On some possible mathematical connections between various equations concerning the Dirichlet boundary conditions of the D-branes and several equations inherent the zeros of certain Dirichlet series

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Abstract

In this paper we have described some possible mathematical connections between various equations concerning the Dirichlet boundary conditions of the D-branes and several equations inherent the zeros of certain Dirichlet series

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https://commons.wikimedia.org/wiki/File:AnatolyA.Karatsuba.jpg

We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

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We have that:

$$\varkappa = \frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1},$$

[sqrt(((10-2sqrt(5))))-2] / ((((sqrt(5))-1)))

Input:

$$\frac{\sqrt{10-2\sqrt{5}}-2}{\sqrt{5}-1}$$

Decimal approximation:

0.284079043840412296028291832393126169091088088445737582759...

0.28407904384…

Alternate forms:

$$\frac{1}{4}\left(\sqrt{10-2\sqrt{5}}-2\sqrt{5}+\sqrt{5\left(10-2\sqrt{5}\right)}-2\right)$$

-

$$\frac{1}{4}\left(1+\sqrt{5}\right)\left(\sqrt{10-2\sqrt{5}}-2\right)$$

$$\frac{1}{2}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)$$

-

Minimal polynomial:

$$x^4+2x^3-6x^2-2x+1$$

We have that

$$L(s,\chi_1) = \sum_{n=1}^{\infty} \chi_1(n) n^{-s}, \qquad \operatorname{Re} s > 0.$$

For s = 2 and n = 2

$$\chi_1 = \chi_1(n)$$

$$\chi_1(2) = i,\ i^2 = -1,$$

$$f(s) = \frac{1 - i\varkappa}{2} L(s, \chi_1) + \frac{1 + i\varkappa}{2} L(s, \overline{\chi}_1),$$

we obtain:

(1-i*0.28407904384)/2*((((2^-2)*i))) + (1+i*0.28407904384)/2*(((((2^-2)*i))))

Input interpretation:

$$\left(\frac{1}{2}(1 + i \times (-0.28407904384))\right) \times \frac{i}{2^2} + \left(\frac{1}{2}(1 + i \times 0.28407904384)\right) \times \frac{i}{2^2}$$

i is the imaginary unit

Result:

0.25 i

0.25i = *f(s)*

$$f(s) = \sum_{n=1}^{\infty} r(n) n^{-s},$$

$$r(1) = 1, \quad r(2) = \varkappa, \quad r(3) = -\varkappa, \quad r(4) = -1, \quad r(5) = 0$$

$$r(n) = r(n + 5).$$

$$g(s) = g(1 - s), \qquad g(s) = \left(\frac{\pi}{5}\right)^{-s/2} \Gamma\left(\frac{s + 1}{2}\right) f(s).$$

$$g(s) = \left(\frac{\pi}{5}\right)^{-s/2} \Gamma\left(\frac{s + 1}{2}\right) f(s).$$

(5/Pi) * gamma(3/2) * 0.25i

Input:

$$\frac{5}{\pi}\,\Gamma\left(\frac{3}{2}\right)\times 0.25\,i$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$0.352618\ldots\,i$

0.352618…i = *g(s)*

Polar coordinates:

$r = 0.352618$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi} = \frac{1.25\,i\,e^{-\log G(3/2)+\log G(5/2)}}{\pi}$$

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi} = \frac{1.25\,i\,\frac{1}{2}!}{\pi}$$

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi} = \frac{1.25\,i\,\Gamma\left(\frac{3}{2},0\right)}{\pi}$$

$\log G(z)$ gives the logarithm of the Barnes G-function

$n!$ is the factorial function

$\Gamma(a, x)$ is the incomplete gamma function

Series representations:

$$\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi} = \frac{1.25\,i\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}{\pi} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi} = \frac{1.25\,i}{\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times0.25\,i}{\pi}=\frac{1.25\,i}{\pi}\int_0^\infty e^{-t}\sqrt{t}\,dt$$

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times0.25\,i}{\pi}=\frac{1.25\,i}{\pi}\int_0^1\sqrt{\log\left(\frac{1}{t}\right)}\,dt$$

- $$\frac{\Gamma\left(\frac{3}{2}\right)5\times0.25\,i}{\pi}=\frac{1.25\exp\left(\int_0^1\frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)i}{\pi}$$

log(x) is the natural logarithm

From *f(s)* + *g(s)*, we obtain:

0.25i + ((((5/Pi) * gamma(3/2) * 0.25i)))

Input:

$$0.25\,i+\frac{5}{\pi}\,\Gamma\left(\frac{3}{2}\right)\times0.25\,i$$

Γ(x) is the gamma function

i is the imaginary unit

Result:

0.602618… i

0.602618…i

Polar coordinates:

$r=0.602618$ (radius), $\theta=90°$ (angle)

Alternative representations:

- $$0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times0.25\,i}{\pi}=0.25\,i+\frac{1.25\,i\,e^{-\log G(3/2)+\log G(5/2)}}{\pi}$$

- $$0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times0.25\,i}{\pi}=0.25\,i+\frac{1.25\,i\,\frac{1}{2}!}{\pi}$$

-

$$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = 0.25\,i + \frac{1.25\,i\,\Gamma\left(\frac{3}{2}, 0\right)}{\pi}$$

$\log G(z)$ gives the logarithm of the Barnes G-function

$n!$ is the factorial function

$\Gamma(a, x)$ is the incomplete gamma function

Series representations:

$$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = 0.25\,i + \frac{1.25\,i \sum_{k=0}^{\infty} \frac{\left(\frac{3}{2} - z_0\right)^k \Gamma^{(k)}(z_0)}{k!}}{\pi} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

- $$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = \frac{0.25\,i \left(5 + \sum_{k=0}^{\infty} \left(\frac{3}{2} - z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}\right)}{\sum_{k=0}^{\infty} \left(\frac{3}{2} - z_0\right)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2} \pi (-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j! (-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = 0.25\,i + \frac{1.25\,i}{\pi} \int_0^{\infty} e^{-t} \sqrt{t}\, dt$$

- $$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = 0.25\,i + \frac{1.25\,i}{\pi} \int_0^1 \sqrt{\log\left(\frac{1}{t}\right)}\, dt$$

- $$0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right) 5 \times 0.25\,i}{\pi} = 0.25\,i + \frac{1.25 \exp\left(\int_0^1 \frac{\frac{1}{2} - \frac{3x}{2} + x^{3/2}}{(-1+x)\log(x)}\, dx\right) i}{\pi}$$

$\log(x)$ is the natural logarithm

and:

-1/((((((0.25i + ((((5/Pi) * gamma(3/2) * 0.25i))))))))

Input:

$$-\frac{1}{0.25\,i+\frac{5}{\pi}\,\Gamma\left(\frac{3}{2}\right)\times 0.25\,i}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.65942… i

1.65942…i

Polar coordinates:

$r = 1.65942$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}}=-\frac{1}{0.25\,i+\frac{1.25\,i\,\frac{1}{2}!}{\pi}}$$

-

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}}=-\frac{1}{0.25\,i+\frac{1.25\,i\,e^{-\log G(3/2)+\log G(5/2)}}{\pi}}$$

-

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}}=-\frac{1}{0.25\,i+\frac{1.25\,i\,\Gamma\left(\frac{3}{2},0\right)}{\pi}}$$

$n!$ is the factorial function

$\log G(z)$ gives the logarithm of the Barnes G-function

$\Gamma(a, x)$ is the incomplete gamma function

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Series representations:

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}}=-\frac{4\pi}{i\pi+5\,i\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$ for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

-

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = -\frac{4\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k \sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}{i\left(5+\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = -\frac{4\pi}{5\exp\left(\int_0^1 \frac{\frac{1}{2}-\frac{3x}{2}+x^{3/2}}{(-1+x)\log(x)}\,dx\right)i+i\,\pi}$$

-

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = -\frac{1}{0.25\,i+\frac{2.5\,i\,\mathcal{A}}{\oint_L \frac{e^t}{t^{3/2}}\,dt}}$$

$$-\frac{1}{0.25\,i+\frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = -\frac{1}{0.25\,i+\frac{1.25\exp\left(-\frac{3\gamma}{2}+\int_0^1\frac{-1+x^{3/2}-\log\left(x^{3/2}\right)}{(-1+x)\log(x)}\,dx\right)i}{\pi}}$$

((((((((1728)^1/3)))/10^3)))i-1/((((((0.25i + ((((5/Pi) * gamma(3/2) * 0.25i))))))))

Where 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729

Input:

$$\frac{\sqrt[3]{1728}}{10^3}\,i-\frac{1}{0.25\,i+\frac{5}{\pi}\,\Gamma\left(\frac{3}{2}\right)\times 0.25\,i}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$1.67142\ldots\,i$

1.67142…i

Polar coordinates:

$r = 1.67142$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5 \times 0.25\,i}{\pi}} = \frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{1.25\,i\,\frac{1}{2}!}{\pi}}$$

-

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5 \times 0.25\,i}{\pi}} = \frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{1.25\,i\,e^{-\log G(3/2)+\log G(5/2)}}{\pi}}$$

-

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5 \times 0.25\,i}{\pi}} = \frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{1.25\,i\,\Gamma\left(\frac{3}{2},0\right)}{\pi}}$$

