De Broglie's matter-wave: concept and issues

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Summary

This paper explores the assumptions underpinning de Broglie's concept of a wavepacket and the various conceptual questions and issues. It also explores how the alternative – the ring current model of an electron (or of matter-particles in general) – relates to Louis de Broglie's λ = *h*/*p* relation and rephrases the theory in terms of the wavefunction as well as the wave equation(s) for an electron in free space.

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De Broglie's wavelength and the Compton radius

De Broglie's ideas on the matter-wave are oft-quoted and are usually expressed in de Broglie's $\lambda = h/p$ relation. However, there is remarkably little geometric or physical interpretation of it[1]: what *is* that wavelength, *exactly*? The relation itself is easy enough to *read*: λ goes to infinity as p goes to zero. In contrast, for $p = \mathrm{m}v$ going to $p = \mathrm{m}c$, this length becomes the *Compton* wavelength $\lambda = h/p = h/\mathrm{m}c$. This must mean *something*, obviously, but what *exactly*?

Mainstream theory does not answer this question because the Heisenberg interpretation of quantum mechanics essentially refuses to look into a geometric or physical interpretation of de Broglie's relation and/or the underlying concept of the *matter-wave* or wavefunction which, *lest we forget*, must *somehow* represent the particle itself. In contrast, we will request the reader to think of the (elementary) wavefunction as representing a current ring.

To be precise, we request the reader to think of the (elementary) wavefunction $\boldsymbol{r} = \psi = a \cdot e^{i\theta}$ as representing the *physical* position of a pointlike elementary charge – pointlike but *not dimensionless*[2] – moving at the speed of light around the center of its motion in a space that is defined by the electron's Compton radius $a = \hbar/\mathrm{m}c$. This radius – which effectively doubles up as the *amplitude* of the wavefunction – can easily be derived from (1) Einstein's mass-energy equivalence relation, (2) the Planck-Einstein relation, and (3) the formula for a tangential velocity, as shown below:

$$\left.\begin{array}{r} \left.\begin{array}{r} \mathrm{E} = \mathrm{m}c^2 \\ \mathrm{E} = \hbar\omega \end{array}\right\} \Rightarrow \mathrm{m}c^2 = \hbar\omega \\ c = a\omega \Leftrightarrow a = \dfrac{c}{\omega} \Leftrightarrow \omega = \dfrac{c}{a} \end{array}\right\} \Rightarrow \mathrm{m}a^2\omega^2 = \hbar\omega \Longrightarrow \mathrm{m}\frac{c^2}{\omega^2}\omega^2 = \hbar\frac{c}{a} \Leftrightarrow a = \frac{\hbar}{\mathrm{m}c}$$

This easy derivation[3] already gives a more precise explanation of Prof. Dr. Patrick R. LeClair's interpretation of the Compton wavelength as "the scale above which the particle can be localized in a particle-like sense"[4], but we may usefully further elaborate the details by *visualizing* the model (Figure

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[1] Wikipedia offers an overview of the mainstream view(s) in regard to a physical interpretation of the matter-wave and/or the *de Broglie* wavelength by quoting from the papers by Erwin Schrödinger, Max Born and Werner Heisenberg at the occasion of the 5th Solvay Conference (1927). These views are part of what is rather loosely referred to as the Copenhagen interpretation of quantum mechanics.

[2] The *non*-zero dimension of the elementary charge explains the small anomaly in the magnetic moment which is, therefore, not anomalous at all. For more details, see our paper on the electron model.

[3] It is a derivation one can also use to derive a *theoretical* radius for the proton (or for any *elementary* particle, really). It works perfectly well for the muon, for example. However, for the proton, an additional assumption in regard to the proton's angular momentum and magnetic moment is needed to ensure it fits the experimentally established radius. We shared the derivation with Prof. Dr. Randolf Pohl and the PRad team but we did not receive any substantial comments so far, except for the PRad spokesman (Prof. Dr. Ashot Gasparan) confirming the Standard Model does not have any explanation for the proton radius from first principles and, therefore, encouraging us to continue our theoretical research. In contrast, Prof. Dr. Randolf Pohl suggested the concise calculations come across as numerological only. We hope this paper might help to make him change his mind!

[4] Prof. Dr. Patrick LeClair, *Introduction to Modern Physics*, Course Notes (PH253), 3 February 2019, p. 10.

1) and exploring how it fits de Broglie's intuitions in regard to the matter-wave, which is what we set out to do in this paper.[5]

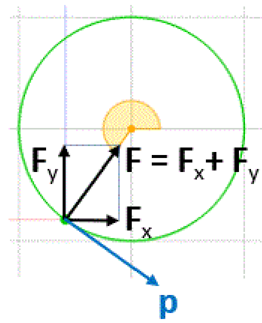

Figure 1: The ring current model of an electron

Of course, the reader will, most likely, not be familiar with the ring current model or – using the term Erwin Schrödinger coined for it – the *Zitterbewegung* model and we should, therefore, probably quote an unlikely authority on it so as to establish some early credentials[6]:

> "The variables [of Dirac's wave equation] give rise to some rather unexpected phenomena concerning the motion of the electron. These have been fully worked out by Schrödinger. It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Indeed, the *dual* radius of the electron (Thomson versus Compton radius) and the *Zitterbewegung* model combine to explain the wave-particle duality of the electron and, therefore, diffraction and/or interference as well as Compton scattering itself. We will not dwell on these aspects of the ring current electron model because we have covered them in (too) lengthy papers before. Indeed, we will want to stay focused on the prime objective of this paper, which is a geometric or *physical* interpretation of the matter-wave.

Before we proceed, we must note the momentum of the pointlike charge – which we denote by p in the illustration – must be distinguished from the momentum of the electron as a whole. The momentum of

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[5] We will analyze de Broglie's views based on his paper for the 1927 Solvay Conference: Louis de Broglie, *La Nouvelle Dynamique des Quanta* (the new quantum dynamics), 5th Solvay Conference, 1927. This paper has the advantage of being concise and complete at the same time. Indeed, its thirty pages were written well after the publication of his thesis on the new *mécanique ondulatoire* (1924) but the presentation helped him to secure the necessary fame which would then lead to him getting the 1929 Nobel Prize for Physics.

[6] For an overview of other eminent views, we refer to our paper on the 1921 and 1927 Solvay Conferences.

the pointlike charge will always be equal to $\boldsymbol{p} = m\boldsymbol{c}$.[7] The rest mass of the pointlike charge must, therefore, be *zero*. However, its velocity give its an *effective* mass which one can calculate to be equal to $m_{eff} = m_e/2$.[8]

Let us now review de Broglie's youthful intuitions.[9]

De Broglie's dissipating wavepacket

The ring current model of an electron incorporates the wavelike nature of an electron: the frequency of the oscillation is the frequency of the circulatory or oscillatory motion (*Zitterbewegung*) of the pointlike electric charge. Hence, the *intuition* of the young Louis de Broglie that an electron must have a frequency was, effectively, a stroke of genius. However, as the magnetic properties of an electron were, by then, not well established and this may explain why Louis de Broglie is either not aware of it or refuses to further build on it.[10]

Let us have a closer look at his paper for the 1927 Solvay Conference, titled *La Nouvelle Dynamique des Quanta*, which we may translate as *The New Quantum Dynamics*. The logic is, by now, well known: we think of the particle as a wave *packet* composed of waves of slightly different frequencies ν_i.[11] This leads to a necessary distinction between the *group* and *phase* velocities of the wave. The group velocity corresponds to the classical velocity *v* of the particle, which is often expressed as a fraction or *relative*

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[7] We consciously use a vector notation to draw attention to the rather particular direction of $\boldsymbol{p}$ and $\boldsymbol{c}$: they must be analyzed as *tangential* vectors in this model.

[8] We may refer to one of our previous papers here (Jean Louis Van Belle, *An Explanation of the Electron and Its Wavefunction*, 26 March 2020). The calculations involve a relativistically correct analysis of an oscillation in *two* independent directions: we effectively interpret circular motion as a two-dimensional oscillation. Such oscillation is, mathematically speaking, self-evident (Euler's function is geometric sum of a sine and a cosine) but its *physical* interpretation is, obviously, *not* self-evident at all!

