Black Holes and Anti-gravity

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Abstract

In the framework of general relativity the new model of a black hole was presented. A description of the gravitational field under the event horizon became possible when modifications of previously known concepts appeared. The black hole is a homogeneous ball with a radius smaller than the Schwarzschild radius, but not less than half the Schwarzschild radius. Equations describing gravitational acceleration inside a black hole correspond to Gaussian gravitational law. The gravitational acceleration outside the black hole is directed from its center. At distances from the center larger than the Schwarzschild radius, gravitational acceleration is directed to the center.

Keywords: general relativity, black hole, Schwarzschild radius, anti-gravity

01. Introduction

A description of the gravitational field under the event horizon became possible when modifications of previously known concepts appeared.

1. Scalar (dot) product [3]

If we want to get local basis vectors with real values in physical spacetime, then we need to adopt new relation between those vectors and components of a metric tensor.

$$\boxed{\mathbf{e}_\mu \cdot \mathbf{e}_\nu = -\left(\mathrm{sgn} ds^2\right) g_{\mu\nu} \geq 0, \quad (ds)^2 \neq 0, \quad (\mu, \nu = 1,2,3,4)}.$$

It causes a necessity to change a definition of scalar (dot) product and associated terms. Those changes, forced by physics, will cause only small complications of some formulas.

In cases when

$$(ds)^2 < 0 \quad \text{and} \quad g_{\mu\nu} \geq 0,$$

above modifications do not lead to any changes, because we have then

$$\mathbf{e}_\mu \cdot \mathbf{e}_\nu = g_{\mu\nu} \geq 0.$$

The scalar (dot) product of vectors $\mathbf{A} = A^\mu \mathbf{e}_\mu$ and $\mathbf{B} = A^\nu \mathbf{e}_\nu$ is the expression

$$\boxed{\mathbf{A}\cdot\mathbf{B} = A^{\mu}B^{\nu}\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu}},$$

taking into account that

$$\mathbf{e}_{\mu}\cdot\mathbf{e}_{\nu} = -\left(\operatorname{sgn} ds^2\right)g_{\mu\nu} \ge 0, \quad (ds)^2 \ne 0, \quad (\mu,\nu = 1,2,3,4),$$

we get

$$\boxed{A\cdot B = -\left(\operatorname{sgn} ds^2\right)g_{\mu\nu}A^{\mu}B^{\nu}}.$$

2. Four-dimensional equations of test particle motion [3]
Components of the four-acceleration of a test particle with mass (m) at a given point of curved Riemann spacetime are described by equations:

$$\boxed{\frac{\tilde{F}^{\alpha}}{m} = \tilde{a}^{\alpha}_{\text{force}} \overset{\text{df}}{=} \left(\operatorname{sgn} ds^2\right)c^2\left(\frac{d^2x^{\alpha}}{ds^2} + \tilde{k}\Gamma^{\alpha}_{\mu\nu}\frac{dx^{\mu}}{ds}\frac{dx^{\nu}}{ds}\right), \quad ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} \ne 0, \quad \left(\operatorname{sgn} ds^2\right)g_{\nu\nu} \le 0},$$

where

$(\tilde{F}^{\alpha})$ – components of the resultant four-force, with ommision of „gravitational" and „inertial" forces,
$(g_{\mu\nu})$ – components of the spacetime metric tensor (which are the solutions of the field equations),

$$\tilde{k} = \begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}.$$

3. Two-potentiality of the gravitational field [3, 5, 8]
From classical physics we know, that absolute value of gravitational field strength in center of homogeneous ball (that has constant density) is equal to zero. Together with growth of distance from the center – strength grows linearly, reaching its maximal value on the surface of a ball. With further growth of distance – it decreases inversely squared.