$n!$ is the factorial function

$\log G(z)$ gives the logarithm of the Barnes G-function

$\Gamma(a, x)$ is the incomplete gamma function

Series representations:

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5 \times 0.25\,i}{\pi}} = \frac{3\,i}{250} - \frac{4\,\pi}{i\,\pi + 5\,i\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

-

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5 \times 0.25\,i}{\pi}} = \left(0.012\left(5\,i^2 - 333.333\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!} + i^2\sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)\right) \Big/ \left(i\left(5 + \sum_{k=0}^{\infty}\left(\frac{3}{2}-z_0\right)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)\right)$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = \frac{3\,i}{250} - \frac{4\,\pi}{i\,\pi + 5\,i\int_0^\infty e^{-t}\sqrt{t}\,dt}$$

- $$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = \frac{3\,i}{250} - \frac{4\,\pi}{i\,\pi + 5\,i\int_0^1 \sqrt{\log\left(\frac{1}{t}\right)}\,dt}$$

- $$\frac{i\sqrt[3]{1728}}{10^3} - \frac{1}{0.25\,i + \frac{\Gamma\left(\frac{3}{2}\right)5\times 0.25\,i}{\pi}} = \frac{3\,i}{250} - \frac{1}{0.25\,i + \frac{1.25\exp\left(\int_0^1 \frac{-1-\frac{3}{2}(-1+x)+x^{3/2}}{(-1+x)\log(x)}\,dx\right)i}{\pi}}$$

$\log(x)$ is the natural logarithm

For k = 5 and 0.5 mod 5 = 0.5 = χ

$$a = (\chi(1) - \chi(-1))/2;$$

((((0.5(1)-0.5(-1))))/2

Input:

$$\frac{1}{2}(0.5\times 1 - 0.5\times(-1))$$

Result:

0.5

0.5 = a

((((2^-2)*i))) = *L(s, χ)*

$$\xi(s,\chi) = \left(\frac{\pi}{k}\right)^{-(s+a)/2}\Gamma\left(\frac{s+a}{2}\right)L(s,\chi),$$

(Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i)))

Input:

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right)\times\frac{i}{2^2}}{\left(\frac{\pi}{5}\right)^{1.25}}$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

0.405076… i

0.405076…i

Polar coordinates:

$r = 0.405076$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{0.24134\, i}{1.06504 \left(\frac{\pi}{5}\right)^{1.25}}$$

•

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{i}{4\, e^{0.0982718} \left(\frac{\pi}{5}\right)^{1.25}}$$

•

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{i\, 0.25!}{4 \left(\frac{\pi}{5}\right)^{1.25}}$$

$n!$ is the factorial function

Series representations:

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{1.86919\, i \sum_{k=0}^{\infty} \frac{(1.25-z_0)^k\, \Gamma^{(k)}(z_0)}{k!}}{\pi^{1.25}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

•

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{1.86919\, i}{\pi^{0.25} \sum_{k=0}^{\infty} (1.25 - z_0)^k \sum_{j=0}^{k} \frac{(-1)^j\, \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{1.86919\, i}{\pi^{1.25}} \int_0^\infty e^{-t}\, t^{0.25}\, dt$$

- $$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{1.86919\, i}{\pi^{1.25}} \int_0^1 \log^{0.25}\left(\frac{1}{t}\right) dt$$

- $$\frac{\Gamma\left(\frac{2+0.5}{2}\right) i}{\left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{1.86919\, e^{\int_0^1 \frac{0.25-1.25 x+x^{1.25}}{(-1+x)\log(x)} dx}\, i}{\pi^{1.25}}$$

$\log(x)$ is the natural logarithm

and:

4(89+55)/144 (((((Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i))))))

Input:

$$4 \times \frac{89+55}{144}\left(\frac{\Gamma\left(\frac{2+0.5}{2}\right)}{\left(\frac{\pi}{5}\right)^{1.25}} \times \frac{i}{2^2}\right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

1.62030... i

1.62030…i

Polar coordinates:

$r = 1.6203$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{139.012\,i}{1.06504\times 144\left(\frac{\pi}{5}\right)^{1.25}}$$

- $$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{144\,i}{144\,e^{0.0982718}\left(\frac{\pi}{5}\right)^{1.25}}$$

- $$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{144\,i\,0.25!}{144\left(\frac{\pi}{5}\right)^{1.25}}$$

$n!$ is the factorial function

Series representations:

$$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{7.47674\,i\sum_{k=0}^{\infty}\frac{(1.25-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\pi^{1.25}}\quad\text{for }(z_0\notin\mathbb{Z}\text{ or }z_0>0)$$

- $$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=$$
$$\frac{7.47674\,i}{\pi^{0.25}\sum_{k=0}^{\infty}(1.25-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{7.47674\,i}{\pi^{1.25}}\int_0^{\infty}e^{-t}\,t^{0.25}\,dt$$

- $$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{7.47674\,i}{\pi^{1.25}}\int_0^{1}\log^{0.25}\left(\frac{1}{t}\right)dt$$

- $$\frac{\left(4\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(89+55)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{7.47674\,e^{\int_0^1\frac{0.25-1.25x+x^{1.25}}{(-1+x)\log(x)}dx}\,i}{\pi^{1.25}}$$

or:

Pi(8+34+55+89)*1/144 (((((Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i))))))

Input:

$$\pi(8+34+55+89)\times\frac{1}{144}\left(\frac{\Gamma\left(\frac{2+0.5}{2}\right)}{\left(\frac{\pi}{5}\right)^{1.25}}\times\frac{i}{2^2}\right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$1.64375\ldots\, i$

1.64375…i

Polar coordinates:

$r = 1.64375$ (radius), $\theta = 90°$ (angle)

$1.64375 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternative representations:

$$\frac{(\pi(8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right)i}{144\left(\frac{\pi}{5}\right)^{1.25}2^2} = \frac{44.8892\, i\,\pi}{1.06504\times 144\left(\frac{\pi}{5}\right)^{1.25}}$$

- $$\frac{(\pi(8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right)i}{144\left(\frac{\pi}{5}\right)^{1.25}2^2} = \frac{93\, i\,\pi}{2\times 144\, e^{0.0982718}\left(\frac{\pi}{5}\right)^{1.25}}$$

- $$\frac{(\pi(8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right)i}{144\left(\frac{\pi}{5}\right)^{1.25}2^2} = \frac{93\, i\,\pi\, 0.25!}{2\times 144\left(\frac{\pi}{5}\right)^{1.25}}$$

$n!$ is the factorial function

Series representations:

$$\frac{(\pi (8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right) i}{144 \left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{2.41437\, i \sum_{k=0}^{\infty} \frac{(1.25-z_0)^k\, \Gamma^{(k)}(z_0)}{k!}}{\pi^{0.25}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

- $$\frac{(\pi (8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right) i}{144 \left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{2.41437\, i\, \pi^{0.75}}{\sum_{k=0}^{\infty} (1.25-z_0)^k \sum_{j=0}^{k} \frac{(-1)^j \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2 z_0)\right) \Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{(\pi (8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right) i}{144 \left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{2.41437\, i}{\pi^{0.25}} \int_0^{\infty} e^{-t}\, t^{0.25}\, dt$$

- $$\frac{(\pi (8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right) i}{144 \left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{2.41437\, i}{\pi^{0.25}} \int_0^{1} \log^{0.25}\left(\frac{1}{t}\right) dt$$

- $$\frac{(\pi (8+34+55+89))\,\Gamma\left(\frac{2+0.5}{2}\right) i}{144 \left(\frac{\pi}{5}\right)^{1.25} 2^2} = \frac{2.41437\, e^{\int_0^1 \frac{0.25-1.25 x+x^{1.25}}{(-1+x)\log(x)} dx}\, i}{\pi^{0.25}}$$

$\log(x)$ is the natural logarithm

(55/10^3-2/10^3)i-8/10^3i+Pi(1+5+89*2)/144 (((((Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i))))))

Input:

$$\left(\frac{55}{10^3} - \frac{2}{10^3}\right) i - \frac{8}{10^3}\, i + \pi \left(\frac{1}{144}\,(1+5+89\times 2)\right) \left(\frac{\Gamma\left(\frac{2+0.5}{2}\right)}{\left(\frac{\pi}{5}\right)^{1.25}} \times \frac{i}{2^2}\right)$$

$\Gamma(x)$ is the gamma function

i is the imaginary unit

Result:

$1.6710783237794470773775616753839881060096598137636241168000\ldots i$

1.67107832…i

Polar coordinates:

$r = 1.67108$ (radius), $\theta = 90°$ (angle)

Alternative representations:

$$\left(\frac{55}{10^3} - \frac{2}{10^3}\right)i - \frac{i\,8}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{44.4065\, i\,\pi}{1.06504\times 144\left(\frac{\pi}{5}\right)^{1.25}} + \frac{45\, i}{10^3}$$

- $$\left(\frac{55}{10^3} - \frac{2}{10^3}\right)i - \frac{i\,8}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{46\, i\,\pi}{144\, e^{0.0982718}\left(\frac{\pi}{5}\right)^{1.25}} + \frac{45\, i}{10^3}$$

- $$\left(\frac{55}{10^3} - \frac{2}{10^3}\right)i - \frac{i\,8}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{46\, i\,\pi\, 0.25!}{144\left(\frac{\pi}{5}\right)^{1.25}} + \frac{45\, i}{10^3}$$

