Diagram of space-time. Quantum equation of the Doppler shift

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Abstract

Using a diagram of space-time (which represents intersections of the spheres depicting the front of an electromagnetic wave in the resting and moving frames with a plane going through the centers of the spheres) and the assumption that a quantum (photon) of spherical electromagnetic wave maintains its entity not only at the moments of emission and absorption, but also during the propagation of the wave, it is derived a quantum equation of the Doppler effect: $\sqrt{1-\beta^2}\,\tau^2 - 2\cos\gamma\,\tau_0\tau + \sqrt{1-\beta^2}\,\tau_0^2 = 0$, where τ_0 and τ are the periods of an electromagnetic wave in the source's and the receiver's frames, β is the relative speed of the frames divided by the speed of light in free space, and γ is the angle of aberration. The question of the angles of disappearance of the geometric factor of the Doppler shift is discussed. It is maintained that despite the differences between the obtained equation and Einstein's expression for Doppler shift, the equation of time dilation doesn't need any alteration.

1. The simplest form of space-time diagram

Einstein's theory of special relativity (SR), the mathematics of which is based on the Lorenz transformation, has two major faults: it considers electromagnetic (EM) waves as plane waves (while they are in fact spherical) and it doesn't take into account quantum nature of EM emission. This paper aims at making some corrections in consideration of those two factors. For that purpose, a so-called space-time diagram is used.

The simplest space-time diagram represents intersections of two spheres with a plane going through their centers (Fig. 1). The spheres depict the same front of a spherical EM wave in two different frames, from which one is resting and the other is moving with constant speed. It is implied that the centers of the spheres coincide in point O when emission starts from one of them. The radii of the spheres are proportional to the elapsed times in the corresponding frames. Because of that, those spheres may be considered also as time spheres without connection to any kind of waves. The main feature of this type of space-time diagram (the one without acceleration and gravity) is that the radii of the time spheres and the distance between their centers satisfy the Pythagorean theorem; i.e. their connection gives the equation of time dilation.

Intersections of a line of an arbitrary slope passing through the center of the moving sphere (it is shown below that the slope is the same in both frames, as opposed to the SR) with the spheres give the same point of the front of the EM wave in those frames. In Fig. 1 the points A and B of the front of an EM wave in the resting frame are the same as the points A' and B' respectively in the moving frame. To show this, imagine that the line with slope α in Fig. 1 is an imaginary tube in which a ray of an EM wave is traveling. Since the speed of light is independent of the speed of the emitter, $|OA| = ct$ and $|O'A'| = ct'$, where t and t' are the readings of the clocks in the resting O and moving O' points respectively, and c is the speed of light in free space; i.e. those segments are equal to the distances covered by the same point of the wavefront in respective frames.