If we want to get the same result, within the frames of Einstein's general theory of relativity, then we have to notice that stationary gravitational field is a two-potential field.

$$\boxed{\begin{matrix}\dfrac{\partial\mathbf{E}}{\partial t} = 0, \quad \operatorname{rot}\mathbf{E} = 0 \\ \operatorname{rot}\operatorname{grad}\varphi = 0\end{matrix}} \Rightarrow \boxed{\begin{matrix}\mathbf{E}^{in} = \operatorname{grad}\varphi^{in} = -\tilde{k}\operatorname{grad}\varphi^{in}, & 0 \le r < R, & \lim\limits_{r\to 0}\varphi^{in} = 0 \\ \mathbf{E}^{ex} = -\operatorname{grad}\varphi^{ex} = -\tilde{k}\operatorname{grad}\varphi^{ex}, & r \ge R, & \lim\limits_{r\to\infty}\varphi^{ex} = 0\end{matrix}}$$

$$E^{in}_{r} = -\frac{4}{3}\pi G\rho r, \quad \varphi^{in} = -\frac{2}{3}\pi G\rho r^2, \quad E^{ex}_{r} = -\frac{GM}{r^2}, \quad \varphi^{ex} = -\frac{GM}{r}.$$

On the surface of a ball, we have

$$\varphi^{in} - \varphi^{ex} = \frac{GM}{2R}, \quad \mathbf{E}^{in} - \mathbf{E}^{ex} = 0,$$

where

$(\mathbf{E}^{in})$, $(\mathbf{E}^{ex})$ – gravitational field strenght respectively inside and outside of a ball,
(φ^{in}), (φ^{ex}) – gravitational field potential respectively inside and outside of a ball,
(M) – mass of a ball, (R) – radius of a ball, (ρ) – density of a ball,

$$\tilde{k} = \begin{cases} +1 & \text{outside of a mass source} \\ -1 & \text{inside of a mass source} \end{cases}.$$

4. Relativistic energies: resting, kinetic and total [3, 9, 11, 12]
Rest energy (E_0), relativistic kinetic energy (E_k) and relativistic total energy (E) are given respectively by:

$$\boxed{E_0 = \frac{1}{2}mc^2},$$

$$\boxed{E_k = \frac{1}{2}m\gamma^2 v^2},$$

$$\boxed{E = \frac{1}{2}m\gamma^2 c^2},$$

where

$$\boxed{\gamma = \left(1 - v^2c^{-2}\right)^{-\frac{1}{2}}},$$

(γ) – Lorentz factor,
(m) – mass (resting),
(v) – tree-dimensional speed,
(c) – maximum value of signal propagation speed.

5. Energy-momentum tensor [3, 11]
The energy-momentum (momentum-energy) tensor has the form

$$\boxed{T_{\alpha\alpha} = -k\rho c^2 g_{\alpha\alpha}, \quad T_{\mu\nu} = 0, \quad (\alpha, \mu, \nu = 1,2,3,4; \quad \mu \neq \nu), \quad 0.5 \leq k < 1}$$

6. Gravity field equations [1, 3, 4, 10, 11, 13]
The equations of the gravitational field

$$\boxed{R_{\alpha\beta} = -\kappa\left(T_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}T\right)}, \quad \kappa = \frac{8\pi G}{c^4} = 2.073 \cdot 10^{-43}\frac{s^2}{kg \cdot m},$$

we will write in a different form.

Taking into account that

$$g_{11}=\frac{1}{g_{44}},\quad g_{22}=r^2,\quad g_{33}=r^2\sin^2\theta,\quad g^{\alpha\alpha}=\frac{1}{g_{\alpha\alpha}},$$

$$\boxed{T_{\alpha\alpha}=-k\rho c^2 g_{\alpha\alpha}},\quad \boxed{T\overset{df}{=}\sum_{\alpha=1}^{4}g^{\alpha\alpha}T_{\alpha\alpha}=-4\rho c^2},\quad \boxed{T_{\alpha\alpha}-\frac{1}{2}g_{\alpha\alpha}T=k\rho c^2 g_{\alpha\alpha}},$$

$$R_{12}=R_{21}=R_{13}=R_{31}=R_{14}=R_{41}=R_{23}=R_{32}=R_{24}=R_{42}=R_{34}=R_{43}=0,$$

$$R_{11}=\frac{1}{g_{44}}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right),$$

$$R_{22}=-1+g_{44}+r\frac{\partial g_{44}}{\partial r},$$

$$R_{33}=\sin^2\theta\left(-1+g_{44}+r\frac{\partial g_{44}}{\partial r}\right),$$

$$R_{44}=g_{44}\left(\frac{1}{r}\frac{\partial g_{44}}{\partial r}+\frac{1}{2}\frac{\partial^2 g_{44}}{\partial r^2}\right),$$

we get

$$\boxed{R_{\alpha\alpha}=-\kappa k\rho c^2 g_{\alpha\alpha},\quad R_{\mu\nu}=0,\quad (\alpha,\mu,\nu=1,2,3,4;\quad \mu\neq\nu),\quad \kappa=\frac{8\pi G}{c^4},\quad 0.5\leq k<1}$$