$n!$ is the factorial function

Series representations:

$$\left(\frac{55}{10^3} - \frac{2}{10^3}\right)i - \frac{i\,8}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = 0.045\, i + \frac{2.3884\, i \sum_{k=0}^{\infty} \frac{(1.25-z_0)^k\, \Gamma^{(k)}(z_0)}{k!}}{\pi^{0.25}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

- $$\left(\frac{55}{10^3} - \frac{2}{10^3}\right)i - \frac{i\,8}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{2.3884\, i\left(\pi^{0.75} + 0.018841 \sum_{k=0}^{\infty}(1.25-z_0)^k \sum_{j=0}^{k} \frac{(-1)^j\, \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2 z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}\right)}{\sum_{k=0}^{\infty}(1.25-z_0)^k \sum_{j=0}^{k} \frac{(-1)^j\, \pi^{-j+k} \sin\left(\frac{1}{2}\pi(-j+k+2 z_0)\right)\Gamma^{(j)}(1-z_0)}{j!\,(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\left(\frac{55}{10^3}-\frac{2}{10^3}\right)i-\frac{i\,8}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=0.045\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^\infty e^{-t}\,t^{0.25}\,dt$$

- $$\left(\frac{55}{10^3}-\frac{2}{10^3}\right)i-\frac{i\,8}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}= 0.045\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^1 \log^{0.25}\left(\frac{1}{t}\right)dt$$

- $$\left(\frac{55}{10^3}-\frac{2}{10^3}\right)i-\frac{i\,8}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}= 0.045\,i+\frac{2.3884\,e^{\int_0^1 \frac{0.25-1.25\,x+x^{1.25}}{(-1+x)\log(x)}dx}\,i}{\pi^{0.25}}$$

log(x) is the natural logarithm

-8/10^3i+Pi(1+5+89*2)/144 (((((Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i))))))

Input:

$$-\frac{8}{10^3}\,i+\pi\left(\frac{1}{144}(1+5+89\times 2)\right)\left(\frac{\Gamma\left(\frac{2+0.5}{2}\right)}{\left(\frac{\pi}{5}\right)^{1.25}}\times\frac{i}{2^2}\right)$$

Γ(x) is the gamma function

i is the imaginary unit

Result:

1.618078323779447077377561675383988106009659813763624116800... i

1.61807832…i

Polar coordinates:

$r = 1.61808$ (radius), $\theta = 90°$ (angle)

1.61808 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{44.4065\,i\,\pi}{1.06504\times 144\left(\frac{\pi}{5}\right)^{1.25}}-\frac{8\,i}{10^3}$$

-

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{46\,i\,\pi}{144\,e^{0.0982718}\left(\frac{\pi}{5}\right)^{1.25}}-\frac{8\,i}{10^3}$$

-

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=\frac{46\,i\,\pi\,0.25!}{144\left(\frac{\pi}{5}\right)^{1.25}}-\frac{8\,i}{10^3}$$

$n!$ is the factorial function

Series representations:

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=-0.008\,i+\frac{2.3884\,i\sum_{k=0}^{\infty}\frac{(1.25-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\pi^{0.25}}$$

for $(z_0\notin\mathbb{Z}$ or $z_0>0)$

-

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=$$

$$\frac{2.3884\,i\left(\pi^{0.75}-0.00334952\sum_{k=0}^{\infty}(1.25-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)}{\sum_{k=0}^{\infty}(1.25-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=-0.008\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^{\infty}e^{-t}\,t^{0.25}\,dt$$

-

$$\frac{i(-8)}{10^3}+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=-0.008\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^{1}\log^{0.25}\left(\frac{1}{t}\right)dt$$

-

$$\frac{i(-8)}{10^3} + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = -0.008\, i + \frac{2.3884\, e^{\int_0^1 \frac{0.25-1.25x+x^{1.25}}{(-1+x)\log(x)}dx}\, i}{\pi^{0.25}}$$

log(x) is the natural logarithm

(21/10^3-3/10^3)i+Pi(1+5+89*2)/144 (((((Pi/5)^-1.25 * gamma(((2+0.5)/2))) * ((((2^-2)*i))))))

Input:

$$\left(\frac{21}{10^3} - \frac{3}{10^3}\right)i + \pi\left(\frac{1}{144}(1+5+89\times 2)\right)\left(\frac{\Gamma\left(\frac{2+0.5}{2}\right)}{\left(\frac{\pi}{5}\right)^{1.25}} \times \frac{i}{2^2}\right)$$

Γ(x) is the gamma function

i is the imaginary unit

Result:

1.64408… i

1.64408…i

Polar coordinates:

$r = 1.64408$ (radius), $\theta = 90°$ (angle)

$1.64408 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternative representations:

$$\left(\frac{21}{10^3} - \frac{3}{10^3}\right)i + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{44.4065\, i\,\pi}{1.06504\times 144\left(\frac{\pi}{5}\right)^{1.25}} + \frac{18\, i}{10^3}$$

- $$\left(\frac{21}{10^3} - \frac{3}{10^3}\right)i + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{46\, i\,\pi}{144\, e^{0.0982718}\left(\frac{\pi}{5}\right)^{1.25}} + \frac{18\, i}{10^3}$$

- $$\left(\frac{21}{10^3} - \frac{3}{10^3}\right)i + \frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25} 2^2\right)144} = \frac{46\, i\,\pi\, 0.25!}{144\left(\frac{\pi}{5}\right)^{1.25}} + \frac{18\, i}{10^3}$$

n! is the factorial function

Series representations:

$$\left(\frac{21}{10^3}-\frac{3}{10^3}\right)i+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=0.018\,i+\frac{2.3884\,i\sum_{k=0}^{\infty}\frac{(1.25-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\pi^{0.25}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

•

$$\left(\frac{21}{10^3}-\frac{3}{10^3}\right)i+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=$$
$$\frac{2.3884\,i\left(\pi^{0.75}+0.00753641\sum_{k=0}^{\infty}(1.25-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)}{\sum_{k=0}^{\infty}(1.25-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\,\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}}$$

$\mathbb{Z}$ is the set of integers

Integral representations:

$$\left(\frac{21}{10^3}-\frac{3}{10^3}\right)i+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=0.018\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^{\infty}e^{-t}\,t^{0.25}\,dt$$

•

$$\left(\frac{21}{10^3}-\frac{3}{10^3}\right)i+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=0.018\,i+\frac{2.3884\,i}{\pi^{0.25}}\int_0^{1}\log^{0.25}\left(\frac{1}{t}\right)dt$$

•

$$\left(\frac{21}{10^3}-\frac{3}{10^3}\right)i+\frac{\left(\pi\left(\Gamma\left(\frac{2+0.5}{2}\right)i\right)\right)(1+5+89\times 2)}{\left(\left(\frac{\pi}{5}\right)^{1.25}2^2\right)144}=0.018\,i+\frac{2.3884\,e^{\int_0^1\frac{0.25-1.25x+x^{1.25}}{(-1+x)\log(x)}dx}\,i}{\pi^{0.25}}$$

$\log(x)$ is the natural logarithm

From

$$f(s) = \sum_{n=1}^{\infty} r(n) n^{-s},$$

$$r(2) = \varkappa,$$

$$\varkappa = \frac{\sqrt{10 - 2\sqrt{5}} - 2}{\sqrt{5} - 1},$$

we obtain:

2^-2 * (((((([sqrt(((10-2sqrt(5))))-2] *1/ ((((sqrt(5))-1))))))

Input:

$$\frac{\left(\sqrt{10 - 2\sqrt{5}} - 2\right) \times \frac{1}{\sqrt{5} - 1}}{2^2}$$

Result:

$$\frac{\sqrt{10 - 2\sqrt{5}} - 2}{4(\sqrt{5} - 1)}$$

Decimal approximation:

0.071019760960103074007072958098281542272772022111434395689...

0.07101976…

Alternate forms:

$$\frac{1}{16}\left(\sqrt{10 - 2\sqrt{5}} - 2\sqrt{5} + \sqrt{5\left(10 - 2\sqrt{5}\right)} - 2\right)$$

- $$\frac{1}{8}\left(-1 - \sqrt{5} + \sqrt{2\left(5 + \sqrt{5}\right)}\right)$$
- $$\frac{\sqrt{2(5 - \sqrt{5})} - 2}{4(\sqrt{5} - 1)}$$
-

Minimal polynomial:

$256 x^4 + 128 x^3 - 96 x^2 - 8 x + 1$

and:

sqrt((((((colog((((((2^-2 * ((((([sqrt(((10-2sqrt(5))))-2] *1/ ((((sqrt(5))-1))))))))))))))))))

Input:

$$\sqrt{-\log\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\times\frac{1}{\sqrt{5}-1}}{2^2}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{4\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation:

1.626283221731029060431182422919583254374345669824004146502...

1.62628322…

Property:

$$\sqrt{-\log\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}\right)}$$ is a transcendental number

(34+8+3)/10^3+sqrt((((((colog((((((2^-2 * ((((([sqrt(((10-2sqrt(5))))-2] *1/ ((((sqrt(5))-1))))))))))))))))))

Input:

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\times\frac{1}{\sqrt{5}-1}}{2^2}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{9}{200}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{4\left(\sqrt{5}-1\right)}\right)}$$

Decimal approximation:

1.6712832217310290604311824229195832543743456698240041465502...

1.67128322...