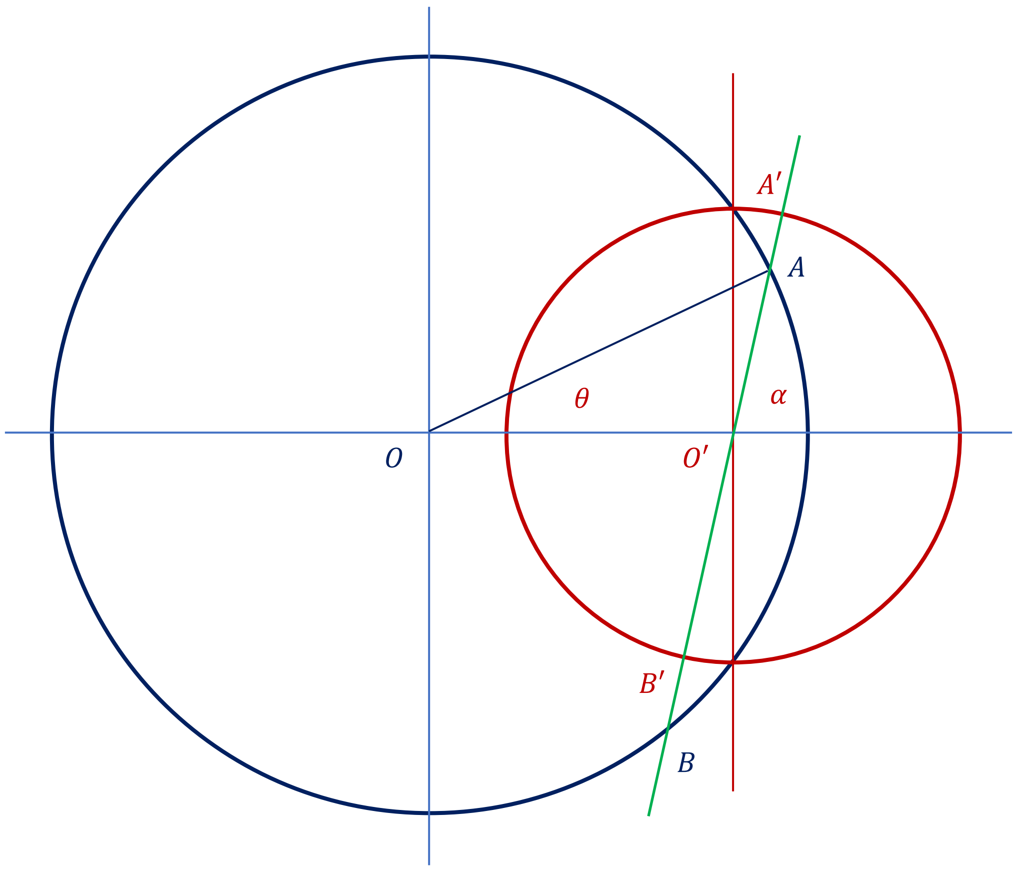

Fig 1. The simplest type of space-time diagram

Angle α is sometimes called the angle of emission: the slope of a ray in the source's frame. The slope of the same ray in the resting frame is θ. It is sometimes called the angle of reception: the observer in point A has to direct a spyglass in that direction in order to see the point O from which the emission has started. In other words, θ is the angle the line connecting the source and the receiver is making with the relative velocity of the frames at the start of the emission of the wave, and α is the same angle at the moment the wave arrives at the receiver.

When a resting observer measures distances from the moving point of reference O', that seems as if the observer is "teleported" into the moving frame. Because of this, the picture the observer gets now should be the same as the one observed within the moving frame. The velocity of the EM wave relative to point O' is $(\vec{c}-\vec{v})$ (where $\vec{c}$ is the velocity of light in direction θ, and $\vec{v}$ is the velocity of the moving frame), which is directed as $\overrightarrow{O'A}$. Since the speed of light is constant, $|\vec{c}-\vec{v}|=c$.

Fig. 2 shows several (three) consecutive wavefronts with the phase delays of 2π in the frames of the resting receiver and the moving source. The segment $|O'A|$ in the resting frame contains the same number of wavelengths as the segment $|O'A'|$ in the moving frame. This means that the lengths measured from moving points undergo changes. In particular, the units for measuring distances with slope of α from the *moving* point O' to the wave's front and the units for measuring the same distances from the *stationary* point O', must have the ratios of $|O'A|:|O'A'|$ and $|O'B|:|O'B'|$ in opposite directions, so that the speed of light remains the same in both frames.

