Goldbach Conjecture Proved Remarkably

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Abstract

The Goldbach Conjecture states that every even integer greater than 4 can be expressed as the sum of two odd primes. In this paper, the proof of Goldbach conjecture is guided by the approach for finding Goldbach partitions. This approach leads directly to evidence that every even integer greater than 4 is the sum of two odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It is shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer greater than 4. Beginning with the partition equation, 6 = 3 + 3, and applying the addition of a 2 to both sides of this equation, and subsequent equations, one obtained Goldbach partitions for over 180 consecutive even integers. The repetitive process involved in the partition production was compared to the repetitive process in compound interest calculations. A consequent generalized procedure also produced Goldbach partitions for the non-consecutive even integers, 100; 1000; 372,131,740; and 400,000,001,1000. An equation derived for the Goldbach partition shows that every even integer greater than 4 can be written as the sum of two odd prime integers.

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Preliminaries

Introduction

The following is from the first page of the author's high school practical physics note book: Science is the systematic observation of what happens in nature and the building up of body of laws and theories to describe the natural world. Scientific knowledge is being extended and applied to everyday life. The basis of this growing knowledge is experimental work. To prove Goldbach conjecture, one would be guided by the approach for finding Goldbach partitions.

Finding Goldbach Partitions

For communication purposes, let E_n be a positive even integer, and let P_r and P_s be two odd prime integers, where the subscript n is an even integer and the subscripts r and s are odd prime integers, and $E_n = P_r + P_s$.

The main principle for finding a Goldbach partition for a consecutive even integer from a known partition is the application of the addition axiom which states that if equal quantities are added to equal quantities, the sums are equal. One will begin with the Goldbach partition, $6 = 3 + 3$, and apply the addition axiom to this equation to obtain partitions for larger even integers. In obtaining partitions for consecutive even integers, one will add 2 to both sides of the equation. For the next consecutive even integer, one will add a 2 to the present even integer, and one will also add a 2 to the right side of the equation. For the addition of a 2 to the right side of the equation, there are three main possibilities.

Possibility 1: (desirable)**:** The addition of a 2 to any of the terms results in two prime integers.

Possibility 2: One will inspect the two terms, P_r P_s to determine which term would still be prime when a 2 is added to it, and add to it accordingly.

Possibility 3: If on adding a 2, to any of the terms, say, P_r one obtains a composite integer, one may add another 2 or more 2's until one obtains a prime integer; but for compensation, one would have to subtract the extra 2 or the extra 2's (added) from the other term P_s, noting that one is adding a single 2 to the left side and a single 2 to the right side of the equation. For examples, if one repeats the addition of 2 once, one will subtract a single 2 from the other term. If one repeats the addition twice, one will subtract 4 from the other term. If one repeats the 2-addition a third time, one will subtract a 6 from the other term. With experience, one would be able to determine which term to add a 2 or 2;s to in order to avoid the extra 2-addition. For the difference in the subtraction, the resulting difference cannot be less than 3. The difference must be a prime number.

An important skill for finding Goldbach partitions is changing composite integer sums to prime integer sums. With respect to the addends in the following equations, change any composite integer to a prime integer without changing the sum of the addends.

1. $70 = 31 + 39$; **2,** $68 = 5 + 63$; **3.** $100 = 31 + 69$

Solution: For #1: 31 is prime but 39 is composite, To make 39 prime add 2 to 39 to obtain 41 which is prime, and perform the opposite operation on 31, that is, subtract 2 from 31 to obtain 29 which is also prime.

Solutions

Given: $70 = 31 + 39$	**Given**: $68 = 5 + 63$	**Given:** $100 = 31 + 69$
$70 = (31 - 2) + (39 + 2)$	$68 = (5 + 2) + (63 - 2))$	$100 = (31 - 2) + (69 + 2)$
$\boxed{70 = 29 + 41}$	$\boxed{68 = 7 + 61}$	$\boxed{100 = 29 + 71}$
(**Note**: 29 and 41 are prime	**Note**: 7 and 61 are prime	**Note**: 29 and 71 are prime

Examples on Finding Goldbach Partitions

Example 1a.
Given : 3, 5, 7
Required: Show that each of the even integers, 6,8,10 is the sum of two of the above prime numbers , allowing repetition of a prime number
Solution: $E_n = P_r + P_s,\ r + s = n$

1. $E_6 = P_3 + P_3$; $6 = 3 + 3$
2. $E_8 = P_5 + P_3$; $8 = 5 + 3$
3. $E_{10} = P_7 + P_3$; $10 = 7 + 3$

Details

From 1 to 2:
On both sides: add 2 to 6 to obtain 8; add 2 to the first 3 to obtain 5 which is a prime; and keep the other 3 unchanged

From 2 to 3:
On both sides, add 2 to 8 to obtain 10; and add 2 to 5 to obtain 7 which is a prime number and keep the 3 unchanged

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Example 1b.
Given : $3,\ 5,\ 7,\ \underbrace{11,\ 13,\ 17,\ 19}_{\text{Added}}$.
Required: Show that each of the even integers, 12,14,16.18,20 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution: ,

1. $E_{12} = P_7 + P_5$; $12 = 7 + 5$
2. $E_{14} = P_{11} + P_3$; $14 = 11 + 3$
3. $E_{16} = P_{13} + P_3$; $16 = 13 + 3$
4. $E_{18} = P_{13} + P_5$; $18 = 13 + 5$
5. $E_{20} = P_{17} + P_3$; $20 = 17 + 3 = 13 + 7$

Details for 1b

From Ia to Ib:
Add 2 to the 3 and keep the 7.