Property:

$\frac{9}{200}+\sqrt{-\log\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)}$ is a transcendental number

Alternate forms:

$$\frac{9}{200}+\sqrt{-\log\left(\frac{1}{8}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)\right)}$$

- $$\frac{9}{200}-i\sqrt{\log\left(\sqrt{10-2\sqrt{5}}-2\right)-\log\left(4\left(\sqrt{5}-1\right)\right)}$$

- $$\frac{1}{200}\left(9+200\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{4\left(\sqrt{5}-1\right)}\right)}\right)$$

Alternative representations:

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2\left(\sqrt{5}-1\right)}\right)}=\frac{45}{10^3}+\sqrt{-\log_e\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}\right)}$$

- $$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2\left(\sqrt{5}-1\right)}\right)}=\frac{45}{10^3}+\sqrt{-\log(a)\log_a\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}\right)}$$

- $$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2\left(\sqrt{5}-1\right)}\right)}=\frac{45}{10^3}+\sqrt{\mathrm{Li}_1\left(1-\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}\right)}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=\frac{9}{200}+\sqrt{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)^k}{k}}$$

•

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=$$
$$\frac{9}{200}+\sqrt{-2\,i\,\pi\left\lfloor\frac{\arg\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-x\right)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-x\right)^k x^{-k}}{k}}$$

for $x < 0$

•

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=$$
$$\frac{9}{200}+\sqrt{-2\,i\,\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-z_0\right)^k z_0^{-k}}{k}}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representation:

$$\frac{34+8+3}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=\frac{9}{200}+\sqrt{-\int_1^{\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}}\frac{1}{t}\,dt}$$

-8/10^3+sqrt((((((colog((((((2^-2 * (((((([sqrt(((10-2sqrt(5))))-2] *1/ ((((sqrt(5))-1))))))))))))))))))

Input:

$$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\left(\sqrt{10-2\sqrt{5}}-2\right)\times\frac{1}{\sqrt{5}-1}}{2^2}\right)}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{4\left(\sqrt{5}-1\right)}\right)}-\frac{1}{125}$$

Decimal approximation:

1.618283221731029060431182422919583254374345669824004146502...

1.61828322…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Property:

$$-\frac{1}{125}+\sqrt{-\log\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}\right)}$$ is a transcendental number

Alternate forms:

$$\sqrt{-\log\left(\frac{1}{8}\left(-1-\sqrt{5}+\sqrt{2\left(5+\sqrt{5}\right)}\right)\right)}-\frac{1}{125}$$

- $$-\frac{1}{125}-i\sqrt{\log\left(\sqrt{10-2\sqrt{5}}-2\right)-\log\left(4\left(\sqrt{5}-1\right)\right)}$$

- $$\frac{1}{125}\left(125\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{4\left(\sqrt{5}-1\right)}\right)}-1\right)$$

Alternative representations:

$$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=-\frac{8}{10^3}+\sqrt{-\log_e\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)}$$

- $$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=-\frac{8}{10^3}+\sqrt{-\log(a)\log_a\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)}$$

- $$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=-\frac{8}{10^3}+\sqrt{\text{Li}_1\left(1-\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=-\frac{1}{125}+\sqrt{\sum_{k=1}^{\infty}\frac{(-1)^k\left(-1+\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}\right)^k}{k}}$$

- $$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=-\frac{1}{125}+\sqrt{-2i\pi\left\lfloor\frac{\arg\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-x\right)}{2\pi}\right\rfloor-\log(x)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-x\right)^k x^{-k}}{k}} \quad \text{for } x<0$$

- $$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2(\sqrt{5}-1)}\right)}=$$
$$-\frac{1}{125}+\sqrt{-2i\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-\log(z_0)+\sum_{k=1}^{\infty}\frac{(-1)^k\left(\frac{-2+\sqrt{10-2\sqrt{5}}}{4(-1+\sqrt{5})}-z_0\right)^k z_0^{-k}}{k}}$$

arg(z) is the complex argument

⌊x⌋ is the floor function

Integral representation:

$$-\frac{8}{10^3}+\sqrt{-\log\left(\frac{\sqrt{10-2\sqrt{5}}-2}{2^2\left(\sqrt{5}-1\right)}\right)} = -\frac{1}{125}+\sqrt{-\int_1^{\frac{-2+\sqrt{10-2\sqrt{5}}}{4\left(-1+\sqrt{5}\right)}}\frac{1}{t}\,dt}$$

Thence:

Z(t) = exp(i*1.7917594692) zeta(1/2+3i)

Input interpretation:

$$\exp(i\times 1.7917594692)\,\zeta\left(\frac{1}{2}+3\,i\right)$$

$\zeta(s)$ is the Riemann zeta function

i is the imaginary unit

Result:

$-0.039781315591\ldots+0.53707584880\ldots\,i$

Polar coordinates:

$r = 0.538547713854$ (radius), $\theta = 94.23617405°$ (angle)

0.538547…

Note that 0.538547*3 = 1.615641, result very near to the value of golden ratio

and:

Result:

$0.25\,i$

0.25i = *f(s)*

We have that:

A. A. Karatsuba, **On zeros of certain Dirichlet series**,
Sovrem. Probl. Mat., 2016, Issue 23, 12–16
DOI: https://doi.org/10.4213/spm57

From:

$$I_1 = \int_t^{t+h} |Z(u)|\, du, \qquad I_2 = \left| \int_t^{t+h} Z(u)\, du \right|.$$

$$F(u) = Z(u)|f(u)|^2,$$

where: $r = 0.53854713854$ (radius), $\theta = 94.23617405°$ (angle)

0.53854713854 * (0.25i)^2

Input interpretation:

$0.53854713854\,(0.25\,i)^2$

i is the imaginary unit

Result:

-0.03365919615875

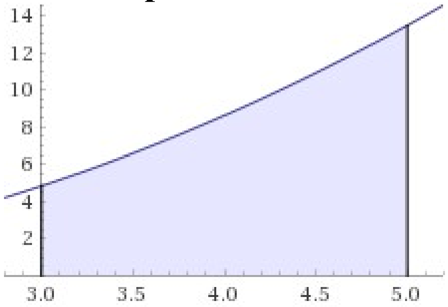
-0.03365919615875

integrate [0.53854713854]x x,[3, 5]

$$\int_3^5 0.53854713854\, x\, x\, dx = 17.592539859$$

17.592539859

Visual representation of the integral:



((((integrate [0.53854713854]x x,[3, 5]))))^1/6

Input interpretation:

$$\sqrt[6]{\int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.61270434959

1.61270434959 result that is a good approximation to the value of the golden ratio 1.618033988749...

Computation result:

$$\sqrt[6]{\int_3^5 0.53854713854\, x\, x\, dx} = 1.6127$$

((((integrate [0.53854713854]x x,[3, 5]))))^1/(24^2/10^2)

Input interpretation:

$$\sqrt[\frac{24^2}{10^2}]{\int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.64514003871

$1.64514003871 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Computation result:

$$\frac{24^2}{10^2}\sqrt{\int_3^5 0.53854713854\, x\, x\, dx} = 1.64514$$

sqrt(((((1/(2Pi) ((((integrate [0.53854713854]x x,[3, 5])))))))))

Input interpretation:

$$\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.67330202895

1.67330202895

Computation result:

$$\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\, x\, x\, dx} = 1.6733$$

-2/10^3+(((((sqrt(((((1/(2Pi) ((((integrate [0.53854713854]x x,[3, 5])))))))))))))

Input interpretation:

$$-\frac{2}{10^3}+\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.67130202895

1.67130202895

Computation result:

$$-\frac{2}{10^3}+\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\, x\, x\, dx} = 1.6713$$

-55/10^3+(((((sqrt(((((1/(2Pi) ((((integrate [0.53854713854]x x,[3, 5])))))))))))))

Input interpretation:

$$-\frac{55}{10^3}+\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.61830202895

1.61830202895 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Computation result:

$$-\frac{55}{10^3}+\sqrt{\frac{1}{2\pi}\int_3^5 0.53854713854\,x\,x\,dx} = 1.6183$$

-1/(((0.53854713854 * (0.25i)^2)))

Input interpretation:

$$-\frac{1}{0.53854713854\,(0.25\,i)^2}$$

i is the imaginary unit

Result:

29.709562738325862426742734429978390132697482329473190036 86...

