On the mathematical connections between some formulas concerning Modular Forms, Elliptic Curves, Ramanujan equations, ϕ, ζ(2) and various topics and parameters of String Theory and Particle Physics. II

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Abstract

In this paper we describe and analyze the mathematical connections between some formulas concerning Modular forms, Ramanujan equations, ϕ, ζ(2) and various topics and parameters of String Theory and Particle Physics.

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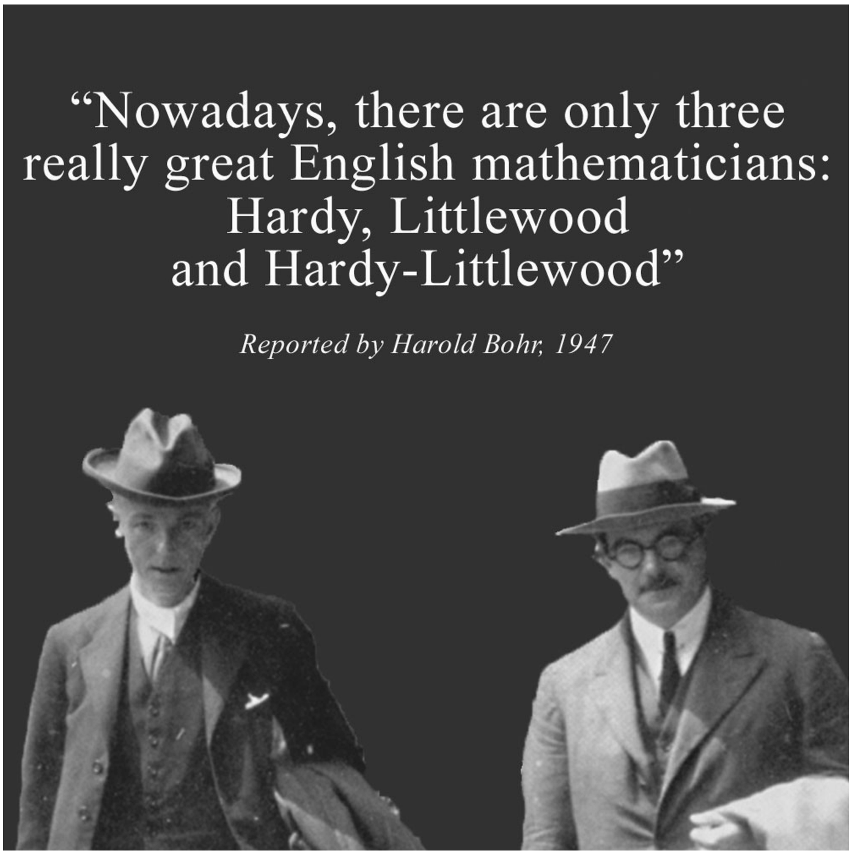

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We want to highlight that the development of the various equations was carried out according an our possible logical and original interpretation

For more information on the data entered for the development of the various equations, see the "Observations" section.

From:

MOCK THETA FUNCTIONS, RANKS, AND MAASS FORMS
KEN ONO - 2000 Mathematics Subject Classification. 11F, 11P, 05A17.

We begin by recalling the notion of a weak Maass form of half-integral weight $k \in \frac{1}{2}\mathbb{Z} \setminus \mathbb{Z}$. If $z = x + iy$ with $x, y \in \mathbb{R}$, then the weight k hyperbolic Laplacian is given by

$$\Delta_k := -y^2 \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) + iky \left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y} \right). \tag{2.1}$$

If v is odd, then define ϵ_v by

$$\epsilon_v := \begin{cases} 1 & \text{if } v \equiv 1 \pmod 4, \\ i & \text{if } v \equiv 3 \pmod 4. \end{cases} \tag{2.2}$$

Lemma 3.3. *Assume the notation and hypotheses above. For $z \in \mathbb{H}$, we have*

$$G\left(\frac{a}{c}; z\right) = \frac{i\ell_c^{\frac{1}{2}} \sin\left(\frac{\pi a}{c}\right)}{\sqrt{3}} \int_0^{i\infty} \frac{\left((-i\ell_c\tau)^{-\frac{3}{2}}\Theta\left(\frac{a}{c}; -\frac{1}{\ell_c\tau}\right), \Theta\left(\frac{a}{c}; \ell_c\tau\right), \right)^T}{\sqrt{-i(\tau + z)}}\, d\tau.$$

Proof of the lemma. For brevity, we only prove the asserted formula for the first component of $G\left(\frac{a}{c}; z\right)$. The proof of the second component follows in the same way.

By analytic continuation and a change of variables (note, we may assume that $z = it$ with $t > 0$), we find that

$$J\left(\frac{a}{c}; \frac{2\pi}{\ell_c t}\right) = \ell_c t \cdot \int_0^{\infty} e^{-3\ell_c\pi t x^2} \cdot \frac{\cosh\left(\left(\frac{3a}{c} - 2\right) 2\pi x\right) + \cosh\left(\left(\frac{3a}{c} - 1\right) 2\pi x\right)}{\cosh(3\pi x)}\, dx.$$

Using the Mittag-Leffler theory of partial fraction decompositions, one finds that

$$\frac{\cosh\left(\left(\frac{3a}{c} - 2\right) 2\pi x\right) + \cosh\left(\left(\frac{3a}{c} - 1\right) 2\pi x\right)}{\cosh(3\pi x)} = \frac{-i}{\sqrt{3}\pi} \sum_{n \in \mathbb{Z}} \frac{(-1)^n \sin\left(\frac{\pi a(6n+1)}{c}\right)}{x - i\left(n + \frac{1}{6}\right)} - \frac{i}{\sqrt{3}\pi} \sum_{n \in \mathbb{Z}} \frac{(-1)^n \sin\left(\frac{\pi a(6n+1)}{c}\right)}{-x - i\left(n + \frac{1}{6}\right)}.$$

From:

$$\frac{-i}{\sqrt{3}\pi}\sum_{n\in\mathbb{Z}}\frac{(-1)^n \sin\left(\frac{\pi a(6n+1)}{c}\right)}{x-i\left(n+\frac{1}{6}\right)}-\frac{i}{\sqrt{3}\pi}\sum_{n\in\mathbb{Z}}\frac{(-1)^n \sin\left(\frac{\pi a(6n+1)}{c}\right)}{-x-i\left(n+\frac{1}{6}\right)}.$$

For

$0 < a < c \quad c > 0.$

a = 1 or 5; c = 2 or 8; n = 3; x = 1/2

for a = 1, c = 2, we obtain:

-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((1/2-i(3+1/6))))-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((-1/2-i(3+1/6))))

Input:

$$-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{\frac{1}{2}-i\left(3+\frac{1}{6}\right)}-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{-\frac{1}{2}-i\left(3+\frac{1}{6}\right)}$$

i is the imaginary unit

Exact result:

$$\frac{38\sqrt{3}}{185\,\pi}$$

Decimal approximation:

0.1132459352758275561158400374913126420167604662374721340 88...

0.113245935…

Property:

$\frac{38\sqrt{3}}{185\,\pi}$ is a transcendental number

Alternative representations:

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} = \frac{i \cos(-9\,\pi)}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)} + \frac{i \cos(-9\,\pi)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}$$

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} = -\frac{i \cos(10\,\pi)}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)} - \frac{i \cos(10\,\pi)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}$$

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} = \frac{i \cosh(-9\,i\,\pi)}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)} + \frac{i \cosh(-9\,i\,\pi)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}$$

Series representations:

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} = -\frac{456\, i^2 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{19\pi}{2}\right)}{(-3 + 19\,i)(3 + 19\,i)\,\pi\sqrt{2}\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}}$$

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} = -\frac{228\, i^2 \sum_{k=0}^{\infty} \frac{(-81)^k \pi^{2k}}{(2k)!}}{(-3 + 19\,i)(3 + 19\,i)\,\pi \exp\left(\pi\,\mathcal{A}\left\lfloor \frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} = -\frac{456\, i^2 \sum_{k=0}^{\infty} (-1)^k J_{1+2k}\left(\frac{19\pi}{2}\right)}{(-3 + 19\, i)(3 + 19\, i)\, \pi \exp\left(\pi\, \mathcal{A} \left\lfloor \frac{\arg(3-x)}{2\pi} \right\rfloor\right) \sqrt{x} \sum_{k=0}^{\infty} \frac{(-1)^k (3-x)^k x^{-k} \left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} = \frac{2166\, i^2}{(9 - 361\, i^2)\sqrt{3}} \int_0^1 \cos\left(\frac{19\, \pi\, t}{2}\right) dt$$

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} = \frac{1083\, i^2 \sqrt{\pi}}{18\, \pi\, \mathcal{A} \sqrt{3} - 722\, i^2\, \pi\, \mathcal{A} \sqrt{3}} \int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma} \frac{e^{-(361\,\pi^2)/(16\,s)+s}}{s^{3/2}}\, ds \quad \text{for } \gamma > 0$$

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} = \frac{114\, i^2 \sqrt{\pi}}{(9 - 361\, i^2)\, \pi^2\, \mathcal{A} \sqrt{3}} \int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma} \frac{\left(\frac{19}{4}\right)^{1-2s} \pi^{1-2s}\, \Gamma(s)}{\Gamma\left(\frac{3}{2} - s\right)}\, ds \quad \text{for } 0 < \gamma < 1$$

Half-argument formula:

$$\frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right)(-i)}{\left(\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi\, 19}{2}\right)\right) i}{\left(-\frac{1}{2} - i\left(3 + \frac{1}{6}\right)\right)\left(\sqrt{3}\, \pi\right)} = \frac{1}{\left(-\frac{1}{2} - \frac{19 i}{6}\right)\pi\sqrt{6}} (-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi) \rfloor}\, i \sqrt{2} \sqrt{\frac{1}{2}(1 - \cos(19\,\pi))} \left(1 - \left(1 + (-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi) \rfloor + \lfloor \mathrm{Re}(19\pi)/(2\pi) \rfloor}\right) \theta(-\mathrm{Im}(19\,\pi))\right) + \frac{1}{\left(\frac{1}{2} - \frac{19 i}{6}\right)\pi\sqrt{6}} (-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi) \rfloor}\, i \sqrt{2} \sqrt{\frac{1}{2}(1 - \cos(19\,\pi))} \left(1 - \left(1 + (-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi) \rfloor + \lfloor \mathrm{Re}(19\pi)/(2\pi) \rfloor}\right) \theta(-\mathrm{Im}(19\,\pi))\right)$$

Multiple-argument formulas:

$$\frac{((-1)^3 \sin(\frac{\pi 19}{2}))(-i)}{(\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} - \frac{((-1)^3 \sin(\frac{\pi 19}{2}))\,i}{(-\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} = \frac{228\, i^2 \sin(\frac{19\pi}{6})(-3 + 4 \sin^2(\frac{19\pi}{6}))}{(-9 + 361\, i^2)\pi \sqrt{3}}$$

$$\frac{((-1)^3 \sin(\frac{\pi 19}{2}))(-i)}{(\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} - \frac{((-1)^3 \sin(\frac{\pi 19}{2}))\,i}{(-\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} = \frac{456\, i^2 \cos(\frac{19\pi}{4}) \sin(\frac{19\pi}{4})}{9\pi\sqrt{3} - 361\, i^2\, \pi\sqrt{3}}$$

$$\frac{((-1)^3 \sin(\frac{\pi 19}{2}))(-i)}{(\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} - \frac{((-1)^3 \sin(\frac{\pi 19}{2}))\,i}{(-\frac{1}{2} - i(3 + \frac{1}{6}))(\sqrt{3}\,\pi)} = \frac{228\, i^2 \sin(\frac{19\pi}{6})(-3 \cos^2(\frac{19\pi}{6}) + \sin^2(\frac{19\pi}{6}))}{(-9 + 361\, i^2)\pi \sqrt{3}}$$

((3/((16[-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((1/2-i(3+1/6)))-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((-1/2-i(3+1/6))))]))))

Input:

$$\frac{3}{16\left(-\frac{i}{\sqrt{3}\,\pi} \times \frac{(-1)^3 \sin\left(\frac{\pi\times 19}{2}\right)}{\frac{1}{2} - i\left(3+\frac{1}{6}\right)} - \frac{i}{\sqrt{3}\,\pi} \times \frac{(-1)^3 \sin\left(\frac{\pi\times 19}{2}\right)}{-\frac{1}{2} - i\left(3+\frac{1}{6}\right)}\right)}$$

i is the imaginary unit

Exact result:

$$\frac{185\sqrt{3}\,\pi}{608}$$

Decimal approximation:

1.655688564391432412960054988472691499042198631071498387238...