[9] We must quality the remark on youthfulness. Louis de Broglie was, obviously, quite young when developing his key intuitions. However, he does trace his own ideas on the matter-wave back to the time of writing of his PhD thesis, which is 1923-1924. Hence, he was 32 years old at the time, not nineteen! The reader will also know that, after WW II, Louis de Broglie would distance him from modern interpretations of his own theory and modern quantum physics by developing a *realist* interpretation of quantum physics himself. This interpretation would culminate in the *de Broglie-Bohm* theory of the pilot wave. We do not think there is any need for such alternative theories: we should just go back to where de Broglie went wrong and connect the dots.

[10] The papers and interventions by Ernest Rutherford at the 1921 Conference do, however, highlight the magnetic dipole property of the electron. It should also be noted that Arthur Compton would highlight in his famous paper on Compton scattering, which he published in 1923 and was an active participant in the 1927 Conference itself. Louis de Broglie had extraordinary exposure to all of the new ideas, as his elder brother – Maurice *Duc* de Broglie – had already engaged him scientific secretary for the very first Solvay Conference in 1911, when Louis de Broglie was just 19 years. More historical research may reveal why Louis de Broglie did not connect the dots. As mentioned, he must have been very much aware of the limited but substantial knowledge on the magnetic moment of an electron as highlighted by Ernest Rutherford and others at the occasion of the 1921 Solvay Conference.

[11] We invite the reader to check our *exposé* against de Broglie's original 1927 paper in the Solvay Conference proceedings. We will try to stick closely to the symbols that are used in this paper, such as the *nu* (ν) symbol for the frequency.

velocity $\beta = v/c$.

The assumption is then that we know how the *phase* frequencies ν_i are related to *wavelengths* λ_i. This is modeled by a so-called dispersion relation, which is usually written in terms of the *angular* frequencies $\omega_i = 2\pi\cdot\nu_i$ and the *wave numbers* $k_i = 2\pi/\lambda_i$.[12] The relation between the frequencies ν_i and the wavelengths λ_i (or between angular frequencies ω_i and wavenumbers k_i) is referred to as the *dispersion* relation because it effectively determines if and how the wave packet will disperse or dissipate. Indeed, wave packets have a rather nasty property: they dissipate away. A real-life electron does not.

Prof. H. Pleijel, then Chairman of the Nobel Committee for Physics of the Royal Swedish Academy of Sciences, dutifully notes this rather inconvenient property in the ceremonial speech for the 1933 Nobel Prize, which was awarded to Heisenberg for nothing less than "*the creation of quantum mechanics*"[13]:

> "Matter is formed or represented by a great number of this kind of waves which have somewhat different velocities of propagation and such phase that they combine at the point in question. Such a system of waves forms a crest which propagates itself with quite a different velocity from that of its component waves, this velocity being the so-called group velocity. Such a wave crest represents a material point which is thus either formed by it or connected with it, and is called a wave packet. […] As a result of this theory on is forced to the conclusion to conceive of matter as not being durable, or that it can have definite extension in space. The waves, which form the matter, travel, in fact, with different velocity and must, therefore, sooner or later separate. Matter changes form and extent in space. The picture which has been created, of matter being composed of unchangeable particles, must be modified."

This should sound very familiar to you. However, it is, obviously, not true: real-life particles – electrons or atoms traveling in space – do not dissipate. Matter may change form and extent in space a little bit – such as, for example, when we are forcing them through one or two slits[14] – but not *fundamentally* so![15]

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[12] The concept of an angular frequency (*radians* per time unit) may be more familiar to you than the concept of a wavenumber (*radians* per distance unit). Both are related through the velocity of the wave (which is the velocity of the *component* wave here, so that is the *phase* velocity v_p):

$$v_p = \nu_i \cdot \lambda_i = \frac{\omega_i}{2\pi} \cdot \frac{2\pi}{k_i} = \frac{\omega_i}{k_i}$$

[13] To be precise, Heisenberg got a postponed prize from 1932. Erwin Schrödinger and Paul A.M. Dirac jointly got the 1933 prize. Prof. Pleijel acknowledges all three in more or less equal terms in the introduction of his speech: "This year's Nobel Prizes for Physics are dedicated to the new atomic physics. The prizes, which the Academy of Sciences has at its disposal, have namely been awarded to those men, Heisenberg, Schrödinger, and Dirac, who have created and developed the basic ideas of modern atomic physics."

[14] The wave-particle duality of the ring current model should easily explain single-electron diffraction and interference (the electromagnetic oscillation which keeps the charge swirling would necessarily interfere with itself when being forced through one or two slits), but we have not had the time to engage in detailed research here.

[15] We will slightly *nuance* this statement later but we will not fundamentally alter it. We think of matter-particles as an electric charge in motion. Hence, as it acts on a charge, the nature of the centripetal force that keeps the particle together must be *electromagnetic*. Matter-particles, therefore, combine wave-particle duality. Of course, it makes a difference when this electromagnetic oscillation, and the electric charge, move through a slit or in free space. We will come back to this later. The point to note is: matter-particles do not dissipate. Feynman actually notes that at the very beginning of his *Lectures* on quantum mechanics, when describing the double-slit

We should let this problem rest. We will want to look a related but somewhat different topic: the wave *equation*. However, before we do so, we should discuss one more conceptual issue with de Broglie's concept of a matter-wave *packet*: the problem of (non-)localization.

De Broglie's non-localized wavepacket

The idea of a particle includes the idea of a more or less well-known position. Of course, we may rightfully assume we cannot know this position *exactly* for the following reasons:

1. The *precision* of our measurements may be limited (Heisenberg referred to this as an *Ungenauigkeit*).
2. Our measurement might *disturb* the position and, as such, *cause* the information to get lost and, as a result, *introduce* an uncertainty (*Unbestimmtheit*).
3. One may also think the uncertainty is *inherent* to Nature (*Ungewissheit*).

We think that – despite all *thought* experiments and Bell's No-Go Theorem[16] – the latter assumption remains non-proven. Indeed, we fully second the crucial comment/question/criticism from H.A. Lorentz – after the presentation of the papers by Louis de Broglie, Max Born and Erwin Schrödinger, Werner Heisenberg, and Niels Bohr at the occasion of the 1927 Solvay Conference here: "*Why should we elevate indeterminism to a philosophical principle*?"[17] However, the *root cause* of the uncertainty does not matter. The point is this: the necessity to model a particle as a wave *packet* rather than as a single wave is usually motivated by the need to confine it to a certain region. Let us, once again, quote Richard Feynman here:

> "If an amplitude to find a particle at different places is given by $e^{i(\omega \cdot t - k \cdot x)}$, whose absolute square is a constant, that would mean that the probability of finding a particle is the same at all points. That means we do not know *where* it is—it can be anywhere—there is a great uncertainty in its location. On the other hand, if the position of a particle is more or less well known and we can predict it fairly accurately, then the probability of finding it in different places must be confined to a certain region, whose length we call Δx. Outside this region, the probability is zero. Now this probability is the absolute square of an amplitude, and if the absolute square is zero, the amplitude is also zero, so that we have a wave train whose length is Δx, and the wavelength (the distance between nodes of the waves in the train) of that wave train is what corresponds to the particle momentum."[18]

Indeed, one of the *properties* of the idea of a particle is that it must be *somewhere* at any point in time, and that somewhere must be defined in terms of one-, two- or three-dimensional physical space. Now, we do not quite see how the idea of a wave train or a wave*packet* solves that problem. A *composite*

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experiment for electrons: *"Electrons always arrive in identical lumps."*

[16] A mathematical proof is only as good as its assumptions and we, therefore, think the uncertainty is, somehow, built into the assumptions of John Stewart Bell's (in)famous theorem. There is ample – but, admittedly, non-conclusive – literature on that so we will let the interested reader *google* and study such metaphysics.

[17] See our paper on the 1921 and 1927 Solvay Conferences in this regard. We translated Lorentz' comment from the original French, which reads as follows: "*Faut-il nécessairement ériger l' indéterminisme en principe*?"

[18] See: *Probability wave amplitudes*, in: Feynman's *Lectures on Physics*, Vol. III, Chapter 2, Section 1.

wave with a finite or infinite number of component waves with (phase) frequencies ν_i and wavelengths λ_i is still what it is: an oscillation which repeats itself in space and in time. It is, therefore, all over the place, *unless you want to limit its domain* to some randomly or non-randomly Δx space.

We will let this matter rest too. Let us look at the concept of concepts: the wave equation.

