\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^8\left(29+47\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{131072}+\frac{27\left(-1+\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)^{16}\left(59+89\sqrt{4}\sum_{k=0}^{\infty}4^{-k}\binom{\frac{1}{2}}{k}\right)}{137438953472}\right)^2$$

$$\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)^2 =$$

$$4\pi^2\left(-1+5\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}+\frac{\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^8\left(29+47\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{131072}+\frac{27\left(-1+\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^{16}\left(59+89\sqrt{4}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{4}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)}{137438953472}\right)^2$$

$$\left(2\pi\left(\left(5\sqrt{5}-1\right)+\frac{1}{64}\left(\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^8\right)\left(47\sqrt{5}+29\right)+\frac{\left(\left(\frac{3}{8}\right)^3\left(\frac{1}{2}\left(\sqrt{5}-1\right)\right)^{16}\right)\left(89\sqrt{5}+59\right)}{64^2}\right)\right)^2 =$$

$$4\pi^2\left(-1+5\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}+\frac{1}{131072}\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^8\left(29+47\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)+\frac{1}{137438953472}\,27\left(-1+\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)^{16}\left(59+89\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(5-z_0)^k z_0^{-k}}{k!}\right)\right)^2$$

for (not $(z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0)$)

Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{\Gamma(s)\Gamma(-a-s)}{z^s}\,ds}{(2\pi i)\Gamma(-a)} \quad \text{for } (0<\gamma<-\mathrm{Re}(a) \text{ and } |\arg(z)|<\pi)$$

We have the following mathematical connection:

$$\left[\left(\frac{\frac{1}{2\times 5}-\frac{5}{2}}{-\frac{1}{2(5-3)}+\frac{1}{2}(5-3)}\left(-\frac{1}{2(5-2)}+\frac{1}{2}(5-2)\right)(-3\times 5)\right)^2\right] = 4096$$

$$\left[4\left(-1+5\sqrt{5}+\frac{(\sqrt{5}-1)^8(29+47\sqrt{5})}{131072}+\frac{27(\sqrt{5}-1)^{16}(59+89\sqrt{5})}{137438953472}\right)^2\pi^2\right] = 4095.9999\ldots$$

From

Pre – Inflationary Clues from String Theory ?

N. Kitazawa and A. Sagnotti - arXiv:1402.1418v2 [hep-th] 12 Mar 2014

Now, we have that:

$$V(\varphi) = V_0 \left(e^{2\varphi} + e^{2\gamma\varphi}\right) . \qquad (2.14)$$

in [14]. As argued in [16, 17], all these branes ought to have been generically present in the vacuum at very early epochs, close to the initial singularity. In particular, an NS fivebrane wrapped on a small internal cycle, corresponding to $p = 4$ and $\alpha = 2$, would yield a "mild" exponential term with $\gamma = \frac{1}{12}$, while its instability in orientifold models and its consequent decay could perhaps account for the eventual graceful exit of the Universe from the inflationary phase.

8 = x(e^(2*40) + e^(2*1/12*40))

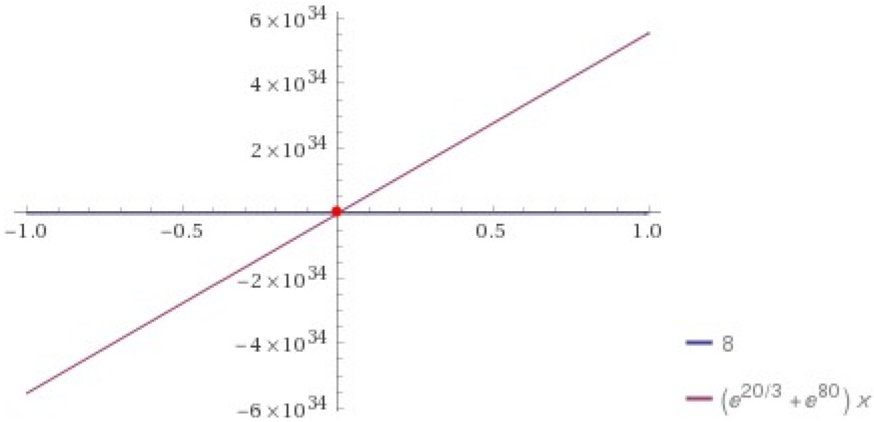
Input:

$$8 = x\left(e^{2\times 40} + e^{2\times 1/12\times 40}\right)$$

Exact result:

$$8 = \left(e^{20/3} + e^{80}\right) x$$

Plot:



Alternate forms:

$$-e^{80}\,x - e^{20/3}\,x + 8 = 0$$

$$8 = e^{20/3}\left(1+e^{4/3}\right)\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}\right)\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}-e^{20/3}+e^{8}-e^{28/3}+e^{32/3}-e^{12}+e^{40/3}\right)\left(1+e^{4/3}-e^{20/3}-e^{8}+e^{40/3}-e^{16}-e^{20}+e^{68/3}+e^{80/3}+e^{92/3}-e^{100/3}-e^{112/3}+e^{40}-e^{136/3}-e^{140/3}+e^{52}+e^{160/3}\right)x$$

Expanded form:

$$8 = e^{80}\,x + e^{20/3}\,x$$

Solution:

$$x = \frac{8}{e^{20/3} + e^{80}}$$

8/(e^(20/3) + e^80) * (e^(2*40) + e^(2*1/12*40))

Input:

$$\frac{8}{e^{20/3}+e^{80}}\left(e^{2\times 40}+e^{2\times 1/12\times 40}\right)$$

Exact result:

8

8

Alternative representation:

$$\frac{\left(e^{2\times 40}+e^{(2\times 40)/12}\right)8}{e^{20/3}+e^{80}} = \frac{\left(\exp^{2\times 40}(z)+\exp^{\frac{2\times 40}{12}}(z)\right)8}{\exp^{\frac{20}{3}}(z)+\exp^{80}(z)} \quad \text{for } z = 1$$

Where

8/(e^(20/3) + e^80)

Input:

$$\frac{8}{e^{20/3}+e^{80}}$$

Decimal approximation:

$$1.443881110276332137849702685880001459790801473100191 7\ldots \times 10^{-34}$$

$1.44388111027\ldots * 10^{-34}$

Property:

$\frac{8}{e^{20/3}+e^{80}}$ is a transcendental number

Alternate forms:

$$\frac{8}{e^{20/3}\left(1+e^{220/3}\right)}$$

$$8\Big/\Big(e^{20/3}\left(1+e^{4/3}\right)\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}\right)\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}-e^{20/3}+e^{8}-e^{28/3}+e^{32/3}-e^{12}+e^{40/3}\right)\left(1+e^{4/3}-e^{20/3}-e^{8}+e^{40/3}-e^{16}-e^{20}+e^{68/3}+e^{80/3}+e^{92/3}-e^{100/3}-e^{112/3}+e^{40}-e^{136/3}-e^{140/3}+e^{52}+e^{160/3}\right)\Big)$$

$$\frac{8}{e^{20/3}}-\frac{8}{55\left(1+e^{4/3}\right)}+\frac{8\left(-4+3e^{4/3}-2e^{8/3}+e^{4}\right)}{55\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}\right)}+\frac{8\left(1-2e^{4/3}+3e^{8/3}-4e^{4}+5e^{16/3}-6e^{20/3}-4e^{8}+3e^{28/3}-2e^{32/3}+e^{12}\right)}{55\left(1-e^{4/3}+e^{8/3}-e^{4}+e^{16/3}-e^{20/3}+e^{8}-e^{28/3}+e^{32/3}-e^{12}+e^{40/3}\right)}-\Big(8\big(-4-5e^{4/3}+9e^{20/3}-e^{8}-11e^{28/3}-14e^{40/3}+16e^{16}+19e^{20}+e^{68/3}+22e^{24}-24e^{80/3}-27e^{92/3}+29e^{100/3}-e^{112/3}-33e^{116/3}-34e^{40}+38e^{136/3}+39e^{140/3}+e^{52}\big)\Big)\Big/\Big(55\big(1+e^{4/3}-e^{20/3}-e^{8}+e^{40/3}-e^{16}-e^{20}+e^{68/3}+e^{80/3}+e^{92/3}-e^{100/3}-e^{112/3}+e^{40}-e^{136/3}-e^{140/3}+e^{52}+e^{160/3}\big)\Big)$$

Alternative representation:

$$\frac{8}{e^{20/3}+e^{80}}=\frac{8}{\exp^{\frac{20}{3}}(z)+\exp^{80}(z)} \quad \text{for } z=1$$

Series representations:

$$\frac{8}{e^{20/3}+e^{80}}=\frac{8}{\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{20/3}+\left(\sum_{k=0}^{\infty}\frac{1}{k!}\right)^{80}}$$

$$\frac{8}{e^{20/3}+e^{80}}=\frac{8}{\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{20/3}+\left(\sum_{k=0}^{\infty}\frac{(-1+k)^2}{k!}\right)^{80}}$$

$$\frac{8}{e^{20/3}+e^{80}}=\frac{8}{\left(\frac{1}{\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}}\right)^{20/3}+\frac{1}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k}{k!}\right)^{80}}}$$

From

$$\frac{8}{e^{20/3} + e^{80}}$$

we obtain also:

colog((8/(e^(20/3) + e^80)))

Input:

$$-\log\left(\frac{8}{e^{20/3} + e^{80}}\right)$$

$\log(x)$ is the natural logarithm

Decimal approximation:

77.920558458320164071748303635625484477790242711055955531264...