1.6556885643… result very near to the 14th root of the following Ramanujan's class invariant $Q = (G_{505}/G_{101/5})^3 = 1164.2696$ i.e. 1.65578...

Property:

$\frac{185\sqrt{3}\,\pi}{608}$ is a transcendental number

Alternative representations:

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=\frac{3}{16\left(\frac{i\cos(-9\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cos(-9\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)}$$

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=\frac{3}{16\left(-\frac{i\cos(10\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}-\frac{i\cos(10\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)}$$

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=\frac{3}{16\left(\frac{i\cosh(-9i\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cosh(-9i\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)}$$

Series representations:

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=-\frac{(-3+19i)(3+19i)\pi\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}}{2432\,i^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{19\pi}{2}\right)}$$

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=-\frac{(-3+19i)(3+19i)\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{1216\,i^2\sum_{k=0}^{\infty}\frac{(-81)^k\pi^{2k}}{(2k)!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)}=-\frac{(-3+19i)(3+19i)\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{2432\,i^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{19\pi}{2}\right)}$$

for ($x \in \mathbb{R}$ and $x < 0$)

Integral representations:

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = -\frac{(-3+19\,i)(3+19\,i)\sqrt{3}}{11552\,i^2\int_0^1\cos\left(\frac{19\pi t}{2}\right)dt}$$

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = \frac{\left(9-361\,i^2\right)\pi\,\mathcal{A}\sqrt{3}}{2888\,i^2\sqrt{\pi}\int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma}\frac{e^{-\left(361\pi^2\right)/(16 s)+s}}{s^{3/2}}\,ds} \quad \text{for } \gamma > 0$$

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = -\frac{(-3+19\,i)(3+19\,i)\pi^2\,\mathcal{A}\sqrt{3}}{608\,i^2\sqrt{\pi}\int_{-\mathcal{A}\,\infty+\gamma}^{\mathcal{A}\,\infty+\gamma}\frac{\left(\frac{19}{4}\right)^{1-2s}\pi^{1-2s}\,\Gamma(s)}{\Gamma\left(\frac{3}{2}-s\right)}\,ds} \quad \text{for } 0 < \gamma < 1$$

Half-argument formula:

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = 3\Bigg/\left(16\left(\frac{1}{\left(-\frac{1}{2}-\frac{19 i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(19\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(19\pi))\right)+\frac{1}{\left(\frac{1}{2}-\frac{19 i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(19\pi))}\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(19\pi))\right)\right)\right)$$

Multiple-argument formulas:

$$\frac{3}{16\left(\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = \frac{\left(9-361\,i^2\right)\pi\sqrt{3}}{2432\,i^2\cos\left(\frac{19\pi}{4}\right)\sin\left(\frac{19\pi}{4}\right)}$$

$$\frac{3}{16\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = \frac{(-3+19\,i)(3+19\,i)\,\pi\sqrt{3}}{1216\,i^2 \sin\left(\frac{19\pi}{6}\right)\left(-3+4\sin^2\left(\frac{19\pi}{6}\right)\right)}$$

$$\frac{3}{16\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)} - \frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)} = \frac{\left(9-361\,i^2\right)\pi\sqrt{3}}{1216\,i^2\, U_{\frac{17}{2}}(\cos(\pi))\sin(\pi)}$$

we have also:

(((2*(log base 3(5)))/((16[-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((1/2-i(3+1/6))))-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((-1/2-i(3+1/6))))]))))

where 2*(log base 3(5) = 2*1.4649, where 1.4649 is a Hausdorff dimension

Input:

$$\frac{2\log_3(5)}{16\left(-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{\frac{1}{2}-i\left(3+\frac{1}{6}\right)} - \frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{-\frac{1}{2}-i\left(3+\frac{1}{6}\right)}\right)}$$

$\log_b(x)$ is the base- b logarithm

i is the imaginary unit

Exact result:

$$\frac{185\,\pi\log(5)}{304\sqrt{3}\,\log(3)}$$

$\log(x)$ is the natural logarithm

Decimal approximation:

1.617026603592618133766840717920890229818732014251198612088...

1.61702660359…. result that is a very good approximation to the value of the golden ratio 1.618033988749...

Alternative representations:

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{2\log_3(5)}{16\left(\frac{i\cos(-9\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cos(-9\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)}$$

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{2\log_3(5)}{16\left(-\frac{i\cos(10\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}-\frac{i\cos(10\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)}$$

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{2\log(5)}{\log(3)\left(16\left(\frac{i\cos(-9\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cos(-9\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)\right)}$$

Series representations:

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\pi\left(\log(4)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{4}\right)^k}{k}\right)}{304\sqrt{3}\left(\log(2)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{2}\right)^k}{k}\right)}$$

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\pi\left(2\pi\left\lfloor\frac{\arg(5-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(5-x)^k x^{-k}}{k}\right)}{304\sqrt{3}\left(2\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor-i\log(x)+i\sum_{k=1}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}}{k}\right)}\quad\text{for } x<0$$

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\pi\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k(5-z_0)^k z_0^{-k}}{k}\right)}{304\sqrt{3}\left(2\pi\left\lfloor\frac{\pi-\arg\left(\frac{1}{z_0}\right)-\arg(z_0)}{2\pi}\right\rfloor-i\log(z_0)+i\sum_{k=1}^{\infty}\frac{(-1)^k(3-z_0)^k z_0^{-k}}{k}\right)}$$

Integral representations:

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\,\pi\int_1^5\frac{1}{t}\,dt}{304\sqrt{3}\int_1^3\frac{1}{t}\,dt}$$

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\,\pi\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{4^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}{304\sqrt{3}\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{2^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds}$$

for $-1<\gamma<0$

Half-argument formula:

$$\frac{2\log_3(5)}{16\left(-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}-\frac{i\left((-1)^3\sin\left(\frac{\pi 19}{2}\right)\right)}{\left(\sqrt{3}\,\pi\right)\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)}\right)}=\frac{185\,\pi\,(-\log(2)+\log(10))}{304\sqrt{3}\,(-\log(2)+\log(6))}$$

and again:

1+1/[-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((1/2-i(3+1/6))))-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((-1/2-i(3+1/6))))]^Pi

Input:

$$1+\frac{1}{\left(-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{\frac{1}{2}-i\left(3+\frac{1}{6}\right)}-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times 19}{2}\right)}{-\frac{1}{2}-i\left(3+\frac{1}{6}\right)}\right)^{\pi}}$$

i is the imaginary unit

Exact result:

$$1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}$$

Decimal approximation:

938.2934420718423164453689407839688855234460969372834952306...

938.29344207…. result practically equal to the proton mass 938.272046

Alternate form:

$$3^{-\pi/2}\times 38^{-\pi}\left(3^{\pi/2}\times 38^{\pi}+(185\,\pi)^{\pi}\right)$$

Alternative representations:

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+\frac{1}{\left(\frac{i\cos(-9\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cos(-9\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)^{\pi}}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+\frac{1}{\left(-\frac{i\cos(10\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}-\frac{i\cos(10\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)^{\pi}}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+\frac{1}{\left(\frac{i\cosh(-9i\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cosh(-9i\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)^{\pi}}$$

Series representations:

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$
$$1+\left(\frac{18\,741\,610\,000}{1\,172\,889}\right)^{\sum_{k=0}^{\infty}e^{i k\pi}/(1+2k)}\left(\sum_{k=0}^{\infty}\frac{e^{i k\pi}}{1+2k}\right)^{4\sum_{k=0}^{\infty}e^{i k\pi}/(1+2k)}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$
$$1+3^{1/2\sum_{k=0}^{\infty}\left(4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)\right)/(1+2k)}\times$$
$$38^{\sum_{k=0}^{\infty}\left(4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)\right)/(1+2k)}\times$$
$$185^{\sum_{k=0}^{\infty}-\left(4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)\right)/(1+2k)}$$
$$\left(\sum_{k=0}^{\infty}-\frac{4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{1+2k}\right)^{\sum_{k=0}^{\infty}-\left(4(-1)^k\,1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)\right)/(1+2k)}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$

$$3^{-1/2\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}\times 38^{-\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}\left(3^{1/2\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}\times 38^{\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}+185^{\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}\left(\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)\right)^{\sum_{k=0}^{\infty}(-1/4)^k\left(1/(1+2k)+2/(1+4k)+1/(3+4k)\right)}\right)$$

Integral representations:

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$

$$1+\left(\frac{34\,225}{1083}\right)^{\int_0^{\infty}1/(1+t^2)\,dt}\left(\int_0^{\infty}\frac{1}{1+t^2}\,dt\right)^{2\int_0^{\infty}1/(1+t^2)\,dt}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$

$$1+\left(\frac{34\,225}{1083}\right)^{\int_0^{\infty}\sin(t)/t\,dt}\left(\int_0^{\infty}\frac{\sin(t)}{t}\,dt\right)^{2\int_0^{\infty}\sin(t)/t\,dt}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$

$$1+\left(\frac{34\,225}{1083}\right)^{\int_0^{1}1/\sqrt{1-t^2}\,dt}\left(\int_0^{1}\frac{1}{\sqrt{1-t^2}}\,dt\right)^{2\int_0^{1}1/\sqrt{1-t^2}\,dt}$$

Half-argument formula:

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=$$

$$1+\left(\frac{1}{\left(-\frac{1}{2}-\frac{19i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor\operatorname{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(19\pi))}\right.$$
$$\left(1-\left(1+(-1)^{\lfloor-\operatorname{Re}(19\pi)/(2\pi)\rfloor+\lfloor\operatorname{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\operatorname{Im}(19\pi))\right)+$$
$$\frac{1}{\left(\frac{1}{2}-\frac{19i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor\operatorname{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\sqrt{\frac{1}{2}(1-\cos(19\pi))}$$
$$\left.\left(1-\left(1+(-1)^{\lfloor-\operatorname{Re}(19\pi)/(2\pi)\rfloor+\lfloor\operatorname{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\operatorname{Im}(19\pi))\right)\right)^{-\pi}$$

Multiple-argument formulas:

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+228^{-\pi}\left(\frac{i^2\sin\left(\frac{19\pi}{6}\right)\left(-3+4\sin^2\left(\frac{19\pi}{6}\right)\right)}{\left(-9+361\,i^2\right)\pi\sqrt{3}}\right)^{-\pi}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+456^{-\pi}\left(\frac{i^2\cos\left(\frac{19\pi}{4}\right)\sin\left(\frac{19\pi}{4}\right)}{9\pi\sqrt{3}-361\,i^2\,\pi\sqrt{3}}\right)^{-\pi}$$

$$1+\frac{1}{\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)^{\pi}}=1+228^{-\pi}\left(\frac{i^2\,U_{\frac{17}{2}}(\cos(\pi))\sin(\pi)}{9\pi\sqrt{3}-361\,i^2\,\pi\sqrt{3}}\right)^{-\pi}$$

((13/3)+1/8)*[-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((1/2-i(3+1/6))))-i/((sqrt3)Pi)*((((-1)^3*sin((Pi*19)/2))))/(((-1/2-i(3+1/6))))]+1/golden ratio*1/10^2

Input:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times19}{2}\right)}{\frac{1}{2}-i\left(3+\frac{1}{6}\right)}-\frac{i}{\sqrt{3}\,\pi}\times\frac{(-1)^3\sin\left(\frac{\pi\times19}{2}\right)}{-\frac{1}{2}-i\left(3+\frac{1}{6}\right)}\right)+\frac{1}{\phi}\times\frac{1}{10^2}$$

i is the imaginary unit

ϕ is the golden ratio

Exact result:

$$\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}$$

Decimal approximation:

0.511068467992230136165166035492425243501926837106787559765...

