Quantum Behavior

Jean Louis Van Belle

Abstract

The special problem we try to get at with this paper is to maintain the interest of the very enthusiastic and rather smart people trying to understand physics. They have heard a lot about how interesting and exciting physics is — the theory of relativity, quantum mechanics, and other modern ideas — and spend many years studying textbooks, following online courses, and blogging about their experiences. Many are discouraged because there are really very few grand, new, modern ideas presented to them. The problem is whether or not we can make a course which would save them by maintaining their enthusiasm. This paper is a draft first chapter of such course.

Lectures on Physics
Chapter I : Quantum Behavior

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Preface

The special problem we try to get at with these lectures is to maintain the interest of the very enthusiastic and rather smart people trying to understand physics. They have heard a lot about how interesting and exciting physics is—the theory of relativity, quantum mechanics, and other modern ideas—and spend many years studying textbooks, following online courses, and reading and writing blogs about their experiences. Many are discouraged because there are really very few grand, new, modern ideas presented to them. The problem is whether or not we can make a course which would save them by maintaining their enthusiasm.

The lectures here are not in any way meant to be a survey course, but are serious. I thought it would to re-write *Feynman's Lectures* in a way that makes sure most would be able to encompass (almost) everything that is in the lectures. This is the link to Feynman's Preface to his lectures, so you can see how my approach differs from his.

Of course, because we are re-writing Feynman's lectures on *quantum* mechanics, we must assume you have gone through a course in classical mechanics and electromagnetic theory. In fact, we will assume you are already familiar with the basics of *mainstream* quantum mechanics. We cannot make any compromises in this regard: you must cover your bases. Good luck—please do mail your thoughts!

1-1 Particle mechanics

Newton thought that light was made up of particles, but then it was discovered that it behaves like a wave. Later, however (in the beginning of the twentieth century), it was found that light did indeed sometimes behave like a particle. Historically, the electron, for example, was thought to behave like a particle, and then it was found that in many respects it behaved like a wave.

Hence, the challenge is to find a description that takes account both of the wave- as well as of the particle-like character of both matter- as well as light-particles. We may refer to both as wavicles but – for historical reasons – this term did not become household language. Light-particles are known as photons. Photons carry electromagnetic energy, but they do *not* carry charge. In contrast, matter-particles always carry charge. If they are neutral – think of a neutron or an atom – they will carry both positive and negative charges. We should, therefore, think of them as composite particles.

Elementary particles are stable. Composite particles consist of elementary particles and may be stable or unstable. An atom is an example of a stable composite particle. A neutron is stable inside of the nucleus but unstable as a free particle: it spontaneously disintegrates into a proton and an electron. This process involves the emission of a neutrino, which ensures energy is conserved. We think of a neutrino as a lightlike particle: it also carries energy but no charge.[1]

Electrons and protons are elementary matter-particles. They are stable but not indestructible. High-energy collisions between protons – or between protons and anti-protons – yield unstable particles which disintegrate back into stable particles. Because they are unstable, such particles should not be referred to as *particles* but as *transients* or, when very short-lived, as *resonances*.

[1] The *nature* of this energy is *not* electromagnetic, however. Electromagnetic energy is related to electromagnetic forces. We may, therefore, think of the energy of a neutrino as being related to the stronger force inside of a proton or a neutron.

The Higgs particle is an example of an extremely short-lived resonance: its lifetime is of the order of 10^{-22} seconds. Even at the speed of light – which an object with an estimated *rest* mass of 125 GeV/c^2 can never aspire to attain – it can, therefore, not travel any further than 0.3 *femto*meter (0.3×10^{-15} m) before it disintegrates. Such distance is *smaller* than the radius of a proton, which is in the range of 0.83 to 0.84 fm. Labelling it as a particle is, therefore, hugely misleading. Likewise, quarks have also never been directly observed or isolated. Their existence is and remains, therefore, a mere hypothesis, which we will not entertain in these lectures because we have no need for it.[2]

These high-energy collisions show that protons must have some internal *structure*. We think of such structure in terms of the motion of the elementary charge. Paul Dirac wrote the following on that:

> "Quantum mechanics may be defined as the application of equations of motion to particles. […] The domain of applicability of the theory is mainly the treatment of electrons and other charged particles interacting with the electromagnetic field—a domain which includes most of low-energy physics and chemistry.
>
> Now there are other kinds of interactions, which are revealed in high-energy physics and are important for the description of atomic nuclei. These interactions are not at present sufficiently well understood to be incorporated into a system of equations of motion. Theories of them have been set up and much developed and useful results obtained from them. But in the absence of equations of motion these theories cannot be presented as a logical development of the principles set up in this book. We are effectively in the pre-Bohr era with regard to these other interactions. It is to be hoped that with increasing knowledge a way will eventually be found for adapting the high-energy theories into a scheme based on equations of motion, and so unifying them with those of low-energy physics."[3]

These words were written in 1958 but still ring true today. High-energy physics study disintegration processes: these involve non-equilibrium states, which we will not study in these lectures.

1-2 The nature of space and time

When Max Born, Werner Heisenberg, Erwin Schrödinger, Louis de Broglie, and Niels Bohr had presented their seminal papers at the 1927 Solvay Conference, Hendrik Antoon Lorentz remarked the following:

> "I would like to draw your attention to the difficulties in these theories. We are trying to represent phenomena. We try to form an image of them in our mind. Till now, we always tried to do using the ordinary notions of space and time. These notions may be innate; they result, in any case, from our personal experience, from our daily observations. To me, these notions are clear, and I admit I am not able to have any idea about physics without those notions. The image I want to have when thinking physical phenomena has to be clear and well defined, and it seems

[2] We think the quark hypothesis results from an unproductive approach to analyzing disintegration processes. To explain the disintegration of K-mesons, Gell-Man and Kazuhiko Nishijima in the 1950s invented new quantities that are supposedly being conserved while unstable transients or resonances disintegrate into more stable constituents. One of these quantities was referred to as strangeness (see the analysis of K-mesons in [Feynman's *Lectures*]). These strange new concepts then started to lead an even stranger life of their own.

[3] Paul A.M. Dirac, *The Principles of Quantum Mechanics*, 4th edition (1958), p. 312.

> to me that cannot be done without these notions of a system defined in space and in time.
>
> To me, the electron is a particle which, at any moment, must be at some specific point in space, and if I think it should be somewhere else at the next moment, then I need to be able to think of its trajectory, which is a line in space. And if that electron meets an atom and penetrates it and if it, after several adventures, leaves that atom, then I need to have some theory in which that electron conserves its individuality. In other words, I actually think of a trajectory of the same electron *within* the atom. It may be difficult to develop such theory but, *a priori*, this should not be impossible."[4]

On the Uncertainty Principle in particular, he remarked that all of the actual or theoretical experiments[5] only proved that we have indeterminism *from a practical point of view*. He argued that he would keep his deterministic belief in fundamental phenomena as a scientific principle, at least. It is probably useful to literally quote his last words here: "Can we not keep determinism as an object of faith? Why do we have to elevate indeterminism to a philosophical principle?"[6]

We agree with Lorentz. Heisenberg himself initially preferred to use the German term *Ungenauigkeit* to describe the apparent randomness in our mathematical description of Nature. *Ungenauigkeit* translates as imprecision and it is a concept that is valid in classical as well as in quantum mechanics. It is just what it: an imprecision inherent to measurement—as opposed to the weird metaphysical quality which Heisenberg would later claim it to be and which, without any precise definition, physicists now refer to as 'Uncertainty', with a capital letter that is used like the capital letter in 'God'. We, therefore, will re-state Lorentz's question as an affirmative statement: *there is no need whatsoever to elevate indeterminism to a philosophical principle*. In fact, without the assumption of determinism at the most elementary level of analysis, science would basically not be science: we would relegate it to the realm of beliefs. We all need to believe in something, when doing physics or whatever else, but we should not confuse these beliefs with the scientific approach. We will not engage in much philosophy here but just make a few remarks.

First, it is quite obvious that relativity theory has profoundly changed the *meaning* of these ideas. We now know that they are *related*. To be precise, they are related through the more fundamental idea of *motion*. However, while related, space and time are still very different, and we agree with Lorentz: both are essential when expressing or explaining an idea in physics. So what can we say about them?

Stating that the concepts of space and time are related because of the more fundamental idea of

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[4] The proceedings of the Solvay Conferences were published by the Free University of Brussels. There is, however, no fully complete English translation of the earlier and most crucial conferences, notably those of 1921 and 1927. While the reader will be able to find English translations of the papers, we find the questions and answers the most interesting. We highlighted some of these in our paper on the history of quantum-mechanical ideas.

[5] Scientific lore has it that Einstein lost out to Bohr when discussing a series of hypothetical experiments—thought experiments, as they are usually referred to. We think there is no clear winner in these discussions and that, in any case, Einstein's private correspondence indicates he might have gotten tired of them. He probably had better things to do: there were so many personal and historical events calling for his attention in the 1930s.

[6] These sentences were really one of his last contributions to science: he died a few months later. the 1927 conference proceedings have both the sad announcement of his demise as well his interventions—such was the practice of actually physically printing stuff at the time.

motion is stating the obvious: we already had such relation in Galilean relativity, so we should be more specific. We will briefly say something about Einstein's relativity theory in a moment so you should be a little bit patient as for now. Here we will only make a very simple remark on the *arrow* of time in the context of Galilean relativity. We should probably not be making this remark but we feel there is so much *humbug* around the possibility of time reversal that we feel we should.

Spacetime trajectories – or, to put it more simply[7], *motions* – need to be described by well-defined functions: for every value of *t* (time), we should have one, and only one, value of *x* (space).[8] The reverse, of course, is *not* true: a particle can travel back to where it was (or, if there is no motion, just stay where it is). Hence, it is easy to see that the concepts of motion and time must be related because this logic imposes the use of the well-behaved functions to describe reality. This is illustrated below: a pointlike particle which moves like what is show on the right-hand side cannot exist because there are a few occasions here where the particle occupies multiple positions in space *at the same point in time*. Now, some physicists may honestly believe that should actually be possible, but we do not want to entertain such ideas and, therefore, wish them all the best.[9]

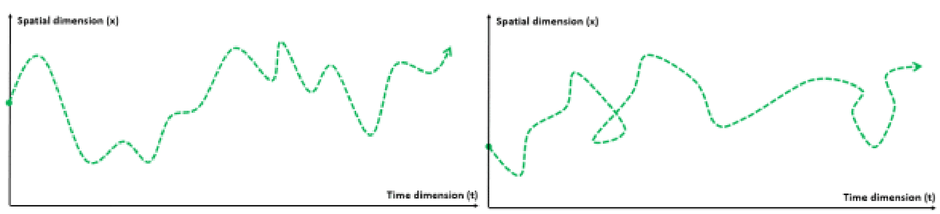

Figure 1: A well- and a not-well behaved trajectory in spacetime[10]

This shows that time must go in one direction only. We can play a movie backwards, but we cannot reverse time. We know that because a movie in which two like charges (say, two electrons) would attract rather than repel each other does not make sense. This intuition contrasts with the erroneous suggestion of Richard Feynman that we may want to think, perhaps, of matter-particles as particles that travel back in time. We will come back to this as part of our discussions on the physical meaning of the

[7] We do not like the use of the term *spacetime* because it is usually not very clearly defined. We may use it as a shorthand to refer to four-vector algebra.