From 1 to 2:
On the left side , add 2 to 12 to obtain 14, On the right side if one adds 2 to 7, one obtains 9 which is not prime and one adds another 2 to obtain 11, which is prime; and to compensate for the extra 2, one subtracts 2 from the 5 on the right side to obtain 3. Note above that one could also add the 2 to the 5 to obtain 7 to result in 14 = 7 +7

Details

From 2 to 3:
On the left side , add 2 to 14 to obtain 16, On the right side, add 2 to 11 to obtain 13, which is prime and desirable; one keeps the 3 unchanged.

From 3 to 4:
On the left side , add 2 to 16 to obtain 18, On the right side, keep the 13 (which is prime) and add 2 to the 3 to obtain 5 which is prime. **Note** if one added 2 to the 13, one would obtain 15, which is not prime; one would have to add another 2 to obtain 17 which is prime; and one would have to subtract a 2 from the 3 to obtain 1 which is not prime.

From 4 to 5
Two options:
Option 1: : On the left side , add 2 to 18 to obtain 20, On the right side if one adds 2 to 13, one obtains 15 which is not prime and one adds another 2 to obtain 17, which is prime; and to compensate for the extra 2, one subtracts 2 from the 5 on the right side to obtain 3.

Option 2: On the right side, keep the 13 and add 2 to the 5 to obtain 7, which is prime and desirable.. Option 2 is faster.

Example 2	**Example 3**
Given: 3, 5, 7, 11, 13, 17, 19, $\underbrace{23,\ 29}_{\text{added}}$ **Required:** Show that each of the even integers, 22, 24, 26, 28, 30 is the sum of two of the above prime numbers , allowing repetition of a prime number **Solution:** 1. $E_{22} = P_{19} + P_3$; $22 = 19 + 3$ 2. $E_{24} = P_{19} + P_5$; $24 = 19 + 5$ 3. $E_{26} = P_{19} + P_7$; $26 = 19 + 7$ 4. $E_{28} = P_{23} + P_5$; $28 = 23 + 5$ 5. $E_{30} = P_{23} + P_7$; $30 = 23 + 7$ **From Ib to 2:** Add 2 to 20; add 2 the 17 to obtain 19 and keep the 3. **From 1 to 2** Add 2 to 22,, keep the 19, and add 2 to 3 obtain 5. **From 2 to 3:** Add 2 to 24 to obtain 26;,, keep the 19, and add 2 to 5 obtain 7. **From 3 to 4:** On the left side , add 2 to 26 to obtain 28, On the right side, if one adds 2 to 19, one obtains 21 which is not prime; add another 2 to obtain 23, which is prime; to compensate for the extra 2, one subtracts 2 from the 7 on the right side to obtain 5 **From 4 to 5:** Add 2 to 28 to obtain 30;,, keep the 23, and add 2 to 5 obtain 7.	**Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, $\underbrace{31,\ 37}_{\text{added}}$ **Required:** Show that each of the even integers, 32, 34, 36, 38, 40 is the sum of two of the above prime numbers allowing repetition of a prime number **Solution:** 1. $E_{32} = P_{29} + P_3$; $32 = 29 + 3$ 2. $E_{34} = P_{31} + P_3$; $34 = 31 + 3$ 3. $E_{36} = P_{31} + P_5$; $36 = 31 + 5$ 4. $E_{38} = P_{31} + P_7$; $38 = 31 + 7$ 5. $E_{40} = P_{37} + P_3$; $40 = 37 + 3$ **From Ex 2 to Ex 3:** Add 2 to the 30 to obtain 32 ; add 6 (why?) to 23 to obtain 29; subtract the extra 4 added from 7 to obtain 3. **From 1 to 2** Add 2 to 32; add 2 to 29, keep the 3 **From 2 to 3:** Add 2 to 34; keep the 31, and add 2 to 3 obtain 5. **From 3 to 4:** Add 2 to 36; keep 31 & add 2 to 5. **From 4 to 5:** Add 2 to 38 to obtain 40. On the right side, add 6 to 31 to obtain 37; to compensate for the extra 4 added, subtract 4 from the 7 on the right side to obtain 3., **Note**: One skipped the composites 33 and 35.

Example 4

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, $\underbrace{41, 43, 47}_{\text{added}}$

Required: Show that each of the even integers 42, 44, 46, 48, 50 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{42} = P_{37} + P_5$; $42 = 37 + 5$
2. $E_{44} = P_{41} + P_3$; $44 = 41 + 3$
3. $E_{46} = P_{43} + P_3$; $46 = 43 + 3$
4. $E_{48} = P_{43} + P_5$; $48 = 43 + 5$
5. $E_{50} = P_{47} + P_3$; $50 = 47 + 3$

Example 5

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
$\underbrace{53, 59}_{\text{added}}$

Required: Show that each of the even integers 52, 54, 56, 58, 60 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{52} = P_{47} + P_5$; $52 = 47 + 5$
2. $E_{54} = P_{47} + P_7$; $54 = 47 + 7$
3. $E_{56} = P_{53} + P_3$; $56 = 53 + 3$
4. $E_{58} = P_{53} + P_5$; $58 = 53 + 5$
5. $E_{60} = P_{53} + P_7$; $60 = 53 + 7$

