

Variation of the energy scale: an alternative to dark matter

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Abstract

We put forward the conjecture that the energy scale can vary from location to location. This simple idea is all that we need to explain those astronomical situations where dark matter is assumed to exist. So, without invoking dark matter, or postulating the existence of exotic particles, or changing the laws of gravity, we can explain: galaxy rotation curves; the high velocities of galaxies in clusters; collisions between clusters of galaxies; gravitational lensing by galaxy clusters; the fluctuations in the cosmic microwave background; physical cosmology; the growth of structure. If variations of the energy scale exist, then we can make a series of predictions, many of which can be tested now. We have nothing to say about dark energy.

1 Introduction

Many astronomical scenarios cannot be explained in terms of the observed baryonic matter and our laws of gravity. These scenarios include: galaxy rotation curves; fluctuations in the cosmic microwave background (CMB); gravitational lensing. Currently there are two options to solve these problems:

- (1) postulate the existence of around 5 times as much dark matter as baryonic matter;
- (2) modify Newton's or Einstein's law of gravity.

A simple summary of dark matter is provided by the Wikipedia article "Dark Matter"

(https://en.wikipedia.org/wiki/Dark_matter) .

Dark matter is a crucial ingredient of the Λ CDM model of the Universe (Λ =cosmological constant; CDM=cold dark matter), where it accounts for around 25% of the current energy density (5% normal matter; 70% dark energy). This enables convincing explanations to be made of most astronomical scenarios where there is a missing mass problem. However, it has difficulties in explaining the detailed nature of the rotation curves of disk galaxies. A simple summary of the Λ CDM model is provided by the Wikipedia article "Lambda-CDM model"

(https://en.wikipedia.org/wiki/Lambda-CDM_model) .

If dark matter exists, then it must exist as some type of particle, presumably within the standard model of particle physics or some extension of it. Several options have been put forward, including axions and sterile neutrinos. Currently, the most popular particle is the so-called

WIMP (weak interacting massive particle), which acts as a generic label for the, as yet, unidentified particle. A simple summary of WIMPs is provided by the Wikipedia article "Weakly interacting massive particles"

(https://en.wikipedia.org/wiki/Weakly_interacting_massive_particles).

A major problem with a dark matter particle is that, despite extensive searches over many decades, nothing has been detected in any experiment. No dark matter particles have been detected in any physical accelerator (including the Large Hadron Collider, LHC), nothing has been detected in other ground-based experiments, and nothing has been detected by any space-borne experiments. Serious searches have been made but nothing has been found.

Our best theory of gravity is Einstein's general theory of relativity. Attempts have been made to modify general relativity, but none have been particularly successful. This is because it is extremely difficult to modify general relativity in one area without breaking it somewhere else.

MOND (modified Newtonian dynamics) is the best-known hypothesis for modifying Newton's law of gravity. MOND provides an empirical expression for the gravitational acceleration and this is better than the Λ CDM model at predicting the shape of the rotation curves of disk galaxies. MOND can predict & explain the shape of galaxy rotation curve, whereas Λ CDM can only explain it. However, MOND is much less successful in other scenarios; in particular, it has trouble explaining the observed peaks in the power spectrum of the cosmic microwave background (CMB).

A good overall description of the science surrounding dark matter is given by Sanders (2010).

In this paper we present a third option, namely the conjecture that

(3) the energy scale can vary from location to location.

This simple conjecture is completely different from both dark matter and modifications to gravity. It requires neither the addition of dark matter (nor any other hypothetical particle), nor the modification of our existing laws of gravity.

It turns out that a variation of the energy scale can provide an explanation to all those scenarios where dark matter is currently invoked. So not just galaxy rotation curves, but the whole gamut including gravitational lensing, and the peaks in the power spectrum of the CMB.

The idea that the energy scale might vary was put forward in JoKe1 (2015) to explain the rotation curves of disk galaxies. This was followed by a series of papers that covered other scenarios, which means the various explanations are scattered across a number of individual papers. The aim of this paper is to consolidate all the material into one single place. This should make it simpler for everyone to understand the concept and the general thrust of the argument.

Whether or not the energy scale varies from location to location can only be decided by observation and experiment. It cannot be decided by theoretical physics or abstract mathematics. We can make a series of predictions. Some of these can be tested now, which make it possible to falsify (or support) our conjecture.

2 The conjecture

The conjecture for variations in the energy scale can be stated very simply as:

The energy scale can vary from location to location

This conjecture was first put forward in JoKe1 (2015) and used in subsequent papers to provide an alternative to dark matter in astronomical scenarios. All those situations where dark matter is invoked can be explained without any dark matter, by using a variation of the energy scale instead. The idea of variation of the physical scales is discussed in much greater (qualitative) detail in "Welcome to the Museum of Dark Matter" (2018).

Mathematically the above conjecture can be written as

$$\xi_A E_{AX} = \xi_B E_{BX} = \xi_X E_{XX} \quad (1)$$

where ξ is the dimensionless function of location that describes the energy scale variation; E is the energy.

The first subscript denotes the location of the observer; the second subscript the location of the quantity (object).

E_{AX} is interpreted as the energy at X as measured by an observer at A .

E_{XX} is interpreted as the "intrinsic energy", as it is the energy at X as measured by an observer at X , i.e. both object and observer are at the same location. Generally, we can drop the suffixes for intrinsic values.

In most situations it is found that equation (1) is used in the form

$$E_{AX} = E_{XX} \left(\frac{\xi_X}{\xi_A} \right) \quad (2)$$

This means the observed energy can be replaced by the intrinsic energy multiplied by the ratio of the ξ values.

Physics operates with a number of scales, principally: length; time; mass; electric charge. We are switching to working with energy rather than mass, and we are assuming that it is only the energy scale that varies. So, the length, time, and electric charge scales are fixed. By extension this means all quantities that are made up of the units of length, time, or electric charge do not vary; e.g. the speed of light does not vary.

Perhaps we should say something about what a variation of the energy scale means, and what it does not mean. If the energy scale is higher in a remote location, then all energies of physical processes will be higher there than here on Earth. So, photons there will appear to have higher energies. But when those photons arrive on Earth, they have exactly the same energy as those processes here. This follows because, by assumption, the length and time scales do not vary, and so the wavelength and frequency of the photons match those on Earth. Variations of the energy scale cannot be detected in particle accelerators or other similar pieces of equipment. This follows because whenever particles collide, they must be together in the same location and, by assumption, variations of the energy scale require interactions at different locations. The only interactions that we are aware of that meet our requirements are gravitational interactions. We note that every scenario where dark matter is needed involves

a gravitational interaction between separate locations. Longer discussions on the nature of variations of the energy scale are given in "Welcome to the Museum of Dark Matter" (2018).

3 A new expression for mass

Mass is associated with energy through Einstein's equation

$$E = M c^2 \quad (3)$$

Using the notation of equation (1) the right-hand side becomes

$$\xi_A (M_{AX} c^2) = \xi_B (M_{BX} c^2) = \xi_X (M_{XX} c^2) \quad (4)$$

We are assuming that it is only the energy scale that varies, and that the speed of light is an absolute constant. Hence, for mass we then have

$$\xi_A M_{AX} = \xi_B M_{BX} = \xi_X M_{XX} \quad (5)$$

or in practical situations

$$M_{AX} = M_{XX} \left(\frac{\xi_X}{\xi_A} \right) \quad (6)$$

For an axisymmetric (disk) or spherical (sphere) distribution of matter, the mass inside radius r is usually given by the integral

$$M(r) = \int_0^r dM(x) \quad (7)$$

where $dM(x)$ is the increment in mass.

For our conjecture and equation (6) this integral is replaced by

$$M_{eff}(r) = \frac{1}{\xi(r)} \int_0^r \xi(x) dM(x) \quad (8)$$

where $M_{eff}(r)$ is the 'effective' mass inside radius r .

Equation (8) can be interpreted as a weighted mass. Each increment of mass is weighted by the value of the ξ function at its location. The whole is then divided by the value of the ξ function at the observer.

The form of equation (8) may look a little unusual. However, there are (at least) two physical situations where similar equations arise.

Particle horizon distance in cosmology.

If we define the function $\eta(x)$ as

$$\eta(x) = \frac{1}{\xi(x)} \quad (9)$$

then equation (8) becomes

$$M_{eff}(r) = \eta(r) \int_0^r \frac{dM(x)}{\eta(x)} \quad (10)$$

Equation (10) can be compared to the equation for the particle horizon distance for a cosmology based on the Robertson-Walker metric (Ryden 2017)

$$D_{hor}(t) = a(t) \int_0^t \frac{c dt'}{a(t')} \quad (11)$$

where $a(t)$ is the scale factor.

Equations (10) and (11) have the same form. In equation (11) it is the length scale of the Universe that is varying. In our equation (10) it is the energy (mass) scale that is varying.

Bayes' theorem in probability.

For a single point mass, $M(A)$, at location A , and an observer at location B , equation (6) can be written in terms of η rather than ξ (i.e. using equation (9)) as

$$M(B) = M(A) \left(\frac{\eta(B)}{\eta(A)} \right) \quad (12)$$

In probability theory Bayes' Theorem is

$$P(B|A) = P(A|B) \left(\frac{P(B)}{P(A)} \right) \quad (13)$$

where $P(B|A)$ is the conditional probability of B given A; $P(A|B)$ the conditional probability of A given B; $P(A)$ the probability of A; $P(B)$ the probability of B.

Again, there is a clear similarity between the form of equations (12) and (13).

In those situations where dark matter is thought to play a part, dark matter is always additive. The effective mass of a system is always the baryonic matter plus the dark matter.

$$M_{eff}(r) = M(r) + M_{dark}(r) \quad (14)$$

We can integrate equation (8) by parts to give

$$M_{eff}(r) = M(r) - \frac{1}{\xi(r)} \int_0^r M(x) \frac{d\xi}{dx} dx \quad (15)$$

If $\xi(x)$ is a decreasing function of distance, then the integral is negative and equation (15) becomes

$$M_{eff}(r) = M(r) + \text{some additional mass} \quad (16)$$

or

$$M_{eff}(r) = \gamma M(r) \quad (17)$$

where

$$\gamma > 1 \quad (18)$$

Our equation (16) has the same form as equation (14) and shows that it has the same effect as dark matter. It gives a good idea of how our conjecture can explain various astronomical scenarios without the need for dark matter.

4 Energy and momentum: an example

We fix our ideas of what it means for the energy (mass) scale to vary with the simple example of two colliding masses. We remember that we are only changing the energy scale so there are no changes to lengths or times.

We consider the elastic collision between two identical masses. We start with a mass m at location A moving with speed u towards a second mass m at location B moving with identical speed u towards mass A . An observer at O measures

$$m_{OA} = \left(\frac{\xi_A}{\xi_O}\right) m_{AA} = \left(\frac{\xi_A}{\xi_O}\right) m \quad (19)$$

and

$$m_{OB} = \left(\frac{\xi_B}{\xi_O}\right) m_{BB} = \left(\frac{\xi_B}{\xi_O}\right) m \quad (20)$$

where ξ is the parameter describing the energy scale.

he total mass M as measured by O is

$$M = \left(\frac{\xi_A}{\xi_O}\right) m + \left(\frac{\xi_B}{\xi_O}\right) m = \left(\frac{\xi_A}{\xi_O} + \frac{\xi_B}{\xi_O}\right) m \quad (21)$$

he total momentum p as measured by O is

$$p = \left(\frac{\xi_A}{\xi_O}\right) m u - \left(\frac{\xi_B}{\xi_O}\right) m u = \left(\frac{\xi_A}{\xi_O} - \frac{\xi_B}{\xi_O}\right) m u \quad (22)$$

So, although the intrinsic masses are the same and the speeds are equal & opposite, the total momentum is not zero.