[8] The *x* may or should be a vector, of course: ***x*** = (*x*, *y*, *z*). However, we may consider motion in one dimension only, or choose our reference frame such that the direction of motion coincides with the *x*-axis. That is done quite often to simplify the calculations. The result can usually be generalized quite easily to also encompass two- and three-dimensional motion. There is no such thing as four-dimensional *physical* space. *Mathematical* spaces may have any number of dimensions but the notion of *physical* space is a category of our mind, and it is three-dimensional: left or right, up or down, front or back. You can try to invent something else but it will always be some combination of these innate notions. If you find something else, please let me know.

[9] We do not want to use not-so-well-behaved functions to arrive at some kind of description of reality. The matter is quite serious because [it drove some of the brightest minds on Earth to madness].

[10] We actually do not like the concept of spacetime very much: time and space are *related* (through special and general relativity theory, to be precise) but they are *not* the same. Nor are they *similar*. We do, therefore, *not* think that some 'kind of union of the two' will replace the separate concepts of space and time any time soon, despite Minkowski's stated expectations in this regard back in 1908. Grand statements and generalizations are not often useful in physics.

wavefunction.

It should also provide a fresh perspective on the discussions on symmetry-breaking, but these discussions are technical and, therefore, difficult, so we will leave them for the time being. Instead, we will use your current attention span for another, more important, technical matter: non-Galilean relativity.

1-3 Electromagnetism and relativity

Relativity theory is *not* easy: the ideas of relativistic mass, relativistic length contraction and time dilation are all related and profound. We must assume you know the formulas and that you have done your utmost to try to understand these as best as you can—which is all you can do: more evolved minds might find it easy to work with the formulas but they never become intuitive. We cannot dwell on this. We just want to think about the relativity of electromagnetic fields and – if possible – to review your courses on four-vector algebra in the context of electromagnetism.[11] Here, we will only make a few introductory remarks—because this is an introductory chapter, after all!

The fundamental idea is this: we do not want to invent new concepts and, therefore, we will want to analyze what might be going on inside of the atomic nucleus in terms of *electromagnetic interactions only*. We do not say it can be done: we are just saying we want to try harder. Such analysis may be simply referred to as an electromagnetic theory of nuclear interaction and offer an easy explanation of the attractive force between protons in terms of the electromagnetic force between ring currents—an idea which is at the very core of our understanding of quantum mechanics. We will come back to this idea of ring currents in a moment.[12] Here, we just want to note that these analyses are based on a more coherent and complete conceptualization of what might going on inside of the nucleus by also focusing on the *magnetic* forces resulting from the *regular motion* of one or more elementary (electric) charges inside, as opposed to a narrow focus on the electrostatic Coulomb force resulting from *static* charges only. The fundamental idea here is that a nuclear lattice structure would not only arise from the mere *presence* of the charges inside but also from the pattern of their *motion*.

Of course, your immediate reaction should be this: we are 2020 now—why wasn't this done before? Our honest answer is: we do not know. We suspect magnetic forces have traditionally been neglected[13] in the analysis because the *magnitude* of the magnetic field – and, therefore, of the force – is only $1/c$

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[11] If you need a reference, we recommend chapters 25 to 29 of Feynman's lectures on electromagnetism (Volume II of his *Lectures*).

[12] Easily accessible references are, for example, Bernard Schaeffer (2016) or Paolo Di Sia (2018). Di Sia relates the approach to the new nuclear lattice effective field theory (NLEFT) but a good understanding of NLEFT requires very advanced mathematical skills and a lot of time, which we do not have. The reader may verify this by having a quick look at Timo Lähde and Ulf-G. Meißner's new book: Nuclear Lattice Effective Field Theory (2019).

[13] We may not only refer to rather mundane examples here – such as, for example, Feynman's treatment of polarized light, which considers the electric field vector only. A historically more spectacular example of neglect is Hideki Yukawa's invention of a new potential and, therefore, of a new *force* that is supposed to be stronger than the electrostatic (repulsive) force between protons. Of course, the invention of a new force and a new potential should also come with the invention of a new charge so as to explain the origin of this potential, but physicists preferred the quark-gluon model instead.

times that of the *electric* field. That is a mistake[14] which becomes obvious when considering the following:

(1) We can use natural time and distance units to ensure the *numerical* value of lightspeed does not distort calculations in regard to the relative strength of both forces. Indeed, the large *numerical* value (299,792,458) of c using the second and meter as time and distance units effectively indicates we may want to think of the second as a rather large unit as compared to the meter when thinking about the relativity of electric and magnetic fields[15];

(2) "Magnetism and electricity are not independent things: they should always be taken together as one complete electromagnetic field"[16];

(3) The idea of a pointlike charge having no other attributes than its charge – a *naked* charge with zero *rest* mass, in other words – implies that it must, theoretically, whizz around at the speed of light.[17] This ensures that the $\beta = v/c$ factor in the transformation formulas we use when going from one reference frame to another is always equal to 1 and, therefore, eliminate the need to think about measurement units when discussing the relativity of electric and magnetic fields. In fact, the idea of the electric or magnetic force being more or less important than the other completely vanishes here!

The point is this: such electromagnetic theories of nuclear interaction require us to think of the magnetic moment of protons – and of neutrons too, of course – as being generated by an actual electric current—as opposed to some *intrinsic* property requiring no further explanation or detail. It is, in fact, at this point *exactly*, where such theories start to diverge from the mainstream interpretation of quantum

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[14] We think of it as a *mistake* rather than just an oversight or something incomplete because an analysis of polarization which would also include the behavior of the magnetic field vector might provide a classical explanation to [one-photon Mach-Zehnder interference]().

[15] The definition of the speed of light as being equal to c = 299,792,458 m/s *exactly* results from the re-definition of both the meter and the second in terms of the wavelength and cycle time of electromagnetic radiation. Of course, such re-definition still requires a reference to a real-life electromagnetic oscillation. This reference is the frequency of the light which is emitted or absorbed as a result of the transition of an electron between the two hyperfine states that define the unperturbed ground state of the cesium-133 atom, which is defined as f = 9 192 631 770 Hz, *exactly* ([BIPM, 2019]()). We may, therefore, think of the cycle time ($T = 1/f$) and the wavelength ($\lambda = c \cdot T = c/f$) of this radiation as the true *base* units of time and distance. The definition of Planck's constant as being equal to h = $6.62607015 \times 10^{-34}$ J·s, *exactly*, may then be combined with the natural time unit to define a natural unit for energy which, in turn, may be combined with the natural distance unit to define a natural unit for force (energy is defined as a force over some distance). Needless to say, if we have a natural unit for energy, we get a natural unit for mass from Einstein's mass-energy equivalence relation. We then need to add the defined value of the elementary charge ($1.602176634 \times 10^{-19}$ C) so as to be able to *calculate* all other constants that one might need in quantum mechanics.

[16] Richard Feynman, Lectures on Physics, II-13-6 ([the relativity of magnetic and electric fields]()).

[17] Newton's relativistically correct force law $F = m \cdot a$ implies even the smallest of small forces will give it infinite acceleration. Such charge will, therefore, be photon-like in the sense that all of its mass will result from its motion. However, photons do not carry charge, and we think of local circular motion here—as opposed to the linear motion of light photons. This distinguishes matter from light.

physics[18]: they require us to think of electrons and protons as tiny ring currents with an actual radius and circumference in space, which is what we shall do.[19]

It must also result in a dynamic view of the fields surrounding charged particles. Potential barriers – or their corollary: potential *wells* – should, therefore, not be thought of as static fields: they vary in time. They result from two or more charges moving around and creating some *joint* or superposed field *which varies in time*. Hence, a particle breaking through a 'potential wall' or coming out of a potential 'well' may just be using a temporary opening corresponding to a very classical trajectory in space and in time. That is why we think there is no need to invoke an Uncertainty Principle.

All of the considerations above may come across as being rather elementary. Taken together, however, they amount to a coherent re-formulation of the basic principles of quantum physics – or a common-sense or *realist* interpretation of the same – which we will try to summarize below.

1-4 Matter-particles

Electrons

The internal structure of the electron was revealed by Arthur Holly Compton's 1923 scattering experiment, which showed photons can either scatter *elastically* or *inelastically* from an electron. When scattering is inelastic, the outgoing photon has a different frequency than the incoming photon. The difference in energy is absorbed by the electron as *kinetic* energy. The effective area of interference between photons and electrons is defined by the Compton radius of the electron, which is defined by the motion of the elementary charge. The elementary charge has a radius itself, which defines the effective area of interference for elastic scattering, which is also referred to as Thomson scattering: there is no frequency change here. Think of it as some *core* within the electron.

The idea of an electron must, therefore, combine the idea of a charge and its motion. This combined idea effectively accounts for both the particle- as well as the wave-like character of matter-particles. It also explains the magnetic moment of the electron. In fact, the *ring current* or *magneton* model of an electron was first suggested by the British chemist and physicist Alfred Lauck Parson (1915) to do exactly that.[20] A theoretical local oscillatory motion also emerged from Dirac's wave equation for an electron, which he wrote down in 1927. Erwin Schrödinger referred to it as a *Zitterbewegung* [21], and Dirac

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[18] The Copenhagen interpretation of quantum mechanics is, perhaps, not well defined but we interpret it as having the Uncertainty Principle at its core, which amounts to saying we should not even try to look for hidden variables to explain intrinsic properties – most notably inertial mass and magnetic moment – of elementary particles. The *Zitterbewegung* interpretation of quantum mechanics explains the mass of an elementary particle as the equivalent mass of the energy in the oscillation, and the magnetic moment as the magnetic moment of a ring current.

[19] It also requires us to think of neutrons as, somehow, combining a proton and an electron—to explain why they are electrically neutral but do have a magnetic moment of their own. The non-stability of neutrons outside of the nucleus and other considerations make this a plausible hypothesis. We will come back to this so the reader should not worry about it now.

[20] We refer to Ernest Rutherford's remarks on Parson's '*électron annulaire*' (ring electron) and the magnetic properties of the electron in his lecture on '*The Structure of the Electron*' at the 1921 Solvay Conference.