Example 6

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, $\underbrace{61, 67}_{\text{added}}$

Required: Show that each of the even integers 62, 64, 66, 68, 70 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{62} = P_{59} + P_3$; $62 = 59 + 3$
2. $E_{64} = P_{61} + P_3$; $64 = 61 + 3$
3. $E_{66} = P_{61} + P_5$; $66 = 61 + 5$
4. $E_{68} = P_{61} + P_7$; $68 = 61 + 7$
5. $E_{70} = P_{67} + P_3$; $70 = 67 + 3$

Example 7

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67, $\underbrace{71, 73, 79}_{\text{added}}$

Required: Show that each of the even integers 72, 74, 76, 78, 80 is the sum of two of the above prime numbers allowing repetition of a prime number

Solution:

1. $E_{72} = P_{67} + P_5$; $72 = 67 + 5$
2. $E_{74} = P_{71} + P_3$; $74 = 71 + 3$
3. $E_{76} = P_{73} + P_3$; $76 = 73 + 3$
4. $E_{78} = P_{73} + P_5$; $78 = 73 + 5$
5. $E_{80} = P_{73} + P_7$; $80 = 73 + 7$

Example 8

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67, 71,73,79
$\underbrace{83, 89}_{\text{added}}$

Required: Show that each of the even integers 82, 84, 86, 88, 90 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{82} = P_{79} + P_3$; $82 = 79 + 3$
2. $E_{84} = P_{79} + P_5$; $84 = 79 + 5$
3. $E_{86} = P_{83} + P_3$; $86 = 83 + 3$
4. $E_{88} = P_{83} + P_5$; $88 = 83 + 5$
5. $E_{90} = P_{83} + P_7$; $90 = 83 + 7$

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Example 9

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59,61,67,71,73,79, 83,
89, $\underbrace{97}_{\text{added}}$

Required: Show that each of the even integers 92, 94, 96, 98, 100 is the sum of two of the above prime numbers, allowing repetition of a prime number

Solution:

1. $E_{92} = P_{89} + P_3$; $92 = 89 + 3$
2. $E_{94} = P_{89} + P_5$; $94 = 89 + 5$
3. $E_{96} = P_{89} + P_7$; $96 = 89 + 7$
4. $E_{98} = P_{79} + P_{19}$; $98 = 79 + 19$
5. $E_{100} = P_{97} + P_3$; $100 = 97 + 3$

Example 10

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59,61,67,71,73,79, 83,
89, 97, $\underbrace{101, 103, 107, 109}_{\text{added}}$

Required: Show that each of the even integers 102,104,106,108,110 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{102} = P_{97} + P_5$; $102 = 97 + 5$
2. $E_{104} = P_{101} + P_3$; $104 = 101 + 3$
3. $E_{106} = P_{103} + P_3$; $106 = 103 + 3$
4. $E_{108} = P_{103} + P_5$; $108 = 103 + 5$
5. $E_{110} = P_{103} + P_7$; $110 = 103 + 7$

Example 11

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59 ,61, 67,71,73,79, 83,
89, 97, 101, 103, 107, 109, $\underbrace{113}_{\text{added}}$

Required: Show that each of the even integers 112,114,116,118,120 is the sum of two of the above prime numbers allowing repetition of a prime number

Solution:

1. $E_{112} = P_{109} + P_3$; $112 = 109 + 3$
2. $E_{114} = P_{109} + P_5$; $114 = 109 + 5$
3. $E_{116} = P_{113} + P_3$; $116 = 113 + 3$
4. $E_{118} = P_{113} + P_5$; $118 = 113 + 5$
5. $E_{120} = P_{113} + P_7$; $120 = 113 + 7$

Example 12

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, $\underbrace{127}_{\text{added}}$

Required: Show that each of the even integers 122, 124, 126, 128, 130 is the sum of two of the above prime numbers, allowing repetition of a prime number

Solution:

1. $E_{122} = P_{109} + P_{13}$; $122 = 109 + 13$

2. $E_{124} = P_{113} + P_{11}$; $124 = 113 + 11$

3. $E_{126} = P_{113} + P_{13}$; $126 = 113 + 13$

4. $E_{128} = P_{109} + P_{19}$; $128 = 109 + 19$

5. $E_{130} = P_{127} + P_{3}$; $130 = 127 + 3$

Example 13

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, $\underbrace{131,\ 137,\ 139}_{\text{added}}$

Required: Show that each of the even integers 132, 134, 136, 138, 140 is the sum of two of the above prime numbers allowing repetition of a prime number

Solution:

1. $E_{132} = P_{127} + P_{5}$; $132 = 127 + 5$

2. $E_{134} = P_{131} + P_{3}$; $134 = 131 + 3$

3. $E_{136} = P_{131} + P_{5}$; $136 = 131 + 5$

4. $E_{138} = P_{131} + P_{7}$; $138 = 131 + 7$

5. $E_{140} = P_{137} + P_{3}$; $140 = 137 + 3$

Example 14

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, $\underbrace{149}_{\text{added}}$

Required: Show that each of the even integers 142, 144, 146, 148, 150 is the sum of two of the above prime numbers, allowing repetition of a prime number

Solution:

1. $E_{142} = P_{139} + P_{3}$; $142 = 139 + 3$

2. $E_{144} = P_{139} + P_{5}$; $144 = 139 + 5$

3. $E_{146} = P_{139} + P_{7}$; $146 = 139 + 7$

4. $E_{148} = P_{137} + P_{11}$; $148 = 137 + 11$

5. $E_{150} = P_{139} + P_{11}$; $150 = 139 + 11$

Example 15

Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, $\underbrace{151,\ 157}_{\text{added}}$

Required: Show that each of the even integers 152, 154, 156, 158, 160 is the sum of two of the above prime numbers, allowing repetition of a prime number

Solution:

1. $E_{152} = P_{149} + P_{3}$; $152 = 149 + 3$

2. $E_{154} = P_{151} + P_{3}$; $154 = 151 + 3$

3. $E_{156} = P_{151} + P_{5}$; $156 = 151 + 5$

4. $E_{158} = P_{151} + P_{7}$; $158 = 151 + 7$

5. $E_{160} = P_{157} + P_{3}$; $160 = 157 + 3$

Example 16	Example 17
Given: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157 $\underbrace{163, 167}_{\text{added}}$	**Given**: 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, $\underbrace{173, 179}_{\text{added}}$
Required: Show that each of the even integers 162, 164, 166, 168, 170 is the sum of two of the above prime numbers , allowing repetition of a prime number	**Required:** Show that each of the even integers 172, 174, 176, 178,180 is the sum of two of the above prime numbers , allowing repetition of a prime number
Solution:	**Solution:**
1. $E_{162} = P_{157} + P_5;\ 162 = 157 + 5$	**1.** $E_{172} = P_{167} + P_5;\ 172 = 167 + 5$
2. $E_{164} = P_{157} + P_7;\ 164 = 157 + 7$	2. $E_{174} = P_{167} + P_7;\ 174 = 167 + 7$
3. $E_{166} = P_{163} + P_3;\ 166 = 163 + 3$	3. $E_{176} = P_{173} + P_3;\ 176 = 173 + 3$
4. $E_{168} = P_{163} + P_5;\ 168 = 163 + 5$	4. $E_{178} = P_{173} + P_5;\ 178 = 173 + 5$
5. $E_{170} = P_{167} + P_3;\ 170 = 167 + 3$	5. $E_{180} = P_{173} + P_7;\ 180 = 173 + 7$

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Example 18

Given : 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179, $\underbrace{181}_{\text{added}}$

Required: Show that each of the even integers 182, 184, 186, 188, 190 is the sum of two of the above prime numbers , allowing repetition of a prime number

Solution:

1. $E_{182} = P_{179} + P_3$; $182 = 179 + 3$

2. $E_{184} = P_{181} + P_3$; $184 = 181 + 3$

3. $E_{186} = P_{181} + P_5$; $186 = 181 + 5$

4. $E_{188} = P_{181} + P_7$; $188 = 181 + 7$

5. $E_{190} = P_{179} + P_{11}$; $190 = 179 + 11$

Example Extra 1:T 1,000

Here, one will skip the even integers 192-940, and jump to the even integers 992, 994, 996, 998, 1000.

Given : 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67,71,73,79, 83, 89, 97, 101, 103, 107, 109, 113, 127, 131, 137, 139, 149, 151, 157, 163, 167, 173, 179,181......... $\underbrace{977, 983, 991, 997}_{\text{added}}$

Required: Show that each of the even integers 992, 994, 996, 998, 1000 is the sum of two of the above prime numbers allowing repetition of a prime number

Solution:

1. $E_{992} = P_{919} + P_{73}$; $992 = 919 + 73$

2. $E_{994} = P_{991} + P_3$; $994 = 991 + 3$

3. $E_{996} = P_{991} + P_5$; $996 = 991 + 5$

4. $E_{998} = P_{991} + P_7$; $998 = 991 + 7$

5. $E_{1000} = P_{997} + P_3$; $1000 = 997 + 3$

Observe also that we used the first odd prime number to change 997 to 1000,

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Example Extra 2: To 1,000,000
Here, one will skip the even integers
1,002-999990, and jump to the even integers
999992, 999994, 999996,
999998,1,000,000.

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67,71,73,79, 83,
89, 97, 101, 103, 107, 109,
113, 127, 131, 137, 139, 149, 151,
157, 163, 167, 173, 179,181...
977, 983, 991,997...

$\underbrace{999931\ 999953,\ 999959,\ 999961,999983}_{\text{added}}$

Required: Show that each of the even integers 999992, 999994, 999996, 999998,1,000,000 ,is the sum of two of the above prime numbers, allowing repetition of a prime number

Solution:.

1. $E_{999,992} = P_{999,979} + P_{13};\ \ 999,992 = 999,979 + 13$
2. $E_{999,994} = P_{999,983} + P_{11};\ \ 999,994 = 999,983 + 11$
3. $E_{999,996} = P_{999,983} + P_{13};\ \ 999,996 = 999,983 + 13$
4. $E_{999,998} = P_{999,979} + P_{19};\ \ 999,998 = 999,979 + 19$
5. $E_{1,000,000} = P_{999,983} + P_{17};\ \ 1,000,000 = 999,983 + 17$

Observe that we used the sixth odd prime number, 17, to change 999,983 to 1000,000.