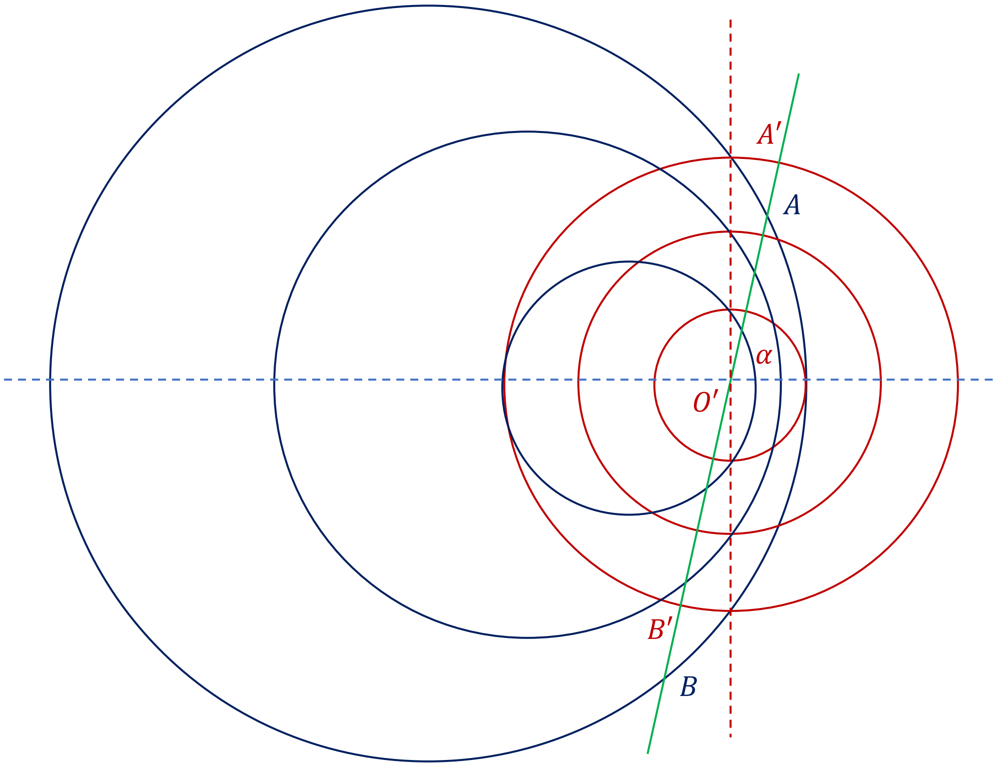

Fig. 2. Space-time diagram for several (three) wavelengths

It should be noted also that $\frac{|A'B'|}{|AB|}=\sqrt{\frac{1-\beta^2}{1-\beta^2\sin^2\alpha}}$, which represents the equation of Lorenz-Fitzgerald contraction. [1]

2. Doppler shift: the distance between the source and the receiver is equal to one wavelength

In Fig. 3, the source of an EM wave, S', is moving to the right from point S with velocity $v = \beta c$. At the moment, when points S and S' coincide, the clocks in those points have the same readings $t_0 = t_0' = 0$. Let τ_0 be the period of an EM wave in the source's frame (which is equal to the period of the source's internal oscillations). Due to time dilation, in the observer's frame the period of the source's internal oscillations is $\tau_0' = \frac{\tau_0}{\sqrt{1-\beta^2}}$. The period of the wave in the receiver's frame depends on the angle of propagation of a ray (e.g. angle θ or θ_1, which in the receiver's frame are the slopes of two rays traveling in opposite directions in the source's frame.)

Let's consider the moment of time when the clock in point S shows $t = \tau_0'$, and the clock in point S' reads $t' = \tau_0$. A wavefront has just arrived from point S at the resting receiver in point O. The next wavefront with phase delay of 2π starts emitting from point S', and will arrive at the receiver after the time interval of $\tau = \frac{|S'O|}{c}$. Thus, τ is equal of the wave period measured by the resting observer in point O.