0.511068467…. result practically equal to the electron mass 0.51099895

Property:

$\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}$ is a transcendental number

Alternate forms:

$$\frac{1}{200}\left(\sqrt{5}-1\right)+\frac{2033}{740\sqrt{3}\,\pi}$$

$$\frac{10\,165\,\phi+37\sqrt{3}\,\pi}{3700\sqrt{3}\,\pi\,\phi}$$

$$\frac{1}{50\left(1+\sqrt{5}\right)}+\frac{2033}{740\sqrt{3}\,\pi}$$

Alternative representations:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=\frac{1}{\phi\,10^2}+\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{i\cos(-9\,\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cos(-9\,\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3\sin\left(\frac{\pi\,19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=\frac{1}{\phi\,10^2}+\left(\frac{13}{3}+\frac{1}{8}\right)\left(-\frac{i\cos(10\,\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}-\frac{i\cos(10\,\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$\frac{1}{\phi\,10^2}+\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{i\cosh(-9\,i\,\pi)}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}+\frac{i\cosh(-9\,i\,\pi)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\pi\sqrt{3}\right)}\right)$$

Series representations:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$-\left(\left(203\,300\,\phi\,i^2\sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{19\,\pi}{2}\right)+9\,\pi\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}-361\,i^2\,\pi\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\Big/\left(100\,\phi\,(-3+19\,i)(3+19\,i)\,\pi\sqrt{2}\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\right)$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi\,19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$-\left(\left(101\,650\,\phi\,i^2\sum_{k=0}^{\infty}\frac{(-81)^k\,\pi^{2k}}{(2k)!}+9\,\pi\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}-361\,i^2\,\pi\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/\left(100\,\phi\,(-3+19\,i)(3+19\,i)\,\pi\exp\left(\pi\,\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\,\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right)\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$

$$-\left(\left(203\,300\,\phi\,i^2 \sum_{k=0}^{\infty}(-1)^k J_{1+2k}\left(\frac{19\pi}{2}\right)+ 9\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!} - 361\,i^2\,\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\Big/ \left(100\,\phi\,(-3+19\,i)(3+19\,i)\,\pi\exp\left(\pi\mathcal{A}\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k (3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}\right)\right) \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

Integral representations:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}= \frac{965\,675\,\phi\,i^2\int_0^1\cos\left(\frac{19\pi t}{2}\right)dt+9\sqrt{3}-361\,i^2\sqrt{3}}{900\,\phi\sqrt{3}-36\,100\,\phi\,i^2\sqrt{3}}$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=\frac{1}{100\,\phi}+ \int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma}\left(\frac{2033\,e^{-(361\pi^2)/(16s)+s}\,i\sqrt{\pi}}{192\left(-\frac{1}{2}-\frac{19i}{6}\right)\pi\,s^{3/2}\,\mathcal{A}\sqrt{3}}+\frac{2033\,e^{-(361\pi^2)/(16s)+s}\,i\sqrt{\pi}}{192\left(\frac{1}{2}-\frac{19i}{6}\right)\pi\,s^{3/2}\,\mathcal{A}\sqrt{3}}\right)ds \quad \text{for } \gamma>0$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right) i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}= \frac{1}{100\,\phi}+\int_{-\mathcal{A}\infty+\gamma}^{\mathcal{A}\infty+\gamma}\left(\frac{107\times 4^{-3+2s}\times 19^{1-2s}\,i\,\pi^{-1-2s}\,\Gamma(s)\sqrt{\pi}}{3\left(-\frac{1}{2}-\frac{19i}{6}\right)\mathcal{A}\,\Gamma\left(\frac{3}{2}-s\right)\sqrt{3}}+ \frac{107\times 4^{-3+2s}\times 19^{1-2s}\,i\,\pi^{-1-2s}\,\Gamma(s)\sqrt{\pi}}{3\left(\frac{1}{2}-\frac{19i}{6}\right)\mathcal{A}\,\Gamma\left(\frac{3}{2}-s\right)\sqrt{3}}\right)ds \quad \text{for } 0<\gamma<1$$

Half-argument formula:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$

$$\frac{1}{100\,\phi}+\frac{107}{24}\left(\frac{1}{\left(-\frac{1}{2}-\frac{19 i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\,\sqrt{\frac{1}{2}(1-\cos(19\pi))}\right.$$
$$\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(19\pi))\right)+$$
$$\frac{1}{\left(\frac{1}{2}-\frac{19 i}{6}\right)\pi\sqrt{6}}(-1)^{\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\,i\sqrt{2}\,\sqrt{\frac{1}{2}(1-\cos(19\pi))}$$
$$\left.\left(1-\left(1+(-1)^{\lfloor -\mathrm{Re}(19\pi)/(2\pi)\rfloor+\lfloor \mathrm{Re}(19\pi)/(2\pi)\rfloor}\right)\theta(-\mathrm{Im}(19\pi))\right)\right)$$

Multiple-argument formulas:

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$\frac{203\,300\,\phi\,i^2\cos\left(\frac{19\pi}{4}\right)\sin\left(\frac{19\pi}{4}\right)+9\pi\sqrt{3}-361\,i^2\pi\sqrt{3}}{900\,\phi\,\pi\sqrt{3}-36\,100\,\phi\,i^2\pi\sqrt{3}}$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$\frac{304\,950\,\phi\,i^2\sin\left(\frac{19\pi}{6}\right)-406\,600\,\phi\,i^2\sin^3\left(\frac{19\pi}{6}\right)+9\pi\sqrt{3}-361\,i^2\pi\sqrt{3}}{900\,\phi\,\pi\sqrt{3}-36\,100\,\phi\,i^2\pi\sqrt{3}}$$

$$\left(\frac{13}{3}+\frac{1}{8}\right)\left(\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)(-i)}{\left(\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}-\frac{\left((-1)^3 \sin\left(\frac{\pi 19}{2}\right)\right)i}{\left(-\frac{1}{2}-i\left(3+\frac{1}{6}\right)\right)\left(\sqrt{3}\,\pi\right)}\right)+\frac{1}{10^2\,\phi}=$$
$$\frac{101\,650\,\phi\,i^2\,U_{\frac{17}{2}}(\cos(\pi))\sin(\pi)+9\pi\sqrt{3}-361\,i^2\pi\sqrt{3}}{900\,\phi\,\pi\sqrt{3}-36\,100\,\phi\,i^2\pi\sqrt{3}}$$

From the ratio of the two expressions, that are about equal to the Proton and Electron masses, we obtain:

[1 + 3^(-π/2) 38^(-π) (185 π)^π]/[1/(100 *golden ratio) + 2033/(740 sqrt(3) π)]

Input:

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}}$$

ϕ is the golden ratio

Decimal approximation:

1835.944693981995689716265099832553867301809164293450374138...

1835.9446939….

Alternate forms:

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{200} (\sqrt{5} - 1) + \frac{2033}{740 \sqrt{3} \pi}}$$

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{50 (1+\sqrt{5})} + \frac{2033}{740 \sqrt{3} \pi}}$$

$$\frac{925 \times 2^{3-\pi} \times 3^{1/2-\pi/2} \times 19^{-\pi} \pi (3^{\pi/2} \times 38^{\pi} + (185 \pi)^{\pi}) \phi}{10\,165 (1 + \sqrt{5}) + 74 \sqrt{3} \pi}$$

Series representations:

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} = \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi} \pi^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}}$$

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} = \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi} \pi^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}$$

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}} = \frac{1+3^{-\pi/2}\times 38^{-\pi}\times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi}+\frac{2033\sqrt{\pi}}{370\,\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

From which:

(([1 + 3^(-π/2) 38^(-π) (185 π)^π]/[1/(100 *golden ratio) + 2033/(740 sqrt(3) π)]))-107

where 107 is an Eisenstein prime number

Input:

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107$$

ϕ is the golden ratio

Exact result:

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107$$

Decimal approximation:

1728.944693981995689716265099832553867301809164293450374138...

1728.9446939… ≈ 1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternate forms:

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{200}\left(\sqrt{5}-1\right)+\frac{2033}{740\sqrt{3}\,\pi}}-107$$

$$\frac{925 \times 2^{3-\pi} \times 3^{1/2-\pi/2} \times 19^{-\pi}\, \pi \left(3^{\pi/2} \times 38^{\pi} + (185\,\pi)^{\pi}\right) \phi}{10\,165 \left(1+\sqrt{5}\right) + 74\sqrt{3}\,\pi} - 107$$

$$-107 + \frac{1}{\frac{1}{50\left(1+\sqrt{5}\right)} + \frac{2033}{740\sqrt{3}\,\pi}} + \frac{3^{-\pi/2}\left(\frac{185\,\pi}{38}\right)^{\pi}}{\frac{1}{50\left(1+\sqrt{5}\right)} + \frac{2033}{740\sqrt{3}\,\pi}}$$

Series representations:

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\sqrt{3}\,\pi}} - 107 = -107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\,\pi\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}}$$

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\sqrt{3}\,\pi}} - 107 = -107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi} + \frac{2033\sqrt{\pi}}{370\,\pi\sum_{j=0}^{\infty}\text{Res}_{s=-\frac{1}{2}+j}\,2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\sqrt{3}\,\pi}} - 107 = -107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\,\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(3-z_0)^{k}\,z_0^{-k}}{k!}}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \leq 0$))

and:

(((([1 + 3^(-π/2) 38^(-π) (185 π)^π]/[1/(100 *golden ratio) + 2033/(740 sqrt(3) π)]))-107))^1/15

Input:

$$\sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi} + \frac{2033}{740\sqrt{3}\,\pi}} - 107}$$

ϕ is the golden ratio

Exact result:

$$\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107}$$

Decimal approximation:

1.6438117232838444682746126446723652923557228969740666641274...

1.643811723…. ≈ ζ(2) = $\frac{\pi^2}{6}$ = 1.644934 ...

Alternate forms:

$$\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{200}\left(\sqrt{5}-1\right)+\frac{2033}{740\sqrt{3}\,\pi}}-107}$$

$$\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{50\left(1+\sqrt{5}\right)}+\frac{2033}{740\sqrt{3}\,\pi}}-107}$$

$$\frac{1}{\sqrt[15]{\frac{10\,165\left(1+\sqrt{5}\right)+74\sqrt{3}\,\pi}{925\times 2^{3-\pi}\times 3^{1/2-\pi/2}\times 19^{-\pi}\,\pi\left(3^{\pi/2}\times 38^{\pi}+(185\,\pi)^{\pi}\right)\phi-107\left(10\,165\left(1+\sqrt{5}\right)+74\sqrt{3}\,\pi\right)}}}$$

All 15th roots of (1 + 3^(-π/2) 38^(-π) (185 π)^π)/(1/(100 ϕ) + 2033/(740 sqrt(3) π)) - 107:

$$e^{0}\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107}\approx 1.6438 \text{ (real, principal root)}$$

$$e^{(2 i \pi)/15}\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107}\approx 1.5017+0.6686\,i$$

$$e^{(4 i \pi)/15}\sqrt[15]{\frac{1+3^{-\pi/2}\times 38^{-\pi}\,(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}-107}\approx 1.0999+1.2216\,i$$

$$e^{(2 i \pi)/5} \sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} - 107} \approx 0.5080 + 1.5634 i$$

$$e^{(8 i \pi)/15} \sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} - 107} \approx -0.1718 + 1.6348 i$$

Series representations:

$$\sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} - 107} = \sqrt[15]{-107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi} \pi^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \pi \sqrt{2} \sum_{k=0}^{\infty} 2^{-k} \binom{\frac{1}{2}}{k}}}}$$

$$\sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} - 107} = \sqrt[15]{-107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi} \pi^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \pi \sqrt{2} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{2}\right)^k \left(-\frac{1}{2}\right)_k}{k!}}}}$$

$$\sqrt[15]{\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} - 107} = \sqrt[15]{-107 + \frac{1 + 3^{-\pi/2} \times 38^{-\pi} \times 185^{\pi} \pi^{\pi}}{\frac{1}{100 \phi} + \frac{2033 \sqrt{\pi}}{370 \pi \sum_{j=0}^{\infty} \text{Res}_{s=-\frac{1}{2}+j} 2^{-s} \Gamma\left(-\frac{1}{2}-s\right) \Gamma(s)}}}$$

Integral representation:

$$(1+z)^a = \frac{\int_{-i \infty+\gamma}^{i \infty+\gamma} \frac{\Gamma(s) \Gamma(-a-s)}{z^s} \, ds}{(2 \pi i) \Gamma(-a)} \quad \text{for } (0 < \gamma < -\text{Re}(a) \text{ and } |\arg(z)| < \pi)$$

(([1 + 3^(-π/2) 38^(-π) (185 π)^π]/[1/(100 *golden ratio) + 2033/(740 sqrt(3) π)]))+34-1/2

where 34 is a Fibonacci number

Input:

$$\frac{1 + 3^{-\pi/2} \times 38^{-\pi} (185 \pi)^{\pi}}{\frac{1}{100 \phi} + \frac{2033}{740 \sqrt{3} \pi}} + 34 - \frac{1}{2}$$

ϕ is the golden ratio

Exact result:

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}+\frac{67}{2}$$

Decimal approximation:

1869.444693981995689716265099832553867301809164293450374138...

