Conservation Property of Standing Wave

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The standing wave exists in a microwave resonator if the length of the resonator cavity is equal to multiple half-wavelengths of microwave. The stationary interference of standing wave will travel in another inertial reference frame. The vibrating pattern of the standing wave is conserved. The existence of nodes in all reference frames requires the wavelength of the microwave to be conserved in all inertial reference frames. The angular frequency of microwave is different in every reference frame. Hence, the apparent velocity of the microwave depends on the choice of reference frame while the elapsed time remains invariant in all reference frames.

I. INTRODUCTION

A standing wave exists between the wave transmitter and the reflector if the distance between the transmitter and the reflector is equal to multiple half-wavelengths. The standing wave exhibits the first harmonic if the distance is equal to one half of the wavelength. The standing wave exhibits the second harmonic if the distance is equal to the wavelength.

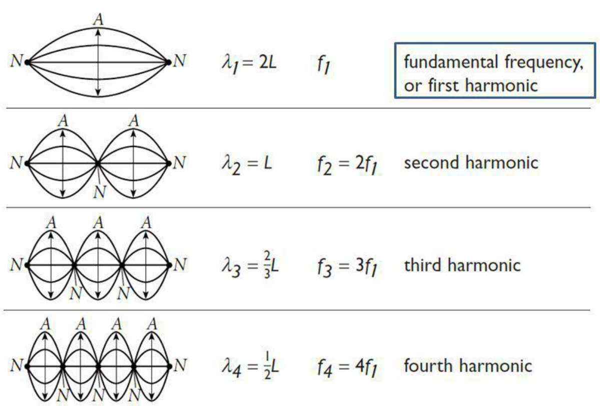

FIG. 1. Standing Wave harmonic

The stationary pattern of standing wave will move in another inertial reference frame. The frequency and the wavelength of the microwave may depend on the choice of reference frame. However, the interference pattern of standing wave is conserved in all inertial reference frames. The nodes of the standing wave exist in all inertial reference frames. Therefore, the new frequency and wavelength in a new inertial reference frame can be derived from the conservation of node precisely.

II. PROOF

Consider one dimensional motion.

A. Standing Wave

The interference pattern of standing wave created by two identical waves travelling in the opposite direction can be represented as

$$sin(k'x' - w't') + sin(-k'x' - w't') \tag{1}$$

Let F_2 be an inertial reference frame moving at the velocity of -V relative to F_1, the rest frame of all nodes. In F_2, the standing wave can be represented as

$$sin(k_1x - W_1t) + sin(-k_2x - W_2t) \tag{2}$$

Define W_+ and W_- as

$$W_+ = \frac{W_1 + W_2}{2} \tag{3}$$

$$W_- = \frac{W_1 - W_2}{2} \tag{4}$$

From equations (2,3,4), the standing wave is

$$sin(k_1x - W_+t - W_-t) \tag{5}$$

$$+sin(-k_2x - W_+t + W_-t) \tag{6}$$

Define k_+ and k_- as

$$k_+ = \frac{k_1 + k_2}{2} \tag{7}$$

$$k_- = \frac{k_1 - k_2}{2} \tag{8}$$

From equations (5,6,7,8), the standing wave is

$$sin(k_+x + k_-x - W_+t - W_-t) \tag{9}$$

$$+sin(-k_+x + k_-x - W_+t + W_-t) \tag{10}$$

Apply trigonometry identity.

$$sin(A + B) = sin(A)cos(B) + sin(B)cos(A) \tag{11}$$

From equations (9,10,11), the standing wave in F_2 is

$$2sin(k_-x - W_+t)cos(k_+x - W_-t) \tag{12}$$

From equations (1,11), the standing wave in F_1 is

$$2sin(-w't')cos(k'x') \tag{13}$$

$2sin(-w't')$ is the amplitude of the static wave $cos(k'x')$.

In F_2, all nodes travel at the velocity of V. The static wave of equation (13) can be represented in F_2 as

$$2sin(-Wt)cos(kx - wt) \tag{14}$$

$$\frac{w}{k} = V \tag{15}$$

From equations (12,14) with the initial condition that $0 = x = x' = t = t'$,

$$k_- = 0 \tag{16}$$

$$W = W_+ \tag{17}$$

$$k = k_+ \tag{18}$$

$$w = W_- \tag{19}$$

B. Microwave Resonance

Let a microwave transmitter and a reflector plate be stationary relative to F_1. The microwave is emitted in the positive x direction toward the reflector which is in the y-z plane. A standing wave can be formed by adjusting the distance between the transmitter and the reflector until the distance is equal to multiple half wavelengths[1].