29.7095627…

((((integate (((0.53854713854 * (0.25i)^2)))x, [3, 13]))))^1/2

Input interpretation:

$$\sqrt{\int_3^{13}\left(0.53854713854\,(0.25\,i)^2\right)x\,dx}$$

i is the imaginary unit

Result:

$1.00476\times10^{-16}+1.64096\,i$

Computation result:

$$\sqrt{\int_3^{13}\left(0.53854713854\,(0.25\,i)^2\right)x\,dx} = 1.00476\times10^{-16}+1.64096\,i$$

Alternate form:

$1.64096\, i$

$1.64096i \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$ (with imaginary unit)

(34/10^3-3/10^3)i+((((integate (((0.53854713854 * (0.25i)^2)))x, [3, 13]))))^1/2

Input interpretation:

$$\left(\frac{34}{10^3} - \frac{3}{10^3}\right) i + \sqrt{\int_3^{13} \left(0.53854713854 \left(0.25\, i\right)^2\right) x \, dx}$$

i is the imaginary unit

Result:

$1.00476 \times 10^{-16} + 1.67196\, i$

Computation result:

$$\left(\frac{34}{10^3} - \frac{3}{10^3}\right) i + \sqrt{\int_3^{13} \left(0.53854713854 \left(0.25\, i\right)^2\right) x \, dx} = 1.00476 \times 10^{-16} + 1.67196\, i$$

Alternate form:

$1.67196\, i$

1.67196i

(-55/10^3-2/10^3+34/10^3)i+((((integate (((0.53854713854 * (0.25i)^2)))x, [3, 13]))))^1/2

Input interpretation:

$$\left(-\frac{55}{10^3} - \frac{2}{10^3} + \frac{34}{10^3}\right) i + \sqrt{\int_3^{13} \left(0.53854713854 \left(0.25\, i\right)^2\right) x \, dx}$$

i is the imaginary unit

Result:

$1.00476 \times 10^{-16} + 1.61796\, i$

Computation result:

$$\left(-\frac{55}{10^3} - \frac{2}{10^3} + \frac{34}{10^3}\right) i + \sqrt{\int_3^{13} \left(0.53854713854 \left(0.25\, i\right)^2\right) x \, dx} = 1.00476 \times 10^{-16} + 1.61796\, i$$

Alternate form:

$1.61796\,i$

1.61796i result that is a very good approximation to the value of the golden ratio 1.618033988749... with imaginary unit

Now, from the sum of I_1 and I_2, we obtain:

2* integrate [0.53854713854]x x,[3, 5]

Definite integral:

$$2\int_3^5 0.53854713854\,x^2\,dx = 35.1851$$

35.1851

and:

((((((2* integrate [0.53854713854]x x,[3, 5]))))))^1/7

Input interpretation:

$$\sqrt[7]{2\int_3^5 0.53854713854\,x\,x\,dx}$$

Result:

1.66306170311

1.66306170311 result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Computation result:

$$\sqrt[7]{2\int_3^5 0.53854713854\,x\,x\,dx} = 1.66306$$

8/10^3+((((((2* integrate [0.53854713854]x x,[3, 5]))))))^1/7

Input interpretation:

$$\frac{8}{10^3} + \sqrt[7]{2\int_3^5 0.53854713854\,x\,x\,dx}$$

Result:

1.67106170311

1.67106170311

Computation result:

$$\frac{8}{10^3} + \sqrt[7]{2 \int_3^5 0.53854713854\, x\, x\, dx} = 1.67106$$

-(21/10^3-3/10^3)+((((((2* integrate [0.53854713854]x x,[3, 5]))))))^1/7

Input interpretation:

$$-\left(\frac{21}{10^3} - \frac{3}{10^3}\right) + \sqrt[7]{2 \times \int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.64506170311

1.64506170311 ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Computation result:

$$-\left(\frac{21}{10^3} - \frac{3}{10^3}\right) + \sqrt[7]{2 \times \int_3^5 0.53854713854\, x\, x\, dx} = 1.64506$$

-(34/10^3+8/10^3+3/10^3)+((((((2* integrate [0.53854713854]x x,[3, 5]))))))^1/7

Input interpretation:

$$-\left(\frac{34}{10^3} + \frac{8}{10^3} + \frac{3}{10^3}\right) + \sqrt[7]{2 \int_3^5 0.53854713854\, x\, x\, dx}$$

Result:

1.61806170311

1.61806170311 result that is a very good approximation to the value of the golden ratio 1.618033988749...

Computation result:

$$-\left(\frac{34}{10^3}+\frac{8}{10^3}+\frac{3}{10^3}\right)+\sqrt[7]{2\int_3^5 0.53854713854\,x\,x\,dx} = 1.61806$$

D-BRANES (Dirichlet boundary conditions)

From:

STRING THEORY VOLUME II - Superstring Theory and Beyond
JOSEPH POLCHINSKI
Institute for Theoretical Physics - University of California at Santa Barbara

We have that:

The scattering amplitudes of closed strings from the D-brane are invariant only under these 16 supersymmetries.

To see the significance of this, consider first the conservation of momentum. There is a nonzero amplitude for a closed string to reflect backwards from the D-brane, which clearly does not conserve momentum in the direction orthogonal to the D-brane. This occurs because the Dirichlet boundary conditions explicitly break translational invariance. However,

from the spacetime point of view the breaking is spontaneous: we are expanding around a D-brane in some definite location, but there are degenerate states with the D-brane translated by any amount.[1] For a spontaneously broken symmetry the consequences are more subtle than for an unbroken symmetry: the apparent violation of the conservation law is related to the amplitude to emit a long-wavelength Goldstone boson. For the D-brane, as for any extended object, the Goldstone bosons are the collective coordinates for its motion. In fact, the nonconservation of momentum is measured by the integral of the corresponding current over the world-sheet boundary,

$$\frac{1}{2\pi\alpha'}\int_{\partial M} ds\, \partial_n X'^9 \,, \tag{13.2.3}$$

which up to normalization is just the (0 picture) vertex operator for the collective coordinate, with zero momentum in the Neumann directions.

We conclude by analogy that the D-brane also spontaneously breaks 16 of the 32 spacetime supersymmetries, the ones that are explicitly broken by the open string boundary conditions. The integrals

$$\int_{\partial M} ds\, \mathscr{V}'_\alpha = -\int_{\partial M} ds\, (\beta^9 \tilde{\mathscr{V}}')_\alpha \,, \tag{13.2.4}$$

which measure the breaking of supersymmetry, are just the vertex operators for the fermionic open string state (13.2.1). Thus this state is a *goldstino*, the Goldstone state associated with spontaneously broken supersymmetry.

We observe the following possible mathematical connection, between the following integrals concerning the Dirichlet series and the integrals of the eq. (13.2.4):

$$I_1 = \int_t^{t+h} |Z(u)|\, du, \qquad I_2 = \left| \int_t^{t+h} Z(u)\, du \right|.$$

$$\int_{\partial M} ds\, \mathcal{V}'_\alpha = - \int_{\partial M} ds\, (\beta^9 \tilde{\mathcal{V}}')_\alpha \, ,$$

From the sum of I_1 and I_2, we obtain:

$$\int_t^{t+h} |Z(u)| du + \int_t^{t+h} |Z(u)| du = 35.1851$$

Thence:

$$\int_t^{t+h} |Z(u)| du = 35.1851 - \int_t^{t+h} |Z(u)| du$$

But:

$$\int_t^{t+h} |Z(u)| du = 17.592539859$$

Thence:

$$\int_t^{t+h} |Z(u)| du = 35.1851 - 17.592539859 = 17.592560141$$

This is a possible solution to the eq. (13.2.4). Dividing by 48, i.e. 32 + 16, that are the spacetime supersymmetries, we obtain:

sqrt(((((((48/((((((-17.592539859+[((((((2* integrate [0.53854713854]x x,[3, 5])))))]))))))))))))))

Input interpretation:

$$\sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}}$$

Result:

1.6517957595

1.6517957595 result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Computation result:

$$\sqrt{\frac{48}{-17.592539859 + 2 \int_3^5 0.53854713854\, x\, x\, dx}} = 1.6518$$

and:

2sqrt(((((((6*sqrt((((((((48/((((((-17.592539859+[((((((2* integrate [0.53854713854]x x,[3, 5])))))])))))))))))))))))))))))

Input interpretation:

$$2\sqrt{6\sqrt{\frac{48}{-17.592539859 + 2 \int_3^5 0.53854713854\, x\, x\, dx}}}$$

Result:

6.2962765368

Computation result:

$$2\sqrt{6\sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}}} = 6.29628$$

6.2962765368 $\approx 2\pi r$, where r :

Input interpretation:

$$2\sqrt{6\sqrt{\frac{48}{-17.592539859+2\times\int_3^5 0.53854713854\,x\,x\,dx}}\times\frac{1}{2\pi}}$$

Result:

1.00208353390

1.00208353390

Possible closed form:

$$\sqrt{\frac{1}{13}(-162+5e+60\pi-13\log(8))}\approx 1.00208353389882$$

We have that:

1/1.00208353390

Input interpretation:

$$\frac{1}{1.00208353390}$$

Result:

0.997920798187461365689645327082047964983530800635182455820...