7. Schwarzschild external (vacuum) solution and anti-gravity [14]
Precise external (vacuum) solution of gravitational field equations was served in 1916 by Carl Schwarzschild (1873-1916):

$$\boxed{g_{44}=\frac{1}{g_{11}}=1-\frac{r_S}{r},\quad r\geq R},$$

where

(M) – mass of a homogeneous ball with constant density,
(R) – radius of a ball,

$$\boxed{r_S=\frac{2GM}{c^2}}\ \text{– Schwarzschild radius.}$$

Anti-gravity is hidden in this solution.

It is easy to see that, for $r>r_S$ sign of quadratic differential form of spacetime

$$(ds)^2=g_{11}(dr)^2+g_{44}(dx^4)^2$$

is negative, and for $r<r_S$ this sign is positive.

8. Speed of light in the gravitational field [3, 7, 11]
If the source of the field is a mass uniformly distributed in the ball area, the spacetime metric has the form

$$\boxed{(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + g_{44}\left(dx^4\right)^2, \quad x^4 \equiv ict},$$

$$g_{11} = \frac{1}{g_{44}}, \quad g_{22} = r^2, \quad g_{33} = r^2\sin^2\theta, \quad g^{11} = \frac{1}{g_{11}}, \quad g^{22} = \frac{1}{g_{22}}, \quad g^{33} = \frac{1}{g_{33}}, \quad g^{44} = \frac{1}{g_{44}}.$$

This metric, for

$$\theta = \text{const}, \quad d\theta = 0, \quad \varphi = \text{const}, \quad d\varphi = 0,$$

reduces itself to

$$\boxed{(ds)^2 = g_{11}(dr)^2 - g_{44}c^2(dt)^2}.$$

We will designate speed (v_{light}) of light propagation in virtual vacuum tunnel from condition

$$\boxed{(ds)^2 = 0}$$

or equivalent

$$\boxed{v_{light}^2 = \left(\frac{dr}{dt}\right)^2 = c^2(g_{44})^2 \le c^2}.$$

9. Photon energy in the gravitational field [3]
We assume that energy of photon depends on a point in spacetime, where that photon was emitted and it stays constant during the movement. It means that photons have „memory", or more scholarly – energy of photon is invariant. Wherein, in stronger gravitational field, given source should send photons with lower energy than the same source in weaker gravitational field.

Photon energy, emitted in certain point of spacetime, is given by equation:

$$\boxed{E = \sqrt{|g_{44}|}\,E_{max}},$$

where

(E_{max}) – photon energy emitted in nondeformed spacetime,
(g_{44}) – time-time component of metric tensor in photon's emission point.

10. Redshift [3, 6, 11]
The redshift (z^*) of light reaching Earth (among others) from distant galaxies is defined below.

$$\boxed{z^* \equiv \frac{E_{lab}}{E_{out}} - 1},$$

where

$$E_{lab} = \sqrt{g_{44}^{lab}} E_{max}, \quad E_{out} = \sqrt{g_{44}^{out}} E_{max},$$

(E_{lab}) – photon energy emitted from a source that is in laboratory,
(E_{out}) – photon energy emitted from a source that is outside laboratory,
(E_{max}) – photon energy emitted from a source in the absence of gravitational field,
(g_{44}^{lab}) – time-time component of metric tensor in laboratory in a place of photon detection,
(g_{44}^{out}) – time-time component of metric tensor outside of laboratory in a place of photon emission.

02. Black hole

Black hole is a homogeneous ball with mass (M) and radius (R) smaller than the Schwarzschild radius (r_S), but not less than half the Schwarzschild radius:

$$\boxed{0.5 r_S \le R < r_S = \frac{2GM}{c^2}, \quad R = k r_S, \quad 0.5 \le k < 1},$$

where (c) is maximum value of signal propagation speed, (G) – gravitational constant.

03. Energy density in the area of the black hole

According to General Relativity, spacetime metric is determined by the spatial density distribution of all energies (including mass equivalent energy) [10].