The total kinetic energy E as measured by O is

$$E = \frac{1}{2} \left(\frac{\xi_A}{\xi_0} \right) m u^2 + \frac{1}{2} \left(\frac{\xi_B}{\xi_0} \right) m u^2 = \frac{1}{2} \left(\frac{\xi_A}{\xi_0} + \frac{\xi_B}{\xi_0} \right) m u^2 \quad (23)$$

The position of the centre of mass as measured by \mathbf{O} is given by

$$\left(\frac{\xi_A}{\xi_0} \right) m r_A = \left(\frac{\xi_B}{\xi_0} \right) m r_B \quad (24)$$

or

$$\frac{r_A}{r_B} = \left(\frac{\xi_B}{\xi_A} \right) \quad (25)$$

So, although the masses have the same intrinsic value, the centre of mass is not the mid-point between the masses.

The two masses meet at location \mathbf{X} , exactly halfway between \mathbf{A} and \mathbf{B} . The total mass \mathbf{M}_X as measured by \mathbf{O} is

$$\mathbf{M}_X = \left(\frac{\xi_X}{\xi_0} \right) m + \left(\frac{\xi_X}{\xi_0} \right) m = 2 \left(\frac{\xi_X}{\xi_0} \right) m \quad (26)$$

The total momentum \mathbf{p}_X as measured by \mathbf{O} is

$$\mathbf{p}_X = \left(\frac{\xi_X}{\xi_0} \right) m u - \left(\frac{\xi_X}{\xi_0} \right) m u = \mathbf{0} \quad (27)$$

So, when the two masses collide, the total momentum is zero.

The total kinetic energy \mathbf{E}_X as measured by \mathbf{O} is

$$\mathbf{E}_X = \frac{1}{2} \left(\frac{\xi_X}{\xi_0} \right) m u^2 + \frac{1}{2} \left(\frac{\xi_X}{\xi_0} \right) m u^2 = \left(\frac{\xi_X}{\xi_0} \right) m u^2 \quad (28)$$

For an elastic collision, the two masses bounce back with a new velocity, \mathbf{v} . Conservation of momentum gives

$$\left(\frac{\xi_X}{\xi_0} \right) m u - \left(\frac{\xi_X}{\xi_0} \right) m u = \left(\frac{\xi_X}{\xi_0} \right) m v - \left(\frac{\xi_X}{\xi_0} \right) m v \quad (29)$$

or

$$m u - m u = m v - m v = \mathbf{0} \quad (30)$$

Conservation of energy gives

$$\left(\frac{\xi_X}{\xi_O}\right) m u^2 = \left(\frac{\xi_X}{\xi_O}\right) m v^2 \quad (31)$$

or

$$u^2 = v^2 \quad (32)$$

Neither equations (30) or (32) involve the energy scale factor ξ ; all the factors cancel out. This confirms that, at a local level, all the normal conservation laws hold.

And when the masses move back to their original locations, the observer at O measures exactly the same values for the masses, energies, momenta, but with the sense of the velocity reversed. So, the before-collision measurements are in complete agreement with the after-collision measurements.

5 A new expression for gravity

Newton's law of gravity gives the gravitational acceleration at location P , for a mass, M , at location A , as simply

$$\ddot{r} = - \frac{G M}{r^2} \quad (33)$$

In our new notation, this becomes for an observer at P

$$\ddot{r}_{PP} = - \frac{G_{PP} M_{PA}}{r_{PP}^2} \quad (34)$$

G_{PP} is the gravitational constant at X measured by an observer at X . So G_{PP} is the "intrinsic" value and we can drop the subscripts. Similarly, for \ddot{r}_{PP} and r_{PP}^2 . Equation (34) becomes

$$\ddot{r} = - \frac{G M_{PA}}{r^2} \quad (35)$$

Finally, we apply equation (6) to express M_{PA} in terms of the intrinsic mass

$$\ddot{r} = - \frac{G}{r^2} M_{AA} \left(\frac{\xi_A}{\xi_P}\right) = - \frac{G M}{r^2} \left(\frac{\xi_A}{\xi_P}\right) \quad (36)$$

If we have a distribution of matter, rather than a single point mass, then we have to replace equation (36) with an integral over the mass distribution. For a spherically-symmetric distribution (and our observer at r , rather than A), equation (36) is replaced by

$$\ddot{r} = - \frac{G}{r^2} \frac{1}{\xi(r)} \int_0^r \xi(x) dM(x) \quad (37)$$

where $\xi(\mathbf{x})$ is the dimensionless function that describes the energy scale variation; $dM(\mathbf{x})$ is the incremental mass of the spherical shell.

Using equation (8), equation (37) becomes

$$\ddot{r} = - \frac{G}{r^2} M_{eff}(r) \quad (38)$$

and we are back with Newton's acceleration as given by equation (33), albeit with a different expression for the mass.

Equation (8) for the effective mass of an object is a key equation for us, as is equation (37) which defines the gravitational acceleration. Both these equations point the way to how variations of the energy scale affect interactions involving gravity and hence can replace dark matter as the explanation for many astronomical scenarios. We must remember that equation (37) only applies for spherical-symmetry, whereas equation (8) applies for both spherical-symmetry and axial-symmetry.

Later we will look at the rotation curves of disk galaxies. Disks are not spheres and so we should not be using equation (37). However, disk galaxies are (approximately) axisymmetric and are observed to have a density distribution that decreases exponentially away from the centre. For this situation Binney & Tremaine (2008) show that the difference in the rotation curve between a spherically symmetric galaxy and an axisymmetric galaxy is only a few per cent. This means that, to good approximation, we can replace equation (37) with

$$\ddot{r} = - \frac{G}{r^2} \frac{1}{\xi(r)} \int_0^r \xi(x) dM_e(x) \quad (39)$$

where $dM_e(\mathbf{x})$ is the increment in the "effective mass". This is discussed further in section 9 "Galaxy Rotation Curves".

6 The virial theorem

The following notes on the virial theorem is based on material presented by Ryden (2017) and D'Inverno (1995). We start by looking at the basic results without our conjecture of variations of the energy scale. We then look at how these results change when we add in energy scale variations.

No variations of the energy scale.

The virial theorem for a relaxed system of gravitating masses is

$$2T + V = 0 \quad (40)$$

where T is the kinetic energy; V is the potential energy.

The kinetic energy, T , for n masses is

$$T = \frac{1}{2} \sum_{i=1}^n m_i v_i^2 \quad (41)$$

where m_i is the mass of the i th galaxy; v_i . the velocity.

With variations of the energy scale.

For our conjecture of energy scale variations, we have to write equation (41) as

$$T_X = \frac{1}{2} \sum_{i=1}^n \left(\frac{\xi_i}{\xi_X} \right) m_i v_i^2 \quad (42)$$

where T_X is the kinetic energy measured by an observer at X ; ξ the factor describing the energy scale variation.

We can write this is the form

$$T_X = \frac{1}{2} \frac{\langle \xi \rangle}{\xi_X} M \langle v^2 \rangle \quad (43)$$

where $\langle \xi \rangle$ is some mean value of ξ_i ; M is the total (intrinsic) mass; $\langle v^2 \rangle$ is the mean square velocity.

Equation (43) also requires

$$M = \sum_{i=1}^n m_i \quad (44)$$

and

$$\langle \xi \rangle M \langle v^2 \rangle = \sum_{i=1}^n \xi_i m_i v_i^2 \quad (45)$$

No variations of the energy scale.

The potential energy, V , of the n point masses is

$$V = -\frac{G}{2} \sum_{\substack{j,k=1 \\ j \neq k}}^n \frac{m_j m_k}{|\bar{r}_j - \bar{r}_k|} \quad (46)$$

where \bar{r} is the position vector.

With variations of the energy scale.

For our conjecture of energy scale variations, we have to write equation (46) as

$$V_X = -\frac{G}{2} \sum_{\substack{j,k=1 \\ j \neq k}}^n \frac{\left(\frac{\xi_j}{\xi_X} m_j \right) \left(\frac{\xi_k}{\xi_X} m_k \right)}{|\bar{r}_j - \bar{r}_k|} \quad (47)$$

where \mathbf{V}_X is the position vector.

We can write this in the form

$$\mathbf{V}_X = -\frac{G}{2 \xi_X^2} \frac{\langle \xi \rangle M \langle \xi \rangle M}{\langle R \rangle} \quad (48)$$

where $\langle R \rangle$ is a characteristic distance.

Equation (48) also requires

$$\frac{\langle \xi \rangle M \langle \xi \rangle M}{\langle R \rangle} = \sum_{\substack{j,k=1 \\ j \neq k}}^n \frac{(\xi_j m_j) (\xi_k m_k)}{|\bar{r}_j - \bar{r}_k|} \quad (49)$$

Using equations (43) & (48), the virial theorem becomes

$$2 \frac{1}{2 \xi_X} \langle \xi \rangle M \langle v^2 \rangle - \frac{G}{2 \xi_X^2} \frac{\langle \xi \rangle M \langle \xi \rangle M}{\langle R \rangle} = 0 \quad (50)$$

or

$$\langle v^2 \rangle = \frac{G}{2} \frac{\langle \xi \rangle M}{\xi_X \langle R \rangle} \quad (51)$$

This can be compared to the standard result for no variations in the energy scale

$$\langle v^2 \rangle = \frac{G}{2} \frac{M}{\langle R \rangle} \quad (52)$$

By comparing equations (51) and (52), it is clear that a variation of the energy scale can lead to a larger effective mass and thence to higher velocities of the gravitating masses. This result is important to us when considering clusters of galaxies.

7 The principle of least action

We look at how the principle of least action can be applied to the rotation curves of disk galaxies. Much of the following is based on material presented in Coopersmith (2017).

We consider a small mass, m , in orbit about a large mass, M , and where M produces a radial gravitational potential, $V(r)$. We work in polar coordinates. The Lagrangian is (Coopersmith, 2017)

$$L = T - V \quad (53)$$

where the kinetic energy, T , is given by

$$T = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) \quad (54)$$

and the potential energy, V , has the unknown form

$$V = V(r) \quad (55)$$

So, we have for the Lagrangian

$$L = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2) - V(r) \quad (56)$$

The Euler-Lagrange equation for θ

$$\frac{\partial L}{\partial \theta} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) = 0 \quad (57)$$

which gives us

$$\frac{d}{dt} (m r^2 \dot{\theta}) = 0 \quad (58)$$

which is simply the conservation of angular momentum.

The Euler-Lagrange equation for r

$$\frac{\partial L}{\partial r} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{r}} \right) = 0 \quad (59)$$

which gives us

$$m r \dot{\theta}^2 - \frac{\partial V}{\partial r} - m \ddot{r} = 0 \quad (60)$$

Although we do not have a simple algebraic form for the gravitational potential, we do know the gradient of the potential and that is given by equation (36). So

$$-\frac{\partial V}{\partial r} = -\frac{G m M}{r^2} \left(\frac{\xi_0}{\xi} \right) \quad (61)$$

where ξ_0 is the value of the energy scale at the galaxy centre; ξ the value at the star.

For circular orbits

$$r = \text{constant} \quad (62)$$

and the circular velocity, v , is

$$v = r \dot{\theta} \quad (63)$$

Substituting into equation (60) finally leads to

$$v^2 = -\frac{GM}{r} \left(\frac{\xi_0}{\xi} \right) \quad (64)$$

which is the standard result for circular orbits around a central gravitational force. So, our conjecture of variations of the energy scale is consistent with the principle of least action (for disk galaxies).

8 The Friedmann equation

Much of cosmology is explained using the Friedmann equation. From Ryden (2017) for a matter-only universe (i.e. with no radiation and no cosmological constant) the Friedman equation can be written as

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \{ \rho_b + \rho_{DM} \} \quad (65)$$

where a is the scale factor; the dot indicates differentiation with respect to time; ρ_b the density of baryonic matter; ρ_{DM} the density of dark matter. This equation should hold for the universe from when matter dominated radiation (~100,000 yrs), through the time of the cosmic microwave background (~300,000 yrs), and on to at least 5 billion years. The dark matter scenario requires there to be around 5 times as much dark matter as baryonic matter.