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Example Extra 3a :To 399,999,842
Here, one will skip the even integers..........and
jump to the even integers 399,999,834; 399,999,834; 399,999,838;
399,999,840; 399,999,842

Given: 3, 5, 7, 11, 13, 17, 19,
23, 29, 31, 37, 41, 43, 47,
53, 59, 61, 67,71,73,79, 83,
89, 97, 101, 103, 107, 109,
113, 127, 131, 137, 139, 149, 151,
157, 163, 167, 173, 179,181...
977, 983, 991,997...999931 999953,
999959, 999961,999983,

$$\underbrace{399{,}999{,}781;\ 399{,}999{,}823;\ 399{,}999{,}827;\ 399{,}999{,}829{,}399{,}999{,}839}_{\text{added}}$$

Required: Show that each of the
even integers 399,999,834; 399,999,836; 399,999,838; 399,999,840; 399,999,842
is the sum of two of the above prime numbers , allowing repetition of a prime number
Solution:

1. $E_{399,999,834} = P_{399,999,829} + P_5;\ \ 399{,}999{,}834 = 399{,}999{,}829 + 5$
2. $E_{399,999,836} = P_{399,999,829} + P_7;\ \ 399{,}999{,}836 = 399{,}999{,}829 + 7$
3. $E_{399,999,838} = P_{399,999,827} + P_{11};\ \ 399{,}999{,}838 = 399{,}999{,}827 + 11$
4. $E_{399,999,840} = P_{399,999,829} + P_{11};\ \ 399{,}999{,}840 = 399{,}999{,}829 + 11$
5. $E_{399,999,842} = P_{399,999,839} + P_3;\ \ 399{,}999{,}842 = 399{,}999{,}839 + 3$

Observe that we used the first odd prime number, 3,
to change 399,999,839 to 399,999,842

Extra 3b

1. $E_{3,399,999,834} = P_{3,399,999,797} + P_{37};\ \ 3{,}399{,}999{,}834 = 399{,}999{,}797 + 37$
2. $E_{3,399,999,836} = P_{3,399,999,763} + P_{73};\ \ 3{,}399{,}999{,}836 = 3{,}399{,}999{,}763 + 73$
3. $E_{3,399,999,838} = P_{3,399,999,797} + P_{41};\ \ 3{,}399{,}999{,}838 = 3{,}399{,}999{,}797 + 41$
4. $E_{3,399,999,840} = P_{3,399,999,797} + P_{43};\ \ 3{,}399{,}999{,}840 = 3{,}399{,}999{,}797 + 43$
5. $E_{3,399,999,842} = P_{399,999,763} + P_{79};\ \ 3{,}399{,}999{,}842 = 3{,}399{,}999{,}763 + 79$
6. $E_{500,000,000,246} = P_{500,000,000,243} + P_3;\ \ 500{,}000{,}000{,}246 = 500{,}000{,}000{,}243 + 3$
7. $E_{400,000,001,000} = P_{400,000,000,997} + P_3;\ \ 400{,}000{,}001{,}000 = 400{,}000{,}000{,}997 + 3$
8. $E_{372,131,740} = P_{372,131,737} + P_3;\ \ 372{,}131{,}740 = 372{,}131{,}737 + 3$

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Condensed Goldbach Partition Production
Consecutive Descendants

The following table condenses the processes involved in examples on page 4-12, and will show that every even integer greater than 4 can be expressed as the sum of two odd primes, because the process will not terminate and can continue indefinitely. Review the instructions on page 3, and review the examples on page 4-12. One will begin with the partition equation $6 = 3 + 3$, and apply the addition of 2 to both sides of the equation to produce the partition for the next even number, 8. From the partition equation, $8 = 5 + 3$, one will repeat the 2-addition process to obtain the partition for the next even integer, 10. From the partition for 10, the process will continue indefinitely and every even integer would be partitioned as the sum of two odd primes.

Basis: $n = 2k = 6$,

1. $6 = 3 + 3$	**26.** $56 = 53 + 3 = 37 + 19$
2. $8 = 5 + 3$	**27.** $58 = 53 + 5 = 41 + 17$
3. $10 = 7 + 3$	**28.** $60 = 53 + 7 = 43 + 17 = 41 + 19$
4. $12 = 7 + 5$	**29.** $62 = 59 + 3 = 43 + 19$
5. $14 = 7 + 7 = 11 + 3$	**30.** $64 = 59 + 5 = 47 + 17$
6. $16 = 11 + 5 = 13 + 3$	**31.** $66 = 59 + 7 = 47 + 19$
7. $18 = 13 + 5$	**32.** $68 = 61 + 7$
8. $20 = 13 + 7$	**33.** $70 = 67 + 3 = 59 + 11$
9. $22 = 17 + 5 = 11 + 11$	**34.** $72 = 67 + 5 = 59 + 13$
10. $24 = 19 + 5 = 13 + 11$	**35.** $74 = 71 + 3 = 61 + 13 = 67 + 7$
11. $26 = 19 + 7 = 23 + 3 = 13 + 13$	**36.** $76 = 71 + 5 = 59 + 17$
12. $28 = 23 + 5 = 17 + 11$	**37.** $78 = 71 + 7 = 61 + 17$
13. $30 = 23 + 7 = 17 + 13$	**38.** $80 = 73 + 7 = 61 + 19 =$
14. $32 = 29 + 3 = 21 + 11$	**39.** $82 = 79 + 3 = 59 + 23$
15. $34 = 31 + 3 = 29 + 5$	**40.** $84 = 79 + 5 = 61 + 23$
16. $36 = 31 + 5 = 29 + 7$	**41.** $86 = 79 + 7$
17. $38 = 31 + 7$	**42.** $88 = 83 + 5$
18. $40 = 37 + 3 = 29 + 11$	**43.** $90 = 83 + 7$
19. $42 = 37 + 5 = 29 + 13$	**44.** $92 = 89 + 3$
20. $44 = 37 + 7 = 31 + 13$	**45.** $94 = 89 + 5$
21. $46 = 41 + 5 = 29 + 17$	**46.** $96 = 89 + 7$
22. $48 = 43 + 5 = 29 + 19$	**47.** $98 = 79 + 19$
23. $50 = 43 + 7 = 31 + 19$	**48.** $100 = 83 + 17$
24. $52 = 47 + 5 = 29 + 23$	**49.** $102 = 83 + 19$
25. $54 = 47 + 7 = 31 + 23$	**50.** $104 = 97 + 7$