1869.44469398… result practically equal to the rest mass of D meson 1869.62

Alternate forms:

$$\frac{67}{2}+\frac{1+3^{-\pi/2}\times 38^{-\pi}(185\,\pi)^{\pi}}{\frac{1}{200}(\sqrt{5}-1)+\frac{2033}{740\sqrt{3}\,\pi}}$$

$$\frac{925\times 2^{3-\pi}\times 3^{1/2-\pi/2}\times 19^{-\pi}\,\pi\left(3^{\pi/2}\times 38^{\pi}+(185\,\pi)^{\pi}\right)\phi}{10\,165\,(1+\sqrt{5})+74\sqrt{3}\,\pi}+\frac{67}{2}$$

$$\frac{67}{2}+\frac{1}{\frac{1}{50(1+\sqrt{5})}+\frac{2033}{740\sqrt{3}\,\pi}}+\frac{3^{-\pi/2}\left(\frac{185\,\pi}{38}\right)^{\pi}}{\frac{1}{50(1+\sqrt{5})}+\frac{2033}{740\sqrt{3}\,\pi}}$$

Series representations:

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}+34-\frac{1}{2}=\frac{67}{2}+\frac{1+3^{-\pi/2}\times 38^{-\pi}\times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\,\pi\sqrt{2}\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^{k}\left(-\frac{1}{2}\right)_{k}}{k!}}}$$

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}+34-\frac{1}{2}=\frac{67}{2}+\frac{1+3^{-\pi/2}\times 38^{-\pi}\times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi}+\frac{2033\sqrt{\pi}}{370\,\pi\sum_{j=0}^{\infty}\operatorname{Res}_{s=-\frac{1}{2}+j}2^{-s}\,\Gamma\left(-\frac{1}{2}-s\right)\Gamma(s)}}$$

$$\frac{1+3^{-\pi/2}\times 38^{-\pi}(185\,\pi)^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\sqrt{3}\,\pi}}+34-\frac{1}{2}=\frac{67}{2}+\frac{1+3^{-\pi/2}\times 38^{-\pi}\times 185^{\pi}\,\pi^{\pi}}{\frac{1}{100\,\phi}+\frac{2033}{740\,\pi\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^{k}\left(-\frac{1}{2}\right)_{k}(3-z_0)^{k}\,z_0^{-k}}{k!}}}$$

for (not ($z_0 \in \mathbb{R}$ and $-\infty < z_0 \le 0$))

Now, we have that:

After a long calculation, one can show that this Maass-Poincaré series has the Fourier expansion

$$(3.41)\quad P_{\frac{1}{2}}\left(\frac{3}{4};z\right)=\left(1-\pi^{-\frac{1}{2}}\cdot\Gamma\left(\frac{1}{2},\frac{\pi y}{6}\right)\right)\cdot q^{-\frac{1}{24}}+\sum_{n=-\infty}^{0}\gamma_y(n)q^{n-\frac{1}{24}}+\sum_{n=1}^{\infty}\beta(n)q^{n-\frac{1}{24}},$$

where for positive integers n we have

$$(3.42)\quad \beta(n)=\pi(24n-1)^{-\frac{1}{4}}\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor\frac{k+1}{2}\rfloor}A_{2k}\left(n-\frac{k(1+(-1)^k)}{4}\right)}{k}\cdot I_{\frac{1}{2}}\left(\frac{\pi\sqrt{24n-1}}{12k}\right),$$

and for non-positive integers n we have

$$\gamma_y(n)=\pi^{\frac{1}{2}}|24n-1|^{-\frac{1}{4}}\cdot\Gamma\left(\frac{1}{2},\frac{\pi|24n-1|\cdot y}{6}\right)$$
$$\times\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor\frac{k+1}{2}\rfloor}A_{2k}\left(n-\frac{k(1+(-1)^k)}{4}\right)}{k}\cdot J_{\frac{1}{2}}\left(\frac{\pi\sqrt{|24n-1|}}{12k}\right).$$

From

$$\beta(n)=\pi(24n-1)^{-\frac{1}{4}}\sum_{k=1}^{\infty}\frac{(-1)^{\lfloor\frac{k+1}{2}\rfloor}A_{2k}\left(n-\frac{k(1+(-1)^k)}{4}\right)}{k}\cdot I_{\frac{1}{2}}\left(\frac{\pi\sqrt{24n-1}}{12k}\right)$$

For n = 4, we obtain:

Pi*(24*4-1)^(-0.25) sum 1/k*((-1)^((k+1)/2)) (4-((k(1+(-1)^k)/4)) (((Pi/(12k)*(sqrt(24*4-1)))))), k = 1..infinity

Input interpretation:

$$\frac{\pi\sum_{k=1}^{\infty}\frac{1}{k}(-1)^{(k+1)/2}\left(4-\left(k\left(\frac{1}{4}\left(1+(-1)^k\right)\right)\right)\right)\left(\frac{\pi}{12\,k}\sqrt{24\times4-1}\right)}{(24\times4-1)^{0.25}}$$

Result:

$-3.16132-0.950047\,i$

Input interpretation:

$-3.16132 + i\times(-0.950047)$

i is the imaginary unit

Result:

$-3.16132\ldots - 0.950047\ldots\, i$

Polar coordinates:

$r = 3.30099$ (radius), $\theta = -163.273°$ (angle)

3.30099

From

$$\gamma_y(n) = \pi^{\frac{1}{2}}|24n-1|^{-\frac{1}{4}} \cdot \Gamma\left(\frac{1}{2}, \frac{\pi|24n-1|\cdot y}{6}\right) \times \sum_{k=1}^{\infty} \frac{(-1)^{\lfloor\frac{k+1}{2}\rfloor} A_{2k}\left(n - \frac{k(1+(-1)^k)}{4}\right)}{k} \cdot J_{\frac{1}{2}}\left(\frac{\pi\sqrt{|24n-1|}}{12k}\right).$$

For n = 4, y = 1/4 , we obtain:

Pi^(1/2) (24*4-1)^(-0.25) * gamma (1/2 , 1/6*Pi*(24*4-1)*1/4) * sum 1/k*((-1)^((k+1)/2)) (4-((k(1+(-1)^k)/4)) (((Pi/(12k)*(sqrt(24*4-1)))))), k = 1..infinity

Input interpretation:

$$\frac{\sqrt{\pi}\, \Gamma\left(\frac{1}{2}, \frac{1}{6}\pi(24\times4-1)\times\frac{1}{4}\right) \sum_{k=1}^{\infty} \frac{1}{k}(-1)^{(k+1)/2}\left(4-\left(k\left(\frac{1}{4}\left(1+(-1)^k\right)\right)\right)\left(\frac{\pi}{12\,k}\sqrt{24\times4-1}\right)\right)}{(24\times4-1)^{0.25}}$$

$\Gamma(a, x)$ is the incomplete gamma function

Result:

$-1.93787\times10^{-6} - 5.82374\times10^{-7}\, i$

Input interpretation:

$-1.93787\times10^{-6}-5.82374\times10^{-7}\,i$

i is the imaginary unit

Result:

$-1.93787...\times10^{-6}-$
$5.82374...\times10^{-7}\,i$

Polar coordinates:

$r=2.02349\times10^{-6}$ (radius), $\theta=-163.273°$ (angle)

2.02349*10^-6

From the ratio of the two results, we obtain:

3.30099 / 2.02349×10^-6

Input interpretation:

$$\frac{3.30099}{2.02349\times10^{-6}}$$

Result:

$1.631334970768326011000795655031653232780987304113190 57...\times10^{6}$

$1.6313349707...*10^{6}$

From which:

(((3.30099 / 2.02349×10^-6)^1/30))

Input interpretation:

$$\sqrt[30]{\frac{3.30099}{2.02349\times10^{-6}}}$$

Result:

1.610960053721008574512565471714658498438570650432823712070...

1.610960053…. result that is a good approximation to the value of the golden ratio 1.618033988749...

From

ELLIPTIC CURVES AND LOWER BOUNDS FOR CLASS NUMBERS
MICHAEL GRIFFIN AND KEN ONO - arXiv:2001.07332v3 [math.NT] 29 Apr 2020

We have that:

We derive lower bounds in terms of $\Omega_r := \pi^{\frac{r}{2}}/\Gamma\left(\frac{r}{2}+1\right)$, the volume of the $\mathbb{R}^r$-unit ball, the regulator $R_{\mathbb{Q}}(E)$, the diameter $d(E)$ (see (3.3)), and the torsion subgroup $E_{\mathrm{tor}}(\mathbb{Q})$. We define

$$c(E) := \frac{|E_{\mathrm{tor}}(\mathbb{Q})|}{\sqrt{R_{\mathbb{Q}}(E)}} \cdot \Omega_r, \tag{1.4}$$

Remark. *A classical theorem of Hooley (see Ch. IV of [13]) gives asymptotic formulas for the number of square-free values of irreducible cubic polynomials $f(t) \in \mathbb{Z}[t]$. Namely, it is generally the case (i.e. barring trivial obstructions arising from congruence conditions) that a positive proportion of the values of f at integer arguments are square-free. Using this fact we can quantify the frequency with which Theorem 1.2 improves on (1.1) for large $-D_E(t)$ (i.e. $t \to +\infty$) when $r(E) \geq 3$. A famous example of Elkies [5] has $r(E) \geq 28$, and so we obtain the effective lower bound*

$$h(-D) \gg_{\varepsilon} (\log D)^{14-\varepsilon}$$

which holds for $\gg_{\varepsilon} X^{\frac{1}{3}}$ many explicit fundamental discriminants $-X < -D < 0$.

Example. *For $E:\ y^2 = x^3 - 16x + 1$, we have[3] $|E_{\mathrm{tor}}(\mathbb{Q})| = 1$, $r(E) = 3$, and $R_E(\mathbb{Q}) \sim 0.930\ldots$. Therefore, for large fundamental discriminants of the form $-D_E(t) = -4(t^3 - 16t - 1)$, we have*

$$h(-D_E(t)) > \frac{1}{20} \cdot (\log(D_E(t))^{\frac{3}{2}}.$$

We give infinite families of $E/\mathbb{Q}$ using the discriminant $\Delta_{a,b} := -16(27b^4 - 4a^6)$ curves

$$E_{a,b}\ :\ y^2 = x^3 - a^2x + b^2. \tag{1.8}$$

For integers t, we let $D_{a,b}(t) := 4(t^3 - a^2t - b^2)$. For positive integers a, b, we let

$$c_{a,b}^{(2)} := \frac{\Omega_2}{12 \cdot \widehat{h}(P_{\max}^{(2)})} \quad \text{and} \quad c_{a,b^3}^{(3)} := \frac{\Omega_3}{24\sqrt{3} \cdot \widehat{h}(P_{\max}^{(3)})^{\frac{3}{2}}}, \tag{1.9}$$

where $P_{\max}^{(2)} \in \{(0,b), (-a,b)\} \subset E_{a,b}(\mathbb{Q})$ and $P_{\max}^{(3)} \in \{(0,b^3), (-a,b^3), (-b^2, ab)\} \subset E_{a,b^3}(\mathbb{Q})$ are chosen to have the largest canonical height.

Theorem 1.3. *If a and b are positive integers, then the following are true:*

(1) If $a \gg_b 1$ (resp. $b \gg_a 1$), then $r(E_{a,b}(\mathbb{Q})) \geq 2$. Moreover, if $\varepsilon > 0$, then for sufficiently large fundamental discriminants $-D_{a,b}(t) < 0$ in absolute value we have

$$h(-D_{a,b}(t)) \geq (c_{a,b}^{(2)} - \varepsilon) \cdot \log(D_{a,b}(t)).$$

(2) If $a \gg_b 1$ (resp. $b \gg_a 1$), then $r(E_{a,b^3}(\mathbb{Q})) \geq 3$. Moreover, if $\varepsilon > 0$, then for sufficiently large fundamental discriminants $-D_{a,b^3}(t) < 0$ in absolute value we have

$$h(-D_{a,b^3}(t)) \geq (c_{a,b^3}^{(3)} - \varepsilon) \cdot \log(D_{a,b^3}(t))^{\frac{3}{2}}.$$

From

$$h(-D_E(t)) > \frac{1}{20} \cdot (\log(D_E(t))^{\frac{3}{2}}.$$

and

$$-D_E(t) = -4(t^3 - 16t - 1),$$

we obtain:

1+1/2*1/(((-4(4^3-16*4-1))))

Input:

$$1 + \frac{1}{2}\left(-\frac{1}{4(4^3 - 16 \times 4 - 1)}\right)$$

Exact result:

$$\frac{9}{8}$$

Decimal form:

1.125

1.125

and:

1+1/2*1/(((1/20*(ln(4(4^3+16*4+1)))^(3/2))))

Input:

$$1 + \frac{1}{2} \times \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1))}$$

$\log(x)$ is the natural logarithm

Exact result:

$$1 + \frac{10}{\log^{3/2}(516)}$$

Decimal approximation:

1.640598466792149425627232451710086376470664375377164940673...