77.9205584…

Alternate forms:

$$\log\left(e^{20/3} + e^{80}\right) - \log(8)$$

$$\frac{20}{3} - 3\log(2) + \log\left(1 + e^{220/3}\right)$$

$$\frac{1}{3}\left(20 - 9\log(2) + 3\log\left(1 + e^{220/3}\right)\right)$$

Alternative representations:

$$-\log\left(\frac{8}{e^{20/3} + e^{80}}\right) = -\log_e\left(\frac{8}{e^{20/3} + e^{80}}\right)$$

$$-\log\left(\frac{8}{e^{20/3} + e^{80}}\right) = -\log(a)\log_a\left(\frac{8}{e^{20/3} + e^{80}}\right)$$

$$-\log\left(\frac{8}{e^{20/3} + e^{80}}\right) = \text{Li}_1\left(1 - \frac{8}{e^{20/3} + e^{80}}\right)$$

Series representations:

$$-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)=\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{8}{e^{20/3}+e^{80}}\right)^k}{k}$$

$$-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)=-2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{e^{20/3}+e^{80}}-x\right)}{2\,\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}}-x\right)^k x^{-k}}{k}$$

for $x < 0$

$$-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)=$$
$$-2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}}-z_0\right)^k z_0^{-k}}{k}$$

Integral representation:

$$-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)=-\int_1^{\frac{8}{e^{20/3}+e^{80}}}\frac{1}{t}\,dt$$

From the formula of coefficients of the '5th order' mock theta function ψ_1(q):
(A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 84

sqrt(golden ratio) * exp(Pi*sqrt(84/15)) / (2*5^(1/4)*sqrt(84))

Input:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{84}{15}}\right)}{2\sqrt[4]{5}\sqrt{84}}$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{2\sqrt{7/5}\,\pi}\sqrt{\frac{\phi}{21}}}{4\sqrt[4]{5}}$$

Decimal approximation:

78.57518744959091921978556483268167026458376263454526334024...

78.5751874495… that is very near to the result of the previous expression

Property:

$$\frac{e^{2\sqrt{7/5}\,\pi}\sqrt{\frac{\phi}{21}}}{4\sqrt[4]{5}} \text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{4}\sqrt{\frac{1}{210}\left(5+\sqrt{5}\right)}\,e^{2\sqrt{7/5}\,\pi}$$

$$\frac{\sqrt{\frac{1}{42}\left(1+\sqrt{5}\right)}\,e^{2\sqrt{7/5}\,\pi}}{4\sqrt[4]{5}}$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{84}{15}}\right)}{2\sqrt[4]{5}\sqrt{84}} = \frac{\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{28}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}}{2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(84-z_0)^k z_0^{-k}}{k!}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{84}{15}}\right)}{2\sqrt[4]{5}\sqrt{84}} = \left(\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{28}{5}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{28}{5}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Bigg/ \left(2\sqrt[4]{5}\exp\left(i\pi\left\lfloor\frac{\arg(84-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(84-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{84}{15}}\right)}{2\sqrt[4]{5}\sqrt{84}} =$$

$$\left(\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg\left(\frac{28}{5}-z_0\right)/(2\pi)\rfloor} z_0^{1/2\left(1+\lfloor\arg\left(\frac{28}{5}-z_0\right)/(2\pi)\rfloor\right)} \sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{28}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\right.$$
$$\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(84-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(84-z_0)/(2\pi)\rfloor+1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}$$
$$\left.\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Bigg/\left(2\sqrt[4]{5}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(84-z_0)^k z_0^{-k}}{k!}\right)$$

and again:

11*2(((colog((8/(e^(20/3) + e^80))))))+11+4

Input:

$$11\times 2\left(-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)\right)+11+4$$

log(x) is the natural logarithm

Exact result:

$$15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)$$

Decimal approximation:

1729.252286083043609578462679983760658511385339643231016878...

1729.25228608…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$\frac{485}{3}-66\log(2)+22\log\left(1+e^{220/3}\right)$$

$$15 - 22\left(\log(8) - \log\left(e^{20/3} + e^{80}\right)\right)$$

$$\frac{1}{3}\left(485 - 198\log(2) + 66\log\left(1 + e^{220/3}\right)\right)$$

Alternative representations:

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 - 22\log_e\left(\frac{8}{e^{20/3} + e^{80}}\right)$$

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 - 22\log(a)\log_a\left(\frac{8}{e^{20/3} + e^{80}}\right)$$

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 + 22\,\text{Li}_1\left(1 - \frac{8}{e^{20/3} + e^{80}}\right)$$

Series representations:

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 + 22\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1 + \frac{8}{e^{20/3}+e^{80}}\right)^k}{k}$$

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 - 44 i\pi\left\lfloor\frac{\arg\left(\frac{8}{e^{20/3}+e^{80}} - x\right)}{2\pi}\right\rfloor - 22\log(x) + 22\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}} - x\right)^k x^{-k}}{k} \quad \text{for } x < 0$$

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 - 44 i\pi\left\lfloor\frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2\pi}\right\rfloor - 22\log(z_0) + 22\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}} - z_0\right)^k z_0^{-k}}{k}$$

Integral representation:

$$(11 \times 2)(-1)\log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4 = 15 - 22\int_1^{\frac{8}{e^{20/3}+e^{80}}}\frac{1}{t}\,dt$$

From which:

[11*2(((colog((8/(e^(20/3) + e^80))))))+11+4]^1/15

Input:

$$\sqrt[15]{11\times 2\left(-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)\right)+11+4}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}$$

Decimal approximation:

1.643831218086257705699543144757843120327944923967051447744...

1.64383121808….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms:

$$\sqrt[15]{15-22\left(\log(8)-\log\left(e^{20/3}+e^{80}\right)\right)}$$

$$\sqrt[15]{\frac{1}{3}\left(485-198\log(2)+66\log\left(1+e^{220/3}\right)\right)}$$

$$\sqrt[15]{15-66\log(2)+22\log\left(e^{20/3}+e^{80}\right)}$$

All 15th roots of 15 - 22 log(8/(e^(20/3) + e^80)):

$$e^0\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}\approx 1.64383 \text{ (real, principal root)}$$

$$e^{(2i\pi)/15}\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}\approx 1.50171+0.6686\,i$$

$$e^{(4i\pi)/15}\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}\approx 1.0999+1.2216\,i$$

$$e^{(2 i \pi)/5} \sqrt[15]{15 - 22 \log\left(\frac{8}{e^{20/3} + e^{80}}\right)} \approx 0.5080 + 1.5634 i$$

$$e^{(8 i \pi)/15} \sqrt[15]{15 - 22 \log\left(\frac{8}{e^{20/3} + e^{80}}\right)} \approx -0.17183 + 1.63483 i$$

Alternative representations:

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} = \sqrt[15]{15 - 22 \log_e\left(\frac{8}{e^{20/3} + e^{80}}\right)}$$

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} = \sqrt[15]{15 - 22 \log(a) \log_a\left(\frac{8}{e^{20/3} + e^{80}}\right)}$$

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} = \sqrt[15]{15 + 22 \operatorname{Li}_1\left(1 - \frac{8}{e^{20/3} + e^{80}}\right)}$$

Series representations:

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} = \sqrt[15]{15 + 22 \sum_{k=1}^{\infty} \frac{(-1)^k \left(-1 + \frac{8}{e^{20/3} + e^{80}}\right)^k}{k}}$$

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} =$$
$$\sqrt[15]{15 - 22 \left(2 i \pi \left\lfloor \frac{\arg\left(\frac{8}{e^{20/3} + e^{80}} - x\right)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{8}{e^{20/3} + e^{80}} - x\right)^k x^{-k}}{k}\right)} \quad \text{for } x < 0$$

$$\sqrt[15]{(11 \times 2)(-1) \log\left(\frac{8}{e^{20/3} + e^{80}}\right) + 11 + 4} =$$
$$\sqrt[15]{15 - 22 \left(2 i \pi \left\lfloor \frac{\pi - \arg\left(\frac{1}{z_0}\right) - \arg(z_0)}{2 \pi} \right\rfloor + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k \left(\frac{8}{e^{20/3} + e^{80}} - z_0\right)^k z_0^{-k}}{k}\right)}$$

Integral representation:

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4} = \sqrt[15]{15-22\int_1^{\frac{8}{e^{20/3}+e^{80}}}\frac{1}{t}\,dt}$$

and:

[11*2(((colog((8/(e^(20/3) + e^80)))))+11+4]^1/15 - (21+5)1/10^3

Input:

$$\sqrt[15]{11\times 2\left(-\log\left(\frac{8}{e^{20/3}+e^{80}}\right)\right)+11+4}-(21+5)\times\frac{1}{10^3}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}-\frac{13}{500}$$

Decimal approximation:

1.617831218086257705699543144757843120327944923967051447744...

1.61783121808…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\sqrt[15]{15-22\left(\log(8)-\log\left(e^{20/3}+e^{80}\right)\right)}-\frac{13}{500}$$

$$\frac{1}{500}\left(500\sqrt[15]{15-22\log\left(\frac{8}{e^{20/3}+e^{80}}\right)}-13\right)$$

$$\frac{500\sqrt[15]{485-198\log(2)+66\log\left(1+e^{220/3}\right)}-13\sqrt[15]{3}}{500\sqrt[15]{3}}$$

Alternative representations:

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=\sqrt[15]{15-22\log_e\left(\frac{8}{e^{20/3}+e^{80}}\right)}-\frac{26}{10^3}$$

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=$$
$$\sqrt[15]{15-22\log(a)\log_a\left(\frac{8}{e^{20/3}+e^{80}}\right)}-\frac{26}{10^3}$$

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=$$
$$\sqrt[15]{15+22\,\mathrm{Li}_1\left(1-\frac{8}{e^{20/3}+e^{80}}\right)}-\frac{26}{10^3}$$

Series representations:

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=$$
$$-\frac{13}{500}+\sqrt[15]{15+22\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{8}{e^{20/3}+e^{80}}\right)^k}{k}}$$

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=-\frac{13}{500}+$$
$$\sqrt[15]{15-22\left(2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{8}{e^{20/3}+e^{80}}-x\right)}{2\,\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}}-x\right)^k x^{-k}}{k}\right)}$$
for $x<0$

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=-\frac{13}{500}+$$
$$\sqrt[15]{15-22\left(2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\,\pi}\right\rfloor+\log(z_0)-\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{8}{e^{20/3}+e^{80}}-z_0\right)^k z_0^{-k}}{k}\right)}$$

Integral representation:

$$\sqrt[15]{(11\times 2)(-1)\log\left(\frac{8}{e^{20/3}+e^{80}}\right)+11+4}-\frac{21+5}{10^3}=$$

$$-\frac{13}{500}+\sqrt[15]{15-22\int_1^{\frac{8}{e^{20/3}+e^{80}}}\frac{1}{t}\,dt}$$

From

The no-boundary proposal in biaxial Bianchi IX minisuperspace
O. Janssen, J. J. Halliwell, and T. Hertog - arXiv:1904.11602v1 [gr-qc] 25 Apr 2019