[21] *Zitter* refers to a rapid trembling or shaking motion in German.

highlighted its significance at the occasion of his Nobel Prize lecture:

> "It is found that an electron which seems to us to be moving slowly, must actually have a very high frequency oscillatory motion of small amplitude superposed on the regular motion which appears to us. As a result of this oscillatory motion, the velocity of the electron at any time equals the velocity of light. This is a prediction which cannot be directly verified by experiment, since the frequency of the oscillatory motion is so high, and its amplitude is so small. But one must believe in this consequence of the theory, since other consequences of the theory which are inseparably bound up with this one, such as the law of scattering of light by an electron, are confirmed by experiment." (Paul A.M. Dirac, *Theory of Electrons and Positrons*, Nobel Lecture, December 12, 1933)

Unfortunately, Dirac confuses the idea of the electron and the naked charge here. The *Zitterbewegung* charge is a charge only: it has no other properties. It has no rest mass, for example, and it must, therefore, effectively move at the speed of light: Newton's law tells us the slightest force on it will give it infinite acceleration. The *Zitterbewegung* of the pointlike charge is possibly chaotic but can be modelled by the formula for circular or tangential velocity:

$$c = a \cdot \omega \Leftrightarrow a = \frac{c}{\omega}$$

The illustration provides the simplest of simple visualizations of what might be going on. We effectively think of an electron (and a proton) as consisting of a pointlike elementary charge – pointlike but *not dimensionless*[22] - moving about at the speed of light around the center of its motion. This pointlike charge must have some momentum—if only by virtue of its motion. This is *not* the classical moment of the electron or proton as it is moving about classically—with an equally classical velocity *v*. No. The momentum in this illustration is equal to $p = m_\gamma c$: m_γ is the *relativistic* mass of the pointlike charge, whose *rest* mass itself is zero because the naked charge itself has no other properties but its charge. We can use the idea of a centripetal force (**F**) keeping this charge in its orbit to prove this effective mass will be *half* of the electron mass. The other half of the mass or energy of the electron is in the oscillating field which must keep this charge, somehow, in its orbital motion.[23]

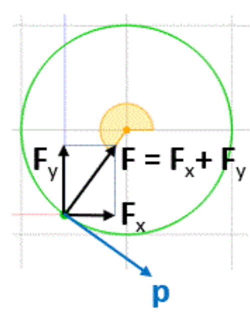

Figure 2: The ring current model

[22] We will come back to this when discussing the (anomalous) magnetic moment.

[23] We know this must sound rather fantastical but we ask you to just go along with it for the time being. We will think about the nature of this force in later developments.

Any (regular) oscillation has a frequency and a cycle time T = 1/*f* = 2π/ω. The Planck-Einstein relation (E = *h*·*f* = *ħ*·ω) relates *f* and T to the energy (E) through Planck's constant *h* (or, when using the reduced form of Planck's equation, *ħ*). This frequency formula then allows us to use the tangential velocity formula to calculate the radius of this orbital motion:

$$a = c/\omega = c\frac{\hbar}{\mathrm{E}} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar}{\mathrm{m}c} \approx 0.38616\ \mathrm{pm}$$

This effectively corresponds to what we refer to as the Compton *radius* of an electron[24], which, paraphrasing Prof. Dr. Patrick LeClair, we can now understand as "the scale above which the electron can be localized in a particle-like sense."[25] It is now obvious that Louis de Broglie's intuition in regard to the wave nature of matter-particles was correct. However, we should think of de Broglie's concept of the matter-wave as corresponding to orbital rather than linear motion.

Of course, we can also measure the scale of the pointlike charge because of the phenomenon of elastic scattering (Thomson scattering) but we cannot precisely *localize* the pointlike charge because of its lightlike velocity. To be precise, this *Thomson* radius is the classical electron radius and is expressed as a fraction of the *Compton* radius: α·*a* ≈ 2.818 fm. This radius formula introduces the fine-structure constant α ≈ 0.0073. The fine-structure constant pops up in other places too and we will, therefore, quickly make some calculations which you should find interesting. Before we do these calculations, you should just note that the fine-structure constant α and the electric and magnetic constants ε_0 and μ_0 are related to each other and to the fine-structure constant as follows:

$$(1)\ \varepsilon_0\mu_0 = \frac{1}{c^2} \qquad (2)\ \varepsilon_0 = \frac{\mathrm{q_e^2}}{2\alpha h c} \qquad (3)\ \mu_0 = \frac{2\alpha h}{\mathrm{q_e^2}c}$$

The reader can easily *google* these results. We get the second and third equation from combining the first and the definition of the fine-structure constant. The reader should not confuse the magnetic *constant* μ_0 with μ, which is a symbol we will use for the magnetic *moment* of an electron in the following section.

The anomalous magnetic moment

A ring current generates a magnetic moment. We have a formula for that from electromagnetic theory: the magnetic moment (μ) is equal to the current (I) times the area of the loop (πa^2).

The current here is the elementary charge ($\mathrm{q_e}$) times the frequency (*f*), and the radius of the loop is the Compton radius (*a* = *ħ*/m*c*), so we can write:

$$\mu = \mathrm{I}\pi a^2 = \mathrm{q_e} f \pi a^2 = \mathrm{q_e}\frac{c}{2\pi a}\pi a^2 = \frac{\mathrm{q_e}c}{2}a = \frac{\mathrm{q_e}c}{2}\frac{\hbar}{\mathrm{m}c} = \frac{\mathrm{q_e}\hbar}{2\mathrm{m}}$$

We used the *f* = *c*/2π*a* formula for the frequency: the frequency is the velocity divided by the circumference of the loop. We can also use the Planck-Einstein relation to get the same result:

[24] The Compton *radius* is equal to the reduced Compton wavelength: *a* = λ/2π.

[25] See: http://pleclair.ua.edu/PH253/Notes/compton.pdf, p. 10.

$$\mu = I\pi a^2 = q_e f \pi a^2 = q_e \frac{E}{\hbar}\pi a^2 = \frac{q_e m c^2}{2\pi\hbar}\frac{\pi\hbar^2}{m^2 c^2} = \frac{q_e}{2}\frac{\hbar}{m} = \frac{q_e\hbar}{2m}$$

We can now use the values for q_e, $\hbar$ and m to calculate the magnetic moment.[26] If you do that, you will get this:

$$\mu = 9.27401\ldots\times10^{-24}\ \mathrm{J{\cdot}T^{-1}}$$

However, the CODATA value – which represents the actually *measured* value – for the magnetic moment is slightly larger:

$$\mu_{\mathrm{CODATA}} = 9.2847647043(28)\times10^{-24}\ \mathrm{J{\cdot}T^{-1}}$$

The difference is the so-called anomaly[27], which we can easily calculate as follows[28]:

$$\frac{\mu_{\mathrm{CODATA}} - \mu}{\mu} = 0.00115965\ \ldots$$

The reader may or may not recognize this value: it is, effectively, equal to about 99.85% of Schwinger's factor: $\alpha/2\pi$ = 0.00116141… So why is that the fine-structure constant pops up here once again? The formulas above assume all of the charge is concentrated in one mathematical point. In other words, they assume the *Zitterbewegung* charge has no *spatial* dimension whatsoever. However, we know that is a mathematical idealization only: the phenomenon of elastic scattering of photons tells us the radius of the charge must be of the order of $\alpha{\cdot}\hbar/mc$. We should, therefore, distinguish between a theoretical and an effective radius of the electron. The illustration below shows why: If the *zbw* charge is effectively whizzing around at the speed of light, and we think of it as a charged sphere or shell, then its *center of charge* will *not* coincide with a point on its orbit. The *effective* radius of the orbit will, therefore, be slightly larger than the theoretical radius a.

[26] You should use the CODATA values as published by the US National Institute of Standards and Technology (NIST). These take into account the latest measurements as well as the 2018 revision of the system of SI units.

[27] After reading this section, you will understand the anomalous magnetic moment is not 'anomalous' at all: one would expect to see it pop up in any interpretation of quantum mechanics —in any interpretation which does *not* assume charges have no dimension whatsoever. The mathematical assumption that a pointlike object has no dimension is useful (it allows us to calculate stuff and get meaningful results) but it is what it is: a mathematical idealization only!

[28] You should watch out with the minus signs here – and you may want to think why you put what in the denominator – but it all works out!

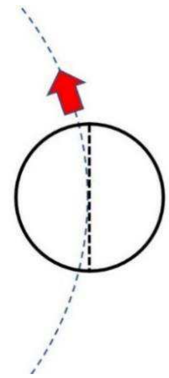

Figure 3: The effective and geometric center of a charge in orbital motion

In fact, relativity theory tells us a sphere or shell will appear as a disk or a hoop because of relativistic length contraction. Hence, the drawing is actually not correct: the plane of the disk should be perpendicular to the direction of motion. If we denote the effective radius by *r*, we should also associate a velocity *v* with it—and this velocity too will be slightly larger than the theoretical velocity *c*. We can now apply the usual formulas for the magnetic moment to get the following result:

$$\frac{\mu_r}{\mu_a} = \frac{\frac{q_e v r}{2}}{\frac{q_e}{2m}\hbar} = \frac{v \cdot r}{c \cdot a} = \frac{\omega \cdot r^2}{\omega \cdot a^2} = \frac{r^2}{a^2} \approx 1 + \frac{\alpha}{2\pi} \Leftrightarrow r = \sqrt{1 + \frac{\alpha}{2\pi}} \cdot a \approx 1.00058 \cdot \frac{\hbar}{mc}$$

There is a crucial step here: we equated the anomaly to 1 + $\alpha/2\pi$. Is that a good approximation? In a first-order approximation, it is. In fact, the reader will probably have heard that Schwinger's $\alpha/2\pi$ factor explains about 99.85% of the anomaly, but it is actually better than. When using the CODATA value for μ_r, we get a μ_r/μ_a ratio that is equal to 99.99982445% of 1 + $\alpha/2\pi$. We think that is good enough to validate our model. You can also calculate the velocity *v* from the equation. We do *not* have an effective frequency, of course: do not forget we talk about the same electron here, so *v* and *r* are not really different things: they are just a different way of looking at the same thing. Hence, you should be able to see the logic of the following calculation and be able to explain its result:

$$1 = \frac{\omega}{\omega} = \frac{v/r}{c/a} \Leftrightarrow v = \frac{r}{a} \cdot c = \frac{\sqrt{1 + \frac{\alpha}{2\pi}} \cdot a}{a} \cdot c = \sqrt{1 + \frac{\alpha}{2\pi}} \cdot c \approx 1.00058 \cdot c$$

The proton

A proton also carries the elementary charge but with a positive sign. When applying the $a = \hbar/mc$ radius formula to a proton, we get a value which is 1/4 of the *measured* proton radius: about 0.21 fm, as opposed to the 0.83-0.84 fm charge radius which was established by Professors Pohl, Gasparan and others over the past decade.[29] We get the right radius from using a modified Planck-Einstein relation (E = 4*hf* = 4ħω) for the orbital frequency of the charge:

[29] For the exact references and contextual information on the (now solved) 'proton radius puzzle', see our paper on it: https://vixra.org/abs/2002.0160.

$$a = c/\omega = c\frac{4\hbar}{\mathrm{E}} = \frac{4\hbar c}{\mathrm{m}c^2} = \frac{4\hbar}{\mathrm{m}c} \approx 0.84\ \mathrm{fm}$$