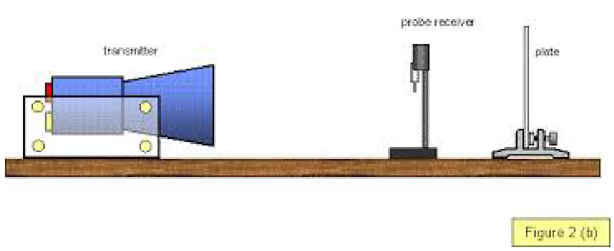

FIG. 2. Microwave Transmitter and Reflector Plate

In F_2, the standing wave and the reflector are moving at the velocity of V. One microwave is represented by k_1 and W_1. The other microwave is represented by k_2 and W_2.

From equations (7,8,16,18),

$$k_1 = k_2 = k \tag{20}$$

The wavelengths of both microwaves are identical in F_2.

From equations (3,17),

$$W_1 + W_2 = 2W \tag{21}$$

From equations (4,19),

$$W_1 - W_2 = 2w \tag{22}$$

From equations (21,22),

$$W_1 = W + w \tag{23}$$

$$W_2 = W - w \tag{24}$$

One microwave travels at the speed of v_1.

$$v_1 = \frac{W_1}{k_1} \tag{25}$$

The other microwave travels at the speed of v_2.

$$v_2 = \frac{W_2}{k_2} \tag{26}$$

From equations (15,20,22,25,26),

$$v_1 - v_2 = 2V \tag{27}$$

Upon reflection, the microwave changes its speed by twice the speed of the reflector.

C. Angular Frequency Transformation

If the amplitude of the standing wave diminishes in one reference frame, it also diminishes in all reference frames. From equations (13,14),

$$w't' = Wt + 2n\pi \tag{28}$$

$$w'dt' = Wdt \tag{29}$$

$$\frac{w'}{W} = \frac{dt}{dt'} \tag{30}$$

Let there be two more standing waves in F_1 represented by w'_a and w'_b. All three standing waves should satisfy equation (30).

$$\frac{w'}{W} = \frac{dt}{dt'} = \frac{w'_a}{W_a} = \frac{w'_b}{W_b} = Q \tag{31}$$

Therefore, Q is a constant independent of both w' and W.

$$w' = QW \tag{32}$$

If V is reduced to zero, F_2 becomes F_1. The frequencies of both waves are identical in F_1.

$$W = W_1 = W_2 = w' \tag{33}$$

From equations (32,33),

$$Q = 1 \tag{34}$$

From equations (23,24,32,34),

$$W_1 = w' + w \tag{35}$$

$$W_2 = w' - w \tag{36}$$

The angular frequency depends on the choice of reference frame. This is commonly known as the Doppler effect[2].

D. Velocity Transformation

From equation (27),

$$v_1 - V = v_2 + V \tag{37}$$

Define C as

$$C = v_1 - V = v_2 + V \tag{38}$$

$$v_1 = C + V \tag{39}$$

$$v_2 = C - V \tag{40}$$

From equations (15,20,25,35),

$$v_1 = \frac{w'}{k} + V \tag{41}$$

From equations (39,41),

$$C = \frac{w'}{k} \tag{42}$$

From equations (15,42),

$$\frac{dC}{dV} = \frac{dw'}{dw} = 0 \tag{43}$$

C is independent of V. Set V to zero to obtain C. F_2 becomes F_1. From equations (1,38), C is the speed of both waves in F_1.

$$C = v_1 = v_2 = \frac{w'}{k'} \tag{44}$$

From equations (39,44), the velocity of a microwave in F_2 is the sum of its velocity in F_1 and the velocity of F_2 relative to F_1.

E. Wavelength Transformation

The transformation rule for the wavelength follows from equations (42,44),

$$k' = k \tag{45}$$

$$\lambda' = \lambda \tag{46}$$

The wavelength is conserved in both F_1 and F_2. The wavelength of microwave is invariant in all reference frames.

F. Time Transformation

The transformation rule for the elapsed time follows directly from equations (31,34),

$$dt' = dt \tag{47}$$

The elapsed time is conserved in inertial reference frames. If the initial condition is set as

$$0 = t' = t \tag{48}$$

From equations (47,48)

$$t' = t \tag{49}$$

The time becomes invariant in all reference frames.

III. CONCLUSION

The velocity of microwave in the rest frame of the observer is equal to the sum of the velocity of microwave in the rest frame of the wave source and the relative velocity between the observer and the wave source.

The node of the standing wave exits in all reference frames. The conservation of node results in the conservation of wavelength. The angular frequency of the microwave depends on the choice of reference frame. Hence, the microwave travels at a different velocity in a different reference frame.

The conservation of interference pattern generates the conservation of elapsed time which is also a property of parity symmetry[3,4].

The standing wave can be observed in a typical household microwave oven by examining the melted area on chocolate or cheese. The existence of melted spots in all reference frames proves to be an excellent experimental evidence for the conservation of wavelength. Consequently, the apparent speed of microwave depends on the rest frame of the observer.

[1] R. E. Collin, Foundations for Microwave Engineering, 2nd edition, IEEE Press, New York, NY, 2001.

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