For the rest energy (E_0) of the ball with mass (M), volume (V), density (ρ) and radius (R) we will assume the expression [3, 9, 11, 12]:

$$\boxed{E_0 = \frac{1}{2} \rho V c^2}.$$

A homogeneous ball resting energy density (ε_0) is:

$$\boxed{\varepsilon_0 = 0.5 \rho c^2}.$$

We will generally assume for energy density (ε) that

$$\boxed{0.5 \rho c^2 \le \varepsilon < \rho c^2, \quad \varepsilon = k \rho c^2, \quad 0.5 \le k < 1}.$$

04. Spacetime metric under and above the event horizon

Spacetime metric under and above the event horizon, when the source of the gravitational field is a black hole, can be described by equations [11, 13]:

$$\boxed{R_{\alpha\alpha} = -\kappa k \rho c^2 g_{\alpha\alpha}, \quad R_{\mu\nu} = 0, \quad (\alpha, \mu, \nu = 1,2,3,4; \quad \mu \ne \nu), \quad \kappa = \frac{8\pi G}{c^4}, \quad 0.5 \le k < 1}$$

NOTE
All mixed components of Ricci tensor are identically equal to zero. Set of remaining equations can be reduced to only two independent ones.

$$\boxed{\begin{aligned} R_{11} &= -\kappa k\rho c^2 g_{11} \\ R_{22} &= -\kappa k\rho c^2 g_{22} \end{aligned}} \quad \Rightarrow \quad \boxed{\begin{aligned} &\frac{\partial g_{44}}{\partial r} + \frac{r}{2}\frac{\partial^2 g_{44}}{\partial r^2} = -\kappa k\rho c^2 r \\ &-1 + g_{44} + r\frac{\partial g_{44}}{\partial r} = -\kappa k\rho c^2 r^2 \end{aligned}}$$

Spacetime described by above equations, in which every component of Ricci tensor is proportional to adequate component of metric tensor is an Einstein's space [13].

These equations are fulfilled when

$$0 \le r < R\,, \quad \rho = \text{const} > 0\,, \quad g_{44} = 1 - \frac{k r_S}{R}\frac{r^2}{R^2}\,, \quad 0.5 \le k < 1\,,$$

$$r \ge R\,, \quad \rho = 0\,, \quad g_{44} = 1 - \frac{r_S}{r}\,, \quad r \ne r_S\,.$$

05. Exterior Schwarzschild metric

Spacetime metric outside mass source ($r \ge R$, $\rho = 0$) is being described by exterior Schwarzschild metric [14]:

$$\boxed{(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1}(dr)^2 + r^2(d\theta)^2 + r^2\sin^2\theta\,(d\varphi)^2 + \left(1 - \frac{r_S}{r}\right)\left(dx^4\right)^2, \quad x^4 = ict, \quad r \ne r_S = \frac{2GM}{c^2}}\,.$$

06. Speed of light propagation and exterior Schwarzschild metric

Exterior Schwarzschild metric, for

$$\theta = \text{const}\,, \quad d\theta = 0\,, \quad \varphi = \text{const}\,, \quad d\varphi = 0\,,$$

reduces itself to [3, 7, 11]

$$\boxed{(ds)^2 = \left(1 - \frac{r_S}{r}\right)^{-1}(dr)^2 - \left(1 - \frac{r_S}{r}\right)c^2(dt)^2}\,.$$

We designate speed (v_{lihgt}) of light propagation from condition

$$\boxed{(ds)^2 = 0}$$

or equivalent

$$\boxed{0 < v_{light}^2 = \left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{r_S}{r}\right)^2 \le c^2}\,.$$

$$\lim_{r\to 0.5 r_S} v_{light} = c\,,\qquad \lim_{r\to r_S} v_{light} = 0\,,\qquad \lim_{r\to\infty} v_{light} = c\,.$$

Note that

$$\left[0 < \left(\frac{dr}{dt}\right)^2 \le c^2\right] \Leftrightarrow \left[r \ge \frac{1}{2} r_S,\quad r \ne r_S\right].$$

It means that exterior Schwarzschild metric is correct if and only if

$$\boxed{r \ge \frac{1}{2} r_S,\quad r \ne r_S}\,.$$

07. Gravitational acceleration of free fall outside mass source

We will designate radial component of gravitational acceleration of free falling test particle by equation of motion [3, 11]:

$$\tilde{a}^r = \tilde{a}^1 = -\tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right),\qquad r \ne r_S\,,\qquad (ds)^2 \ne 0\,.$$