Writing the Planck-Einstein relation using an integer multiple of *h* or *ħ* (E = *n*·*h*·*f* = *n*·*ħ*·ω) is not uncommon. You should have encountered this relation when studying the black-body problem, for example, and it is also commonly used in the context of Bohr orbitals of electrons. But why is *n* equal to 4 here? Why not 2, or 3, or 5 or some other integer? We do not know: all we know is that the proton is very different. It is definitely not the antimatter counterpart of an electron—a positron. It is much smaller (smaller than the radius we calculated for the *Zitterbewegung* charge) and – somewhat counterintuitive, perhaps – its mass is about 1,836 times that of the electron. Why is that so? Why is a proton even smaller than the $\alpha \cdot a_e$? Again, we do not know. If we think in terms of some force holding the pointlike charge in its orbit, then we calculate this force for the electron as being equal to about 0.106 N. This is the formula[30]:

$$\mathrm{F} = \frac{1}{2} \cdot \frac{\mathrm{m_e}^2 c^3}{\hbar} \approx 0.106\ \mathrm{N}$$

That is a *huge* force at the sub-atomic scale: it is equivalent to a force that gives a mass of about 106 *gram* (1 g = 10^{-3} kg) an acceleration of 1 m/s per second! We will now give you a small exercise and something to reflect about. The electron also comes in a more massive but unstable version: the muon-electron.[31] The muon energy is about 105.66 MeV, so that is about 207 times the electron energy. Its lifetime is much shorter than that of a free neutron[32] but longer than that of other unstable particles: about 2.2 *micro*seconds (10^{-6} s). Now that is fairly long as compared to other non-stable particles – all is relative! – and that may explain why we also get a sensible result when using the Planck-Einstein relation to calculate its frequency and radius.[33]

$$a = c/\omega = c\frac{\hbar}{\mathrm{E}} = \frac{\hbar c}{\mathrm{m}c^2} = \frac{\hbar}{\mathrm{m}c} = \approx 1.87\ \mathrm{fm}$$

The CODATA value for the Compton *wavelength* of the muon is the following:

$$1.173444110 \times 10^{-14}\ \mathrm{m}\ \pm 0.000000026 \times 10^{-14}\ \mathrm{m}$$

If you divide this by 2π - to get a *radius* instead of a wavelength, you get the same value: about 1.87×10^{-15} m. Hence, our oscillator model seems to work for a muon as well! Why, then, is it not stable? We think it

[30] We have derived this formula elsewhere. The ½ factor is there because we think of the *zbw* charge as having an *effective* (relativistic) mass that is 1/2 (*half*) of the total electron mass. We will come back to this.

[31] You may also have heard about the tau-electron but that is just a resonance with an extremely short lifetime, so the Planck-Einstein relation does not apply: it is *not* an equilibrium state. To be precise, the energy of the tau electron (or tau-*particle* as it is more commonly referred to[31]) is about 1776 MeV, so that is almost 3,500 times the electron mass. Its lifetime, in contrast, is *extremely* short: 2.9×10^{-13} s only. We think the conceptualization of both the muon- as well as the tau-electron in terms of particle *generations* is unproductive: stable and unstable particles are, generally speaking, very different animals!

[32] The mean lifetime of a neutron in the open (outside of the nucleus) is almost 15 minutes!

[33] This presumed longevity of the muon-electron should not be exaggerated, however: the mean lifetime of charged pions, for example, is about 26 *nano*seconds (10^{-9} s), so that's only 85 times less.

is because the oscillation is *almost* on, but not quite.[34] In contrast, the exercise for the tau-electron does *not* yield such sensible result: the theoretical $a = \hbar/mc$ radius does *not* match the CODATA.[35] We think this confirms our interpretation of the Planck-Einstein relation as modelling stable particles.

Why this digression? We can also use our model to calculate the centripetal force which must keep the charge in its orbit for the muon-electron and – an even more interesting exercise – the *ratio* of this force for the electron and the muon:

$$\frac{\mathrm{F_\mu}}{\mathrm{F_e}} = \frac{\frac{\mathrm{m_\mu^2}c^3}{2\hbar}}{\frac{\mathrm{m_e^2}c^3}{2\hbar}} = \frac{\mathrm{m_\mu^2}}{\mathrm{m_e^2}} \approx 42{,}753$$

If a force of 0.106 N is pretty humongous, then a force that is 42,753 times as strong, may surely be called a strong force, right? What about the force inside of a proton? The proton mass is about 8.88 times that of the muon, and it is about 2.22 times smaller. Once again, we get this strange 1/4 factor. The Planck-Einstein relation gives us a frequency, but it also gives us the angular momentum of a (stable) particle. Hence, a proton is also definitely *not* some more massive or stable antimatter version of a muon-electron—if only the Planck-Einstein relation suggests its momentum is four times that of an electron or a muon-electron!

To calculate those forces, we used an oscillator model which we will come back to later. We'll just jot down the formula for the proton:

$$\mathrm{F_p} = \frac{1}{2}\mathrm{m_p}a\omega^2 = \frac{1}{2}\mathrm{m_p}a\frac{c^2}{a^2} = \frac{1}{2}\frac{\mathrm{m_p}c^2}{a} = \frac{1}{2}\frac{\mathrm{m_p}^2\mathrm{c}^3}{4\hbar} = \frac{1}{8}\frac{\mathrm{m_p}^2\mathrm{c}^3}{\hbar} \approx 89{,}349\ \mathrm{N}$$

A force of 4,532 N inside of a muon and a force of 89,349 N inside of a proton? We get nonsensical results here, don't we? Maybe. Maybe not. A few back-of-the-envelope calculations reveal we should not be too worried we are modelling a black hole here.[36] We will come back to these calculations later because – for the time being – these are the questions to which we have no real answer to them. Here we will just note these calculations crucially depend on the concept of the *effective* mass of the charge inside of these particles. The effective mass is purely relativistic: it is the mass which the charge acquires because (and *only* because) of its velocity (lightspeed). As such, the pointlike charge inside of particles makes one think of a photon (a photon also acquires an effective mass because of its sheer velocity) but such comparison does not go far: photons do *not* carry charge. Hence, you should probably forget about this comparison for the time being—until we talk about the photon, which we will do soon enough.

For the time being, you should note various considerations suggests this effective mass is equal to *half* of

[34] The reader can verify this by re-calculating the Compton wavelength from the radius we obtain and the exact CODATA values (with or without the last digits for the uncertainties) for the constants and variables. The reader will see that the value thus obtained falls within the uncertainty interval of the CODATA value for the Compton wavelength. The reader think about this result and its meaning as part of the exercise.

[35] CODATA/NIST values for the properties of the tau-electron can be found here: https://physics.nist.gov/cgi-bin/cuu/Results?search_for=tau. To go from wavelength to radius and vice versa, one should divide or multiply by 2π.

[36] The more informed reader should calculate the Schwarzschild radius of a proton. He or she may also calculate the related energy *densities* and electromagnetic field strength. They are humongous but not impossible.

the total mass of the particle. Hence, the effective mass of the charge in an electron is half of the mass of an electron, and it is half of the mass of the muon inside of a muon, and half of the mass inside of a proton. Why is that so? The intuitive argument here is the energy equipartition theorem: half of the energy of an elementary particle is in the kinetic energy of this charge, and the other half must then be in the oscillation – electromagnetic, strong, or whatever it might be – which keeps this charge in its orbital motion. Does this sound mysterious? It should—because it is!

We need to move on, but we want to make one final remark on this concept of an effective mass. You may or may not have come across the idea of Wheeler's idea of 'mass without mass'. If so, one of such 'mass without mass' models is the idea of the mass of an electron being all electromagnetic. The basic idea is to 'assemble' the elementary charge by bringing infinitesimally small charge *fractions* together. The calculation involves an easy integral which we will not repeat here.[37] We will just present the grand result. When calculating the electromagnetic mass or energy of a sphere of charge with radius *a*, you get the following interesting formula[38]:

$$\mathrm{U} = \frac{1}{2}\frac{\mathrm{e}^2}{a} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r_\mathrm{e}} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{\mathrm{m}c}{\alpha\hbar} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{4\pi\varepsilon_0\hbar\mathrm{m}c^2}{\mathrm{q_e^2}\hbar} = \frac{1}{2}\mathrm{m}c^2$$

Feynman was puzzled by that ½ factor: where is the other half of the energy (or the mass) of the electron? Our ring current model shows the ½ factor is quite logical: Feynman is assembling the *zbw* charge here—*not* the electron as a whole. Hence, the missing mass is in the *Zitterbewegung* or orbital/circular motion of the *zbw* charge. We can now *derive* the classical electron radius from the formula above:

$$\mathrm{U} = \frac{1}{2}\frac{\mathrm{e}^2}{r_\mathrm{e}} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0}\frac{1}{r_\mathrm{e}} \Leftrightarrow r_\mathrm{e} = \frac{1}{2}\frac{\mathrm{q_e^2}}{4\pi\varepsilon_0\mathrm{U}} = \alpha\frac{\hbar c}{2\mathrm{m}_\gamma c^2} = \alpha\frac{\hbar}{\mathrm{m_e}c} = \alpha r_\mathrm{C} \approx 2.82\times 10^{-15}\ \mathrm{m}$$

As mentioned, we will come back to all of this. For the time being, we just want to plant these ideas in your head so you can start thinking through them for yourself.

The neutron

We think of photons, electrons, and protons – and neutrinos – as elementary particles. Elementary particles are, obviously, stable. They would not be elementary, otherwise. Again, we repeat that the difference between photons and neutrinos on the one hand, and electrons, protons, and other matter-particles on the other, is that we think all matter-particles carry charge—even if they are neutral. A neutron is an example of a neutral matter-particle. We know it is unstable outside of the nucleus but its longevity – as compared to other non-stable particles – is remarkable. Let us explore what it might be—if only to provide some kind of model for analyzing other unstable particle, perhaps.

We should first note that the neutron radius is about the same as that of a proton. How do we know this? NIST only gives the *rms* charge radius for a proton based on the various proton radius measurements. There is only a CODATA value for the Compton wavelength for a neutron, which is more

[37] We may refer to one of Feynman's lectures on electromagnetism here.

[38] You need the $\alpha = \frac{\mathrm{q_e^2}}{2\varepsilon_0 hc} = \frac{\mathrm{q_e^2}}{4\pi\varepsilon_0\hbar c}$ and $a = r_\mathrm{e} = \alpha\frac{\hbar}{\mathrm{m}c}$ formulas here.

or less the same as that for the proton. To be precise, the two values are this.

$$\lambda_{neutron} = 1.31959090581(75)\times10^{-15}\ \text{m}$$

$$\lambda_{proton} = 1.32140985539(40)\times10^{-15}\ \text{m}$$

These values are just mechanical calculations based on the *mass* or energy of protons and neutrons respectively: the Compton wavelength is, effectively, calculated as $\lambda = h/mc$.[39] However, you should, of course, not only rely on CODATA values only: you should *google* for experiments measuring the size of a neutron directly or indirectly to get an idea of what is going on here.

