Beal's Conjecture is Tenable

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Abstract

In this article, first classify A, B and C according to their odevity, and thereby get rid of two kinds of $A^X+B^Y \neq C^Z$. Then, exemplify $A^X+B^Y=C^Z$ under the given requirements. After that, divide $A^X+B^Y \neq C^Z$ into 4 inequalities under the known requirements, and that apply the mathematical induction, the odd-even relations on the symmetry or the method that takes apart integers to prove each inequality. Finally, reach the conclusion that Beal's conjecture is tenable via the comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y \neq C^Z$ under the given requirements.

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Contents

1. Introduction

The Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

The conjecture was discovered by Andrew Beal in 1993. Later, the conjecture was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. Yet it is still both unproved and un-negated a conjecture hitherto.

Let us regard limits of values of A, B, C, X, Y and Z in the indefinite equation $A^X+B^Y=C^Z$ as given requirements for indefinite equations and inequalities concerned after this.

In addition to this, in order to avoid misunderstanding, we might as well stipulate that in this article all numbers concern merely positive integers, and that the exponent of any integer is directed to the greatest common divisor of exponents of distinct prime factors of the integer.

2. Choices for combinations of values of A, B and C

First, classify A, B and C according to their respective odevity, and thereby exclude following two kinds of $A^X+B^Y\neq C^Z$:

1) A, B and C, all are odd numbers.

2) A, B and C are two even numbers and an odd number.

After that, merely continue to have following two kinds which contain $A^X+B^Y=C^Z$ under the given requirements:

1) A, B and C, all are even numbers.

2) A, B and C are two odd numbers and an even number.

3. Exemplify $A^X+B^Y=C^Z$ under the given requirements

For the indefinite equation $A^X+B^Y=C^Z$ which satisfies aforesaid either qualification, in fact, it has many sets of the solution with A, B and C as integers, and illustrate with examples as follows respectively.

When A, B and C all are even numbers, let A=B=C=2, X=Y≥3, and Z=X+1, then $A^X+B^Y=C^Z$ are changed into $2^X+2^X=2^{X+1}$.So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 2, 2 and 2, and that A, B and C have the common prime factor 2.

In addition, let A=B=162, C=54, X=Y=3 and Z=4, then $A^X+B^Y=C^Z$ are changed into $162^3+162^3=54^4$. So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 162, 162 and 54, and that A, B and C have common prime factors 2 and 3.

When A, B and C are two odd numbers and an even number, let A=C=3, B=6, X=Y=3 and Z=5, then $A^X+B^Y=C^Z$ are changed into $3^3+6^3=3^5$. So $A^X+B^Y=C^Z$ at here have a set of the solution with A, B and C as integers 3, 6 and 3, and that A, B and C have the common prime factor 3.

In addition, let A=B=7, C=98, X=6, Y=7 and Z=3, then $A^X+B^Y=C^Z$ are changed into $7^6+7^7=98^3$. So $A^X+B^Y=C^Z$ at here has a set of the solution with A, B and C as integers 7, 7 and 98, and that A, B and C have the common prime factor 7.

Thus it can be seen, the indefinite equation $A^X+B^Y=C^Z$ under the given requirements plus aforementioned either qualification is able to hold water, but A, B and C must have at least a common prime factor.

4. Divide $A^X+B^Y\neq C^Z$ into 4 inequalities under the known requirements

As mentioned above, if can prove $A^X+B^Y\neq C^Z$ under the given requirements plus the qualification that A, B and C have not a common prime factor, then the conjecture is tenable doubtlessly.

Since A, B and C have the common prime factor 2 where A, B and C all are even numbers, then these circumstances that A, B and C have not a common prime factor can only occur in which case A, B and C are two odd numbers and an even number.

If A, B and C have not a common prime factor, then any two of them have not a common prime factor either, because in case any two have a common prime factor, yet another has not, then it will lead up to $A^X+B^Y\neq$

C^Z according to the unique factorization theorem of natural number. Undoubtedly, following two inequalities add together, be able to replace fully $A^X+B^Y \neq C^Z$ under the given requirements plus the qualification that A, B and C without common prime factor are two odd numbers and an even number.