The wavefunction, the wave equation and Heisenberg's uncertainty

With the benefit of hindsight, we now know the 1927 and later Solvay Conferences pretty much settled the battle for ideas in favor of the *new physics*. At the occasion of the 1948 Solvay Conference, it is only Paul Dirac who seriously challenges the approach based on perturbation theory which, at the occasion, is powerfully presented by Robert Oppenheimer. Dirac makes the following comment:

> "All the infinities that are continually bothering us arise when we use a perturbation method, when we try to expand the solution of the wave equation as a power series in the electron charge. Suppose we look at the equations without using a perturbation method, then there is no reason to believe that infinities would occur. The problem, to solve the equations without using perturbation methods, is of course very difficult mathematically, but it can be done in some simple cases. For example, for a single electron by itself one can work out very easily the solutions without using perturbation methods and one gets solutions without infinities. I think it is true also for several electrons, and probably it is true generally : we would not get infinities if we solve the wave equations without using a perturbation method."

However, Dirac is very much aware of the problem we mentioned above: the wavefunctions that come out as solutions dissipate away. Real-life electrons – any real-life matter-particle, really – do not do that. In fact, we refer to them as being *particle-like* because of their integrity—an integrity that is modeled by the Planck-Einstein relation in Louis de Broglie's earliest papers too. Hence, Dirac immediately adds the following, recognizing the problem:

> "If we look at the solutions which we obtain in this way, we meet another difficulty: namely we have the run-away electrons appearing. Most of the terms in our wave functions will correspond to electrons which are running away[19], in the sense we have discussed yesterday and cannot correspond to anything physical. Thus nearly all the terms in the wave functions have to be discarded, according to present ideas. Only a small part of the wave function has a physical meaning."[20]

In our interpretation of matter-particles, this small part of the wavefunction is, of course, the real electron, and it is the ring current or *Zitterbewegung* electron! It is the trivial solution that Schrödinger had found, and which Dirac mentioned very prominently in his 1933 Nobel Prize lecture.[21] The other part of the solution(s) is (are), effectively, bizarre oscillations which Dirac here refers to as 'run-away

[19] This corresponds to wavefunctions dissipating away. The matter-particles they purport to describe obviously do *not*.

[20] See pp. 282-283 of the report of the 1948 Solvay Conference, *Discussion du rapport de Mr. Oppenheimer.*

[21] See the quote from Dirac's 1933 Nobel Prize speech in this paper.

electrons'. With the benefit of hindsight, one wonders why Dirac did not see what we see now.[22]

When discussing wave equations, it is always useful to try to imagine what they might be modeling. Indeed, if we try to imagine what the wavefunction might actually *be*, then we should also associate some (physical) *meaning* with the wave equation: what could it *be*? In physics, a wave *equation* – as opposed to the wavefunctions that are a *solution* to the wave equation (usually a second-order linear differential equation) – are used to model the properties of the *medium* through which the waves are traveling. If we are going to associate a physical meaning with the wavefunction, then we may want to assume the medium here would be the same medium as that through which electromagnetic waves are traveling, so that is the *vacuum*. Needless to say, we already have a set of wave equations here: those that come out of Maxwell's equations! Should we expect contradictions here?

We hope not, of course—but then we cannot be sure. An obvious candidate for a wave equation for matter-waves in free space is Schrödinger equation's *without* the term for the electrostatic potential around a positively charged nucleus[23]:

$$\frac{\partial \psi}{\partial t} = i\frac{\hbar}{2m_{eff}}\nabla^2\psi = i\frac{\hbar}{m}\nabla^2\psi$$

What is m_{eff}? It is the concept of the *effective* mass of an electron which, in our ring current model, corresponds to the relativistic mass of the electric charge as it *zitters* around at lightspeed and so we can effectively substitute $2m_{eff}$ for the mass of the electron $m = m_e = 2m_{eff}$.[24] So far, so good. The question now is: are we talking one wave or many waves? A wave packet or the elementary wavefunction? Let us first make the analysis for one wave only, assuming that we can write ψ as some *elementary* wavefunction $\psi = a \cdot e^{i\theta} = a \cdot e^{i\cdot(k(x-\omega t)}$.

Now, two complex numbers $a + i\cdot b$ and $c + i\cdot d$ are equal if, and only if, their real and imaginary parts are the same, and the $\partial\psi/\partial t = i\cdot(\hbar/m)\cdot\nabla^2\psi$ equation amounts to writing something like this: $a + i\cdot b = i\cdot(c + i\cdot d)$. Remembering that $i^2 = -1$, you can then easily figure out that $i\cdot(c + i\cdot d) = i\cdot c + i^2\cdot d = -d + i\cdot c$. The $\partial\psi/\partial t = i\cdot(\hbar/m)\cdot\nabla^2\psi$ wave equation therefore corresponds to the following *set* of equations[25]:

[22] One of our correspondents wrote us this: "Remember these scientists did not have all that much to work with. Their experiments were imprecise – as measured by today's standards – and tried to guess what is at work. Even my physics professor in 1979 believed Schrödinger's equation yielded the *exact* solution (electron orbitals) for hydrogen." Hence, perhaps we should not be surprised. In light of the caliber of these men, however, we are.

[23] For Schrödinger's equation in free space or the same equation with the Coulomb potential see Chapters 16 and 19 of Feynman's *Lectures* on Quantum Mechanics respectively. Note that we moved the imaginary unit to the right-hand side, as a result of which the usual *minus* sign disappears: $1/i = -i$.

[24] See Dirac's description of Schrödinger's *Zitterbewegung* of the electron for an explanation of the lightspeed motion of the charge. For a derivation of the $m = 2m_{eff}$ formula, we refer the reader to our paper on the ring current model of an electron, where we write the effective mass as $m_{eff} = m_\gamma$. The *gamma* symbol (γ) refers to the photon-like character of the charge as it zips around some center at lightspeed. However, unlike a photon, a charge carries charge. Photons do not.

[25] We invite the reader to double-check our calculations. If needed, we provide some more detail in one of our physics blog posts on the geometry of the wavefunction.

- $Re(\partial\psi/\partial t) = -(\hbar/m)\cdot Im(\nabla^2\psi) \Leftrightarrow \omega\cdot\cos(kx - \omega t) = k^2\cdot(\hbar/m)\cdot\cos(kx - \omega t)$
- $Im(\partial\psi/\partial t) = (\hbar/m)\cdot Re(\nabla^2\psi) \Leftrightarrow \omega\cdot\sin(kx - \omega t) = k^2\cdot(\hbar/m)\cdot\sin(kx - \omega t)$

It is, therefore, easy to see that ω and k must be related through the following dispersion relation[26]:

$$\omega = \frac{\hbar \mathrm{k}^2}{\mathrm{m}} = \frac{\hbar c^2 \mathrm{k}^2}{\mathrm{E}}$$

So far, so good. In fact, we can easily verify this makes sense if we substitute the energy E using the Planck-Einstein relation $E = \hbar\cdot\omega$ and assuming the wave velocity is equal to *c*, which should be the case if we are talking about the same vacuum as the one through which Maxwell's electromagnetic waves are supposed to be traveling[27]:

$$\omega = \frac{\hbar \mathrm{k}^2}{\mathrm{m}} = \frac{\hbar c^2 \mathrm{k}^2}{\mathrm{E}} = \frac{\hbar c^2 \mathrm{k}^2}{\hbar\omega} = \frac{c^2 \mathrm{k}^2}{\omega} \Leftrightarrow \frac{\omega^2}{\mathrm{k}^2} = \frac{(2\pi f)^2}{(2\pi/\lambda)^2} = (f\lambda)^2 = c^2 \Leftrightarrow c = f\lambda$$

We know need to think about the question we started out with: one wave or many component waves? It is fairly obvious that if we think of many component waves, each with their own frequency, then we need to think about different values m_i or E_i for the mass and/or energy of the electron as well! How can we motivate or justify this? The electron mass or energy is known, isn't it? This is where the *uncertainty* comes in: the electron may have some (classical) velocity or momentum for which we may not have a *definite* value. If so, we may assume different values for its (kinetic) energy and/or its (linear) momentum may be possible. We then effectively get various *possible* values for m, E, and p which we may denote as m_i, E_i and p_i, respectively. We can, then, effectively write our dispersion relation and, importantly, *the condition for it to make physical sense* as:

$$\omega_i = \frac{\hbar \mathrm{k}^2}{\mathrm{m}_i} = \frac{\hbar c^2 \mathrm{k}_i^2}{\mathrm{E}_i} = \frac{\hbar c^2 \mathrm{k}_i^2}{\mathrm{E}_i} = \frac{\hbar c^2 \mathrm{k}_i^2}{\hbar\omega_i} = \frac{c^2 \mathrm{k}_i^2}{\omega_i} \Leftrightarrow \frac{\omega_i^2}{\mathrm{k}_i^2} = c^2 \Leftrightarrow c = f_i\lambda_i$$

Of course, the $c = f_i\lambda_i$ makes *a lot of* sense: we would not want the properties of the medium in which matter-particles move to be different from the medium through which electromagnetic waves are travelling: lightspeed should remain lightspeed, and waves – *matter*-waves included – should not be traveling faster.