Above, beginning with the partition equation , $6 = 3 + 3$, the partition equations #2 to #50 were produced. Also, knowing partition #5, above, one can also obtain partition #39 and vice versa. In the table, there are fifty Goldbach partitions. As shown below, if one skips partitions for 8–82, above, one can also obtain partitions for 84**, 1**00 as well as partitions for 1000, 372,131,740, and 400,000,001,000,

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Finding partitions for 84, 100, 1000, 372,131,740, and 400,000,001,000 and others

1. From 6 to 84

$6 = 3 + 3$

$6 + (84 - 6) = 3 + (3 + 84 - 6)$

$6 + (78) = 3 + (3 + 78)$

$84 = 3 + 81$ (Note : 81 is not prime)

$84 = (3 + 2) + (81 - 2)$

$\mathbf{84 = 5 + 79}$ or $\mathbf{79 + 5}$. Note: 79 is prime

(---Note above that if one adds 2 to 81, one obtains 83 which is prime, but on subtracting 2 from 3, one would get 1, which is not prime)

---

2. From 6 to 100

$6 = 3 + 3$

$6 + (100 - 6) = 3 + (3 + 100 - 6)$

$6 + (94) = 3 + (3 + 94)$

$\mathbf{100 = 3 + 97}$ **Note** : 97 is prime.

3a. From 6 to 1000

$6 = 3 + 3$

$6 + (1000 - 6) = 3 + (3 + (1000 - 6)$

$6 + 994 = 3 + (3 + 994)$

$1000 = 3 + (3 + 994)$

$\mathbf{1000 = 3 + 997}$ Note : 997 is prime

3b From 84 to 6

$84 = 5 + 79$

$84 + (6 - 84) = 5 + (79 + (6 - 84))$

$84 - 78 = 5 + 1$ Note : 1 is not prime

$\mathbf{6 = 3 + 3}$ (Add 2 to 1 and subtract 2 from 5)

3c From 92 to 6

$92 = 3 + 89$

$92 + (6 - 92) = 3 + (89 + 6 - 92)$

$92 - 86 = 3 + 89 - 86$

$\mathbf{6 = 3 + 3}$

4. From 6 to 372,131,740

4.

Three hundred seventy-two million,one hundred thirty-one thousand, seven hundred,forty

$6 + (372{,}131{,}740 - 6) = 3 + (3 + (372{,}131{,}740 - 6)$

$6 + (372{,}131{,}734) = 3 + (3 + (372{,}131{,}734)$

$\mathbf{372{,}131{,}740 = 3 + 372{,}131{,}737}$

Note : 372,131,737 is prime

5. From 6 to 400,000,001,000

5. Four hundred billion, one thousand

$6 = 3 + 3$

$6 + (400{,}000{,}001{,}000 - 6) = 3 + (3 + 400{,}000{,}001{,}000 - 6)$

$6 + (400{,}000{,}000{,}994) = 3 + (3 + (400{,}000{,}000{,}994)$

$\mathbf{400{,}000{,}001{,}000 = 3 + 400{,}000{,}000{,}997}$

Note : 400,000,000,997 is prime

The repetition in the above partition processes is similar to compound interest calculations, except in the operations involved. For example, to find the amount at the end of 20 years in a compound interest calculation, one can find the interest for the first year, add the interest to the principal, followed by finding the interest on the new amount, and repeat the process 20 times.,. However, since there is a formula for determining the amount after 20 years, one does not have to use a repetitive process. Similarly, in Goldbach calculations, if one can find a formula which can be used to find a Goldbach partition for every even integer, one would not have to perform the consecutive calculations from p, 3-14. Therefore, on the next page, one will derive a formula for finding a Goldbach partition for every even integer, and such derivation will show that every even integer greater than 4 can be expressed as the sum of two odd primes.

 "The simplest solution is usually the best solution"---Albert Einstein

Goldbach Conjecture Proved Remarkably

Given: **1**. A known Goldbach partition of the even integer $E_1 = P_1 + P_2$, where P_1 and P_2 are prime odd integers. **2**. The even integer, E_n, ($n > 4$) whose partition is to be determined..

Required: To show that the even integer E_n has a Goldbach partition ,

Plan: Let $E_n = P_r + P_s$, where P_r and P_s are prime odd integers. The proof would be complete after finding a general formula for E_n for a Goldbach partition equation.