1).$A^X+B^Y \neq (2W)^Z$, i.e. $A^X+B^Y \neq 2^Z W^Z$;

2).$A^X+(2W)^Y \neq C^Z$, i.e. $A^X+2^Y W^Y \neq C^Z$.

In above 2 inequalities, A, B and C are odd numbers; X, Y and $Z \geq 3$; $W \geq 1$; and that three terms in each inequality have not a common prime factor.

Continue to divide $A^X+B^Y \neq 2^Z W^Z$ into following two inequalities:

(1) $A^X+B^Y \neq 2^Z$;

(2) $A^X+B^Y \neq 2^Z O^Z$.

Continue to divide $A^X+2^Y W^Y \neq C^Z$ into following two inequalities:

(3) $A^X+2^Y \neq C^Z$;

(4) $A^X+2^Y O^Y \neq C^Z$.

In above 4 inequalities, A, B, C and O are odd numbers; X, Y and $Z \geq 3$; and that three terms in each inequality have not a common prime factor.

Regard above these qualifications as known requirements for inequalities or indefinite equations concerned after this.

Therefore, the proof of $A^X+B^Y\neq C^Z$ under the given requirements can be transformed to prove above 4 inequalities under the known requirements.

5. Mainstays that prove preceding inequalities

Before the proof begins, be necessary to expound basic conceptions concerned, so as to regard them as mainstays that prove these inequalities.

What first expounded is that at positive half line of the number axis, regard any even point as a symmetric center, then odd points on the left side of the symmetric center and odd points concerned on the right side are one-to-one bilateral symmetries.

Like that, in the sequence of natural numbers, regard any even number as a symmetric center, then odd numbers which are smaller than the even number and part odd numbers which are greater than the even number are one-to-one bilateral symmetries too, [2].

Regard any one of $2^{H-1}W^V$ as a symmetric center, then two distances from the symmetric center to each other's symmetric odd numbers are two equilong line segments at the number axis or two same differences in the sequence of natural numbers, where $W\geq1$, $H\geq3$ and $V\geq1$.

We thereby deduce several conclusions from the above-mentioned interrelation relation *inter* 3 integers in the sequence of natural numbers:

Conclusion 1˙ The sum of bilateral symmetric two odd numbers is equal to the double of even number as the symmetric center, in the sequence of natural numbers.

Conclusion 2˙ The sum of two non-symmetric odd numbers is unequal to the double of the even number as the symmetric center, in the sequence of natural numbers.

Conclusion 3˙ If the sum of two odd numbers is equal to the double of an even number, then the two odd numbers are in the symmetry whereby the even number to act as symmetric center, in the sequence of natural numbers.

Conclusion 4˙ If the sum of two odd numbers is unequal to the double of an even number, then two such odd numbers are not in the symmetry whereby the even number to act as symmetric center, in the sequence of natural numbers.

Moreover, any odd number is able to be expressed into one of O^V, where O is an odd number and $V\geq1$. Yet, when V=1 or 2, write O^V to $O^{1\sim2}$.

Thereinafter, let us set about proving aforesaid 4 inequalities, one by one.

6. Proving $A^X+B^Y\neq2^Z$ under the known requirements

Regard 2^{Z-1} as symmetric center of odd numbers concerned to prove $A^X+B^Y\neq2^Z$ under the known requirements by the mathematical induction.

(1) When Z-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of symmetric centers 2^{Z-1} are listed below successively.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

As listed above, it is observed that there are not two of O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^{Z-1} to act as symmetric center where Z-1=2, 3, 4, 5 and 6.

So there are $A^X+B^Y\neq 2^3$, $A^X+B^Y\neq 2^4$, $A^X+B^Y\neq 2^5$, $A^X+B^Y\neq 2^6$ and $A^X+B^Y\neq 2^7$ under the known requirements, according to have got Conclusion 2.