In the next section, we will show how one relate the uncertainties in the (kinetic) energy and the (linear) momentum of our particle using the relativistically correct energy-momentum relation and also taking into account that linear momentum is a *vector* and, hence, we may have uncertainty in both its *direction* as well as its magnitude. Such explanations also provide for a geometric interpretation of the *de Broglie* wavelength. At this point, however, we should just note the key conclusions from our analysis so far:

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[26] If you *google* this (check out the Wikipedia article on the dispersion relation, for example), you will find this relation is referred to as a *non-relativistic* limit of a supposedly relativistically correct dispersion relation, and the various authors of such accounts will usually also add the 1/2 factor because they conveniently (but *wrongly*) forget to distinguish between the *effective* mass of the *Zitterbewegung* charge and the total energy or mass of the electron as a whole.

[27] We apologize if this sounds slightly ironic but we are actually astonished Louis de Broglie does not mind having to assume superluminal speeds for wave velocities, even if it is for *phase* rather than group velocities.

1. If there is a matter-wave, then it must travel at the speed of light and *not*, as Louis de Broglie suggests, at some superluminal velocity.

2. If the matter-wave is a wave *packet* rather than a single wave with a precisely defined frequency and wavelength, then such wave packet will represent *our limited knowledge* about the momentum and/or the velocity of the electron. The uncertainty is, therefore, not inherent to Nature, but to our limited knowledge about the initial conditions.

We will now look at a moving electron in more detail. Before we do so, we should address a likely and very obvious question of the reader: why did we choose Schrödinger's wave equation as opposed to, say, Dirac's wave equation for an electron in free space? It is *not* a coincidence, of course! The reason is this: Dirac's equation obviously does *not* work! It produces 'run-away electrons' only.[28] The reason is simple: Dirac's equation comes with a nonsensical dispersion relation. Schrödinger's original equation does *not*, which is why it works so well for *bound* electrons too![29] We refer the reader to the Annex to this paper for a more detailed discussion on this.

The wavefunction and (special) relativity

Let us consider the idea of a particle traveling in the positive x-direction at constant speed v. This idea implies a pointlike concept of position: we think the particle will be *somewhere* at some point in time. The *somewhere* in this expression does not mean that we think the particle itself is dimensionless or pointlike: we think it is *not*. It just implies that we can associate the ring current with some *center* of the oscillation. The oscillation itself has a *physical* radius, which we referred to as the Compton radius of the electron and which illustrates the *quantization* of space that results from the Planck-Einstein relation.

Two extreme situations may be envisaged: $v = 0$ or $v = c$. However, let us consider the more general case inbetween. In our reference frame[30], we will have a position – a mathematical *point* in space, that is[31] – which is a function of time: $x(t) = v{\cdot}t$. Let us now denote the position and time in the reference frame of the particle itself by x' and t'. Of course, the position of the particle in its own reference frame will be equal to $x'(t') = 0$ for all t', and the position and time in the two reference frames will be related by Lorentz's equations[32]:

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[28] We offer a non-technical historical discussion in our paper on the metaphysics of modern physics.

[29] It is a huge improvement over the Rutherford-Bohr model as it explains the finer structure of the hydrogen spectrum. However, Schrödinger's model of an atom is incomplete as well because it does *not* the hyperfine splitting, the Zeeman splitting (anomalous or not) in a magnetic field, or the (in)famous Lamb shift. These are to be explained not only in terms of the magnetic moment of the electron but also in terms of the magnetic moment of the nucleus and its constituents (protons and neutrons)—or of the *coupling* between those magnetic moments. The coupling between magnetic moments is, in fact, the only *complete and correct* solution to the problem, and it cannot be captured in a wave equation: one needs a more sophisticated analysis in terms of (a more refined version of) Pauli matrices to do that.

[30] We conveniently choose our x-axis so it coincides with the direction of travel. This does not have any impact on the generality of the argument.

[31] We may, of course, also think of it as a position *vector* by relating this point to the chosen origin of the reference frame: a point can, effectively, only be defined in terms of other points.

[32] These are the Lorentz equations in their simplest form. We may refer the reader to any textbook here but, as usual, we like Feynman's lecture on it (chapters 15, 16 and 17 of the first volume of Feynman's *Lectures on*

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence, if we denote the energy and the momentum of the electron in our reference frame as E_v and p = $\gamma m_0 v$, then the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows[33]:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

We have just shown that **the argument of the wavefunction is relativistically invariant**: E_0 is, obviously, the rest energy and, because p' = 0 in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself.

Note that, in the process, we also demonstrated the relativistic invariance of the Planck-Einstein relation! This is why we feel that the argument of the wavefunction (and the wavefunction itself) is more *real* – in a *physical* sense – than the various wave equations (Schrödinger, Dirac, or Klein-Gordon) for which it is some solution. Let us further explore this by trying to think of the physical meaning of the *de Broglie* wavelength λ = *h*/*p*. How should we think of it? What does it *represent*?

We have been interpreting the wavefunction as an implicit function: for each *x*, we have a *t*, and vice versa. There is, in other words, no uncertainty here: we think of our particle as being *somewhere* at any point in time, and the relation between the two is given by *x*(*t*) = *v*·*t*. We will get some linear motion. If we look at the $\psi = a\cdot\cos(p\cdot x/\hbar - E\cdot t/\hbar) + i\cdot a\cdot\sin(p\cdot x/\hbar - E\cdot t/\hbar)$ once more, we can write $p\cdot x/\hbar$ as Δ and think of it as a phase factor. We will, of course, be interested to know for what *x* this phase factor $\Delta = p\cdot x/\hbar$ will be equal to 2π. Hence, we write:

$$\Delta = p\cdot x/\hbar = 2\pi \Leftrightarrow x = 2\pi\cdot\hbar/p = h/p = \lambda$$

What *is* it this λ? If we think of our *Zitterbewegung* traveling in a space, we may think of an image as the one below, and it is tempting to think the *de Broglie* wavelength must be the distance between the crests (or the troughs) of the wave.[34]

Physics).

[33] One can use either the general E = mc^2 or – if we would want to make it look somewhat fancier – the p*c* = E*v*/*c* relation. The reader can verify they amount to the same.

[34] We have an oscillation in two dimensions here. Hence, we cannot really talk about crests or troughs, but the reader will get the general idea. We should also note that we should probably *not* think of the plane of oscillation as being *perpendicular* to the plane of motion: we think it is moving about in space itself as a result of past interactions or events (think of photons scattering of it, for example).

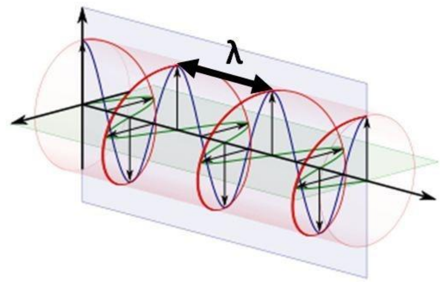


Figure 2: An interpretation of the *de Broglie* wavelength?

However, that would be too easy: note that for $p = mv = 0$ (or $v \to 0$), we have a division by zero and we, therefore, get an infinite value for $\lambda = h/p$. We can also easily see that for $v \to c$, we get a λ that is equal to the Compton wavelength h/mc. How should we interpret that? We may get some idea by playing some more with the relativistically correct equation for the argument of the wavefunction. Let us, for example, re-write the argument of the wavefunction as a function of *time* only:

$$\theta = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(t - \frac{v}{c^2}vt\right) = \frac{1}{\hbar}\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}\left(1-\frac{v^2}{c^2}\right)t = \sqrt{1-\frac{v^2}{c^2}}\cdot\frac{\mathrm{E}_0}{\hbar}t$$

We recognize the *inverse* Lorentz factor here, which goes from 1 to 0 as v goes from 0 to c, as shown below.

