Decays of the Hagen-Hurley bosons: possible compositeness of the W boson

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Abstract

We continue our study of the Hagen-Hurley equations describing spin 1 bosons. Recently, we have demonstrated that it is possible to describe the decay of a Hagen-Hurley boson into a lepton and a neutrino. However, it was necessary to assume that the spin of the boson is in the $0 \oplus 1$ space. We have suggested that this Hagen-Hurley boson can be identified with the W boson mediating weak interactions. The mixed beta decays have been explained by a mechanism of spin 1 and spin 0 mixing of the virtual W boson. In this work, we study the top quark decay involving the real W boson. We substantiate the view that the real W boson is a mixture of spin 1 and spin 0 states.

Keywords: W boson, top quark, beta decay

1 Introduction

The intermediate vector bosons, $W^{\pm}$ as well as Z^0, are extremely short-lived particles with a half-life of about 3×10^{-25} s, see [1] for decay widths. This property is justifiable in all these cases when W bosons are virtual particles. On the other hand, it is quite surprising that the real W, as it appears in the top quark decay, is such an ephemeral particle. Decays of the W bosons, especially in the case of mixed beta decay, also lead to some interpretational difficulties [2, 3]. Moreover, the possibility of composite W and Z bosons has been suggested, see [4, 5], and references therein.

Recently, we have addressed some of these problems describing decays of the virtual W boson within the Hagen-Hurley formalism [6–10]. More precisely, we have assumed that the spin of the virtual W is partly undefined – belonging to the $0 \oplus 1$ space [2, 3]. However, in the top quark decay, the W boson is real. In the present work, we investigate further the possibility, put forward in [3], that the real W boson can be a mixture of spin 1 and spin 0 states. Accordingly, compositeness of the W boson is suggested and the problem of experimental verification is addressed in the last Section.

2 The Hagen-Hurley equations

In what follows, tensor indices are denoted with Greek letters, $\mu = 0, 1, 2, 3$. The metric tensor is assumed as $g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$, and we always sum over repeated indices. Four-momentum operators are defined in natural units ($c = 1$, $\hbar = 1$) as $P^\mu = i\frac{\partial}{\partial x_\mu}$ while non-operator four-vectors are denoted as p^μ, x^μ, etc. For elements of the spinor calculus, see [11–14].

The spin 1 Hagen-Hurley equations can be written in spinor formalism as [13, 15, 16]:

$$\left.\begin{array}{l} P_A{}^{\dot{B}}\zeta_{C\dot{B}} = m\eta_{AC}, \quad \eta_{AC} = \eta_{CA} \\ P^C{}_{\dot{B}}\eta_{AC} = -m\zeta_{A\dot{B}} \end{array}\right\}, \tag{1}$$

$$\left.\begin{array}{l} P^A{}_{\dot{B}}\zeta_{A\dot{D}} = m\chi_{\dot{B}\dot{D}}, \quad \chi_{\dot{B}\dot{D}} = \chi_{\dot{D}\dot{B}} \\ P_A{}^{\dot{D}}\chi_{\dot{B}\dot{D}} = -m\zeta_{A\dot{B}} \end{array}\right\}, \tag{2}$$

where spin 1 conditions follow from the symmetry of the spinors: $\eta_{AC} = \eta_{CA}$ and $\chi_{\dot{B}\dot{D}} = \chi_{\dot{D}\dot{B}}$. Solutions of, for example, Eqs. (2) are of form $\zeta_{A\dot{B}} = \hat{\zeta}_{A\dot{B}}e^{-ip\cdot x}$, $\chi_{\dot{B}\dot{D}} = \hat{\chi}_{\dot{B}\dot{D}}e^{-ip\cdot x}$, where $\hat{\zeta}_{A\dot{B}}$, $\hat{\chi}_{\dot{B}\dot{D}}$ are constant spinors and

$$p^\mu p_\mu = m^2. \tag{3}$$

Equations (1), (2) can also be written in tensor formalism with 7×7 matrices β^μ [9, 10]:

$$\beta_\mu P^\mu \Psi = m\Psi. \tag{4}$$

Eq. (4) describes a particle with definite mass if β^μ matrices obey the commutation relations [9, 10, 17–19]:

$$\sum\nolimits_{\lambda,\mu,\nu} \beta^\lambda \beta^\mu \beta^\nu = \sum\nolimits_{\lambda,\mu,\nu} g^{\lambda\mu}\beta^\nu, \tag{5}$$

and the sum is over all permutations of λ, μ, ν.

