

**Further mathematical connections between some Ramanujan formulas,  $\phi$ ,  $\zeta(2)$  and various topics and parameters of LQG, Open Strings and Particle Physics.  
VI**

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**Abstract**

*In this paper we continue to describe and analyze some Ramanujan expressions. Furthermore, we have obtained several mathematical connections with  $\phi$ ,  $\zeta(2)$  and various topics and parameters of LQG, Open Strings and Particle Physics.*

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<https://www.giornalettismo.com/la-verita-dietro-al-numero-segreto-di-futurama/>

**We want to highlight that the development of the various equations was carried out according to our possible logical and original interpretation**

From:

### **LQG vertex with finite Immirzi parameter**

*Jonathan Engle, Etera Livine, Roberto Pereira, Carlo Rovelli - arXiv:0711.0146v2*

[gr-qc] 13 Dec 2007

We have that:

$$k^2 = \left( \frac{2j^-}{1-\gamma} \right)^2 = \left( \frac{2j^+}{1+\gamma} \right)^2, \quad (26)$$

The solutions are substantially different for  $\gamma < 1$  and for  $\gamma > 1$  (the value  $\gamma = 1$  is the natural turning point in the euclidean setting since it corresponds to a pure self-dual connection):

$$k = \begin{cases} j^+ + j^- & 0 < \gamma < 1, \\ j^+ - j^- & \gamma > 1. \end{cases} \quad (27)$$

$$Area = \sqrt{A_3} = 8\pi\hbar G \gamma \sqrt{k(k+1)}. \quad (48)$$

which is *exactly* the spectrum of LQG. This spectrum can be compared with the continuous spectrum

$$Area \sim \frac{1}{2} \sqrt{4k(k+1) - n^2 + \rho^2 + 4}. \quad (49)$$

that was previously obtained in covariant LQG, before imposing the second class constraints (see [9]). Remarkably, imposing the simplicity constraints (17) and (22) reduces the continuous spectrum (49) to the exact discrete LQG spectrum (48).

For  $k = 1/2 - (-5/2) = 1/2 + 5/2 = 6/2 = 3$ ;  $\gamma > 1 = \sqrt{2}$

From (48)

$$Area = \sqrt{A_3} = 8\pi\hbar G \gamma \sqrt{k(k+1)}.$$

we obtain:

$$((8\pi * 1.054571e-34 * 6.67430e-11 * (\sqrt{2}))) * \sqrt{3(3+1)}$$

**Input interpretation:**

$$(8\pi \times 1.054571 \times 10^{-34} \times 6.67430 \times 10^{-11} \sqrt{2}) \sqrt{3(3+1)}$$

**Result:**

$$8.6661664965108968590357230301939611783720215608502843... \times 10^{-43}$$

8.66616649651... \* 10<sup>-43</sup> that is exactly the spectrum of LQG

From which:

$$((( -\ln[ ((8\pi * 1.054571e-34 * 6.67430e-11 * (\sqrt{2}))) * \sqrt{3(3+1)}) ] - 8 )) * 1/55$$

**Input interpretation:**

$$\left( -\log\left( (8\pi \times 1.054571 \times 10^{-34} \times 6.67430 \times 10^{-11} \sqrt{2}) \sqrt{3(3+1)} \right) - 8 \right) \times \frac{1}{55}$$

log(x) is the natural logarithm

**Result:**

$$1.6154860...$$

1.6154860.... result that is a good approximation to the value of the golden ratio

$$1.618033988749...$$

Now:

$(n, \rho)$ , where  $n$  is a positive integer and  $\rho$  real

$$n = 3; \rho = \sqrt{3}$$

We have also:

$$\text{Area} \sim \frac{1}{2} \sqrt{4k(k+1) - n^2 + \rho^2 + 4}. \quad (49)$$

from which, we obtain:

$$\frac{1}{2} * \text{sqrt}(4*3(3+1)-3^2+3+4)$$

**Input:**

$$\frac{1}{2} \sqrt{4 \times 3(3+1) - 3^2 + 3 + 4}$$

**Result:**

$$\sqrt{\frac{23}{2}}$$

**Decimal approximation:**

3.391164991562634069532278163312984552597874161961644116375...

3.3911649915...

**Alternate form:**

$$\frac{\sqrt{46}}{2}$$

From which:

$$\text{sqrt}[1+\frac{1}{2}((\frac{1}{2} * \text{sqrt}(4*3(3+1)-3^2+3+4)))]$$

**Input:**

$$\sqrt{1 + \frac{1}{2} \left( \frac{1}{2} \sqrt{4 \times 3(3+1) - 3^2 + 3 + 4} \right)}$$

**Result:**

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}}$$

**Decimal approximation:**

1.641822918521153131599341730921279378718770584571369312189...

$$1.64182291852\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Alternate forms:**

$$\frac{\sqrt{4 + \sqrt{46}}}{2}$$

$$\frac{\sqrt{1 - \frac{1}{2}i} \sqrt{\frac{15}{2}} + \sqrt{\frac{1}{2}i} \left( \sqrt{\frac{15}{2}} + -2i \right)}{\sqrt{2}}$$

**Minimal polynomial:**

$$8x^4 - 16x^2 - 15$$

$$\text{sqrt}[1+1/2((1/2 * \text{sqrt}(4*3(3+1)-3^2+3+4)))]-24/10^3$$

**Input:**

$$\sqrt{1 + \frac{1}{2} \left( \frac{1}{2} \sqrt{4 \times 3 (3 + 1) - 3^2 + 3 + 4} \right) - \frac{24}{10^3}}$$

**Result:**

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2} - \frac{3}{125}}$$

**Decimal approximation:**

1.617822918521153131599341730921279378718770584571369312189...

1.61782291852.... result that is a very good approximation to the value of the golden ratio 1.618033988749...

**Alternate forms:**

$$\frac{1}{250} \left( 125 \sqrt{4 + \sqrt{46}} - 6 \right)$$

$$\frac{\sqrt{4 + \sqrt{46}}}{2} - \frac{3}{125}$$

$$\frac{125 \sqrt{1 - \frac{1}{2} i \sqrt{\frac{15}{2}}} + \sqrt{2} \left( -3 + \frac{125}{2} \sqrt{i \left( \sqrt{\frac{15}{2}} + -2 i \right)} \right)}{125 \sqrt{2}}$$

**Minimal polynomial:**

$$1953125000x^4 + 187500000x^3 - 3899500000x^2 - 187392000x - 3664358727$$

From

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} - \frac{3}{125}$$

we obtain:

$$-3/x + \text{sqrt}(1 + \text{sqrt}(23/2)/2) = 1.61782291852$$

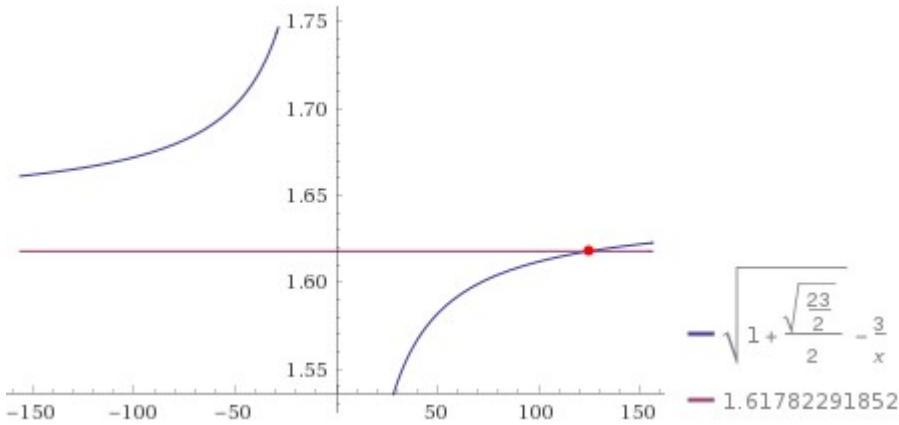
**Input interpretation:**

$$-\frac{3}{x} + \sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} = 1.61782291852$$

**Result:**

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} - \frac{3}{x} = 1.61782291852$$

**Plot:**



**Alternate form assuming x is real:**

$$\frac{125.0000000}{x} = 1.000000000$$

**Alternate forms:**

$$(1.00000000 + 0. \times 10^{-10} i)x = 125.000000 + 0. \times 10^{-8} i \quad (\text{for } x \neq 0)$$

$$\frac{\sqrt{4 + \sqrt{46}}}{2} - \frac{3}{x} = 1.61782291852$$

$$\frac{\sqrt{4 + \sqrt{46}}}{2x} x - 6 = 1.61782291852$$

**Alternate form assuming x is positive:**

$$1.000000000 x = 125.0000000 \quad (\text{for } x \neq 0)$$

**Solution:**

$$x \approx 125.0000000$$

125 result very near to the Higgs boson mass 125.18 GeV

and:

$$-3/(x-(11+3)) + \text{sqrt}(1 + \text{sqrt}(23/2)/2) = 1.61782291852$$

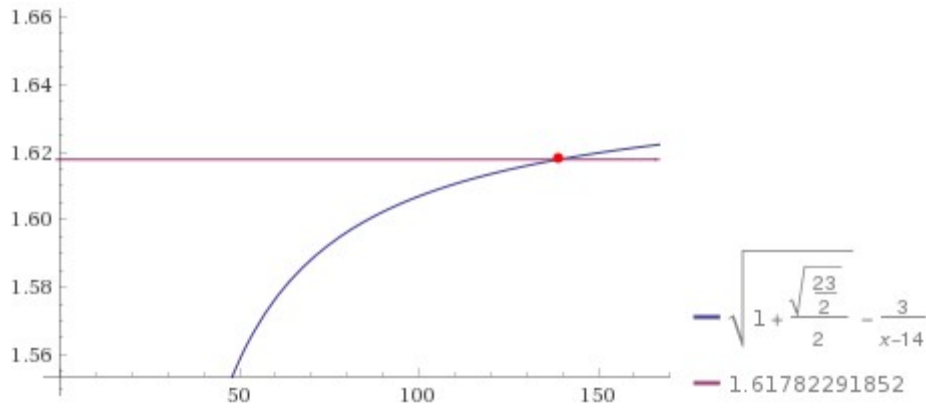
**Input interpretation:**

$$-\frac{3}{x - (11 + 3)} + \sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} = 1.61782291852$$



**Result:**

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} - \frac{3}{x-14} = 1.61782291852$$

**Plot:****Alternate forms:**

$$(1.00000000 + 0. \times 10^{-10} i)x = 139.000000 + 0. \times 10^{-8} i \text{ (for } x \neq 14)$$

$$\frac{\sqrt{4 + \sqrt{46}}}{2} - \frac{3}{x-14} = 1.61782291852$$

$$\frac{\sqrt{4 + \sqrt{46}} x - 14 \sqrt{4 + \sqrt{46}} - 6}{2(x-14)} = 1.61782291852$$

**Alternate form assuming x is positive:**

$$-\frac{125.00000}{14.00000000 - 1.00000000 x} = 1.0000000$$

**Solution:**

$$x \approx 139.0000000$$

139 result practically equal to the rest mass of Pion meson 139.57 MeV

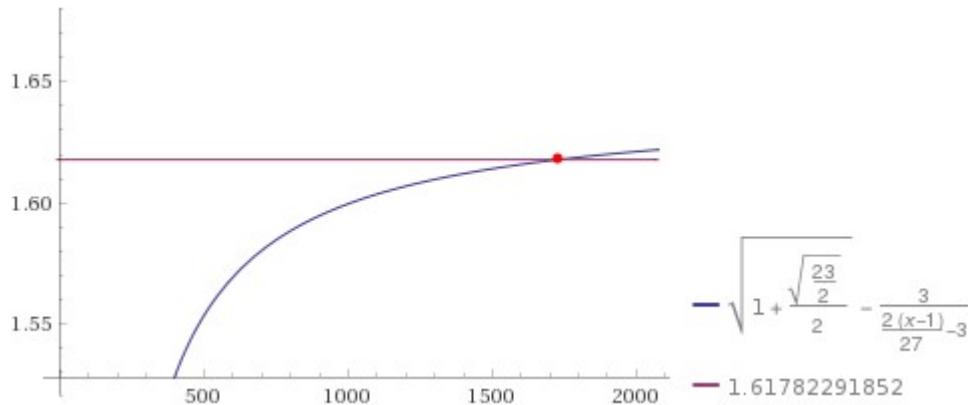
$$((-3/(((2/27))(x-1)-3))) + \text{sqrt}(1 + \text{sqrt}(23/2)/2) = 1.61782291852$$

**Input interpretation:**

$$-\frac{3}{\frac{2}{27}(x-1)-3} + \sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} = 1.61782291852$$

**Result:**

$$\sqrt{1 + \frac{\sqrt{\frac{23}{2}}}{2}} - \frac{3}{\frac{2(x-1)}{27} - 3} = 1.61782291852$$

**Plot:****Alternate forms:**

$$(1.00000000 + 0. \times 10^{-10} i)x = 1729.00000 + 0. \times 10^{-7} i \quad (\text{for } x \neq \frac{83}{2})$$

$$\frac{81}{83 - 2x} + \frac{\sqrt{4 + \sqrt{46}}}{2} = 1.61782291852$$

$$\frac{\sqrt{4 + \sqrt{46}}}{2} - \frac{3}{\frac{2(x-1)}{27} - 3} = 1.61782291852$$

**Alternate form assuming x is positive:**

$$-\frac{1687.5000}{41.5000000 - 1.00000000x} = 1.00000000$$

**Solution:**

$$x \approx 1729.000000$$

1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From:

### Open Strings

*C. Angelantonj and A. Sagnotti - arXiv:hep-th/0204089v2 4 Jul 2002*

Now, we have:

$$\begin{aligned} N &= -8192 \epsilon, & R_N &= 0, \\ D &= -8192 \epsilon, & R_D &= 0, \end{aligned}$$

thus also requiring that  $\epsilon = -1$ .