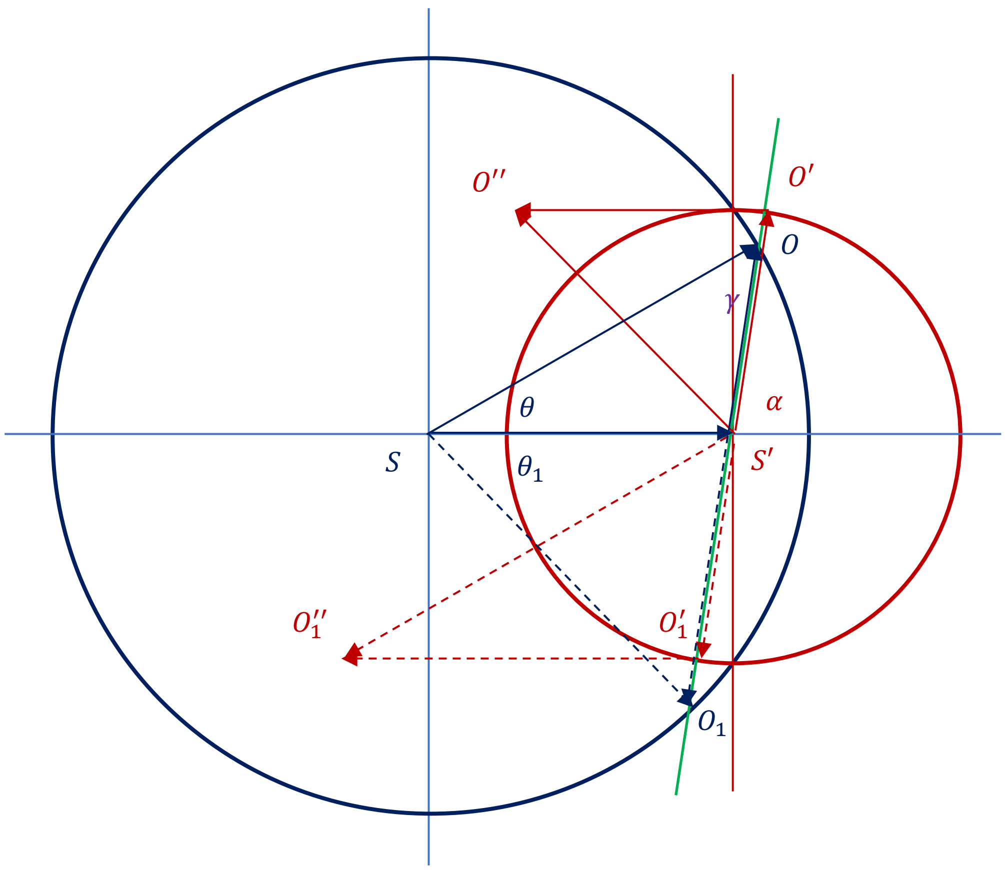

Fig 3. The distance between the source and the receiver is equal to one wavelength

Sometimes it is convenient to consider not the wave itself, but a sequence of short pulses emitted by the source with the same frequency. In Fig. 3, out of two consecutive pulses one is emitted from point S and the other - from point S'. Both pulses are received by the receiver in point O. The period of time between the emission of those pulses in the source's frame is τ_0, and the time period between the received pulses measured by the observer shall be $\tau = \frac{|S'O|}{c}$.

In the figure, $|SO| = c\tau_0'$, $|SS'| = \beta c\tau_0'$, and $|S'O| = c\tau$. It is important to note that the angle α is not only the slope of the moving tube in which the first pulse (or a front point of the wave) is traveling, but also the slope of the direction of the second pulse (or a rear point of the wave) in the receiver's frame.

From $\Delta SOS'$:

$$\sqrt{1-\beta^2}\,\tau^2 + 2\beta\cos\alpha\,\tau_0\tau - \sqrt{1-\beta^2}\,\tau_0^2 = 0\,. \tag{1}$$

The solutions are:

$$\tau = \tau_0 \frac{\left(-\beta\cos\alpha \pm \sqrt{1-\beta^2\sin^2\alpha}\right)}{\sqrt{1-\beta^2}}\;; \tag{2}$$

or for the first solution:

$$\tau = \tau_0 \frac{\sqrt{1-2\beta\cos\theta+\beta^2}}{\sqrt{1-\beta^2}}\,. \tag{3}$$

Now let's see how all this looks in the source's frame.