For our conjecture of variations of the energy scale, we need to replace equation (65) with an equation of the form

$$\left(\frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \gamma \rho_b \quad (66)$$

where γ is the dimensionless multiplicative factor representing the energy scale variation. This γ factor must have a value of around 6 in order to match the contribution from dark matter.

In JoKe17 (2018) we put forward some arguments in support of equation (66), without giving a rigorous derivation. We now give a better justification of equation (65) based on the presentation of Newtonian cosmology by D'Inverno (1995, section 22.3). We consider a finite universe containing a system of point mass galaxies that are expanding uniformly.

For a uniform expansion we have for the distance, r , and velocity, v

$$r_i = r_i(t) = a(t) r_i(t_0) = a r_{i0} \quad (67)$$

and

$$v_i = \dot{r}_i(t) = \dot{a}(t) r_i(t_0) = \dot{a} r_{i0} \quad (68)$$

where \mathbf{a} is the scale factor; the second 0 subscript denotes time t_0 .

We also add the condition that the particles (galaxies) appear to have the same mass. In terms of our conjecture of energy scale variations, this means

$$\xi_i m_i = \bar{\xi} \bar{m} \quad (69)$$

where the bars represent mean values, i.e.

$$\sum_{i=1}^n \xi_i m_i = n \bar{\xi} \bar{m} \quad (70)$$

It is clear this gives us some 'wiggle room' in that we can make \bar{m} smaller by making $\bar{\xi}$ larger.

The kinetic energy for our system of particles is essentially the same as equation (41), which now becomes

$$\xi_X T_X = \frac{1}{2} \bar{\xi} \bar{m} \sum_{i=1}^n \dot{a}^2 r_{i0}^2 \quad (71)$$

$$= A \bar{\xi} \bar{m} \dot{a}^2 \quad (72)$$

where the constant, \mathbf{A} , is defined by

$$A = \frac{1}{2} \sum_{i=1}^n r_{i0}^2 \quad (73)$$

Similarly, the potential energy for our system of particles is essentially the same as equation (46), which now becomes

$$\xi_X V_X = -\frac{G}{\xi_X} (\bar{\xi} \bar{m}) (\bar{\xi} \bar{m}) \frac{1}{a} \sum_{\substack{j,k=1 \\ j \neq k}}^n \frac{1}{|\bar{r}_j - \bar{r}_k|} \quad (74)$$

$$= -B \frac{G}{\xi_X} (\bar{\xi} \bar{m}) (\bar{\xi} \bar{m}) \frac{1}{a} \quad (75)$$

where the constant, \mathbf{B} , is defined by

$$B = \sum_{\substack{j,k=1 \\ j \neq k}}^n \frac{1}{|\bar{r}_j - \bar{r}_k|} \quad (76)$$

For a flat homogeneous universe, we impose the zero-energy condition

$$\mathbf{T} + \mathbf{V} = \mathbf{0} \quad (77)$$

which for our conjecture is

$$\xi_X T_X + \xi_X V_X = 0 \quad (78)$$

Substituting in from equations (72) & (75)

$$A \dot{a}^2 - B \frac{G}{\xi_X} (\bar{\xi} \bar{m}) \frac{1}{a} = 0 \quad (79)$$

or

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{B}{A} G \left(\frac{\bar{\xi}}{\xi_X}\right) \frac{1}{a^3} \quad (80)$$

B/A has the units of density. So, we somewhat arbitrarily define the density as

$$\rho = \frac{3}{8\pi} \frac{B}{A} \frac{1}{a^3} \quad (81)$$

Finally we have

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \left(\frac{\bar{\xi}}{\xi_X}\right) \rho \quad (82)$$

which is the form of Friedmann's equation we wanted to reach.

We are now in a position where we can examine how our conjecture, that the energy scale can vary from location to location, explains all those scenarios where dark matter is invoked.

The following sections cover

- 9) Galaxy rotation curves
- 10) Clusters of galaxies
- 11) Cluster collisions
- 12) Gravitational lensing
- 13) Physical cosmology
- 14) Cosmic microwave background
- 15) Growth of structure

9 Galaxy rotation curves

The work in this section has been presented in much greater detail in paper "An analysis of the rotation curves of disk galaxies using the SPARC catalogue" (2019, viXra:1903.0109). This section summarises that work.

The problem with the rotation curves of disk galaxies is illustrated in the top left panel of Figure 1. The black diamonds show the observed rotation curve, and the solid blue line shows the

expected curve, as calculated from the measured mass distribution (Lelli et al, 2016). The discrepancy between the two curves becomes very apparent at large distances.

Disk galaxies are not spherical and so none of the equations derived earlier for spherical systems can be applied, such as equation (37). However, if we know the mass distribution, then we can solve Poisson's equation for the gravitational potential and thence get at the gravitational acceleration. This is the approach adopted by Lelli et al (2016) in the SPARC catalogue, where the expected circular velocities for the observed mass distributions are presented.

The difference between the observed velocity and the expected velocity is usually attributed to a spherically-symmetric halo of dark matter:

$$v(r)^2 = u(r)^2 + w(r)^2 \quad (83)$$

where $v(r)$ is the observed velocity; $u(r)$ the expected velocity; $w(r)$ the velocity for the dark matter halo. As the dark matter is in a spherical halo, its contribution to the rotation curve is given by

$$w(r)^2 = \frac{G}{r} \int_0^r dM_d(x) \quad (84)$$

where $dM_d(x)$ is the mass of an elemental spherical shell of dark matter.

Given the observed and expected velocities, we can always use equation (83) to define the contribution required from dark matter. The mass distribution of dark matter then follows by solving equation (84). It turns out that around five times as much dark matter (than normal baryonic matter) is needed to explain the observed rotation curves.

Spiral galaxies are observed to approximate well to exponential disks, where the density decreases exponentially away from the galaxy centre. Although we cannot in general approximate disks to spheres, Binney & Tremaine (2008) have shown that the circular velocities for an exponential disk are very close to those of a sphere with a similar mass. So, to within a few percent, we can introduce an 'effective mass' defined by

$$u(r)^2 = \frac{G}{r} \int_0^r dM_e(x) \quad (85)$$

where $dM_e(x)$ is the increment of the 'effective mass'. Having calculated the expected velocity, $u(r)$, from the observed mass distribution, we can then derive the 'effective mass' distribution by solving equation (85).

NGC 5055

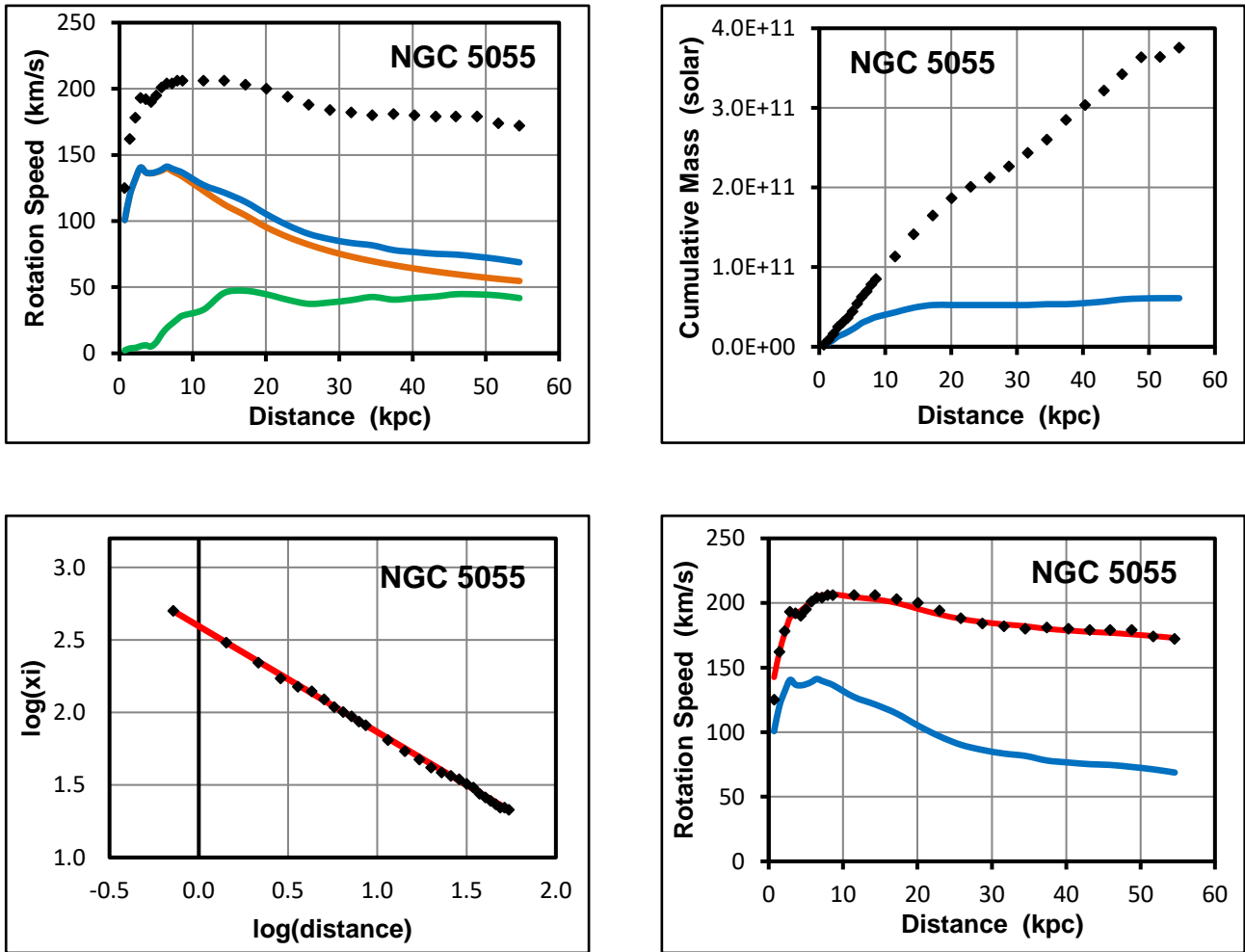


Figure 1. Rotation curve plots for spiral galaxy NGC 5055.

Top left: Rotation curves from SPARC data. Black diamonds: observed velocities; green line: contribution from gas; orange line: contribution from stars; blue line total of gas and stars.

Top right: Cumulative mass. Black diamonds: mass corresponding to black diamonds in top left panel; blue line: mass corresponding to blue line in top left panel.

Bottom left: ξ -function. Black diamonds: derived by solving equation (86); red line: straight line fit to black diamonds.

Bottom right: Fitted rotation curve. Black diamonds and blue line are the same as in the top left panel; red line is the fit calculated from equation (86), using the blue line mass data from top right panel and the red line ξ -function from bottom left panel.

So, for our conjecture of variations of the energy scale, we replace equation (37) with

$$v(r)^2 = \frac{G}{r} \frac{1}{\xi(r)} \int_0^r \xi(x) dM_e(x) \quad (86)$$

where $\xi(x)$ is the dimensionless function that defines the shape of the energy scale variation across the galaxy. This means we are working with the 'effective mass', as defined by equation (85), rather than the actual mass.

It can be noted that in deriving the expected velocity, $u(r)$, we have to use the mass-to-light ratio, which is only known to within a factor of 2 (~200%). This means that the errors of a few percent that are introduced by using equation (86) can be neglected safely.

If we know the distribution of baryonic matter across the galaxy and the shape of the energy scale variation, then we can use equation (86) to calculate the rotational velocity. Conversely, if we know the rotational velocity and the baryonic matter distribution then we can solve equation (86) for the shape of the energy scale variation.

The SPARC catalogue of disk galaxies (Lelli et al, 2016) provides data on both the observed and the expected rotational velocities for 175 disk galaxies. This data set can be used to get at the shape of the energy scale variation. The process is illustrated in Figure 1 for the Sunflower galaxy, NGC 5055.

The bottom left panel of Figure 1 shows a good linear relationship between the logarithms of the $\xi(x)$ function and the distance. This is an observational result and is completely unexpected. All the galaxies in the SPARC catalogue show a similar linear relationship for the majority of their rotation curves. There are often deviations in the galactic centre and at the outer limits, but a straight line is a good approximation for most of the curve.