Taking into account, that

$$g_{44} = 1 - \frac{r_S}{r}\,,\qquad r_S = \frac{2GM}{c^2}\,,\qquad \tilde{k} = +1\,,$$

$$\Gamma^1_{11} = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r} = -\left(1-\frac{r_S}{r}\right)^{-1}\cdot\frac{GM}{c^2 r^2}\,,\qquad \Gamma^1_{44} = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r} = -\left(1-\frac{r_S}{r}\right)\cdot\frac{GM}{c^2 r^2}\,,$$

$$1 = \left(1-\frac{r_S}{r}\right)^{-1}\left(\frac{dr}{ds}\right)^2 + \left(1-\frac{r_S}{r}\right)\left(\frac{dx^4}{ds}\right)^2,$$

we get

$$\boxed{\tilde{a}^r = \tilde{a}^1 = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r} = \left(\operatorname{sgn} ds^2\right)\frac{GM}{r^2}}\,.$$

Physical (true) component of gravitational acceleration of free fall

$$\hat{a}^r \overset{df}{=} \sqrt{-\left(\operatorname{sgn} ds^2\right)g_{rr}}\;\tilde{a}^r\,,$$

where

$$g_{rr} = g_{11} = \left(1-\frac{r_S}{r}\right)^{-1},$$

finally can be saved in the form:

$$\boxed{\hat{a}^r = \sqrt{-\left(\operatorname{sgn} ds^2\right) g_{rr}}\left(\operatorname{sgn} ds^2\right)\frac{GM}{r^2}}.$$

08. Gravity and anti-gravity

Above equation has interesting physical interpretation. For $r > r_S$ it describes gravity and for $\frac{1}{2} r_S \le r < r_S$ – anti-gravity [3, 11].

Gravity

$$r > r_S = \frac{2GM}{c^2}, \quad g_{rr} = \left(1 - \frac{r_S}{r}\right)^{-1} > 0, \quad (ds)^2 < 0, \quad \hat{a}^r = -\frac{GM}{r^2} \cdot \frac{1}{\sqrt{1 - \frac{r_S}{r}}}$$

Anti-gravity

$$\frac{1}{2} r_S \le r < r_S = \frac{2GM}{c^2}, \quad g_{rr} = \left(1 - \frac{r_S}{r}\right)^{-1} < 0, \quad (ds)^2 > 0, \quad \hat{a}^r = +\frac{GM}{r^2} \cdot \frac{1}{\sqrt{\frac{r_S}{r} - 1}}$$

09. Main hypothesis

Anti-gravity works in such a way that free test particle located in external gravitational field in certain area gets acceleration directed from the center of that mass source [3, 11].

In areas, where $g_{\mu\nu} \ge 0, \quad (ds)^2 < 0, \quad (\mu, \nu = 1,2,3,4)$, graviy occurs.	In areas, where $g_{\mu\nu} \le 0, \quad (ds)^2 > 0, \quad (\mu, \nu = 1,2,3,4)$, anti-graviy occurs.

10. Spacetime metric inside mass source

Spacetime metric inside mass source ($0 \le r < R$, $\rho = \text{const} > 0$) is given by [3, 7, 11]:

$$\boxed{(ds)^2 = g_{11}(dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta\,(d\varphi)^2 + g_{44}\left(dx^4\right)^2},$$

where

$$x^4 = ict, \quad g_{11} = \frac{1}{g_{44}}, \quad g_{44} == 1 - \frac{kr_S}{R}\frac{r^2}{R^2}, \quad 0.5 \le k < 1, \quad r_S = \frac{2GM}{c^2}.$$

11. Speed of light propagation in virtual vacuum tunnel that is inside black hole

Spacetime metric inside black hole, for

$$\theta = \text{const}, \quad d\theta = 0, \quad \varphi = \text{const}, \quad d\varphi = 0, \quad R = kr_S, \quad 0.5 \le k < 1,$$

reduces itself to the form [3, 7, 11]:

$$(ds)^2 = g_{11}(dr)^2 - g_{44}c^2(dt)^2, \quad g_{11} = \frac{1}{g_{44}}, \quad g_{44} = 1 - \frac{r^2}{R^2}.$$