$1.640598466\ldots. \approx \zeta(2) = \frac{\pi^2}{6} = 1.644934 \ldots$

Property:

$1 + \frac{10}{\log^{3/2}(516)}$ is a transcendental number

Alternate forms:

$$\frac{10 + \log^{3/2}(516)}{\log^{3/2}(516)}$$

$$1 + \frac{10}{(2 \log(2) + \log(129))^{3/2}}$$

$$\frac{10 + (2 \log(2) + \log(3) + \log(43))^{3/2}}{(2 \log(2) + \log(3) + \log(43))^{3/2}}$$

Alternative representations:

$$1 + \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1)) 2} = 1 + \frac{1}{\frac{2}{20} \log_e^{3/2}(4(65 + 4^3))}$$

$$1 + \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1)) 2} = 1 + \frac{1}{\frac{2}{20} (\log(a) \log_a(4(65 + 4^3)))^{3/2}}$$

$$1 + \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1)) 2} = 1 + \frac{1}{\frac{2}{20} (-\text{Li}_1(1 - 4(65 + 4^3)))^{3/2}}$$

Series representations:

$$1 + \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1)) 2} = 1 + \frac{10}{\left(\log(515) - \sum_{k=1}^{\infty} \frac{\left(-\frac{1}{515}\right)^k}{k}\right)^{3/2}}$$

$$1 + \frac{1}{\frac{1}{20} \log^{3/2}(4(4^3 + 16 \times 4 + 1)) 2} =$$
$$1 + \frac{10}{\left(2 i \pi \left\lfloor \frac{\arg(516 - x)}{2 \pi} \right\rfloor + \log(x) - \sum_{k=1}^{\infty} \frac{(-1)^k (516 - x)^k x^{-k}}{k}\right)^{3/2}} \quad \text{for } x < 0$$

$$1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}=$$

$$1+\frac{10}{\left(\log(z_0)+\left\lfloor\frac{\arg(516-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k(516-z_0)^k z_0^{-k}}{k}\right)^{3/2}}$$

Integral representations:

$$1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}=1+\frac{10}{\left(\int_1^{516}\frac{1}{t}\,dt\right)^{3/2}}$$

$$1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}=1+\frac{20\sqrt{2}\,\pi^{3/2}}{\left(-i\int_{-i\infty+\gamma}^{i\infty+\gamma}\frac{515^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/2}}$$

for $-1<\gamma<0$

and also:

[1+1/2*1/(((1/20*(ln(4(4^3+16*4+1))^(3/2))))+((1+1/2*1/(((-4(4^3-16*4-1))))))]^1/2

Input:

$$\sqrt{1+\frac{1}{2}\times\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))}+\left(1+\frac{1}{2}\left(-\frac{1}{4(4^3-16\times 4-1)}\right)\right)}$$

log(x) is the natural logarithm

Exact result:

$$\sqrt{\frac{17}{8}+\frac{10}{\log^{3/2}(516)}}$$

Decimal approximation:

1.663008859505008597904347894923863133618463837199130168288...

1.6630088595…. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3$ = 1164.2696 i.e. 1.65578...

Property:

$\sqrt{\frac{17}{8} + \frac{10}{\log^{3/2}(516)}}$ is a transcendental number

Alternate forms:

$$\frac{\sqrt{\frac{1}{2}\left(80 + 17\log^{3/2}(516)\right)}}{2\log^{3/4}(516)}$$

$$\frac{\sqrt{\frac{1}{2}\left(80 + 17\left(2\log(2) + \log(129)\right)^{3/2}\right)}}{2\left(2\log(2) + \log(129)\right)^{3/4}}$$

$$\sqrt{\frac{17}{8} + \frac{10}{\left(2\log(2) + \log(3) + \log(43)\right)^{3/2}}}$$

All 2nd roots of 17/8 + 10/(log^(3/2)(516)):

$$e^{0}\sqrt{\frac{17}{8} + \frac{10}{\log^{3/2}(516)}} \approx 1.66301 \text{ (real, principal root)}$$

$$e^{i\pi}\sqrt{\frac{17}{8} + \frac{10}{\log^{3/2}(516)}} \approx -1.6630 \text{ (real root)}$$

Alternative representations:

$$\sqrt{1 + \frac{1}{\frac{1}{20}\log^{3/2}(4(4^3 + 16\times 4 + 1))\,2} + \left(1 + -\frac{1}{(4(4^3 - 16\times 4 - 1))\,2}\right)} = \sqrt{2 + \frac{1}{2(-4(-65 + 4^3))} + \frac{1}{\frac{2}{20}\log_e^{3/2}(4(65 + 4^3))}}$$

$$\sqrt{1 + \frac{1}{\frac{1}{20}\log^{3/2}(4(4^3 + 16\times 4 + 1))\,2} + \left(1 + -\frac{1}{(4(4^3 - 16\times 4 - 1))\,2}\right)} = \sqrt{2 + \frac{1}{2(-4(-65 + 4^3))} + \frac{1}{\frac{2}{20}\left(\log(a)\log_a(4(65 + 4^3))\right)^{3/2}}}$$

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}+\left(1+-\frac{1}{(4(4^3-16\times 4-1))\,2}\right)}=$$
$$\sqrt{2+\frac{1}{2(-4(-65+4^3))}+\frac{1}{\frac{2}{20}\left(-\mathrm{Li}_1(1-4(65+4^3))\right)^{3/2}}}$$

Series representations:

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}+\left(1+-\frac{1}{(4(4^3-16\times 4-1))\,2}\right)}=$$
$$\sqrt{\frac{17}{8}+\frac{10}{\left(\log(515)-\sum_{k=1}^{\infty}\frac{\left(-\frac{1}{515}\right)^k}{k}\right)^{3/2}}}$$

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}+\left(1+-\frac{1}{(4(4^3-16\times 4-1))\,2}\right)}=$$
$$\sqrt{\frac{17}{8}+\frac{10}{\left(2 i \pi \left\lfloor\frac{\arg(516-x)}{2\pi}\right\rfloor+\log(x)-\sum_{k=1}^{\infty}\frac{(-1)^k (516-x)^k x^{-k}}{k}\right)^{3/2}}} \quad \text{for } x<0$$

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}+\left(1+-\frac{1}{(4(4^3-16\times 4-1))\,2}\right)}=$$
$$\sqrt{\frac{17}{8}+\frac{10}{\left(\log(z_0)+\left\lfloor\frac{\arg(516-z_0)}{2\pi}\right\rfloor\left(\log\left(\frac{1}{z_0}\right)+\log(z_0)\right)-\sum_{k=1}^{\infty}\frac{(-1)^k (516-z_0)^k z_0^{-k}}{k}\right)^{3/2}}}$$

Integral representations:

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times 4+1))\,2}+\left(1+-\frac{1}{(4(4^3-16\times 4-1))\,2}\right)}=$$
$$\sqrt{\frac{17}{8}+\frac{10}{\left(\int_1^{516}\frac{1}{t}\,dt\right)^{3/2}}}$$

$$\sqrt{1+\frac{1}{\frac{1}{20}\log^{3/2}(4(4^3+16\times4+1))2}+\left(1+-\frac{1}{(4(4^3-16\times4-1))2}\right)} =$$

$$\sqrt{\frac{17}{8}+\frac{20\sqrt{2}\,\pi^{3/2}}{\left(-i\int_{-i\,\infty+\gamma}^{i\,\infty+\gamma}\frac{515^{-s}\,\Gamma(-s)^2\,\Gamma(1+s)}{\Gamma(1-s)}\,ds\right)^{3/2}}} \quad \text{for } -1<\gamma<0$$

Now, we have:

$\Omega_r := \pi^{\frac{r}{2}}/\Gamma\left(\frac{r}{2}+1\right)$, the volume of the $\mathbb{R}^r$-unit ball,

$$(1.9) \qquad c^{(2)}_{a,b} := \frac{\Omega_2}{12\cdot\widehat{h}(P^{(2)}_{\max})} \quad \text{and} \quad c^{(3)}_{a,b^3} := \frac{\Omega_3}{24\sqrt{3}\cdot\widehat{h}(P^{(3)}_{\max})^{\frac{3}{2}}},$$

where $P^{(2)}_{\max} \in \{(0,b),(-a,b)\} \subset E_{a,b}(\mathbb{Q})$ and $P^{(3)}_{\max} \in \{(0,b^3),(-a,b^3),(-b^2,ab)\} \subset E_{a,b^3}(\mathbb{Q})$ are chosen to have the largest canonical height.

$$r(E_{a,b}(\mathbb{Q})) \geq 2.$$

$$r(E_{a,b^3}(\mathbb{Q})) \geq 3.$$

Pi / gamma (2)

Input:

$$\frac{\pi}{\Gamma(2)}$$

$\Gamma(x)$ is the gamma function

Exact result:

π

Decimal approximation:

3.141592653589793238462643383279502884197169399375105820974...

3.14159265358…

Property:

π is a transcendental number

Pi^(3/2) / gamma (3/2+1)

Input:

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{4\pi}{3}$$

Decimal approximation:

4.18879020478639098461685784437267051226289253250014109463 3...

4.1887902047…

Property:

$\frac{4\pi}{3}$ is a transcendental number

Alternative representations:

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{\frac{3}{2}!}$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{e^{-\log G(5/2)+\log G(7/2)}}$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{(1)_{\frac{3}{2}}}$$

Series representations:

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{16}{3}\sum_{k=0}^{\infty}\frac{(-1)^k}{1+2k}$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \sum_{k=0}^{\infty}-\frac{16(-1)^k\, 1195^{-1-2k}\left(5^{1+2k}-4\times 239^{1+2k}\right)}{3(1+2k)}$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{4}{3}\sum_{k=0}^{\infty}\left(-\frac{1}{4}\right)^k\left(\frac{1}{1+2k}+\frac{2}{1+4k}+\frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{16}{3}\int_0^1\sqrt{1-t^2}\,dt$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \sqrt{3}+32\int_0^{\frac{1}{4}}\sqrt{-(-1+t)t}\,dt$$

$$\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)} = \frac{8}{3}\int_0^1\frac{1}{\sqrt{1-t^2}}\,dt$$

Now, from:

$$c_{a,b}^{(2)} := \frac{\Omega_2}{12\cdot\widehat{h}(P_{\max}^{(2)})}$$

we obtain:

(Pi / gamma (2)) / (12*2)

Input:

$$\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{\pi}{24}$$

Decimal approximation:

0.130899693899574718269276807636645953508215391640629409207...

0.13089969389...

Property:

$\frac{\pi}{24}$ is a transcendental number

Alternative representations:

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{\pi}{24}$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{\pi}{24\,e^0}$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{\pi}{24 \times 1!}$$

Series representations:

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{1}{6}\sum_{k=0}^{\infty} \frac{(-1)^k}{1+2k}$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \sum_{k=0}^{\infty} -\frac{(-1)^k\, 1195^{-1-2k} \left(5^{1+2k} - 4 \times 239^{1+2k}\right)}{6\,(1+2k)}$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{1}{24}\sum_{k=0}^{\infty} \left(-\frac{1}{4}\right)^k \left(\frac{1}{1+2k} + \frac{2}{1+4k} + \frac{1}{3+4k}\right)$$

Integral representations:

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{1}{6}\int_0^1 \sqrt{1-t^2}\,dt$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{1}{12}\int_0^1 \frac{1}{\sqrt{1-t^2}}\,dt$$

$$\frac{\pi}{(12 \times 2)\,\Gamma(2)} = \frac{1}{12}\int_0^{\infty} \frac{1}{1+t^2}\,dt$$

and from

$$c_{a,b^3}^{(3)} := \frac{\Omega_3}{24\sqrt{3}\cdot \widehat{h}(P_{\max}^{(3)})^{\frac{3}{2}}},$$

we obtain:

((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))

Input:

$$\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}$$

$\Gamma(x)$ is the gamma function

Result:

0.0193925472443814397435965640943179190382541320949080606 23...

0.0193925472…

Alternative representations:

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{\frac{3}{2}!\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{\pi^{3/2}}{(1)_{\frac{3}{2}}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

Series representations:

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\,\pi^{3/2}}{\sqrt{2}\left(\sum_{k=0}^{\infty} 2^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\,\pi^{3/2}}{\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x \in \mathbb{R}$ and $x < 0)$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\,\pi^{3/2}}{\sqrt{2}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{2}\right)^k\left(-\frac{1}{2}\right)_k}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0 > 0)$

Integral representations:

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\,\pi^{3/2}}{\sqrt{3}\int_0^{\infty}e^{-t}\,t^{3/2}\,dt}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\,\pi^{3/2}}{\sqrt{3}\int_0^{1}\log^{3/2}\left(\frac{1}{t}\right)dt}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)} = \frac{0.00801875\exp\left(-\int_0^1\frac{\frac{3}{2}-\frac{5x}{2}+x^{5/2}}{(-1+x)\log(x)}\,dx\right)\pi^{3/2}}{\sqrt{3}}$$

We obtain also:

(((((Pi / gamma (2)) / (12*2))))) * [((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))]

Input:

$$\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}\times\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}$$

$\Gamma(x)$ is the gamma function

Result:

0.0025384784982225716612228629114907796129922066376648124003...