The slope of the line is clearly negative, so we can write the relationship as

$$\log\{\xi(x)\} = -\alpha \log\{x\} \quad (87)$$

where $-\alpha$ is the slope of the line.

Figures 2 & 3 show plots for spiral galaxies NGC 3109 & NGC 6503. The bottom left graphs clearly show the same linear relationship for the ξ -function that we obtained for NGC 5055. In fact, all the galaxies in the SPARC catalogue show this same logarithmic relationship, but with different values for the slope. This means

$$\frac{\xi(x)}{\xi_0} = \left\{ \frac{r_0}{x} \right\}^\alpha \quad (88)$$

appears to be a general relationship that applies to all disk galaxies.

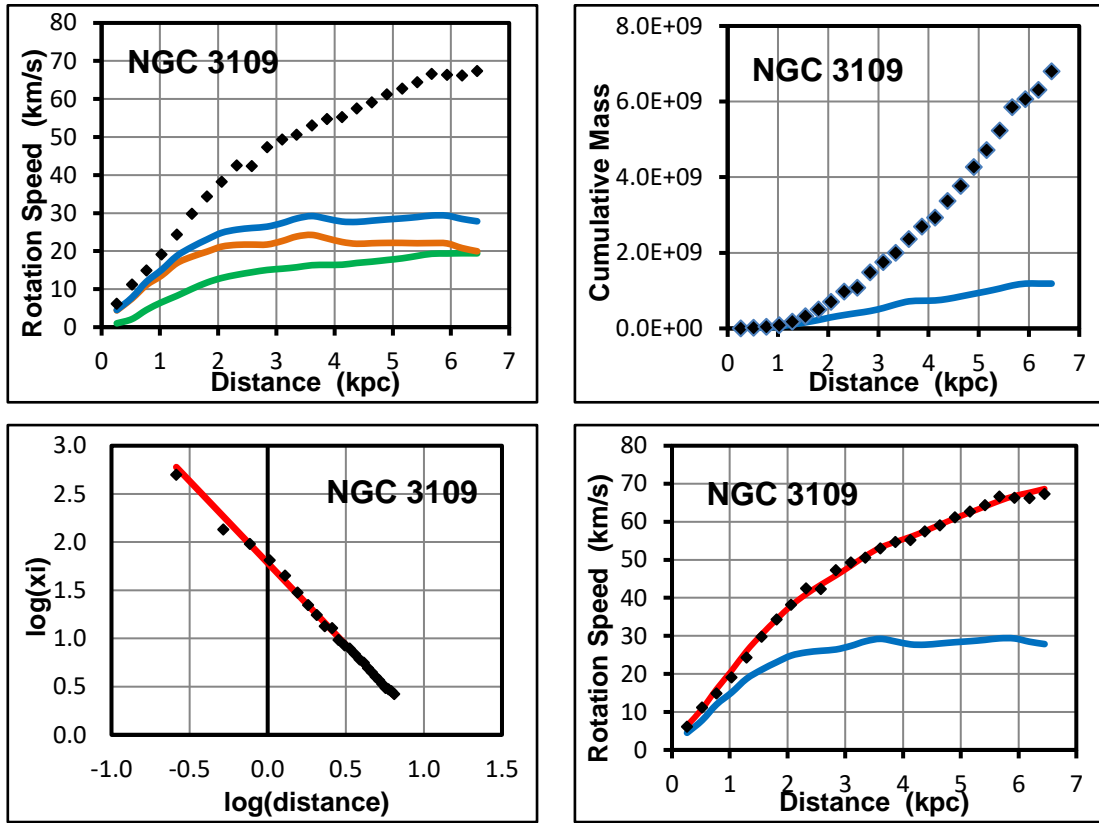


Figure 2. Rotation curve plots for spiral galaxy NGC 3109.

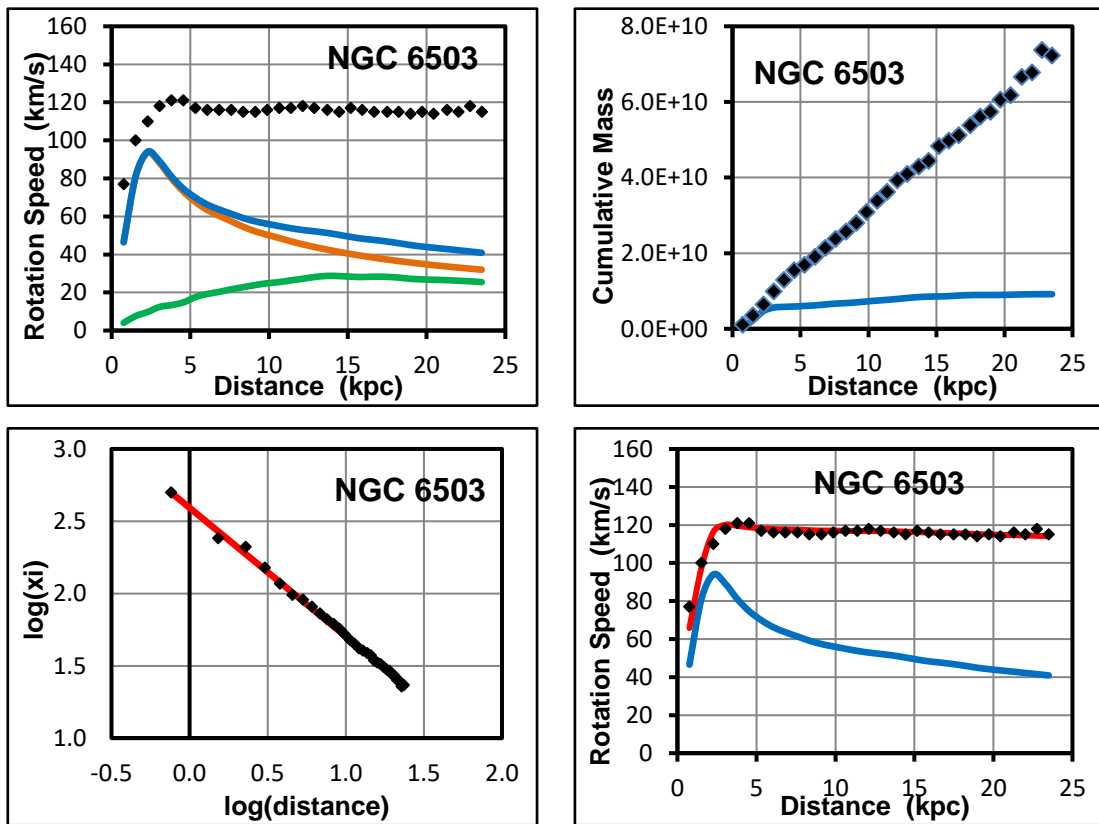


Figure 3. Rotation curve plots for spiral galaxy NGC 6503.

Applying equation (88) to equation (86) leads to

$$v^2(r) = \frac{G}{r} r^\alpha \int_{x=0}^r \frac{1}{x^\alpha} dM_e(x) = \frac{G}{r} \int_{x=0}^r \left\{ \frac{r}{x} \right\}^\alpha dM_e(x) \quad (89)$$

This is a remarkably simple relationship with only one adjustable parameter, the inverse slope α .

For the galaxies in the SPARC catalogue, the α exponent is found to satisfy

$$+0.5 \leq \alpha \leq +1.8 \quad (90)$$

Equation (89) for the rotation curves of disk galaxies has been derived from the observed data in the SPARC catalogue and the assumption that equation (86) holds.

Equation (89) is only expected to hold out to a limiting distance R beyond which the rotation curve reverts to the usual Newtonian decline. The ξ function reaches a terminal (intergalactic) value, ξ_T , and the effective mass reaches a terminal value, M_T

$$\int_{x=0}^R \xi(x) dM(x) = M_T = \text{constant} \quad (91)$$

At large distances the equation for the rotation curve transitions from equation (89) to

$$v^2(r) = \left\{ \frac{G M_T}{\xi_T} \right\} \frac{1}{r} \quad (92)$$

The bracketed term on the right-hand side is constant. We are now back with the rotation curve declining with the inverse root of the distance. So, at large distances, we have the normal fall off for Newtonian gravity, but at a higher level.

To summarise, if we know the distribution of baryonic matter in a disk galaxy, then we can predict the observed rotation curve using equation (89), together with a suitable choice of the α parameter. This has been shown to work for most of the galaxies in the SPARC catalogue (Lelli et al, 2016). We do not need any dark matter in disk galaxies, a variation of the energy scale does the same job.

10 Clusters of galaxies

This section summarises and extends work that was originally presented in JoKe4 (2015, "Clusters of galaxies"). Clusters of galaxies are thought to be embedded in dark matter halos for three reasons (Sanders, 2010; Ryden, 2017).

Firstly, the velocities of the individual galaxies are far too high to be maintained by the observed baryonic mass (galaxies and hot gas). When the virial theorem is applied to the cluster members it is found that the cluster must contain at least five times the observed mass,

otherwise the cluster would have dispersed a long time ago. This additional mass is thought to be provided by the dark matter halo.

Secondly, X-ray observations of clusters of galaxies reveal that most of the baryonic mass is in the form of hot gas. Perhaps as much as 90% of the baryonic mass is in the hot gas. The gas is thought to be in hydrostatic equilibrium in which case the cluster mass must be at least five times the observed baryonic mass. Again, this additional mass would come from a dark matter halo.

Thirdly, strong gravitational lensing of remote galaxies by the clusters can only be explained if the mass of the cluster is at least five times the observed baryonic mass. This additional mass would be provided by the dark matter halo.

All three cases provide a consistent picture in that they all require the mass of the cluster of galaxies to be at least five times the baryonic mass. There is no doubt that dark matter does a good job at explaining the observations.

We take a different view in that dark matter does not exist and that the correct explanation comes from variations of the energy scale. JoKe4 (2015) showed that a Gaussian-shaped energy scale variation can explain the observations of galaxy clusters. An energy scale variation with a modest central peak gives rise to higher than expected orbital velocities of cluster members, exactly as observed. The effective mass of the cluster is also increased in such a way that the hot gas can be maintained in hydrostatic equilibrium.

Section 6 "The virial theorem" (above) derived the equations for the expected velocities of galaxies in a cluster of galaxies.

For dark matter, equation (52) gives us

$$\langle v^2 \rangle = \frac{G}{2} \frac{(M_b + M_d)}{\langle R \rangle} \quad (93)$$

where M_b is the mass of the baryonic matter (galaxies & gas); M_d is the mass of the dark matter.

For our conjecture of variations of the energy scale, equation (51) gives us

$$\langle v^2 \rangle = \frac{G}{2} \frac{M_b}{\langle R \rangle} \frac{\langle \xi \rangle}{\xi_X} \quad (94)$$

It is clear that a suitable choice of the ξ function, describing the energy scale variation is capable of explaining the high velocities of cluster galaxies in exactly the same way as dark matter.

Some simple calculations were carried out to justify the assertions that variations in the energy scale can explain the observations of galaxy clusters. However, to date no detailed models have been made of any cluster, simply because we do not have access to the data. It should be possible to construct models of individual clusters that are in full agreement with the observational data of galaxy positions and velocities.