( $\epsilon = 1, -1$ )

$$\phi_{++} = 1.25 \quad \phi_{+-} = -0.75$$

$$\phi_{-+} = 1.5477867 \quad \phi_{--} = -0.32304191 \quad v = 1 / \text{sqrt}(0.9991104684)$$

$$n_1 + n_2 + n_3 + n_4 = 32,$$

$$n_1 = 16 = n_4, \quad n_2 = 0 = n_3,$$

and

$$R_N = i(n - \bar{n}),$$

$$R_D = i(d_i - \bar{d}_i), \quad (\text{see 315})$$

$$N = D = 8192; \quad \text{Sqrt}(v_4) = (1/\text{sqrt}(0.9630552342))^{0.5}; \quad Q_0 = Q_v = 8;$$

$$R_N = R_D = 16$$

$$\mathcal{F} = \left\{ -\frac{1}{2} < \tau_1 \leq \frac{1}{2}, |\tau| \geq 1 \right\} \tag{7}$$

$$\left\{ -\frac{1}{2} < \tau_1 \leq \frac{1}{2}, \epsilon < \tau_2 < \infty \right\}$$

$$\tau_1 = \frac{1}{2}.$$

$$0.767550\dots = P_m$$

$$0.0294961 = W_n$$

$$\eta = 2, 3, 7 \text{ and } \vartheta = 1, 2, 2,$$

$$T = e^{-in\pi/12} \text{diag} (1, -1, e^{in\pi/4}, e^{in\pi/4})$$

$$T = (0.965925826289 - 0.2588190451i) \approx 1$$

$$v = 1.000445063$$

From:

$$\begin{aligned} \tilde{\mathcal{K}} = \frac{2^5}{8} & \left[ \left( v_1 v_2 v_3 W_1^e W_2^e W_3^e + \frac{v_k}{2v_l v_m} W_k^e P_l^e P_m^e \right) T_{oo} \right. \\ & \left. + 2\epsilon_k \left( v_k W_k^e + \epsilon \frac{P_k^e}{v_k} \right) T_{ok} \left( \frac{2\eta}{\vartheta_2} \right)^2 \right], \end{aligned} \quad (382)$$

we obtain:

$$(2^5)/8 * ((((((1.000445063^3 * 0.0294961^3 + (1.000445063/(2*1.000445063^2))*0.0294961*0.767550^2))))+2(1.000445063*0.0294961+0.767550/1.000445063)*4^2))))$$

**Input interpretation:**

$$\frac{2^5}{8} \left( \left( 1.000445063^3 \times 0.0294961^3 + \frac{1.000445063}{2 \times 1.000445063^2} \times 0.0294961 \times 0.767550^2 \right) + 2 \left( 1.000445063 \times 0.0294961 + \frac{0.767550}{1.000445063} \right) \times 4^2 \right)$$

**Result:**

$$102.0147163271381382082681869715311393123361987852878444418\dots$$

$$102.0147163\dots$$

and adding 26 (The number of spacetime dimensions in bosonic string theory):

$$26 + (2^5)/8 * ((((((1.000445063^3 * 0.0294961^3 + (1.000445063/(2*1.000445063^2))*0.0294961*0.767550^2))))+2(1.000445063*0.0294961+0.767550/1.000445063)*4^2)))$$

**Input interpretation:**

$$26 + \frac{2^5}{8} \left( \left( 1.000445063^3 \times 0.0294961^3 + \frac{1.000445063}{2 \times 1.000445063^2} \times 0.0294961 \times 0.767550^2 \right) + 2 \left( 1.000445063 \times 0.0294961 + \frac{0.767550}{1.000445063} \right) \times 4^2 \right)$$

**Result:**

128.0147163271381382082681869715311393123361987852878444418...

128.01471632...

Thence:

$$1/2 * (((26 + (2^5)/8 * ((((((1.000445063^3 * 0.0294961^3 + (1.000445063/(2*1.000445063^2))*0.0294961*0.767550^2))))+2(1.000445063*0.0294961+0.767550/1.000445063)*4^2))))))$$

**Input interpretation:**

$$\frac{1}{2} \left( 26 + \frac{2^5}{8} \left( \left( 1.000445063^3 \times 0.0294961^3 + \frac{1.000445063}{2 \times 1.000445063^2} \times 0.0294961 \times 0.767550^2 \right) + 2 \left( 1.000445063 \times 0.0294961 + \frac{0.767550}{1.000445063} \right) \times 4^2 \right) \right)$$

**Result:**

64.00735816356906910413409348576556965616809939264392222092...

64.007358163.... ≈ 64

From which:

$$27 * 1/2 * (((26 + (2^5)/8 * ((((((1.000445063^3 * 0.0294961^3 + (1.000445063/(2*1.000445063^2))*0.0294961*0.767550^2))))+2(1.000445063*0.0294961+0.767550/1.000445063)*4^2)))))) + 1$$

**Input interpretation:**

$$27 \times \frac{1}{2} \left( 26 + \frac{2^5}{8} \left( \left( 1.000445063^3 \times 0.0294961^3 + \frac{1.000445063}{2 \times 1.000445063^2} \times 0.0294961 \times 0.767550^2 \right) + 2 \left( 1.000445063 \times 0.0294961 + \frac{0.767550}{1.000445063} \right) \times 4^2 \right) \right) + 1$$

**Result:**

1729.198670416364865811620524115670380716538683601385899965...

1729.19867041...

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

and:

$$[27 \times \frac{1}{2} \times (((26 + (2^5)/8 \times ((((((1.000445063^3 \times 0.0294961^3 + (1.000445063/(2 \times 1.000445063^2)) \times 0.0294961 \times 0.767550^2)))) + 2(1.000445063 \times 0.0294961 + 0.767550/1.000445063) \times 4^2)))))) + 1]^{1/15}$$

**Input interpretation:**

$$\left( 27 \times \frac{1}{2} \left( 26 + \frac{2^5}{8} \left( \left( 1.000445063^3 \times 0.0294961^3 + \frac{1.000445063}{2 \times 1.000445063^2} \times 0.0294961 \times 0.767550^2 \right) + 2 \left( 1.000445063 \times 0.0294961 + \frac{0.767550}{1.000445063} \right) \times 4^2 \right) \right) + 1 \right)^{(1/15)}$$

**Result:**

1.643828...

$$1.643828 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, we have that:

The transverse-channel annulus amplitude is

$$\begin{aligned}
\tilde{\mathcal{A}} = & \frac{2^{-5}}{8} \left\{ \left( N_o^2 v_1 v_2 v_3 W_1 W_2 W_3 + \frac{D_{k;o}^2 v_k}{2v_l v_m} W_k P_l P_m \right) T_{oo} \right. \\
& + 4 \left[ (N_k^2 + D_{k;k}^2) v_k W_k + D_{l \neq k;k}^2 \frac{P_k}{v_k} \right] T_{ko} \left( \frac{2\eta}{\vartheta_4} \right)^2 \\
& + 2N_o D_{k;o} v_k W_k \tilde{T}_{ok}^{(\epsilon_k)} \left( \frac{2\eta}{\vartheta_2} \right)^2 + 2N_k D_{k;k} v_k W_k \tilde{T}_{kk}^{(\epsilon_k)} \left( \frac{2\eta}{\vartheta_3} \right)^2 \\
& + 4N_l D_{k \neq l;l} \tilde{T}_{lk}^{(\epsilon_k)} \frac{8\eta^3}{\vartheta_2 \vartheta_3 \vartheta_4} + D_{k;o} D_{l;o} \frac{P_m}{v_m} \tilde{T}_{om}^{(\epsilon_k \epsilon_l)} \left( \frac{2\eta}{\vartheta_2} \right)^2 \\
& \left. + D_{k;m} D_{l;m} \frac{P_m}{v_m} \tilde{T}_{mm}^{(\epsilon_k \epsilon_l)} \left( \frac{2\eta}{\vartheta_3} \right)^2 + 4D_{k;k} D_{l;k} \tilde{T}_{km}^{(\epsilon_k \epsilon_l)} \frac{8\eta^3}{\vartheta_2 \vartheta_3 \vartheta_4} \right\}, \quad (385)
\end{aligned}$$

where  $N_o$ ,  $D_{g;o}$ ,  $D_{f;o}$  and  $D_{h;o}$  count the numbers of D9 branes and of the three sets of D5 or  $\bar{D}5$  branes wrapped around the first, second and third torus, denoted for brevity  $5_{1,2,3}$  or  $\bar{5}_{1,2,3}$  in the following. In a similar fashion,  $N_k$ ,  $D_{g;k}$ ,  $D_{f;k}$  and  $D_{h;k}$  ( $k = g, f, h$ ) parametrize the breakings induced by the three orbifold operations  $g$ ,  $f$  and  $h$ . Notice

From

$$\begin{aligned}
\tilde{\mathcal{A}} = & \frac{2^{-5}}{8} \left\{ \left( N_o^2 v_1 v_2 v_3 W_1 W_2 W_3 + \frac{D_{k;o}^2 v_k}{2v_l v_m} W_k P_l P_m \right) T_{oo} \right. \\
& + 4 \left[ (N_k^2 + D_{k;k}^2) v_k W_k + D_{l \neq k;k}^2 \frac{P_k}{v_k} \right] T_{ko} \left( \frac{2\eta}{\vartheta_4} \right)^2 \\
& + 2N_o D_{k;o} v_k W_k \tilde{T}_{ok}^{(\epsilon_k)} \left( \frac{2\eta}{\vartheta_2} \right)^2 + 2N_k D_{k;k} v_k W_k \tilde{T}_{kk}^{(\epsilon_k)} \left( \frac{2\eta}{\vartheta_3} \right)^2 \\
& + 4N_l D_{k \neq l;l} \tilde{T}_{lk}^{(\epsilon_k)} \frac{8\eta^3}{\vartheta_2 \vartheta_3 \vartheta_4} + D_{k;o} D_{l;o} \frac{P_m}{v_m} \tilde{T}_{om}^{(\epsilon_k \epsilon_l)} \left( \frac{2\eta}{\vartheta_2} \right)^2 \\
& \left. + D_{k;m} D_{l;m} \frac{P_m}{v_m} \tilde{T}_{mm}^{(\epsilon_k \epsilon_l)} \left( \frac{2\eta}{\vartheta_3} \right)^2 + 4D_{k;k} D_{l;k} \tilde{T}_{km}^{(\epsilon_k \epsilon_l)} \frac{8\eta^3}{\vartheta_2 \vartheta_3 \vartheta_4} \right\},
\end{aligned}$$

$$N = D = 8192; \quad \eta = 2, 3, 7 \quad \text{and} \quad \vartheta = 1, 2, 2,$$

$$T = e^{-in\pi/12} \text{diag} \left( 1, -1, e^{in\pi/4}, e^{in\pi/4} \right)$$

$$T = (0.965925826289 - 0.2588190451i) \approx 1$$





$$v = 1.000445063; 0.767550\dots = P_m; 0.0294961 = W_n$$

$$2*8192^2*1.000445063*0.0294961*4^2+2*8192^2*1.000445063*0.0294961*4^2+4*8192^2*8*2+8192^2*(0.767550/1.000445063)*4^2+8192^2*(0.767550/1.000445063)*4^2+4*8192^2*8*2$$

**Input interpretation:**

$$2 \times 8192^2 \times 1.000445063 \times 0.0294961 \times 4^2 + 2 \times 8192^2 \times 1.000445063 \times 0.0294961 \times 4^2 + 4 \times 8192^2 \times 8 \times 2 + 8192^2 \times \frac{0.767550}{1.000445063} \times 4^2 + 8192^2 \times \frac{0.767550}{1.000445063} \times 4^2 + 4 \times 8192^2 \times 8 \times 2$$

**Result:**

$$1.0364243562123441679073235497918076487124410948290121\dots \times 10^{10}$$

$$1.0364243562123441679\dots \times 10^{10}$$

$$(2^{(-5)/8}) * (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10})$$

**Input interpretation:**

$$\frac{1}{2^5 \times 8} (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10})$$

**Result:**

$$5.434939862121473312109375 \times 10^7$$

$$5.434939862\dots * 10^7 \text{ (final result)}$$

From which:

$$7((((2^{(-5)/8}) * (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10}))))^{1/8 - (2 \pi)/\sqrt{53}}$$

Where 7 is a Lucas number and 53 is a Eisenstein prime

**Input interpretation:**

$$7 \sqrt[8]{\frac{1}{2^5 \times 8} (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10})} - \frac{2 \pi}{\sqrt{53}}$$

**Result:**

63.99999060314199...

63.99999... ≈ 64

From which:

$$27 * (((7((((2^{(-5)}/8) * (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10}))))))^{1/8} - (2 \pi) / \sqrt{53}))) + 1$$

**Input interpretation:**

$$27 \left( 7 \sqrt[8]{\frac{1}{2^5 \times 8} (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10})} - \frac{2 \pi}{\sqrt{53}} \right) + 1$$

**Result:**

1728.999746284834...

1728.999746284834... ≈ 1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

With regard 27 (From Wikipedia):

*“The fundamental group of the complex form, compact real form, or any algebraic version of E<sub>6</sub> is the cyclic group  $\mathbb{Z}/3\mathbb{Z}$ , and its outer automorphism group is the cyclic group  $\mathbb{Z}/2\mathbb{Z}$ . Its fundamental representation is 27-dimensional (complex), and a basis is given by the 27 lines on a cubic surface. The dual representation, which is inequivalent, is also 27-dimensional. In particle physics, E<sub>6</sub> plays a role in some grand unified theories”.*

$$[((((27 * (((7 * (((2^{-5})/8) * (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10}))))^{1/8} - (2\pi)/\sqrt{53})))))) + 1]^{1/15}$$

**Input interpretation:**

$$\left( 27 \left[ 7 \sqrt[8]{\frac{1}{2^5 \times 8} (3.54920248490753 \times 10^9 + 1.0364243562123441679 \times 10^{10}) - \frac{2\pi}{\sqrt{53}}} \right] + 1 \right)^{(1/15)}$$

**Result:**

1.64381521266772244...

$$1.64381521266772244 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, from:

$$\mathcal{M} = \frac{1}{8} \left\{ N_o P_1 P_2 P_3 \hat{T}_{oo} \quad N_o P_k C_k \hat{T}_{ok} \left( \frac{2\hat{\eta}}{\hat{\nu}_2} \right)^2 \mid \frac{1}{2} D_{k;o} P_k W_l W_m C_k \hat{T}_{co}^{(\epsilon_k)} \right. \\ \left. - \left( D_{l;o} \epsilon_k W_m \hat{T}_{om}^{(\epsilon_l)} + D_{k;o} P_k \hat{T}_{ok}^{(\epsilon_k)} \right) \left( \frac{2\hat{\eta}}{\hat{\nu}_2} \right)^2 \right\}. \quad (392)$$

$$N = D = 8192; \quad \eta = 2, 3, 7 \quad \text{and} \quad \vartheta = 1, 2, 2,$$

$$T = (0.965925826289 - 0.2588190451i) \approx 1$$

$$v = 1.000445063; \quad 0.767550 \dots = P_m; \quad 0.0294961 = W_n$$

we obtain:

$$-1/8[8192*0.767550*3-8192*0.767550*4^2+1/2*8192*0.767550*0.0294961^2-(8192*0.0294961+8192*0.767550)*4^2]$$

**Input interpretation:**

$$-\frac{1}{8} \left( 8192 \times 0.767550 \times 3 + 8192 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 8192 \times 0.767550 \times 0.0294961^2 - (8192 \times 0.0294961 + 8192 \times 0.767550) \times 4^2 \right)$$

**Result:**

23276.086997101609249024

23276.08699710...