0.99792079818…

Note that, this result is an excellent approximation to the result of the following wonderful Ramanujan formula, that link π, e and ϕ:

$$\frac{1}{1+\frac{e^{-2\pi}}{1+\frac{e^{-4\pi}}{1+\frac{e^{-6\pi}}{1+\frac{e^{-8\pi}}{1+\frac{e^{-10\pi}}{\ddots}}}}}}=\left(\sqrt{\frac{5+\sqrt{5}}{2}}-\frac{1+\sqrt{5}}{2}\right)e^{2\pi/5}$$

0.9981360044598509332150024459047074735311382994763043982185… =

0.9981360044598509332150024459047074735311382994763043982185… ≈

$\approx 0.99792079818746136568964532708204796498353080063518245582 0...$

In conclusion, we obtain:

$$\int_{\partial M} ds\, \mathcal{V}'_\alpha = -\int_{\partial M} ds\, (\beta^9 \tilde{\mathcal{V}}')_\alpha\,, \qquad \Rightarrow$$

$$\Rightarrow \int_t^{t+h} |Z(u)|du = 35.1851 - \int_t^{t+h} |Z(u)|du$$

$$\int_t^{t+h} |Z(u)|du = 35.1851 - 17.592539859 = 17.592560141$$

From the result, dividing by 48 and computing the square root, we obtain:

$$\sqrt{48/\int_t^{t+h} |Z(u)|du = 48/\left(35.1851 - \int_t^{t+h} |Z(u)|du\right)} = 1.6517957595$$

result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

We also obtain:

21/10^3+sqrt((((((((48/((((((-17.592539859+[((((((2* integrate [0.53854713854]x x,[3, 5]))))))])))))))))))))

Input interpretation:

$$\frac{21}{10^3} + \sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}}$$

Result:

1.6727957595

1.6727957595

Computation result:

$$\frac{21}{10^3} + \sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}} = 1.6728$$

and:

[(((-34-8-5-2))/10^3)]+sqrt(((((((48/(((((-17.592539859+[((((((2* integrate [0.53854713854]x x,[3, 5]))))))])))))))))))))

Input interpretation:

$$\frac{-34-8-5-2}{10^3} + \sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}}$$

Result:

1.6027957595

1.6027957595

Computation result:

$$\frac{-34-8-5-2}{10^3} + \sqrt{\frac{48}{-17.592539859 + 2 \times \int_3^5 0.53854713854\, x\, x\, dx}} = 1.6028$$

From:

A. A. Karatsuba, **Euler and Number Theory**, Sovrem. Probl. Mat., 2008, Issue 11, 19–37
DOI: https://doi.org/10.4213/spm21

N = 21, $p_1 = 3$ $p_2 = 5$ $p_3 = 13$

$\alpha = 1 - 0.00003$ (Хохайзель, 1930 г.) $\alpha = 1 - 0.004$ (Хейльбронн, 1933 г.) $\alpha = 1 - 0.25$ (Чудаков, 1936 г.) $\alpha = 1 - \frac{3}{8}$ (Ингам, 1937 г.) $\alpha = 1 - \frac{2}{5}$ (Монтгомери, 1969 г.) $\alpha = 1 - \frac{5}{12}$ (Хаксли, 1972 г.).

$$p_1 + p_2 + p_3 = N. \qquad (15)$$

$$\sigma(N) > 1,$$

$$I(N) \sim \frac{N^2}{2(\ln N)^3}\,\sigma(N),$$

$$I(N) = \int_0^1 T^3(\alpha)\, e^{-2\pi i \alpha N}\, d\alpha,$$

$$T(\alpha) = \sum_{p \leqslant N} e^{2\pi i \alpha p}.$$

We obtain:

((((21^2*2))) / ((((2(ln21)^3))))

Input:

$$\frac{21^2 \times 2}{2 \log^3(21)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{441}{\log^3(21)}$$

Decimal approximation:

15.62719574714896294408343576998644036034904541127061744027...

15.6271957…

Property:

$\frac{441}{\log^3(21)}$ is a transcendental number

-

Alternate form:

$$\frac{441}{(\log(3) + \log(7))^3}$$

Alternative representations:

$$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{2 \times 21^2}{2 \log_e^3(21)}$$

- $$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{2 \times 21^2}{2 (\log(a) \log_a(21))^3}$$

- $$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{2 \times 21^2}{2 (-\text{Li}_1(-20))^3}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{441}{\left(\log(20) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{20}\right)^k}{k}\right)^3}$$

- $$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{441}{\left(2 i \pi \left\lfloor \frac{\arg(21-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21-x)^k x^{-k}}{k}\right)^3} \quad \text{for } x < 0$$

- $$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{441}{\left(\log(z_0) + \left\lfloor \frac{\arg(21-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (21-z_0)^k z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{21^2 \times 2}{2 \log^3(21)} = \frac{441}{\left(\int_1^{21} \frac{1}{t} \, dt\right)^3}$$

- $$\frac{21^2 \times 2}{2 \log^3(21)} = -\frac{3528\, i\, \pi^3}{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{20^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^3} \text{ for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

and:

1/(3Pi) ((((21^2*2))) / ((((2(ln21)^3))))

Input:

$$\frac{1}{3\pi} \times \frac{21^2 \times 2}{2 \log^3(21)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{147}{\pi \log^3(21)}$$

Decimal approximation:

1.658096966548934673456691819261555085659590816762242191423...

1.65809696…. result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Alternate form:

$$\frac{147}{\pi (\log(3) + \log(7))^3}$$

Alternative representations:

$$\frac{21^2 \times 2}{(2 \log^3(21))(3\pi)} = \frac{2 \times 21^2}{(3\pi)(2 \log_e^3(21))}$$

-

$$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{2 \times 21^2}{(3\pi)\left(2(\log(a)\log_a(21))^3\right)}$$

- $$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{2 \times 21^2}{(3\pi)\left(2(-\text{Li}_1(-20))^3\right)}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{147}{\pi\left(\log(20) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{20}\right)^k}{k}\right)^3}$$

- $$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{147}{\pi\left(2 i \pi \left\lfloor \frac{\arg(21-x)}{2\pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (21-x)^k x^{-k}}{k}\right)^3} \quad \text{for } x < 0$$

- $$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{147}{\pi\left(\log(z_0) + \left\lfloor \frac{\arg(21-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) - \sum_{k=1}^{\infty} \frac{(-1)^k (21-z_0)^k z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x \rfloor$ is the floor function

Integral representations:

$$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{147}{\pi\left(\int_1^{21} \frac{1}{t}\, dt\right)^3}$$

- $$\frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = -\frac{1176\, i \pi^2}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma} \frac{20^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\, ds\right)^3} \quad \text{for } -1 < \gamma < 0$$

$\Gamma(x)$ is the gamma function

13/10^3+1/(3Pi) ((((21^2*2))) / ((((2(ln21)^3))))

Input:

$$\frac{13}{10^3} + \frac{1}{3\pi} \times \frac{21^2 \times 2}{2\log^3(21)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{13}{1000} + \frac{147}{\pi \log^3(21)}$$

Decimal approximation:

1.671096966548934673456691819261555085659590816762242191423...

1.67109696…

Alternate forms:

$$\frac{147000 + 13\pi \log^3(21)}{1000\pi \log^3(21)}$$

- $$\frac{147000 + 13\pi (\log(3) + \log(7))^3}{1000\pi (\log(3) + \log(7))^3}$$

$$\frac{13}{1000} + \frac{147}{\pi (\log(3) + \log(7))^3}$$

Alternative representations:

$$\frac{13}{10^3} + \frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{13}{10^3} + \frac{2 \times 21^2}{(3\pi)(2\log_e^3(21))}$$

- $$\frac{13}{10^3} + \frac{21^2 \times 2}{(2\log^3(21))(3\pi)} = \frac{13}{10^3} + \frac{2 \times 21^2}{(3\pi)(2(\log(a)\log_a(21))^3)}$$

- $$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{10^3}+\frac{2\times 21^2}{(3\pi)\left(2(-\text{Li}_1(-20))^3\right)}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{1000}+\frac{147}{\pi\left(\log(20)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20}\right)^k}{k}\right)^3}$$

- $$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{1000}+\frac{147}{\pi\left(2i\pi\left\lfloor\frac{\arg(21-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-x)^k x^{-k}}{k}\right)^3}\quad\text{for } x<0$$

- $$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{1000}+\frac{147}{\pi\left(\log(z_0)+\left\lfloor\frac{\arg(21-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-z_0)^k z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{1000}+\frac{147}{\pi\left(\int_1^{21}\frac{1}{t}\,dt\right)^3}$$

- $$\frac{13}{10^3}+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=\frac{13}{1000}-\frac{1176\,i\,\pi^2}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{20^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3}\quad\text{for } -1<\gamma<0$$

Γ(x) is the gamma function

-(13/10^3+1/10^3)+ 1/(3Pi) ((((21^2*2))) / ((((2(ln21)^3))))

Input:

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{1}{3\pi}\times\frac{21^2\times 2}{2\log^3(21)}$$

log(x) is the natural logarithm

Exact result:

$$\frac{147}{\pi\log^3(21)}-\frac{7}{500}$$

Decimal approximation:

1.644096966548934673456691819261555085659590816762242191423...

$1.64409696 \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Alternate forms:

$$-\frac{7(\pi\log^3(21)-10500)}{500\pi\log^3(21)}$$

- $$\frac{7(10500-\pi(\log(3)+\log(7))^3)}{500\pi(\log(3)+\log(7))^3}$$

$$\frac{147}{\pi(\log(3)+\log(7))^3}-\frac{7}{500}$$

Alternative representations:

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=-\frac{14}{10^3}+\frac{2\times 21^2}{(3\pi)(2\log_e^3(21))}$$

-

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{14}{10^3}+\frac{2\times 21^2}{(3\pi)\left(2\left(\log(a)\log_a(21)\right)^3\right)}$$

-

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{14}{10^3}+\frac{2\times 21^2}{(3\pi)\left(2\left(-\mathrm{Li}_1(-20)\right)^3\right)}$$

$\log_b(x)$ is the base- b logarithm

$\mathrm{Li}_n(x)$ is the polylogarithm function

Series representations:

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{7}{500}+\frac{147}{\pi\left(\log(20)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20}\right)^k}{k}\right)^3}$$

-

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} =$$
$$-\frac{7}{500}+\frac{147}{\pi\left(2\,i\,\pi\left\lfloor\frac{\arg(21-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-x)^k x^{-k}}{k}\right)^3} \quad \text{for } x<0$$

-

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} =$$
$$-\frac{7}{500}+\frac{147}{\pi\left(\log(z_0)+\left\lfloor\frac{\arg(21-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-z_0)^k z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{7}{500}+\frac{147}{\pi\left(\int_1^{21}\frac{1}{t}\,dt\right)^3}$$

-

$$-\left(\frac{13}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{7}{500}-\frac{1176\,i\,\pi^2}{\left(\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{20^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3} \text{ for } -1<\gamma<0$$

$\Gamma(x)$ is the gamma function

-(34/10^3+5/10^3+1/10^3)+ 1/(3Pi) ((((21^2*2))) / ((((2(ln21)^3))))

Input:

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{1}{3\pi}\times\frac{21^2\times 2}{2\log^3(21)}$$

$\log(x)$ is the natural logarithm

Exact result:

$$\frac{147}{\pi\log^3(21)}-\frac{1}{25}$$

Decimal approximation:

1.618096966548934673456691819261555085659590816762242191423...