Now, we have that

$$N_s = 3\left(\pm\sqrt{Q/3-1}-i\right)$$

Q = 100; 3(sqrt((100/3)-1)-i)

Input:

$$3\left(\sqrt{\frac{100}{3}-1}-i\right)$$

i is the imaginary unit

Result:

$$3\left(\sqrt{\frac{97}{3}}+-i\right)$$

Decimal approximation:

17.058722109231980803379478538021109976184359384734035116... − 3 i

Polar coordinates:

$r \approx 17.3205$ (radius), $\theta \approx -9.97422°$ (angle)

$N_s = 17.3205$

Alternate forms:

$\sqrt{291} - 3i$

$\sqrt{6\left(47 - i\sqrt{291}\right)}$

Minimal polynomial:

$x^4 - 564x^2 + 90000$

Continued fraction:

$$(17-3i)+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{17+\cfrac{1}{34+\cfrac{1}{\cdots}}}}}}}}}}}}}}}$$

(using the Hurwitz expansion)

Now, we have that:

A. Action

Here instead we are doing the quantum-cosmological analogue of quantum field theory in a (fixed) curved background spacetime [12]. In our case the background is complex and lives on a compact four-manifold. The (bulk, Lorentzian) action for a massless minimally coupled scalar $\phi(\tau, \Omega)$ on an anisotropic background specified by $(p(\tau), q(\tau), N_s)$ reads

$$S_\phi = -\frac{1}{2}\int \mathrm{d}^4x\,\sqrt{-g}\,(\partial\phi)^2$$
$$= \frac{1}{2\pi^2}\int_0^1 \mathrm{d}\tau N_s \frac{p^{3/2}}{\sqrt{q}}\int_{S^3}\mathrm{d}\Omega\sqrt{g_\Omega}\left(\frac{q}{2N_s^2}\dot{\phi}^2 + \frac{1}{2p}\,\phi\nabla^2\phi\right)$$
$$= \frac{1}{2\pi^2}\int_0^1 \mathrm{d}\tau N_s \int_{S^3}\mathrm{d}\Omega\sqrt{g_\Omega(\alpha\equiv 0)}\left(\frac{pq}{2N_s^2}\dot{\phi}^2 + \frac{1}{2}\,\phi\nabla^2\phi\right). \quad (9.1)$$

Here Ω stands for the Euler angles $\theta \in [0,\pi], \phi \in [0, 2\pi), \psi \in [0, 4\pi)$ on S^3 (i.e. the coordinates used in Eq. (2.1) but not in Eq. (C1))[17] and $(g_\Omega)_{ij}$ is the rescaled spatial part of the metric (2.1),

$$4\,(g_\Omega)_{ij}\mathrm{d}\Omega^i\mathrm{d}\Omega^j = \sigma_1^2 + \sigma_2^2 + \frac{1}{1+\alpha(\tau)}\sigma_3^2\,, \quad (9.2)$$

where $\alpha(\tau) \equiv p(\tau)/q(\tau) - 1$ and $(p(\tau), q(\tau), N_s)$ is one of the complex, no-boundary background solutions discussed in Sections IV and V. For clarity, the Laplacian in Eq. (9.1) is with respect to the τ-dependent metric g_Ω given in Eq. (9.2). We have

$$\sqrt{g_\Omega} \equiv \sqrt{\det\left[(g_\Omega)_{ij}\right]} = \frac{\sin\theta}{8\sqrt{1+\alpha}} = \frac{\sin\theta}{8}\sqrt{\frac{q}{p}}\,. \quad (9.3)$$

For $\theta = \pi/2$, $p = 2$ and $q = 3$, from (9.3) we obtain:

1/8 sin(Pi/2) * sqrt(3/2)

Input:

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}$$

Exact result:

$$\frac{\sqrt{\frac{3}{2}}}{8}$$

Decimal approximation:

0.153093108923948631137330254669118211997871717541041883027...

0.1530931089…

Alternate form:

$$\frac{\sqrt{6}}{16}$$

Alternative representations:

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{8} \cos(0) \sqrt{\frac{3}{2}}$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{8} \cosh(0) \sqrt{\frac{3}{2}}$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{\sqrt{\frac{3}{2}}}{8 \sec(0)}$$

Series representations:

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{8} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2}\, 2^{-1-2k_1-k_2}\, \pi^{1+2k_1} \left(-\frac{1}{2}\right)_{k_2}}{(1+2k_1)!\, k_2!}$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{4} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2}\, 2^{-k_2}\, J_{1+2k_1}\left(\frac{\pi}{2}\right) \left(-\frac{1}{2}\right)_{k_2}}{k_2!}$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{4} \exp\left(i \pi \left\lfloor \frac{\arg\left(\frac{3}{2} - x\right)}{2 \pi} \right\rfloor\right) \sqrt{x} \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \frac{(-1)^{k_1+k_2} \left(\frac{3}{2} - x\right)^{k_2} x^{-k_2}\, J_{1+2k_1}\left(\frac{\pi}{2}\right) \left(-\frac{1}{2}\right)_{k_2}}{k_2!} \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

Integral representations:

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{1}{16} \pi \sqrt{\frac{3}{2}} \int_0^1 \cos\left(\frac{\pi t}{2}\right) dt$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{\sqrt{\frac{3}{2}} \sqrt{\pi}}{64 i} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{e^{-\pi^2/(16 s)+s}}{s^{3/2}}\, ds \quad \text{for } \gamma > 0$$

$$\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}} = \frac{\sqrt{\frac{3}{2}} \sqrt{\pi}}{16 i \pi} \int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{4^{-1+2s}\, \pi^{1-2s}\, \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}\, ds \quad \text{for } 0 < \gamma < 1$$

Half-argument formulas:

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}} = \frac{\sqrt{3}\sqrt{\frac{1}{2}(1-\cos(\pi))}}{8\sqrt{2}}$$

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}} = \frac{(-1)^{\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{3}\sqrt{\frac{1}{2}(1-\cos(\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)}{8\sqrt{2}}$$

Multiple-argument formulas:

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}} = \frac{1}{4}\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}$$

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}} = \frac{1}{8}\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)\sqrt{\frac{3}{2}}$$

$$\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}} = \frac{1}{8}U_{-\frac{1}{2}}(\cos(\pi))\sin(\pi)\sqrt{\frac{3}{2}}$$

From which:

1+1/2sqrt[11((1/8 sin(Pi/2) * sqrt(3/2)))]

Input:

$$1+\frac{1}{2}\sqrt{11\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}$$

Exact result:

$$1+\frac{\sqrt[4]{3}\sqrt{11}}{4\times 2^{3/4}}$$

Decimal approximation:

1.648849789659254444323354613882736436020782027459183742855...

$1.64884978965.... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 ...$

Alternate forms:

$$\frac{1}{8}\left(8+\sqrt[4]{6}\sqrt{11}\right)$$

$$\frac{1}{8}\left(\sqrt[4]{6}\ \sqrt{11}\right)+1$$

$$\frac{4\times 2^{3/4}+\sqrt[4]{3}\ \sqrt{11}}{4\times 2^{3/4}}$$

Minimal polynomial:

$$2048\,x^4-8192\,x^3+12\,288\,x^2-8192\,x+1685$$

Alternative representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{8}\cos(0)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{8}\cosh(0)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}}{8\sec(0)}}$$

Series representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{8}\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{4}\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}(-1)^k\,J_{1+2k}\left(\frac{\pi}{2}\right)}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{8}\,\pi\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}+2k\right)\left(\left(\frac{1}{2}\right)_k\right)^3}{(k!)^3}}$$

Integral representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11}{16}\,\pi\sqrt{\frac{3}{2}}\int_0^1\cos\left(\frac{\pi t}{2}\right)dt}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}=1+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}\sqrt{\pi}}{64\,i}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\pi^2/(16s)+s}}{s^{3/2}}\,ds}\quad \text{for } \gamma>0$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)} = 1+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}\sqrt{\pi}}{16\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{4^{-1+2s}\,\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds}$$

for $0<\gamma<1$

Half-argument formula:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)} = 1+\frac{\sqrt{\frac{11}{4}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}}}{2\sqrt{2}}$$

Multiple-argument formulas:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)} = 1+\frac{1}{2}\sqrt{\frac{11}{8}}\sqrt{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)} = \frac{1}{2}\left(2+\sqrt{\frac{11}{8}\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)\sqrt{\frac{3}{2}}}\right)$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)} = \frac{1}{2}\left(2+\sqrt{\frac{11}{4}\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}}\right)$$

and:

1+1/2sqrt[11((1/8 sin(Pi/2) * sqrt(3/2)))]-(29+2)1/10^3

Input:

$$1+\frac{1}{2}\sqrt{11\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-(29+2)\times\frac{1}{10^3}$$

Exact result:

$$\frac{969}{1000}+\frac{\sqrt[4]{3}\sqrt{11}}{4\times 2^{3/4}}$$

Decimal approximation:

1.617849789659254444323354613882736436020782027459183742855...