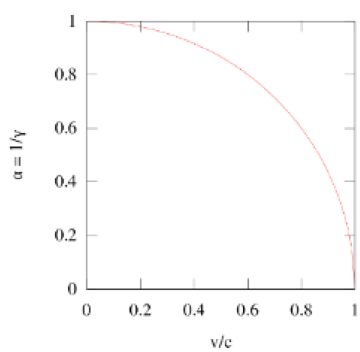


Figure 3: The inverse Lorentz factor as a function of (relative) velocity (v/c)

Note the shape of the function: it is a simple circular arc. This result should not surprise us, of course, as we also get it from the Lorentz formula:

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{t - \frac{v^2}{c^2}t}{\sqrt{1-\frac{v^2}{c^2}}} = \sqrt{1-\frac{v^2}{c^2}}\cdot t$$

This formula gives us the relation between the coordinate time and proper time which – by taking the derivative of one to the other – we can write in terms of the Lorentz factor:

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}} = \frac{\mathrm{dt}}{\mathrm{d\tau}}$$

We introduced a different symbol here: the time in *our* reference frame (t) is the *coordinate time*, and the time in the reference frame of the object itself (τ) is referred to as the *proper time*. Of course, τ is just *t'*, so why are we doing this? What does it all mean? We need to do these gymnastic because we want to introduce a not-so-intuitive but very important result: the Compton radius becomes a wavelength when *v* goes to *c*.[35]

We will be *very* explicit here and go through a simple numerical example to think through that formula above. Let us assume that, for example, that we are able to speed up an electron to, say, about one tenth of the speed of light. Hence, the Lorentz factor will then be equal to $\gamma = 1.005$. This means we added 0.5% (about 2,500 eV) – to the rest energy E_0: $E_v = \gamma E_0 \approx 1.005 \cdot 0.511$ MeV ≈ 0.5135 MeV. The relativistic momentum will then be equal to $m_v v = (0.5135\ \mathrm{eV}/c^2)\cdot(0.1\cdot c) = 5.135\ \mathrm{eV}/c$. We get:

$$\theta = \frac{\mathrm{E}_0}{\hbar}t' = \frac{1}{\hbar}(\mathrm{E}_v t - \mathrm{p}x) = \frac{1}{\hbar}\left(\frac{\mathrm{E}_0}{\sqrt{1-\frac{v^2}{c^2}}}t - \frac{\mathrm{E}_0 v}{c^2\sqrt{1-\frac{v^2}{c^2}}}x\right) = 0.955\frac{\mathrm{E}_0}{\hbar}t$$

This is interesting because we can see these equations are not all that abstract: we effectively get an explanation for relativistic time dilation out of them. An equally interesting question is this: what happens to the *radius* of the oscillation for larger (classical) velocity of our particle? Does it change? It must. In the moving reference frame, we measure higher mass and, therefore, higher energy – as it includes the kinetic energy. The $c^2 = a^2\cdot\omega^2$ identity must now be written as $c^2 = a'^2\cdot\omega'^2$. Instead of the rest mass m_0 and rest energy E_0, we must now use $m_v = \gamma m_0$ and $E_v = \gamma E_0$ in the formulas for the Compton radius and the Einstein-Planck frequency[36], which we just write as m and E in the formula below:

$$\mathrm{m}a'^2\omega'^2 = \mathrm{m}\frac{\hbar^2}{\mathrm{m}^2c^2}\frac{\mathrm{m}^2c^4}{\hbar^2} = \mathrm{m}c^2$$

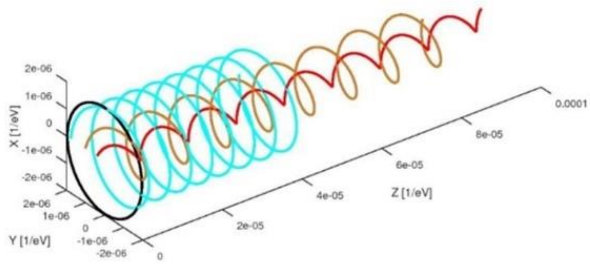
This is easy to understand intuitively: we have the mass factor in the denominator of the formula for the Compton radius, so it must increase as the mass of our particle increases with speed. Conversely, the mass factor is present in the numerator of the *zbw* frequency, and this frequency must, therefore, increase with velocity. It is interesting to note that we have a simple (inverse) proportionality relation here. The idea is visualized in the illustration below[37]: the *radius* of the circulatory motion must effectively diminish as the electron gains speed.[38]

[35] To be precise, the Compton radius multiplied by 2π becomes a wavelength, so we are talking the Compton circumference , or whatever you want to call it.

[36] Again, the reader should note that both the formula for the Compton radius or wavelength as well as the Planck-Einstein relation are relativistically invariant.

[37] We thank Prof. Dr. Giorgio Vassallo and his publisher to let us re-use this diagram. It originally appeared in an article by Francesco Celani, Giorgio Vassallo and Antonino Di Tommaso (*Maxwell's equations and Occam's Razor*, November 2017).

[38] Once again, however, we should warn the reader that he or she should imagine the plane of oscillation to rotate or oscillate itself. He should not think of it of being *static* – unless we think of the electron moving in a magnetic



Zitterbewegung trajectories for different electron speeds: v/c = 0, 0.43, 0.86, 0.98

Figure 4: The Compton radius must decrease with increasing velocity

Can the velocity go to *c*? In the limit, yes. This is very interesting because we can see that the circumference of the oscillation effectively turns into a *linear* wavelength in the process![39] This rather remarkable geometric property related our *zbw* electron model with our photon model, which we will not talk about here, however.[40] Let us quickly make some summary remarks here, before we proceed to what we wanted to present here: a geometric interpretation of the *de Broglie* wavelength:

1. The center of the *Zitterbewegung* was plain nothingness and we must, therefore, assume some two-dimensional oscillation makes the charge go round and round. This is, in fact, the biggest mystery of the model and we will, therefore, come back to it later. As for now, the reader should just note that the angular frequency of the *Zitterbewegung* rotation is given by the Planck-Einstein relation ($\omega = E/\hbar$) and that we get the *Zitterbewegung* radius (which is just the Compton radius $a = r_C = \hbar/mc$) by equating the $E = m \cdot c^2$ and $E = m \cdot a^2 \cdot \omega^2$ equations. The energy and, therefore, the (equivalent) mass is in the oscillation and we, therefore, should associate the momentum $p = E/c$ with the electron as a whole or, if we would really like to associate it with a single mathematical *point* in space, with the center of the oscillation – as opposed to the rotating massless *charge*.

2. We should note that the distinction between the pointlike *charge* and the electron is subtle but essential. The electron is the *Zitterbewegung* as a whole: the pointlike charge has no rest mass, but the electron as a whole does. In fact, that is the whole point of the whole exercise: we explain the rest mass of an electron by introducing a *rest matter oscillation*.

3. As Dirac duly notes, the model cannot be verified *directly* because of the extreme frequency ($f_e = \omega_e/2\pi = E/h \approx 0.123 \times 10^{-21}$ Hz) and the sub-atomic scale ($a = r_C = \hbar/mc \approx 386 \times 10^{-15}$ m). However, it can be verified *indirectly* by phenomena such as Compton scattering, the interference of an electron with itself as it goes through a one- or double-slit experiment, and other *indirect* evidence. In addition, it is *logically* consistent as it generates the right values for the angular momentum ($L = \hbar/2$), the magnetic moment ($\mu = (q_e/2m) \cdot \hbar$, and other intrinsic properties of the electron.[41]

field, in which case we should probably think of the plane of oscillation as being *parallel* to the direction of propagation. We will let the reader think through the geometric implications of this.

[39] We may, therefore, think of the Compton wavelength as a *circular* wavelength: it is the length of a circumference rather than a linear feature!

[40] We may refer the reader to [our paper on *Relativity, Light and Photons*](#).

[41] The two results that we gave also show we get the gyromagnetic factor (g = 2). We have also demonstrated that

We are now ready to finally give you a geometric interpretation of the *de Broglie* wavelength.

