Justifying the Twin Prime Conjecture
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Abstract: Let P_n be the n_th prime. For twin primes $P_n - P_{n-1} = 2$. Let X be the number of $(6j-1, 6j+1)$ pairs in the closed interval $[P_n, P_n^2]$. The number of twin primes (TPA_n) in $[P_n, P_n^2]$ is

$$((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X).\ P_3=5,\ 1.7 < a_n, a_{n-1,\ldots,} a_3 < 2.3.$$

We exhibit a formula showing as P_n increases, the number of twin primes in the interval $[P_n, P_n^2]$ also increases.

Let $P_n - P_{n-1} = c$. For $n \geq 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Section 1. Calculating the number of $(6j-1, 6j+1)$ pairs (F_n) with no factor $< P_{n+1}$ in $[1, J_n+1]$.

All primes greater than or equal to five are of the form $6j-1$ or $6j+1$.

$m=1$ to n $\prod P_m = J_n$ is the product of the first n primes. $m=3$ to n $\prod (1/6)(P_m-2)/P_m)(J_n) = F_n$ (proof by induction) is true for $n=3$. Assume the formula is true for n. For each $(6j-1, 6j+1)$ pair with no factor less than P_{n+1} in $[1, J_n+1]$ there are pairs $(6j-1+mJ_n, 6j+1+mJ_n)$ for $m = 0$ to P_n in $[1, J_{n+1}+1]$. P_{n+1} and J_n are relatively prime. P_{n+1} divides $6j-1+mJ_n$ and $6j+1+mJ_n$ each for exactly one different value of $0 \leq m \leq P_n$. Thus, for $m=3$ to $n+1$ $\prod (1/6)(P_m-2)/P_m)(J_{n+1}) = F_{n+1}$.

Determining the number of twin primes pairs in the closed interval (P_n, P_n^2) TPA_n.

All the $(6j-1, 6j+1)$ pairs with no factor less than P_n in which $6j < P_n^2$ are twin primes.

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$. X_m is the number of $(6j-1, 6j+1)$ pairs in the interval $[P_n, P_n^2]$ with no factor in the interval $[P_m, P_n]$. $n \geq m \geq 3$. $(X_{m+1})(P_m - a_m) / P_m = X_m$

$TPA_n = ((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$ for $n \geq m \geq 3$, $P_3=5$, $1.7 < a_m < 2.3$.

The absolute value of $P_m - 2$ for $n \geq m \geq 3$ used in calculating the number of $(6j-1, 6j+1)$ pairs in $[1, J_n+1]$ with no factor less than P_{n+1} sets a range for the values of a_m at $2 - .3 < a_m < 2 + .3$.

Graph 1 (P_m in descending order) plot a_m **values for** 44 **equally spaced** P_m in the intervals $[5, 743]$, $[5, 3011]$, $[5, 10007]$ and $[5, 19993]$ to illustrate the above formula for $[743, 743^2]$, $[3011, 3011^2]$, $[10007, 10007^2]$ and $[19993, 19993^2]$. For selected P_m they show the values of a_m of $(P_m - a_m)$. Since the primes $< P_{n+1}$ are removed in a linear fashion, similar a_m patterns to the four plots in graph 1 are found in all sufficiently large $[P_n, P_n^2]$ intervals $(P_n > 500)$.

Graph 1 – a_m values for 44 selected P_m From $743, 3011, 10007, 19993$ down to 5