It was noticed in Ref. [21] that β^μ matrices can be realized in the form:

$$\beta^\mu = \frac{1}{2}\left(\gamma^\mu \otimes I_{4\times4} + I_{4\times4} \otimes \gamma^\mu\right). \tag{6}$$

It turns out that such β^μ obey simpler but more restrictive commutation relations [20, 21]:

$$\beta^\lambda\beta^\mu\beta^\nu + \beta^\nu\beta^\mu\beta^\lambda = g^{\lambda\mu}\beta^\nu + g^{\nu\mu}\beta^\lambda, \tag{7}$$

for which Eq. (4) yields the Duffin-Kemmer-Petiau (DKP) theory of spin 0 and 1 mesons, see [20–22]. This reducible 16-dimensional representation (6) of β^μ matrices (denoted as $\mathbf{16}$) can be decomposed as $\mathbf{16} = \mathbf{10} \oplus \mathbf{5} \oplus \mathbf{1}$. Explicit formulas for the corresponding 10×10 (spin 1 case) and 5×5 (spin 0) matrices are given in [9, 10, 21], while the one-dimensional representation $\mathbf{1}$ is trivial, i.e. all $\beta^\mu = 0$. In the case of more general Eqs. (5) there are also other representations of β^μ matrices, see [9, 10] for a review. For example, there are two representations $\mathbf{7}$ for which the corresponding 7×7 matrices β^μ yield the Hagen-Hurley equations for spin 1 bosons [6–8]. We have demonstrated that the Hagen-Hurley equations can be obtained by splitting the 10×10 spin 1 DKP equations [14].

3 Rearrangement of the Hagen-Hurley equations

We shall now rewrite the Hagen-Hurley equations (2). Substituting expressions for $P_A{}^{\dot{B}}$ and $P^C{}_{\dot{B}}$ into Eqs. (2), cf. [14], we obtain a system of eight equations:

$$\left.\begin{array}{rcr} -\left(P^1+iP^2\right)\chi_{\dot{1}\dot{1}}-\left(P^0-P^3\right)\chi_{\dot{2}\dot{1}} & = & -m\zeta_{1\dot{1}} \\ \left(P^0+P^3\right)\chi_{\dot{1}\dot{1}}+\left(P^1-iP^2\right)\chi_{\dot{2}\dot{1}} & = & -m\zeta_{2\dot{1}} \\ -\left(P^1-iP^2\right)\zeta_{1\dot{1}}-\left(P^0-P^3\right)\zeta_{2\dot{1}} & = & m\chi_{\dot{1}\dot{1}} \\ \left(P^0+P^3\right)\zeta_{1\dot{1}}+\left(P^1+iP^2\right)\zeta_{2\dot{1}} & = & m\chi_{\dot{2}\dot{1}} \end{array}\right\}, \tag{8a}$$

$$\left.\begin{array}{rcr} -\left(P^1+iP^2\right)\chi_{\dot{1}\dot{2}}-\left(P^0-P^3\right)\chi_{\dot{2}\dot{2}} & = & -m\zeta_{1\dot{2}} \\ \left(P^0+P^3\right)\chi_{\dot{1}\dot{2}}+\left(P^1-iP^2\right)\chi_{\dot{2}\dot{2}} & = & -m\zeta_{2\dot{2}} \\ -\left(P^1-iP^2\right)\zeta_{1\dot{2}}-\left(P^0-P^3\right)\zeta_{2\dot{2}} & = & m\chi_{\dot{1}\dot{2}} \\ \left(P^0+P^3\right)\zeta_{1\dot{2}}+\left(P^1+iP^2\right)\zeta_{2\dot{2}} & = & m\chi_{\dot{2}\dot{2}} \end{array}\right\}, \tag{8b}$$

where the equations are arranged into two subsets (8a), (8b). Note that each of these subsets is the Dirac equation with the same set of γ^μ matrices [23]. Alternatively, equations (8) can be written as one Dirac equation with generalized matrix solution $\left(\zeta_{A\dot{B}},\ \chi_{\dot{C}\dot{D}}\right)^T$ where ^{T} stands for transposition [2,23]. Moreover, the condition $\chi_{\dot{B}\dot{D}}=\chi_{\dot{D}\dot{B}}$ entails that the spin equals one. Indeed, it follows from the fourth equation in (8a), the third equation in (8b), and $\chi_{\dot{B}\dot{D}}=\chi_{\dot{D}\dot{B}}$ that:

$$\begin{gathered} \left(P^0+P^3\right)\zeta_{1\dot{1}}+\left(P^1+iP^2\right)\zeta_{2\dot{1}}+\left(P^1-iP^2\right)\zeta_{1\dot{2}}+\left(P^0-P^3\right)\zeta_{2\dot{2}}= \\ =P^{A\dot{B}}\zeta_{A\dot{B}}=0. \end{gathered} \tag{9}$$

The spinor equation $P^{A\dot{B}}\zeta_{A\dot{B}}=0$ is equivalent to $P^\mu\zeta_\mu=0$, i.e. to the spin 1 condition, see definitions of the spinors $\zeta_{A\dot{B}}$, $p^{C\dot{D}}$ in Section 3 in [14].

4 Reduction of the Hagen-Hurley equations and decay of bosons

The coupled Dirac equations (8) are non-standard because they are concerned with higher-order spinors rather than spinors ξ_A, $\eta_{\dot{B}}$. Equations (8) can be decoupled and cast into a standard form by the following substitution:

$$\chi_{\dot{B}\dot{D}}(x) = \eta_{\dot{B}}(x)\,\alpha_{\dot{D}}(x)\,, \tag{10a}$$

$$\zeta_{A\dot{B}}(x) = \xi_A(x)\,\alpha_{\dot{B}}(x)\,, \tag{10b}$$

where $\alpha_{\dot{A}}(x)$ is the Weyl spinor, describing massless neutrinos, while $\eta_{\dot{B}}(x)$, $\xi_A(x)$ are the Dirac spinors. Although neutrinos are massive [24], their masses are tiny; therefore, this approximation should not lead to significant errors.

Note that now $\chi_{\dot{1}\dot{2}}\neq\chi_{\dot{2}\dot{1}}$ and, accordingly, the spin is not determined – more exactly, the spin is in the $0\oplus 1$ space. Accordingly, we consider not real but virtual (off-shell) bosons [25] (note, however, that in the case of top quark decay, discussed in Section 6, the W boson is real). The substitution (10) was inspired by the method of fusion of de Broglie [11, 26] (see also [27] where a similar formula was used in the $s = 0$ case). After the substitution of (10) into Eqs. (8) we obtain two equations:

$$\left.\begin{array}{rcl} -\left(P^1 + iP^2\right)\eta_{\dot{1}}\alpha_{\dot{A}} - \left(P^0 - P^3\right)\eta_{\dot{2}}\alpha_{\dot{A}} & = & -m\xi_1\alpha_{\dot{A}} \\ \left(P^0 + P^3\right)\eta_{\dot{1}}\alpha_{\dot{A}} + \left(P^1 - iP^2\right)\eta_{\dot{2}}\alpha_{\dot{A}} & = & -m\xi_2\alpha_{\dot{A}} \\ -\left(P^1 - iP^2\right)\xi_1\alpha_{\dot{A}} - \left(P^0 - P^3\right)\xi_2\alpha_{\dot{A}} & = & m\eta_{\dot{1}}\alpha_{\dot{A}} \\ \left(P^0 + P^3\right)\xi_1\alpha_{\dot{A}} + \left(P^1 + iP^2\right)\xi_2\alpha_{\dot{A}} & = & m\eta_{\dot{2}}\alpha_{\dot{A}} \end{array}\right\}, \tag{11}$$

where $\dot{A} = \dot{1}, \dot{2}$, and, after substituting solution of the Weyl equation

$$P^{A\dot{B}}\alpha_{\dot{B}} = 0, \tag{12}$$

$\alpha_{\dot{A}}(x) = \hat{\alpha}_{\dot{A}}e^{-ik\cdot x}$, $k^\mu k_\mu = 0$, we get a single Dirac equation for spinors $\xi_A(x)$, $\eta_{\dot{B}}(x)$:

$$\left.\begin{array}{rcl} -\left(\tilde{P}^1 + i\tilde{P}^2\right)\eta_{\dot{1}} - \left(\tilde{P}^0 - \tilde{P}^3\right)\eta_{\dot{2}} & = & -m\xi_1 \\ \left(\tilde{P}^0 + \tilde{P}^3\right)\eta_{\dot{1}} + \left(\tilde{P}^1 - i\tilde{P}^2\right)\eta_{\dot{2}} & = & -m\xi_2 \\ -\left(\tilde{P}^1 - i\tilde{P}^2\right)\xi_1 - \left(\tilde{P}^0 - \tilde{P}^3\right)\xi_2 & = & m\eta_{\dot{1}} \\ \left(\tilde{P}^0 + \tilde{P}^3\right)\xi_1 + \left(\tilde{P}^1 + i\tilde{P}^2\right)\xi_2 & = & m\eta_{\dot{2}} \end{array}\right\}, \tag{13}$$

with rescaled momentum operators $\tilde{P}^\mu = P^\mu + k^\mu = i\frac{\partial}{\partial x_\mu} + k^\mu$.

Indeed, the first term in the first of equations (11), for example, can be written as:

$$\begin{array}{l} -\alpha_{\dot{A}}\left(P^1 + iP^2\right)\eta_{\dot{1}} - \eta_{\dot{1}}\left(P^1 + iP^2\right)\alpha_{\dot{A}} = \\ -\alpha_{\dot{A}}\left(P^1 + iP^2\right)\eta_{\dot{1}} - \eta_{\dot{1}}\left(k^1 + ik^2\right)\alpha_{\dot{A}} = \\ -\alpha_{\dot{A}}\left(\tilde{P}^1 + i\tilde{P}^2\right)\eta_{\dot{1}} \end{array} \tag{14}$$

and thus, Eqs. (11) reduce to a single Dirac equation (13) for spinors $\xi_A(x)$, $\eta_{\dot{B}}(x)$ since components $\alpha_{\dot{1}}(x)$, $\alpha_{\dot{2}}(x)$ cancel out.

Summing up, the Hagen-Hurley equations have been reduced to the Weyl equation (12) and the Dirac equation (13) with rescaled momentum operators $\tilde{P}^\mu = P^\mu + k^\mu$ [2]. This transformation has been carried out at the cost of relaxing the condition that spin s of the Hagen-Hurley boson equals one. Indeed, we had to assume that $s \in 0 \oplus 1$. Note that there is an invertible operator transforming spin 0 into spin 1 states [3], and it seems that mixing of spin 0 and spin 1 states is possible.

Equations (12), (13) describe a pair of spin $\frac{1}{2}$ particles, one massless and another massive, with total spin $s = 0$ or $s = 1$. The transformation of Eqs. (8) into equations (12), (13) corresponds to a decay of a virtual W^- boson into a lepton and antineutrino, for example [2]:

$$W^- \longrightarrow e + \bar{\nu}_e. \tag{15}$$

Note that the W boson appears exclusively as an intermediate particle in weak decays. In the products of decay, there must also be at least a third particle that accounts for energy, momentum, and angular momentum conservation. An example is furnished by the case of a mixed beta decay [28]:

$$n(\uparrow) \longrightarrow \begin{cases} p(\downarrow) + [e(\uparrow)\,\bar{\nu}_e(\uparrow)] & \text{Gamow-Teller transition} \\ p(\uparrow) + [e(\uparrow)\,\bar{\nu}_e(\downarrow)] & \text{Fermi transition} \end{cases} \tag{16}$$

where products of the W^- boson decay (see [1]) are shown in square brackets and $(\uparrow)$ denotes spin $\frac{1}{2}$ – this seems to correspond well to the proposed transition from Eq. (2) to Eqs. (12), (13). To describe the decay, we had to accept spin 1 and spin 0 mixing, see the beginning of this Section and Refs. [2, 3]. However, this agrees well with decay products of the W^- meson with spins coupling to $s = 1$ (Gamow-Teller transition) or $s = 0$ (Fermi transition) with the spin change absorbed by a spin-flip of the proton.