Let us look at the energies. The neutron's energy is about 939,565,420 eV. The proton energy is about 938,272,088 eV. Hence, the difference is about 1,293,332 eV. This mass difference, combined with the fact that neutrons spontaneously decay into protons but – conversely – there is no such thing as spontaneous proton decay[40], makes us think a neutron must, somehow, combine a proton and an electron. The mass of an electron is 0.511 MeV/c^2, so that is only about 40% of the energy difference, but the kinetic and *binding* energy could make up for the remainder.[41]

So, yes, we will want to think of a neutron as carrying both positive and negative charge inside. These charges balance each other out (there is no *net* electric charge) but their respective motion still yields a small magnetic moment, which we think of as some *net* result from the motion of the positive and negative charge inside. We will come back to this later. As mentioned above, we want to limit ourselves to rather short introductory descriptions of the stuff we are going to look at in much more detail later. Just make a mental note of what you instinctively may or may not agree with, and why, and then you will see it will all fall into place later. Or not. 😊

We should probably write about some other composite matter-particles here – like a nucleus or an atom – but we will save that for later chapters because we want you to read the other chapters of this (future) book too. 😊 However, before we move to the next section, we should say a few words about the concept of *spin*. It is one of those ill-defined concepts in physics and we should, therefore, tell you what we think about it.

Spin

The concepts of spin, magnetic moment and angular momentum and the gyromagnetic ratio are closely related. Indeed, if we think of our charge having some *effective* or *relativistic* mass, then the particle as a whole will have some *real* angular momentum. The gyromagnetic ratio can then be defined as the ratio between the magnetic moment (μ) and this angular moment (L). The same kind of geometric argument

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[39] The reader should note that the Compton wavelength and, therefore, the Compton radius is *inversely* proportional to the mass: a more massive particle is, therefore, associated with a *smaller* radius. This is somewhat counterintuitive but it is what it is.

[40] None of the experiments (think of the Super-Kamiokande detector here) found any evidence of proton decay so far.

[41] The reader should note that the mass of a proton and an electron add up to *less* than the mass of a neutron, which is why it is only logical that a neutron should decay into a proton and an electron. Binding energies – think of [Feynman's calculations of the radius of the hydrogen atom](), for example – are usually *negative*.

we have used to derive the force can then be used to calculate this g-ratio.[42] Of course, we need to get the units right, so what is it this *unit* for the g-ratio? If you do a dimensional analysis[43], you will find it is the charge per unit mass. We can, therefore, *define* the g-ratio like this:

$$\frac{\mu}{L} = g \cdot \frac{q}{m}$$

We can now use the theoretical values we found for the electron to calculate this g-ratio for an electron. It turns out to be equal to 1/2:

$$\mu_e = g_e \cdot \frac{q_e}{m_e} \cdot L_e \Leftrightarrow \frac{q_e}{2m_e}\hbar \ = g_e \cdot \frac{q_e}{m_e} \cdot \hbar \Leftrightarrow g_e = \frac{1}{2}$$

We can repeat this exercise for the muon and the proton and – despite that weird ¼ or 4 factor for the proton – you will find you get the same ratio. Let us put them all together here.

$$\frac{\mu}{L} = g \cdot \frac{q}{m} = \frac{1}{2} \cdot \frac{q}{m} \Leftrightarrow \frac{m \cdot \mu}{q \cdot L} = \frac{1}{2}$$

$$electron\text{:}\ \frac{m_e \cdot \mu_e}{q_e \cdot L_e} = \frac{m_e \cdot \frac{q_e\hbar}{2m_e}}{q_e \cdot \hbar} = \frac{1}{2}$$

$$muon\text{:}\ \frac{m_\mu \cdot \mu_\mu}{q_e \cdot L_\mu} = \frac{m_\mu \cdot \frac{q_e\hbar}{2m_\mu}}{q_e \cdot \hbar} = \frac{1}{2}$$

$$proton\text{:}\ \frac{m_p \cdot \mu_p}{q_e \cdot L_p} = \frac{m_p \cdot \frac{2q_e\hbar}{m_p}}{q_e \cdot 4\hbar} = \frac{1}{2}$$

We may, therefore, say that the only *meaningful* g-factor that can be defined is really this: q/2m. It is, effectively, the ratio between the magnetic moment and the angular momentum for all of the matter-particles we looked at here, which are the electron, the muon, and the proton. That is why matter-particles are referred to as spin-1/2 particles. It is weird but not as weird as some would like you to believe: there is no magic here![44]

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[42] The reader should note this simpler definition of the g-ratio differs from the mainstream definition by a factor ½ because of the definition of the *Bohr magneton*, which also has this ½ factor. The origin of this factor is, once again, related to the concept of the effective or relativistic mass of the charge, which is half of the mass of the particle. In the context of this discussion (spin and g-ratios), we think this factor confuses rather than clarifies the matter – literally 😊 – which is why we omit it. The mainstream physicist will cry wolf here but he should not: the Bohr magneton will pop up again in a moment and we will see that it is, effectively, a meaningful unit. We just feel one should not insert the Bohr magneton at the *start* of the analysis but present it as a *result* of the analysis instead.

[43] This may sound formidable but you should just write out the units for μ and L and see what you are missing to be able to write g as a number *without* any physical dimension.

[44] There is enough magic elsewhere so we would rather tell you upfront what we do and what we do not understand.

We should note something else that is oft forgotten. Spin is an axial vector. Read this again: it is a *vector*, and it is an *axial* vector.[45] We note this because it is related to another remark. We tend to think of the Planck-Einstein relation as a scalar equation. Indeed, the E = $h \cdot f$ equation involves scalar quantities only, notably energy and frequency.[46] However, in their reduced form, we should probably think of Planck's quantum and (angular) frequency as vectors, so we get a *vector* dot product[47]:

$$E = \boldsymbol{\hbar} \cdot \boldsymbol{\omega}$$

This brings us, in turn, to the relation between the Planck-Einstein relation ($E = h \cdot f = \hbar \cdot \omega$) and the *de Broglie* relation ($\lambda = h/p \Leftrightarrow h = p \cdot \lambda$). We can, effectively, write Planck's quantum of action as the product of some energy and a cycle time ($E = h \cdot f \Leftrightarrow h = E/f = E \cdot T$) but we can also write it as the product of some momentum (p) and some *length*—linear or circular, perhaps: think of the wavelength of light, or the circumference of the orbital motion of the pointlike charge. If we denote such length by λ and, keeping in mind that (linear) momentum is a vector too, we can write the *de Broglie* relation as a vector relation[48]:

$$\mathbf{p} = \frac{\boldsymbol{h}}{\lambda}$$

In a similar vein, we will actually want to think of the reduced Planck constant ($\boldsymbol{\hbar} = \boldsymbol{h}/2\pi$) as a proper angular momentum, which can and should be written as $\boldsymbol{\hbar} = I \cdot \boldsymbol{\omega}$: the product of an *angular mass* (the rotational inertia I) and an orbital angular frequency ($\boldsymbol{\omega}$). This, then, also gives meaning to the concept of *spin* (which is either up or down).

You should not there is no uncertainty in these concepts except for the uncertainty in regard to the *plane* of oscillation (which is given by the *direction* of $\boldsymbol{\hbar}$ and $\boldsymbol{\omega}$) *in the absence of an external electromagnetic field*. Indeed, the oscillatory motion of the charge generates a classical magnetic moment which – equally classically – will precess in an external electromagnetic field. Hence, it is only in the *absence* of an electromagnetic field that we cannot know what the plane of oscillation will be. This is quite consistent from an epistemological point of view: how would we define up or down, left or right, back and front – space itself, actually – in the absence of an electromagnetic field?

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[45] The reader should be familiar with the concepts of axial and polar vectors. Polar vectors (think of a position or radius vector, for example) are sometimes referred to as real vectors, while axial vectors (think of angular momentum or the magnetic moment) are often referred to as pseudovectors, but they are equally *real* in a physical sense, of course! Their mathematical behavior is different, however. That is because they can be written as the cross-product of two other vectors. Real vectors reverse sign when a coordinate axis is reversed. Pseudovectors do *not*. We will let the reader *google* the nitty-gritty here. The definition crucially matters because it is used in *core* quantum-mechanical arguments. We may refer, for example, to Feynman's chapter on symmetries and conservation laws.

[46] With some imagination, you may think of *potential* energy as a quantity also involving the idea of direction because we *measure* or *define* it as an energy difference between two points: we move a charge with or against a force. However, this idea is not very productive in this context.

[47] The boldface notation is subtle but powerful!

[48] The symbol is obvious in the context of a linear wavelength but much less so when denoting a circumference—which we want you to think of as a *circular* wavelength!

Let us move to a brief discussion of the nature of light-particles. In fact, we only *know* one: the photon. We effectively think neutrinos are light-like too: we think that, just like photons, they carry energy too, but the nature of this energy is apparently related to some strong force of which we know very little (we refer to the discussion of forces inside particles as part of our proton model here).[49]

1-5 Light-particles

Dirac was of the opinion that a lot of what we write above could not be verified directly by experiment because "the frequency of the oscillatory motion is so high, and its amplitude is so small." We are effectively reflecting on very small dimensions, both in space as well as in time: distances as short as 0.2 or 0.3 femtometer (10^{-15} m), and times as short as 10^{-22} or 10^{-23} seconds. You will often read that we cannot *imagine* how short such distances or time intervals actually *are*. We disagree. Imaging a particle with *no dimension whatsoever* is an impossible task, but thinking of very short time or distance intervals should not be a problem: if you can, somehow, imagine a radio wave, then you can also imagine the extreme frequencies of gamma-rays or the *Zitterbewegung* of the electric charge.

So that is what we ask you do: think about extremely small or extremely large values, but do not try to imagine zero-dimensional or infinite stuff. The finite speed of light (c) probably tells us the mathematical concept of infinity is useful as a limiting idea but that, in Nature, actual infinities do probably *not* exist. Likewise, forget about the idea of the electric charge having no dimension whatsoever. We already talked about this. In fact, if you wonder what you should remember from this lecture, it is this: *matter*-particles carry electric charge and they are *not* pointlike. If they have a shape, then it is probably disk-like.

But we have light-particles also, and if there is one particle which we should probably think of as being pointlike, then it is the photon (and the neutrino—which we think of as the photon of the strong force[50]), although you will soon see why we may think of them as being linear rather than pointlike too![51] So let us quickly tell you how we think of the photon as a model of a light-particle—*the* light-particle, in fact. We refer to it as the one-cycle photon model and the argument goes like this.