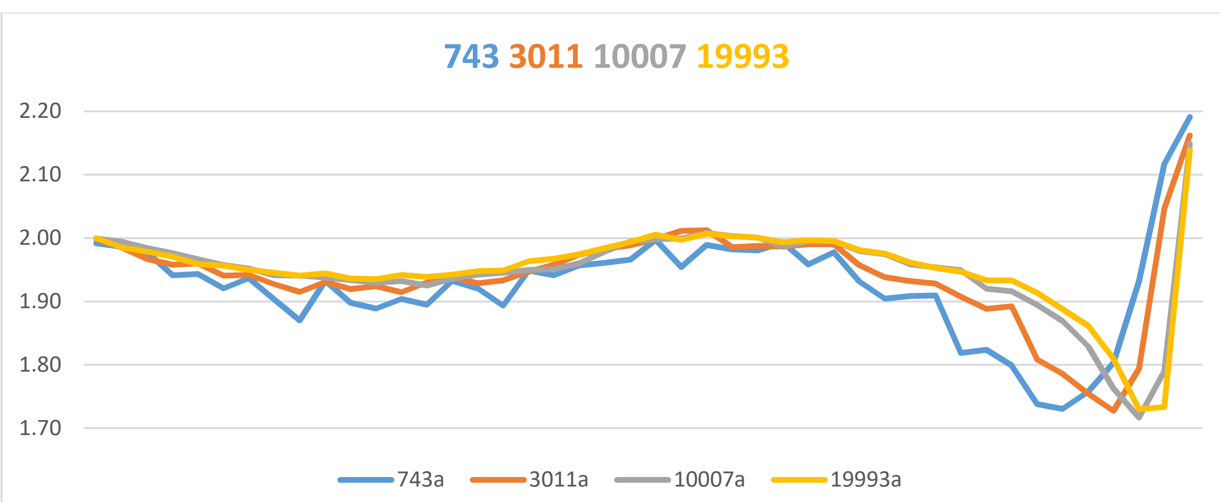


Table 1 shows the number of twin primes calculated ***(TPC$_n$)*** in **[*P_n, P_n^2*]** for ***347 ≤ P_n ≤ 31153,*** when the calculated **average *a_m*** equals ***2.04*** and ***2.06,*** Comparing ***TPC_n*** with ***TPA_n*** shows the average value for ***P_m - a_m*** starts out near ***P_m – 2.02*** for ***P_n = 347*** and decreases to slightly less than ***P_m – 2.06*** for ***P_n = 31153.***

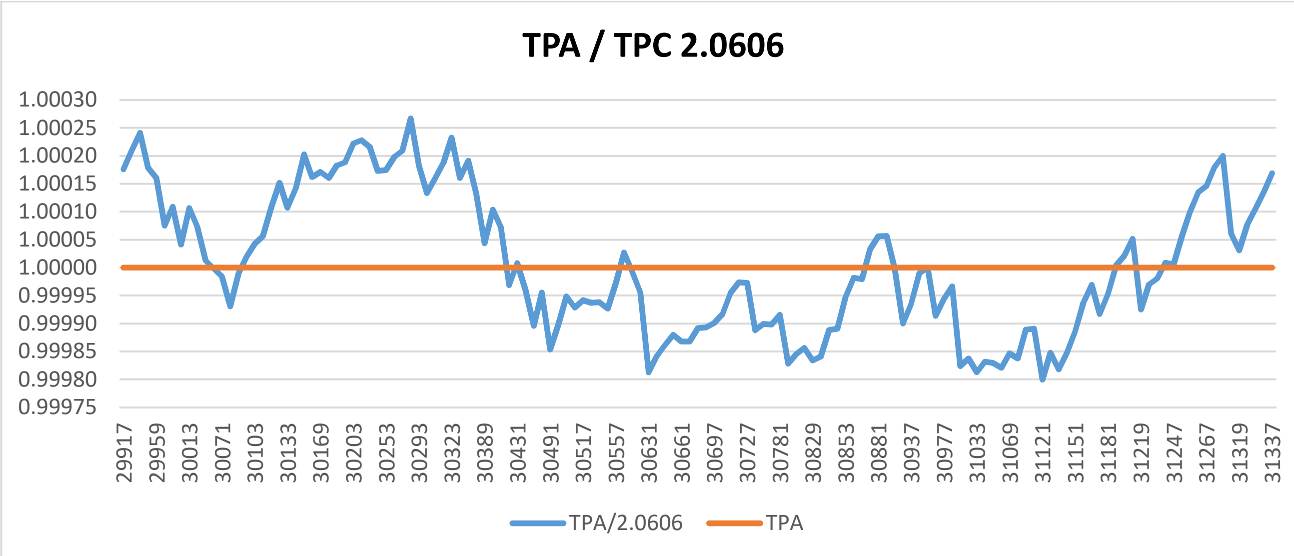
Table 1 – Twin Primes in the interval [P_n, P_n^2] for $347 \le P_n \le 31153$

$TPA_n = ((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$. For ***$3 \le m \le n$***