5 Kinematics of decay of the real Hagen-Hurley bosons

Assume now that the Hagen-Hurley boson is a real (on-shell) particle and interpret Eq. (13) involving a fixed four-momentum k^μ of the Weyl particle. Solution of this equation is of form $\Psi = (\xi_1, \xi_2, \eta_{\dot{1}}, \eta_{\dot{2}})^T = \hat{\Psi}\, e^{-iq\cdot x}$, where $\hat{\Psi}$ is a constant bispinor, and thus:

$$(q_\mu + k_\mu)(q^\mu + k^\mu) = q_\mu q^\mu + 2q_\mu k^\mu + k_\mu k^\mu = m^2, \tag{17}$$

where m is a mass of the Hagen-Hurley boson. It should be kept in mind that in the case of unstable particles, the mass is not sharply defined. Note that due to $p_\mu p^\mu = m^2$, cf. Eq. (3), we get conservation of the four-momentum, i.e. $q^\mu + k^\mu = p^\mu$. In particular, we get the momentum conservation:

$$|\overrightarrow{q}|^2 + \left|\overrightarrow{k}\right|^2 + 2\overrightarrow{q} \cdot \overrightarrow{k} = |\overrightarrow{p}|^2 . \tag{18}$$

Moreover, we have

$$\begin{aligned} q_\mu q^\mu &= \left(q^0\right)^2 - |\overrightarrow{q}|^2 = \tilde{m}^2, & (19) \\ k_\mu k^\mu &= \left(k^0\right)^2 - \left|\overrightarrow{k}\right|^2 = 0, & (20) \end{aligned}$$

where $\tilde{m}$ is a well-defined mass of the lepton, while a neutrino – the Weyl particle – is massless. We thus have

$$\tilde{m}^2 + 2\left(q^0 k^0 - \overrightarrow{q} \cdot \overrightarrow{k}\right) = m^2. \tag{21}$$

We expect, of course, that $\tilde{m} < m$. Indeed, this inequality follows from Eq. (21) if we choose the following solutions of Eqs. (19), (20): $q^0 = +\sqrt{|\overrightarrow{q}|^2 + \tilde{m}^2}$, $k^0 = +\left|\overrightarrow{k}\right|$, respectively. Finally, we obtain:

$$q^0 k^0 - \overrightarrow{q} \cdot \overrightarrow{k} = \sqrt{\left|\overrightarrow{q}\right|^2 + \tilde{m}^2}\,\left|\overrightarrow{k}\right| - \left|\overrightarrow{q}\right| \left|\overrightarrow{k}\right| \cos\varphi > 0, \tag{22}$$

and hence $\tilde{m} < m$.

We shall analyse the consequences of condition (21), bearing at mind that, while k^μ and $\tilde{m}$ are fixed, the mass m is not well defined with the mass indeterminacy $|\Delta m| < \frac{1}{2}\Gamma$ where Γ is the decay width. Moreover, assume also that $|\overrightarrow{q}|$ is fixed, while $\frac{\overrightarrow{q}}{|\overrightarrow{q}|}$ variable. Therefore, Eqs. (21), (22) lead to two main cases:

$$\begin{aligned} \sqrt{\left|\overrightarrow{q}\right|^2 + \tilde{m}^2} + \left|\overrightarrow{q}\right| \left|\cos\varphi_{(1)}\right| &= \tfrac{1}{2}\frac{m_{(1)}^2 - \tilde{m}^2}{\left|\overrightarrow{k}\right|}, \ \cos\varphi_{(1)} < 0, && (23) \\ \sqrt{\left|\overrightarrow{q}\right|^2 + \tilde{m}^2} - \left|\overrightarrow{q}\right| \cos\varphi_{(2)} &= \tfrac{1}{2}\frac{m_{(2)}^2 - \tilde{m}^2}{\left|\overrightarrow{k}\right|}, \ \cos\varphi_{(2)} > 0, && (24) \end{aligned}$$

and thus

$$m_{(1)}^2 - m_{(2)}^2 = 2\left|\overrightarrow{q}\right| \left|\overrightarrow{k}\right| \left(-\cos\varphi_{(1)} + \cos\varphi_{(2)}\right) > 0. \tag{25}$$

Equation (25) can also be written as

$$\Delta m \equiv m_{(1)} - m_{(2)} = \frac{\left|\overrightarrow{q}\right|\left|\overrightarrow{k}\right|}{\overline{m}} \left(-\cos\varphi_{(1)} + \cos\varphi_{(2)}\right) > 0 \tag{26}$$

where $\overline{m} = \frac{1}{2}\left(m_{(1)} + m_{(2)}\right)$. The mass difference $m_{(1)} - m_{(2)}$ should be smaller than $\frac{1}{2}\Gamma$.