0.002538478498…

Alternative representations:

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{\pi\,\pi^{3/2}}{24\times 1!\times \frac{3}{2}!\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{\pi\,\pi^{3/2}}{24\,e^{0}\,e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{\pi\,\pi^{3/2}}{24\,(1)_1\,(1)_{\frac{3}{2}}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

Series representations:

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\,\pi^{5/2}}{\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{x}\,\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}} \quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\,\pi^{5/2}\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\,\pi^{5/2}}{\sqrt{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\left(\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$

for $(z_0 \notin \mathbb{Z}$ or $z_0>0)$

Integral representations:

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\exp\left(-\int_0^1\frac{\frac{5}{2}-\frac{9x}{2}+x^2+x^{5/2}}{(-1+x)\log(x)}\,dx\right)\pi^{5/2}}{\sqrt{3}}$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\exp\left(\frac{9\gamma}{2} - \int_0^1 \frac{2-x^2-x^{5/2}+\log(x^2)+\log\left(x^{5/2}\right)}{\log(x)-x\log(x)}\,dx\right)\pi^{5/2}}{\sqrt{3}}$$

$$\frac{\pi^{3/2}\,\pi}{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\Gamma(2)\,(12\times 2)} = \frac{0.000334115\,\pi^{5/2}}{\left(\int_0^1 \log\left(\frac{1}{t}\right)dt\right)\left(\int_0^1 \log^{3/2}\left(\frac{1}{t}\right)dt\right)\sqrt{3}}$$

γ is the Euler-Mascheroni constant

From which:

sqrt((((1024*((((((((((Pi / gamma (2)) / (12*2))))) * [(((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5))))))])))))))

where 1024 = 64*16

Input:

$$\sqrt{1024\left(\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}\times\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}\right)}$$

$\Gamma(x)$ is the gamma function

Result:

1.612266101541526978305847340126360650919169998304658289395...

1.6122661015…. result that is a good approximation to the value of the golden ratio 1.618033988749...

All 2nd roots of 2.5994:

$1.61227\,e^{0} \approx 1.6123$ (real, principal root)

$1.61227\,e^{i\pi} \approx -1.6123$ (real root)

Alternative representations:

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{24\times 1!\times\frac{3}{2}!\,\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{24\,e^{0}\;e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{24\,(1)_1\,(1)_{\frac{3}{2}}\,\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

Series representations:

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{-1+\frac{0.342133\,\pi^{5/2}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{3}}}\sum_{k=0}^{\infty}\binom{\frac{1}{2}}{k}\left(-1+\frac{0.342133\,\pi^{5/2}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{3}}\right)^{-k}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{-1+\frac{0.342133\,\pi^{5/2}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{3}}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(-1+\frac{0.342133\,\pi^{5/2}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{3}}\right)^{-k}}{k!}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{z_0}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\left(\frac{0.342133\,\pi^{5/2}}{\Gamma(2)\,\Gamma\left(\frac{5}{2}\right)\sqrt{3}}-z_0\right)^k z_0^{-k}}{k!}$$

$\text{for (not } (z_0\in\mathbb{R} \text{ and } -\infty< z_0\le 0))$

Integral representations:

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}}=\sqrt{\frac{0.342133\,\exp\left(-\int_0^1\frac{\frac{5}{2}-\frac{9x}{2}+x^2+x^{5/2}}{(-1+x)\log(x)}\,dx\right)\pi^{5/2}}{\sqrt{3}}}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}} = \sqrt{\frac{0.342133\exp\left(\frac{9\gamma}{2}-\int_0^1 \frac{2-x^2-x^{5/2}+\log(x^2)+\log(x^{5/2})}{\log(x)-x\log(x)}\,dx\right)\pi^{5/2}}{\sqrt{3}}}$$

$$\sqrt{\frac{1024\,\pi\,\pi^{3/2}}{(\Gamma(2)\,(12\times 2))\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)}} = \sqrt{\frac{0.342133\,\pi^{5/2}}{\left(\int_0^1 \log\left(\frac{1}{t}\right)dt\right)\left(\int_0^1 \log^{3/2}\left(\frac{1}{t}\right)dt\right)\sqrt{3}}}$$

γ is the Euler-Mascheroni constant

and:

(((((Pi / gamma (2)) / (12*2))))) *1/ [(((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))]

Input:

$$\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}\times\frac{1}{\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}}$$

$\Gamma(x)$ is the gamma function

Result:

6.75

6.75

Alternative representations:

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{\pi}{\frac{24\times 1!\,\pi^{3/2}}{\frac{3}{2}!\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{\pi}{\frac{24\,e^{0}\,\pi^{3/2}}{e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{\pi}{\frac{24\,(1)_1\,\pi^{3/2}}{(1)_{\frac{3}{2}}\left(24\times 3^{1.5}\sqrt{3}\right)}}$$

Series representations:

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}{\sqrt{\pi}\;\Gamma(2)}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\,\Gamma\left(\frac{5}{2}\right)\left(\frac{1}{z_0}\right)^{1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{1/2+1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}{\sqrt{\pi}\;\Gamma(2)}$$

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\sqrt{2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{5}{2}-z_0\right)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!}}{\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

Integral representations:

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\exp\left(\int_0^1 -\frac{1+\sqrt{x}-2x^2}{2\log(x)+2\sqrt{x}\log(x)}\,dx\right)\sqrt{3}}{\sqrt{\pi}}$$

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\exp\left(-\frac{\gamma}{2}+\int_0^1\frac{x^2-x^{5/2}-\log\left(x^2\right)+\log\left(x^{5/2}\right)}{\log(x)-x\log(x)}\,dx\right)\sqrt{3}}{\sqrt{\pi}}$$

$$\frac{\pi}{\frac{\pi^{3/2}\,\Gamma(2)\,(12\times 2)}{\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}} = \frac{5.19615\sqrt{3}\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt}{\sqrt{\pi}\int_0^1\log\left(\frac{1}{t}\right)dt}$$

γ is the Euler-Mascheroni constant

[((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))] *1/ (((((Pi / gamma (2)) / (12*2)))))

Input:

$$\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}} \times \frac{1}{\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}}$$

$\Gamma(x)$ is the gamma function

Result:

0.148148148148148148148148148148148148148148148148148148148148...

Repeating decimal:

$0.\overline{148}$ (period 3)

0.148…

Rational approximation:

$$\frac{4}{27}$$

Alternative representations:

$$\frac{\pi^{3/2}}{\frac{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}{\Gamma(2)(12\times 2)}} = \frac{\pi^{3/2}}{\frac{\frac{3}{2}!\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\times 1!}}$$

$$\frac{\pi^{3/2}}{\frac{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}{\Gamma(2)(12\times 2)}} = \frac{\pi^{3/2}}{\frac{e^{-\log G(5/2)+\log G(7/2)}\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\,e^{0}}}$$

$$\frac{\pi^{3/2}}{\frac{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}{\Gamma(2)(12\times 2)}} = \frac{\pi^{3/2}}{\frac{(1)_{\frac{3}{2}}\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\,(1)_1}}$$

Series representations:

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\sqrt{\pi}\;\Gamma(2)}{\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for ($x \in \mathbb{R}$ and $x < 0$)

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\sqrt{\pi}\;\Gamma(2)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{\Gamma\left(\frac{5}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}$$

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\sqrt{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}}$$

for ($z_0 \notin \mathbb{Z}$ or $z_0 > 0$)

Integral representations:

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\exp\left(\int_0^1\frac{1+\sqrt{x}-2x^2}{2\log(x)+2\sqrt{x}\log(x)}\,dx\right)\sqrt{\pi}}{\sqrt{3}}$$

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\exp\left(\frac{\gamma}{2}+\int_0^1\frac{x^2-x^{5/2}-\log\left(x^2\right)+\log\left(x^{5/2}\right)}{(-1+x)\log(x)}\,dx\right)\sqrt{\pi}}{\sqrt{3}}$$

$$\frac{\frac{\pi^{3/2}}{\pi\,\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)}}{\Gamma(2)(12\times 2)} = \frac{0.19245\sqrt{\pi}\int_0^1\log\left(\frac{1}{t}\right)dt}{\sqrt{3}\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt}$$

γ is the Euler-Mascheroni constant

From which:

11*((([((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))] *1/ (((((Pi / gamma (2)) / (12*2))))))))

where 11 is a Lucas number

Input:

$$11\left(\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}\times\frac{1}{\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}}\right)$$

$\Gamma(x)$ is the gamma function

Result:

1.629629629629629629629629629629629629629629629629629629629...

Repeating decimal:

$1.\overline{629}$ (period 3)

1.629…

Alternative representations:

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\,3^{1.5}\right)\right)\pi}{\Gamma(2)\,(12\times 2)}}=\frac{11\,\pi^{3/2}}{\frac{\frac{3}{2}!\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\times 1!}}$$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\,3^{1.5}\right)\right)\pi}{\Gamma(2)\,(12\times 2)}}=\frac{11\,\pi^{3/2}}{\frac{e^{-\log G(5/2)+\log G(7/2)}\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\,e^{0}}}$$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\,3^{1.5}\right)\right)\pi}{\Gamma(2)\,(12\times 2)}}=\frac{11\,\pi^{3/2}}{\frac{(1)_{\frac{3}{2}}\,\pi\left(24\times 3^{1.5}\sqrt{3}\right)}{24\,(1)_1}}$$

Series representations:

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\,3^{1.5}\right)\right)\pi}{\Gamma(2)\,(12\times 2)}}=\frac{2.11695\sqrt{\pi}\,\Gamma(2)}{\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}$$

for $(x\in\mathbb{R}$ and $x<0)$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\,3^{1.5}\right)\right)\pi}{\Gamma(2)\,(12\times 2)}}=\frac{2.11695\sqrt{\pi}\,\Gamma(2)\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{\Gamma\left(\frac{5}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}$$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\pi}{\Gamma(2)(12\times 2)}} = \frac{2.11695\sqrt{\pi}\;\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}}{\sqrt{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\,\Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

Integral representations:

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\pi}{\Gamma(2)(12\times 2)}} = \frac{2.11695\exp\left(\int_0^1 \frac{1+\sqrt{x}-2x^2}{2\log(x)+2\sqrt{x}\,\log(x)}\,dx\right)\sqrt{\pi}}{\sqrt{3}}$$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\pi}{\Gamma(2)(12\times 2)}} = \frac{2.11695\exp\left(\frac{\gamma}{2}+\int_0^1 \frac{x^2-x^{5/2}-\log\left(x^2\right)+\log\left(x^{5/2}\right)}{(-1+x)\log(x)}\,dx\right)\sqrt{\pi}}{\sqrt{3}}$$

$$\frac{11\,\pi^{3/2}}{\frac{\left(\Gamma\left(\frac{3}{2}+1\right)\left(24\sqrt{3}\;3^{1.5}\right)\right)\pi}{\Gamma(2)(12\times 2)}} = \frac{2.11695\sqrt{\pi}\;\int_0^1\log\left(\frac{1}{t}\right)dt}{\sqrt{3}\;\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt}$$

γ is the Euler-Mascheroni constant

[((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))] + (((((Pi / gamma (2)) / (12*2)))))

Input:

$$\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}} + \frac{\frac{\pi}{\Gamma(2)}}{12\times 2}$$

$\Gamma(x)$ is the gamma function

Result:

0.1502922...