(2) When Z-1=K with $K\geq 6$, we suppose that there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements.

(3) When Z-1=K+1, it needs us to prove that there are $A^X+B^Y\neq 2^{K+2}$ under the known requirements.

Proof· Suppose that A^X and B^Y are two bilateral symmetric odd numbers whereby 2^K to act as symmetric center, then there are $A^X+B^Y=2^{K+1}$ according to have got Conclusion 1.

While, there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements in line with second step of the mathematical induction. Namely there are not two of O^V with $V\geq 3$ on two places of every pair of bilateral symmetric odd numbers whereby 2^K to act as symmetric center.

So, let us tentatively regard A^X as one of O^V with $V\geq 3$, and regard B^Y as one of $O^{1\sim 2}$, i.e. let $X\geq 3$ and Y=1 or 2.

Taken one with another, the existence of the equality $A^X+B^Y=2^{K+1}$ must possess two requirements integrally, namely on the one hand, A^X and B^Y must be two bilateral symmetric odd numbers whereby 2^K to act as symmetric center; on the other, at least one of Y and X is equal to 1 or 2.

If you change either requirement therein, even though it is a little bit, then it will lead to $A^X+B^Y\neq 2^{K+1}$ inevitably.

Vice versa, there are surely $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2.

Thereupon, there are $A^X+(A^X+2B^Y)=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2. And that A^X and A^X+2B^Y are two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center, according to have got Conclusion 3.

But then, as stated, there are $A^X+B^Y\neq 2^{K+1}$ under the known requirements, thus there are $A^X+(A^X+2B^Y)\neq 2^{K+2}$ under the known requirements. And that A^X and A^X+2B^Y are not two symmetric odd numbers whereby 2^{K+1} to act as symmetric center, according to have got Conclusion 4.

In any case, the sum of A^X+2B^Y is an odd number, so let $A^X+2B^Y=O^E$, where O is yet an odd number, and E is its exponent.

After pass the substitution, on the one hand, there are $A^X+(A^X+2B^Y)= A^X+O^E=2^{K+2}$ under the known requirements except for Y, and Y=1 or 2, and that A^X and O^E are two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center.

On the other hand, there are $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ under the known requirements, and that A^X and O^E are not two bilateral symmetric odd numbers whereby 2^{K+1} to act as symmetric center. In which case, no matter what integer which E equals, including $E\geq3$, all are able to satisfy $A^X+O^E\neq2^{K+2}$.

Although two of O^E derive from A^X+2B^Y, but two limits of values of Y are not alike absolutely, i.e. $Y\geq3$ in $A^X+(A^X+2B^Y)=A^X+O^E\neq2^{K+2}$ and Y=1 or 2 in $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$, therefore A^X+2B^Y within $A^X+(A^X+2B^Y)= A^X+O^E\neq2^{K+2}$ are greater than A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$.

That is to say, O^E within $A^X+O^E\neq2^{K+2}$ be greater than O^E within $A^X+O^E=2^{K+2}$.

When A^X within $A^X+O^E\neq2^{K+2}$ and A^X within $A^X+O^E=2^{K+2}$ are one and the same, additionally O within $A^X+O^E\neq2^{K+2}$ be equal to O within $A^X+O^E=2^{K+2}$, as thus, E within $A^X+O^E\neq2^{K+2}$ be greater than E within $A^X+O^E=2^{K+2}$ surely.

Thus it can be seen, values of E within $A^X+O^E\neq2^{K+2}$ both contain $E\geq3$ and are greater than values of E within $A^X+O^E=2^{K+2}$.

As has been mentioned, there are $A^X+B^Y=2^{K+1}$ under the known requirements except for Y, and Y=1 or 2, likewise deduce $A^X+O^E=2^{K+2}$ under the known requirements except for E, and E=1 or 2, by the same way of doing thing.