The geometric interpretation of the *de Broglie* wavelength

We should refer the reader to Figure 4 to ensure an understanding of what happens when we think of an electron in motion. If the tangential velocity remains equal to *c*, and the pointlike charge has to cover some horizontal distance as well, then the circumference of its rotational motion *must* decrease so it can cover the extra distance. Our formula for the *zbw* or Compton radius was this:

$$a = \frac{\hbar}{\mathrm{m}c} = \frac{\lambda_C}{2\pi}$$

The λ_C is the *Compton* wavelength. We may think of it as a *circular* rather than a *linear* length: it is the circumference of the circular motion.[42] How can it decrease? If the electron moves, it will have some kinetic energy, which we must add to the *rest energy*. Hence, the mass m in the denominator (m*c*) increases and, because $\hbar$ and *c* are physical constants, *a* must decrease.[43] How does that work with the frequency? The frequency is proportional to the energy ($E = \hbar\cdot\omega = h\cdot f = h/T$) so the frequency – in whatever way you want to measure it – must also *increase*. The *cycle* time T must, therefore, *decrease*. We write:

$$\theta = \omega t = \frac{\mathrm{E}}{\hbar}t = \frac{\gamma \mathrm{E}_0}{\hbar}t = 2\pi \cdot \frac{\mathrm{t}}{\mathrm{T}}$$

Hence, our Archimedes' screw gets stretched, so to speak. Let us think about what happens here. We get the following formula for the λ wavelength in Figure 2:

$$\lambda = v \cdot \mathrm{T} = \frac{v}{f} = v \cdot \frac{\mathrm{h}}{\mathrm{E}} = v \cdot \frac{\mathrm{h}}{\mathrm{m}c^2} = \frac{v}{c} \cdot \frac{\mathrm{h}}{\mathrm{m}c} = \beta \cdot \lambda_C$$

It is now easy to see that, if we let the velocity go to *c*, the circumference of the oscillation will effectively become a *linear* wavelength![44] We can now relate this classical velocity (*v*) to the equally classical *linear* momentum of our particle and provide a geometric interpretation of the *de Broglie* wavelength, which we'll denote by using a separate subscript: $\lambda_p = h/p$. It is, obviously, *different* from the λ wavelength in Figure 2. In fact, we have *three* different wavelengths now: the *Compton* wavelength λ_C (which is a circumference, actually), that weird horizontal distance λ, and the *de Broglie* wavelength λ_p. It is easy to make sense of them by relating all three. Let us first re-write the *de Broglie*

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we can easily explain the anomaly in the magnetic moment of the electron by assuming a non-zero physical dimension for the pointlike charge (see our paper on *The Electron and Its Wavefunction*).

[42] Hence, the *C* subscript stands for the *C* of Compton, not for the speed of light (*c*).

[43] We advise the reader to always think about proportional and inversely proportional relations ($y = kx$ versus $y = x/k$) throughout the *exposé* because these relations are *not* always intuitive. The *inverse* proportionality relation between the Compton radius and the mass of a particle is a case in point in this regard: a more *massive* particle has a *smaller* size!

[44] This is why we think of the Compton wavelength as a *circular* wavelength. However, note that the idea of *rotation* does not disappear: it is what gives the electron angular momentum—regardless of its (linear) velocity! As mentioned above, these rather remarkable geometric properties relate our *zbw* electron model with our photon model, which we have detailed in another paper.

wavelength in terms of the Compton wavelength ($\lambda_C = h/mc$), its (relative) velocity $\beta = v/c$, and the Lorentz factor γ:

$$\lambda_p = \frac{h}{p} = \frac{h}{mv} = \frac{hc}{mcv} = \frac{h}{mc\beta} = \frac{\lambda_C}{\beta} = \frac{1}{\gamma\beta}\frac{h}{m_0 c} = \frac{1}{\gamma\beta}2\pi a_0$$

It is a curious function, but it helps us to see what happens to the *de Broglie* wavelength as m and *v* both increase as our electron picks up some momentum p = m·*v*. Its wavelength must actually *decrease* as its (linear) momentum goes from zero to some much larger value – possibly infinity as *v* goes to *c* – and the 1/γβ factor tells us *how* exactly. To help the reader, we inserted a simple graph (below) that shows how the 1/γβ factor comes down from infinity (+∞) to zero as *v* goes from 0 to *c* or – what amounts to the same – if the relative velocity β = *v*/*c* goes from 0 to 1. The 1/γ factor – so that is the inverse Lorentz factor) – is just a simple circular arc, while the 1/β function is just a regular inverse function (*y* = 1/*x*) over the domain β = *v*/*c*, which goes from 0 to 1 as *v* goes from 0 to *c*. Their product gives us the green curve which – as mentioned – comes down from +∞ to 0.

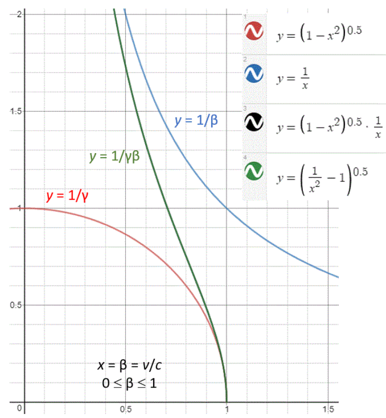

Figure 5: The 1/γ, 1/β and 1/γβ graphs

This analysis yields the following:

1. The *de Broglie* wavelength will be equal to $\lambda_C = h/mc$ for $v = c$:

$$\lambda_p = \frac{h}{p} = \frac{h}{mc}\cdot\frac{1}{\beta} = \lambda_C = \frac{h}{mc} \Leftrightarrow \beta = 1 \Leftrightarrow v = c$$

2. We can now relate both Compton as well as *de Broglie* wavelengths to our new wavelength $\lambda = \beta\cdot\lambda_C$ wavelength—which is that length between the crests or troughs of the wave.[45] We get the following two rather remarkable results:

$$\lambda_p \cdot \lambda = \lambda_p \cdot \beta \cdot \lambda_C = \frac{1}{\beta}\cdot\frac{h}{mc}\cdot\beta\cdot\frac{h}{mc} = \lambda_C^2$$

[45] We should emphasize, once again, that our two-dimensional wave has no real crests or troughs: λ is just the distance between two points whose argument is the same—except for a phase factor equal to *n*·2π (*n* = 1, 2,…).

$$\frac{\lambda}{\lambda_p} = \frac{\beta \cdot \lambda_C}{\lambda} = \frac{p}{h} \cdot \frac{v}{c} \cdot \frac{h}{mc} = \frac{mv^2}{mc^2} = \beta^2$$

The product of the $\lambda = \beta \cdot \lambda_C$ wavelength and *de Broglie* wavelength is the square of the Compton wavelength, and their ratio is the square of the relative velocity $\beta = v/c$. – *always!* – and their ratio is equal to 1 – *always!*

This is all very interesting but *not good enough* yet: the formulas do *not* give us the easy *geometric* interpretation of the *de Broglie* wavelength that we are looking for. We get such easy geometric interpretation only when using natural units.[46] If we re-define our distance, time and force units by equating *c* and *h* to 1, then the Compton wavelength (remember: it is a circumference, really) and the mass of our electron will have a simple inversely proportional relation:

$$\lambda_C = \frac{1}{\gamma m_0} = \frac{1}{m}$$

We get equally simple formulas for the *de Broglie* wavelength and our λ wavelength:

$$\lambda_p = \frac{1}{\beta \gamma m_0} = \frac{1}{\beta m}$$

$$\lambda = \beta \cdot \lambda_C = \frac{\beta}{\gamma m_0} = \frac{\beta}{m}$$

This is quite deep: we have three *lengths* here – defining all of the geometry of the model – and they all depend on the *rest* mass of our object and its relative velocity *only*. They are related through that equation we found above:

$$\lambda_p \cdot \lambda = \lambda_C^2 = \frac{1}{m^2}$$

This is nothing but the *latus rectum* formula for an ellipse, which is illustrated below.[47] The length of the chord – perpendicular to the major axis of an ellipse is referred to as the *latus rectum*. One half of that length is the actual *radius of curvature* of the osculating circles at the endpoints of the major axis.[48] We then have the usual distances along the major and minor axis (*a* and *b*). Now, one can show that the

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[46] Equating *c* to 1 gives us natural distance and time units, and equating *h* to 1 then also gives us a natural force unit—and, because of Newton's law, a natural mass unit as well. Why? Because Newton's ***F*** = m·***a*** equation is relativistically correct: a force is that what gives some mass acceleration. Conversely, mass can be defined of the inertia to a change of its state of motion—because any change in motion involves a force and some acceleration. We, therefore, prefer to write m as the proportionality factor between the force and the acceleration: m = **F**/***a***. This explains why time, distance and force units are closely related.