If in Fig. 3 the direction of time's flow is reversed, the triangle and the equation stay the same, but the source and the receiver swap places: the source (in point O) is stationary now and the receiver (in point S') is moving in the opposite direction; θ becomes the angle of emission and α - the angle of reception. The period between the pulses in the source's frame is τ and the same period measured by the observer is τ_0. Thus, the form of the equation is the same in the source's frame, but τ and τ_0 swap places:

$$\sqrt{1-\beta^2}\,\tau_0^2 + 2\beta\cos\alpha\,\tau\tau_0 - \sqrt{1-\beta^2}\,\tau^2 = 0$$

To get the aforementioned case of direct flow of time in the source's frame (where τ_0 is the period in the source's frame and τ - in the observer's frame), in the last equation the sign before the cosine needs to be changed to its opposite, so that the angle of emission now is not α, but $\alpha_1 = (180° - \alpha)$:

$$\sqrt{1-\beta^2}\,\tau^2 - 2\beta\cos\alpha_1\,\tau_0\tau - \sqrt{1-\beta^2}\,\tau_0^2 = 0\,. \tag{4}$$

This equation is obviously algebraically identical to equation (1), but it refers to $\Delta S'O'O''$, where $|S'O'| = c\tau_0$, $|S'O''| = c\tau'$, $|O'O''| = \beta c\tau'$, and $\tau' = \frac{\tau}{\sqrt{1-\beta^2}}$.

It has to be noted that: $\Delta S'O'O''\sim\Delta O_1S'S$, and the angle α between the tube and the line connecting the centers of the spheres as well as the angle between the rays, γ, is the same in both frames. This means that in Fig. 3 as well as in Fig. 1 the slope of the line representing the tube is the same in both frames, and $\gamma=(\alpha-\theta)=(\alpha_1-\theta_1)$ is the so-called angle of aberration: the difference between the angles of emission and reception. In other words, γ is the angle at the receiver between the directions towards the source at the moments of emission of the first and second pulses (or of two consecutive wavefronts with 2π-phase delay), or the angle at the source between the directions towards the receiver at the moments of reception of the same pulses (or wavefronts).

Using the angle of aberration, both aforementioned equations take the same form:

$$\boldsymbol{\sqrt{1-\beta^2}\,\tau^2-2\cos\gamma\,\tau_0\tau+\sqrt{1-\beta^2}\,\tau_0^2=0\,.} \tag{5}$$

The solutions of this equation are:

$$\boldsymbol{\tau=\tau_o\frac{\cos\gamma\mp\sqrt{\beta^2-\sin^2\gamma}}{\sqrt{1-\beta^2}}}\,. \tag{6}$$

The first solution is for the observer in point O, and the other - for the observer in point O_1. It must be noted that the square root from the product of those solutions gives τ_0, and

$$\frac{1}{\tau}=\frac{1}{\tau_o}\frac{\cos\gamma\pm\sqrt{\beta^2-\sin^2\gamma}}{\sqrt{1-\beta^2}}\,.$$

Eq. (5) represents spherical EM waves' answer to the principle of relativity.

It has to be noted that the form of equations (1) and (4) was predicted in [2] using an entirely different approach.

3. Time Dilation

In the above discussion it was implied that Einstein's equation of time dilation was absolutely correct. Let's ascertain that in fact it does not require alteration.

Consider two very short pulses emitted by a resting source from point S' (Fig. 3). The receiver is moving with velocity $v = \beta c$. It receives the first pulse in point $O'$and the other – in point O''. The time period between the pulses emitted is τ_0 in the source's frame and τ_0' in the receiver's frame; the time period between the received pulses measured by the observer is τ, and the same interval of time measured in the source's frame is τ'. Let's see whether or not the time periods designated by letters without primes and the ones by the same letters with primes have the same values.