[49] The reader may wonder why we do not say anything about the weak force. We do not think very highly of the hypothesis of a weak force. We think a force theory that explains why charges stay together must also explain when and how they separate. More fundamentally, we think a force works through a force *field*: the idea that forces are *mediated* by virtual messenger particles sounds medieval to us: it reflects earlier *aether* theories, which died because of Occam's Razor principle: a theory should be parsimonious. The idea of a medium turned out to be superfluous at the end of the 19th century. We think the idea of gluons and W/Z bosons are headed the same way: no one has ever *observed* these things so why should we assume they actually *exist*?

[50] You may think this contradicts our earlier statements on the idea of force-carrying particles and you are right—to some extent, at least. We do not believe in W and Z bosons and gluons, but we do believe photons carry electromagnetic *energy*: carrying energy and carrying force are slightly different concepts. Likewise, we think the concept of a strong(er) force inside the nucleus is useful and, therefore, we feel the concept of neutrinos carrying some strong(er) energy is useful too.

[51] A photon has a (linear) wavelength, and that is not some mysterious mathematical quantity but an actual *physical* length. However, relativity theory implies such lengths contract when moving at the speed of light: there is, therefore, no contradiction in thinking of photons as being linear and pointlike at the same time!

The size and shape of a photon

1. Photons are very real and carry equally real energy: electromagnetic energy. When an electron goes from one state to another – from one electron orbital to another, for example[52] – it will absorb or emit a photon. Photons make up light: visible light, low-energy radio waves, or high-energy X- and γ-rays. These waves carry energy and, yes, when we look really closely, these waves are made up of photons.[53] So, yes, the energy in the light is carried by the photons.

Saying they carry electromagnetic energy is something else than saying they carry electromagnetic *force* itself. A force acts on a charge: a photon carries no charge. So what *are* they then? How should we *think* of them? Think of it like this: a photon is an *oscillating* electromagnetic field. We describe this field by an electric and a magnetic field vector **E** and **B**. Field vectors do not take up any space: think of them as a force without a charge to act on. Indeed, a non-zero field at some point in space and time – which we describe using the (*x*, *y*, *x*, *t*) coordinates – tell us what the force *would* be *if* we would happen to have a unit charge at the same point in space and in time. You know the formula for the *electromagnetic* force: it is the Lorentz force $\mathbf{F} = q\cdot(\mathbf{E} + \boldsymbol{v}\times\mathbf{B})$. Hence, the electromagnetic force is the sum of two (orthogonal) *component* vectors: $q\cdot\mathbf{E}$ and $q\cdot\boldsymbol{v}\times\mathbf{B}$.

The velocity vector $\boldsymbol{v}$ in the equation shows *both* of these two component force vectors depend on our frame of reference. Hence, we should think of the separation of the electromagnetic force into an 'electric' (or electrostatic) and a 'magnetic' force *component* as being somewhat artificial: the *electromagnetic* force is (very) *real* – because it determines the *motion* of the charge – but our cutting-up of it in two separate components depends on our frame of reference and is, therefore, (very) *relative*. We should refer to our remarks on the relative *strength* of the electric and magnetic field, however: the reader should not think in terms of the electric or magnetic force being more or less important in the analysis and always analyze *both* as aspects of one and the same *reality*.

Let us get back to our photon: we think the photon is pointlike because the **E** and **B** vectors that describe it will be zero at each and every point in time *and* in space *except if our photon happens to be at the* (*x, y, z*) *location at time t.*

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[52] In most cases, we will be talking about atoms emitting or absorbing light but that is not always the case. We think Compton scattering may be explained conceptually by accepting the incoming and outgoing photon are *different* photons (they have different wavelengths so it should not be too difficult to accept this as a logical statement: the wavelength pretty much *defines* the photon—so if it is different, you have a different photon). This, then, leads us to think of an excited electron state, which briefly combines the energy of the stationary electron and the photon it has just *absorbed*. The electron then returns to its equilibrium state by emitting a new photon. The energy difference between the incoming and outgoing photon then gets added to the *kinetic* energy of the electron through the following *law*:

$$\lambda' - \lambda = \Delta f = \frac{h}{\mathrm{m}c}(1 - \cos\theta)$$

This physical law can be easily derived from first principles (see, for example, Patrick R. Le Clair, 2019): the energy and momentum conservation laws, to be precise. More importantly, however, it has been confirmed *experimentally*.

[53] We *know* this because a zillion experiments did confirm the *reality* of the photoelectric effect.

[…]

Please read the above again, and think about it for a while. To help you, we will repeat ourselves: **our photon is pointlike because the electric and magnetic field vectors that describe it are zero everywhere except where our photon happens to be.**

2. At the same time, we know a photon is defined by its *wavelength*. So how does that work? What is the *physical* meaning of the wavelength? It is, quite simply, the distance over which the electric and magnetic field vectors will go through a full *cycle* of their oscillation. That is all there is to it: nothing more, nothing less.

That distance is, of course, a *linear* distance: to be precise, it is the distance Δs between two points (x_1, y_1, z_1) and (x_2, y_2, z_2) where the **E** and **B** vectors have the same value. The photon will need some time Δt to travel between these two points, and these intervals in time and space are related through the (constant) velocity of the wave, which is also the velocity of the pointlike photon. That velocity is, of course, the speed of light, and the time interval is the cycle time T = 1/*f*. We, therefore, get the equation that will be familiar to you:

$$c = \frac{\Delta s}{\Delta t} = \frac{\lambda}{\mathrm{T}}$$

We can now relate this to the Planck-Einstein relation.

3. Any (regular) oscillation has a frequency and a cycle time T = 1/*f* = 2π/ω. The Planck-Einstein relation relates *f* and T to the energy (E) through Planck's constant (*h*):

$$\mathrm{E} = h \cdot f = \hbar \cdot \omega \Leftrightarrow \mathrm{E} \cdot \mathrm{T} = h$$

The Planck-Einstein relation does not only apply to matter-particles but also to a photon. In fact, it was *first* applied to a photon.[54] Think of the photon as *packing* not only the energy E but also an amount of *physical action* that is equal to *h*.

We have not talked much about the meaning of *h* so far, so let us do that now. *Physical action* is a concept that is not used all that often in physics: physicists will talk about energy or momentum rather than about physical action.[55] However, we find the concept as least as useful. Physical action can *express* itself in two ways: as some energy over some time (E·T) or – alternatively – as some momentum over some distance (p·λ). For example, we know the (pushing) momentum of a photon[56] will be equal to p = E/*c*. We can, therefore, write the Planck-Einstein relation for the photon in two equivalent ways:

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[54] The application of the Planck-Einstein relation to matter-particles is *implicit* in the *de Broglie* relation. Unfortunately, Louis de Broglie imagined the matter-wave as a linear instead of a circular or orbital oscillation. He also made the mistake of thinking of a particle as a wave *packet*, rather than as a single wave! The latter mistake then led Bohr and Heisenberg to promote uncertainty to a metaphysical principle.

[55] We think the German term for physical action – *Wirkung* – describes the concept much better than English.

[56] For an easily accessible treatment and calculation of the formula, see: *Feynman's Lectures*, Vol. I, Chapter 34, section 9.

$$\mathrm{E}\cdot\mathrm{T}=\frac{E}{c}\cdot c\mathrm{T}=h\Leftrightarrow \mathrm{p}\cdot\lambda=h$$

We could jot down many more relations, but we should not be too long here.[57] We said the photon *packs* an energy that is given by its frequency (or its wavelength or cycle time through the $c = \lambda f$ relation) through the Planck-Einstein relation. We also said it packs an amount of *physical action* that is equal to *h*. So how should we think of that? Let us connect all of the dots now.

4. The Planck-Einstein relation does not only apply to a photon, but it also applies to electron orbitals—but in a different way. When analyzing the electron orbitals for the simplest of atoms (the one-proton hydrogen atom), the Planck-Einstein rule amounts to saying the electron orbitals are separated by an amount of *physical action* that is equal to $h = 2\pi\cdot\hbar$.[58] Hence, when an electron jumps from one level to the next – say from the second to the first – then the atom will lose one unit of *h*. The photon that is emitted or absorbed will have to pack that somehow. It will also have to pack the related energy, which is given by the Rydberg formula:

$$\mathrm{E}_{n_2}-\mathrm{E}_{n_1}=-\frac{1}{{n_2}^2}\mathrm{E}_R+\frac{1}{{n_1}^2}\mathrm{E}_R=\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)\cdot\mathrm{E}_R=\left(\frac{1}{{n_1}^2}-\frac{1}{{n_2}^2}\right)\cdot\frac{\alpha^2\mathrm{m}c^2}{2}$$

To focus our thinking, let us consider the transition from the second to the first level, for which the $1/1^2 - 1/2^2$ is equal 0.75. Hence, the photon energy should be equal to $(0.75)\cdot\mathrm{E}_R \approx 10.2$ eV. Now, if the total action is equal to *h*, then the cycle time T can be calculated as:

$$\mathrm{E}\cdot\mathrm{T}=h\Leftrightarrow\mathrm{T}=\frac{h}{\mathrm{E}}\approx\frac{4.135\times10^{-15}\mathrm{eV}\cdot\mathrm{s}}{10.2\ \mathrm{eV}}\approx0.4\times10^{-15}\ \mathrm{s}$$

This corresponds to a wave train with a length of $(3\times10^8\ \mathrm{m/s})\cdot(0.4\times10^{-15}\ \mathrm{s}) = 122$ nm. It is, in fact, the wavelength of the light ($\lambda = c/f = c\cdot\mathrm{T} = \mathrm{h}\cdot c/\mathrm{E}$) that we would associate with this photon energy.

The reader may think all of the above is rather trivial. If so, then that is good: the reader should just consider it as a warm-up for the math that follows. If not, then it is also good: it then means it was useful to take you through this. Before we move on to the next, we should make a final remark in regard to the *spin* of a photon.

Polarization and photon spin states

You may have heard that photons are part of a more general category of particles which are referred to

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[57] We may refer the reader to our manuscript, our paper on the meaning of the fine-structure constant, or various others papers in which we explore the nature of light. We just like to point out one thing that is quite particular for the photon: the reader should note that the $\mathrm{E} = \mathrm{m}c^2$ mass-energy equivalence relation and the $\mathrm{p} = \mathrm{m}c = \mathrm{E}/c$ can be very easily related when discussing photons. There is an easy *mathematical* equivalence here. That is not the case for matter-particles: the *de Broglie* wavelength can be interpreted geometrically but the analysis is somewhat more complicated—not impossible (not at all, actually) but just a bit more convoluted because of its circular (as opposed to linear) nature.

[58] The model of the atom here is the Bohr model. It does *not* take incorporate the finer structure of electron orbitals and energy states. That finer structure is explained by differences in magnetic energies due to the *spin* (angular momentum) of the electron. We will come back to this. Also note we take the most general of cases: a photon being emitted or absorbed by an atom. Photons can also be emitted by free electrons in an excited state. The basics of the analysis remain the same.

as *bosons*, while the above-mentioned matter-particles are generally referred to as *fermions*. The difference between these two is either described in terms of their amplitudes having to be added with a plus or a minus sign[59] or, else, in terms of their spin number being 1 or 1/2. In addition, mainstream quantum physicists will tell you bosons *mediate* forces and should, therefore, be thought of as "force-carriers." Finally, you will also have heard about (1) the Pauli exclusion principle, which applies to fermions only and which states two fermions can only be in the same *quantum state* if their spin is opposite and (2) its opposite, which states any number of identical bosons can occupy the same state. Let us go over these statements one by one.