0.1502922…

Alternative representations:

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\frac{\pi}{24\times 1!}+\frac{\pi^{3/2}}{\frac{3}{2}!\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\frac{\pi}{24\,e^{0}}+\frac{\pi^{3/2}}{e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\frac{\pi}{24\,(1)_1}+\frac{\pi^{3/2}}{(1)_{\frac{3}{2}}\left(24\times 3^{1.5}\sqrt{3}\right)}$$

Series representations:

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\frac{\pi}{24\,\Gamma(2)}+\frac{0.00801875\,\pi^{3/2}}{\exp\left(i\pi\left\lfloor\frac{\arg(3-x)}{2\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k(3-x)^k x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\frac{\pi}{24\,\Gamma(2)}+\frac{0.00801875\,\pi^{3/2}\left(\frac{1}{z_0}\right)^{-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor} z_0^{-1/2-1/2\lfloor\arg(3-z_0)/(2\pi)\rfloor}}{\Gamma\left(\frac{5}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k(3-z_0)^k z_0^{-k}}{k!}}$$

$$\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}=\left(0.00801875\,\pi\left(\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}+5.19615\sqrt{2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{5}{2}-z_0\right)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!}\right)\right)\Bigg/\left(\sqrt{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\left(\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)$$

for $(z_0\notin\mathbb{Z}$ or $z_0>0)$

Integral representations:

$$\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\Gamma(2)}=\frac{0.00710644\oint_L \frac{e^t}{t^{5/2}}\,dt+0.0360844\oint_L \frac{e^t}{t^2}\,dt}{i\sqrt{3}}$$

$$\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\Gamma(2)}=\frac{0.00801875\,\pi\left(\sqrt{\pi}\int_0^1\log\left(\frac{1}{t}\right)dt+5.19615\sqrt{3}\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt\right)}{\left(\int_0^1\log\left(\frac{1}{t}\right)dt\right)\left(\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt\right)\sqrt{3}}$$

$$\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\Gamma(2)}=\frac{0.00801875\,\pi\left(\sqrt{\pi}\int_0^\infty e^{-t}\,t\,dt+5.19615\sqrt{3}\int_0^\infty e^{-t}\,t^{3/2}\,dt\right)}{\left(\int_0^\infty e^{-t}\,t\,dt\right)\left(\int_0^\infty e^{-t}\,t^{3/2}\,dt\right)\sqrt{3}}$$

11 * ((([((((Pi^(3/2) / gamma (3/2+1))) / ((24*sqrt3*(3)^(1.5)))))] + (((((Pi / gamma (2)) / (12*2))))))))

where 11 is a Lucas number and dimensions number of M-Theory

Input:

$$11\left(\frac{\frac{\pi^{3/2}}{\Gamma\left(\frac{3}{2}+1\right)}}{24\sqrt{3}\times 3^{1.5}}+\frac{\frac{\pi}{\Gamma(2)}}{12\times 2}\right)$$

$\Gamma(x)$ is the gamma function

Result:

1.653214652583517738141607089040602598011164761090912168136...

1.653214652583517…. result very near to the 14th root of the following Ramanujan's class invariant $Q=\left(G_{505}/G_{101/5}\right)^3=1164.2696$ i.e. 1.65578...

Alternative representations:

$$11\left(\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\Gamma(2)}\right)=11\left(\frac{\pi}{24\times 1!}+\frac{\pi^{3/2}}{\frac{3}{2}!\,(24\times 3^{1.5}\sqrt{3})}\right)$$

$$11\left(\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$11\left(\frac{\pi}{24\,e^{0}}+\frac{\pi^{3/2}}{e^{-\log G(5/2)+\log G(7/2)}\left(24\times 3^{1.5}\sqrt{3}\right)}\right)$$

$$11\left(\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=11\left(\frac{\pi}{24\,(1)_1}+\frac{\pi^{3/2}}{(1)_{\frac{3}{2}}\left(24\times 3^{1.5}\sqrt{3}\right)}\right)$$

Series representations:

$$11\left(\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$\frac{11\,\pi}{24\,\Gamma(2)}+\frac{0.0882063\,\pi^{3/2}}{\exp\left(i\,\pi\left\lfloor\frac{\arg(3-x)}{2\,\pi}\right\rfloor\right)\Gamma\left(\frac{5}{2}\right)\sqrt{x}\ \sum_{k=0}^{\infty}\frac{(-1)^k\,(3-x)^k\,x^{-k}\left(-\frac{1}{2}\right)_k}{k!}}\quad \text{for } (x\in\mathbb{R} \text{ and } x<0)$$

$$11\left(\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$\frac{11\,\pi}{24\,\Gamma(2)}+\frac{0.0882063\,\pi^{3/2}\left(\frac{1}{z_0}\right)^{-1/2\,\lfloor\arg(3-z_0)/(2\,\pi)\rfloor}z_0^{-1/2-1/2\,\lfloor\arg(3-z_0)/(2\,\pi)\rfloor}}{\Gamma\left(\frac{5}{2}\right)\sum_{k=0}^{\infty}\frac{(-1)^k\left(-\frac{1}{2}\right)_k\,(3-z_0)^k\,z_0^{-k}}{k!}}$$

$$11\left(\frac{\pi^{3/2}}{\left(24\sqrt{3}\ 3^{1.5}\right)\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$\left(0.0882063\,\pi\left(\sqrt{\pi}\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}+5.19615\right.\right.$$
$$\left.\left.\sqrt{2}\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{2^{-k_1}\binom{\frac{1}{2}}{k_1}\left(\frac{5}{2}-z_0\right)^{k_2}\Gamma^{(k_2)}(z_0)}{k_2!}\right)\right)\Bigg/$$
$$\left(\sqrt{2}\left(\sum_{k=0}^{\infty}2^{-k}\binom{\frac{1}{2}}{k}\right)\left(\sum_{k=0}^{\infty}\frac{(2-z_0)^k\,\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}-z_0\right)^k\Gamma^{(k)}(z_0)}{k!}\right)$$
$$\text{for } (z_0\notin\mathbb{Z} \text{ or } z_0>0)$$

Integral representations:

$$11\left(\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\,\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=\frac{0.0781708\oint_L \frac{e^t}{t^{5/2}}\,dt+0.396928\oint_L \frac{e^t}{t^2}\,dt}{i\sqrt{3}}$$

$$11\left(\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\,\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$\frac{0.0882063\,\pi\left(\sqrt{\pi}\int_0^1\log\left(\frac{1}{t}\right)dt+5.19615\sqrt{3}\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt\right)}{\left(\int_0^1\log\left(\frac{1}{t}\right)dt\right)\left(\int_0^1\log^{3/2}\left(\frac{1}{t}\right)dt\right)\sqrt{3}}$$

$$11\left(\frac{\pi^{3/2}}{(24\sqrt{3}\ 3^{1.5})\,\Gamma\left(\frac{3}{2}+1\right)}+\frac{\pi}{(12\times 2)\,\Gamma(2)}\right)=$$
$$\frac{0.0882063\,\pi\left(\sqrt{\pi}\int_0^\infty e^{-t}\,t\,dt+5.19615\sqrt{3}\int_0^\infty e^{-t}\,t^{3/2}\,dt\right)}{\left(\int_0^\infty e^{-t}\,t\,dt\right)\left(\int_0^\infty e^{-t}\,t^{3/2}\,dt\right)\sqrt{3}}$$

From:

Superstringhe oltre un loop - Tesi di Laurea in Fisica – *Candidato Giovanni RICCO – Relatore Prof. Augusto SAGNOTTI* **-** Anno Accademico 2004-2005 – arXiv:hep-th/0507184v1 19 Jul 2005

The Shapiro-Virasoro amplitude is:

$$C\sim\frac{\Gamma(-1-\alpha't/4)\Gamma(-1-\alpha'u/4)\Gamma(-1-\alpha's/4)}{\Gamma(-2-\alpha't/4-\alpha'u/4)\Gamma(-2-\alpha't/4-\alpha's/4)\Gamma(-2-\alpha'u/4-\alpha's/4)}\ . \quad (3.85)$$

For:

$$\alpha's,\alpha't,\alpha'u=-4,0,4,8,\ldots\ , \quad (3.82)$$

For: $\alpha' t = 4$; $\alpha' s = 0$; $\alpha' u = 8$

From

$$C \sim \frac{\Gamma(-1-\alpha' t/4)\Gamma(-1-\alpha' u/4)\Gamma(-1-\alpha' s/4)}{\Gamma(-2-\alpha' t/4-\alpha' u/4)\Gamma(-2-\alpha' t/4-\alpha' s/4)\Gamma(-2-\alpha' u/4-\alpha' s/4)} .$$

we obtain:

(((gamma (-1-1) gamma (-1-2) gamma (-1))))) / (((gamma (-2-1-2) gamma (-2-1)) gamma (-2-2)))))

Input:

$$\frac{-(\Gamma(-1-1)\,\Gamma(-1-2)\,\Gamma(-1))}{-((\Gamma(-2-1-2)\,\Gamma(-2-1))\,\Gamma(-2-2))}$$

Input:

$$\frac{\Gamma(-2)\,\Gamma(-3)\,\Gamma(-1)}{(\Gamma(-5)\,\Gamma(-3))\,\Gamma(-4)}$$

Multiplying by – 1 , thus changing the sign, we obtain:

(((gamma (2) gamma (3) gamma (1)))) / (((gamma (5) gamma (3)) gamma (4)))

Input:

$$\frac{\Gamma(2)\,\Gamma(3)\,\Gamma(1)}{(\Gamma(5)\,\Gamma(3))\,\Gamma(4)}$$

$\Gamma(x)$ is the gamma function

Exact result:

$$\frac{1}{144}$$

Decimal approximation:

0.006944444444444444444444444444444444444444444444444444...

0.00694444…

Alternative representations:

$$\frac{\Gamma(2)\,(\Gamma(3)\,\Gamma(1))}{(\Gamma(5)\,\Gamma(3))\,\Gamma(4)} = \frac{2}{\frac{3456}{12}}$$

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \frac{0!\times 1!\times 2!}{2!\times 3!\times 4!}$$

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \frac{(e^0)^2 e^{\log(2)}}{e^{\log(2)} e^{-\log(2)+\log(12)} e^{-\log(12)+\log(288)}}$$

Series representations:

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \frac{\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}\frac{(1-z_0)^{k_1}(2-z_0)^{k_2}\Gamma^{(k_1)}(z_0)\Gamma^{(k_2)}(z_0)}{k_1!k_2!}}{\left(\sum_{k=0}^{\infty}\frac{(4-z_0)^k\Gamma^{(k)}(z_0)}{k!}\right)\sum_{k=0}^{\infty}\frac{(5-z_0)^k\Gamma^{(k)}(z_0)}{k!}} \quad \text{for } (z_0 \notin \mathbb{Z} \text{ or } z_0 > 0)$$

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \left(\sum_{k_1=0}^{\infty}\sum_{k_2=0}^{\infty}(4-z_0)^{k_1}(5-z_0)^{k_2}\sum_{j_1=0}^{k_1}\sum_{j_2=0}^{k_2}\left((-1)^{j_1+j_2}\pi^{-j_1-j_2+k_1+k_2}\sin\left(\frac{1}{2}\pi(-j_1+k_1+2z_0)\right)\sin\left(\frac{1}{2}\pi(-j_2+k_2+2z_0)\right)\Gamma^{(j_1)}(1-z_0)\Gamma^{(j_2)}(1-z_0)\right)\Big/(j_1!j_2!(-j_1+k_1)!(-j_2+k_2)!)\right)\Big/ \left(\left(\sum_{k=0}^{\infty}(1-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)\sum_{k=0}^{\infty}(2-z_0)^k\sum_{j=0}^{k}\frac{(-1)^j\pi^{-j+k}\sin\left(\frac{1}{2}\pi(-j+k+2z_0)\right)\Gamma^{(j)}(1-z_0)}{j!(-j+k)!}\right)$$

Integral representations:

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \int_0^1\int_0^1\log\left(\frac{1}{t_2}\right)dt_2\,dt_1$$

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = e^{\int_0^1 -\frac{-6+x+2x^2+2x^3+x^4}{\log(x)}dx}$$

$$\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)} = \exp\left(6\gamma + \int_0^1\frac{x+x^2-x^4-x^5-\log(x)-\log(x^2)+\log(x^4)+\log(x^5)}{(-1+x)\log(x)}dx\right)$$

γ is the Euler-Mascheroni constant

From

$$\frac{\Gamma(2)\,(\Gamma(3)\,\Gamma(1))}{(\Gamma(5)\,\Gamma(3))\,\Gamma(4)} = \exp\left(6\,\gamma + \int_0^1 \frac{x + x^2 - x^4 - x^5 - \log(x) - \log(x^2) + \log(x^4) + \log(x^5)}{(-1 + x)\log(x)}\,dx\right)$$

we obtain also:

-(8*(sqrt(3/5)))/(((integral_0^1 (x + x^2 - x^4 - x^5 - log(x) - log(x^2) + log(x^4) + log(x^5))/((-1 + x) log(x)) dx)))

where 8 is a Fibonacci number

Input:

$$-\frac{8\sqrt{\frac{3}{5}}}{\int_0^1 \frac{x+x^2-x^4-x^5-\log(x)-\log(x^2)+\log(x^4)+\log(x^5)}{(-1+x)\log(x)}\,dx}$$

log(x) is the natural logarithm

Computation result:

$$-\frac{8\sqrt{\frac{3}{5}}}{\int_0^1 \frac{x+x^2-x^4-x^5-\log(x)-\log(x^2)+\log(x^4)+\log(x^5)}{(-1+x)\log(x)}\,dx} = -\frac{8\sqrt{\frac{3}{5}}}{-6\,\gamma - \log(1440)}$$

γ is the Euler-Mascheroni constant

Decimal approximation:

0.577212268519696854810822850701945598731646592054165732301...