In the figure: $|S'O'| = c\tau_0$, $|S'O''| = c\tau'$, $|O'O''| = \beta c\tau'$, and γ is the angle between the directions of the pulses. From $\Delta S'O'O''$:

$$(1-\beta^2)\tau'^2 - 2\tau_0\tau' \cos\gamma + \tau_0^2 = 0 \,. \tag{7}$$

In order to get the form of an appropriate equation in the receiver's frame, let's use the method of reversing time. In that case all velocities reverse their directions and the source and the receiver swap places; i.e. in $\Delta S'O'O''$, the moving source emits the first pulse from point O'' and the second one from O'. Those pulses are received by the receiver in point S'. Now: $|O''S'| = c\tau_0'$, $|O''O'| = \beta c\tau_0'$, $|O'S'| = c\tau$, and

$$\tau^2 - 2\tau_0'\tau\cos\gamma + (1-\beta^2)\tau_0'^2 = 0 \,, \tag{8}$$

In general, the Eqs. (7) and (8) are different, which makes it possible to determine whether the source or the receiver is resting or moving. If there is no difference between the values with and without primes (as for sound waves, for example), and Eq. (7) holds true, the source of waves is resting and the receiver is moving relative to the medium in which the wave propagates, and if Eq. (8) holds - the opposite is true. When considering EM waves in free space, it is obvious that Eqs. (7) and (8) must have the same form. That happens when the values with and without primes have the following relations: $\tau_0' = \frac{\tau_0}{\sqrt{1-\beta^2}}$ and $\tau' = \frac{\tau}{\sqrt{1-\beta^2}}$, which, in both cases, give Eq. (5) that is in full compliance with the principle of relativity. It can be proved that for the events having physical sense, those relations are unique.

It should also be noted that Eq. (5) applies to all four triangles in Fig. 3.

Thus, there is no reason to doubt absolute correctness of Einstein's equation for time dilation, which cannot be said about his equations for the Doppler shift and aberration.

4. The distance between the source and the receiver is greater than one wavelength: quantum equation of the Doppler shift

Until now the discussion didn't contain anything that would contradict the rules of the classical physics. The contradiction starts when the distance between the source and the receiver becomes greater than one wavelength.