We will designate speed (v_{light}) of light propagation in virtual vacuum tunnel from condition

$$(ds)^2 = 0$$

or equivalent

$$\boxed{0 < v_{light}^2 = \left(\frac{dr}{dt}\right)^2 = c^2\left(1 - \frac{r^2}{R^2}\right)^2 \le c^2}.$$

$$\lim_{r\to 0} v_{light} = c, \quad \lim_{r\to R} v_{light} = 0.$$

Notice that

$$R = kr_S, \quad 0.5 \le k < 1, \quad 0 < v_{light}^2 \le c^2.$$

It means that spacetime metric inside black hole is correct if and only if

$$\boxed{\frac{1}{2}r_S \le R < r_S, \quad r < R}.$$

12. Gravitational acceleration of free fall inside black hole with anti-gravity halo

Radial component of gravitational acceleration of freely falling test particle inside virtual vacuum tunnel, which is located inside black hole with anti-gravity halo, we will get from equation of motion [3, 11]

$$\tilde{a}^r = \tilde{a}^1 = -\tilde{k}\left(\operatorname{sgn} ds^2\right)c^2\left(\Gamma^1_{11}\frac{dr}{ds}\cdot\frac{dr}{ds} + \Gamma^1_{44}\frac{dx^4}{ds}\cdot\frac{dx^4}{ds}\right), \quad 0 \le r < R, \quad (ds)^2 \ne 0.$$

Taking into account, that

$$\tilde{k} = -1, \quad \operatorname{sgn} ds^2 = -1, \quad R = kr_S, \quad 0.5 \le k < 1, \quad r_S = \frac{2GM}{c^2},$$

$$\Gamma^1_{11} = -\frac{1}{2g_{44}}\frac{\partial g_{44}}{\partial r}, \quad \Gamma^1_{44} = -\frac{1}{2}g_{44}\frac{\partial g_{44}}{\partial r}, \quad g_{44} = 1 - \frac{r^2}{R^2}, \quad 1 = g_{44}^{-1}\left(\frac{dr}{ds}\right)^2 + g_{44}\left(\frac{dx^4}{ds}\right)^2,$$

we get

$$\boxed{\tilde{a}^r = \tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{2}\frac{\partial g_{44}}{\partial r} = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\frac{c^2}{R^2}r = -\frac{c^2}{R^2}r}.$$

Physical (true) component of gravitational acceleration of free fall

$$\hat{a}^r \overset{df}{=} \sqrt{-\left(\operatorname{sgn} ds^2\right) g_{rr}}\ \tilde{a}^r ,$$

where

$$g_{rr} = g_{11} = \left(1 - \frac{r^2}{R^2}\right)^{-1} ,$$

in the end, can be written in the form:

$$\boxed{\hat{a}^r = -\tilde{k}\left(\operatorname{sgn} ds^2\right)\sqrt{-\left(\operatorname{sgn} ds^2\right) g_{rr}}\ \frac{c^2}{R^2} r = -\frac{c^2}{R^2} r \cdot \frac{1}{\sqrt{1 - \frac{r^2}{R^2}}}} .$$

13. Graphic analysis of the full solution

Time-time component of metric tensor and physical (true) component of gravitational acceleration of free fall, in three distance intervals from the center of black hole, are given by below relations.

GRAVITY

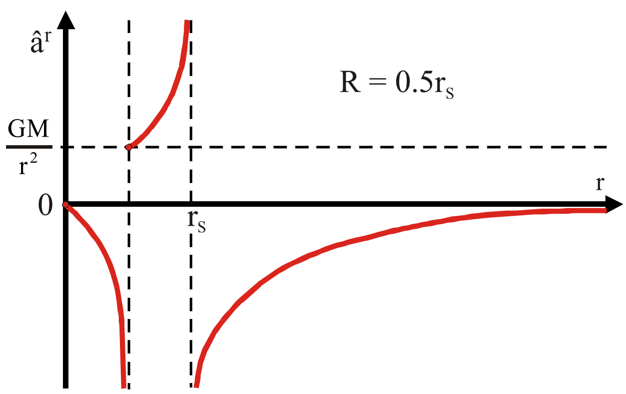
$$0 \le r < R = kr_S , \quad g_{44} = \left(1 - \frac{r^2}{R^2}\right) > 0 , \quad \hat{a}^r = -\frac{c^2}{R^2} r \cdot \frac{1}{\sqrt{1 - \frac{r^2}{R^2}}}$$

ANTI-GRAVITY

$$k\, r_S = R \le r < r_S , \quad g_{44} = \left(1 - \frac{r_S}{r}\right) < 0 , \quad \hat{a}^r = +\frac{GM}{r^2} \cdot \frac{1}{\sqrt{\frac{r_S}{r} - 1}}$$

GRAVITY

$$r > r_S , \quad g_{44} = \left(1 - \frac{r_S}{r}\right) > 0 , \quad \hat{a}^r = -\frac{GM}{r^2} \cdot \frac{1}{\sqrt{1 - \frac{r_S}{r}}}$$



Above we presented charts of dependence of radial component ($\hat{a}^r$) of physical (true) gravitational acceleration of free fall on the distance (r) from the center of black hole with anti-gravity halo.