Or rather, E within $A^X+O^E=2^{K+2}$ can only be 1 or 2, due to have supposed $X\geq3$. Yet, for E within $A^X+O^E\neq2^{K+2}$ under the known requirements, if A^X and O^E are not in the symmetry whereby 2^{K+1} to act as symmetric center, then it can be any positive integer; if A^X and O^E are in the symmetry whereby 2^{K+1} to act as symmetric center, then there are merely $E\geq3$ in which case $X\geq3$, since when E=1 or 2, there are probably $A^X+O^E=2^{K+2}$.

For $A^X+O^E\neq2^{K+2}$, substitute B for O, since B and O, both can express every odd number; in addition, substitute Y for E where E≥3, then get $A^X+B^Y\neq2^{K+2}$ under the known requirements.

In this proof, if B^Y is one of O^V with V≥3, then A^X is surely one of $O^{1\sim2}$, or suppose A^X and B^Y be two of $O^{1\sim2}$, yet a conclusion concluded finally from this is one and the same with $A^X+B^Y\neq2^{K+2}$ under the known requirements.

So much for, the author has proven that when Z-1=K+1 with K≥6, there are $A^X+B^Y\neq2^{K+2}$ under the known requirements.

By the preceding way of doing thing, can continue to prove that when Z-1=K+2, K+3…up to each and every integer >7, there are $A^X+B^Y\neq2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to $A^X+B^Y\neq2^Z$ under the known requirements.

7. Proving $A^X+B^Y\neq2^ZO^Z$ under the known requirements

Regard $2^{Z-1}O^Z$ as symmetric center of odd numbers concerned to prove $A^X+B^Y\neq2^ZO^Z$ under the known requirements by the mathematical induction successively, and point out O≥3 emphatically.

(1) When O=1, $2^{Z-1}O^Z$ i.e. 2^{Z-1}, as has been proved, there are $A^X+B^Y\neq2^Z$ under the known requirements in №6 section.

(2) When O=J and J≥1, $2^{Z-1}O^Z$ i.e. $2^{Z-1}J^Z$, we suppose that there are $A^X+B^Y\neq2^ZJ^Z$ under the known requirements.

(3) When O=K and K=J+2, $2^{Z-1}O^Z$ i.e. $2^{Z-1}K^Z$, it needs us to prove that there are $A^X+B^Y\neq2^ZK^Z$ under the known requirements.

Proof· Suppose that A^X and B^Y are two bilateral symmetric odd numbers whereby $2^{Z-1}J^Z$ to act as symmetric center, and X≥3, then there are

$A^X+B^Y=2^ZJ^Z$ according to have got Conclusion 1.

And yet, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements in line with second step of the mathematical induction.

Thus, there are surely $A^X+B^Y=2^ZJ^Z$ under the known requirements except for Y, and Y=1 or 2.

Thereupon there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2.

So A^X and $B^Y+2^Z(K^Z-J^Z)$ are two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center according to have got Conclusion 3.

As stated, there are $A^X+B^Y\neq 2^ZJ^Z$ under the known requirements, hereby conclude $A^X+[B^Y+2^Z(K^Z-J^Z)]=(A^X+B^Y)+2^ZK^Z-2^ZJ^Z\neq 2^ZK^Z$ under the known requirements. Accordingly, A^X and $B^Y+2^Z(K^Z-J^Z)$ are not two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center according to have got Conclusion 4.

Such being the case, let the odd number $B^Y+2^Z (K^Z-J^Z)$ be equal to D^E where D is an odd number, and E is its exponent.

After pass the substitution, on the one hand, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$ under the known requirements except for Y, and Y=1 or 2, and that A^X and D^E are two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center.

On the other hand, there are $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$ under the known requirements, and that A^X and D^E are not two bilateral symmetric odd numbers whereby $2^{Z-1}K^Z$ to act as symmetric center. In which case, no matter what integer which E equals, including E≥3, all are able to satisfy $A^X+D^E\neq 2^ZK^Z$.