1.61809696…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$-\frac{\pi\log^3(21)-3675}{25\pi\log^3(21)}$$

- $$\frac{3675-\pi(\log(3)+\log(7))^3}{25\pi(\log(3)+\log(7))^3}$$

$$\frac{147}{\pi(\log(3)+\log(7))^3}-\frac{1}{25}$$

Alternative representations:

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{40}{10^3}+\frac{2\times 21^2}{(3\pi)(2\log_e^3(21))}$$

-

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{40}{10^3}+\frac{2\times 21^2}{(3\pi)\left(2(\log(a)\log_a(21))^3\right)}$$

- $$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{40}{10^3}+\frac{2\times 21^2}{(3\pi)\left(2(-\text{Li}_1(-20))^3\right)}$$

$\log_b(x)$ is the base- b logarithm

$\text{Li}_n(x)$ is the polylogarithm function

Series representations:

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{1}{25}+\frac{147}{\pi\left(\log(20)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{20}\right)^k}{k}\right)^3}$$

- $$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{1}{25}+\frac{147}{\pi\left(2 i\pi\left\lfloor\frac{\arg(21-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-x)^k x^{-k}}{k}\right)^3} \quad \text{for } x<0$$

- $$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{1}{25}+\frac{147}{\pi\left(\log(z_0)+\left\lfloor\frac{\arg(21-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(21-z_0)^k z_0^{-k}}{k}\right)^3}$$

$\arg(z)$ is the complex argument

$\lfloor x\rfloor$ is the floor function

Integral representations:

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)} = -\frac{1}{25}+\frac{147}{\pi\left(\int_1^{21}\frac{1}{t}\,dt\right)^3}$$

-

$$-\left(\frac{34}{10^3}+\frac{5}{10^3}+\frac{1}{10^3}\right)+\frac{21^2\times 2}{(2\log^3(21))(3\pi)}=-\frac{1}{25}-\frac{1176\,i\,\pi^2}{\left(\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{20^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^3}$$

for $-1<\gamma<0$

$\Gamma(x)$ is the gamma function

As:

$$I(N)\sim\frac{N^2}{2(\ln N)^3}\,\sigma(N),\quad =$$

= 15.62719574714896294408343576998644036034904541127061744027...

$$I(N)=\int_0^1 T^3(\alpha)\,e^{-2\pi i\alpha N}\,d\alpha,$$
$$T(\alpha)=\sum_{p\leqslant N}e^{2\pi i\alpha p}.$$

we have also that:

$$I(N)=\frac{1}{2\pi i}\int_{|z|=R<1}\frac{F(z)}{z^{N+1}}\,dz. \qquad (13)$$

Using special transformations, which later served to create the “circular method”, Hardy and Ramanujan found the asymptotic behaviour of the integral (13) corresponding to f (x) and, thus, found the asymptotic formula for p(n):

$$p(n)=\frac{e^{k\,\lambda_n}}{4\sqrt{3}\,\lambda_n^2}\left(1+O\left(\frac{1}{\lambda_n}\right)\right),$$
$$k=\pi\sqrt{\frac{2}{3}},\quad \lambda_n=\sqrt{n-\frac{1}{24}},\quad n\geqslant 1.$$

With $O=(\pi^2/6)^4$ and $n=1$, we obtain:

e^((((Pi*(2/3)^(1/2)*(1-1/24)^(1/2))))) / ((4sqrt(3) * (1-1/24))) * (1+(Pi^2/(6))^4(1/(1-1/24)^(1/2)))

Input:

$$\frac{e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}\left(1+\left(\frac{\pi^2}{6}\right)^4\times\frac{1}{\sqrt{1-\frac{1}{24}}}\right)$$

Exact result:

$$\frac{2}{23}\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^8}{108\sqrt{138}}\right)$$

Decimal approximation:

15.730904179584451320769566032042711055541630174251886299 45...

15.7309041795… a result very near to the previous 15.6271957...

We note that 15,627195 + 15,730904 = 31,358099 and 31,358099 ÷ 2 = 15,6790495 that is an excellent approximation to the black hole entropy value 15.6730. Furthermore, 15.7309041798... is very near to the black hole entropy value 15.8174. The mean between the two entropies is 15.7452

Alternate forms:

$$\frac{e^{\left(\sqrt{23}\,\pi\right)/6}\left(108\sqrt{138}+\pi^8\right)}{1242\sqrt{46}}$$

$$\frac{e^{\left(\sqrt{23}\,\pi\right)/6}\left(14904+\sqrt{138}\,\pi^8\right)}{57132\sqrt{3}}$$

- $$e^{\left(\sqrt{23}\,\pi\right)/6}\left(\frac{2\sqrt{3}}{23}+\frac{\pi^8}{1242\sqrt{46}}\right)$$

Series representations:

$$\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^4}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=\frac{6\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^8}{108\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}$$

- $$\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^4}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=\frac{6\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^8}{108\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

- $$\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^4}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=\frac{e^{\left(\sqrt{23}\,\pi\right)/6}\left(14904+\sqrt{138}\,\pi^8\right)\sqrt{\pi}}{28566\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Now, we have also that:

-288+3+e^((((Pi*(2/3)^(1/2)*(1-1/24)^(1/2))))) / ((4sqrt(3) * (1-1/24))) * (1+(Pi^2/(6))^14(1/(1-1/24)^(1/2)))

Input:

$$-288+3+\frac{e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}\left(1+\left(\frac{\pi^2}{6}\right)^{14}\times\frac{1}{\sqrt{1-\frac{1}{24}}}\right)$$

Exact result:

$$\frac{2}{23}\sqrt{3}\, e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)-285$$

Decimal approximation:

1729.358638618564572637680114413329991669316638384459714974...

1729.3586386…

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$-285+\frac{2}{23}\sqrt{3}\, e^{\left(\sqrt{23}\,\pi\right)/6}+\frac{e^{\left(\sqrt{23}\,\pi\right)/6}\,\pi^{28}}{75098990592\sqrt{46}}$$

- $$\frac{-984547766661120+300395962368\sqrt{3}\, e^{\left(\sqrt{23}\,\pi\right)/6}+\sqrt{46}\, e^{\left(\sqrt{23}\,\pi\right)/6}\,\pi^{28}}{3454553567232}$$

- $$\frac{\sqrt{3}\, e^{\left(\sqrt{23}\,\pi\right)/6}\left(6530347008\sqrt{138}+\pi^{28}\right)-21403212318720\sqrt{138}}{75098990592\sqrt{138}}$$

Series representations:

$$-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=-285+\frac{6\, e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}$$

-

$$-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=-285+\frac{6\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}$$

-

$$-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}=$$
$$-285+\frac{12\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)\sqrt{\pi}}{23\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}$$

and:

((((((-288+3+e^((((Pi*(2/3)^(1/2)*(1-1/24)^(1/2))))) / ((4sqrt(3) * (1-1/24))) * (1+(Pi^2/(6))^14(1/(1-1/24)^(1/2)))))))))^1/15

Input:

$$\sqrt[15]{-288+3+\frac{e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}\left(1+\left(\frac{\pi^2}{6}\right)^{14}\times\frac{1}{\sqrt{1-\frac{1}{24}}}\right)}$$

Exact result:

$$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)-285}$$

Decimal approximation:

1.643837957823887142232218981647618542436598049609781269445...