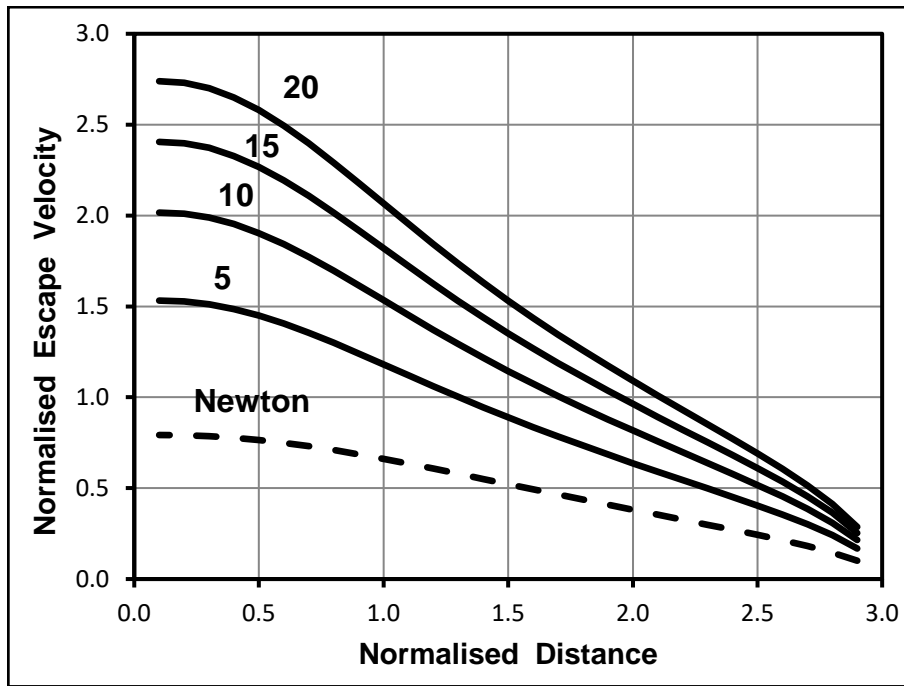


Figure 4. Escape velocities for a galaxy cluster having a Gaussian density distribution and a Gaussian energy scale variation. High values of the energy scale variation lead to high escape velocities, which lead to high velocities of galaxy members.

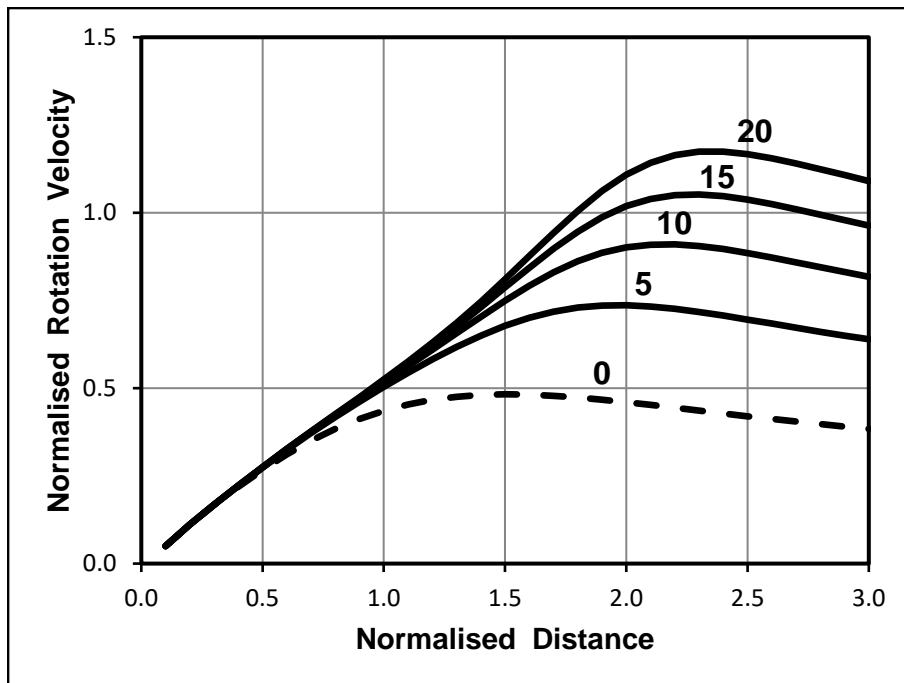


Figure 5. Circular velocities (rotation curves) for a galaxy cluster having a Gaussian density distribution and a Gaussian energy scale variation. High values of the energy scale variation lead to high circular velocities.

Figure 4 (above) is taken from JoKe4 (2015) and shows the escape velocity for a cluster with a Gaussian density distribution and a Gaussian energy scale variation. The curves are for different strengths of the energy scale variation and demonstrate that increases in the strength of the energy scale variation lead to increases in the escape velocities, which in turn lead to higher than expected velocities of galaxy members.

Figure 5 (above) is also taken from JoKe4 (2015) and shows the circular velocities for a cluster with a Gaussian density distribution and a Gaussian energy scale variation. These are essentially the expected rotation curves for different strengths of the energy scale variation and demonstrate that increases in the energy scale variation lead to increases in the rotational velocities. Although the galaxies in a cluster of galaxies do not move in circular orbits, this nevertheless supports the higher than expected velocities of galaxy members.

Models of individual clusters would provide a good test of the conjecture of energy scale variations. This idea forms the basis of one of the tests presented later in this paper.

Separately paper JoKe7 (2016, "Gravitational Lensing") showed that the enhanced effective mass of the cluster, from an energy scale variation, is sufficient to account for the observed gravitational lensing. This is covered in section 12 "Gravitational lensing", below.

To summarise, we do not need any dark matter in a cluster of galaxies, a variation of the energy scale is capable of explaining the observed data.

11 Cluster collisions

This section summarises and extends work that was originally presented in "Collisions between clusters of galaxies" (JoKe5, 2015).

Weak-gravitational lensing observations of galaxy clusters that have collided show a clear separation between the location of the gas and the location of the galaxies, with the strongest effects centred on the galaxies. In the dark matter scenario this is interpreted as demonstrating that the dark matter in the clusters has passed straight through the collision and taken the galaxies with it. The gas, on the other hand, has collided and got left behind.

The following two diagrams illustrate the same scenario but from the point of view of energy scale variations.

Before collision (Figure 6, below), the energy scale variation provides sufficient gravity to hold the galaxies and gas together, as described in section 7 (above). It also gives rise to the high velocities of the galaxy members.

During the collision the galaxies behave like solid particles and pass straight through. It is also assumed that the energy scale variations pass straight through, possibly in the manner of waves.

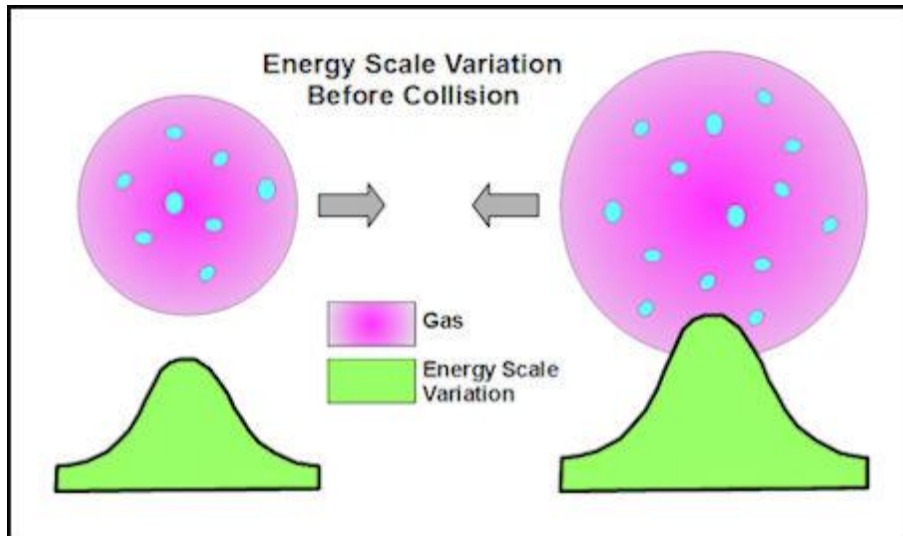


Figure 6. Two clusters of galaxies before collision. The galaxies (blue) are surrounded by hot gas (pink) and the whole embedded in energy scale variations (indicated by the green Gaussians). There is no dark matter.

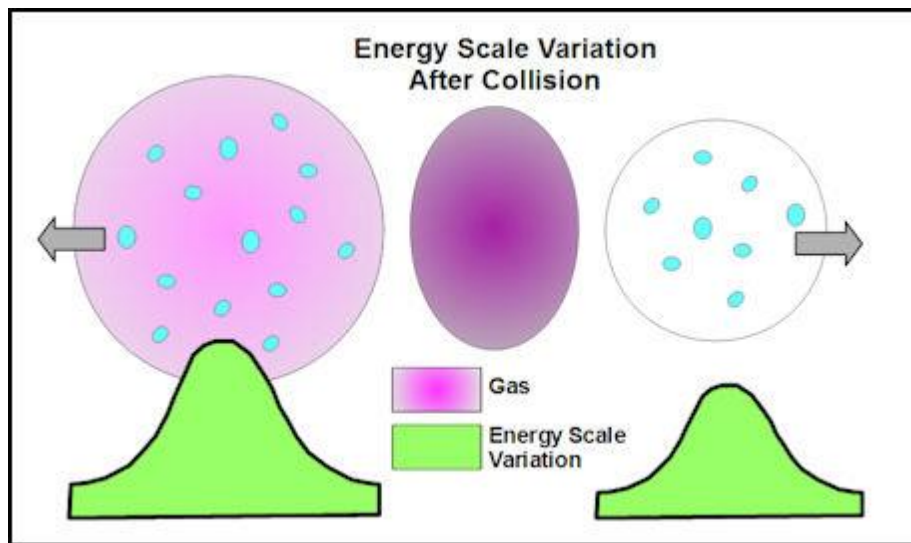


Figure 7. Two clusters of galaxies after collision. The galaxies (blue) remain embedded in the energy scale variations (indicated by the green Gaussians). The hot gas (pink) tends to get stripped out and left behind.

The hot gas in the two clusters collides and interacts strongly. The gas tends to get stripped out and left behind. So, the collision results in the observed separation of galaxies and hot gas.

In the dark matter scenario, the collision results in the loss of the gas but the dark matter and the galaxies remain. So, the overall mass of the cluster is reduced by at most 10%. This should be sufficient to keep the cluster intact.

In our energy scale variation scenario, the collision results in at least 50% of the mass (the hot gas) being lost. This means the cluster does not have sufficient mass to remain intact and should start to disperse.

So, a prediction of energy scale variations is that galaxy clusters should show signs of break up following collisions. This should be clear from the velocities of the galaxy members, which should be too high for the cluster to stay together. Also, any weak-gravitational lensing should be much weaker than in the dark matter scenario.

12 Gravitational lensing

This section summarises and extends work that was originally presented in "Gravitational Lensing" (JoKe7, 2015).

Currently, the most common form of gravitational lensing is that produced by a foreground cluster of galaxies that images remote galaxies. Both strong lensing, which images the remote galaxies into arcs of light, and weak lensing, which distorts the shape of remote galaxies, are observed. Examples of galaxy clusters showing strong lensing are: Abell 2218; SDSS J1038+4849; Abell 370; GC 0024+1654.

The lensed images enable an estimate to be made of the mass of the cluster of galaxies. This is always much larger than can be accounted for by the observed baryonic mass. The usual explanation is that there must be around five times the baryonic mass present in the form of dark matter.

We can account for the observations using an energy scale variation as illustrated in the following two diagrams.

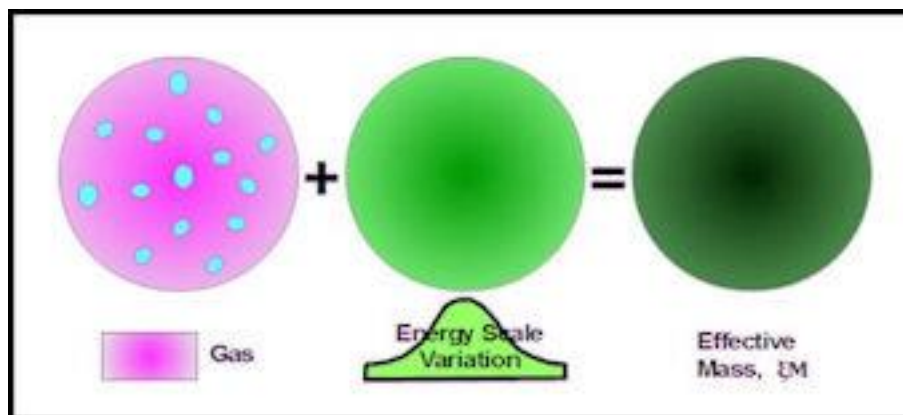


Figure 8: The effective mass of a galaxy within an energy scale variation is the product of the mass of the galaxies & gas and the ξ factor for the energy scale variation.