1.61784978965…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\frac{969+125\sqrt[4]{6}\sqrt{11}}{1000}$$

$$\frac{1}{8}\left(\sqrt[4]{6}\sqrt{11}\right)+\frac{969}{1000}$$

$$\frac{969\times 2^{3/4}+250\sqrt[4]{3}\sqrt{11}}{1000\times 2^{3/4}}$$

Minimal polynomial:

$$1\,000\,000\,000\,000\,x^4-3\,876\,000\,000\,000\,x^3+5\,633\,766\,000\,000\,x^2-3\,639\,412\,836\,000\,x+704\,401\,665\,771$$

Alternative representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=1-\frac{31}{10^3}+\frac{1}{2}\sqrt{\frac{11}{8}\cos(0)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=1-\frac{31}{10^3}+\frac{1}{2}\sqrt{\frac{11}{8}\cosh(0)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=1-\frac{31}{10^3}+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}}{8\sec(0)}}$$

Series representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{8}\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-1-2k}\,\pi^{1+2k}}{(1+2k)!}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{4}\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{8}\pi\sqrt{\frac{3}{2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}+2k\right)\left(\left(\frac{1}{2}\right)_k\right)^3}{(k!)^3}}$$

Integral representations:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{16}\pi\sqrt{\frac{3}{2}}\int_0^1\cos\left(\frac{\pi t}{2}\right)dt}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=$$
$$\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}\sqrt{\pi}}{64 i}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\pi^2/(16 s)+s}}{s^{3/2}}ds} \quad \text{for } \gamma>0$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=$$
$$\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11\sqrt{\frac{3}{2}}\sqrt{\pi}}{16 i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^{-1+2s}\pi^{1-2s}\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}ds} \quad \text{for } 0<\gamma<1$$

Half-argument formula:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{\sqrt{\frac{11}{4}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}}}{2\sqrt{2}}$$

Multiple-argument formulas:

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{8}}\sqrt{\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{8}\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)\sqrt{\frac{3}{2}}}$$

$$1+\frac{1}{2}\sqrt{\frac{11}{8}\left(\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)}-\frac{29+2}{10^3}=\frac{969}{1000}+\frac{1}{2}\sqrt{\frac{11}{4}\cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}}$$

From (9.2), we obtain:

4(((1/8 sin(Pi/2) * sqrt(3/2))))^2

Input:

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2$$

Exact result:

$$\frac{3}{32}$$

Decimal form:

0.09375

0.09375

Alternative representations:

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = 4\left(\frac{1}{8}\cos(0)\sqrt{\frac{3}{2}}\right)^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = 4\left(\frac{1}{8}\cosh(0)\sqrt{\frac{3}{2}}\right)^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = 4\left(\frac{\sqrt{\frac{3}{2}}}{8\sec(0)}\right)^2$$

Series representations:

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{4}\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{16}\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{4}\exp^2\left(i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2 \left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2 \quad \text{for } (x \in \mathbb{R} \text{ and } x < 0)$$

Occurrence in convergents:

$$\frac{2\pi}{67} \approx 0, \frac{1}{10}, \frac{1}{11}, \frac{2}{21}, \frac{3}{32}, \frac{101}{1077}, \frac{609}{6494}, \ldots$$

$$\frac{e^{-\gamma}}{6} \approx 0, \frac{1}{10}, \frac{1}{11}, \frac{3}{32}, \frac{16}{171}, \frac{51}{545}, \ldots$$

(simple continued fraction convergent sequences)

γ is the Euler-Mascheroni constant

Half-argument formulas:

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{\sqrt{3}^2 \sqrt{\frac{1}{2}(1-\cos(\pi))}^2}{16\sqrt{2}^2}$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 =$$
$$\frac{(-1)^{2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(-1+\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2}{16\sqrt{2}^2}$$

Multiple-argument formulas:

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{4}\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{16}\left(-3\sin\left(\frac{\pi}{6}\right)+4\sin^3\left(\frac{\pi}{6}\right)\right)^2\sqrt{\frac{3}{2}}^2$$

$$4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{16}U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{\frac{3}{2}}^2$$

From which, if we perform 24 divided the expression, we obtain:

24*1/(((4(((1/8 sin(Pi/2) * sqrt(3/2))))^2)))

Input:

$$24\times\frac{1}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2}$$

Exact result:

256

256

Alternative representations:

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{24}{4\left(\frac{1}{8}\cos(0)\sqrt{\frac{3}{2}}\right)^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{24}{4\left(\frac{1}{8}\cosh(0)\sqrt{\frac{3}{2}}\right)^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{24}{4\left(-\frac{1}{8}\cos(\pi)\sqrt{\frac{3}{2}}\right)^2}$$

Series representations:

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{96}{\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2 \left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{384}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2 \left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} =$$

$$\frac{96}{\exp^2\left(i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Half-argument formulas:

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{384\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} =$$

$$\frac{384\,(-1)^{-2\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\left(1+(-1)^{\lfloor -\operatorname{Re}(\pi)/(2\pi)\rfloor+\lfloor \operatorname{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\operatorname{Im}(\pi))\right)^2}$$

Multiple-argument formulas:

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{96}{\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{384}{\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)^2\sqrt{\frac{3}{2}}^2}$$

$$\frac{24}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2} = \frac{384}{U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{\frac{3}{2}}^2}$$

and again:

1/4*24*1/(((4(((1/8 sin(Pi/2) * sqrt(3/2))))^2)))

Input:

$$\frac{1}{4}\times 24\times\frac{1}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2}$$

Exact result:

64

64

Alternative representations:

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{24}{4\left(4\left(\frac{1}{8}\cos(0)\sqrt{\frac{3}{2}}\right)^2\right)}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{24}{4\left(4\left(\frac{1}{8}\cosh(0)\sqrt{\frac{3}{2}}\right)^2\right)}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{24}{4\left(4\left(-\frac{1}{8}\cos(\pi)\sqrt{\frac{3}{2}}\right)^2\right)}$$

Series representations:

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{24}{\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2 \left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{96}{\left(\sum_{k=0}^{\infty}\frac{(-1)^k 2^{-1-2k}\pi^{1+2k}}{(1+2k)!}\right)^2 \left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} =$$

$$\frac{24}{\exp^2\left(i\pi\left\lfloor\frac{\arg\left(\frac{3}{2}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}^2\left(\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{\pi}{2}\right)\right)^2\left(\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{3}{2}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)^2}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Half-argument formulas:

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{96\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} =$$

$$\frac{96(-1)^{-2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2}$$

Multiple-argument formulas:

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{24}{\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}^2}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{96}{\left(3\sin\left(\frac{\pi}{6}\right) - 4\sin^3\left(\frac{\pi}{6}\right)\right)^2 \sqrt{\frac{3}{2}}^2}$$

$$\frac{24}{\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)4} = \frac{96}{U_{-\frac{1}{2}}(\cos(\pi))^2 \sin^2(\pi) \sqrt{\frac{3}{2}}^2}$$

(((1/4*24*1/(((4(((1/8 sin(Pi/2) * sqrt(3/2))))^2))))))^2

Input:

$$\left(\frac{1}{4} \times 24 \times \frac{1}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2}\right)^2$$

Exact result:

4096

4096

We have the following connection:

$$\left[\left(\frac{1}{4} \times 24 \times \frac{1}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2}\right)^2\right] = 4096$$

$$\left[4\left(-1 + 5\sqrt{5} + \frac{(\sqrt{5}-1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16}(59 + 89\sqrt{5})}{137438953472}\right)^2 \pi^2\right] = 4095.9999\ldots$$

and:

27*1/4*24*1/(((4(((1/8 sin(Pi/2) * sqrt(3/2))))^2))) + 1

Input:

$$27 \times \frac{1}{4} \times 24 \times \frac{1}{4\left(\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}}\right)^2} + 1$$

Exact result:

1729

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*"The fundamental group of the complex form, compact real form, or any algebraic version of E_6 is the cyclic group **Z**/3**Z**, and its outer automorphism group is the cyclic group **Z**/2**Z**. Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E_6 plays a role in some grand unified theories".*

Alternative representations:

$$\frac{27 \times 24}{4\left(4\left(\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}}\right)^2\right)} + 1 = 1 + \frac{648}{4\left(4\left(\frac{1}{8} \cos(0) \sqrt{\frac{3}{2}}\right)^2\right)}$$

$$\frac{27 \times 24}{4\left(4\left(\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}}\right)^2\right)} + 1 = 1 + \frac{648}{4\left(4\left(\frac{1}{8} \cosh(0) \sqrt{\frac{3}{2}}\right)^2\right)}$$

$$\frac{27 \times 24}{4\left(4\left(\frac{1}{8} \sin\left(\frac{\pi}{2}\right) \sqrt{\frac{3}{2}}\right)^2\right)} + 1 = 1 + \frac{648}{4\left(4\left(-\frac{1}{8} \cos(\pi) \sqrt{\frac{3}{2}}\right)^2\right)}$$

Half-argument formulas:

$$\frac{27\times 24}{4\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)}+1=1+\frac{2592\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2}$$

$$\frac{27\times 24}{4\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)}+1=$$
$$1+\frac{2592(-1)^{-2\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\sqrt{2}^2}{\sqrt{3}^2\sqrt{\frac{1}{2}(1-\cos(\pi))}^2\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(\pi))\right)^2}$$

Multiple-argument formulas:

$$\frac{27\times 24}{4\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)}+1=1+\frac{648}{\cos^2\left(\frac{\pi}{4}\right)\sin^2\left(\frac{\pi}{4}\right)\sqrt{\frac{3}{2}}^2}$$

$$\frac{27\times 24}{4\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)}+1=1+\frac{2592}{\left(3\sin\left(\frac{\pi}{6}\right)-4\sin^3\left(\frac{\pi}{6}\right)\right)^2\sqrt{\frac{3}{2}}^2}$$

$$\frac{27\times 24}{4\left(4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2\right)}+1=1+\frac{2592}{U_{-\frac{1}{2}}(\cos(\pi))^2\sin^2(\pi)\sqrt{\frac{3}{2}}^2}$$

From which:

(((27*1/4*24*1/(((4(((1/8 sin(Pi/2) * sqrt(3/2))))^2))) + 1)))^1/15

Input:

$$\sqrt[15]{27\times\frac{1}{4}\times 24\times\frac{1}{4\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)^2}+1}$$

Exact result:

$$\sqrt[15]{1729}$$

Decimal approximation:

1.6438152287487281305800880313247695143292831436999401726 45...