Average *a_m* is set at *2.04* and *2.06*

P_n	TPA_n	TPA_n / TPC_n 2.06	TPC_n 2.06	TPC_n 2.04
347	1405	1.0637	1320.8	1360.2
349	1419	1.0683	1328.2	1368.0
1151	10387	1.0430	9958.4	10293.6
1153	10408	1.0434	9975.2	10311.2
1997	26735	1.0369	25783.3	26690.2
1999	26777	1.0375	25808.3	26716.5
2969	52817	1.0302	51267.4	53125.5
2971	52877	1.0307	51300.9	53160.6
3851	82712	1.0224	80901.1	83885.3
3853	82802	1.0230	80941.9	83928.1
4649	114842	1.0196	112636	116843
4651	114919	1.0198	112683	116892
5849	171367	1.0156	168737	175132
5851	171471	1.0159	168793	175191
6947	231582	1.0123	228770	237533
6949	231708	1.0126	228834	237600
8387	322646	1.0100	319452	331826
8389	322805	1.0103	319526	331903
9677	415267	1.0091	411530	427606
9679	415417	1.0092	411613	427693
10937	515723	1.0078	511751	531882
10939	515884	1.0079	511842	531977
12251	630469	1.0059	626775	651581
12253	630646	1.0060	626874	651685
13997	798218	1.0048	794435	826113
13999	798427	1.0049	794545	826228
15731	982287	1.0039	978483	1017750
15733	982497	1.0040	978604	1017877
17291	1162662	1.0026	1159636	1206394
17293	1162911	1.0027	1159767	1206531
18251	1280482	1.0031	1276491	1328116
18253	1280728	1.0032	1276627	1328259
19991	1506151	1.0015	1503866	1564964
19993	1506427	1.0016	1504011	1565117
21191	1671686	1.0015	1669161	1737182
21193	1671950	1.0016	1669314	1737343
22541	1866304	1.0009	1864560	1940784
22543	1866615	1.0010	1864721	1940953
23831	2061886	1.0010	2059785	2144230
23833	2062203	1.0011	2059953	2144407
26111	2428375	1.0000	2428472	2528479
26113	2428739	1.0000	2428652	2528668
27689	2697588	0.9992	2699839	2811333
27691	2697935	0.9992	2700028	2811532
29207	2968309	0.9994	2970220	3093224
29209	2968674	0.9994	2970418	3093432
31151	3333028	0.9987	3337515	3476151

(Graph 2) Twin Primes in the interval [P_n, P_n^2] for $29917 \le P_n \le 31337$ For $P_n > 29000$ the average a_m values cycle around $(2.0601, 2.0612)$. The graph shows the average a_m ÷ base line 2.0606 ratio.



Section 2

Establishing a bound for the ratio TPA_n / TPA_{n-1}

Let X be the number of $(6j-1, 6j+1)$ pairs in the interval [P_n, P_n^2].

$TPA_n = ((P_n - a_n)/P_n)((P_{n-1} - a_{n-1})/P_{n-1})((P_{n-2} - a_{n-2})/P_{n-2})\ldots((5 - a_3)/5)(X)$ for $n \ge m \ge 3$, $P_3=5$, $1.7 < a_m < 2.3$.

The average value of a_m, $3 \le m \le n$ can be approximated by $P_m - 2$.

X equals $(P_n^2 - P_n)/6+1$. $m=3$ to n $\prod P_m - 2 = T_n$ $m=1$ to n $\prod P_m = J_n$

The number of twin prime pairs in [P_n, P_n^2] is approximately $(T_n)(P_n^2)/J_n$

TPA_n is approximately $(TPA_{n-1})((T_n)(P_n^2)/J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1})$.

TPA_n is greater than $(TPA_{n-1})\left(\left(((T_n)(P_n^2)/J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1})\right)+1\right)/2$.

Calculating $\left(\left(((T_n)(P_n^2)/J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1})\right)+1\right)/2$.

Let $P_n - P_{n-1} = c$.