From the charts we can see that:

$0 \le r < R = k \cdot r_S \quad \Rightarrow \quad$ gravity,

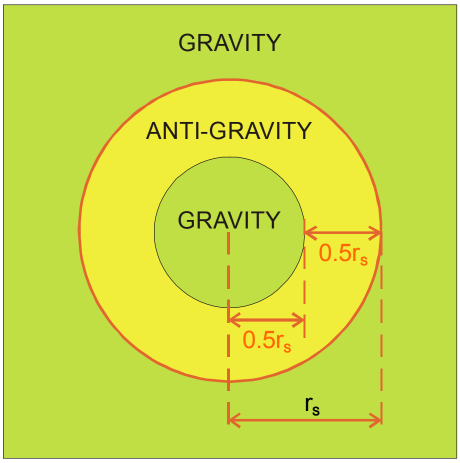
$r = R = k \cdot r_S \quad \Rightarrow \quad$ transition from gravity to anti-gravity,

$k \cdot r_S = R < r < r_S \quad \Rightarrow \quad$ anti-gravity,

$r = r_S \quad \Rightarrow \quad$ transition from anti-gravity to gravity,

$r > r_S \quad \Rightarrow \quad$ gravity.

Anti-gravity halo thickness is half the Schwarzschild radius. Gravity and anti-gravity has layer-like nature.



NOTE

For $\varepsilon = \rho c^2$, k = 1 it is impossible to black hole formation, and thus the appearance of anti-gravity.

14. Radius and density of the black hole

The radius and density of the black hole will be determined using the following relations

$$R = kr_S, \quad 0.5 \le k < 1,$$

$$r_S = \frac{2GM}{c^2},$$

$$M = \rho\left(\frac{4}{3}\pi R^3\right),$$

$$\kappa = \frac{8\pi G}{c^4}.$$

As a result we get:

$$R^2 = \frac{3}{k} \cdot \frac{c^2}{8\pi G \rho},$$

$$R^2 = \frac{3}{k} \cdot \frac{1}{c^2 \kappa \rho},$$

$$R = \sqrt{\frac{3}{k}} \cdot \frac{1}{c\sqrt{\kappa\rho}},$$

$$\rho = \frac{3}{k} \cdot \frac{1}{8\pi G R^2},$$

$$\rho = \frac{3}{k} \cdot \frac{1}{c^2 \kappa R^2}.$$

$$\rho = \frac{3}{k} \cdot \frac{1}{8\pi G R^2},$$

$$\rho = \frac{3}{k} \cdot \frac{1}{c^2 \kappa R^2}.$$

15. Final remarks

The use of the traditional relation for resting energy density $\boxed{\varepsilon_0 = \rho c^2}$ does not describe a black hole, and even more so a phenomenon of anti-gravity.

The model of the rotating black hole that I propose (apart from the name) has little to do with the commonly accepted models of this object.

Points 9 and 10 from the introduction were used in [3] to explain, among others, in the framework the black-hole model of the Universe
A. a non-linear rapid increase in the redshift of light reaching the Earth from very distant sources such as galaxies,
B. existence of a distance from Earth, where redshift changes the sign from negative to positive.

16. Propositions

I propose calling the fourth order mixed curvature tensor $R^{\alpha}_{\beta\mu\nu}$ as Grossmann curvature tensor. The concept of mixed tensor was first introduced by Marcel Grossmann (1878-1936) when considering the curvature of spacetime [2].

Below, we recall the known theorems about flatness and curvature of space [10].

Space is flat if and only if all the components of the tensor $R^{\alpha}_{\beta\mu\nu}$ are zero.

Space is curved if and only if at least one of the components of the tensor $R^{\alpha}_{\beta\mu\nu}$ is different from zero.

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