[47] Source: Wikimedia Commons (By Ag2gaeh - Own work, CC BY-SA 4.0, https://commons.wikimedia.org/w/index.php?curid=57428275).

[48] The endpoints are also known as the *vertices* of the ellipse. As for the concept of an osculating circle, that is the circle which, among all tangent circles at the given point, which approaches the curve most tightly. It was named *circulus osculans* – which is Latin for 'kissing circle' – by Gottfried Wilhelm Leibniz. Apart from being a polymath and a philosopher, he was also a great mathematician. In fact, he may be credited for inventing differential and integral calculus.

following formula has to be true:

$$a \cdot p = b^2$$

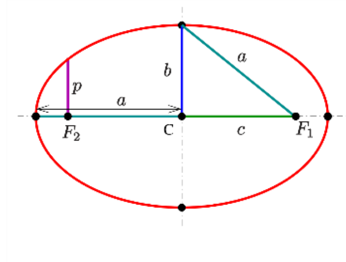

Figure 6: The latus rectum formula

The reader can now easily verify that our three wavelengths obey the same *latus rectum* formula, which we think of as a rather remarkable result. We must now proceed and offer some final remarks on a far more difficult question.

The real mystery of quantum mechanics

We think we have sufficiently demonstrated the theoretical attractiveness of the historical ring current model. This is why we shared it as widely as we could. We usually get positive comments. However, when we first submitted our thoughts to Prof. Dr. Alexander Burinskii, who is leading the research on possible Dirac-Kerr-Newman geometries of electrons[49], he wrote us this:

> "I know many people who considered the electron as a toroidal photon[50] and do it up to now. I also started from this model about 1969 and published an article in JETP in 1974 on it: "Microgeons with spin". [However] There was [also] this key problem: *what keeps [the pointlike charge] in its circular orbit*?"

This question still puzzles us, and we do not have a definite answer to it. As far as we are concerned, it is, in fact, the only *remaining* mystery in quantum physics. What can we say about it? Let us elaborate Burinskii's point:

1. The centripetal force must, obviously, be electromagnetic because it only has a pointlike charge to grab onto, and comparisons with superconducting persistent currents are routinely made. However, such comparisons do not answer this pertinent question: in free space, there is nothing to effectively hold the pointlike charge in place and it must, therefore, spin away!

[49] We will let the reader *google* the relevant literature on electron models based on Dirac-Kerr-Newman geometries here. The mentioned email exchange between Dr. Burinskii and the author of this paper goes back to 22 December 2018.

[50] This was Dr. Burinskii's terminology at the time. It does refer to the *Zitterbewegung* electron: a pointlike charge with no mass in an oscillatory motion—orbiting at the speed of light around some center. Dr. Burinskii later wrote saying he does not like to refer to the pointlike charge as a toroidal *photon* because a photon does not carry any charge. The pointlike charge inside of an electron does: that is why matter-particles are matter-particles and photons are photons. Matter-particles carry charge (we think of neutrons as carrying equal positive and negative charge).

2. In addition, the analogy with superconducting persistent currents also does not give any *unique* Compton radius: the formulas work for any mass. It works, for example, for a muon-electron, and for a proton. The question then becomes: what makes an electron an electron—and what makes a muon a muon? Or a proton a proton?

For the time being, we must simply accept an electron is what it is. In other words, we must take *both* the elementary charge and the mass of the electron (and its more massive variant(s), as well as the mass of the proton) as given by *Nature*. In the longer run, however, we should probably abandon an explanation of the ring current model in terms of Maxwell's equations in favor for what it is, effectively, the more mysterious explanation of a two-dimensional oscillation. However, we have not advanced very far in our thinking on these issues[51] and we, therefore, welcome any suggestion that the reader of this paper might have in this regard.

Jean Louis Van Belle, 9 May 2020

[51] For some speculative thoughts, we may refer the reader to previous references, such as our electron paper or the Annex to our more general paper on classical quantum physics. We calculate the rather enormous force inside of muon and proton in these papers and conclude they may justify the concept of a strong(er) force.

Annex: The wave equations for matter-particles in free space

We will not spend any time on Dirac's wave equation for a very simple reason: it does not work. We quoted Dirac himself on that so we will not even bother to present and explain it. Nor will we present wave equations who further build on it: trying to fix something that did not work in the first place suggests poor problem-solving tactics. We are amateur physicists only[52] and, hence, we are then left with two basic choices: Schrödinger's wave equation in free space[53] and the Klein-Gordon equation. Before going into detail, let us quickly jot them down and offer some brief introductory comments:

1. Schrödinger's wave equation in free space is the one we used in our paper and which mainstream physicists, unfortunately and *wrongly so* (in our not-so-humble view, at least), consider to be *not* relativistically correct:

$$\frac{\partial \psi}{\partial t} = i\frac{\hbar}{2m_{eff}}\nabla^2\psi$$

The reader should note that the concept of the *effective* mass in this equation (m_{eff}) of an electron emerges from an analysis of the motion of an electron through a crystal lattice (or, to be *very* precise, its motion in a linear array or a *line* of atoms). We will look at this argument in a moment. You should just note here that Richard Feynman and all academics who produced textbooks based on his rather shamelessly substitute the efficient mass (m_{eff}) for m_e rather than by $m_e/2$. They do so by noting, *without any explanation at all*[54], that the effective mass of an electron becomes the free-space mass of an electron outside of the lattice.

We think this is totally unwarranted too. The ring current model explains the ½ factor by distinguishing between (1) the effective mass of the pointlike *charge* inside of the electron (while its *rest* mass is zero, it acquires a relativistic mass equal to half of the total mass of the electron) and (2) the *total* (rest) mass of the electron, which consists of *two* parts: the (kinetic) energy of the pointlike charge and the (potential) energy in the field that sustains that motion.[55] Schrödinger's wave equation for a charged particle in free space, which he wrote down in 1926, which Feynman describes – with the hyperbole we all love – as "the great historical moment marking the birth of the quantum mechanical description of matter occurred when Schrödinger first wrote down his equation in 1926", therefore reduces to this:

$$\frac{\partial \psi}{\partial t} = i\frac{\hbar}{m}\nabla^2\psi$$

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[52] The author's we (*pluralis modestiae*) sounds somewhat weird but, fortunately, we did talk this through with some other amateur physicists, and they did not raise any serious objections to *my* thoughts here.

[53] Schrödinger's equation in free space is just Schrödinger's equation without the V(*r*) term: this term captures the electrostatic potential from the positively charged nucleus. If we drop it, logic tells us that we should, effectively, get an equation for a *non*-bound electron: an equation for its motion in free space (the vacuum).

[54] See: Richard Feynman's move from equation 16.12 to 16.13 in his *Lecture* on the dependence of amplitudes on position.

[55] One can show this in various ways (see our paper on the ring current model for an electron) but, for our purposes here, we may simply remind the reader of the energy equipartition theorem: one may think of half of the energy being kinetic while the other half is potential. The kinetic energy is in the *motion* of the pointlike charge, while its potential energy is *field* energy.

As mentioned above, we think this is the right wave equation because it produces a sensible dispersion relation: one that does *not* lead to the dissipation of the particles that it is supposed to describe. The Nobel Prize committee should have given Schrödinger *all* of the 1933 Nobel Prize, rather than splitting it half-half between him and Paul Dirac.

However, for some reason, physicists did *not* think of the *Zitterbewegung* of a charge or some ring current model and, therefore, dumped Schrödinger equation for something fancier.

2. The Gordon-Klein equation, which Feynman, somewhat hastily, already writes down as part of a discussion on *classical* dispersion equations for his sophomore students simply because he 'cannot resist' writing down this 'grand equation', which 'corresponds to the dispersion equation for *quantum-mechanical* waves'[56]:

$$\frac{1}{c^2}\frac{\partial^2\psi}{\partial \mathrm{t}^2} - \frac{\mathrm{m}^2 c^2}{h^2}\psi = \nabla^2\psi$$

In fact, because his students are – at that point – not yet familiar with differential calculus for *vector* fields (and, therefore, not with the *Laplacian* operator ∇^2), Feynman just writes it like this[57]:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = \frac{\mathrm{m}^2 c^2}{\hbar^2}\phi$$

For some reason we do not quite understand, Feynman does not replace the $\mathrm{m}^2c^2/\hbar^2$ by the inverse of the squared Compton radius $a = \hbar/\mathrm{m}c$: why did he *not* connect the dots here?[58] It is what it is. We are, in any case, very fortunate that Feynman does go through the trouble of developing *both* Schrödinger's as well as the more popular Gordon-Klein wave equation for the propagation of quantum-mechanical probability amplitudes.[59] Let us look at them in more detail now.