1.6438379578.... ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms:

$$\sqrt[15]{e^{(\sqrt{23}\,\pi)/6}\left(\frac{2\sqrt{3}}{23}+\frac{\pi^{28}}{75\,098\,990\,592\sqrt{46}}\right)-285}$$

- $$\frac{\sqrt[15]{\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\,(6\,530\,347\,008\sqrt{138}+\pi^{28})-21\,403\,212\,318\,720\sqrt{138}}}{2^{23/30}\times 3^{9/10}\,\sqrt[10]{23}}$$

$$\frac{\sqrt[15]{-984\,547\,766\,661\,120+300\,395\,962\,368\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}+\sqrt{46}\,e^{(\sqrt{23}\,\pi)/6}\,\pi^{28}}}{2^{4/5}\times 3^{13/15}\times 23^{2/15}}$$

All 15th roots of 2/23 sqrt(3) e^((sqrt(23) π)/6) (1 + π^28/(6530347008 sqrt(138))) - 285:

- Polar form

$$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\sqrt{138}}\right)-285}\;e^{0}\approx 1.6438 \text{ (real, principal root)}$$

- $$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\sqrt{138}}\right)-285}\;e^{(2i\pi)/15}\approx 1.5017+0.6686\,i$$

- $$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\sqrt{138}}\right)-285}\;e^{(4i\pi)/15}\approx 1.0999+1.2216\,i$$

- $$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\sqrt{138}}\right)-285}\;e^{(2i\pi)/5}\approx 0.5080+1.5634\,i$$

- $$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{(\sqrt{23}\,\pi)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\sqrt{138}}\right)-285}\;e^{(8i\pi)/15}\approx -0.1718+1.6348\,i$$

Series representations:

$$\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = \sqrt[15]{-285+\frac{6\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}}$$

- $$\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = \sqrt[15]{-285+\frac{6\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}$$

- $$\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = \sqrt[15]{-285+\frac{12\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)\sqrt{\pi}}{23\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\operatorname{Res}_{z=z_0} f$ is a complex residue

Integral representation:

$$(1+z)^a = \frac{\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma} \frac{\Gamma(s)\,\Gamma(-a-s)}{z^s}\,ds}{(2\,\pi\,i)\,\Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

Re(z) is the real part of z

arg(z) is the complex argument

$|z|$ is the absolute value of z

i is the imaginary unit

(((21+5+2)/10^3))+((((((-288+3+e^((((Pi*(2/3)^(1/2)*(1-1/24)^(1/2))))) / ((4sqrt(3) * (1-1/24))) * (1+(Pi^2/(6))^14(1/(1-1/24)^(1/2))))))))^1/15

Input:

$$\frac{21+5+2}{10^3} + \sqrt[15]{-288+3+\frac{e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}\left(1+\left(\frac{\pi^2}{6}\right)^{14}\times\frac{1}{\sqrt{1-\frac{1}{24}}}\right)}$$

Exact result:

$$\frac{7}{250}+\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6\,530\,347\,008\,\sqrt{138}}\right)-285}$$

Decimal approximation:

1.671837957823887142232218981647618542436598049609781269445...

1.6718379578…

Alternate forms:

$$\frac{7}{250}+\sqrt[15]{e^{\left(\sqrt{23}\,\pi\right)/6}\left(\frac{2\sqrt{3}}{23}+\frac{\pi^{28}}{75\,098\,990\,592\,\sqrt{46}}\right)-285}$$

- $$\frac{7}{250}+\frac{\sqrt[15]{\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(6\,530\,347\,008\,\sqrt{138}+\pi^{28}\right)-21\,403\,212\,318\,720\,\sqrt{138}}}{2^{23/30}\times 3^{9/10}\,\sqrt[10]{23}}$$

$$\frac{1}{17250}\left(483+125\sqrt[5]{2}\ 3^{2/15}\times 23^{13/15}\right.$$
$$\left.\sqrt[15]{-984547766661120+300395962368\sqrt{3}\ e^{\left(\sqrt{23}\ \pi\right)/6}+\sqrt{46}\ e^{\left(\sqrt{23}\ \pi\right)/6}\ \pi^{28}}\right)$$

Series representations:

$$\frac{21+5+2}{10^3}+\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}}=$$
$$\frac{7}{250}+\sqrt[15]{-285+\frac{6\,e^{\left(\sqrt{23}\ \pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\ \sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}}$$

-

$$\frac{21+5+2}{10^3}+\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}}=$$
$$\frac{7}{250}+\sqrt[15]{-285+\frac{6\,e^{\left(\sqrt{23}\ \pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\ \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}}}$$

-

$$\frac{21+5+2}{10^3}+\sqrt[15]{-288+3+\frac{\left(1+\frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right)e^{\pi\sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}}=$$
$$\frac{7}{250}+\sqrt[15]{-285+\frac{12\,e^{\left(\sqrt{23}\ \pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)\sqrt{\pi}}{23\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\ 2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\underset{z=z_0}{\operatorname{Res}} f$ is a complex residue

and, in conclusion:

(((-21-4)/10^3))+((((((-288+3+e^((((Pi*(2/3)^(1/2)*(1-1/24)^(1/2))))) / ((4sqrt(3) * (1-1/24))) * (1+(Pi^2/(6))^14(1/(1-1/24)^(1/2))))))))^1/15

Input:

$$\frac{-21-4}{10^3}+\sqrt[15]{-288+3+\frac{e^{\pi\sqrt{2/3}\sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}\left(1+\left(\frac{\pi^2}{6}\right)^{14}\times\frac{1}{\sqrt{1-\frac{1}{24}}}\right)}$$

Exact result:

$$\sqrt[15]{\frac{2}{23}\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(1+\frac{\pi^{28}}{6530347008\sqrt{138}}\right)-285}-\frac{1}{40}$$

Decimal approximation:

1.618837957823887142232218981647618542436598049609781269445...

1.6188379578…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternate forms:

$$\sqrt[15]{e^{\left(\sqrt{23}\,\pi\right)/6}\left(\frac{2\sqrt{3}}{23}+\frac{\pi^{28}}{75098990592\sqrt{46}}\right)-285}-\frac{1}{40}$$

- $$\frac{\sqrt[15]{\sqrt{3}\,e^{\left(\sqrt{23}\,\pi\right)/6}\left(6530347008\sqrt{138}+\pi^{28}\right)-21403212318720\sqrt{138}}}{2^{23/30}\times 3^{9/10}\sqrt[10]{23}}-\frac{1}{40}$$

$$\frac{1}{2760}\left(20 \sqrt[5]{2}\ 3^{2/15} \times 23^{13/15} \sqrt[15]{-984547766661120 + 300395962368 \sqrt{3}\ e^{\left(\sqrt{23}\ \pi\right)/6} + \sqrt{46}\ e^{\left(\sqrt{23}\ \pi\right)/6}\ \pi^{28}} - 69\right)$$

Series representations:

$$\frac{-21-4}{10^3} + \sqrt[15]{-288 + 3 + \frac{\left(1 + \frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right) e^{\pi \sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = -\frac{1}{40} + \sqrt[15]{-285 + \frac{6\, e^{\left(\sqrt{23}\ \pi\right)/6}\left(1 + \frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\ \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}}$$

$$\frac{-21-4}{10^3} + \sqrt[15]{-288 + 3 + \frac{\left(1 + \frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right) e^{\pi \sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = -\frac{1}{40} + \sqrt[15]{-285 + \frac{6\, e^{\left(\sqrt{23}\ \pi\right)/6}\left(1 + \frac{\pi^{28}}{6530347008\sqrt{138}}\right)}{23\sqrt{2}\ \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}$$

- $$\frac{-21-4}{10^3} + \sqrt[15]{-288 + 3 + \frac{\left(1 + \frac{\left(\frac{\pi^2}{6}\right)^{14}}{\sqrt{1-\frac{1}{24}}}\right) e^{\pi \sqrt{2/3}\ \sqrt{1-1/24}}}{4\sqrt{3}\left(1-\frac{1}{24}\right)}} = -\frac{1}{40} + \sqrt[15]{-285 + \frac{12\, e^{\left(\sqrt{23}\ \pi\right)/6}\left(1 + \frac{\pi^{28}}{6530347008\sqrt{138}}\right)\sqrt{\pi}}{23 \sum_{j=0}^{\infty} \operatorname{Res}_{s=-\frac{1}{2}+j}\ 2^{-s}\ \Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$\binom{n}{m}$ is the binomial coefficient

$n!$ is the factorial function

$(a)_n$ is the Pochhammer symbol (rising factorial)

$\Gamma(x)$ is the gamma function

$\underset{z=z_0}{\text{Res}} f$ is a complex residue

Inserting the value of the entropy 15.7309041795 in the Hawking radiation calculator, we obtain the mass, the radius and the temperature:

Mass = 3.695563e-8

Radius = 5.487362e-35

Temperature = 3.320748e+30

From the Ramanujan-Nardelli mock formula, we obtain:

sqrt[[[[1/((((((((4*1.962364415e+19)/(5*0.0864055^2)))*1/(3.695563e-8)* sqrt[[-((((3.320748e+30 * 4*Pi*(5.487362e-35)^3-(5.487362e-35)^2))))) / ((6.67*10^-11))]]]]]

Input interpretation:

$$\sqrt{\left(1\Big/\left(\frac{4\times 1.962364415\times 10^{19}}{5\times 0.0864055^2}\times\frac{1}{3.695563\times 10^{-8}}\sqrt{-\frac{3.320748\times 10^{30}\times 4\pi\,(5.487362\times 10^{-35})^3-(5.487362\times 10^{-35})^2}{6.67\times 10^{-11}}}\right)\right)}$$

Result:

1.618249292694184034104092724032297083348105028254480058182...

1.618249292….

Observations

We observe that there is a strong connection between the equations concerning the Dirichlet boundary conditions of the D-branes and the equations inherent to the Dirichlet series of the A. A. Karatsuba's paper. Furthermore, we obtain: a) class invariant solutions, b) a circumference of radius 1.00208353390, whose reciprocal provides a value very close to that of the wonderful Ramanujan formula, that link π, *e* and ϕ, c) the values without exponent of the proton mass and of the elementary charge. We note that both the elementary charge and the class invariant type solution and the proton mass belong to the golden ratio range. It is possible to hypothesize, at least from the mathematical point of view, that the D-branes are circles of almost unitary radius, as they are subject to vibrations that slightly alter their shape, and are of the fermionic type

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