Figure 8 (above) illustrates how an energy scale variation works to give a much increased "effective mass" for the galaxy cluster. If the cluster is embedded in an energy scale variation, then the gravitational mass felt by a particle or light ray is magnified. Section 4 above (A new equation for mass) explains how this comes about.

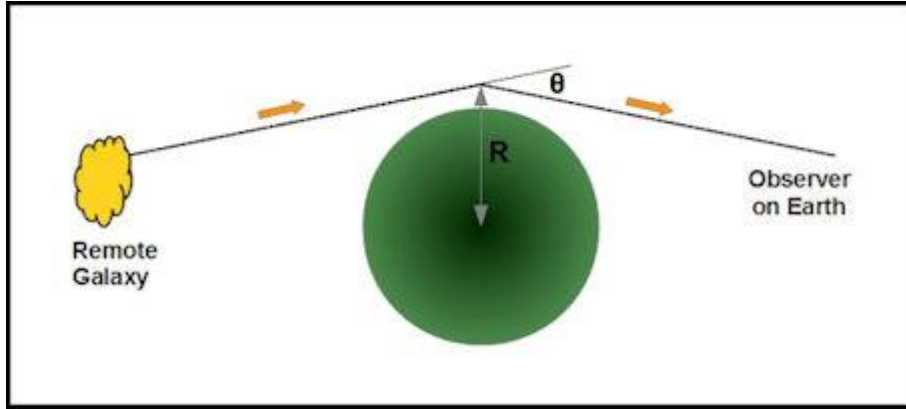


Figure 9: Illustration of the bending of light from a remote galaxy by an intervening cluster of galaxies with an energy scale variation.

Figure 9 illustrates how a light ray from a remote galaxy is bent by the foreground galaxy cluster.

Dark matter. The standard result for the bending of light by a gravitational field is (Ryden, 2017)

$$\theta = \frac{4 G (M_b + M_d)}{R c^2} \quad (95)$$

where M_b is the baryonic mass (galaxies & gas); M_d is the dark matter mass.

Variation of the energy scale. For our conjecture of variations of the energy scale, equation (95) becomes

$$\theta = \frac{4 G M_b}{R c^2} \gamma \quad (96)$$

where γ is the factor describing the energy scale variation (see equation (17)).

Once again, we see that a variation of the energy scale is capable of explaining the gravitational lensing effects observed in clusters of galaxies. There is no need for any dark matter.

13 Physical Cosmology

This section summarises and extends work that was originally presented in "Cosmology with no dark matter " (JoKe26, 2019). We show how the basic results of cosmology can be explained by variations in the energy scale, rather than by dark matter.

The currently accepted model for explaining the evolution of the Universe is the Λ CDM model (Λ =cosmological constant, CDM=cold dark matter). This assumes the Universe is made up of four components:

- a) radiation (photons and neutrinos)
- b) baryonic matter
- c) cold dark matter (CDM)
- d) cosmological constant (Λ).

It also assumes the Universe is flat, which means the energy density is the critical energy density.

The Friedmann equation for such a Universe is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \{ \epsilon_r + (\epsilon_b + \epsilon_d) + \epsilon_\Lambda \} = \frac{8\pi G}{3c^2} \epsilon_c \quad (97)$$

where

$$\epsilon_b + \epsilon_d = \epsilon_m \quad (98)$$

where H is the Hubble parameter; a the scale factor; ϵ_r the energy density of radiation; ϵ_b the energy density of baryonic matter; ϵ_d the energy density of dark matter; ϵ_Λ the energy density of a cosmological constant; ϵ_m the energy density of matter; ϵ_c the critical energy density of radiation.

The Friedmann equation is often written in terms of the dimensionless density parameter, Ω , where

$$\Omega_x = \frac{\epsilon_x}{\epsilon_c} \quad (99)$$

where the x subscript denotes the component. Hence

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2} \{ \Omega_r + \Omega_m + \Omega_\Lambda \} \epsilon_c \quad (100)$$

where

$$\Omega_m = \Omega_b + \Omega_d \quad (101)$$

In section 7 (above) we showed that, for our conjecture of energy scale variations and a (baryonic) matter-only universe, the Friedmann equation can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} \gamma \rho_b = \frac{8\pi G}{3c^2} \gamma \epsilon_b \quad (102)$$

where γ is the dimensionless multiplicative factor representing the energy scale variation.

In deriving equation (102) we assumed that $\gamma \rho_b$ was constant, whilst allowing the density to have an inhomogeneous lumpy distribution. Although the matter has a lumpy distribution, we expect the radiation to be uniform and the cosmological constant, by definition, to have a uniform (constant) value. So, we have to rewrite equation (97) as

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8 \pi G}{3 c^2} \{ \epsilon_r + \gamma \epsilon_b + \epsilon_\Lambda \} = \frac{8 \pi G}{3 c^2} \epsilon_c \quad (103)$$

or, in terms of the density parameter

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8 \pi G}{3 c^2} \{ \Omega_r + \gamma \Omega_b + \Omega_\Lambda \} \epsilon_c \quad (104)$$

So, the γ factor affects only the baryonic matter component and not the radiation nor the cosmological constant components.

We arrive at the following two rules for explaining physical cosmology in terms of energy scale variations.

Rule 1: Local processes. These include nuclear reactions, atomic reactions, scattering, etc, i.e. all processes where the components must be in the same locations to interact. Working with the baryonic density, ρ_b , or the baryonic density parameter, Ω_b , we have

$$\rho_m = \rho_b + \rho_d \Rightarrow \rho_b \quad (105)$$

and

$$\Omega_m = \Omega_b + \Omega_d \Rightarrow \Omega_b \quad (106)$$

where the double-headed arrow means 'is replaced by'.

So, (for local processes) expressions using matter as the sum of baryonic matter plus dark matter should change to work with just the baryonic matter.

Rule 2: Non-local processes. These include gravitational interactions, equations where the Hubble parameter or Friedmann equation is involved.

$$\rho_m = \rho_b + \rho_d \Rightarrow \gamma \rho_b \quad (107)$$

and

$$\Omega_m = \Omega_b + \Omega_d \Rightarrow \gamma \Omega_b \quad (108)$$

So, (for non-local processes), expressions using matter as the sum of baryonic matter plus dark matter should change to work with the baryonic matter multiplied by the γ factor for the energy scale variation.

For example, Rule 2 is used on the Friedmann equation to go from equations (97) & (99) to equations (103) & (104).

Armed with these two rules, we can now look at how variations in the energy scale explain the main results of physical cosmology.

Big Bang Nucleosynthesis (BBN) occurred around 200 seconds after the Big Bang when the light elements up to beryllium (${}^8\text{Be}$) were created out of the existing protons and neutrons (Weinberg, 2008; Ryden 2017). Observations of the primordial abundances of hydrogen (${}^1\text{H}$), deuterium (${}^2\text{D}$), helium (${}^3\text{He}$), and lithium (${}^7\text{Li}$), fix the baryon-to-photon ratio, η , at

$$\eta = 6.1 \times 10^{-10} \quad (109)$$

This number is roughly constant and its value during BBN is expected to be close to its value today. This follows since baryons are conserved and the photons are dominated by the radiation from the CMB (cosmic microwave background). Our conjecture of energy scale variations has no impact on this number.

Material presented in Weinberg (2008), Ryden (2017) (and elsewhere), support the following argument.

The current temperature of the cosmic microwave background, 2.73 K, defines the number density of photons. Combining this with the baryon-to-photon ratio, η , (see 13.4 above) gives the current energy density of baryons, $\epsilon_{b,0}$. Red shift and distance observations of remote objects, including type Ia supernovae, fix the Hubble constant at ~ 68 km/s/Mpc. The Friedmann equation then gives the critical energy density, $\epsilon_{c,0}$.

These lead directly to the current energy density of baryons, $\epsilon_{b,0}$, being only 4.8% of the critical energy density, $\epsilon_{c,0}$, required for a flat Universe

$$\Omega_{b,0} = \frac{\epsilon_{b,0}}{\epsilon_{c,0}} = 0.048 \quad (110)$$

where $\Omega_{b,0}$ is the current density parameter for baryons.

This number does not depend in any way on our conjecture of energy scale variations. In fact, it is independent of the existence of both dark matter and a cosmological constant. It places a solid constraint, not only on the Λ CDM model, but on all cosmological models and hypotheses that attempt to explain how the Universe has evolved. So, along with the Λ CDM model, our conjecture of variations of the energy scale supports the notion that the current baryonic density is only 4.8% of the critical density.

The Λ CDM model has the current epoch dominated by matter and a cosmological constant. The Friedmann equation for this is (after Ryden 2017):

$$H^2 = \left\{ \frac{\dot{a}}{a} \right\}^2 = \frac{8 \pi G}{3 c^2} \left\{ \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} \right\} \epsilon_{c,0} \quad (111)$$

where H is the Hubble parameter; a the scale factor; $\epsilon_{c,0}$ the current critical density; $\Omega_{m,0}$ the current density parameter for matter; $\Omega_{\Lambda,0}$ the current density parameter for a cosmological constant.

Observations of type Ia supernovae (and other standard candles) out to red-shifts a little beyond $z=1.0$ have shown that the best fit to equation (111) is achieved with the values

$$\Omega_{m,0} \approx 0.3 \quad (112)$$

$$\Omega_{\Lambda,0} \approx 0.7 \quad (113)$$

For the Λ CDM model, equations (101) & (99) lead to a current density parameter, $\Omega_{d,0}$, for dark matter of

$$\Omega_{d,0} \approx 0.25 \quad (114)$$

So, for the Λ CDM model, the 30% matter is made up of 5% baryonic matter and 25% dark matter.

For our conjecture of energy scale variations, equations (110) and (108) lead to matter being entirely baryonic matter with a value for the γ factor of

$$\gamma \approx 6 \quad (115)$$

It is generally accepted that the early Universe was dominated by radiation and matter alone, i.e. the cosmological constant played no part. The Friedmann Equation for a flat Universe containing only radiation and matter is (after Ryden 2017)

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8 \pi G}{3 c^2} \left\{ \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} \right\} \epsilon_{c,0} \quad (116)$$

For our conjecture of energy scale variations this becomes

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8 \pi G}{3 c^2} \left\{ \frac{\Omega_{r,0}}{a^4} + \frac{\gamma \Omega_{b,0}}{a^3} \right\} \epsilon_{c,0} \quad (117)$$

Radiation-matter equality occurs when the bracketed terms in equations (116) & (117) are the same. For equation (116), i.e. dark matter, this happens when

$$a_{rm} = \frac{\Omega_{r,0}}{\Omega_{m,0}} \quad (118)$$

where a_{rm} is the scale factor for radiation-matter equality.

For equation (117), i.e. energy scale variations, this happens when

$$a_{rm} = \frac{\Omega_{r,0}}{\gamma \Omega_{b,0}} \quad (119)$$

But these two expressions, for the scale factor when radiation and matter had equal energy densities, are the same by Rule 2.

So, both dark matter and energy scale variations have radiation-matter equality occurring when

$$a_{rm} \approx 2.9 \times 10^{-4} \quad (120)$$

This corresponds to a time of around 50,000 y after the Big Bang: a red shift of

$$z \approx 3450 \quad (121)$$

and a temperature of

$$T \approx 9400K \quad (122)$$

(JoKe26, 2019).