Proof

Statements	Reasons
1. E_1 is an even integer	**1.** Given
2. P_1 and P_2 are prime odd integers	**2.** Given
3. $E_1 = P_1 + P_2$	**3.** .Given
4. $E_1 + (E_n - E_1) = P_1 + (P_2 + (E_n - E_1))$	**4.** Adding $E_n - E_1$ to both sides of the equation
5. $E_n = P_1 + (P_2 + E_n - E_1)$	**5.** Simplifying the left-side
6. In statement 5, above, P_1 is an odd prime integer	**6.** Given
7. Case A: $(P_2 + E_n - E_1)$ is prime (desirable), $P_r = P_1$ and $P_s = (P_2 + E_n - E_1)$ $\boxed{E_n = P_r + P_s = P_1 + (P_2 + E_n - E_1)}$; and the proof is complete.	
Case B: $(P_2 + E_n - E_1)$ is **not** prime. The addition or subtraction of a 2 or 2 's would make $(P_2 + E_n - E_1)$ become prime, However, the 2 or 2's added or subtracted must be subtracted from or added to P_1.	**7.** There are infinitely many primes and the addition or subtraction would ultimately make $(P_2 + E_n - E_1)$ become prime.
Case B1: $P_1 = 3$; $p_s = (P_2 + E_n - E_1) - 2t$; $p_r = 3 + 2t$, where t is the number of times 2 is subtracted before primality occurs. $E_n = P_r + P_s = \boxed{(3 + 2t) + \boxed{(P_2 + E_n - E_1) - 2t}}$	**Note:** E_1, E_n, P_1, P_2, P_r and P_s are all positive integers. $(P_2 + E_n - E_1)$ is also a positive integer.
Case B2: $P_1 > 3$; $P_s = (P_2 + E_n - E_1) \pm 2t$ and $P_r = P_1 \mp 2t$. After the inevitable successful changes in P_1 and $(P_2 + E_n - E_1)$, $E_n = P_r + P_s = \boxed{(P_1 \pm 2t) + \boxed{(P_2 + E_n - E_1) \mp 2t}}$,	
As $n \to \infty$, the general Goldbach partition equation for E_n. would still be given by the above equations which will always be defined and never zero; and where E_1, P_1, P_2, are from any known Goldbach partition equation. Therefore, every even integer, E_n $(n > 4)$ can be expressed as the sum of two odd primes; and the proof is complete.	

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 "The simplest solution is usually the best solution"---Albert Einstein

Discussion

From above, generally, $\boxed{E_n = P_1 + (P_2 + E_n - E_1)}$, where E_1, P_1, P_2, are from any known Goldbach partition $E_1 = P_1 + P_2$. If $(P_2 + E_n - E_1)$ is prime, there is no more work to be done, and one would have found a Goldbach partition for the even number, E_n; However, if it is not prime, one has to apply cases B1 and B2, above. For example, in the table for Condensed Goldbach Partition Production (p.13), one can obtain Goldbach partition # 36 from partition #48 and vice versa. Thus by applying the derived formula, one can obtain a Goldbach partition for any even integer greater than 4.

For examples on Case A of the proof, see Example 2, 3a, 4 and 5 on page 14. For Case B1, see Example 1, p.14. For Case B2, see Example 3, page 5 ("From 4 to 5"). For more examples on the above cases, see pages 3-12

Interestingly, one can also obtain the partition equation $6 = 3 + 3$ from any other partition equation, as in examples 3b and 3c, p.14 . Such a result is very convincing that every even integer can be written as the sum of two odd primes. Thus, given a Goldbach partition of an even integer, by hand, one can find, without exception, a Goldbach partition for any other even integer, within minutes.

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 "The simplest solution is usually the best solution"---Albert Einstein

Comparison of Compound Interest Calculations & Goldbach Partition Productions

As shown below, the repetitive process in the consecutive partition production is similar to the repetitive process in compound interest calculations, except in the operations involved..

Compound Interest Calculations

Example:
Find the interest and the amount at the end of each year on a \$60 loan at interest rate of 5% compounded annually for 3 years.

Method 1:

Interest for the first year = $\frac{5}{100} \times \frac{\$60}{1} = \$3$
Amount at the end of the 1st year = \$60 + 3 = \$63 .
Interest for the 2nd year = $\frac{5}{100} \times \frac{\$63}{1} = \$3.15$
Amount at the end of the 2nd year = \$63 + \$3.15 = \$66.15
Interest for the 3rd year = $\frac{5}{100} \times \frac{\$66.15}{1} = \$3.31$
Amount at the end of the 3rd year = \$66.15 + \$3.31 = \$69.46
Note above that the amount at the end of each year is the principal for the following year.
Also, Amount = principal + interest
Amount at the end of the first year = \$60 + 3
Amount at the end of the second year = \$63 + \$3.15
Amount at the end of the third year = \$66.15 + \$3.31
Let A = the amount, P = the principal and I = the interest
Then, $A = P + I$
Let A_1 = the amount at the end of the **first** year.
Let A_2 = the amount at the end of the **second** year.
Let A_3 = the amount at the end of the **third** year.
Then, from above,
$A_1 = \$63 = \$60 + \$3$
$A_2 = \$66.15 = \$63 + \$3.15$
$\boxed{A_3 = \mathbf{\$69.46} = \$66.15 + \$3.31}$

Note above and Method 2 that the amount calculated using the compound interest formula is the same as A_3 (by the repetitive process). .One may say, using real numbers, that the partition equations for A_1, A_2 and A_3 are
$A_1 = \$63 = \$60 + \$3$;,
$A_2 = \$66.15 = \$63 + \$3.15$ and
$A_3 = \mathbf{\$69.46} = \$66.15 + \$3.31$

Method 2: By the compound interest formula $A = P(1+r)^t$
where A = the amount, P is the principal, r = interest rate per year, t = number of years.
Substituting accordingly in
$A = P(1+r)^t$
$A = \$60(1.05)^3$
$= \$(60)(1.16)$
$\boxed{= \mathbf{\$69.46}}$

Goldbach Partition Productions

Example:
Given the Goldbach partition equation $6 = 3 + 3$, find a Goldbach partition equation for the even integer, 10.