Although two of D^E derive from $B^Y+2^Z(K^Z-J^Z)$, but two limits of values of Y are not alike absolutely, i.e. Y≥3 in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^HK^Z$ and Y=1 or 2 in $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$, therefore $B^Y+2^Z(K^Z-J^Z)$ within $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E\neq 2^ZK^Z$ are greater than $B^Y+2^Z(K^Z-J^Z)$ within $A^X+[B^Y+2^Z(K^Z-J^Z)]=A^X+D^E=2^ZK^Z$.

Namely D^E within $A^X+D^E\neq 2^ZK^Z$ be greater than D^E within $A^X+D^E=2^ZK^Z$.

When A^X within $A^X+D^E\neq 2^ZK^Z$ and A^X within $A^X+D^E=2^ZK^Z$ are one and the same, additionally D within $A^X+D^E\neq 2^ZK^Z$ be equal to D within $A^X+D^E=2^ZK^Z$, as thus, E within $A^X+D^E\neq 2^ZK^Z$ be greater than E within $A^X+D^E=2^ZK^Z$ surely.

Thus it can be seen, values of E within $A^X+D^E\neq 2^ZK^Z$ both contain E≥3 and are greater than values of E within $A^X+D^E=2^ZK^Z$.

As has been mentioned, there are $A^X+B^Y=2^ZJ^Z$ under the known requirements except for Y, and Y=1 or 2, likewise deduce $A^X+D^E=2^ZK^Z$ under the known requirements except for E, and E=1 or 2, by the same way of doing thing.

Or rather, E within $A^X+D^E=2^ZK^Z$ can only be 1 or 2, due to have supposed X≥3. Yet, for E within $A^X+D^E\neq 2^ZK^Z$ under the known requirements, if A^X and D^E are not in the symmetry whereby $2^{Z-1}K^Z$ to act as symmetric center, then it can be any positive integer; if A^X and O^E are in the symmetry whereby $2^{Z-1}K^Z$ to act as symmetric center, then there are merely $E\geq 3$ in which case $X\geq 3$, since when E=1 or 2, there are probably $A^X+D^E=2^ZK^Z$.

For $A^X+D^E\neq 2^ZK^Z$, substitute B for D, since B and D express every odd number; in addition, substitute Y for E where $E\geq 3$ and $Y\geq 3$, then get $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

In this proof, if B^Y is one of O^V with $V\geq 3$, then A^X is one of $O^{1\sim 2}$ surely, or suppose A^X and B^Y be two of $O^{1\sim 2}$, yet a conclusion concluded finally from this is one and the same with $A^X+B^Y\neq 2^ZK^Z$ under the known requirements.

On balance, the author has proven $A^X+B^Y\neq 2^ZK^Z$ with K=J+2 under the known requirements.

By the preceding way of doing thing, can continue to prove that when O=J+4, J+6…up to each and every odd number >3, there are $A^X+B^Y\neq 2^Z(J+4)^Z$, $A^X+B^Y\neq 2^Z(J+6)^Z$…up to $A^X+B^Y\neq 2^ZO^Z$ under the known requirements.

8. Proving $A^X+2^Y\neq C^Z$ under the known requirements

***Proof*·** If you think to turn A^X+2^Y into one of O^V with $V\geq 3$, a visual way is exactly that divide either item therein into the sum of some same powers, after that, apportion averagely another item in accordance with the amount of the same powers to every power. If there is no fractional part after the apportionment, then it can form one of O^V with $V\geq 3$. If there is fractional part after the apportionment, then it can not form one of O^V with $V\geq 3$.

First, be necessary to confirm a point, namely A^X and 2^Y have not a common prime divisor.

From $A^X= A^{X-K}+ A^{X-K}+ A^{X-K}+\ldots = A^K (A^{X-K})$ where $k\geq 1$, it is observed that the amount of A^{X-K} is A^K, yet A^K is an odd number.

