$A^x + B^y = C^z$

Part 2: Another version of my theorem and infinite ascent

Nguyen Van Quang
Hue - Vietnam, 4-2020

Abstract

We give another version of the theorem that was submitted in the previous article [7], which is the theorem leading to the proof of the beal's conjecture and the Fermat- Catalan conjecture. We also give a view to prove whether the equation has infinitely many solutions in integer or not related to parametric solution and infinite ascent.

It is known that
The equation $A^2 + B^2 = C^k$ for any positive integer k has solutions in integer such that [5]
$A = [\frac{(s+it)^k + (s-it)^k}{2}]^2, B^2 = -[\frac{(s+it)^k - (s-it)^k}{2}]^2, C = s^2 + t^2$
That means:
$C = \sqrt[k]{A^2 + B^2} = s^2 + t^2$

and for some equations below:

$$A^2 + AB + B^2 = C^3$$

$$A^2 + B^4 = C^3$$

$$A^3 + B^3 = C^2$$

All unknowns can be expressed as parametric solutions with fixed integer-coefficients, however for the cases below:

1 The theorem

Theorem 1. *(denoted by QuG - theorem)*
For all positive integers n and x_i, k_i, all integers $A_i \neq \pm 1$ and $(A_1, A_2, ..., A_n) = 1, a_k, a_{k-1}, ..., a_1, a_0$ are fixed numbers, for any s,t coprime integers.

$$\sqrt[x_n]{A_1^{x_1}+A_2^{x_2}+...+A_{n-1}^{x_{n-1}}} = a_k s^k + a_{k-1}s^{k-1}t + a_{k-2}s^{k-2}t^2 + ... + a_1 st^{k-1} + a_0 s^k \Rightarrow$$

$$\frac{1}{x_1}+\frac{1}{x_2}+...+\frac{1}{x_{n-1}}+\frac{1}{x_n}+\frac{n}{LCM(x_1,x_2,...,x_{n-1},x_n)} > 1$$

.

Theorem 2. *(denoted by QuS - theorem) For positive integers x, y, z, k_i and A,B coprime integers, and $a_k, a_{k-1}, ..., a_1, a_0$ are fixed numbers.*
$\sqrt[z]{A^x \pm B^y} = a_k s^k + a_{k-1}s^{k-1}t + a_{k-2}s^{k-2}t^2 + ... + a_1 st^{k-1} + a_0 s^k$ *for any s,t coprime integers.*

$$\Rightarrow \frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{3}{LCM(x,y,z)} > 1$$

In other worlds, the $C = \sqrt[z]{A^x \pm B^y}$ $(A, B, C \neq 1,$ coprime)can not be expressed as $a_k s^k + a_{k-1}s^{k-1}t + a_{k-2}s^{k-2}t^2 + ... + a_1 st^{k-1} + a_0 s^k$ with all fixed coefficients , s,t coprime integer if

$$\frac{1}{x}+\frac{1}{y}+\frac{1}{z}+\frac{3}{LCM(x,y,z)} \leqslant 1$$

Notes:
LCM (x,y,z): least common multiples of x,y and z.
Except $A = 3, B = 2, \sqrt[z]{3^2 - 2^3} = 1$

We consider the cases of coefficients :

1. If all coefficients are integers, then the equation has infinitely many solutions in integer.

2. If all coefficients are not simultaneously integers, we can not conclude that the equation has a solution, infinitely many solutions, or no solution in integer.

For example:

$C = s^3 + 3\sqrt{2}s^2t - \sqrt{2}st^2 + t^3$
$C = s^3 + \sqrt{2}st(3s - t) + t^3$
C is integer if $3s - t = 0 \Rightarrow s = 1, t = 3$
then $C = 1 + 3^3 = 28$, since (s,t) = 1, the equation has only one positive solution in integer.

3. If there dose not exist the set of coefficients, then the equation has no solution in integer.

The other way to prove the equation has infinitely many solutions integer is INFINITE ASCENT. *The technique is that if the equation has a solution , then it will have a larger solution in integer, since the set of integers is not upper bound , so the equation has infinitely many solution in integer.*

We known the equation $A^4 + B^2 = C^4, A^4 + B^4 = C^2$ has no solution in integer . Fermat have proved it by INFINITE DESCENT, however the equations below have infinitely many solutions in integer :

$A^4 + B^2 = kC^4, A^4 + B^4 = kC^2$ for some k fixed integer.

They can be proved by INFINITE ASCENT

For example:

The equation $A^4 + B^2 = 5C^4$
has a smallest solution : A = 1, B = 2, C = 1,
and using this solution, we found other one A = 11, B = 358, C = 13
and then A = 2291, B = 7297558, C = 2005 and so on.

The equation $A^3 + B^3 = C^3$

If it has a none- trial solution, it will has a larger solution by :

$[A(A^3 \pm 2B^3)]^3 \mp [B(A^3 \pm 2B^3)]^3 \equiv (A^3 \pm B^3)(A^3 \mp B^3)^3$

The proof of the theorem above will be posted later.

References

[1] Beal conjecture - Wikipedia

[2] Fermat Catalan conjecture - Wikipedia

[3] Catalan's conjecture - Wikipedia

[4] Euler's Sum of powers conjecture - Wikipedia

[5] Quang N V, Theorem for W^n and Fermat's Last theorem Vixra: 1811.0072 v2(NT)

[6] Quang N V, A proof of the four color theorem by induction Vixra: 1601.0247 (CO)

[7] Quang N V, $A^x + B^y = C^z$ Part 1: My theorem Vixra: 1910.0563 (NT)

Email:
nguyenvquang67@gmail.com
quangnhu67@yahoo.com.vn