Multiplying by  $\pi$  and adding 322 and 47 that are two Lucas number, we obtain:

$$\begin{aligned} & \pi * (((-1/8[8192*0.767550*3- \\ & 8192*0.767550*4^2+1/2*8192*0.767550*0.0294961^2- \\ & (8192*0.0294961+8192*0.767550)*4^2])))+322+47 \end{aligned}$$

**Input interpretation:**

$$\begin{aligned} & \pi \left( -\frac{1}{8} \left( 8192 \times 0.767550 \times 3 + 8192 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 8192 \times 0.767550 \times \right. \right. \\ & \left. \left. 0.0294961^2 - (8192 \times 0.0294961 + 8192 \times 0.767550) \times 4^2 \right) \right) + 322 + 47 \end{aligned}$$

**Result:**

73493.0...

73493

**Alternative representations:**

$$\begin{aligned} & \frac{1}{8} \pi(-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - \right. \\ & \left. (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = \\ & 369 - \frac{180}{8} \circ (18\ 863.3 + 3143.88 \times 0.0294961^2 - 12\ 817.2 \times 4^2) \end{aligned}$$

$$\begin{aligned} & \frac{1}{8} \pi(-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - \right. \\ & \left. (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = \\ & 369 + \frac{1}{8} i \log(-1) (18\ 863.3 + 3143.88 \times 0.0294961^2 - 12\ 817.2 \times 4^2) \end{aligned}$$

$$\begin{aligned} & \frac{1}{8} \pi(-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - \right. \\ & \left. (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = \\ & 369 - \frac{1}{8} \cos^{-1}(-1) (18\ 863.3 + 3143.88 \times 0.0294961^2 - 12\ 817.2 \times 4^2) \end{aligned}$$

**Series representations:**

$$\frac{1}{8} \pi (-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = 369 + 93\,104.3 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{1}{8} \pi (-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = -46\,183.2 + 46\,552.2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$\frac{1}{8} \pi (-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = 369 + 23\,276.1 \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

**Integral representations:**

$$\frac{1}{8} \pi (-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = 369 + 46\,552.2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$\frac{1}{8} \pi (-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = 369 + 93\,104.3 \int_0^1 \sqrt{1-t^2} dt$$

$$\frac{1}{8} \pi(-1) \left( 8192 \times 0.76755 \times 3 - 8192 \times 0.76755 \times 4^2 + \frac{8192 \times 0.76755 \times 0.0294961^2}{2} - (8192 \times 0.0294961 + 8192 \times 0.76755) 4^2 \right) + 322 + 47 = 369 + 46552.2 \int_0^\infty \frac{\sin(t)}{t} dt$$

Note that 73493 is connected with

$$\left( I_{21} \ll \int_{-\infty}^{+\infty} \exp\left(-\left(\frac{t}{H}\right)^2\right) \left| \sum_{\lambda \leq p^{1-\varepsilon_2}} \frac{a(\lambda)}{\sqrt{\lambda}} B(\lambda) \lambda^{-i(T+t)} \right|^2 dt \ll \right. \\ \left. \ll H \left\{ \left( \frac{4}{\varepsilon_2 \log T} \right)^{2r} (\log T) (\log X)^{-2\beta} + (\varepsilon_2^{-2r} (\log T)^{-2r} + \varepsilon_2^{-r} h_1^r (\log T)^{-r} \right) T^{-\varepsilon_1} \right\} \right) \\ / (26 \times 4)^2 - 24 = \left( \frac{7.9313976505275 \times 10^8}{(26 \times 4)^2 - 24} \right) = 73493.30662\dots$$

That is the Karatsuba's equation concerning the zeros of a special type of function

We obtain also:

$$1/27 * 2 \left( \left( -1/8 [8192 * 0.767550 * 3 - 8192 * 0.767550 * 4^2 + 1/2 * 8192 * 0.767550 * 0.0294961^2 - (8192 * 0.0294961 + 8192 * 0.767550) * 4^2] \right) \right) + 5$$

**Input interpretation:**

$$\frac{1}{27} \times 2 \left( -\frac{1}{8} \left( 8192 \times 0.767550 \times 3 + 8192 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 8192 \times 0.767550 \times 0.0294961^2 - (8192 \times 0.0294961 + 8192 \times 0.767550) \times 4^2 \right) \right) + 5$$

**Result:**

1729.154592377896981409185185185185185185185185185185185185185...  
1729.15459237...

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

From which:

$$\left( \left( \left( \left( \frac{1}{27} \times 2 \left( \left( -\frac{1}{8} [8192 \times 0.767550 \times 3 - 8192 \times 0.767550 \times 4^2 + \frac{1}{2} \times 8192 \times 0.767550 \times 0.0294961^2 - (8192 \times 0.0294961 + 8192 \times 0.767550) \times 4^2] \right) \right) \right) + 5 \right) \right) \right)^{1/15}$$

**Input interpretation:**

$$\left( \frac{1}{27} \times 2 \left( -\frac{1}{8} \left( 8192 \times 0.767550 \times 3 + 8192 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 8192 \times 0.767550 \times 0.0294961^2 - (8192 \times 0.0294961 + 8192 \times 0.767550) \times 4^2 \right) \right) + 5 \right)^{(1/15)}$$

**Result:**

1.643825...

$$1.643825 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

While, from:

$$\mathcal{M} = -\frac{1}{8} \left\{ N_o P_1 P_2 P_3 \hat{T}_{oo} - N_o P_k \epsilon_k \hat{T}_{ok} \left( \frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^2 + \frac{1}{2} D_{k;o} P_k W_l W_m \epsilon_k \hat{T}_{oo}^{(\epsilon_k)} - \left( D_{l;o} \epsilon_k W_m \hat{T}_{om}^{(\epsilon_l)} + D_{k;o} P_k \hat{T}_{ok}^{(\epsilon_k)} \right) \left( \frac{2\hat{\eta}}{\hat{\vartheta}_2} \right)^2 \right\}, \quad (392)$$

for  $N = D = 32$ , we obtain:

$$-1/8[32*0.767550*3-32*0.767550*4^2+1/2*32*0.767550*0.0294961^2-(32*0.0294961+32*0.767550)*4^2]$$

**Input interpretation:**

$$-\frac{1}{8} \left( 32 \times 0.767550 \times 3 + 32 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 32 \times 0.767550 \times 0.0294961^2 - (32 \times 0.0294961 + 32 \times 0.767550) \times 4^2 \right)$$

**Result:**

90.922214832428161129

90.922214832428161129

$$-1/8[32*0.767550*3-32*0.767550*4^2+1/2*32*0.767550*0.0294961^2-(32*0.0294961+32*0.767550)*4^2]-27$$

**Input interpretation:**

$$-\frac{1}{8} \left( 32 \times 0.767550 \times 3 + 32 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 32 \times 0.767550 \times 0.0294961^2 - (32 \times 0.0294961 + 32 \times 0.767550) \times 4^2 \right) - 27$$

**Result:**

63.922214832428161129

63.922214832428161129 ≈ 64

From which:

$$27 * ((((-1/8[32*0.767550*3-32*0.767550*4^2+1/2*32*0.767550*0.0294961^2-(32*0.0294961+32*0.767550)*4^2]-27)))) + \pi$$

**Input interpretation:**

$$27 \left( -\frac{1}{8} \left( 32 \times 0.767550 \times 3 + 32 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 32 \times 0.767550 \times 0.0294961^2 - (32 \times 0.0294961 + 32 \times 0.767550) \times 4^2 \right) - 27 \right) + \pi$$



**Result:**

1729.04...

1729.04...

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Alternative representations:**

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - \frac{(32 \times 0.0294961 + 32 \times 0.76755) 4^2}{(-1) - 27} \right) + \pi = 180^\circ + 27 \left( -27 - \frac{1}{8} (73.6848 + 12.2808 \times 0.0294961^2 - 50.0671 \times 4^2) \right)$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - \frac{(32 \times 0.0294961 + 32 \times 0.76755) 4^2}{(-1) - 27} \right) + \pi = -i \log(-1) + 27 \left( -27 - \frac{1}{8} (73.6848 + 12.2808 \times 0.0294961^2 - 50.0671 \times 4^2) \right)$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - \frac{(32 \times 0.0294961 + 32 \times 0.76755) 4^2}{(-1) - 27} \right) + \pi = \cos^{-1}(-1) + 27 \left( -27 - \frac{1}{8} (73.6848 + 12.2808 \times 0.0294961^2 - 50.0671 \times 4^2) \right)$$

**Series representations:**

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) + \pi = 1725.9 + 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k} \right)$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) (-1) - 27 \right) + \pi = 1723.9 + 2 \sum_{k=1}^{\infty} \frac{2^k}{\binom{2k}{k}}$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) (-1) - 27 \right) + \pi = 1725.9 + \sum_{k=0}^{\infty} \frac{2^{-k} (-6 + 50k)}{\binom{3k}{k}}$$

### Integral representations:

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) (-1) - 27 \right) + \pi = 1725.9 + 2 \int_0^{\infty} \frac{1}{1+t^2} dt$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) (-1) - 27 \right) + \pi = 1725.9 + 4 \int_0^1 \sqrt{1-t^2} dt$$

$$27 \left( \frac{1}{8} \left( 32 \times 0.76755 \times 3 - 32 \times 0.76755 \times 4^2 + \frac{32 \times 0.76755 \times 0.0294961^2}{2} - (32 \times 0.0294961 + 32 \times 0.76755) 4^2 \right) (-1) - 27 \right) + \pi = 1725.9 + 2 \int_0^{\infty} \frac{\sin(t)}{t} dt$$

$$\left(\left(\left(27 \cdot \left(\left(-\frac{1}{8} \left[32 \cdot 0.767550 \cdot 3 - 32 \cdot 0.767550 \cdot 4^2 + \frac{1}{2} \cdot 32 \cdot 0.767550 \cdot 0.0294961^2 - (32 \cdot 0.0294961 + 32 \cdot 0.767550) \cdot 4^2\right] - 27\right)\right) + \pi\right)\right)\right)^{1/15}$$

**Input interpretation:**

$$\left(27 \left(-\frac{1}{8} \left(32 \times 0.767550 \times 3 + 32 \times 4^2 \times (-0.767550) + \frac{1}{2} \times 32 \times 0.767550 \times 0.0294961^2 - (32 \times 0.0294961 + 32 \times 0.767550) \times 4^2\right) - 27\right) + \pi\right)^{(1/15)}$$

**Result:**

1.643818...

$$1.643818 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

Now, we have that:

$$\begin{aligned} \mathcal{M} = & -\frac{1}{4} \left[ (\hat{Q}_o + \hat{Q}_v)(0; 0) [(n + \bar{n})P_1P_2 + (d + \bar{d})W_1W_2] \right. \\ & - (m + \bar{m})(\hat{Q}_o + \hat{Q}_v)(2\zeta_1\tau; 2\zeta_2\tau) \frac{2k_1\hat{\eta}}{\hat{\vartheta}_1(2\zeta_1\tau)} \frac{2k_2\hat{\eta}}{\hat{\vartheta}_1(2\zeta_2\tau)} \\ & - (n + \bar{n} + d + \bar{d})(\hat{Q}_o - \hat{Q}_v)(0; 0) \left(\frac{2\hat{\eta}}{\hat{\vartheta}_2(0)}\right)^2 \\ & \left. - (m + \bar{m})(\hat{Q}_o - \hat{Q}_v)(2\zeta_1\tau; 2\zeta_2\tau) \frac{2\hat{\eta}}{\hat{\vartheta}_2(2\zeta_1\tau)} \frac{2\hat{\eta}}{\hat{\vartheta}_2(2\zeta_2\tau)} \right]. \end{aligned} \quad (428)$$

For

$$\text{Sqrt}(v_4) = (1/\text{sqrt}(0.9630552342))^{0.5}; \quad Q_0 = Q_v = 8$$

$$N = D = 8192; \quad \eta = 2, 3, 7 \quad \text{and} \quad \vartheta = 1, 2, 2,$$

$$T = (0.965925826289 - 0.2588190451i) \approx 1$$

$$v = 1.000445063; \quad 0.767550 \dots = P_m; \quad 0.0294961 = W_n$$

$$m + \bar{m} + n + \bar{n} = 32,$$

$$k_1 k_2 (m + \bar{m}) + d + \bar{d} = 32$$

$$4+4+12+12 = 32; \quad 1*2(4+4)+8+8 = 32$$

we obtain:

$$-1/4[16*(24*0.767550^2+16*0.0294961^2)-8*8*2*(2*\zeta(2))^4/(2*\zeta(2))^8/(2*\zeta(2))]$$

**Input interpretation:**

$$-\frac{1}{4} \left( 16 (24 \times 0.767550^2 + 16 \times 0.0294961^2) - 8 \times 8 \times 2 (2 \zeta(2)) \times \frac{4}{2 \zeta(2)} \times \frac{8}{2 \zeta(2)} \right)$$

$\zeta(s)$  is the Riemann zeta function

**Result:**

254.646...

254.646...