Similar arguments apply to the era of recombination, when neutral hydrogen was formed. This period lasted over 100,000 y and its end is characterised by the time of last scattering, which resulted in the cosmic background radiation. Both dark matter and energy scale variations give rise to exactly the same values (JoKe26, 2019)

$$T \approx 2970 \quad (123)$$

$$z \approx 1090 \quad (124)$$

$$a_{ls} \approx 9.2 \times 10^{-4} \quad (125)$$

In summary, we assert that the physical cosmological results that come from the Λ CDM model can be reproduced using our conjecture of variations of the energy scale. The two Λ CDM parameters of (a) baryonic density Ω_b , and (b) dark matter density Ω_d , are replaced by (a) baryonic density Ω_b , and (b) energy scale variation factor γ .

Two other areas of physical cosmology, (1) cosmic microwave background, and (2) growth of structure, are covered in separated sections below.

14 Cosmic microwave background

This section summarises & extends work that was originally presented in "Primordial Density Perturbations" (JoKe8, 2016) & "The Friedmann Equation and the Cosmic Microwave Background" (JoKe17, 2018).

It was shown in section 13 "Physical Cosmology" (above) that our conjecture of variations of the energy scale leads to the same basic results for the cosmic microwave background (CMB) as those given by the Λ CDM model.

For the epoch of the CMB

- (a) a temperature of 2790K,
- (b) a red-shift of 1090
- (c) a scale factor of 9.2×10^{-4} .

For the current epoch

- (a) a temperature of 2.73K,
- (b) a red-shift of zero
- (c) a scale factor of unity.

So, with our conjecture of variations of the energy scale, we can explain the current CMB temperature and its near uniformity across the sky.

However, it is also important to explain the acoustic peaks in the power spectrum of the CMB. This can be achieved by applying our Rule 1 & Rule 2 as set out in section 13 "Physical Cosmology" above. As an example, we look at how these explain the location of the first peak (see JoKe26, 2019).

The horizon distance at time t for a universe containing matter and radiation is given by (Ryden, 2017)

$$D_h(t) = \frac{2c a_{rm} a(t)}{H_0 \sqrt{\Omega_{r,0}}} \left[\left(1 + \frac{a(t)}{a_{rm}} \right)^{1/2} - 1 \right] \quad (126)$$

where $a(t)$ is the scale factor at time t ; a_{rm} the scale factor at radiation-matter equality; H_0 the Hubble constant; $\Omega_{r,0}$ the density parameter for radiation at the current epoch. As explained above in section 13 "Physical Cosmology", the various parameters in equation (126) take exactly the same values under our conjecture of variations of the energy scale as under the Λ CDM model.

At the time of last scattering the sound speed of the baryon-photon fluid, c_s , can be taken to be the same as that of a photon gas

$$c_s = \frac{c}{\sqrt{3}} \quad (127)$$

leading to the sound horizon distance at last scattering, $S_h(t_{ls})$, being

$$S_h(t_{ls}) = \frac{D_h(t_{ls})}{\sqrt{3}} \quad (128)$$

where $D_h(t_{ls})$ is the horizon distance at the time of last scattering, given by equation (126).

At the current epoch this has expanded to

$$S_h(t_0) = (1 + z_{ls}) S_h(t_{ls}) \quad (129)$$

The angular size (in degrees) on the sky now of the sound horizon at last scattering is

$$\theta_s = \frac{S_h(t_0)}{D_h(t_0)} \frac{180}{\pi} = \frac{D_h(t_{ls})}{D_h(t_0)} \frac{(1 + z_{ls})}{\sqrt{3}} \frac{180}{\pi} \quad (130)$$

where $D_h(t_0)$ is the current horizon distance ($\sim 14,000$ Mpc).

The sound horizon, equation (128), is around 0.15 Mpc corresponding to an angular size on the sky, equation (130), of about 0.7 degrees. This is in reasonable agreement with the first peak of the CMB power spectrum, which occurs at an angular size of around 0.8 degrees. The small discrepancy arises because our analysis is overly simple and omits other factors that should be considered, as explained by Weinberg (2008) and Lyth & Liddle (2009).

If we look at the physics and equations behind the acoustic peaks (Weinberg 2008; Lyth & Liddle 2009), then we see that they contain terms involving Ω_m and ρ_b (etc). It is clear that our Rule 1 and Rule 2 leave these equations essentially unchanged and so we end up with exactly the same acoustic peaks as the Λ CDM model. In practice the values of Ω_m and ρ_b are chosen to give the best fit to the observed peaks. So, we are also allowed to choose our own values of γ and ρ_b .

We can summarise this section by stating that our conjecture of variations of the energy scale has the potential to explain all observed aspects of the CMB.

15 Growth of structure

This section summarises and extends work that was originally presented in "Structure Formation" (JoKe14, 2018).

It is usually argued that, for baryonic matter alone, there time for gravity, working on the minute density fluctuations, to create the observed structures of galaxies, clusters of galaxies, and super-clusters. This problem is solved by the addition of large amounts of dark matter. Dark matter would have started forming gravitational wells from the Big Bang and these would dominate & control the collapse of baryonic matter from the near uniform density at the time of the cosmic microwave background (CMB). The gravitational collapse of dark matter controls the formation of structures. One consequence is that all galaxies must have a dark matter halo, as must clusters of galaxies, and super-clusters. This is all part of the Λ CDM model.

For our conjecture of variations of the energy scale, it is the gravitational wells formed by such variations that accelerate the gravitational collapse and form the observed structures within the observed time scales.

JoKe14 (1018) carried out a simple simulation to demonstrate how a variation of the energy scale can reduce the time to form structure. 250 unit mass points were randomly placed on a square grid with small random velocities. A single fixed mass of 10 units was placed at the centre. The system was then allowed to evolve under the mutual gravitational attraction of the masses. Two simulations were run side by side: (a) with a Gaussian energy scale variation, and (b) with no energy scale variation. Many runs were made by varying the locations and velocities of the masses.

Figure 11 shows the evolution of a typical simulation. The right frames are for simple Newtonian gravitation. The left frames are for an energy scale variation and it is clear that this evolves significantly faster.

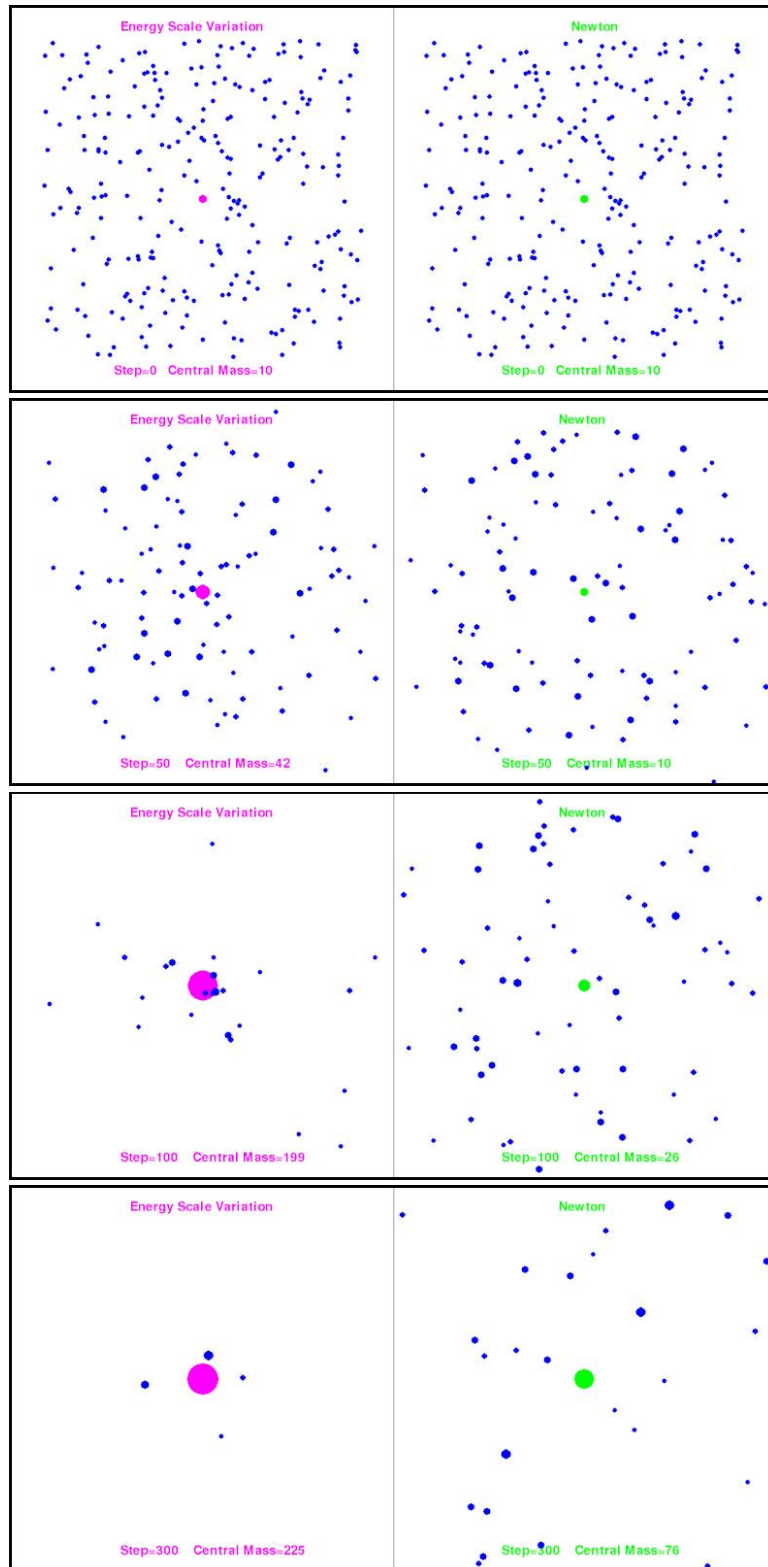


Figure 11. Evolution of 250 randomly placed masses with random speeds. Right side (green) is simple Newtonian gravity; left side (purple) includes an energy scale variation. The simulation shows that structure forms much faster in the presence of an energy scale variation.

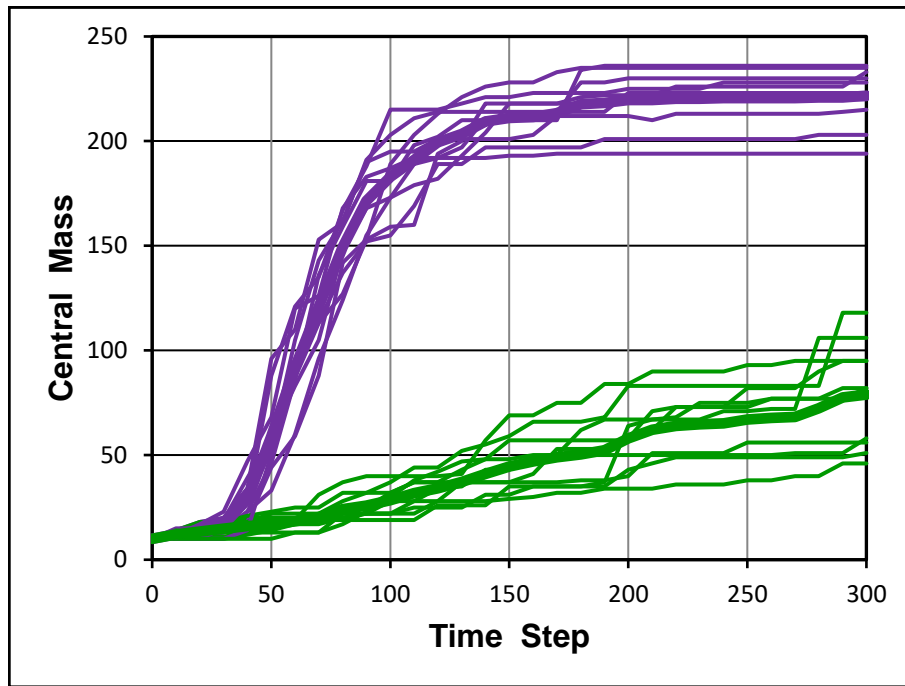


Figure 12. Growth of the central mass for 10 runs of the numerical simulation. The green lines are for Newtonian gravitational; the purple lines are for Newtonian gravity plus an energy scale variation. The thick lines are the averages of the runs.