6 Discussion and conclusions

Consider the important channel of the top quark decay, $t \longrightarrow b + W^+ \left(\to l^+ + \nu_l\right)$. Since the mass of the top quark is larger than the mass of the W boson plus the mass of the bottom quark b, this decay should involve a real, on-shell, boson W. We assume that the Hagen-Hurley equations (1), (2) describe the W boson and thus analyse consequences of results described in Section 5.

To this end, we recall some properties of beta decay predicted by the Standard Model. Namely, in the case of Gamow-Teller transition, spins of lepton and neutrino are parallel, and their momenta are anti-aligned, $\overrightarrow{q} \cdot \overrightarrow{k} < 0$, while in the case of Fermi transition spins are anti-parallel, and momenta are aligned, $\overrightarrow{q} \cdot \overrightarrow{k} > 0$, see [29–31] and references therein.

Let us further assume that decay of the spin 1 W^+ boson, formed in the top quark decay, can be described as in Section 4, with the b quark accounting for the conservation of energy, momentum, and angular momentum. It follows from Section 4 that the Hagen-Hurley boson can decay into a lepton and a neutrino, if we assume that its spin is partly undefined – it is in the $0 \oplus 1$ space. It is important that we have demonstrated the possibility of spin 0 and spin

1 mixing [3]. Moreover, this view is supported by the existence of mixed beta decays, where the W boson is a virtual particle. We thus expect that our theory can indeed describe the decay of the real W boson – a product of the top quark decay. However, since the W boson is highly unstable, its decay width Γ is quite large since $\Gamma = 1/\tau$, where τ is the mean lifetime [1]. Hence, the mass of the W is not sharply defined. We can now make several predictions concerning this decay.

1. The real W boson is described by the Hagen-Hurley equations and due to mechanism of decay (which requires that its spin belongs to the $0 \oplus 1$ space [2]) due to mixing of spin 0 and spin 1 states [3]) decays as a linear combination of these. In the top quark decay (this is the only case when the W boson is real) both channels

$$\begin{array}{ll} t\,(\uparrow) \rightarrow b\,(\downarrow) + W\,(\Uparrow) \rightarrow b\,(\downarrow) + [l\,(\uparrow)\,\nu_l\,(\uparrow)] & \text{Gamow-Teller} \\ t\,(\uparrow) \rightarrow b\,(\uparrow) + W\,(\circ) \rightarrow b\,(\uparrow) + [l\,(\uparrow)\,\nu_l\,(\downarrow)] & \text{Fermi} \end{array} \tag{27}$$

   should be observed [3], where $(\Uparrow)$, $(\uparrow)$, $(\circ)$ denote spin 1, $\frac{1}{2}$ and 0, respectively. Note that spins of the top quark and a lepton are correlated [32]. Conservation of the total momentum is secured by the b quark, carrying the missing momentum.

   While the Fermi-type decay of the t quark is hypothetical, it can be experimentally tested, see the end of Discussion in [3].

2. In the case of Gamow-Teller decay (27), we have $\overrightarrow{q} \cdot \overrightarrow{k} < 0$ (momenta of a lepton and a neutrino anti-aligned, parallel spins) while in the hypothetical Fermi-type decay (27) $\overrightarrow{q} \cdot \overrightarrow{k} > 0$ (aligned momenta of a lepton and a neutrino, anti-parallel spins). It follows from Section 5 that these two decays occur with different masses of the W boson ($m_{(1)}$ in the case of Gamow-Teller mechanism, $m_{(2)}$ in the Fermi case), the mass difference given by Eq. (26).

3. The mass difference, Δm, provides energy $\Delta m\, c^2$ (in standard units) for the reconstruction $[l\,(\uparrow)\,\nu_l\,(\uparrow)] \longrightarrow [l\,(\uparrow)\,\nu_l\,(\downarrow)]$.

4. It seems that the W boson decays in a state with partly undefined mass and spin. The mass indeterminacy stems from the fact that the W is a resonance with decay width Γ. The spin mixing, if confirmed, would be a signature of a mild Lorentz symmetry breaking or a non-elementary character of the W boson, which would be a complex state $l\,(\uparrow)\,\nu_l\,(\uparrow)$.

Summing up, it seems that the W bosons, if described by the Hagen-Hurley equations (1) or (2), are composite, as suggested by Suzuki [4,5]. The possibility of experimental verification of these predictions was pointed out in Ref. [3]. More exactly, the detection of the Fermi channel in the top quark decay 27 would confirm the non-elementary nature of the W boson.

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