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[56] See: Richard Feynman, *Waves in three dimensions*, *Lectures*, Vol. I, Chapter 48.

[57] We are not ashamed to admit Feynman's early introduction of this equation in this three volumes of lectures on physics which, as he clearly states in his preface, were written "to maintain the interest [in physics] of the very enthusiastic and rather smart students coming out of the high schools" did not miss their effect on us: I wrote this equation on a piece of paper on the backside of my toilet of my student room when getting my first degree (in economics) and vowed that, one day, I would understand it "in the way I would like to understand it."

[58] One of the fellow amateur physicists who stimulates our research remarks that we may simply be the first to think of deriving the Compton *radius* of an electron from the more familiar concept of the Compton *wavelength*. When *googling* for the Compton radius of an electron (https://www.google.com/search?q=Compton+radius+of+an+electron), we effectively note our blog posts on it (https://readingfeynman.org/tag/compton-radius/) pop up rather prominently.

[59] This is one of the reasons why we still prefer this 1963 textbook over modern textbooks. Another reason is its usefulness as a *common reference* when discussing physics with other amateur physicists. Finally, when going through the course on quantum mechanics that my son had to go through last year as part of getting his degree as a civil engineer, I must admit I hate the level of abstraction in modern-day textbooks on physics: my son passed with superb marks on it (he is *much* better in math than I am) but, frankly, he admitted he had absolutely no clue of whatever it was he was studying. As a proud father, I like to think my *common-sense* remarks on Pauli matrices and quantum-mechanical oscillators did help him to get his 19/20 score, even as he vowed he would never ever look at 'that weird stuff' (his words) ever again. It made me think physics – as a field of science – may effectively have some problem attracting the brightest of minds, which is *very* unfortunate.

Schrödinger's wave equation in free space

Feynman's derivation – or whatever it is – of Schrödinger's equation in free space is, without any doubt, outright brilliant but – as he admits himself – it is rather *heuristic*. Indeed, Feynman himself writes the following on his own logic:

> "We do not intend to have you think we have derived the Schrödinger equation but only wish to show you one way of thinking about it. When Schrödinger first wrote it down, he gave a kind of derivation based on some heuristic arguments and some brilliant intuitive guesses. Some of the arguments he used were even false, but that does not matter; the only important thing is that the ultimate equation gives a correct description of nature."[60]

We find this very ironic because we actually think Feynman's derivation is essentially correct except for the last-minute substitution of the *effective* mass of an electron by the mass of an electron *tout court*. Indeed, we think Feynman discards Schrödinger's equation for the wrong reason:

> "In principle, Schrödinger's equation is capable of explaining all atomic phenomena except those involving magnetism and relativity. […] The Schrödinger equation as we have written it does not take into account any magnetic effects. It is possible to take such effects into account in an approximate way by adding some more terms to the equation. However, as we have seen in Volume II, magnetism is essentially a relativistic effect, and so a correct description of the motion of an electron in an arbitrary electromagnetic field can only be discussed in a proper relativistic equation. The correct relativistic equation for the motion of an electron was discovered by Dirac a year after Schrödinger brought forth his equation, and takes on quite a different form. We will not be able to discuss it at all here."[61]

We do not want to shamelessly copy stuff here, so we will refer the reader to Feynman's *heuristic* derivation of Schrödinger's wave equation for the motion of an electron through a line of atoms, which we interpret as the description of the *linear* motion of an electron—in a crystal lattice *and* in free space.[62] As mentioned above, we think the argument he labels as being intuitive or heuristic himself is essentially correct except for the inexplicable substitution of the concept of the *effective* mass of the pointlike elementary *charge* (m_{eff}) by the total (rest) mass of an *electron* (m_e). We really wonder why this brilliant physicist did not bother to distinguish the concept of *charge* with that of a *charged particle*. Indeed, when everything is said and done, the ring current model of a particle had been invented in 1915 and got considerable attention from most of the attendees of the 1921 and 1927 Solvay Conferences.[63] Hence, we just repeat the implied dispersion relation, which we derived in the body of

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[60] *Lectures*, Vol. III, Chapter 16, p. 16-4.

[61] *Lectures*, Vol. III, Chapter 16, p. 16-13. We have re-read Feynman's *Lectures* many times now and, in discussions with fellow amateur physicists, we sometimes joke that Feynman must have had a secret copy of the truth. He clearly doesn't bother to develop Dirac's equation because – having worked with Robert Oppenheimer on the Manhattan project – he knew Dirac's equation only produces non-sensical 'run-away electrons'. In contrast, while noting Schrödinger's equation is non-relativistic, it is the only one he bothers to explore extensively. Indeed, while claiming the Klein-Gordon equation is the 'right one', he hardly devotes any space to it.

[62] See: Richard Feynman, 1963, *Amplitudes on a line*.

[63] For a (brief) account of these conferences – which effectively changed the course of mankind's intellectual history and future – see our paper on a (brief) history of quantum-mechanical ideas.

our paper using the simple definition for equality of complex-valued numbers:

$$\omega = \frac{\hbar \mathrm{k}^2}{\mathrm{m}} = \frac{\hbar c^2 \mathrm{k}^2}{\mathrm{E}}$$

The Klein-Gordon equation

In contrast, the Klein-Gordon wave equation is based on a very different dispersion relation:

$$\frac{\hbar^2 \omega^2}{c^2} - \hbar^2 \mathrm{k}^2 = \mathrm{m}c^2$$

We know this sounds extremely arrogant but this dispersion relation results from a rather naïve substation of the relativistic energy-momentum relationship:

$$\mathrm{E}^2 - \mathrm{p}^2 c^2 = \mathrm{m}^2 c^4 \Leftrightarrow \hbar^2 \omega^2 - \frac{h^2}{\lambda^2} c^2 = \hbar^2 \omega^2 - \frac{h^2}{(2\pi / \mathrm{k})^2} c^2 = \hbar^2 \omega^2 - \hbar^2 \mathrm{k}^2 c^2 = \mathrm{m}^2 c^4$$

We impolitely refer to his substitution as rather naïve because it fails to distinguish between the (angular) momentum of the pointlike charge inside of the electron and the (linear) momentum of the electron as a whole. We are tempted to be very explicit now – read: copy great stuff – but we will, once again, defer to the Master of Masters for further detail.[64] The gist of the matter is this:

1. There is no need for the Uncertainty Principle in the ring current model of an electron.[65]
2. There is no need to assume we must represent a particle travels through space as a wave *packet*: modeling charged particles as a simple two-dimensional oscillation in space does the trick.

It is hard to believe geniuses like de Broglie, Rutherford, Compton, Einstein, Schrödinger, Bohr, Jordan, De Donder, Brillouin, Born, Heisenberg, Oppenheimer, Feynman, Schwinger,… – we will stop our listing here – failed to see this. We are totally fine with the reader – amateur or academic – switching off here: this is utter madness. Regardless, we do invite him or her to think about it. When everything is said and done, truth is always personal: it arises when our mind has an explanation for our experience.[66] However, we would like to ask the reader this: why do we need a wave equation? Is this not some rather desperate attempt to revive the idea of an *aether*?

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[64] See: Richard Feynman, 1963, *Probability Amplitudes for Particles*.

[65] Richard Feynman himself actually insisted on 'the lack of a need for the Uncertainty Principle' when looking at quantum-mechanical things in a more comprehensive way. See Feynman's Cornell *Messenger Lectures*. Unfortunately, the video rights on these lectures were bought up by Bill Gates, so they are no longer publicly available.

[66] This is why we do not want to write it all out here (we may do so in a future paper): we think the reader should think for himself and, hence, go through the basic equations himself. As Daisetsu Teitaru Suzuki – the man who brought Zen to the West (he published his *Essays in Zen Buddhism* (1927), from which I am quoting here, around the same time): "Zen does not rely on the intellect for the solution of its deepest problems. It is meant to get at the fact at first hand and not through any intermediary. To point to the moon, a finger is needed, but woe to those who take the finger for the moon." We could not agree more: if you want to be enlightened, *think for yourself!*