1. We think the concept of force-carriers is superfluous at best, and plain nonsense at worst. We think Feynman's description of fermions and bosons in terms of adding amplitudes with a plus or a minus sign is nonsense too: we should not expect electrons and photons to perform some kind of a handshake when they meet each other to decide what sign they should apply to their interaction.[60] You should also note that – even if we could imagine some mechanism for such handshakes – the situation becomes extremely complicated in the context of three or more particles.

2. In contrast, the distinction in terms of their spin state makes sense—to some extent, at least. Indeed, we have shown that the g-ratio for matter-particles is effectively equal to 1/2, even if their angular momentum can come as a multiple of $\hbar$ (think of the proton here or, if you prefer not to trust our proton model, the n = 2, 3,… electron orbital states). So what is that spin-*one* state of a photon? It has to do with (1) the polarization of light and (2) the fact that, with light, we get two waves for the prices of one, so to speak: one should analyze the behavior of both the electric as well as the magnetic field vector.

Such analysis requires a bit of imagination but we trust we will be able to *google* a few things here.[61] The one thing you should note, however, is that the mainstream definition of a photon as a spin-one particle is not very consistent. A spin-one particle should have *three* possible states: 1 (*up*), –1 (*down*), or zero (no spin). However, photons – which, we should remind you, are the only bosons *known* to exist from real-life experiments – do *not* have a spin-zero state. Never ever. Their spin is always up *or* down. It is *never* zero. Hence, we think there is no use for this fermion-boson distinction because *the only boson we know – the photon – does not behave like it should behave according to this theory.*

3. The only thing that is left to explain now, is this Pauli exclusion principle and its opposite. Here, we should note that electrons with opposite spin states are *not* identical: their magnetic moment will be opposite and, hence, depending on their *orientation*, these magnetic moments will attract or repel each other. We can, therefore, imagine two electrons with opposite spin states may lower their *joint* energy in some geometric arrangement involving some *alignment* of their magnetic moments.

What about photons? Photons do not carry charge, and the superposition principle tells us we can

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[59] See, for example, Feynman, *Lectures on Quantum Mechanics*, Chapter IV.

[60] We could point out some more contradictions but we do not want to be lengthy here: we may refer the reader to a more detailed analysis of Feynman's Chapter 4.

[61] If you need a reference, we will refer you, once again, to Feynman's lectures, although the lectures on polarized light and polarization states are a bit scattered. 😉

simply add fields. There is, therefore, no *physical* reason why one photon could not occupy the same position as another one. It is not like two electric charges which must repel each other or – in case of an electron and a positron coming together – which must annihilate each other because the charges are opposite.[62]

Is that all there is? For the time being, yes. Again, we have to leave some of the analysis for later, or we would not have anything left to write about! ☺

1-6 Interference experiments

Interference between matter-particles

Most of Feynman's first chapter of lectures on quantum mechanics talks about the double-slit experiment for electrons. When Feynman wrote this, back in 1963, such interference experiments with matter-particles were a *thought* experiment only, and Feynman could not image technology would ever advance enough to actually *do* this experiment. Feynman did coin the word nanotechnology, however—and nanotechnology eventually allowed for such experiments to be actually *demonstrated*.

While previous experiments had provided sufficient *indirect* evidence for electron interference[63], the first experiment which was done *exactly* as Feynman had imagined it, was carried out in 2012 only. It was done by a now rather famous research group at the University of Nebraska–Lincoln (Bach, Pope, Liou and Batelaan, 2012) and we will let you *google* its results and the subsequent analysis of these results. Such analysis is complicated because it is done in terms of quantum-mechanical amplitudes—as opposed to the wave-particle duality as modelled in our realist interpretation of quantum mechanics. Hence, the equivalent of the *Huygens-Fresnel equations*[64] for electron interference is not yet available. Because an analysis of the interactions between the incoming electron and the electrons in the material of the slits is hugely complicated, we doubt whether such analysis will soon be available. However, we do *not* doubt that our conceptualization of matter-particles – which combines both their wave- as well particle-like character of both matter – fits the bill.

Interference between light-particles

The equivalent of the double-slit experiments for electrons going through the slit(s) *one at a time*, is the one-photon Mach-Zehnder interference experiment. This experiment typically involves two beam splitters and two mirrors, as illustrated below.

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[62] The nature of anti-matter is another mystery. We can *clarify* it, to some extent, by [an analysis in terms of opposite spacetime *signatures*] (we will briefly say something about that in the last section of this chapter) but such analysis does not provide a coherent explanation of the mystery of matter-antimatter pair creation and annihilation. It is good some mystery is left—otherwise physics would be dead as a science!

[63] Bach, Pope, Liou and Batelaan (2012) usefully highlight the experiments of Jönsson (1961), Merli, Missiroli and Pozzi (1976), and Tonomura, Matsuda, Kawasaki and Ezawa (1989). However, these experiments did *not* involve the build-up of the double-slit diffraction pattern by measuring (the impact of) *one electron at a time*.

[64] We use this term rather loosely to refer to the principles and equations describing interference, diffraction and wave propagation both in the far- as well as in the near-field. The near-field refers to distances within which one should apply an analysis in terms of light particles, while the far-field allows for a purely classical analysis in terms of wave mechanics only.

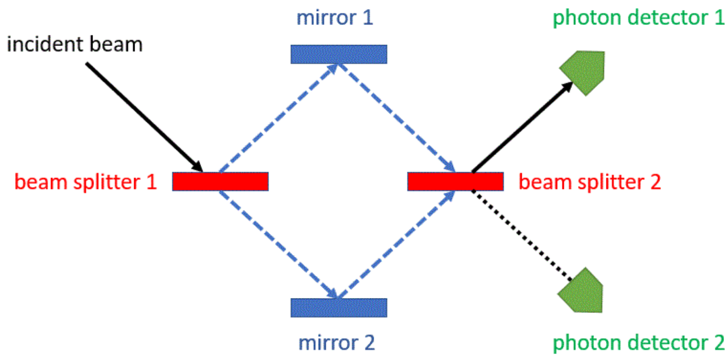

Figure 4: The Mach-Zehnder interferometer

Both popular as well as academic accounts of this experiments wax lyrical about the *mystery* of the results of this experiment but usually forget to mention the obvious: when beam splitters *split* a beam, they produce two *linearly* polarized waves. The energy of the incoming beam is, therefore, also split. There is no reason whatsoever to assume such energy split cannot possibly happen when one single photon goes through the splitter. This provides a rather classical explanation of what might be going on in these experiments:

1. The incoming photon is circularly polarized (left- or right-handed).
2. The first beam splitter splits our photon into two linearly polarized waves.
3. The mirrors reflect those waves and the second beam splitter recombines the two linear waves back into a circularly polarized wave.
4. The positive or negative interference then explains the binary outcome of the Mach-Zehnder experiment – at the level of a photon – *in classical terms*.

Are we sure about this? Our honest answer is this: no, but we are *quite sure*. Why are we so sure? Because of the energy conservation law. However, because we still have more than one chapter to write, we will save the full-blown development of this argument for a later lecture. In the meanwhile, you may want to think about it yourself.[65]

1-7 The meaning of the wavefunction

The ring current model and the elementary wavefunction

We will talk a lot about wavefunctions and probability amplitudes in the next section, so we will be brief here. When looking at **Figure 2**, it is quite obvious that we can use the elementary wavefunction (Euler's formula) to represents the motion of the pointlike charge by interpreting $\boldsymbol{r} = a \cdot e^{i\theta} = a \cdot e^{i\cdot(E\cdot t - \mathbf{k}\cdot\boldsymbol{x})/\hbar}$ as its position vector. The coefficient a is then, equally obviously, nothing but the Compton radius $a = \hbar/mc$.[66]

[65] In case our reader would not be able to wait till we release the next chapter(s) – or would need some inspiration to start thinking about it – we can refer him or her to our paper on Mach-Zehnder interference, which already sketches the contours of the argument.

[66] When discussing the concept of probability amplitudes, we will talk about the need to *normalize* them because the sum of all probabilities – as per our conventions – has to add up to 1. However, the reader may already

The relativistic invariance of the argument of the wavefunction is then easily demonstrated by noting that the position of the pointlike particle in its own reference frame will be equal to $x'(t') = 0$ for all t'.

We can then relate the position and time variables in the reference frame of the particle and in our frame of reference by using Lorentz's equations[67]:

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{vt - vt}{\sqrt{1 - \frac{v^2}{c^2}}} = 0$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

When denoting the energy and the momentum of the electron in our reference frame as E_v and $p = \gamma m_0 v$, the argument of the (elementary) wavefunction $a \cdot e^{i\theta}$ can be re-written as follows[68]:

$$\theta = \frac{1}{\hbar}(E_v t - px) = \frac{1}{\hbar}\left(\frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}t - \frac{E_0 v}{c^2\sqrt{1 - \frac{v^2}{c^2}}}x\right) = \frac{1}{\hbar}E_0\left(\frac{t}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{\frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right) = \frac{E_0}{\hbar}t'$$

E_0 is, obviously, the rest energy and, because $p' = 0$ in the reference frame of the electron, the argument of the wavefunction effectively reduces to $E_0 t'/\hbar$ in the reference frame of the electron itself.

Besides proving that the argument of the wavefunction is relativistically invariant, this calculation also demonstrates the relativistic invariance of the Planck-Einstein relation when modelling elementary particles.[69] This is why we feel that the argument of the wavefunction (and the wavefunction itself) is more *real* – in a *physical* sense – than the various wave equations (Schrödinger, Dirac, or Klein-Gordon) for which it is some solution.

In any case, a wave equation usually models the properties of the medium in which a wave propagates. We do not think the medium in which the matter-wave propagates is any different from the medium in which electromagnetic waves propagate. That medium is generally referred to as the *vacuum* and, whether or not you think of it as true nothingness or some medium, we think Maxwell's equations – which establishes the speed of light as an *absolute* constant – model the properties of it sufficiently well! We, therefore, think superluminal phase velocities are *not* possible, which is why we think de Broglie's conceptualization of a matter particle as a wave*packet* – rather than one *single* wave – is

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appreciate we will want to talk about normalization based on physical realities—as opposed to unexplained mathematical conventions or quantum-mechanical *rules*.

[67] We can use these simplified Lorentz equations if we choose our reference frame such that the (classical) linear motion of the electron corresponds to our *x*-axis.

[68] One can use either the general $E = mc^2$ or – if we would want to make it look somewhat fancier – the $pc = Ev/c$ relation. The reader can verify they amount to the same.