**Alternative representations:**

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-\frac{1}{4} \left( 16 (16 \times 0.0294961^2 + 24 \times 0.76755^2) - 8192 \left( \frac{1}{2 \zeta(2, 1)} \right)^2 \zeta(2, 1) \right)$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-\frac{1}{4} \left( 16 (16 \times 0.0294961^2 + 24 \times 0.76755^2) - 8192 S_{1,1}(1) \left( \frac{1}{2 S_{1,1}(1)} \right)^2 \right)$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-\frac{1}{4} \left( 16 (16 \times 0.0294961^2 + 24 \times 0.76755^2) + \frac{8192 \operatorname{Li}_2(-1) \left( -\frac{1}{\frac{2 \operatorname{Li}_2(-1)}{\frac{1}{2}}} \right)^2}{\frac{1}{2}} \right)$$

**Series representations:**

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{512}{\sum_{k=1}^{\infty} \frac{1}{k^2}}$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 - \frac{256}{\sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}}$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{384}{\sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}}$$

**Integral representations:**

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{768 \Gamma(2)}{\int_0^{\infty} t \operatorname{csch}(t) dt}$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{512 \Gamma(2)}{\int_0^{\infty} \frac{t}{-1+t^2} dt}$$

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{256 \Gamma(3)}{\int_0^{\infty} t^2 \operatorname{csch}^2(t) dt}$$

From:

$$\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - \frac{2 \zeta(2) 8 (8 \times 2 \times 4 \times 8)}{(2 \zeta(2)) (2 \zeta(2))} \right) (-1) =$$

$$-56.6124 + \frac{256 \Gamma(3)}{\int_0^{\infty} t^2 \operatorname{csch}^2(t) dt}$$

we obtain:

**Input interpretation:**

$$-\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - 8 \times 8 \times 2 (2 \zeta(2)) \times \frac{4}{2 \zeta(2)} \times \frac{8}{2 \zeta(2)} \right) =$$

$$-56.6124 + \frac{x \Gamma(3)}{\int_0^\infty t^2 \operatorname{csch}^2(t) dt}$$

$\zeta(s)$  is the Riemann zeta function

$\Gamma(x)$  is the gamma function

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Result:**

$$254.646 = \frac{12x}{\pi^2} - 56.6124$$

**Solution:**

$$x \approx 256.$$

256

from which:

**Input interpretation:**

$$-\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - 8 \times 8 \times 2 (2 \zeta(2)) \times \frac{4}{2 \zeta(2)} \times \frac{8}{2 \zeta(2)} \right) =$$

$$-56.6124 + \frac{4x \Gamma(3)}{\int_0^\infty t^2 \operatorname{csch}^2(t) dt}$$

$\zeta(s)$  is the Riemann zeta function

$\Gamma(x)$  is the gamma function

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Result:**

$$254.646 = \frac{48x}{\pi^2} - 56.6124$$

**Solution:**

$$x \approx 64.$$

64

and

**Input interpretation:**

$$-\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - 8 \times 8 \times 2 (2 \zeta(2)) \times \frac{4}{2 \zeta(2)} \times \frac{8}{2 \zeta(2)} \right) =$$

$$-56.6124 + \frac{4x \times \frac{1}{27} \Gamma(3)}{\int_0^\infty t^2 \operatorname{csch}^2(t) dt}$$

$\zeta(s)$  is the Riemann zeta function

$\Gamma(x)$  is the gamma function

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Result:**

$$254.646 = \frac{16x}{9\pi^2} - 56.6124$$

**Solution:**

$$x \approx 1728.$$

1728

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Input interpretation:**

$$-\frac{1}{4} \left( 16 (24 \times 0.76755^2 + 16 \times 0.0294961^2) - 8 \times 8 \times 2 (2 \zeta(2)) \times \frac{4}{2 \zeta(2)} \times \frac{8}{2 \zeta(2)} \right) =$$

$$-56.6124 + \frac{(4x^{15} \times \frac{1}{27}) \Gamma(3)}{\int_0^\infty t^2 \operatorname{csch}^2(t) dt}$$

$\zeta(s)$  is the Riemann zeta function

$\Gamma(x)$  is the gamma function

$\operatorname{csch}(x)$  is the hyperbolic cosecant function

**Result:**

$$254.646 = \frac{16x^{15}}{9\pi^2} - 56.6124$$

**Alternate forms:**

$$254.646 = 0.180127 x^{15} - 56.6124$$

$$254.646 = -5.73604(9.8696 - 0.0314026 x^{15})$$

$$311.259 - \frac{16 x^{15}}{9 \pi^2} = 0$$

**Real solution:**

$$x \approx 1.64375$$

$$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Complex solutions:**

$$x = -1.60783 - 0.341755 i$$

$$x = -1.60783 + 0.341755 i$$

$$x = -1.32982 - 0.966173 i$$

$$x = -1.32982 + 0.966173 i$$

$$x = -0.821876 - 1.42353 i$$

Now, we have that:

$$\mathcal{M}_0 \sim -\frac{1}{2}(m + \bar{m}) \hat{Q}_v(0) - 2(k_1 k_2 + 1) \frac{n + \bar{n}}{2} \hat{Q}_v(\zeta\tau), \quad (441)$$

For  $n = 12, 14$ ;  $m = 2, 4$ ;  $Q_0 = Q_v = 8$ ;  $k_1 k_2 = 2$ , we obtain:



$$-1/2(2+4)*8 - 2(2+1)*(12+14)/2 * 8 * \text{zeta}(2)$$

**Input:**

$$-\frac{1}{2} (2+4) \times 8 - 2 (2+1) \times \frac{12+14}{2} \times 8 \zeta(2)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$-24 - 104 \pi^2$$

**Decimal approximation:**

$$-1050.43885771329329635878706398711971807262473835304222514\dots$$

$$-1050.4388577\dots$$

**Property:**

$$-24 - 104 \pi^2 \text{ is a transcendental number}$$

**Alternate form:**

$$-8 (3 + 13 \pi^2)$$

**Alternative representations:**

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 624 \zeta(2, 1)$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 624 S_{1,1}(1)$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 + \frac{624 \text{Li}_2(-1)}{\frac{1}{2}}$$

**Series representations:**

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 624 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 + 1248 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 832 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 1664 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 416 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) = -24 - 416 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

Adding 26 and changing the sign, (26 is the number of spacetime dimensions in bosonic string theory) we obtain:

$$-(-1/2(2+4)*8 - 2(2+1)*(12+14)/2 * 8 * \zeta(2) + 26)$$

**Input:**

$$-\left(-\frac{1}{2} (2+4) \times 8 - 2 (2+1) \times \frac{12+14}{2} \times 8 \zeta(2) + 26\right)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$104 \pi^2 - 2$$

**Decimal approximation:**

$$1024.438857713293296358787063987119718072624738353042225146...$$

$$1024.4388577... \approx 1024$$

**Property:**

$$-2 + 104 \pi^2 \text{ is a transcendental number}$$

**Alternate form:**

$$2(52 \pi^2 - 1)$$

**Alternative representations:**

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 624 \zeta(2, 1)$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 624 S_{1,1}(1)$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 - \frac{624 \operatorname{Li}_2(-1)}{\frac{1}{2}}$$

**Series representations:**

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 624 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 - 1248 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 832 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 1664 \left(\int_0^1 \sqrt{1-t^2} dt\right)^2$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 416 \left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^2$$

$$-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} ((12+14) 2) (2+1) (8 \zeta(2)) + 26\right) = -2 + 416 \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^2$$

$$1/16(((-1/2(2+4)*8 - 2(2+1)*(12+14)/2 * 8 * \zeta(2) + 26)))$$

**Input:**

$$\frac{1}{16} \left( -\left( -\frac{1}{2} (2+4) \times 8 - 2 (2+1) \times \frac{12+14}{2} \times 8 \zeta(2) + 26 \right) \right)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{1}{16} (104 \pi^2 - 2)$$

**Decimal approximation:**

64.02742860708083102242419149919498237953904614706513907168...

$$64.0274286\dots \approx 64$$

**Property:**

$\frac{1}{16} (-2 + 104 \pi^2)$  is a transcendental number

**Alternate forms:**

$$\frac{13 \pi^2}{2} - \frac{1}{8}$$

$$\frac{1}{8} (52 \pi^2 - 1)$$

**Alternative representations:**

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = \frac{1}{16} (-2 + 624 \zeta(2, 1))$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = \frac{1}{16} (-2 + 624 S_{1,1}(1))$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = \frac{1}{16} \left( -2 - \frac{624 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)$$

**Series representations:**

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} + 39 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} - 78 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} + 52 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} + 104 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} + 26 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$-\frac{1}{16} \left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) = -\frac{1}{8} + 26 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$27 * 1/16 ((-(-1/2(2+4))*8 - 2(2+1)*(12+14)/2 * 8 * \text{zeta}(2) + 26))$$

**Input:**

$$27 \times \frac{1}{16} \left( -\left( -\frac{1}{2} (2+4) \times 8 - 2(2+1) \times \frac{12+14}{2} \times 8 \zeta(2) + 26 \right) \right)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{27}{16} (104 \pi^2 - 2)$$

**Decimal approximation:**

1728.740572391182437605453170478264524247554245970758754935...

1728.7405723...

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$$\frac{27}{16} (-2 + 104 \pi^2) \text{ is a transcendental number}$$

**Alternate forms:**

$$\frac{351 \pi^2}{2} - \frac{27}{8}$$

$$\frac{27}{8} (52 \pi^2 - 1)$$

**Alternative representations:**

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = \frac{27}{16} (-2 + 624 \zeta(2, 1))$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = \frac{27}{16} (-2 + 624 S_{1,1}(1))$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = \frac{27}{16} \left( -2 - \frac{624 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)$$

**Series representations:**

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} + 1053 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} - 2106 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} + 1404 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} + 2808 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} + 702 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right) = -\frac{27}{8} + 702 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$[27 \cdot \frac{1}{16} ((-\frac{1}{2}(2+4) \cdot 8 - 2(2+1) \cdot \frac{(12+14)}{2} \cdot 8 \cdot \zeta(2) + 26))]^{1/15}$$

**Input:**

$$\sqrt[15]{27 \times \frac{1}{16} \left( -\left( -\frac{1}{2} (2+4) \times 8 - 2 (2+1) \times \frac{12+14}{2} \times 8 \zeta(2) + 26 \right) \right)}$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{\sqrt[5]{3} \sqrt[15]{104 \pi^2 - 2}}{2^{4/15}}$$

**Decimal approximation:**

1.643798784525958647081173524166279275402938807877161680971...

$$1.643798784 \dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Property:**

$$\frac{\sqrt[5]{3} \sqrt[15]{-2 + 104 \pi^2}}{2^{4/15}} \text{ is a transcendental number}$$

**Alternate form:**

$$\sqrt[5]{\frac{3}{2}} \sqrt[15]{52 \pi^2 - 1}$$

**All 15th roots of  $27/16 (104 \pi^2 - 2)$ :**

$$\frac{\sqrt[5]{3} \sqrt[15]{104 \pi^2 - 2} e^0}{2^{4/15}} \approx 1.6438 \text{ (real, principal root)}$$

$$\frac{\sqrt[5]{3} \sqrt[15]{104 \pi^2 - 2} e^{(2i\pi)/15}}{2^{4/15}} \approx 1.5017 + 0.6686 i$$

$$\frac{\sqrt[5]{3} \sqrt[15]{104 \pi^2 - 2} e^{(4i\pi)/15}}{2^{4/15}} \approx 1.0999 + 1.2216 i$$

$$\frac{\sqrt[5]{3} \sqrt[15]{104 \pi^2 - 2} e^{(2i\pi)/5}}{2^{4/15}} \approx 0.5080 + 1.5633 i$$

$$\frac{\sqrt[5]{3} \sqrt[15]{104\pi^2 - 2} e^{(8i\pi)/15}}{2^{4/15}} \approx -0.17182 + 1.6348i$$

### Alternative representations:

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[15]{\frac{27}{16} (-2 + 624\zeta(2, 1))}$$

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[15]{\frac{27}{16} (-2 + 624S_{1,1}(1))}$$

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[15]{\frac{27}{16} \left( -2 - \frac{624 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)}$$

### Series representations:

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 + 312 \sum_{k=1}^{\infty} \frac{1}{k^2}}$$

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 - 624 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}}$$

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 + 416 \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}}$$

### Integral representations:

$$\sqrt[15]{\frac{27}{16} \left( -\left(-\frac{1}{2}(2+4)8 - \frac{1}{2}(12+14)2((2+1)8\zeta(2)) + 26\right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 + 208 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2}$$



$$\sqrt[15]{\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 + 832 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2}$$

$$\sqrt[15]{\frac{27}{16} \left( -\left( -\frac{1}{2} (2+4) 8 - \frac{1}{2} (12+14) 2 ((2+1) 8 \zeta(2)) + 26 \right) \right)} = \sqrt[5]{\frac{3}{2}} \sqrt[15]{-1 + 208 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2}$$

From

$$\mathcal{M}_0^{(r)} \sim -\frac{1}{2}(m + \bar{m})\hat{Q}_v(0) - \frac{1}{2}(2 \cdot 2^{r/2} \cdot k_1 k_2 + 2)(n + \bar{n})\hat{Q}_v(\zeta\tau), \quad (449)$$

$r = 0, 2, 4$  and  $\xi = \pm 1$ .

for  $n = 12, 14$ ;  $m = 2, 4$ ;  $r = 2$ ;  $Q_0 = Q_v = 8$ ;  $k_1 k_2 = 2$ , we obtain:

$$-1/2(2+4)*8 - 1/2(2*2*2+2)(12+14)*8*(\zeta(2))$$

**Input:**

$$-\frac{1}{2} (2+4) \times 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((12+14) \times 8 \zeta(2))$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$-24 - \frac{520 \pi^2}{3}$$

**Decimal approximation:**

-1734.73142952215549393131177331186619678770789725507037524...

**-1734.73142952...**

**Property:**

$-24 - \frac{520\pi^2}{3}$  is a transcendental number

**Alternate form:**

$$-\frac{8}{3}(9 + 65\pi^2)$$

**Alternative representations:**

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - 1040\zeta(2, 1)$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 + \frac{1040 \operatorname{Li}_2(-1)}{\frac{1}{2}}$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - 1040 S_{1,1}(1)$$

**Series representations:**

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - 1040 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 + 2080 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - \frac{4160}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - \frac{8320}{3} \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - \frac{2080}{3} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) = -24 - \frac{2080}{3} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

From which:

$$-1/2(2+4)*8-1/2(2*2*2+2)(12+14)*8*(\zeta(2)) +5$$

**Input:**

$$-\frac{1}{2}(2+4)\times 8 - \frac{1}{2}(2\times 2\times 2+2)((12+14)\times 8\zeta(2)) + 5$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$-19 - \frac{520\pi^2}{3}$$

**Decimal approximation:**

$$-1729.73142952215549393131177331186619678770789725507037524\dots$$

$$-1729.7314295\dots$$

This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson with minus sign. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$$-19 - \frac{520\pi^2}{3} \text{ is a transcendental number}$$

**Alternate form:**

$$\frac{1}{3}(-57 - 520\pi^2)$$

**Alternative representations:**

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2\times 2\times 2+2)(12+14)(8\zeta(2)) + 5 = -19 - 1040\zeta(2, 1)$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2\times 2\times 2+2)(12+14)(8\zeta(2)) + 5 = -19 + \frac{1040\text{Li}_2(-1)}{\frac{1}{2}}$$

$$\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2\times 2\times 2+2)(12+14)(8\zeta(2)) + 5 = -19 - 1040S_{1,1}(1)$$

**Series representations:**

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 - 1040 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 + 2080 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 - \frac{4160}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 - \frac{8320}{3} \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 - \frac{2080}{3} \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{2} ((2+4)8)(-1) - \frac{1}{2} (2 \times 2 \times 2 + 2)(12+14)(8 \zeta(2)) + 5 = -19 - \frac{2080}{3} \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

and:

$$\left( -(-1/2(2+4)*8 - 1/2(2*2*2+2)(12+14)*8*(\zeta(2)) + 5) \right)^{1/15}$$

**Input:**

$$\sqrt[15]{-\left(-\frac{1}{2} (2+4) \times 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((12+14) \times 8 \zeta(2)) + 5\right)}$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}}$$

**Decimal approximation:**

1.643861579151301863171229471586696154153590305813916633339...

$$1.6438615791\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Property:**

$\sqrt[15]{19 + \frac{520 \pi^2}{3}}$  is a transcendental number

**Alternate form:**

$$\sqrt[15]{\frac{1}{3} (57 + 520 \pi^2)}$$

**All 15th roots of  $19 + (520 \pi^2)/3$ :**

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}} e^0 \approx 1.64386 \text{ (real, principal root)}$$