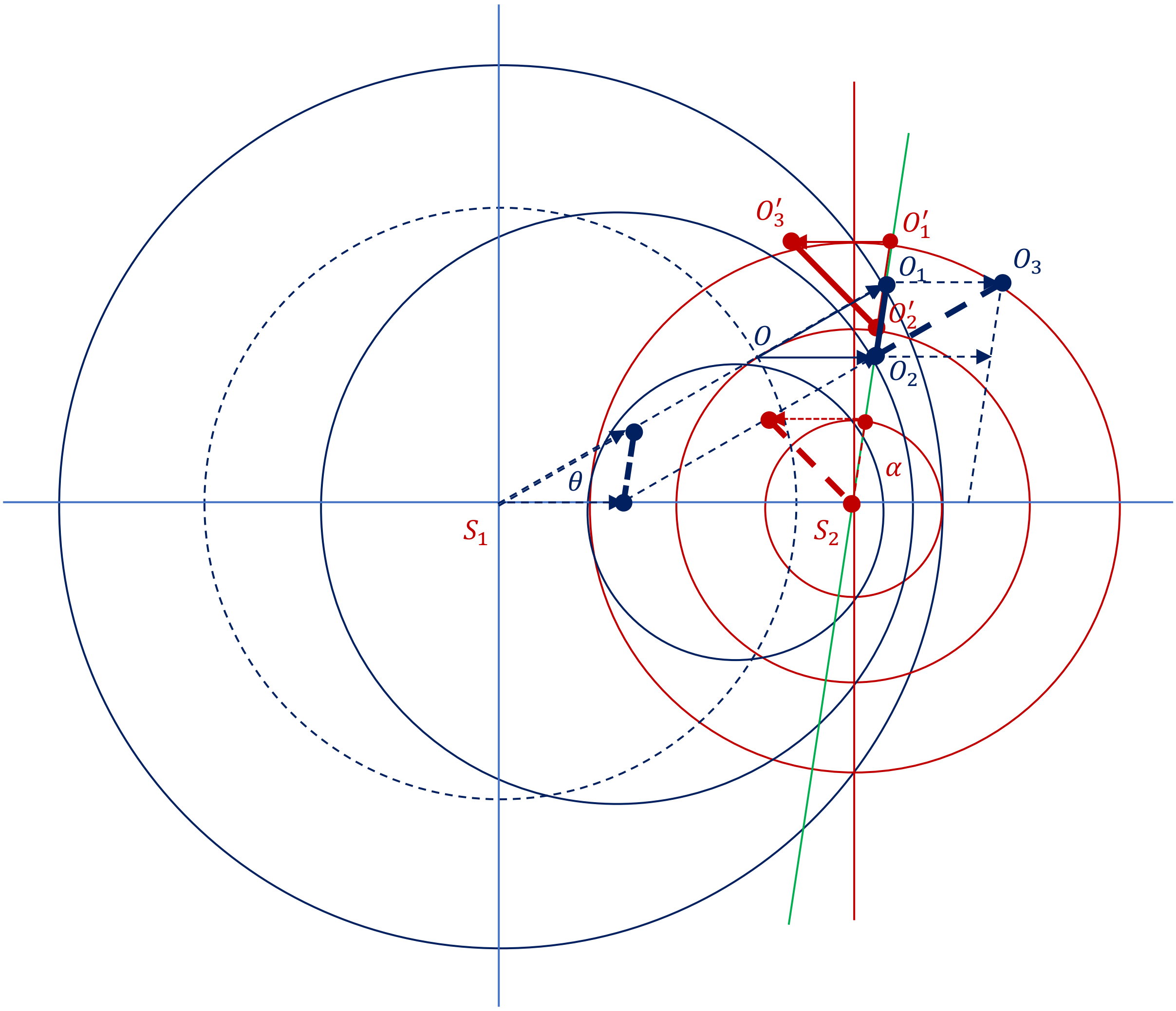

Fig. 4. Space-time diagram: quantum mechanism

According to Plank's hypothesis, the emission of EM waves occurs in the form of quanta of discrete energies. According to Einstein's hypothesis, the absorption of EM waves occurs in the form of photons of discrete energies. Logically, one can assume that during the propagation of EM waves, a quantum (photon) maintains its discrete nature, its entity. This means that if certain points of the wave are associated with some quantum (photon) in a given frame, this association shall be maintained in any other frame. To make a photon (quantum), two consecutive wavefronts with phase delay of 2π must arrive at the receiver. For instance in Fig. 4, if points O_1' and O_2' are the initial and final points of a quantum in the source's frame, then O_1 and O_2 shall be the initial and final points of the same quantum in the receiver's frame. When a photon is absorbed by a receiver in any frame, then all of its points shall be absorbed independent of whether the receiver is resting or moving. The simplest classical model for a quantum is an entity of two very short pulses, the time interval between which is equal to the period of the wave.

For instance, if in the source's frame a *resting* receiver in point O_1' absorbs the photon $[O_1', O_2']$ with a period of τ_0, in the observer's frame this absorption is taking place in the process of displacement of this receiver from point O_1 to point O_3. Thus, in the observer's frame the photon becomes $[O_3, O_2]$, where $|O_3O_2| = \frac{|O_1'O_2'|}{\sqrt{1-\beta^2}} = \frac{c\tau_0}{\sqrt{1-\beta^2}}$.

If in the source's frame the absorption of a photon is taking place during the receiver's displacement from point O_1' to point O_3', then the absorbed photon in the source's frame shall be $[O_3', O_2']$, which in the observer's frame corresponds to the photon $[O_1, O_2]$. Obviously, $|O_1O_2| = |O_1'O_2'|\frac{\left(-\beta\cos\alpha+\sqrt{1-\beta^2\sin^2\alpha}\right)}{\sqrt{1-\beta^2}} = |O_3'O_2'|\sqrt{1-\beta^2} = c\tau.$

According to Huygens' principle, each point of space, at which the front of a wave arrives, becomes itself a source of a secondary spherical wave (so-called wavelet). In Fig. 4, points O_1' and O_3' are the points of the consecutive wavefronts with a phase delay of 2π issued from the same point O_2'. In the observer's frame, this point has moved from O to O_2, and the points of the wavefronts with phase delay of 2π arrive at the receiver in point O_1.