By now, divide 2^Y into A^K parts, i.e. $2^Y/A^K$. It is obvious that $2^Y/A^K$ is not an integer, so it can not turn $A^{X-K} +2^Y/A^K$ into one of O^V with $V\geq 3$.

On the other hand, from $2^Y =2^{Y-K} +2^{Y-K} +2^{Y-K} +\ldots=2^K (2^{Y-K})$ where $k\geq 1$, it is observed that the amount of 2^{Y-K} is 2^K, yet 2^K is an even number.

Like that, divide A^X into 2^K parts, i.e. $A^X/2^K$. It is obvious that $A^X/2^K$ is not an integer either, so it can not turn $2^{Y-K}+A^X/2^K$ into one of O^V with $V\geq 3$ also.

Therefore, there are $A^X+2^Y\neq C^Z$ under the known requirements.

9. Proving $A^X+2^YO^Y\neq C^Z$ under the known requirements

Proof· According to the aforementioned way of doing things, divide either item in $A^X+2^YO^Y$ into the sum of some same powers, after that, apportion averagely another item in accordance with the amount of the same powers to every power. If there is no fractional part after the apportionment, then it can form one of O^V with $V\geq 3$. If there is fractional part after the apportionment, then it can not form one of O^V with $V\geq 3$.

First, be necessary to confirm 2^YO^Y contains the prime divisor 2, yet A^X has not, so the relation of integral multiple exists not *inter se.*

From $A^X= A^{X-K}+ A^{X-K}+ A^{X-K}+\ldots = A^K (A^{X-K})$ where $k\geq 1$, it is observed that the amount of A^{X-K} is A^K, yet A^K is an odd number.

By now, divide 2^YO^Y into A^K parts, i.e. $2^YO^Y/A^K$. It is obvious that $2^YO^Y/A^K$ is not an integer, so it can not turn $A^{X-K}+2^YO^Y/A^K$ into one of O^V

with $V \geq 3$.

On the other hand, from $2^Y O^Y = 2^{Y-K} O^{Y-K} + 2^{Y-K} O^{Y-K} + 2^{Y-K} O^{Y-K} + \ldots = 2^K O^K (2^{Y-K} O^{Y-K})$ where $k \geq 1$, it is observed that the amount of $2^{Y-K} O^{Y-K}$ is $2^K O^K$, yet $2^K O^K$ is an even number.

Like that, divide A^X into $2^K O^K$ parts, i.e. $A^X / 2^K O^K$. It is obvious that $A^X / 2^K O^K$ is not an integer either, so it can not turn $2^{Y-K} O^{Y-K} + A^X / 2^K O^K$ into one of O^V with $V \geq 3$ also.

Therefore, there are $A^X + 2^Y O^Y \neq C^Z$ under the known requirements.

10. Make a summary and reach the conclusion

To sum up, the author has proven every kind of $A^X + B^Y \neq C^Z$ under the given requirements and A, B and C without a common prime factor, in №6, №7, №8 and №9 sections.

In addition, he given examples to have proven $A^X + B^Y = C^Z$ under the given requirements plus which A, B and C have at least a common prime factor in №3 section.

Such being the case, so long as make a comparison between $A^X + B^Y = C^Z$ and $A^X + B^Y \neq C^Z$ under the given requirements, at once inevitably reach such a conclusion that an indispensable prerequisite of existence of $A^X + B^Y = C^Z$ under the given requirements is the very which A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, the Beal's conjecture is tenable.

11. Proving Fermat's last theorem from Beal's conjecture

Fermat's last theorem is a special case of the Beal's conjecture, [3]. If Beal's conjecture is proved to hold water, then let X=Y=Z, so $A^X+B^Y=C^Z$ are changed into $A^X+B^X=C^X$.

Furthermore, divide three terms of $A^X+B^X=C^X$ by greatest common divisor of the three terms, then get a set of solution of positive integers without common prime factor. Obviously, the conclusion is in contradiction with proven Beal's conjecture. As thus, we have proved Fermat's last theorem by reduction to absurdity as easy as pie.

References

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