Figure 12 shows the growth of the central mass for 10 different runs of the simulation. Every run starts with a fresh set of random locations and random velocities. It is clear that the presence of an energy scale variation speeds up the growth of structure. Although the simulation is simple and two-dimensional, it lends support to the claim that our conjecture has the potential to explain the growth of galaxies and clusters of galaxies within the available time frame. There is no need for any dark matter.

16 Predictions

This section summarises and extends work that was originally presented in "Predictions and Tests" (JoKe21, 2019). Up to this point we have demonstrated that our conjecture, of variations of the energy scale, has the potential to explain all those astronomical scenarios where dark matter is invoked. So, we now have two alternative explanations, (a) dark matter, and (b) variations of the energy scale. Clearly it is helpful if we can come up with some predictions that can distinguish between the options. We look at some of these in this section.

Prediction 1: No dark matter particles will ever be detected

This prediction follows from our conjecture of variations of the energy scale. It means, almost by definition, that dark matter particles are not needed to explain those astronomical scenarios where dark matter is invoked. So, dark matter does not exist and will never be detected by any experiment. It is always hard to prove a negative and so this prediction is not useful at a practical level; it does not suggest a test that can be carried out. However, it does provide an explanation as to why decades of experiments have failed to find anything. The prediction also means that, if a new particle is discovered, then it will not explain the dark matter observations.

Prediction 2: No dark matter objects will ever be observed

If the universe contains many times more dark matter than baryonic matter, then we might expect pure dark matter objects to exist. However, we predict that there are no planets, stars, galaxies, or other objects made of nothing but dark matter. Dark matter theory posits the existence of dark matter halos around galaxies and clusters of galaxies. By extension we might expect large objects to exist that contain only dark matter and, occasionally, we might expect these to collide with normal galaxies. However, images of peculiar and colliding galaxies can all be explained on the basis of interactions between normal (baryonic) galaxies. There are no known images of interactions that require the existence of a pure dark matter object. Like Prediction 1, this is a negative prediction and does not give rise to any tests that can be carried out.

Prediction 3: Galaxies with little or no dark matter can exist

The dark matter hypothesis explains galaxy formation as the accretion of baryonic matter into dark matter gravitational wells. Hence, every galaxy must have a dark matter halo. For us, galaxy formation is assisted by the existence of variations of the energy scale. However, our conjecture means the strength of the energy scale variation and the total baryonic mass of the galaxy are independent of one another. So, if we have a sufficiently massive amount of baryonic matter, then this will form a galaxy irrespective of whether or not there is an energy scale variation. Hence, we have no problem in explaining galaxies that appear to have little or no dark matter. And, conversely, we also have no problem in explaining galaxies that appear to have an excessively large amount of dark matter - these would be cases of exceptionally

strong energy scale variations. Our conjecture means that the strength of energy scale variation can vary from zero to a large number, corresponding to galaxies with no dark matter or large amounts. This prediction provides an explanation rather than a test that can be carried out.

Prediction 4: Local phenomena do not require any additional matter

Our conjecture means the gravitational acceleration at location \mathbf{X} produced by a mass \mathbf{M} at location \mathbf{A} is given by equation (36)

$$\ddot{\mathbf{r}} = - \frac{\mathbf{G} \mathbf{M}}{r^2} \left(\frac{\xi_{\mathbf{A}}}{\xi_{\mathbf{X}}} \right) \quad (131)$$

If the two locations are sufficiently close to one another, then there will be no difference between the two ξ values and the ratio in equation (131) will be unity. Hence, we predict that there will be no requirement for any additional mass (or dark matter) to explain local phenomena. This begs the question of what we mean by local, which we can answer by saying that local means any region where the ξ function is not changing. This is a somewhat circular argument. Nevertheless, we predict there is no requirement for dark matter in the solar system, in binary star systems, or in the centres of galaxies.

Prediction 5: Galaxy rotation curves eventually decline and follow Newtonian gravity

This prediction follows from section 9 "Galaxy rotation curves" (above). It is expected that the ξ function, describing the energy scale variation, levels off to a constant value at a modest distance from the galaxy centre. At the same time the baryonic mass of the galaxy converges to an effective total value. Beyond this point it is predicted that the rotation curve will follow the normal $1/\sqrt{r}$ decline of Newtonian gravity. It may be possible to extend observations of galaxy rotation curves to much larger distances and so test this prediction. This would have to be done by using the 21 cm line of neutral hydrogen out to distances beyond 40 kpc from the galaxy centres.

Prediction 6: Galaxy rotation curves can be predicted from the baryonic matter distribution

This prediction follows from section 9 "Galaxy rotation curves" (above) and from JoKe (2019). The analysis of the SPARC catalogue of disk galaxies shows that rotation curve is given by equation (89)

$$v^2(r) = \frac{\mathbf{G}}{r} r^\alpha \int_{x=0}^r \frac{1}{x^\alpha} d\mathbf{M}_e(x) = \frac{\mathbf{G}}{r} \int_{x=0}^r \left\{ \frac{r}{x} \right\}^\alpha d\mathbf{M}_e(x) \quad (132)$$

where α is a constant of order 1.0; $dM_e(x)$ is the increment in 'effective mass'. JoKe (2019) showed there is a correlation between α and the total baryonic mass. So, we can construct the rotation curve if we know the distribution of baryonic mass alone. The dark matter hypothesis cannot predict the rotation curve; it is always the other way round with the rotation curve fixing the dark matter distribution. This prediction provides a clear separation between the hypothesis of dark matter and our conjecture of variations of the energy scale.

Prediction 7: Galaxy interactions are determined solely by the baryonic mass

Consider two disk galaxies, A and B, in orbit around one another, and each its own energy scale variation. The rotational speeds of the stars are determined by the local energy scale variation and, as set out in section 9 "Galaxy rotation curves", we expect to observe flat rotation curves. But what of the orbital velocity of one galaxy around the other. If the ξ values at the centres of both galaxies are the same, then the gravitational acceleration is determined by the baryonic mass of the galaxies alone. The radial acceleration of galaxy A caused by galaxy B is, using equation (36)

$$\ddot{r}_A = - \frac{G M_B}{r^2} \left(\frac{\xi_B}{\xi_A} \right) = - \frac{G M_B}{r^2} \quad (133)$$

Similarly, for the radial acceleration of galaxy B caused by galaxy A. The ratio of the ξ factors cancel out as we expect them to be the same. So, the orbital motions of the two galaxies around one another are determined solely by their baryonic masses. For the dark matter hypothesis, the motions are determined by the total mass (both baryonic matter and dark matter). As the total mass is around 6 times greater than the baryonic mass, we expect the orbital velocities for dark matter to be around $\sqrt{6}$ higher than for our conjecture of variations of the energy scale. All we need to do is measure the masses of a pair of interacting galaxies and follow their motions. Although we can do this in principle, it requires a timescale of tens of millions of years to get the orbital data. However, it may be possible to use computer simulations of galaxy interactions to settle the matter.

One test case is provided by the Andromeda Galaxy and our Milky Way, which are falling towards one another under gravity. If the approach speed comes from the gravitational potential, then this is given by

$$v^2 = \frac{2 G (M_A + M_G)}{r} \quad (134)$$

or

$$(M_A + M_G) = \frac{r v^2}{2 G} \quad (135)$$

Current values for the parameters are: speed v 110 km s⁻¹; distance r 778 kpc; Andromeda mass M_A 1.5×10¹² solar masses; Milky Way mass M_G 1.1×10¹² solar masses (the masses include dark matter). The right-hand side of equation (135) evaluates to around 1.1×10¹², which is lower than the sum of masses and more in keeping with the $1/\sqrt{6}$ reduction that our conjecture expects.

JoKe15 (2018) carried out some simple numerical simulations of interacting galaxies. These showed that tidal tails are produced equally well under variations of the energy scale as under dark matter. Holincheck et al (2016) carried out detailed computations of 62 galaxy interactions and it would be interesting to repeat these with energy scale variations with their lower interaction speeds.

Prediction 8: Clusters of galaxies should disperse after a collision

This prediction follows from section 11 "Cluster collisions" (above). Observations show that, when clusters of galaxies collide, the gas tends to get stripped and left behind. At the same time the individual galaxies pass straight through and continue on their way. As most of the mass is in the gas, collisions should result in a substantial reduction of the baryonic mass of the clusters. For the dark matter hypothesis, the dark matter is unaffected by the collision and continues to provide sufficient gravity to hold the galaxies together. For our conjecture of variations of the energy scale, although we assume the energy scale variations pass straight through, there is insufficient gravity left to hold the galaxies together. We predict that the clusters should show signs of break up with dispersal of the individual galaxies. This should be detectable by carrying out a statistical analysis of the peculiar velocities of the galaxies and comparing this to the mass determined from weak gravitational lensing. However, this result may be difficult to disentangle from the disruptive effects of the collision. Nevertheless, this is a test where there is a clear difference between dark matter and energy scale variations.

Prediction 9: Distribution of gravity across a cluster of galaxies

As mentioned in section 10 "Clusters of galaxies", there are three independent observations that have to be explained

- (a) velocities of galaxy members
- (b) hydrostatic equilibrium of the hot gas
- (c) gravitational lensing

Roughly the same amount of dark matter is needed to explain all three, and so the dark matter hypothesis does a good job in providing a consistent explanation. We expect our conjecture of variations of the energy scale to be able to explain the observations equally well (hopefully, better). However, at the present time no serious calculations have been carried out to demonstrate such a capability. All we have been able to do is show that our conjecture has the potential to explain the observations. Nevertheless, we predict our conjecture will provide a satisfactory explanation for the observations of clusters of galaxies.

17 Discussion

We have put forward the conjecture that the energy scale can vary from location to location. As we have seen this simple idea has the potential to explain all those situations where dark matter is involved. However, we should also point out a few areas where problems remain.

No theory. We have no theory for variations of the energy scale. We are simply stating that they are a phenomenon that exists. We do not know how they behave or interact with one another. If two galaxies merge, then what happens to their combined energy scale variation? When clusters of galaxies collide, do their energy scale variations simply pass straight through like waves? All we have is equation (37), which defines the gravitational acceleration of a remote mass. This is a good start, but it needs backing up with a proper theory.

General relativity. We have said little about general relativity. Our conjecture changes the way energy behaves and we should expect this to affect the energy-momentum tensor. Much of general relativity is concerned with the local effects of remote masses and for these the basic Newtonian gravitational potential is normally used. Clearly, more work has to be done, especially on the Friedmann equation and its application to physical cosmology.

We have not come up with a killer blow that shows the hypothesis of dark matter is wrong. Similarly, we have not found any killer blow that our conjecture is wrong, nor any argument that rules it out. It can be noted that our conjecture not only provides an alternative to dark matter, but also provides a natural explanation as to why no dark matter particles have been found.

The strongest argument in favour of our conjecture comes from the analysis of galaxies in the SPARC catalogue, as set out in section 9 above. The linear relationship for the logarithm of our ξ function (as shown in Figures 1, 2, 3) is an observational fact and was completely unexpected. The linear relationship appears to apply to all spiral galaxies. Whether or not our conjecture is correct, this relationship demands an explanation. An independent analysis should also be carried out to check that it is correct.

In several sections our explanations are not supported as strongly as we would like. For example, we would love to have access to the raw observations of the weak gravitational lensing of clusters of galaxies. We could then analyse the data in terms of variations of the energy scale instead of dark matter.

Similarly with the cosmic microwave background (CMB). We would really like to analyse the CMB data and see whether energy scale variations give different values for the adjustable parameters. In particular, we could possibly end up with a different value for the Hubble constant, which might resolve the tension between the CMB and the distance ladder values.

We have put forward the simple idea that the energy scale can vary from location to location. Our idea has tremendous explanatory power and provides an alternative to the dark matter hypothesis. We have also provided a number of predictions that can be used to test our conjecture. Clearly, there is still much work to be done.

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