From the classical point of view, points O_1' and O_3' belong to the wavelets issued from different points of space because the points of each wavefront arriving at the receiver have to be the closest ones to it (according to Fermat's principle). That seems impossible from quantum viewpoint: a quantum cannot be issued from different points of space, since it must remain a quantum even when flow of time is reversed. Thus, in an EM wave, all points associated with a quantum (photon) must originate from a single point of space in the sources frame and arrive to a single point of space in the receiver's frame, with a phase difference of 2π between the first and the last points of the association.

In the classical physics, there's a difference between space and time pictures of EM waves. In Fig. 2, along the sloping line, i.e. in a given direction from the source (point *S*), the lengths of wave are the same, but a resting receiver if located on this line at different distances from the source shall get different periods (and wavelengths). Namely, the period of the wave (and a photon) measured at different distances from the source varies from $\tau = \tau_0 \frac{\sqrt{1-2\beta\cos\theta+\beta^2}}{\sqrt{1-\beta^2}}$ (when the distance between the source and the receiver is at minimum, i.e. equals to one wavelength) to $\tau = \tau_0 \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}$ (when the distance between the source and the receiver is infinite). That seems to be in conflict with the conservation laws and/or the quantization principle.

From the quantum viewpoint, the time picture is identical to the spatial picture: the period (or wavelength) of an EM wave, and, correspondingly the energy and the momentum of the absorbed photon, for a given direction are the same for any distance between the source and the receiver.

Thus, the equations which are obtained for a single (the first) wavelength of the EM wave remain the same for any number of wavelengths (for any distance between the source and the receiver), which is a logical consequence of Plank's hypothesis.

In contrast with Einstein's equations for the Doppler shift [3], which in our notations may be written this way: $\tau = \tau_0 \frac{1-\beta\cos\theta}{\sqrt{1-\beta^2}}$ and $\tau_0 = \tau \frac{1+\beta\cos\alpha}{\sqrt{1-\beta^2}}$, Eqs. (2) and (3) are asymmetric with respect to the angles of reception and emission. But as it was shown above, they can be brought to the same frame-independent form like Eq. (6) (or Eq. (5)) using the frame-independent angle of aberration, which fully complies with the principle of relativity.

Thus, Eq. (5), as well as Eq. (1), may be considered as a quantum equation of the Doppler effect.

It must be mentioned that only when the relative velocity is directed along the line connecting the source and the receiver, the classical equation of the Doppler shift does not depend on the distance and coincides with the quantum equation.

It seems interesting to consider the angles when the geometric factor of the Doppler shift disappears and only the relativistic one remains. By a simple logic, that must happen when two points belonging to two consecutive wavefronts with a 2π-phase delay take the same time for traveling from the source to the receiver in the given frame.

For the quantum equation, $\tau = \tau_0\sqrt{1-\beta^2}$ when $\cos\alpha = \frac{\beta}{2}$, and $\tau = \frac{\tau_0}{\sqrt{1-\beta^2}}$ when $\cos\theta = \frac{\beta}{2}$, and those relations are valid for any distance between the source and receiver.

By the rules of classical physics, the geometric factor shall disappear when $\cos\alpha = \frac{\beta\tau_0}{2t_0}$ and $\cos\theta = \frac{\beta\tau}{2t}$, where t_0 and t are the times of light's travel from the source to the receiver, and τ_0 and τ the periods of the source's internal oscillations in the sources and receiver's frames respectively. Those relations too are valid for any distances. According to Einstein's equations, which are approximations to the classical ones, the geometric factor disappears at the angles $\alpha = 90°$ and $\theta = 90°$, and that happens in infinity only.

When $\alpha = 90°$ (or $\cos\theta = \beta$), the quantum equation gives: $\tau = \tau_0$. This case is discussed in [4].

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