1.643815228…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Now:

For N_s = 17.3205 ; $\sqrt{g_\Omega}$ = 1/8 sin(Pi/2) * sqrt(3/2) ; p = 2, q = 3 and $\phi = \pi$, from

$$S_\phi = -\frac{1}{2}\int d^4x\,\sqrt{-g}\,(\partial\phi)^2$$

$$= \frac{1}{2\pi^2}\int_0^1 d\tau N_s \int_{S^3} d\Omega\sqrt{g_\Omega(\alpha\equiv 0)}\left(\frac{pq}{2N_s^2}\dot{\phi}^2 + \frac{1}{2}\phi\nabla^2\phi\right)$$

we obtain:

1/(2Pi^2) integrate(17.3205)dx * integrate(((((1/8 sin(Pi/2) * sqrt(3/2))*((6/(2*17.3205^2))*2Pi + (Pi^2)/2))))dx

Input interpretation:

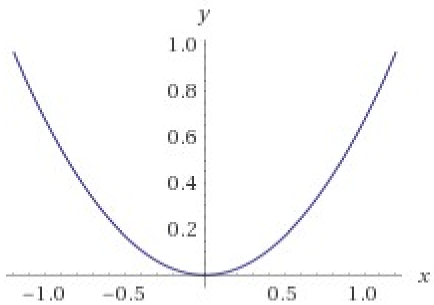
$$\frac{1}{2\pi^2}\left(\int 17.3205\,dx\right)\int\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)\left(\frac{6}{2\times 17.3205^2}\times 2\pi + \frac{\pi^2}{2}\right)dx$$

Result:

$0.671353\,x^2$

0.671353

Plot:



(x from −1.2 to 1.2)

Alternate form assuming x is real:

$0.671353\,x^2 + 0$

For x = 1, summing 1 and multiplying by $1/10^{27}$, we obtain:

1/10^27*(1+0.671353 1^2)

Input interpretation:

$$\frac{1}{10^{27}}\left(1+0.671353\times 1^2\right)$$

Result:

$$1.671353\times 10^{-27}$$

$1.671353*10^{-27}$ result practically equal to the value of the formula:

$$m_{p\prime} = 2\times\frac{\eta}{R}m_P = 1.6714213\times 10^{-27}\ \text{kg}$$

that is the holographic proton mass (N. Haramein)

We have also:

43*1/((((1/(2Pi^2) integrate(17.3205)dx * integrate(((((1/8 sin(Pi/2) * sqrt(3/2))*((6/(2*17.3205^2))*2Pi + (Pi^2)/2))))dx)))))

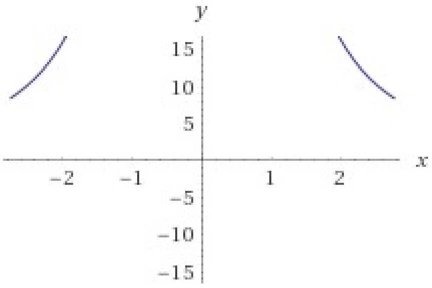
Input interpretation:

$$43\times\frac{1}{\frac{1}{2\pi^2}\left(\int 17.3205\,dx\right)\int\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)\left(\frac{6}{2\times 17.3205^2}\times 2\pi+\frac{\pi^2}{2}\right)dx}$$

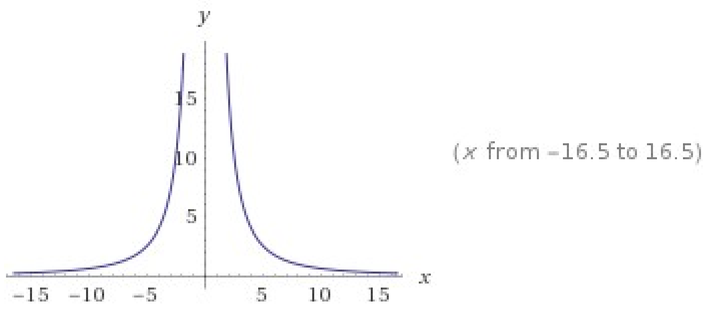
Result:

$$\frac{64.0498}{x^2}$$

Plots:



(x from −2.7 to 2.7)



Alternate form assuming x is real:

$$\frac{64.0498}{x^2}+0$$

For x = 1, we obtain:

Input interpretation:

$$\frac{64.0498}{1^2}$$

Result:

64.0498

64.0498….≈ 64

and:

[43*1/((((1/(2Pi^2) integrate(17.3205)dx * integrate(((((1/8 sin(Pi/2) * sqrt(3/2))*((6/(2*17.3205^2))*2Pi + (Pi^2)/2))))dx)))))]^2-6

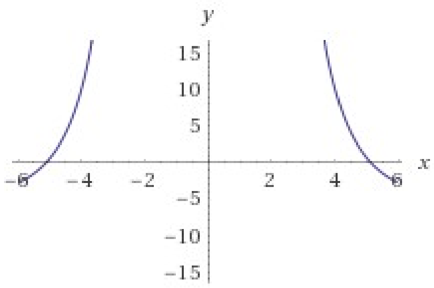
Input interpretation:

$$\left(43\times\frac{1}{\frac{1}{2\pi^2}\left(\int 17.3205\,dx\right)\int\left(\frac{1}{8}\sin\left(\frac{\pi}{2}\right)\sqrt{\frac{3}{2}}\right)\left(\frac{6}{2\times 17.3205^2}\times 2\pi+\frac{\pi^2}{2}\right)dx}\right)^2-6$$

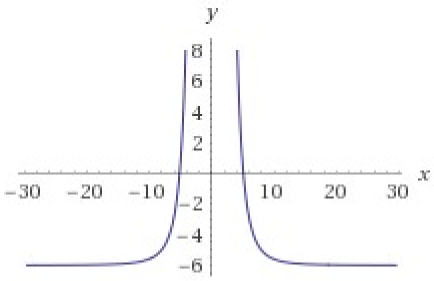
Result:

$$\frac{4102.37}{x^4}-6$$

Plots:



(x from -5.9 to 5.9)



(x from -29.5 to 29.5)

Alternate forms:

$$\frac{6\,(683.729 - x^4)}{x^4}$$

$$-\frac{6\,(x - 5.11353)\,(x + 5.11353)\,(x^2 + 26.1482)}{x^4}$$

Alternate form assuming x is real:

$$\frac{4102.37}{x^4} - 6$$

Indefinite integral assuming all variables are real:

$$-\frac{1367.46}{x^3} - 6\,x + \text{constant}$$

For x = 1, we obtain:

-6 + 4102.37/1^4

Input interpretation:

$$-6 + \frac{4102.37}{1^4}$$

Result:

4096.37

4096.37 ≈ 4096, that can be connected as above with the Ramanujan equation

Now, we have that:

$$\frac{3(N_s - iq)(N_s - 3iq)p}{N_s^3(N_s + 3i)} - \frac{9i(N_s - iq)^2 p}{N_s^3(N_s + 3i)^2} - \frac{4N_s^2}{p} + \frac{6pq}{N_s^2} - \frac{8iN_s}{p} - p + 16 = 0\,, \qquad (5.2)$$

for $N_s = 17.3205$; p = 2, q = 3 , we obtain:

(3(17.3205-i*3)(17.3205-3*i*3)2) / (17.3205^3(17.3205+3*i)))-(9*i(17.3205-i*3)^2*2) /(17.3205^3(17.3205+3*i)^2)-(4*(17.3205^2) / 2) + (6*2*3 / 17.3205^2)-(8*i*17.3205) / 2 -2+16

that is:

(3(17.3205-i*3)(17.3205-3*i*3)2) / (17.3205^3(17.3205+3*i)))-(9*i(17.3205-i*3)^2*2) /(17.3205^3(17.3205+3*i)^2)

Input interpretation:

$$\frac{3\,(17.3205 - i\times 3)\,(17.3205 - 3\,i\times 3)\times 2}{17.3205^3\,(17.3205 + 3\,i)} - \frac{9\,i\,((17.3205 - i\times 3)^2\times 2)}{17.3205^3\,(17.3205 + 3\,i)^2}$$

i is the imaginary unit

Result:

0.0131454... − 0.0191938... i

Polar coordinates:

$r = 0.0232638$ (radius), $\theta = -55.5935°$ (angle)

0.0232638

and:

-(4*(17.3205^2) / 2) + (6*2*3 / 17.3205^2)- (8*i*17.3205) / 2 -2+16

Input interpretation:

$$-\left(4\times\frac{17.3205^2}{2}\right)+6\times2\times\frac{3}{17.3205^2}-\frac{1}{2}\,(8\,i\times17.3205)-2+16$$

i is the imaginary unit

Result:

$-585.879\ldots - 69.282\ldots\, i$

Polar coordinates:

$r = 589.962$ (radius), $\theta = -173.256°$ (angle)

589.962

(0.0131454 – 0.0191938i)+(((-(4*(17.3205^2) / 2) + (6*2*3 / 17.3205^2)-(8*i*17.3205) / 2 -2+16)))

Input interpretation:

$$(0.0131454 + i\times(-0.0191938)) + \left(-\left(4\times\frac{17.3205^2}{2}\right)+6\times2\times\frac{3}{17.3205^2}-\frac{1}{2}\,(8\,i\times17.3205)-2+16\right)$$

i is the imaginary unit

Result:

$-585.866\ldots - 69.3012\ldots\, i$

Polar coordinates:

$r = 589.951$ (radius), $\theta = -173.254°$ (angle)

589.951 (final result)

From the formula of coefficients of the '5th order' mock theta function $\psi_1(q)$: (A053261 OEIS Sequence)

sqrt(golden ratio) * exp(Pi*sqrt(n/15)) / (2*5^(1/4)*sqrt(n)) for n = 144 and subtracting 8, that is a Fibonacci number, we obtain:

sqrt(golden ratio) * exp(Pi*sqrt(144/15)) / (2*5^(1/4)*sqrt(144)) - 8

Input:

$$\sqrt{\phi}\times\frac{\exp\left(\pi\sqrt{\frac{144}{15}}\right)}{2\sqrt[4]{5}\sqrt{144}}-8$$

ϕ is the golden ratio

Exact result:

$$\frac{e^{4\sqrt{3/5}\,\pi}\sqrt{\phi}}{24\sqrt[4]{5}}-8$$

Decimal approximation:

590.2815283793319928532181642137319449412887062023990419819...