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}} e^{(2i\pi)/15} \approx 1.50174 + 0.6686 i$$

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}} e^{(4i\pi)/15} \approx 1.1000 + 1.2216 i$$

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}} e^{(2i\pi)/5} \approx 0.5080 + 1.5634 i$$

$$\sqrt[15]{19 + \frac{520 \pi^2}{3}} e^{(8i\pi)/15} \approx -0.17183 + 1.63486 i$$

**Alternative representations:**

$$\sqrt[15]{-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} (2 \times 2 \times 2 + 2) (12 + 14) (8 \zeta(2)) + 5\right)} = \sqrt[15]{19 + 1040 \zeta(2, 1)}$$

$$\sqrt[15]{-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} (2 \times 2 \times 2 + 2) (12 + 14) (8 \zeta(2)) + 5\right)} = \sqrt[15]{19 + 1040 S_{1,1}(1)}$$

$$\sqrt[15]{-\left(\frac{1}{2} ((2+4) 8) (-1) - \frac{1}{2} (2 \times 2 \times 2 + 2) (12 + 14) (8 \zeta(2)) + 5\right)} = \sqrt[15]{19 - \frac{1040 \operatorname{Li}_2(-1)}{\frac{1}{2}}}$$

**Series representations:**

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 + 1040 \sum_{k=1}^{\infty} \frac{1}{k^2}}$$

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 - 2080 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}}$$

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 + \frac{4160}{3} \sum_{k=0}^{\infty} \frac{1}{(1+2k)^2}}$$

**Integral representations:**

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 + \frac{2080}{3} \left(\int_0^{\infty} \frac{1}{1+t^2} dt\right)^2}$$

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 + \frac{8320}{3} \left(\int_0^1 \sqrt{1-t^2} dt\right)^2}$$

$$\sqrt[15]{-\left(\frac{1}{2}((2+4)8)(-1) - \frac{1}{2}(2 \times 2 \times 2 + 2)(12+14)(8\zeta(2)) + 5\right)} = \sqrt[15]{19 + \frac{2080}{3} \left(\int_0^1 \frac{1}{\sqrt{1-t^2}} dt\right)^2}$$

Now, we have that:

$$\mathcal{M}_0^{(r)} \sim \frac{1}{2}(n_1 + n_2)\hat{Q}_o(0) - \frac{1}{2} [2 \cdot 2^{r/2} \cdot k_1 k_2 + 2] (m + \bar{m})\hat{Q}_v(\zeta\tau). \quad (454)$$

For  $n = 12, 14$ ;  $m = 2, 4$ ;  $r = 2$ ;  $Q_0 = Q_v = 8$ ;  $k_1 k_2 = 2$ , we obtain:

$$\frac{1}{2}(12+14)*8-\frac{1}{2}(2*2*2+2)(2+4)*8*(\zeta(2))$$

**Input:**

$$\frac{1}{2}(12+14)\times 8-\frac{1}{2}(2\times 2\times 2+2)((2+4)\times 8\zeta(2))$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$104-40\pi^2$$

**Decimal approximation:**

$$-290.784176043574344753379639995046045412547976289631625056\dots$$

**-290.784176...**

**Property:**

$104-40\pi^2$  is a transcendental number

**Alternate form:**

$$-8(5\pi^2-13)$$

**Alternative representations:**

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104-240\zeta(2,1)$$

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104-240S_{1,1}(1)$$

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104+\frac{240\operatorname{Li}_2(-1)}{\frac{1}{2}}$$

**Series representations:**

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104-240\sum_{k=1}^{\infty}\frac{1}{k^2}$$

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104+480\sum_{k=1}^{\infty}\frac{(-1)^k}{k^2}$$

$$\frac{1}{2}(12+14)8-\frac{1}{2}(2\times 2\times 2+2)(2+4)(8\zeta(2))=104-320\sum_{k=0}^{\infty}\frac{1}{(1+2k)^2}$$

**Integral representations:**

$$\frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) (2 + 4) (8 \zeta(2)) = 104 - 640 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) (2 + 4) (8 \zeta(2)) = 104 - 160 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) (2 + 4) (8 \zeta(2)) = 104 - 160 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

From which:

$$-1/4(((1/2(12+14)*8-1/2(2*2*2+2)(2+4)*8*(zeta(2))+34)))$$

**Input:**

$$-\frac{1}{4} \left( \frac{1}{2} (12 + 14) \times 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) \times 8 \zeta(2)) + 34 \right)$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\frac{1}{4} (40 \pi^2 - 138)$$

**Decimal approximation:**

64.19604401089358618834490999876151135313699407240790626413...

$$64.196044... \approx 64$$

**Property:**

$$\frac{1}{4} (-138 + 40 \pi^2) \text{ is a transcendental number}$$

**Alternate forms:**

$$10 \pi^2 - \frac{69}{2}$$

$$\frac{1}{2} (20 \pi^2 - 69)$$



**Alternative representations:**

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{1}{4} (138 - 240 \zeta(2, 1))$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{1}{4} (138 - 240 S_{1,1}(1))$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{1}{4} \left( 138 + \frac{240 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)$$

**Series representations:**

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} + 60 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} - 120 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} + 80 \sum_{k=0}^{\infty} \frac{1}{(1 + 2k)^2}$$

**Integral representations:**

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} + 160 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} + 40 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{4} \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) (-1) = -\frac{69}{2} + 40 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$27 \cdot -1/4 \left( \left( \left( \frac{1}{2}(12+14) \cdot 8 - \frac{1}{2}(2 \cdot 2 \cdot 2 + 2)(2+4) \cdot 8 \cdot (\zeta(2)) + 34 \right) \right) \right) - 4$$

**Input:**

$$\frac{27}{4} \times (-1) \left( \frac{1}{2} (12 + 14) \times 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) \times 8 \zeta(2)) + 34 \right) - 4$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$-4 - \frac{27}{4} (138 - 40 \pi^2)$$

**Decimal approximation:**

1729.293188294126827085312569966560806534698839955013469131...

1729.29318829...

This result is very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$-4 - \frac{27}{4} (138 - 40 \pi^2)$  is a transcendental number

**Alternate forms:**

$$270 \pi^2 - \frac{1871}{2}$$

$$\frac{1}{2} (540 \pi^2 - 1871)$$

**Alternative representations:**

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) \cdot 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) \cdot 8 \zeta(2)) + 34 \right) \right) (-1) - 4 = -4 - \frac{27}{4} (138 - 240 \zeta(2, 1))$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-4 - \frac{27}{4} (138 - 240 S_{1,1}(1))$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-4 - \frac{27}{4} \left( 138 + \frac{240 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)$$

### Series representations:

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} + 1620 \sum_{k=1}^{\infty} \frac{1}{k^2}$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} - 3240 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} + 2160 \sum_{k=0}^{\infty} \frac{1}{(1 + 2k)^2}$$

### Integral representations:

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} + 4320 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} + 1080 \left( \int_0^{\infty} \frac{1}{1+t^2} dt \right)^2$$

$$\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4 =$$

$$-\frac{1871}{2} + 1080 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2$$

$$\left(\left(\left(27 \cdot -\frac{1}{4} \left(\left(\frac{1}{2}(12+14) \cdot 8 - \frac{1}{2}(2 \cdot 2 \cdot 2 + 2)(2+4) \cdot 8 \cdot (\zeta(2)) + 34\right)\right) - 4\right)\right)^{1/15}$$

**Input:**

$$\sqrt[15]{\frac{27}{4} \times (-1) \left( \frac{1}{2} (12 + 14) \times 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) \times 8 \zeta(2)) + 34 \right) - 4}$$

$\zeta(s)$  is the Riemann zeta function

**Exact result:**

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)}$$

**Decimal approximation:**

1.643833810173402610581831163516191926995739666197593178056...

$$1.64383381017\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

**Property:**

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} \text{ is a transcendental number}$$

**Alternate forms:**

$$\sqrt[15]{270 \pi^2 - \frac{1871}{2}}$$

$$\sqrt[15]{\frac{1}{2} (540 \pi^2 - 1871)}$$

**All 15th roots of  $-4 - 27/4 (138 - 40 \pi^2)$ :**

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} e^0 \approx 1.64383 \text{ (real, principal root)}$$

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} e^{(2i\pi)/15} \approx 1.5017 + 0.6686 i$$

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} e^{(4i\pi)/15} \approx 1.0999 + 1.2216 i$$

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} e^{(2i\pi)/5} \approx 0.5080 + 1.5634 i$$

$$\sqrt[15]{-4 - \frac{27}{4} (138 - 40 \pi^2)} e^{(8i\pi)/15} \approx -0.17183 + 1.63483 i$$

### Alternative representations:

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-4 - \frac{27}{4} (138 - 240 \zeta(2, 1))}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-4 - \frac{27}{4} (138 - 240 S_{1,1}(1))}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-4 - \frac{27}{4} \left( 138 + \frac{240 \operatorname{Li}_2(-1)}{\frac{1}{2}} \right)}$$

### Series representations:

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-\frac{1871}{2} + 1620 \sum_{k=1}^{\infty} \frac{1}{k^2}}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-\frac{1871}{2} - 3240 \sum_{k=1}^{\infty} \frac{(-1)^k}{k^2}}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} = \sqrt[15]{-\frac{1871}{2} + 2160 \sum_{k=0}^{\infty} \frac{1}{(1 + 2k)^2}}$$

**Integral representations:**

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} =$$
$$\sqrt[15]{-\frac{1871}{2} + 1080 \left( \int_0^\infty \frac{1}{1+t^2} dt \right)^2}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} =$$
$$\sqrt[15]{-\frac{1871}{2} + 4320 \left( \int_0^1 \sqrt{1-t^2} dt \right)^2}$$

$$\sqrt[15]{\frac{1}{4} \left( 27 \left( \frac{1}{2} (12 + 14) 8 - \frac{1}{2} (2 \times 2 \times 2 + 2) ((2 + 4) 8 \zeta(2)) + 34 \right) \right) (-1) - 4} =$$
$$\sqrt[15]{-\frac{1871}{2} + 1080 \left( \int_0^1 \frac{1}{\sqrt{1-t^2}} dt \right)^2}$$

Now, we have that:

Next we let  $n_B = 1$  and  $n_F = 0$ , and demand that, within this single-charge assignment, the contributions in  $\mathcal{A}$  and  $\mathcal{M}$  be equal modulo two. This gives the conditions

$$b_1^2 = a_1^2, \quad \frac{1}{2}(a_1^2 + b_1^2) = a_1 \pmod{2}, \quad \frac{1}{2}(a_1^2 + b_1^2) = b_1 \pmod{2}, \quad (512)$$

that clearly admit the four solutions  $a_1 = b_1 = \pm 1$ ,  $a_1 = -b_1 = \pm 1$ , two of which can be obtained from the others by the usual overall reversal of  $\mathcal{M}$ , while the choice  $n_B = 0$ ,  $n_F = 1$  leads again to these solutions. The general solution can then be obtained *superposing* an independent pair of these, say  $a_1 = b_1 = 1$ ,  $a_2 = -1$ , so that, as before

$$\begin{aligned} \mathcal{A} &= \frac{1}{2}(n_B^2 + n_F^2)(O_8 + V_8) - n_B n_F (S_8 + C_8), \\ \mathcal{M} &= -\frac{1}{2}(n_B - n_F)\hat{O}_8 + \frac{1}{2}(n_B + n_F)\hat{V}_8. \end{aligned} \quad (513)$$

Notice that the restriction to the orientifold can be simply relaxed, so that the oriented D9 brane spectrum of the ‘‘parent’’ 0A model,

$$\begin{aligned} \tilde{\mathcal{A}} &= \frac{2^{-5}}{2} [ |n_B + n_F|^2 V_8 + |n_B - n_F|^2 O_8 ], \\ \mathcal{A} &= (n_B \bar{n}_B + n_F \bar{n}_F)(O_8 + V_8) - (n_B \bar{n}_F + n_F \bar{n}_B)(S_8 + C_8) \end{aligned} \quad (514)$$

can be recovered complexifying all multiplicities, as in the previous section. In analogy with our discussion of subsection 5.12, these amplitudes also describe uncharged branes of the 0B string.

From

$$\begin{aligned} \mathcal{A} &= \frac{1}{2}(n_B^2 + n_F^2)(O_8 + V_8) - n_B n_F (S_8 + C_8), \\ \mathcal{M} &= -\frac{1}{2}(n_B - n_F)\hat{O}_8 + \frac{1}{2}(n_B + n_F)\hat{V}_8. \end{aligned}$$

For:

$$V_8 - S_8 = 5 - 3 = 2; \quad \eta = 2, 3, 7 \quad \text{and} \quad \vartheta = 1, 2, 2$$

$$O_8 = \frac{\vartheta_3^4 + \vartheta_4^4}{2\eta^4} = 1 + 28q + \dots, \quad n_B = 1 \quad \text{and} \quad n_F = 0$$

$$(2^4 + 2^4)/(2 \cdot 2^4) = 1$$

we obtain:

$$1/2(1+5) = 3; \quad \text{(a)}$$

$$-1/2 + 1/2*5$$

$$-\frac{1}{2} + \frac{1}{2} \times 5$$

2

2

and

$$\tilde{A} = \frac{2^{-5}}{2} [ |n_B + n_F|^2 V_8 + |n_B - n_F|^2 O_8 ] ,$$

$$A = (n_B \bar{n}_B + n_F \bar{n}_F)(O_8 + V_8) - (n_B \bar{n}_F + n_F \bar{n}_B)(S_8 + C_8)$$

$$(2^{(-5)})/2 (5+1)$$

$$\frac{1}{2^5 \times 2} (5+1)$$

$$\frac{3}{32}$$

0.09375

0.09375 (b)

$$(1+5)-0 = 6 \quad (c)$$

From the results (a), (b) and (c), we obtain from the following expression:

$$1/2(1+5) * (2^{(-5)})/2 (5+1) + (1+5)$$

**Input:**

$$\frac{1}{2} (1+5) \times \frac{1}{2^5 \times 2} (5+1) + (1+5)$$



**Exact result:**

$$\frac{201}{32}$$

**Decimal form:**

$$6.28125$$

$$6.28125$$

$$2\pi \approx 6.2831853$$

Now, we have that:

$$\tilde{\mathcal{M}} = -(n_1 + n_5 + n_9 + n_{13}) \hat{\chi}_9^{\text{R}} - (n_1 - n_5 + n_9 - n_{13}) \hat{\chi}_9^{\text{NS}}, \quad (530)$$

where we have introduced a minimal set of Chan-Paton multiplicities, that are to be subjected to the R-R tadpole conditions

$$\begin{aligned} n_1 + n_5 + n_9 + n_{13} &= 32, \\ n_1 + n_5 - n_9 - n_{13} &= 0, \\ n_1 - n_5 + n_9 - n_{13} &= 0, \\ n_1 - n_5 - n_9 + n_{13} &= 0. \end{aligned} \quad (531)$$