0.5772122685… result equal to the Euler-Mascheroni Constant

From the initial expression, we obtain also:

12*1/((((((gamma (2) gamma (3) gamma (1)))) / (((gamma (5) gamma (3)) gamma (4))))))+1

Input:

$$12 \times \frac{1}{\frac{\Gamma(2)\,\Gamma(3)\,\Gamma(1)}{(\Gamma(5)\,\Gamma(3))\,\Gamma(4)}} + 1$$

$\Gamma(x)$ is the gamma function

Result:

1729

1729

This result is very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number)

Alternative representations:

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=1+\frac{12}{\frac{2}{\frac{3456}{12}}}$$

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=1+\frac{12}{\frac{0!\times 1!\times 2!}{2!\times 3!\times 4!}}$$

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=1+\frac{12}{\frac{(e^0)^2\,e^{\log(2)}}{e^{\log(2)}\,e^{-\log(2)+\log(12)}\,e^{-\log(12)+\log(288)}}}$$

Integral representations:

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=1+12\,e^{\int_0^1 \frac{-6+x+2x^2+2x^3+x^4}{\log(x)}\,dx}$$

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=$$
$$1+12\exp\left(-6\gamma+\int_0^1 \frac{x+x^2-x^4-x^5-\log(x)-\log(x^2)+\log(x^4)+\log(x^5)}{\log(x)-x\log(x)}\,dx\right)$$

$$\frac{12}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}+1=\frac{\int_0^1\int_0^1 \log\left(\frac{1}{t_2}\right)dt_2\,dt_1+\int_0^1\int_0^1 \log^3\left(\frac{1}{t_1}\right)\log^4\left(\frac{1}{t_2}\right)dt_2\,dt_1}{\left(\int_0^1 1\,dt\right)\int_0^1 \log\left(\frac{1}{t}\right)dt}$$

γ is the Euler-Mascheroni constant

and:

1/((((((gamma (2) gamma (3) gamma (1)))) / (((gamma (5) gamma (3)) gamma (4))))))-5

where 5 is a Fibonacci number

Input:

$$\frac{1}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5$$

$\Gamma(x)$ is the gamma function

Result:

139

139 result practically equal to the rest mass of Pion meson 139.57 MeV

Alternative representations:

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5=-5+\frac{1}{\frac{\frac{2}{3456}}{12}}$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5=-5+\frac{1}{\frac{0!\times 1!\times 2!}{2!\times 3!\times 4!}}$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5=-5+\frac{1}{\frac{(e^0)^2 e^{\log(2)}}{e^{\log(2)} e^{-\log(2)+\log(12)} e^{-\log(12)+\log(288)}}}$$

Integral representations:

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5=-5+\exp\left(\int_0^1 \frac{(-1+x)(6+5x+3x^2+x^3)}{\log(x)}\,dx\right)$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}}-5=$$
$$-5+\exp\left(-6\gamma+\int_0^1 \frac{x+x^2-x^4-x^5-\log(x)-\log(x^2)+\log(x^4)+\log(x^5)}{\log(x)-x\log(x)}\,dx\right)$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 5 = -\frac{\int_0^1\int_0^1 \log\left(\frac{1}{t_2}\right) dt_2\, dt_1 + \int_0^1\int_0^1 \log^3\left(\frac{1}{t_1}\right)\log^4\left(\frac{1}{t_2}\right) dt_2\, dt_1}{\left(\int_0^1 1\, dt\right)\int_0^1 \log\left(\frac{1}{t}\right) dt}$$

γ is the Euler-Mascheroni constant

1/((((((gamma (2) gamma (3) gamma (1)))) / (((gamma (5) gamma (3)) gamma (4))))))-21+2

where 21 and 2 are Fibonacci numbers

Input:

$$\frac{1}{\frac{\Gamma(2)\Gamma(3)\Gamma(1)}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2$$

$\Gamma(x)$ is the gamma function

Result:

125

125 result very near to the Higgs boson mass 125.18 GeV

Alternative representations:

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 = -19 + \frac{1}{\frac{\frac{2}{3456}}{12}}$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 = -19 + \frac{1}{\frac{0!\times 1!\times 2!}{2!\times 3!\times 4!}}$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 = -19 + \frac{1}{\frac{(e^0)^2 e^{\log(2)}}{e^{\log(2)} e^{-\log(2)+\log(12)} e^{-\log(12)+\log(288)}}}$$

Integral representations:

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 = -19 + \exp\left(\int_0^1 \frac{(-1+x)(6+5x+3x^2+x^3)}{\log(x)}\,dx\right)$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 =$$
$$-19 + \exp\left(-6\gamma + \int_0^1 \frac{x + x^2 - x^4 - x^5 - \log(x) - \log(x^2) + \log(x^4) + \log(x^5)}{\log(x) - x\log(x)}\,dx\right)$$

$$\frac{1}{\frac{\Gamma(2)(\Gamma(3)\Gamma(1))}{(\Gamma(5)\Gamma(3))\Gamma(4)}} - 21 + 2 = -\frac{\int_0^1\int_0^1 \log\left(\frac{1}{t_2}\right)dt_2\,dt_1 + \int_0^1\int_0^1 \log^3\left(\frac{1}{t_1}\right)\log^4\left(\frac{1}{t_2}\right)dt_2\,dt_1}{\left(\int_0^1 1\,dt\right)\int_0^1 \log\left(\frac{1}{t}\right)dt}$$

γ is the Euler-Mascheroni constant

Observations

From:
https://www.scientificamerican.com/article/mathematics-ramanujan/?fbclid=IwAR2caRXrn_RpOSvJ1QxWsVLBcJ6KVgd_Af_hrmDYBNyU8mpSjRs1BDeremA

Ramanujan's statement concerned the deceptively simple concept of partitions—the different ways in which a whole number can be subdivided into smaller numbers. Ramanujan's original statement, in fact, stemmed from the observation of patterns, such as the fact that p(9) = 30, p(9 + 5) = 135, p(9 + 10) = 490, p(9 + 15) = 1,575 and so on are all divisible by 5. Note that here the n's come at intervals of five units.

Ramanujan posited that this pattern should go on forever, and that similar patterns exist when 5 is replaced by 7 or 11—there are infinite sequences of p(n) that are all divisible by 7 or 11, or, as mathematicians say, in which the "moduli" are 7 or 11.

Then, in nearly oracular tone Ramanujan went on: "There appear to be corresponding properties," he wrote in his 1919 paper, "in which the moduli are powers of 5, 7 or 11...and no simple properties for any moduli involving primes other than these three." (Primes are whole numbers that are only divisible by themselves or by 1.) Thus, for instance, there should be formulas for an infinity of n's separated by 5^3 = 125 units, saying that the corresponding p(n)'s should all be divisible by 125.
In the past methods developed to understand partitions have later been applied to physics problems such as the theory of the strong nuclear force or the entropy of black holes.

From Wikipedia

In particle physics, Yukawa's interaction or Yukawa coupling, named after Hideki Yukawa, is an interaction between a scalar field ϕ and a Dirac field ψ. The Yukawa interaction can be used to describe the nuclear force between nucleons (which are fermions), mediated by pions (which are pseudoscalar mesons). The Yukawa interaction is also used in the Standard Model to describe the coupling between the Higgs field and massless quark and lepton fields (i.e., the fundamental fermion particles). Through spontaneous symmetry breaking, these fermions acquire a mass proportional to the vacuum expectation value of the Higgs field.

Can be this the motivation that from the development of the Ramanujan's equations we obtain results very near to the dilaton mass calculated as a type of Higgs boson: 125 GeV for T = 0 and to the Higgs boson mass 125.18 GeV and practically equal to the rest mass of Pion meson 139.57 MeV

Note that:

$$g_{22} = \sqrt{(1+\sqrt{2})}.$$

Hence

$$\begin{aligned} 64g_{22}^{24} &= e^{\pi\sqrt{22}} - 24 + 276e^{-\pi\sqrt{22}} - \cdots, \\ 64g_{22}^{-24} &= 4096e^{-\pi\sqrt{22}} + \cdots, \end{aligned}$$

so that

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}.$$

Hence

$$e^{\pi\sqrt{22}} = 2508951.9982\ldots.$$

Thence:

$$64g_{22}^{-24} = 4096e^{-\pi\sqrt{22}} + \cdots$$

And

$$64(g_{22}^{24} + g_{22}^{-24}) = e^{\pi\sqrt{22}} - 24 + 4372e^{-\pi\sqrt{22}} + \cdots = 64\{(1+\sqrt{2})^{12} + (1-\sqrt{2})^{12}\}$$

That are connected with 64, 128, 256, 512, 1024 and 4096 = 64^2

(Modular equations and approximations to π - S. Ramanujan - Quarterly Journal of Mathematics, XLV, 1914, 350 – 372)

All the results of the most important connections are signed in blue throughout the drafting of the paper. We highlight as in the development of the various equations we use always the constants π, ϕ, 1/ϕ, the Fibonacci and Lucas numbers, linked to the *golden ratio, that play a fundamental role in the development, and therefore, in the final results of the analyzed expressions.*

In mathematics, the Fibonacci numbers, commonly denoted F_n, form a sequence, called the Fibonacci sequence, such that each number is the sum of the two preceding ones, starting from 0 and 1. Fibonacci numbers are strongly related to the golden ratio: Binet's formula expresses the nth Fibonacci number in terms of n and the golden ratio, and implies that the ratio of two consecutive Fibonacci numbers tends to the golden ratio as n increases.
Fibonacci numbers are also closely related to Lucas numbers ,in that the Fibonacci and Lucas numbers form a complementary pair of Lucas sequences

The beginning of the sequence is thus:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155...

The Lucas numbers or Lucas series are an integer sequence named after the mathematician François Édouard Anatole Lucas (1842–91), who studied both that sequence and the closely related Fibonacci numbers. Lucas numbers and Fibonacci numbers form complementary instances of Lucas sequences.

The Lucas sequence has the same recursive relationship as the Fibonacci sequence, where each term is the sum of the two previous terms, but with different starting values. This produces a sequence where the ratios of successive terms approach the golden ratio, and in fact the terms themselves are roundings of integer powers of the golden ratio.[1] The sequence also has a variety of relationships with the Fibonacci numbers, like the fact that adding any two Fibonacci numbers two terms apart in the Fibonacci sequence results in the Lucas number in between.

The sequence of Lucas numbers is:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, 1860498, 3010349, 4870847, 7881196, 12752043, 20633239, 33385282, 54018521, 87403803……

All Fibonacci-like integer sequences appear in shifted form as a row of the Wythoff array; the Fibonacci sequence itself is the first row and the Lucas sequence is the *second row. Also like all Fibonacci-like integer sequences, the ratio between two consecutive Lucas numbers converges to the golden ratio.*

A Lucas prime is a Lucas number that is prime. The first few Lucas primes are:
2, 3, 7, 11, 29, 47, 199, 521, 2207, 3571, 9349, 3010349, 54018521, 370248451, 6643838879, ... (sequence A005479 in the OEIS).

In geometry, a golden spiral is a logarithmic spiral whose growth factor is φ, the golden ratio.[1] That is, a golden spiral gets wider (or further from its origin) by a factor of φ for every quarter turn it makes. Approximate logarithmic spirals can occur in nature, for example the arms of spiral galaxies[3] - golden spirals are one special case of these logarithmic spirals

We observe that 1728 and 1729 are results very near to the mass of candidate glueball **f_0(1710) scalar meson.** Furthermore, 1728 occurs in the algebraic formula for the j-invariant of an elliptic curve. As a consequence, it is sometimes called a Zagier as a pun on the Gross–Zagier theorem. The number 1728 is one less than the Hardy–Ramanujan number 1729 (taxicab number).

Furthermore, we obtain as results of our computations, always values very near to the Higgs boson mass 125.18 GeV and practically equals to the rest mass of Pion meson 139.57 MeV. In conclusion we obtain also many results that are very good approximations to the value of the golden ratio 1.618033988749... and to ζ(2) = $\frac{\pi^2}{6} = 1.644934 \ldots$

We note how the following three values: 137.508 (golden angle), 139.57 (mass of the Pion - meson Pi) and 125.18 (mass of the Higgs boson), are connected to each other. In fact, just add 2 to 137.508 to obtain a result very close to the mass of the Pion and subtract 12 to 137.508 to obtain a result that is also very close to the mass of the Higgs boson. We can therefore hypothesize that it is the golden angle (and the related golden ratio inherent in it) to be a fundamental ingredient both in the structures of the microcosm and in those of the macrocosm.

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