590.28152837…. that is very near to the result of the previous expression

Property:

$$-8+\frac{e^{4\sqrt{3/5}\,\pi}\sqrt{\phi}}{24\sqrt[4]{5}}\text{ is a transcendental number}$$

Alternate forms:

$$\frac{1}{24}\sqrt{\frac{1}{10}\left(5+\sqrt{5}\right)}\,e^{4\sqrt{3/5}\,\pi}-8$$

$$\frac{e^{4\sqrt{3/5}\,\pi}\sqrt{\phi}-192\sqrt[4]{5}}{24\sqrt[4]{5}}$$

$$\frac{1}{240}\left(5^{3/4}\sqrt{2\left(1+\sqrt{5}\right)}\,e^{4\sqrt{3/5}\,\pi}-1920\right)$$

Series representations:

$$\frac{\sqrt{\phi}\exp\left(\pi\sqrt{\frac{144}{15}}\right)}{2\sqrt[4]{5}\sqrt{144}}-8=\left(-80\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(144-z_0)^k z_0^{-k}}{k!}+5^{3/4}\exp\left(\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{48}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\Big/\left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(144-z_0)^k z_0^{-k}}{k!}\right)\text{ for (not }(z_0\in\mathbb{R}\text{ and }-\infty<z_0\le 0))$$

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{144}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{144}} - 8 = \left(-80\exp\left(i\pi\left\lfloor\frac{\arg(144-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(144-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} + 5^{3/4}\exp\left(i\pi\left\lfloor\frac{\arg(\phi-x)}{2\pi}\right\rfloor\right)\exp\left(\pi\exp\left(i\pi\left\lfloor\frac{\arg\left(\frac{48}{5}-x\right)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{48}{5}-x\right)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{(-1)^k(\phi-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right) \Big/ \left(10\exp\left(i\pi\left\lfloor\frac{\arg(144-x)}{2\pi}\right\rfloor\right)\sum_{k=0}^{\infty}\frac{(-1)^k(144-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\sqrt{\phi}\,\exp\left(\pi\sqrt{\frac{144}{15}}\right)}{2\sqrt[4]{5}\,\sqrt{144}} - 8 = \left(\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(144-z_0)/(2\pi)\rfloor} z_0^{-1/2\lfloor\arg(144-z_0)/(2\pi)\rfloor}\left(-80\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(144-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(144-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(144-z_0)^k z_0^{-k}}{k!} + 5^{3/4}\exp\left(\pi\left(\frac{1}{z_0}\right)^{1/2\left\lfloor\arg\left(\frac{48}{5}-z_0\right)/(2\pi)\right\rfloor} z_0^{1/2\left(1+\left\lfloor\arg\left(\frac{48}{5}-z_0\right)/(2\pi)\right\rfloor\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{48}{5}-z_0\right)^k z_0^{-k}}{k!}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor} z_0^{1/2\lfloor\arg(\phi-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(\phi-z_0)^k z_0^{-k}}{k!}\right)\right) \Big/ \left(10\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(144-z_0)^k z_0^{-k}}{k!}\right)$$

We note that 589 = 610 – 21 , where 610 and 21 are two Fibonacci numbers.

From the previous expression,

$$(0.0131454 + i\times(-0.0191938)) + \left(-\left(4\times\frac{17.3205^2}{2}\right) + 6\times 2\times\frac{3}{17.3205^2} - \frac{1}{2}(8\,i\times 17.3205) - 2 + 16\right)$$

we obtain also:

((((0.0131454 – 0.0191938i)+(((-(4*(17.3205^2) / 2) + (6*2*3 / 17.3205^2)-(8*i*17.3205) / 2 -2+16))))))^1/13

Input interpretation:

$$\left((0.0131454 + i \times (-0.0191938)) + \left(-\left(4 \times \frac{17.3205^2}{2}\right) + 6 \times 2 \times \frac{3}{17.3205^2} - \frac{1}{2}(8\, i \times 17.3205) - 2 + 16\right)\right)\text{\^{}}(1/13)$$

i is the imaginary unit

Result:

1.589584... –
0.3765594... i

Polar coordinates:

$r = 1.63358$ (radius), $\theta = -13.3272°$ (angle)

$1.63358 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

From:

PHYSICAL REVIEW D VOLUME 28, NUMBER 12
Wave function of the Universe
15 DECEMBER 1983
J. B. Hartle - Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637 and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106
S. W. Hawking - Department ofApplied Mathematics and Theoretical Physics, Silver Street, Cambridge, England and Institute for Theoretical Physics, University of California, Santa Barbara, California 93106 - (Received 29 July 1983)

We have that:

$$\psi_0(a_0) = 2\cos\left[\frac{(H^2 a_0{}^2 - 1)^{3/2}}{3H^2} - \frac{\pi}{4}\right], \quad Ha_0 > 1$$

For H = 1 and $Ha_0 = 8$, we obtain:

2cos [((8^2-1)^1.5)/3 – Pi/4]

Input:

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)$$

Result:

−1.6421824100931121443162008119026536044022403197003927153 0...

$-1.64218241... \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934$ ... with minus sign

Addition formulas:

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\left(\cos(166.682)\cos\left(\frac{\pi}{4}\right)+\sin(166.682)\sin\left(\frac{\pi}{4}\right)\right)$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cos(166.682)\cos\left(-\frac{\pi}{4}\right)-2\sin(166.682)\sin\left(-\frac{\pi}{4}\right)$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cosh\left(\frac{i\pi}{4}\right)\cos(166.682)-2\,i\sinh\left(\frac{i\pi}{4}\right)\sin(166.682)$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\left(\cosh\left(-\frac{i\pi}{4}\right)\cos(166.682)+i\sinh\left(-\frac{i\pi}{4}\right)\sin(166.682)\right)$$

Alternative representations:

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cosh\left(i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cosh\left(-i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=e^{-i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}+e^{i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}$$

Series representations:

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{\pi}{4}\right)^{2k}}{(2k)!}$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=-2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{3\pi}{4}\right)^{1+2k}}{(1+2k)!}$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\propto\frac{2\sum_{k=0}^{\infty}(-1)^k\frac{\partial^{1+2k}}{\partial\left(166.682-\frac{\pi}{4}\right)^{1+2k}}\delta\left(166.682-\frac{\pi}{4}\right)}{\theta\left(166.682-\frac{\pi}{4}\right)}$$

Integral representations:

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2+0.5\,(-666.729+\pi)\int_0^1\sin(-0.25\,(-666.729+\pi)\,t)\,dt$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=-2\int_{\frac{\pi}{2}}^{166.682-\frac{\pi}{4}}\sin(t)\,dt$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(0.015625\,(-666.729+\pi)^2\right)/s+s}}{\sqrt{s}}\,ds\quad\text{for }\gamma>0$$

$$2\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{\sqrt{\pi}}{i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\left(166.682-\frac{\pi}{4}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

For

$$\epsilon_0=-\frac{1}{2}$$

$a_0 = 5$; $H = 1$ and $Ha_0 = 8$; from

$$\psi_0(a_0)=2\left(H^2a_0^4-a_0^2+\epsilon_0+\frac{1}{2}\right)^{-1/4}\times\cos\left[\frac{(H^2a_0^2-1)^{3/2}}{3H^2}-\frac{\pi}{4}\right]. \qquad (6.24)$$

we obtain:

2(2^2*2^4-5^2-1/2+1/2)^(-1/4)* cos [((8^2-1)^1.5)/3 – Pi/4]

Input:

$$2\left(2^2\times2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)$$

Result:

$-0.657136\ldots$

-0.657136…

Addition formulas:

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{2\left(\cos(166.682)\cos\left(-\frac{\pi}{4}\right)-\sin(166.682)\sin\left(-\frac{\pi}{4}\right)\right)}{\sqrt[4]{39}}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{2\left(\cos(166.682)\cos\left(\frac{\pi}{4}\right)+\sin(166.682)\sin\left(\frac{\pi}{4}\right)\right)}{\sqrt[4]{39}}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{2\left(\cosh\left(\frac{i\pi}{4}\right)\cos(166.682)-i\left(\sinh\left(\frac{i\pi}{4}\right)\sin(166.682)\right)\right)}{\sqrt[4]{39}}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{2\left(\cosh\left(-\frac{i\pi}{4}\right)\cos(166.682)+i\sinh\left(-\frac{i\pi}{4}\right)\sin(166.682)\right)}{\sqrt[4]{39}}$$

Alternative representations:

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cosh\left(i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=2\cosh\left(-i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\left(e^{-i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}+e^{i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

Series representations:

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=\frac{2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{\pi}{4}\right)^{2k}}{(2k)!}}{\sqrt[4]{39}}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=-\frac{2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{3\pi}{4}\right)^{1+2k}}{(1+2k)!}}{\sqrt[4]{39}}$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\propto \frac{2\sum_{k=0}^{\infty}(-1)^k\frac{\partial^{1+2k}}{\partial\left(166.682-\frac{\pi}{4}\right)^{1+2k}}\delta\left(166.682-\frac{\pi}{4}\right)}{\sqrt[4]{39}\,\theta\left(166.682-\frac{\pi}{4}\right)}$$

Integral representations:

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)=-\frac{2}{\sqrt[4]{39}}\int_{\frac{\pi}{2}}^{166.682-\frac{\pi}{4}}\sin(t)\,dt$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)= \frac{2}{\sqrt[4]{39}}+\int_0^1(-133.399+0.20008\,\pi)\sin(-0.25\,(-666.729+\pi)\,t)\,dt$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)= \frac{\sqrt{\pi}}{\sqrt[4]{39}\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{e^{-\left(0.015625\,(-666.729+\pi)^2\right)/s+s}}{\sqrt{s}}\,ds\quad\text{for }\gamma>0$$

$$2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)= \frac{\sqrt{\pi}}{\sqrt[4]{39}\,i\,\pi}\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{4^s\left(166.682-\frac{\pi}{4}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

From which:

1-(((2(2^2*2^4-5^2-1/2+1/2)^(-1/4)* cos [((8^2-1)^1.5)/3 – Pi/4])))

Input:

$$1-2\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)$$

Result:

1.657135976287754840265167637951378055386832837775912488046...

1.657135976…. result very near to the 14th root of the following Ramanujan's class invariant $Q = \left(G_{505}/G_{101/5}\right)^3 = 1164.2696$ i.e. 1.65578...