Finally,  $S$  and  $P$  modular transformations and a suitable relabelling of the multiplicities,

$$n_1 = n, \quad n_9 = \bar{n}, \quad n_5 = m, \quad n_{13} = \bar{m}, \quad (532)$$

give the direct-channel amplitudes

$$\begin{aligned} \mathcal{A} &= (n\bar{n} + m\bar{m})\chi_1 + (n\bar{m} + \bar{n}m)\tilde{\chi}_1 \\ &\quad + \frac{1}{2}(n^2 + \bar{n}^2 + m^2 + \bar{m}^2)\chi_9 + (nm + \bar{n}\bar{m})\tilde{\chi}_9, \end{aligned} \quad (533)$$

From (530), we obtain:

$$-32 - 0 = -32$$

From (533), for  $n = 12, 14$ ;  $m = 2, 4$ ; we obtain:

$$(12*14+2*4)+(12*4+14*2)+1/2(12^2+14^2+2^2+4^2)+(12*2+14*4)$$

**Input:**

$$(12 \times 14 + 2 \times 4) + (12 \times 4 + 14 \times 2) + \frac{1}{2} (12^2 + 14^2 + 2^2 + 4^2) + (12 \times 2 + 14 \times 4)$$

**Exact result:**

512

512

We note that  $512 = 64 * 8$  and, dividing (533) by (530), we obtain:

$$(((12*14+2*4)+(12*4+14*2)+1/2(12^2+14^2+2^2+4^2)+(12*2+14*4))) / (-32)$$

**Input:**

$$-\frac{1}{32} \left( (12 \times 14 + 2 \times 4) + (12 \times 4 + 14 \times 2) + \frac{1}{2} (12^2 + 14^2 + 2^2 + 4^2) + (12 \times 2 + 14 \times 4) \right)$$

**Exact result:**

-16

-16

From which, multiplying by -4 (where 4 is a Lucas number):

$$-4 * (((12*14+2*4)+(12*4+14*2)+1/2(12^2+14^2+2^2+4^2)+(12*2+14*4))) / (-32)$$

**Input:**

$$-4 \left( -\frac{1}{32} \left( (12 \times 14 + 2 \times 4) + (12 \times 4 + 14 \times 2) + \frac{1}{2} (12^2 + 14^2 + 2^2 + 4^2) + (12 \times 2 + 14 \times 4) \right) \right)$$

**Exact result:**

64

64

and:

$$27 * -4 * (((12*14+2*4)+(12*4+14*2)+1/2(12^2+14^2+2^2+4^2)+(12*2+14*4))) / (-32) + 1$$

**Input:**

$$27 \times (-4) \left( -\frac{1}{32} \left( (12 \times 14 + 2 \times 4) + (12 \times 4 + 14 \times 2) + \frac{1}{2} (12^2 + 14^2 + 2^2 + 4^2) + (12 \times 2 + 14 \times 4) \right) \right) + 1$$

**Exact result:**

1729

1729

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number, see observations)

and:

$$[27 \cdot -4 \cdot (((12 \cdot 14 + 2 \cdot 4) + (12 \cdot 4 + 14 \cdot 2) + 1/2(12^2 + 14^2 + 2^2 + 4^2) + (12 \cdot 2 + 14 \cdot 4))) / (-32) + 1]^{1/15}$$

**Input:**

$$\left( 27 \times (-4) \left( -\frac{1}{32} \left( (12 \times 14 + 2 \times 4) + (12 \times 4 + 14 \times 2) + \frac{1}{2} (12^2 + 14^2 + 2^2 + 4^2) + (12 \times 2 + 14 \times 4) \right) \right) + 1 \right)^{1/15}$$

**Result:**

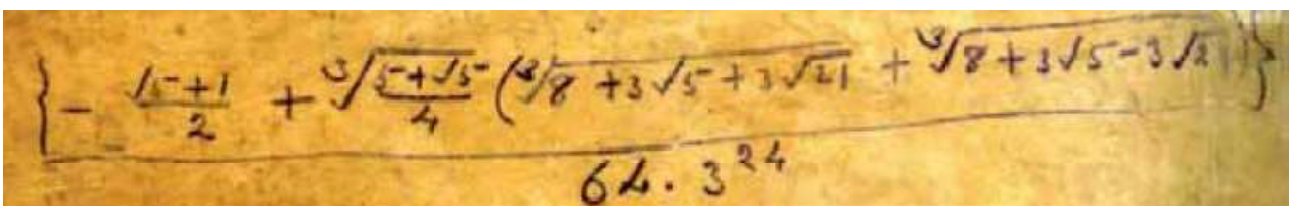
$$\sqrt[15]{1729}$$

**Decimal approximation:**

1.643815228748728130580088031324769514329283143699940172645...

$$1.6438152\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From (Manuscript Book 1 of Srinivasa Ramanujan)



We have:

$$\left[ -\left(\frac{\sqrt{5}+1}{2}\right) + \left(\frac{5+\sqrt{5}}{4}\right)^{1/3} \left( \left(\left(8+3\sqrt{5}+3\sqrt{21}\right)^{1/3} + \left(8+3\sqrt{5}-3\sqrt{21}\right)^{1/3} \right) \right) \right] / (64 \cdot 3^{24})$$

**Input:**

$$\frac{-\left(\frac{1}{2}(\sqrt{5} + 1)\right) + \sqrt[3]{\frac{1}{4}(5 + \sqrt{5})} \left( \sqrt[3]{8 + 3\sqrt{5} + 3\sqrt{21}} + \sqrt[3]{8 + 3\sqrt{5} - 3\sqrt{21}} \right)}{64 \times 3^{24}}$$

**Result:**

$$\frac{\frac{1}{2}(-1 - \sqrt{5}) + \frac{\sqrt[3]{5+\sqrt{5}} \left( \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} \right)}{2^{2/3}}}{18\,075\,490\,334\,784}$$

**Decimal approximation:**

$$1.8279599561322372839255373860016878509597369354090463... \times 10^{-13}$$

$$1.827959956... \cdot 10^{-13}$$

**Alternate forms:**

$$\frac{1}{36\,150\,980\,669\,568} \left( \sqrt[3]{2 \left( 55 + 23\sqrt{5} - 15\sqrt{21} - 3\sqrt{105} \right)} - \sqrt{5} + \sqrt[3]{2 \left( 55 + 23\sqrt{5} + 15\sqrt{21} + 3\sqrt{105} \right)} - 1 \right)$$

$$\text{root of } x^6 + 6x^5 - 31104x + 46656 \text{ near } x = 6.60825$$

$$36\,150\,980\,669\,568$$

root of

$$4784240412380545009888355056253402699267854509007322416 \cdot 933021546481140629504x^6 + 7940432580974931735767800427068528151773115681168876033 \cdot 034682368x^5 - 24100653779712x + 1 \text{ near } x = 1.82796 \times 10^{-13}$$

**Minimal polynomial:**

$$47842404123805450009888355056253402699267854509007322416933 \cdot \\ 021546481140629504x^6 + \\ 7940432580974931735767800427068528151773115681168876033034 \cdot \\ 682368x^5 - 24100653779712x + 1$$

$$\left[ \left( \frac{\sqrt{5}-1}{2} \right) + \left( \frac{5-\sqrt{5}}{4} \right)^{1/3} \left( \left( (8-3\sqrt{5}+3\sqrt{21})^{1/3} + (8-3\sqrt{5}-3\sqrt{21})^{1/3} \right) \right) \right] / (64 \cdot 3^{24})$$

**Input:**

$$\frac{\frac{1}{2}(\sqrt{5}-1) + \sqrt[3]{\frac{1}{4}(5-\sqrt{5})} \left( \sqrt[3]{8-3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8-3\sqrt{5}-3\sqrt{21}} \right)}{64 \times 3^{24}}$$

**Result:**

$$\frac{\frac{1}{2}(\sqrt{5}-1) + \frac{\sqrt[3]{5-\sqrt{5}} \left( \sqrt[3]{8-3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8-3\sqrt{5}+3\sqrt{21}} \right)}{2^{2/3}}}{18075490334784}$$

**Decimal approximation:**

$$2.11608918092794166102589282485086470720058012509155... \times 10^{-13} + \\ 9.81872427763515299793460082073211406094532914183620... \times 10^{-14} i$$

**Polar coordinates:**

$$r \approx 2.33279 \times 10^{-13} \text{ (radius), } \theta \approx 24.8915^\circ \text{ (angle)}$$

$$2.33279 \cdot 10^{-13}$$

**Alternate forms:**

$$\frac{\left( \sqrt[3]{8-3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8-3\sqrt{5}+3\sqrt{21}} \right) \sqrt[3]{2(5-\sqrt{5}) + \sqrt{5}-1}}{36150980669568} \\ \frac{1}{36150980669568} \left( -1 + \sqrt{5} + \sqrt[3]{2(55-23\sqrt{5}+15\sqrt{21}-3\sqrt{105})} + \right. \\ \left. \sqrt[3]{-2(-55+23\sqrt{5}+15\sqrt{21}-3\sqrt{105})} \right) \\ \frac{-1 + \sqrt{5} + \sqrt[3]{2(5-\sqrt{5})(8-3\sqrt{5}+3\sqrt{21})} + \sqrt[3]{-2(5-\sqrt{5})(-8+3\sqrt{5}+3\sqrt{21})}}{36150980669568}$$

**Minimal polynomial:**

1 668 604 915 979 881 695 351 737 454 408 267 987 625 601 632 216 058 736 336 592 √  
 858 529 554 743 319 494 776 204 467 151 611 007 291 101 038 284 070 287 254 603 √  
 970 031 779 306 429 864 069 351 433 163 505 664 x<sup>12</sup> +  
 553 878 722 538 064 302 872 403 744 164 877 015 187 938 094 105 204 645 688 870 √  
 677 855 728 804 290 487 098 365 818 584 744 526 132 984 211 924 514 097 188 √  
 573 936 696 108 322 137 284 018 176 x<sup>11</sup> +  
 45 963 792 318 722 989 345 088 202 385 733 762 972 755 650 778 306 932 667 861 √  
 168 383 615 592 733 329 389 682 591 259 303 075 991 021 838 152 576 559 844 √  
 887 739 913 732 096 x<sup>10</sup> -  
 50 857 588 333 602 736 867 709 001 979 785 042 654 708 519 406 787 959 401 107 √  
 420 464 505 537 713 089 153 096 983 050 287 496 843 127 761 400 761 291 898 √  
 880 x<sup>9</sup> -  
 4 220 432 258 681 281 227 297 818 679 881 133 792 596 343 639 348 511 722 006 √  
 373 401 582 839 175 818 995 723 384 976 627 108 269 239 828 480 x<sup>8</sup> +  
 1 354 237 514 278 969 982 996 680 120 746 381 893 553 208 087 686 925 862 560 √  
 692 836 413 368 416 233 089 301 338 679 934 976 x<sup>7</sup> +  
 604 536 618 508 405 666 324 949 254 490 817 996 507 948 609 575 816 526 060 365 √  
 660 261 335 692 994 412 544 x<sup>6</sup> -  
 3 430 266 874 981 170 509 851 689 784 493 604 161 565 985 974 264 954 446 270 √  
 982 782 976 x<sup>5</sup> -  
 6 404 888 879 159 579 418 700 897 962 727 301 826 902 341 890 248 540 160 x<sup>4</sup> -  
 433 083 549 822 625 895 381 277 739 795 522 608 168 960 x<sup>3</sup> +  
 184 925 416 577 064 307 026 584 887 296 x<sup>2</sup> - 9 471 556 935 426 816 x + 139

**Expanded form:**

$$-\frac{1}{36\,150\,980\,669\,568} + \frac{\sqrt{5}}{36\,150\,980\,669\,568} + \frac{\sqrt[3]{(5-\sqrt{5})(8-3\sqrt{5}+3\sqrt{21})}}{18\,075\,490\,334\,784 \times 2^{2/3}} + \frac{\sqrt[3]{-(5-\sqrt{5})(-8+3\sqrt{5}+3\sqrt{21})}}{18\,075\,490\,334\,784 \times 2^{2/3}}$$

$(2.33279 \times 10^{-13} / 1.827959956132 \times 10^{-13})^2$

**Input interpretation:**

$$\left( \frac{2.33279 \times 10^{-13}}{1.827959956132 \times 10^{-13}} \right)^2$$

**Result:**

1.628613140077691830068197350737835805479086883658031886471...

1.62861314007.... result equal to a golden number

$$(2.33279 \times 10^{-13} / 1.827959956132 \times 10^{-13})^{17} + ((1/18 (2 \pi! + 7 - 2 \pi)))$$

**Input interpretation:**

$$\left( \frac{2.33279 \times 10^{-13}}{1.827959956132 \times 10^{-13}} \right)^{17} + \frac{1}{18} (2 \pi! + 7 - 2 \pi)$$

$n!$  is the factorial function

**Result:**

64.0000...