[69] The relativistic invariance of the Planck-Einstein relation emerges from other problems, of course. However, we see the *added value* of the model here in providing a *geometric* interpretation: the Planck-Einstein relation effectively models the *integrity* of a particle here.

erroneous.[70]

Modelling spin and antimatter

A good theory should respect Occam's Razor—the *lex parsimoniae*: one should not multiply concepts without necessity. The need for new concepts or new principles – such as the conservation of strangeness, or postulating the existence of a new force or a new potential[71] – should, therefore, be continuously questioned. Conversely, when postulating the existence of the positron in 1928 – which directed experimental research to a search for it and which, about five years later, was effectively found to exist – Paul Dirac unknowingly added another condition for a good theory: all of the *degrees of freedom* in the mathematical description should map to a physical *reality*.

It is, therefore, surprising that the mainstream interpretation of quantum mechanics does not integrate the concept of particle spin from the outset because the + or – sign in front of the imaginary unit (*i*) in the elementary wavefunction ($a \cdot e^{-i\cdot\theta}$ or $a \cdot e^{+i\cdot\theta}$) is thought as *a mathematical convention* only. This non-used *degree of freedom* in the mathematical description then leads to the *false* argument that the wavefunction of spin-½ particles has a 720-degree symmetry. Indeed, physicists treat –1 as a *common phase factor* in the argument of the wavefunction.[72] However, we should think of –1 as a complex number itself: the phase factor may be +π or, alternatively, –π: when going from +1 to –1 (or vice versa), it matters how you get there—as illustrated below.[73]

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[70] See our paper on matter-waves, amplitudes and signals.

[71] We think of the *invention* of the concept of strangeness by Murray Gell-Man and Kazuhiko Nishijima in the 1950s here. This concept started a rather strange life of its own and would later serve as the basis for the quark hypothesis which – for a reason we find even stranger than the concept of strangeness itself – was officially elevated to the status of *a scientific dogma* by the Nobel Prize Committee for Physics.

As for the invention of a new force or a new potential, we are, obviously, referring to the Yukawa potential. This hypothesis – which goes back to 1935 – might actually have been productive if it would have led to a genuine exploration of a stronger short-range force on an electric charge—or, if necessary, the invention of a *new* charge. Indeed, if the electromagnetic force acts on an electric charge, it would be more consistent to postulate some new charge – or some new wave equation, perhaps – matching the new force. Unfortunately, theorists took a whole different route. They invented a new *aether* theory instead: it is based on the medieval idea of messenger or *virtual* particles *mediating* forces. The latter led to the invention of *gluons* which – as a concept – we find at least as weird as quarks.

[72] Mainstream physicists therefore think one can just multiply a set of amplitudes – let us say *two* amplitudes, to focus our mind (think of a beam splitter or alternative paths here) – with –1 and get the same *physical states*.

[73] The quantum-mechanical argument is technical, and I did *not* reproduce it in this book. I encourage the reader to glance through it, though. See: *Euler's Wavefunction: The Double Life of – 1*. Note that the $e^{+i\pi} \neq e^{-i\pi}$ expression may look like *horror* to a mathematician! However, if he or she has a bit of a sense for geometry and the difference between identity and equivalence relations, there should be no surprise. If you are an amateur physicist, you should be excited: it is, effectively, the secret key to unlocking the so-called mystery of quantum mechanics. Remember Aquinas' warning: *quia parvus error in principio magnus est in fine*. A small error in the beginning can lead to great errors in the conclusions, and we think of this as a rather serious error in the beginning!

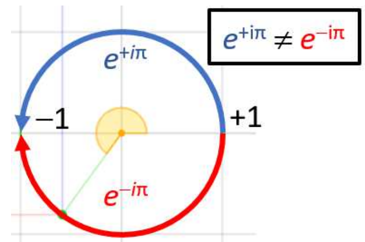

Figure 5: $e^{+i\pi} \neq e^{-i\pi}$

Combining the + and − sign for the imaginary unit with the direction of travel, we get four mutually exclusive structures for the electron wavefunction:

Spin and direction of travel	**Spin up ($J = +\hbar/2$)**	**Spin down ($J = -\hbar/2$)**
Positive *x*-direction	$\psi = \exp[i(kx-\omega t)]$	$\psi^* = \exp[-i(kx-\omega t)] = \exp[i(\omega t-kx)]$
Negative *x*-direction	$\chi = \exp[-i(kx+\omega t)] = \exp[i(\omega t-kx)]$	$\chi^* = \exp[i(kx+\omega t)]$

Table 1: Occam's Razor: mathematical possibilities versus physical realities (1)

We may now combine these four possibilities with the properties of anti-matter. Indeed, we think antimatter is different from matter because of its opposite spacetime signature. The logic here is the following. Consider a particular direction of the elementary current generating the magnetic moment (we effectively define spin as an (elementary) *current*[74]). It is then quite easy to see that the magnetic moment of an electron ($\mu = -q_e\hbar/2m$) and that of a positron ($\mu = +q_e\hbar/2m$) would be opposite. We may, therefore, associate a particular direction of rotation with an angular frequency *vector* **ω** which – depending on the direction of the current – will be up or down with regard to the plane of rotation.[75] We can, therefore, associate this with the spin property, which is also up or down.

We, therefore, have another table with four mutually exclusive possibilities, which we should combine with the possible directions of travel in Table 1[76]:

Matter-antimatter	**Spin up**	**Spin down**
Electron	$\mu_{-e} = -q_e\hbar/2m$	$\mu_{-e} = +q_e\hbar/2m$
Positron	$\mu_{+e} = +q_e\hbar/2m$	$\mu_{+e} = -q_e\hbar/2m$

Table 2: Occam's Razor: mathematical possibilities versus physical realities (2)

Table 2 shows that (1) the ring current model also applies to antimatter but that (2) antimatter has a

[74] We are aware this may sound shocking to those who have been brainwashed in the old culture. If so, make the switch. It should not be difficult: a magnetic moment – *any* magnetic moment, really – is generated by a current. The magnetic moment of elementary particles is no exception.

[75] To determine what is up or down, one has to apply the ubiquitous right-hand rule.

[76] The use of the subscripts in the magnetic moment may be confusing, but should not be: we use −e for an electron and +e for a positron. We do so to preserve the logic of denoting the (positive) elementary charge as q_e (without a + or a − in the subscript here).

different *spacetime* signature. Abusing Minkowski's notation, we may say the spacetime signature of an electron would be + + + + while that of a positron would be + – – –.[77] Table 1 and Table 2, therefore, *complement* each other.

The wavefunction in motion

The geometry of a matter-particle can be related to the geometry of the photon using the following illustration, which shows how the Compton wavelength (the circumference of the *Zitterbewegung* of the pointlike charge) becomes a linear wavelength as the classical velocity goes from 0 to *c*: the *radius* of the circulatory motion must effectively diminish as the electron gains speed. [78]

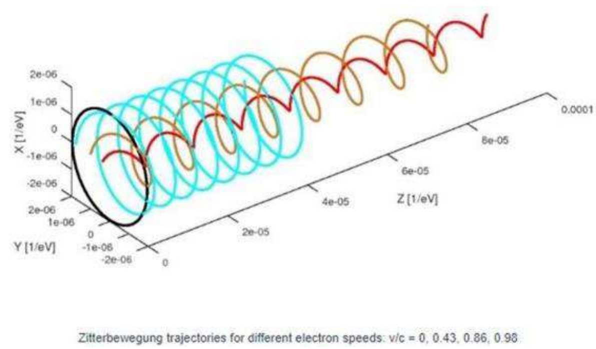

Figure 6: The Compton radius must decrease with increasing velocity

The illustration above is, of course, didactic only: there is no reason to assume perfect perpendicularity between the plane of the ring current and the linear motion of the electron: the plane of oscillation should be taken to be random or – when an external magnetic field is applied – to correspond to Larmor's precessional motion of the angular momentum vector.

We should also reiterate that the fundamental *Zitterbewegung* is probably be chaotic or irregular: perfect circularity or perfect linearity may exist in our mind only. However, our measurements give statistically regular results and, hence, "*though this be madness, yet there is method in 't*"[79] and we can, therefore, meaningfully associate a frequency $\omega = c/a$ and a radius $a = c/\omega$ to the *Zitterbewegung* motion – whether it be perfectly or not-so-perfectly circular or regular – which, using natural time and

[77] In case the reader wonders why we associate the + +++ signature with the *positron* rather than with the electron, the answer is: convention. Indeed, if I am not mistaken (which may or may not be the case), it is the + +++ *metric signature* which is the one which defines the usual righthand rule when dealing with the *direction* of electric currents and magnetic forces.

[78] We thank Prof. Dr. Giorgio Vassallo and his publisher to let us re-use this diagram. It originally appeared in an article by Francesco Celani, Giorgio Vassallo and Antonino Di Tommaso (*Maxwell's equations and Occam's Razor*, November 2017). Once again, however, we should warn the reader that he or she should imagine the plane of oscillation to rotate or oscillate itself. He should not think of it of being *static* – unless we think of the electron moving in a magnetic field, in which case we should probably think of the plane of oscillation as being *parallel* to the direction of propagation. To be precise, he should think of it as *precessing* in the external field.

[79] The reader will recognize the quote from Polonius in *The Tragedy of Hamlet* (Shakespeare), Act 2 Scene 2.

distance units (c = 1) are nothing but the reciprocal of each other.[80]

Conclusions

What we have presented here is definitely *not* mainstream. However, a lot of it is actually already present – implicitly or explicitly – in high-brow scientific articles analyzing various bits and bobs in *much* more detail.[81] We are, therefore, quite confident that we are *not* presenting any nonsense here: we feel we just connected some more dots in a clearly emerging picture of what may, one day, replace what Paul Ehrenfest – in one of his letters to Einstein – referred to as the *'unendlicher Heisenberg-Born-Dirac-Schrödinger Wurstmachinen-Physik-Betrieb'*.[82]

We hope you had as much fun reading this as we had while writing it. If so, please do mail us. Tell us what you think. Tell us what makes sense to you and what does not. It may motivate us to further develop this into an actual (text)book.

Jean Louis Van Belle, 8 June 2020

[80] The reader may also note another inverse proportionality: the $a = \hbar/mc = \hbar c/E$ formula tells us the *Zitterbewegung* or *Compton* radius is *inversely* proportional to the energy (or the equivalent mass) of the elementary particle. This relation is, obviously, *not* as intuitive as the easy $a = c/\omega$ or $\omega = c/a$ relations.

[81] We could, once again, refer to Bernard Schaeffer (2016), Paolo Di Sia (2018) or Celani, Vassallo and Di Tommaso (*Maxwell's equations and Occam's Razor*, November 2017). We could mention many more authors (you may have heard about David Hestenes, for example), but we will do so as we move along.

[82] We will let you *google-translate* that. For the context, we may refer to one of our blog pieces.