Addition formulas:

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1+\frac{-2\cos(166.682)\cos\left(-\frac{\pi}{4}\right)+2\sin(166.682)\sin\left(-\frac{\pi}{4}\right)}{\sqrt[4]{39}}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1-\frac{2\left(\cos(166.682)\cos\left(\frac{\pi}{4}\right)+\sin(166.682)\sin\left(\frac{\pi}{4}\right)\right)}{\sqrt[4]{39}}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1+\frac{-2\cosh\left(\frac{i\pi}{4}\right)\cos(166.682)+2\,i\sinh\left(\frac{i\pi}{4}\right)\sin(166.682)}{\sqrt[4]{39}}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1-\frac{2\left(\cosh\left(-\frac{i\pi}{4}\right)\cos(166.682)+i\sinh\left(-\frac{i\pi}{4}\right)\sin(166.682)\right)}{\sqrt[4]{39}}$$

Alternative representations:

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1-2\cosh\left(i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)= 1-2\cosh\left(-i\left(-\frac{\pi}{4}+\frac{1}{3}\left(-1+8^2\right)^{1.5}\right)\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=$$
$$1-\left(e^{-i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}+e^{i\left(-\pi/4+1/3\left(-1+8^2\right)^{1.5}\right)}\right)\left(4\times 2^4-5^2\right)^{-1/4}$$

Series representations:

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=1-\frac{2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{\pi}{4}\right)^{2k}}{(2k)!}}{\sqrt[4]{39}}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=$$
$$1+\frac{2\sum_{k=0}^{\infty}\frac{(-1)^k\left(166.682-\frac{3\pi}{4}\right)^{1+2k}}{(1+2k)!}}{\sqrt[4]{39}}$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)\propto$$
$$1-\frac{2\sum_{k=0}^{\infty}(-1)^k\frac{\partial^{1+2k}}{\partial\left(166.682-\frac{\pi}{4}\right)^{1+2k}}\delta\left(166.682-\frac{\pi}{4}\right)}{\sqrt[4]{39}\;\theta\left(166.682-\frac{\pi}{4}\right)}$$

Integral representations:

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=$$
$$0.19968-0.20008\,(-666.729+\pi)\int_0^1\sin(-0.25\,(-666.729+\pi)\,t)\,dt$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=1+\frac{2}{\sqrt[4]{39}}\int_{\frac{\pi}{2}}^{166.682-\frac{\pi}{4}}\sin(t)\,dt$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=$$
$$1-\frac{\sqrt{\pi}}{\sqrt[4]{39}\;i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{e^{-\left(166.682-\frac{\pi}{4}\right)^2/(4s)+s}}{\sqrt{s}}\,ds\quad\text{for }\gamma>0$$

$$1-2\left(\left(2^2\times 2^4-5^2-\frac{1}{2}+\frac{1}{2}\right)^{-1/4}\cos\left(\frac{1}{3}\left(8^2-1\right)^{1.5}-\frac{\pi}{4}\right)\right)=$$
$$1-\frac{\sqrt{\pi}}{\sqrt[4]{39}\;i\pi}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^s\left(166.682-\frac{\pi}{4}\right)^{-2s}\Gamma(s)}{\Gamma\left(\frac{1}{2}-s\right)}\,ds\quad\text{for }0<\gamma<\frac{1}{2}$$

From Ramanujan equation (**Modular equations and approximations to π** – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372:

$$G_{505}^2 = (2+\sqrt{5})\sqrt{\left\{\left(\frac{1+\sqrt{5}}{2}\right)(10+\sqrt{101})\right\}} \times \left\{\left(\frac{5\sqrt{5}+\sqrt{101}}{4}\right)+\sqrt{\left(\frac{105+\sqrt{505}}{8}\right)}\right\},$$

we obtain:

(2+sqrt5) (((((1+sqrt5)/2) (10+sqrt101))))^0.5 ((((((5sqrt5+sqrt101)/4+((105+sqrt505)/8)^0.5)))))

Input:

$$\left(2+\sqrt{5}\right)\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(10+\sqrt{101}\right)}\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\sqrt{\frac{1}{8}\left(105+\sqrt{505}\right)}\right)$$

Exact result:

$$\left(2+\sqrt{5}\right)\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)$$

Decimal approximation:

224.36895935132763918399413635761729391464432800073649303810...

224.36895935…

Alternate forms:

$$\sqrt{\text{root of } 256x^8 - 13134080x^7 + 12406662784x^6 + 566469885440x^5 + 8970692383216x^4 + 59000758979200x^3 + 133454526025384x^2 - 21580568998020x + 63001502001 \text{ near } x = 50341.4}$$

$$\frac{1}{4}\left(2+\sqrt{5}\right)\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{4}\left(2+\sqrt{5}\right)\sqrt{\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)\left(105+\sqrt{505}\right)}$$

$$\frac{25}{4}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\frac{5}{2}\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\frac{1}{2}\sqrt{\frac{101}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\frac{1}{4}\sqrt{\frac{505}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\frac{1}{2}\sqrt{\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)\left(105+\sqrt{505}\right)}+\frac{1}{4}\sqrt{5\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)\left(105+\sqrt{505}\right)}$$

Minimal polynomial:

$$256x^{16}-13134080x^{14}+12406662784x^{12}+566469885440x^{10}+8970692383216x^{8}+59000758979200x^{6}+133454526025384x^{4}-21580568998020x^{2}+63001502001$$

Performing the 11th root, we obtain:

((((2+sqrt5) (((((1+sqrt5)/2) (10+sqrt101))))^0.5
((((((5sqrt5+sqrt101)/4+((105+sqrt505)/8)^0.5))))))))^1/11

Input:

$$\sqrt[11]{\left(2+\sqrt{5}\right)\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(10+\sqrt{101}\right)}\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\sqrt{\frac{1}{8}\left(105+\sqrt{505}\right)}\right)}$$

Exact result:

$$\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\ \sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}$$

Decimal approximation:

1.635776213003291789374056840890028295596227184272763857453...

1.635776213….≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms:

$$\frac{\sqrt[22]{\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\ \sqrt[11]{\left(2+\sqrt{5}\right)\left(5\sqrt{5}+\sqrt{101}+\sqrt{2\left(105+\sqrt{505}\right)}\right)}}{2^{5/22}}$$

$$\frac{\frac{1}{2^{5/22}}\sqrt[22]{\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{25+10\sqrt{5}+2\sqrt{101}+\sqrt{505}+2\sqrt{2\left(105+\sqrt{505}\right)}+\sqrt{10\left(105+\sqrt{505}\right)}}}$$

All 11th roots of (2 + sqrt(5)) sqrt(1/2 (1 + sqrt(5)) (10 + sqrt(101))) (1/4 (5 sqrt(5) + sqrt(101)) + 1/2 sqrt(1/2 (105 + sqrt(505)))):

$$\frac{\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}}\, e^{0}$$

≈ 1.63578 (real, principal root)

$$\frac{\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}}\, e^{(2i\pi)/11} \approx 1.3761+0.8844\,i$$

$$\frac{\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}}\, e^{(4i\pi)/11} \approx 0.6795+1.4880\,i$$

$$\frac{\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}}\, e^{(6i\pi)/11} \approx -0.23280+1.61913\,i$$

$$\frac{\sqrt[22]{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}}{\sqrt[11]{\left(2+\sqrt{5}\right)\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)}}\, e^{(8i\pi)/11} \approx -1.0712+1.2362\,i$$

We note that we obtain also:

2((((((2+sqrt5) (((((1+sqrt5)/2) (10+sqrt101))))^0.5 ((((((5sqrt5+sqrt101)/4+((105+sqrt505)/8)^0.5))))))))))+48

Input:

$$2\left(\left(2+\sqrt{5}\right)\sqrt{\left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)\left(10+\sqrt{101}\right)}\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\sqrt{\frac{1}{8}\left(105+\sqrt{505}\right)}\right)\right)+48$$

Exact result:

$$48+\left(2+\sqrt{5}\right)\sqrt{2\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\left(\frac{1}{4}\left(5\sqrt{5}+\sqrt{101}\right)+\frac{1}{2}\sqrt{\frac{1}{2}\left(105+\sqrt{505}\right)}\right)$$

Decimal approximation:

496.7379187026552783679882727152345878292886560014729860763...

496.7379187… ≈ 496

The E_8 x E_8 group, concerning the Heterotic String Theory - two copies of the largest exceptional group - has the dimension equal to 248+248 = 496

Alternate forms:

$$48+\frac{1}{2}\left(2+\sqrt{5}\right)\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}\left(5\sqrt{5}+\sqrt{101}+\sqrt{2\left(105+\sqrt{505}\right)}\right)$$

$$\sqrt{\left(\text{root of } x^8-186\,788\,x^7-2\,385\,736\,088\,x^6-11\,331\,368\,579\,200\,x^5-22\,492\,214\,255\,922\,704\,x^4-1\,490\,945\,227\,125\,102\,080\,x^3+69\,754\,259\,698\,035\,104\,606\,848\,x^2+110\,328\,776\,882\,208\,947\,456\,499\,456\,x+55\,507\,846\,349\,946\,480\,520\,729\,211\,136 \text{ near } x=199\,062.\right)+2304}+48$$

$$48+\frac{25}{2}\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+5\sqrt{\frac{5}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\sqrt{\frac{101}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\frac{1}{2}\sqrt{\frac{505}{2}\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)}+\sqrt{\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)\left(105+\sqrt{505}\right)}+\frac{1}{2}\sqrt{5\left(1+\sqrt{5}\right)\left(10+\sqrt{101}\right)\left(105+\sqrt{505}\right)}$$

Minimal polynomial:

$$x^{16} - 768\,x^{15} + 71\,260\,x^{14} + 75\,976\,320\,x^{13} - 32\,590\,512\,536\,x^{12} + 6\,701\,607\,654\,912\,x^{11} - 874\,485\,415\,645\,312\,x^{10} + 79\,036\,216\,448\,839\,680\,x^{9} - 5\,122\,520\,689\,993\,297\,424\,x^{8} + 239\,641\,688\,731\,285\,788\,672\,x^{7} - 7\,924\,803\,393\,181\,292\,467\,712\,x^{6} + 172\,420\,964\,572\,345\,979\,682\,816\,x^{5} - 1\,881\,835\,874\,446\,610\,544\,530\,816\,x^{4} - 12\,073\,724\,945\,556\,985\,770\,663\,936\,x^{3} + 753\,184\,034\,259\,300\,471\,513\,210\,624\,x^{2} - 10\,591\,562\,580\,692\,058\,955\,823\,947\,776\,x + 55\,507\,846\,349\,946\,480\,520\,729\,211\,136$$

Now, from:

$$\Psi_0^c(a_{(1)}, a_{(2)}) = N\Delta^{-1/2}(a_{(1)}, a_{(2)}) \times \exp\left\{\frac{1}{3H^2}\left[-(1-H^2 a_{(2)}{}^2)^{3/2} + (1-H^2 a_{(1)}{}^2)^{3/2}\right]\right\}.$$

And from:

Black hole pair creation in de Sitter space: a complete one-loop analysis
Mikhail S. Volkov and Andreas Wipf - arXiv:hep-th/0003081v2 29 Jun 2000