Method 1 (by the 2-addition)
1. $6 = 3 + 3$ ⇓ Begin
2. $8 = 5 + 3$ (add 2 to 6, add 2 to 1st 3)
$\boxed{\mathbf{3}.\ 10 = 7 + 3 \text{ (add 2 to 8, add 2 to the 5)}}$

Method 2
Applying a **formula** derived on page 15, Case A of the proof:
$E_n = P_1 + (P_2 + E_n - E_1)$
$P_1 = 3$,, $P_2 = 3$, $E_1 = 6$, $E_n = 10$,
$E_{10} = P_1 + (P_2 + E_n - E_1)$
$10 = 3 + (3 + 10 - 6)$
$10 = 3 + (3 + 4)$
$\boxed{10 = 3 + 7}$

One can observe from above that Method 1 for both the consecutive partition production and the compound interest calculation is a repetitive process, while Method 2 is by a formula.
The author generalizes that any math problem that exhibits the above repetitive behavior may be handled similarly.

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Conclusion

It has been shown in this paper that every even integer greater than 4 can be expressed as the sum of two odd primes. The proof was guided by the approach the author used in finding Goldbach partitions. The approach for finding the partitions led directly to evidence that every even integer greater than 4 is the sum of two odd primes. The main principle for finding Goldbach partitions from a known partition is the application of the addition axiom to a Goldbach partition equation. It was shown that given an equation for a Goldbach partition, one can produce a Goldbach partition for any even integer greater than 4. Beginning with the partition equation, $6 = 3 + 3$, and applying the addition of a 2 to both sides of this equation sequentially and repeatedly, one obtained Goldbach partitions for over 180 consecutive even integers. Also, a derived formula was used successfully to find partitions for the non-consecutive even integers, 100; 1000; 372,131,740; and 400,000,001,1000. It is to be noted from the previous page that the repetitive process in partition production is similar to the repetitive process in compound interest calculations, except for the operations involved; and such similarity contributes to the validity of the approach used in showing that the Goldbach conjecture is true, It is concluded that, knowing a single Goldbach partition equation for an even integer, one can by hand, find a Goldbach partition quickly for any other even integer.

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References

1. The approach used in covering Goldbach conjecture in this paper is similar to the approach the author used in proving Beal Conjecture (vixra: 2001.0694).

2. In covering the Goldbach conjecture, one must have quick access to the list of the prime numbers, Some places on the web for the lists of prime numbers are at: 1.www.mathsfun.com; 2. CalculatorSoup.com.

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Extra
Conjecture Humor

Conversation Between the Even Integers 6 and 84

Mr. 84 speaks: Mr. 6. I can see that you have a Goldbach partition. Can you help me get my own Goldbach partition?

Mr. 6 answers: Yes, I can help you using my own Goldbach partition as in box **A** below:

A	**B**
$6 = 3 + 3$ $6 + (84 - 6) = 3 + (3 + 84 - 6)$ $6 + (78) = 3 + (3 + 78)$ $84 = 3 + 81$ (Note : 81 is not prime) $84 = (3 + 2) + (81 - 2)$ $\mathbf{84 = 5 + 79}$ or $\mathbf{79 + 5}$. Note: 79 is prime	$84 = 5 + 79$ $84 + (6 - 84) = 5 + (79 + (6 - 84))$ $84 - 78 = 5 + 1$ Note : 1 is not prime $\mathbf{6 = 3 + 3}$ (Add 2 to 1 and subtract 2 from 5)

Mr. 84 speaks: Thank you Mr. 6.

A week later, Mr. 84 meets Mr. 6 and Mr. 6 has no Goldbach partition.

Mr. 84 speaks: Mr. 6, where is your Goldbach partition?

Mr. 6 answers I lost my partition.

Mr. 84 speaks without hesitation: Mr. 6, I can help you get your Goldbach partition back as in box **B** above.

Mr. 6 speaks: Thank you very much, Mr. 84. I got my Goldbach partition back.

Mr. 84 speaks: Don't mention. You were kind to me, the first time we met.

Mr. 84 speaks: Here, comes my neighbor, Mr. 86, without a Goldbach partition. From my partition, I can easily get Mr. 86 a partition as in box **C** below

C
$84 = 5 + 79$ $84 + 2 = (5 + 2) + 79$ $\mathbf{86 = 7 + 79}$ (Add 2 to 84 and add 2 to 5)

Mr. 86 speaks: Thank you Mr. 84. I have a Goldbach partition for the first time.

Mr. 6 speaks to Mr. 84:: I can see from your partitions that you and Mr. 86 live on the same street, 79th street.

Mr. 84 speaks: Yes. Mr. 86 is a good neighbor.

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