64

$$\left[ -\left(\frac{\sqrt{5}+1}{2}\right) + \left(\frac{5+\sqrt{5}}{4}\right)^{1/3} \left( \left( (8+3\sqrt{5}+3\sqrt{21})^{1/3} + (8+3\sqrt{5}-3\sqrt{21})^{1/3} \right) \right) \right] / (x \times 3^{24}) = 1.8279599561e-13$$

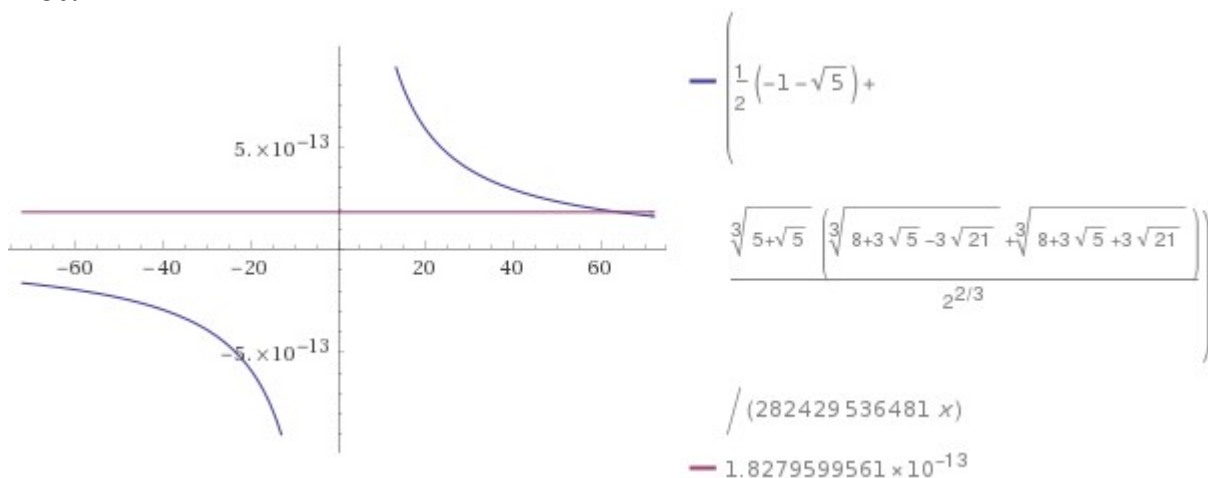
**Input interpretation:**

$$\frac{-\left(\frac{1}{2}(\sqrt{5}+1)\right) + \sqrt[3]{\frac{1}{4}(5+\sqrt{5})} \left( \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} \right)}{x \times 3^{24}} = 1.8279599561 \times 10^{-13}$$

**Result:**

$$\frac{\frac{1}{2}(-1-\sqrt{5}) + \frac{\sqrt[3]{5+\sqrt{5}}}{2^{2/3}} \left( \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} \right)}{282429536481 x} = 1.8279599561 \times 10^{-13}$$

**Plot:**



**Alternate form assuming x is real:**

$$\frac{1.169894372 \times 10^{-11}}{x} = 1.827959956 \times 10^{-13}$$

**Alternate forms:**

$$\frac{\text{root of } x^6 + 6x^5 - 31104x + 46656 \text{ near } x = 6.60825}{564859072962x} = 1.8279599561 \times 10^{-13}$$

$$\frac{\text{root of } 696198609130885597695136021593547814689632716312296141651066450089x^6 + 7395104114874202511988394360121831439224179537192802907x^5 - 376572715308x + 1 \text{ near } x = 1.16989 \times 10^{-11}}{x} = 1.8279599561 \times 10^{-13}$$

$$-\frac{1}{564859072962x} \left( 1 + \sqrt{5} - \sqrt[3]{2(55 + 23\sqrt{5} - 15\sqrt{21} - 3\sqrt{105})} - \sqrt[3]{2(55 + 23\sqrt{5} + 15\sqrt{21} + 3\sqrt{105})} \right) = 1.8279599561 \times 10^{-13}$$

**Alternate form assuming x is positive:**

$$1.827959956 \times 10^{-13} x = 1.169894372 \times 10^{-11} \text{ (for } x \neq 0)$$

**Expanded form:**

$$\frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} + 3\sqrt{21})}}{282429536481 \times 2^{2/3}x} + \frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} - 3\sqrt{21})}}{282429536481 \times 2^{2/3}x} - \frac{1}{564859072962x} = 1.8279599561 \times 10^{-13}$$

**Solution:**

$$x = 64$$

64



$$\left[ -\left(\frac{\sqrt{5}+1}{2}\right) + \left(\frac{5+\sqrt{5}}{4}\right)^{1/3} \left( \left( \left( (8+3\sqrt{5}+3\sqrt{21})^{1/3} + (8+3\sqrt{5}-3\sqrt{21})^{1/3} \right) \right) \right) \right] / \left( \frac{x}{27} \cdot 3^{24} \right) = 1.8279599561e-13$$

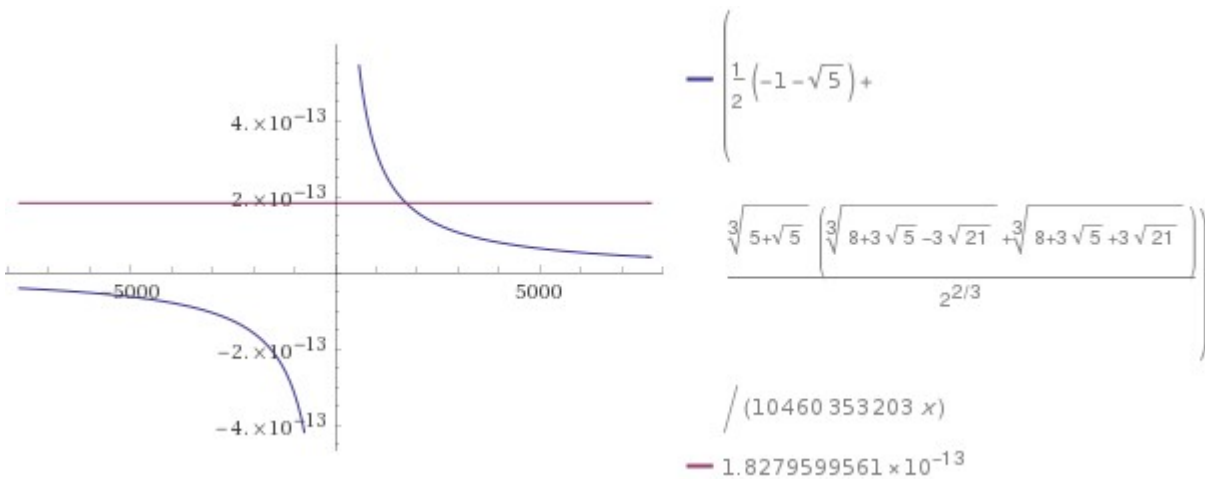
**Input interpretation:**

$$\frac{-\left(\frac{1}{2}(\sqrt{5} + 1)\right) + \sqrt[3]{\frac{1}{4}(5 + \sqrt{5})} \left( \sqrt[3]{8 + 3\sqrt{5} + 3\sqrt{21}} + \sqrt[3]{8 + 3\sqrt{5} - 3\sqrt{21}} \right)}{\frac{x}{27} \times 3^{24}} = 1.8279599561 \times 10^{-13}$$

**Result:**

$$\frac{\frac{1}{2}(-1 - \sqrt{5}) + \frac{\sqrt[3]{5+\sqrt{5}} \left( \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} \right)}{2^{2/3}}}{10460353203x} = 1.8279599561 \times 10^{-13}$$

**Plot:**



**Alternate form assuming x is real:**

$$\frac{3.158714804 \times 10^{-10}}{x} = 1.827959956 \times 10^{-13}$$

**Alternate forms:**

$$\frac{\text{root of } x^6 + 6x^5 - 31104x + 46656 \text{ near } x = 6.60825}{20920706406x} = 1.8279599561 \times 10^{-13}$$

root of

$$1797010299914431210413179829509605039731475627537851106401 \\ x^6 + 515377520732011331036461129765621272702107522001x^5 - \\ 13947137604x + 1 \text{ near } x = 3.15871 \times 10^{-10}$$

$$= 1.8279599561 \times 10^{-13}$$

$$-\frac{1}{20920706406x} \left( 1 + \sqrt{5} - \sqrt[3]{2(55 + 23\sqrt{5} - 15\sqrt{21} - 3\sqrt{105})} - \right. \\ \left. \sqrt[3]{2(55 + 23\sqrt{5} + 15\sqrt{21} + 3\sqrt{105})} \right) = 1.8279599561 \times 10^{-13}$$

**Alternate form assuming x is positive:**

$$1.827959956 \times 10^{-13} x = 3.158714804 \times 10^{-10} \text{ (for } x \neq 0)$$

**Expanded form:**

$$\frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} + 3\sqrt{21})}}{10460353203 \times 2^{2/3} x} + \frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} - 3\sqrt{21})}}{10460353203 \times 2^{2/3} x} - \\ \frac{\sqrt{5}}{20920706406x} - \frac{1}{20920706406x} = 1.8279599561 \times 10^{-13}$$

**Solution:**

$$x = 1728$$

1728

This result is very near to the mass of candidate glueball **f<sub>0</sub>(1710) scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

$$\left[ -\left(\frac{\sqrt{5}+1}{2}\right) + \left(\frac{5+\sqrt{5}}{4}\right)^{1/3} \left( \left( \left( (8+3\sqrt{5}+3\sqrt{21})^{1/3} + (8+3\sqrt{5}-3\sqrt{21})^{1/3} \right) \right) \right) \right] / \left( \left( x^{15} / 27 \right) \times 3^{24} \right) = 1.8279599561e-13$$

**Input interpretation:**

$$\frac{-\left(\frac{1}{2}(\sqrt{5} + 1)\right) + \sqrt[3]{\frac{1}{4}(5 + \sqrt{5})} \left( \sqrt[3]{8 + 3\sqrt{5} + 3\sqrt{21}} + \sqrt[3]{8 + 3\sqrt{5} - 3\sqrt{21}} \right)}{\frac{x^{15}}{27} \times 3^{24}} = 1.8279599561 \times 10^{-13}$$

**Result:**

$$\frac{\frac{1}{2}(-1 - \sqrt{5}) + \frac{\sqrt[3]{5+\sqrt{5}} \left( \sqrt[3]{8+3\sqrt{5}-3\sqrt{21}} + \sqrt[3]{8+3\sqrt{5}+3\sqrt{21}} \right)}{2^{2/3}}}{10460353203 x^{15}} = 1.8279599561 \times 10^{-13}$$

**Alternate form assuming x is real:**

$$\frac{2.301945821 \times 10^9 x^{15} - 3.977762379 \times 10^{12}}{x} = 0$$

**Alternate forms:**

$$\frac{\text{root of } x^6 + 6x^5 - 31104x + 46656 \text{ near } x = 6.60825}{20920706406 x^{15}} = 1.8279599561 \times 10^{-13}$$

$$\frac{\text{root of } 1797010299914431210413179829509605039731475627537851106401 x^6 + 515377520732011331036461129765621272702107522001 x^5 - 13947137604x + 1 \text{ near } x = 3.15871 \times 10^{-10}}{x^{15}}$$

$$= 1.8279599561 \times 10^{-13}$$

$$-\frac{1}{20920706406 x^{15}} \left( 1 + \sqrt{5} - \sqrt[3]{2(55 + 23\sqrt{5} - 15\sqrt{21} - 3\sqrt{105})} - \sqrt[3]{2(55 + 23\sqrt{5} + 15\sqrt{21} + 3\sqrt{105})} \right) = 1.8279599561 \times 10^{-13}$$

**Alternate form assuming x is positive:**

$$2.301945821 \times 10^9 x^{15} = 3.977762379 \times 10^{12} \text{ (for } x \neq 0)$$

**Expanded form:**

$$\frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} + 3\sqrt{21})}}{10460353203 \times 2^{2/3} x^{15}} + \frac{\sqrt[3]{(5 + \sqrt{5})(8 + 3\sqrt{5} - 3\sqrt{21})}}{10460353203 \times 2^{2/3} x^{15}} - \frac{1}{20920706406 x^{15}} = 1.8279599561 \times 10^{-13}$$

**Real solution:**

$$x \approx 1.64375182952$$

$$1.64375182952 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$$

From:

**Modular equations and approximations to  $\pi$**  – *Srinivasa Ramanujan* - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

$$\frac{32}{\pi} = (5\sqrt{5} - 1) + \frac{47\sqrt{5} + 29}{64} \left(\frac{1}{2}\right)^3 \left(\frac{\sqrt{5} - 1}{2}\right)^8 + \frac{89\sqrt{5} + 59}{64^2} \left(\frac{1 \cdot 3}{2 \cdot 4}\right)^3 \left(\frac{\sqrt{5} - 1}{2}\right)^{16} + \dots,$$

$$(5\sqrt{5}-1)+1/64*(47\sqrt{5}+29)*1/8*((\sqrt{5}-1)/2)^8+1/64^2*(89\sqrt{5}+59)*(3/8)^3*((\sqrt{5}-1)/2)^{16}$$

**Input:**

$$(5\sqrt{5} - 1) + \frac{1}{64} (47\sqrt{5} + 29) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{1}{64^2} (89\sqrt{5} + 59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}$$

**Result:**

$$-1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8 (29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16} (59 + 89\sqrt{5})}{137438953472}$$

**Decimal approximation:**

10.18591635745234529933672773439453907823343935074991009694...

10.18591635...

**Alternate forms:**

$$\frac{15(1041875\sqrt{5} - 905609)}{2097152}$$

$$\frac{15628125\sqrt{5}}{2097152} - \frac{13584135}{2097152}$$

$$\frac{15\,628\,125\sqrt{5} - 13\,584\,135}{2\,097\,152}$$

**Minimal polynomial:**

$$1\,099\,511\,627\,776\,x^2 + 14\,243\,997\,941\,760\,x - 259\,165\,682\,844\,975$$

From which:

$$32 / (((5\sqrt{5}-1) + 1/64 * (47\sqrt{5}+29) * 1/8 * ((\sqrt{5}-1)/2)^8 + 1/64^2 * (89\sqrt{5}+59) * (3/8)^3 * ((\sqrt{5}-1)/2)^{16}))$$

**Input:**

$$\frac{32}{(5\sqrt{5} - 1) + \frac{1}{64} (47\sqrt{5} + 29) \times \frac{1}{8} \left(\frac{1}{2} (\sqrt{5} - 1)\right)^8 + \frac{1}{64^2} (89\sqrt{5} + 59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2} (\sqrt{5} - 1)\right)^{16}}$$

**Result:**

$$-1 + 5\sqrt{5} + \frac{32}{131072} \frac{(\sqrt{5}-1)^8 (29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16} (59+89\sqrt{5})}{137438953472}$$

**Decimal approximation:**

3.141592653722094110325513887398231062068309787569351629942...

3.1415926537...  $\approx \pi$

**Alternate forms:**

$$\frac{16\,777\,216(905\,609 + 1\,041\,875\sqrt{5})}{17\,277\,712\,189\,665}$$

$$\frac{15\,193\,597\,804\,544 + 17\,479\,761\,920\,000\sqrt{5}}{17\,277\,712\,189\,665}$$

$$\frac{3\,495\,952\,384\,000\sqrt{5}}{3\,455\,542\,437\,933} + \frac{15\,193\,597\,804\,544}{17\,277\,712\,189\,665}$$

**Minimal polynomial:**

$$259\,165\,682\,844\,975\,x^2 - 455\,807\,934\,136\,320\,x - 1\,125\,899\,906\,842\,624$$

Multiplying by  $2\pi$  the principal expression, we obtain:

$$2\pi * (((5\sqrt{5}-1)+1/64*(47\sqrt{5}+29)*1/8*((\sqrt{5}-1)/2)^8+1/64^2 * (89\sqrt{5}+59)*(3/8)^3*((\sqrt{5}-1)/2)^{16})))$$

**Input:**

$$2\pi \left( (5\sqrt{5}-1) + \frac{1}{64} (47\sqrt{5}+29) \times \frac{1}{8} \left( \frac{1}{2} (\sqrt{5}-1) \right)^8 + \frac{1}{64^2} (89\sqrt{5}+59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5}-1) \right)^{16} \right)$$

**Result:**

$$2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5}-1)^8 (29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16} (59+89\sqrt{5})}{137438953472} \right) \pi$$

**Decimal approximation:**

63.99999999730478877037356118727778112992174448981692286347...