Let us apply these formulas to the $S^2 \times S^2$ background. The volume of the manifold is $V_{S^2\times S^2} = (4\pi)^2/\Lambda^2$, while $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 8\Lambda^2$. As a result,

$$N_0 = \frac{161}{45}, \quad N_1 = \frac{224}{45}, \quad N_2 = \frac{21}{5}. \tag{5.20}$$

Now let us obtain the same result via a direct evaluation of the ζ-functions. First we consider the scalar case. Using the results of Tab.1, the operator $\Delta_0 - 2\Lambda$ has one negative mode, six zero modes, while the rest of the spectrum is positive and gives rise to the ζ-function

$$\zeta_0(s) = 4^s\,(2\,\zeta(2,-9|s) + Z(1,-10|s)). \tag{5.21}$$

Hence the number of all eigenvalues is $7 + \zeta_0(0)$. In order to compute $\zeta_0(0)$, we use the results of the Appendix, where the following values are obtained:

$$\zeta(k,\nu|0) = \frac{1}{12} - \frac{1}{4}\,\nu - k^2\,, \tag{5.22}$$

$$Z(k,\nu|0) = \frac{1}{32}\,\nu^2 - \frac{1}{24}\,\nu + 2k^4 + (\frac{1}{2}\,\nu - \frac{2}{3})\,k^2 + \frac{13}{360}\,. \tag{5.23}$$

This gives for the ζ-functions in (5.15), (5.18), (5.21)

$$\zeta_0(0) = -\frac{154}{45}, \quad \zeta_1(0) = -\frac{18}{5}, \quad \zeta_2(0) = \frac{38}{9}. \tag{5.24}$$

Using these, the number of scalar eigenvalues is $N_0 = 7 - \frac{154}{45} = \frac{161}{45}$, which agrees with (5.20).

Next, the vector operator Δ_1 has 6 zero modes, such that the number of its eigenvalues in the transverse sector is $6 + \zeta_1(0)$. Now, one should take into account also the longitudinal vectors, which are gradients of scalars. It is not difficult to see that if $\nabla_\mu\chi$ is an eigenvector of Δ_1, such that $\Delta_1\nabla_\mu\chi = \sigma\nabla_\mu\chi$, then $(\Delta_0 - 2\Lambda)\chi = \sigma\chi$. We see that the eigenfunctions of $\Delta_0 - 2\Lambda$ are in one-to-one correspondence with the longitudinal vectors. The number of the latter is therefore $N_0 - 1$, where the one is subtracted because the ground state scalar eigenfunction is constant, which vanishes upon differentiation. We therefore conclude that $N_1 = 6 + \zeta_1(0) + N_0 - 1 = 6 - \frac{18}{5} + \frac{161}{45} - 1 = \frac{224}{45}$, which also agrees with (5.20).

Finally, the number of traceless eigenvalues of Δ_2 is $1 + \zeta_2(0)$ (here the one is the contribution of the negative mode) plus the number of longitudinal traceless tensor harmonics $\phi^{\rm L}_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu - \frac{1}{2} g_{\mu\nu} \nabla_\rho \xi^\rho$.

Now, if $\Delta_1 \xi_\mu = \sigma \xi_\nu$ then for $\phi^{\rm L}_{\mu\nu}$ associated with ξ_μ one has $\Delta_2 \phi^{\rm L}_{\mu\nu} = \sigma \phi^{\rm L}_{\mu\nu}$. Hence, the number of longitudinal tensors is determined by the number of vectors, which gives $N_2 = 1 + \zeta_2(0) + (N_1 - 6)$. Here six is subtracted because the six Killing vectors do not contribute to the tensor spectrum, since for Killing vectors one has $\phi^{\rm L}_{\mu\nu} = 0$. We therefore obtain $N_2 = 1 + \frac{38}{9} + \frac{224}{45} - 6 = \frac{21}{5}$, which again is in perfect agreement with (5.20).

From

$$\Psi_0^c(a_{(1)}, a_{(2)}) = N \Delta^{-1/2}(a_{(1)}, a_{(2)}) \times \exp\left[\frac{1}{3H^2} [-(1 - H^2 a_{(2)}{}^2)^{3/2} + (1 - H^2 a_{(1)}{}^2)^{3/2}] \right].$$

For N = 21/5; Δ = 1+(38/9); $a_{(1)} = 2$; $a_{(2)} = 3$; H = 1 and $Ha_0 = 8$; we obtain:

21/5*(1+38/9)^(-1/2) * 6 * exp((((1/3(-(1-64)^1.5 + (1-64)^1.5)))))

Input:

$$\frac{21}{5}\left(1+\frac{38}{9}\right)^{-1/2} \times 6 \exp\left(\frac{1}{3}\left(-(1-64)^{1.5} + (1-64)^{1.5}\right)\right)$$

Result:

11.0274...

11.0274…

For N = 224/45; Δ = 6-(18/5); we obtain:

224/45*(6-18/5)^(-1/2) * 6 * exp((((1/3(-(1-64)^1.5 + (1-64)^1.5)))))

Input:

$$\frac{224}{45}\left(6-\frac{18}{5}\right)^{-1/2}\times 6\exp\left(\frac{1}{3}\left(-(1-64)^{1.5}+(1-64)^{1.5}\right)\right)$$

Result:

19.2789...

19.2789…

For N = 161/45; Δ = 3/5; we obtain:

161/45*(3/5)^(-1/2) * 6 * exp((((1/3(-(1-64)^1.5 + (1-64)^1.5)))))

Input:

$$\frac{161}{45}\left(\frac{3}{5}\right)^{-1/2}\times 6\exp\left(\frac{1}{3}\left(-(1-64)^{1.5}+(1-64)^{1.5}\right)\right)$$

Result:

27.7133...

27.7133…

Performing the square root of the difference of the three results, we obtain:

sqrt(-(27.7133-19.2789-11.0274))

Input interpretation:

$$\sqrt{-(27.7133-19.2789-11.0274)}$$

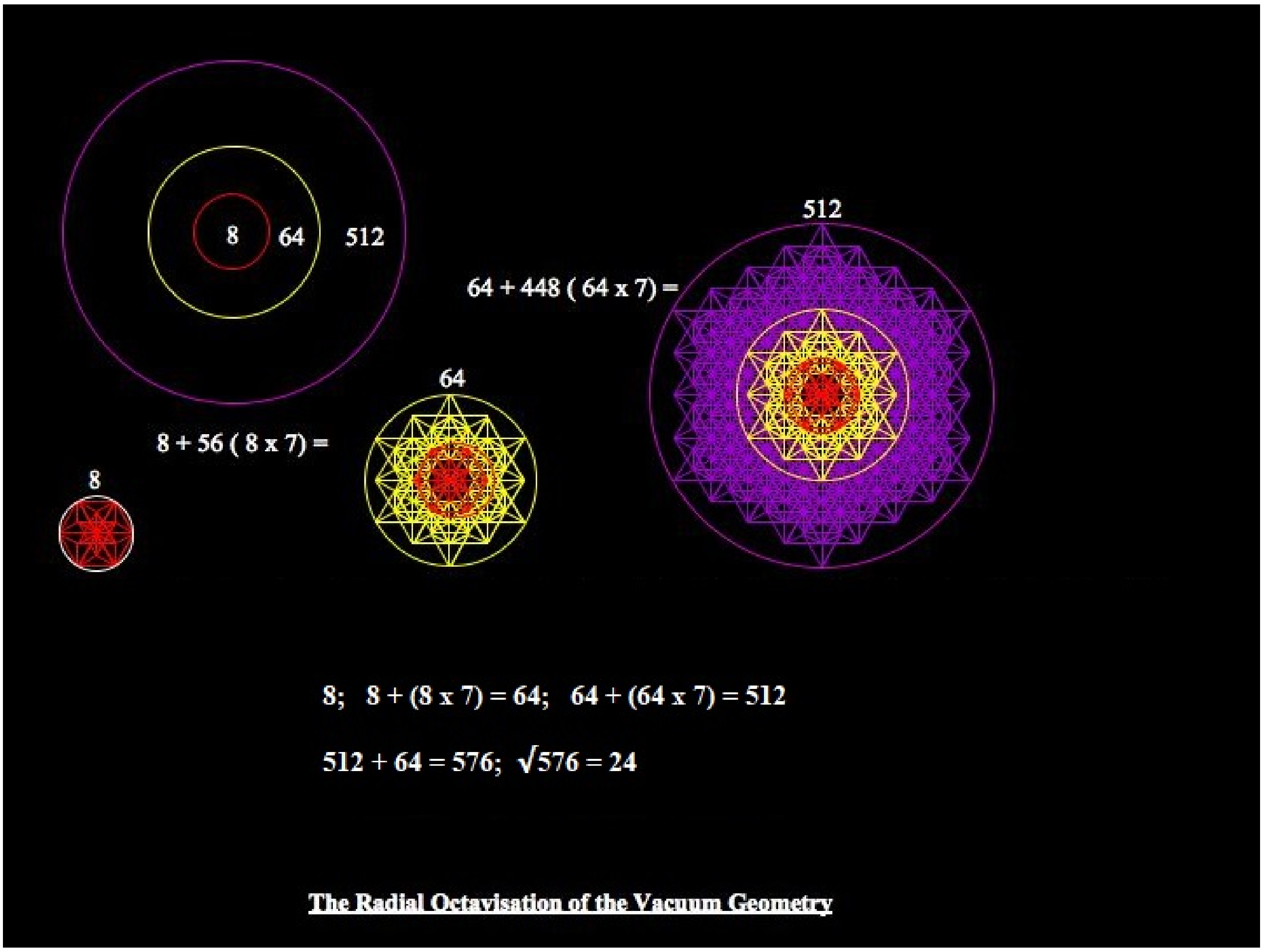
Result:

1.610279478848314703318960810892500084316557145245394642688...

1.61027947884…. result that is a good approximation to the value of the golden ratio 1.618033988749...

Fig. 1

https://www.pinterest.it/pin/570338740293422619/



Conclusion

From what we have described above, it is possible and plausible that the vacuum geometry is strongly connected to the value of the golden ratio and that Ramanujan's mathematics, especially that described in paragraph 5 of the wonderful paper "Modular equations and approximations to π " (precisely the equation where there is 64, a fundamental number in the vacuum geometry), is strictly connected to the quantum gravity, precisely to the mathematical development of this theory.

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

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PHYSICAL REVIEW D VOLUME 28, NUMBER 12
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