$((T_n)(P_n^2)/J_n) / ((T_{n-1})(P_{n-1})^2/J_{n-1}) =$

$\left((T_{n-1})(P_{n-1}+c-2)(P_{n-1}+c)^2/((J_{n-1})(P_{n-1}+c))\right) / \left((T_{n-1})(P_{n-1})^2/J_{n-1}\right) =$

$(P_{n-1}+c-2)(P_{n-1}+c) / P_{n-1}^2 =$

$1+(2c-2)/P_{n-1}+(c^2-2c)/P_{n-1}^2$ **Table 2 below (column D) / (column C)**

For $n \ge 4$, $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

Table 2 (column B)((column D/column C)+1)/2=(column F)

For $n \ge 4$, $TPA_{n-1} < TPA_n$

Table 2 $P_n - P_{n-1} = c.$ $(TPA_{n-1})(1+(2c-2)/2P_{n-1}+(c^2-2c)/2P_{n-1}^2) < TPA_n$

A	B	C	D	E	F	G	H
prime	TPA_{n-1}	$(F_{n-1})(P_{n-1})^2/J_{n-1}$	$(F_n)(P_n)^2/J_n$	$(D/C+1)/2$	$(B)(E)$	TPA_n	F/G
71	120	109.0	112.1	1.01408	121.7	123	0.989483
73	123	112.1	127.9	1.07047	131.7	138	0.954117
1019	8420	8935.3	8952.8	1.00098	8428.2	8450	0.997425
1021	8450	8952.8	9111.3	1.00885	8524.8	8586	0.992872
2087	28819	30850.0	30879.6	1.00048	28832.8	28867	0.998816
2089	28867	30879.6	31146.2	1.00432	28991.6	29106	0.996070
3461	68804	74874.0	74917.3	1.00029	68823.9	68872	0.999302
3463	68872	74917.3	75047.1	1.00087	68931.7	69019	0.998735
4637	114316	125244.7	125298.7	1.00022	114340.6	114394	0.999534
4639	114394	125298.7	125460.9	1.00065	114468.0	114580	0.999023
6299	195208	215150.4	215218.7	1.00016	195239.0	195319	0.999590
6301	195319	215218.7	215833.9	1.00143	195598.2	195879	0.998566
8009	297317	329810.8	329893.1	1.00012	297354.1	297454	0.999664
8011	297454	329893.1	330305.0	1.00062	297639.7	297851	0.999291
9857	428957	476792.2	476889.0	1.00010	429000.5	429089	0.999794
9859	429089	476889.0	477953.7	1.00112	429568.0	430004	0.998986
11777	588001	656535.4	656646.9	1.00008	588050.9	588163	0.999809
11779	588163	656646.9	656981.4	1.00025	588312.8	588502	0.999679
13931	791507	885279.3	885406.4	1.00007	791563.8	791704	0.999823
13933	791704	885406.4	889096.0	1.00208	793353.6	794778	0.998208
16187	1033547	1158651.2	1158794.4	1.00006	1033610.9	1033796	0.999821
16189	1033796	1158794.4	1159223.9	1.00019	1033987.6	1034307	0.999691
18041	1254327	1408473.1	1408629.2	1.00006	1254396.5	1254586	0.999849
18043	1254586	1408629.2	1409097.7	1.00017	1254794.6	1255094	0.999761
20147	1527206	1717720.9	1717891.4	1.00005	1527281.8	1527479	0.999871
20149	1527479	1717891.4	1719767.6	1.00055	1528313.1	1529106	0.999481
21839	1763993	1985940.5	1986122.3	1.00005	1764073.7	1764289	0.999878
21841	1764289	1986122.3	1987759.5	1.00041	1765016.2	1765719	0.999602
23741	2047968	2308071.0	2308265.5	1.00004	2048054.3	2048281	0.999889
23743	2048281	2308265.5	2308848.8	1.00013	2048539.8	2048899	0.999825
26861	2555034	2883638.7	2883853.4	1.00004	2555129.1	2555371	0.999905
26863	2555371	2883853.4	2887074.9	1.00056	2556798.3	2558027	0.999520
28619	2861908	3233814.5	3234040.4	1.00003	2862008.0	2862279	0.999905
28621	2862279	3234040.4	3235170.5	1.00017	2862779.1	2863372	0.999793
31319	3365123	3806114.0	3806357.0	1.00003	3365230.4	3365489	0.999923
31321	3365489	3806357.0	3807572.4	1.00016	3366026.3	3366653	0.999814