63.9999...  $\approx$  64

**Property:**

$$2 \left( -1 + 5\sqrt{5} + \frac{(-1+\sqrt{5})^8 (29+47\sqrt{5})}{131072} + \frac{27(-1+\sqrt{5})^{16} (59+89\sqrt{5})}{137438953472} \right) \pi$$

is a transcendental number

**Alternate forms:**

$$\frac{\pi (15628125\sqrt{5} - 905609)}{1048576}$$

$$\frac{(15628125\sqrt{5} - 13584135)\pi}{1048576}$$

$$\frac{15628125\sqrt{5}\pi}{1048576} - \frac{13584135\pi}{1048576}$$

**Series representations:**

$$2\pi \left( 5\sqrt{5} - 1 \right) + \frac{(47\sqrt{5} + 29) \left( \frac{1}{2}(\sqrt{5} - 1) \right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2}(\sqrt{5} - 1) \right)^{16}}{64^2} =$$

$$2\pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + \frac{\left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^8 \left( 29 + 47\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}{131072} + \frac{27 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^{16} \left( 59 + 89\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}{137438953472} \right)$$

$$2\pi \left( 5\sqrt{5} - 1 \right) + \frac{(47\sqrt{5} + 29) \left( \frac{1}{2}(\sqrt{5} - 1) \right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2}(\sqrt{5} - 1) \right)^{16}}{64^2} =$$

$$2\pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \binom{-\frac{1}{2}}{k}}{k!} + \frac{\left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \binom{-\frac{1}{2}}{k}}{k!} \right)^8 \left( 29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \binom{-\frac{1}{2}}{k}}{k!} \right)}{131072} + \frac{27 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \binom{-\frac{1}{2}}{k}}{k!} \right)^{16} \left( 59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \binom{-\frac{1}{2}}{k}}{k!} \right)}{137438953472} \right)$$

$$\begin{aligned}
& 2\pi \left( (5\sqrt{5}-1) + \frac{(47\sqrt{5}+29)\left(\frac{1}{2}(\sqrt{5}-1)\right)^8}{64 \times 8} + \frac{(89\sqrt{5}+59)\left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16}}{64^2} \right) = \\
& 2\pi \left( -1 + 5\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!} + \right. \\
& \left. \frac{\left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right)^8 \left(29 + 47\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right)}{131072} \right. \\
& \left. + \frac{1}{137438953472} 27 \left(-1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right)^{16} \right. \\
& \left. \left(59 + 89\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5-z_0)^k z_0^{-k}}{k!}\right) \right)
\end{aligned}$$

for (not ( $z_0 \in \mathbb{R}$  and  $-\infty < z_0 \leq 0$ ))

$$27*2\text{Pi}((((5\text{sqrt}5-1)+1/64*(47\text{sqrt}5+29)*1/8*((\text{sqrt}5-1)/2)^8+1/64^2*(89\text{sqrt}5+59)*(3/8)^3*((\text{sqrt}5-1)/2)^16)))$$

**Input:**

$$\begin{aligned}
& 27 \times 2\pi \left( (5\sqrt{5}-1) + \frac{1}{64} (47\sqrt{5}+29) \times \frac{1}{8} \left(\frac{1}{2}(\sqrt{5}-1)\right)^8 + \right. \\
& \left. \frac{1}{64^2} (89\sqrt{5}+59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5}-1)\right)^{16} \right)
\end{aligned}$$

**Result:**

$$54 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5}-1)^8 (29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16} (59+89\sqrt{5})}{137438953472} \right) \pi$$

**Decimal approximation:**

1727.999999927229296800086152056500090507887101225056917313...

$$1727.9999 \approx 1728$$



This result is very near to the mass of candidate glueball  $f_0(1710)$  scalar meson. Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

**Property:**

$$54 \left( -1 + 5\sqrt{5} + \frac{(-1 + \sqrt{5})^8 (29 + 47\sqrt{5})}{131072} + \frac{27(-1 + \sqrt{5})^{16} (59 + 89\sqrt{5})}{137438953472} \right) \pi$$

is a transcendental number

**Alternate forms:**

$$\frac{405 \pi (1041875 \sqrt{5} - 905609)}{1048576}$$

$$\frac{(421959375 \sqrt{5} - 366771645) \pi}{1048576}$$

$$\frac{421959375 \sqrt{5} \pi}{1048576} - \frac{366771645 \pi}{1048576}$$

**Series representations:**

$$27 \times 2 \pi \left( (5\sqrt{5} - 1) + \frac{(47\sqrt{5} + 29) \left(\frac{1}{2}(\sqrt{5} - 1)\right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}}{64^2} \right) =$$

$$54 \pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} + \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{131072} + \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k}\right)}{137438953472} \right)$$

$$27 \times 2 \pi \left( 5\sqrt{5} - 1 \right) + \frac{(47\sqrt{5} + 29) \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16}}{64^2} =$$

$$54 \pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} + \right.$$

$$\left. \frac{\left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} \right)^8 \left( 29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} \right)}{131072} + \right.$$

$$\left. \frac{27 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} \right)^{16} \left( 59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left( -\frac{1}{4} \right)^k \left( -\frac{1}{2} \right)_k}{k!} \right)}{137438953472} \right)$$

$$27 \times 2 \pi \left( 5\sqrt{5} - 1 \right) + \frac{(47\sqrt{5} + 29) \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16}}{64^2} =$$

$$54 \pi \left( -1 + 5\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right.$$

$$\left. \frac{\left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^8 \left( 29 + 47\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)}{131072} \right.$$

$$\left. + \frac{1}{137438953472} 27 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^{16} \right.$$

$$\left. \left( 59 + 89\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left( -\frac{1}{2} \right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right)$$

for (not  $(z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0)$ )

$$[27 \times 2\pi \left( \left( (5\sqrt{5}-1) + \frac{1}{64}(47\sqrt{5}+29) \times \frac{1}{8} \left( \frac{(\sqrt{5}-1)}{2} \right)^8 + \frac{1}{64^2} (89\sqrt{5}+59) \left( \frac{3}{8} \right)^3 \left( \frac{(\sqrt{5}-1)}{2} \right)^{16} \right) \right)^{1/15}]$$

**Input:**

$$\left( 27 \times 2\pi \left( \left( 5\sqrt{5}-1 \right) + \frac{1}{64} \left( 47\sqrt{5}+29 \right) \times \frac{1}{8} \left( \frac{(\sqrt{5}-1)}{2} \right)^8 + \frac{1}{64^2} \left( 89\sqrt{5}+59 \right) \left( \frac{3}{8} \right)^3 \left( \frac{(\sqrt{5}-1)}{2} \right)^{16} \right) \right)^{1/15}$$

**Exact result:**

$$\sqrt[5]{3} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5}-1)^8 (29+47\sqrt{5})}{131072} + \frac{27(\sqrt{5}-1)^{16} (59+89\sqrt{5})}{137438953472} \right) \pi}$$

**Decimal approximation:**

1.643751829512610909819205335727061993363805434231177509417...

$$1.6437518295\dots \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934\dots$$

**Property:**

$$\sqrt[5]{3} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(-1+\sqrt{5})^8 (29+47\sqrt{5})}{131072} + \frac{27(-1+\sqrt{5})^{16} (59+89\sqrt{5})}{137438953472} \right) \pi}$$

is a transcendental number

**Alternate forms:**

$$\sqrt[5]{3} \sqrt[15]{2 \left( \frac{15628125\sqrt{5}}{2097152} - \frac{13584135}{2097152} \right) \pi}$$

$$\frac{\sqrt[5]{3} \sqrt[15]{(15628125\sqrt{5} - 13584135) \pi}}{2 \sqrt[3]{2}}$$

$$\frac{3^{4/15} \sqrt[15]{5(1041875\sqrt{5} - 905609) \pi}}{2 \sqrt[3]{2}}$$

**All 15th roots of  $54(-1 + 5\sqrt{5}) + ((\sqrt{5} - 1)^8(29 + 47\sqrt{5}))/131072 + (27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5}))/137438953472$   $\pi$ :**

$$\sqrt[5]{3}^{15} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8(29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5})}{137438953472} \right) \pi} e^0$$

$\approx 1.64375$  (real, principal root)

$$\sqrt[5]{3}^{15} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8(29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5})}{137438953472} \right) \pi} e^{(2i\pi)/15} \approx 1.5016 + 0.6686i$$

$$\sqrt[5]{3}^{15} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8(29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5})}{137438953472} \right) \pi} e^{(4i\pi)/15} \approx 1.0999 + 1.2215i$$

$$\sqrt[5]{3}^{15} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8(29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5})}{137438953472} \right) \pi} e^{(6i\pi)/15} \approx 0.5079 + 1.5633i$$

$$\sqrt[5]{3}^{15} \sqrt[15]{2 \left( -1 + 5\sqrt{5} + \frac{(\sqrt{5} - 1)^8(29 + 47\sqrt{5})}{131072} + \frac{27(\sqrt{5} - 1)^{16}(59 + 89\sqrt{5})}{137438953472} \right) \pi} e^{(8i\pi)/15} \approx -0.17182 + 1.63475i$$

**Series representations:**

$$\left( 27 \times 2 \pi \left( 5\sqrt{5} - 1 \right) + \frac{(47\sqrt{5} + 29) \left( \frac{1}{2} (\sqrt{5} - 1) \right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left( \frac{3}{8} \right)^3 \left( \frac{1}{2} (\sqrt{5} - 1) \right)^{16}}{64^2} \right)^{\wedge}$$

$$(1/15) = \sqrt[15]{2} \sqrt[5]{3} \left[ \pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right) + \frac{\left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^8 \left( 29 + 47\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}{131072} + \frac{27 \left( -1 + \sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)^{16} \left( 59 + 89\sqrt{4} \sum_{k=0}^{\infty} 4^{-k} \binom{\frac{1}{2}}{k} \right)}{137438953472} \right]^{\wedge (1/15)}$$

$$\begin{aligned}
& \left( 27 \times 2 \pi \left( (5\sqrt{5} - 1) + \frac{(47\sqrt{5} + 29) \left(\frac{1}{2}(\sqrt{5} - 1)\right)^8}{64 \times 8} + \frac{(89\sqrt{5} + 59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}}{64^2} \right) \right)^{\wedge} \\
(1/15) &= \sqrt[15]{2} \sqrt[5]{3} \left( \pi \left( -1 + 5\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!} + \right. \right. \\
& \left. \left. \frac{\left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^8 \left(29 + 47\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{131072} + \right. \\
& \left. \left. \frac{27 \left(-1 + \sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)^{16} \left(59 + 89\sqrt{4} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}\right)^k \left(-\frac{1}{2}\right)_k}{k!}\right)}{137438953472} \right) \right)^{\wedge (1/15)}
\end{aligned}$$

$$\begin{aligned}
& \left( 27 \times 2 \pi \left( (5\sqrt{5} - 1) + \frac{(47\sqrt{5} + 29) \left(\frac{1}{2}(\sqrt{5} - 1)\right)^8}{64 \times 8} + \right. \right. \\
& \left. \left. \frac{(89\sqrt{5} + 59) \left(\frac{3}{8}\right)^3 \left(\frac{1}{2}(\sqrt{5} - 1)\right)^{16}}{64^2} \right) \right)^{\wedge (1/15)} = \\
& \sqrt[15]{2} \sqrt[5]{3} \left( \pi \left( -1 + 5\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} + \right. \right. \\
& \frac{1}{131072} \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^8 \\
& \left( 29 + 47\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) + \\
& \frac{1}{137438953472} 27 \left( -1 + \sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right)^{16} \\
& \left. \left. \left( 59 + 89\sqrt{z_0} \sum_{k=0}^{\infty} \frac{(-1)^k \left(-\frac{1}{2}\right)_k (5 - z_0)^k z_0^{-k}}{k!} \right) \right) \right)^{\wedge} \\
& (1/15) \text{ for (not } (z_0 \in \mathbb{R} \text{ and } -\infty < z_0 \leq 0))
\end{aligned}$$

### Integral representation:

$$(1+z)^a = \frac{\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{\Gamma(s)\Gamma(-a-s)}{z^s} ds}{(2\pi i)\Gamma(-a)} \text{ for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

**We note that the various values, very close to  $\zeta(2)$ , are recurrent and have, although with slight variations, a "repetitive" trend, typical of fractal mathematics.**

## Observations

*From:*

[https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn\\_RpOSvJlQxWsVLBcJ6KVgd\\_Af\\_hrmDYBNyU8mpSjRs1BDeremA](https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJlQxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA)

*Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that  $p(9) = 30$ ,  $p(9 + 5) = 135$ ,  $p(9 + 10) = 490$ ,  $p(9 + 15) = 1,575$  and so on are all divisible by 5. Note that here the  $n$ 's come at intervals of five units.*

*Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of  $p(n)$  that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.*

*Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of  $n$ 's separated by  $5^3 = 125$  units, saying that the corresponding  $p(n)$ 's should all be divisible by 125. In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.*

*From Wikipedia*

*In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field  $\phi$  and a Dirac field  $\psi$ . The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.*

*Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson:*

125 GeV for  $T = 0$  and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1 + \sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \dots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \dots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\dots$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \dots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \dots = 64\{(1 + \sqrt{2})^{12} + (1 - \sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and  $4096 = 64^2$

(Modular equations and approximations to  $\pi$  - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants  $\pi$ ,  $\phi$ ,  $1/\phi$ , the Fibonacci and Lucas numbers, linked to the golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.



*In mathematics, the Fibonacci numbers, commonly denoted  $F_n$ , form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the  $n$ th Fibonacci number in terms of  $n$  and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as  $n$  increases.*

*Fibonacci numbers are also closely related to Lucas numbers, in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences*

*The beginning of the sequence is thus:*

*0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...*

*The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.*

*The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.<sup>[1]</sup> The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.*

*The sequence of Lucas numbers is:*

*2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803.....*

*All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

*A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:*

2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

*In geometry, a golden spiral is a logarithmic spiral whose growth factor is  $\varphi$ , the golden ratio.<sup>[1]</sup> That is, a golden spiral gets wider (or further from its origin) by a factor of  $\varphi$  for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies<sup>[3]</sup> - golden spirals are one special case of these logarithmic spirals*

We observe that 1728 and 1729 are results very near to the mass of candidate glueball  **$f_0(1710)$  scalar meson**. Furthermore, 1728 occurs in the algebraic formula for the  $j$ -invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to  $\zeta(2) = \frac{\pi^2}{6} = 1.644934 \dots$

**We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson  $\pi$ ) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.**

$$9^3 + 10^3 = 12^3 + 1$$

Original Ramanujan's representation of Taxicab number 1729

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